Test Booklet Code **A**

MOCK TEST

for

JEE (Advanced) 2017

Paper-I

INSTRUCTIONS FOR CANDIDATE

1. Read each question carefully.
2. It is mandatory to use blue/black ball point pen to darken the appropriate circle in the answer sheet.
3. Mark should be dark and should completely fill the circle.
4. Rough work must not be done on the answer sheet.
5. Do not use white-fluid or any other rubbing material on answer sheet.
6. Student cannot use log table and calculator or any other material in the examination hall.
7. Before attempting the question paper, student should ensure that the test paper contains all pages and no page is missing.
8. Before handing over the answer sheet to the invigilator, candidate should check that Roll No., Centre Code and Date of Birth have been filled and marked correctly.
9. Immediately after the prescribed examination time is over, the answer sheet to be returned to the invigilator.
10. Pattern of the questions are as under:
    (i) The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part has **3 sections**.
    (ii) **Section-1:** This section contains 5 multiple choice questions which have only one correct answer. Each question carries **+3 marks** each for correct answer and **–1 mark** for wrong answer.
    (iii) **Section-2:** This section contains 8 multiple choice questions which have one or more answers are correct. Each question carries **+4 marks** each for correct answer and **–2 mark** for wrong answer. **Partial +1 mark** given for darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.
    (iv) **Section-3:** This section contains 5 questions. The answers to each of the questions is a single digit integer, ranging from 0 to 9 (both inclusive) without being given any option. Each question carries **+3 marks** for correct answer.

Aakash
Medical | IIT-JEE | Foundations
(Divisions of Aakash Educational Services Pvt. Ltd.)

Time : 3 Hrs. **MOCK TEST** MM : 186

Paper - I (Code-A)

PART - I : PHYSICS

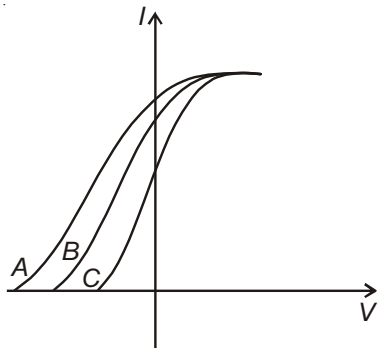
SECTION - 1

Only One Option Correct Type

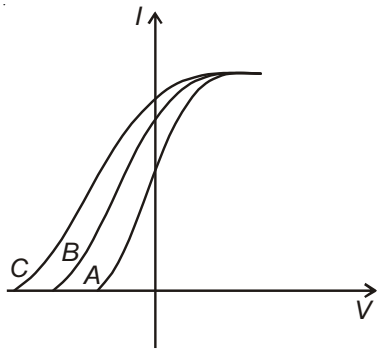
This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. Photoelectric effect experiments are performed using three different metal plates *A*, *B* and *C* having work function ϕ_A = 2 eV, ϕ_B = 3 eV and ϕ_C = 4 eV respectively. A light beam containing wavelength of 450 nm, 350 nm and 250 nm with equal intensities illuminates each of the plates. The correct *I*-*V* graph for the experiment is

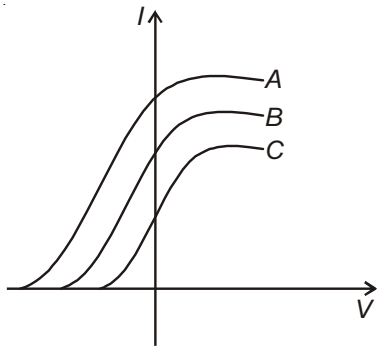
(A)



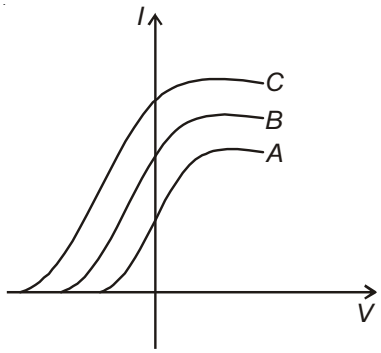
(B)



(C)



(D)



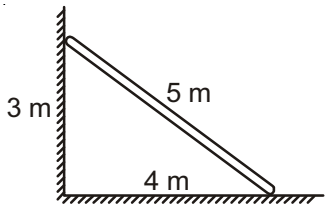
2. A uniform ladder of length 5 m is placed against the wall as shown. If coefficient of friction μ is the same for both the walls, then the minimum value of μ for it not to slip, is

(A) $\mu = \frac{1}{2}$

(B) $\mu = \frac{1}{4}$

(C) $\mu = \frac{1}{3}$

(D) $\mu = \frac{1}{5}$

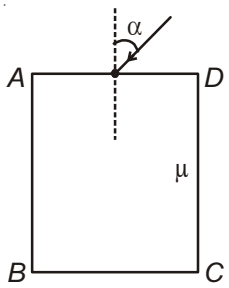


3. A scuba diver is 70 m down, where the pressure is 8 atm. The air she exhales forms bubbles 8.0 mm in radius. The amount of work does each bubble do as it rises to the surface, assuming the bubbles remain at the uniform 300 K temperature of the water is

(A) 0.90 joule (B) 2.37 joule

(C) 1.8 joule (D) 3.56 joule

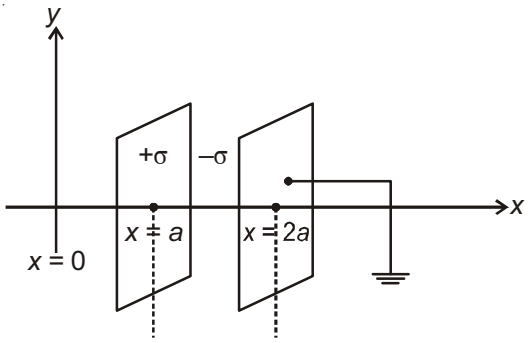
4. A parallel beam of light is incident from air at an angle α on the surface *AD* of a rectangular block of refractive index μ. The value of $\alpha \in (0°, 60°]$ *i.e.,* greater than zero degree but less than or equal to 60°. Calculate the minimum value of refractive index of block so that there is always total internal reflection at the vertical surface *AB*.

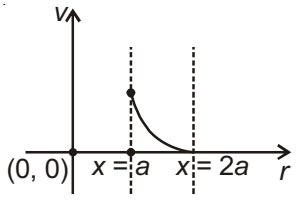


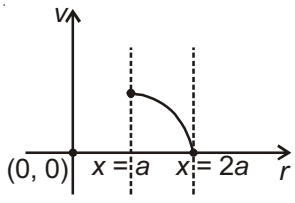
(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$

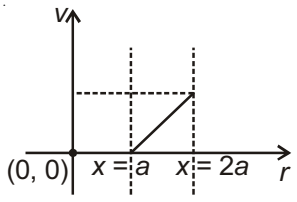
(C) $\frac{\sqrt{5}}{2}$ (D) $\frac{\sqrt{7}}{2}$

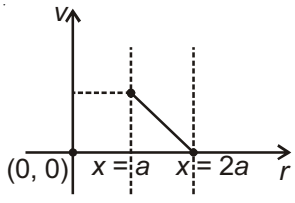
5. Two large parallel conducting sheets charged uniformly with surface charge density σ and $-\sigma$ are located at $x = a$ and $x = 2a$ respectively with negatively charge sheet being earthed. Which of the following graph best represent the variation of electric potential between $x = a$ and $x = 2a$?



(A)


(B)


(C)


(D)


SECTION - 2

One or More Options Correct Type

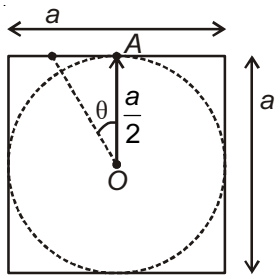
This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONE OR MORE** is/are correct.

6. A hydrogen-like atom (described by Bohr model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between –30.60 eV and –4.896 eV. (including both these values). Which of the following statement(s) is(are) true?

   [Take hc = 1240 eV–nm, ground state energy of hydrogen atom = –13.6 eV]

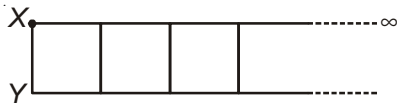
   (A) Atomic number of the atom is 3
   (B) Energy level of energy state –30.60 eV is 3
   (C) Energy level of energy state –4.896 eV is 6
   (D) Smallest wavelength emitted in these transitions is 482 Å

7. A square ground of side $a = \frac{10}{\sqrt{2}}$m has a circular running track of radius $\frac{a}{2}$ with its centre coinciding the centre of the ground. A man is running on the track with an angular velocity ω = 22 rad/s while a car is moving on a road adjacent to ground as shown. The car moves in such a way that the car, the man and the centre of the ground always lie on the same straight line. A source of sound of frequency n = 300 Hz is being placed at the center of the ground. The speed of sound in air is 330 m/s. Which of the following statement(s) is(are) true regarding the sound received by the man in the car? [θ is measured between the line joining car and center with vertical OA]



   (A) The minimum frequency received by the man in the car is 250 Hz
   (B) The minimum frequency received by the man in the car is 200 Hz
   (C) At θ = 45°, minimum frequency is received by the man in the car
   (D) At θ = 30°, minimum frequency is received by the man in the car

8. A large number of identical rods are arranged as shown. Each rod has length l, cross-sectional area A and thermal conductivity of material is k. Ends X and Y are maintained at temperatures θ_1 and θ_2 ($< \theta_1$) respectively. If lateral surface of each rod is thermally insulated, then which of the following statement(s) is(are) true at steady state?



   (A) Equivalent thermal resistance between X and Y = $\frac{l(\sqrt{3}-1)}{kA}$
   (B) Equivalent thermal resistance between X and Y = $\frac{l(\sqrt{3}+1)}{kA}$
   (C) Rate of heat flow between X and Y = $\frac{kA(\theta_1-\theta_2)}{l(\sqrt{3}+1)}$
   (D) Rate of heat flow between X and Y = $\frac{kA(\theta_1-\theta_2)}{l(\sqrt{3}-1)}$

9. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. Which of the following statement(s) is(are) true?

(A) The refractive index of the lens is 1.5

(B) The distance of object from the lens is 12 m

(C) The focal length of the lens is 6 m

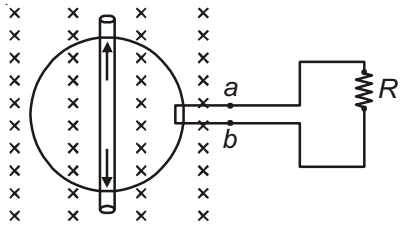
(D) The radius of curvature of the convex surface is 3 m

10. If the time period (T) of vibration of a liquid drop depends on surface tension(S), radius(r) of the drop and density(ρ) of the liquid, then which of the following expression(s) for T is(are) dimensionally correct?

(A) $T = K\sqrt{\frac{\rho^{\frac{1}{2}} r^3}{S}}$ (B) $T = K\sqrt{\frac{\rho r^3}{S}}$

(C) $T = K\sqrt{\frac{\rho^2 \cdot r^3}{S}}$ (D) $T = K\sqrt{\frac{\rho r^3}{S^{\frac{1}{2}}}}$

11. A flexible circular loop 10 cm in diameter lies in a magnetic field of 1.2 T, directed into the plane of paper as shown. The loop is pulled at the points indicated by the arrows, forming a loop of zero area in 0.2 s. If the resistance of external circuit R = 10 Ω and the resistance of the loop is negligible, then which of the following statement(s) is(are) correct?



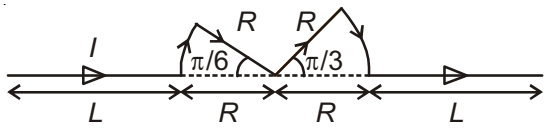
(A) Average induced emf in the circuit is 0.047 volts

(B) Direction of current in R is anticlockwise

(C) Induced charge flowing through R during 0.2 s is equal to 9.42×10^{-4} C

(D) Induced charge flowing through R during 0.2 s is equal to 4.7×10^{-3} C

12. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is(are)



(A) If $\vec{B}$ is along $\hat{Z}$, $F \propto (L + R)$ (B) If $\vec{B}$ is along $\hat{X}$, $F = 0$

(C) If $\vec{B}$ is along $\hat{Y}$, $F \propto (L + R)$ (D) If $\vec{B}$ is along $\hat{Z}$, $F = 0$

13. A container of fixed volume, has a mixture of one mole of oxygen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is(are)

(A) The average energy per mole of the gas mixture is $2RT$

(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\frac{1}{\sqrt{5}}$

(C) The ratio of rms speed of helium atoms to that of oxygen molecules is $\frac{1}{\sqrt{2}}$

(D) The ratio of the rms speed of helium atoms to that of oxygen molecules is $2\sqrt{2}$

SECTION - 3

Integer Value Correct Type

This section contains 5 questions. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9, both inclusive. The answer will have to be appropriately bubbled in the ORS as per the instructions as follows.
Examples- If the correct answer to question numbers X, Y and Z (say) are 6, 0 and 9 respectively, then mark 6, 0 and 9 in ORS respectively.

X	⓪①②③④⑤●⑦⑧⑨
Y	●①②③④⑤⑥⑦⑧⑨
Z	⓪①②③④⑤⑥⑦⑧●

14. Find the quantum number n corresponding to excited state of He^+ ion, if on de-excitation to the ground state, the ion emits two photons in succession with wavelengths 108.72 nm and 30.44 nm. The ionisation energy of H-atom is 13.6 eV. Take hc = 1242 eV-nm.

15.

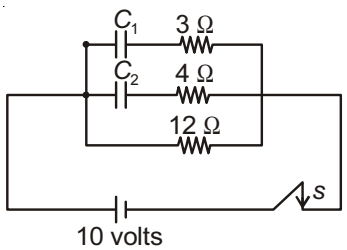
	$A \xrightarrow{\lambda} B \xrightarrow{2\lambda} C$		
No. of active nuclei at $t = 0$	N_0	0	0
No. of active nuclei at time t	N_1	N_2	N_3

The ratio of N_1 to N_2 when N_2 is maximum, is

16. Two spherical stars A and B emit black body radiations. The radius of A is 900 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peak occur in their respective radiation curves is

17. Consider two solid spheres A and B each of density 9 cm/cc and diameters 2 cm and 1 cm respectively. Sphere A is dropped into a liquid of density 1.8 gm/cc and viscosity η = 3 poiseuilles. Sphere B is dropped into a liquid of density 2.6 gm/cc and viscosity η = 2 poiseuilles. The ratio of terminal velocities of A and B is

18. Two capacitors C_1 = 1 μF and C_2 = 2 μF are arranged in given electrical circuit as shown across a battery of 10 volt. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is



PART - II : CHEMISTRY

SECTION - 1

Only One Option Correct Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

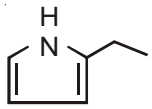
19. A stationary Li^{+2} ion emits a photon corresponding to first line of Lyman series. Photon thus emitted hits on H-atom in ground state. Assume one photon ejects one electron. Kinetic energy of emitted electron will be

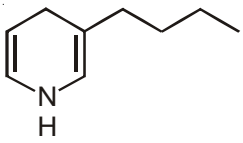
(E_1 for hydrogen = 2.18×10^{-18} J/atom)

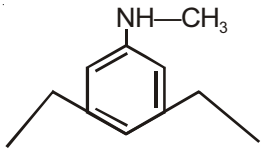
(A) 12.535×10^{-18} J (B) 14.715×10^{-18} J

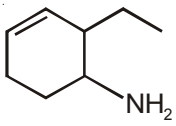
(C) 2.18×10^{-18} J (D) No electron is ejected

20. Which of the following is most basic in nature?

(A) 

(B) 

(C) 

(D) 

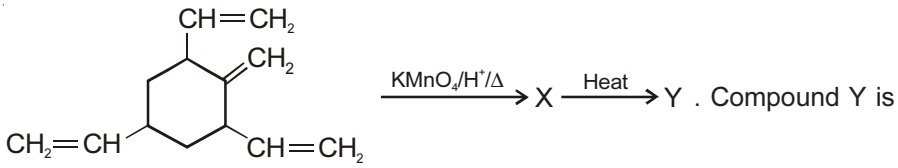
21. The sublimation energy of metal is 80 kJ/mole. Its first and second ionization energies are 300 and 700 kJ/mole respectively. The hydration energy of M^+ is –400 kJ/mole and M^{2+} is –1200 kJ/mole. Then, which of the following is applicable to stability of two ions in water

(A) M^+ and M^{+2} are equally stable in aqueous medium

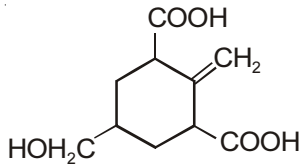
(B) M^{+2} is more stable than M^+ in aqueous medium

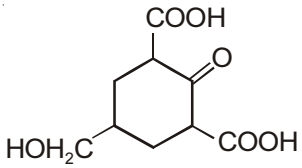
(C) M^+ is more stable than M^{+2} in aqueous medium

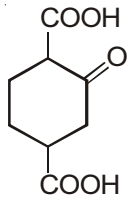
(D) Lesser is hydration energy more is stability

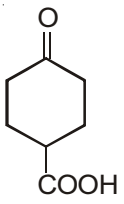
22. 

. Compound Y is

(A) 

(B) 

(C) 

(D) 

23. A 200 mL flask and a 150 mL flask are separated by a stopcock. At 300 K, the nitric oxide exerts pressure of 1.5 atm in larger flask and smaller one contains oxygen at 0.5 atm. At room temperature gases completely react according to the following reaction.

$2NO + O_2 \longrightarrow 2NO_2$

$2NO_2 \longrightarrow N_2O_4$

Dimer N_2O_4 solidify at 262 K. The gases are mixed by opening stopcock and after the end of reaction the flasks are cooled to 220 K. Find the pressure of the mixture after reaction at 220 K (neglect V.P. of solid) assume gases to behave ideally.

(A) 4 atm (B) 0.214 atm

(C) 0.327 atm (D) 0.85 atm

SECTION - 2

One or More Options Correct Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONE OR MORE** is/are correct.

24. Cyclohexene is heated with acidified $KMnO_4$ to form P.

$P \xrightarrow{NH_3, \Delta} Q \xrightarrow{LiAlH_4} R$

Product R is made to react with more of P to give product G then

(A) Product P will give Lucas test

(B) Product R may dissolve in HCl

(C) Product Q can give Hoffmann Bromamide reaction

(D) Product G is a polyamide

25. When XeF_4 is treated with water, then

(A) $XeOF_4$ and HF are formed (B) XeO_3 is formed

(C) Xe and O_2 are formed (D) XeF_4 undergoes oxidation as well as reduction

26. From hard water after evaporation, 4.44 g of solid mixture of $CaCl_2$ and NaCl was obtained. This mixture was treated with Na_2CO_3 to precipitate all calcium ions as calcium carbonate which is strongly heated to give 0.56 g of calcium oxide, then

(A) Percentage of NaCl in mixture is 50% (B) Percentage of calcium chloride is 25%

(C) Mass of NaCl in mixture is 3.33 g (D) Moles of calcium oxide formed are 0.01 mole

27. Which of the following compound(s) contain Aluminium?

(A) Diaspore (B) Corundum

(C) Feldspar (D) Bauxite

28. But-2-ene $\xrightarrow{O_3, H_2O} P \xrightarrow[573K]{MnO} Q \xrightarrow{DIBAL-H} R$

(A) Compound Q gives Fehling solution test

(B) Compound R gives yellow solid with iodine and NaOH

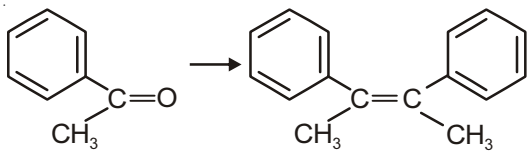
(C) Compound P formed dissolves in $NaHCO_3$ solution

(D) R can be converted to Q using PCC

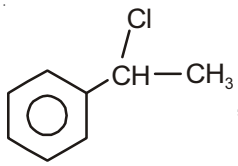
29. The complex/complexes showing spin only magnetic moment of approximately 2.82 BM is/are

(A) $Ni(CO)_4$ (B) $[NiCl_4]^{2-}$

(C) $[Mn(CN)_6]^{3-}$ (D) $[CoF_6]^{3-}$

30. 

The above conversion can be done successfully with

(A)  , PPh_3; BuLi (B) $LiAlH_4$, $SOCl_2$, Na + ether

(C) PCl_5, Na + ether (D) PCl_5, Zn + CH_3OH

31. Which of the following will not be precipitated by H_2S in the presence of acid?

(A) Mn^{+2} (B) Ni^{+2}

(C) Co^{+2} (D) Cd^{+2}

SECTION - 3

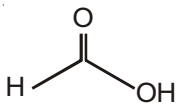
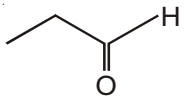
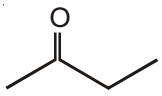
Integer Value Correct Type

This section contains 5 questions. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9, both inclusive. The answer will have to be appropriately bubbled in the ORS as per the instructions as follows. **Examples-** If the correct answer to question numbers X, Y and Z (say) are 6, 0 and 9 respectively, then mark 6, 0 and 9 in ORS respectively.

X	⓪①②③④⑤●⑦⑧⑨
Y	●①②③④⑤⑥⑦⑧⑨
Z	⓪①②③④⑤⑥⑦⑧●

32. How many compounds can give positive test with ammonical silver nitrate?

(a) Glucose (b) Sucrose (c) Fructose

(d)  (e)  (f) 

33. Identify number of complexes/compounds, which are expected to be coloured.

(a) $K_3[VF_6]$ (b) $\left[Cu(NH_2CH_3)\right]^+ BF_4^-$ (c) $KMnO_4$

(d) $Ti(NO_3)_4$ (e) $[Cr(NH_3)_6]Cl_3$ (f) $PbCrO_4$

(g) CrO_2Cl_2

34. o-phenyldiamine react with α-dicarbonyl compounds to give quinoxalines. This reaction is used to identify diamines. In one of such reaction phenanthroquinone (α-dicarbonyl) is made to react with ortho-phenylenediamine to form product. Degree of unsaturation in product is 2x. Then what will be value of x?

35. A given sample of Ferrous oxalate weighing 14.4 gm will require what volume in litres of 0.02 M $KMnO_4$ in acidic medium to get oxidised completely? ($M_{FeC_2O_4}$ = 144 g/mole)

36. How many compounds/salts give basic solution when 0.1 mole of each is dissolved in 1000 ml of water?

(a) NaCl (b) $NaClO_4$ (c) Na_2O (d) KNO_2

(e) NaI (f) Sodium propanoate (g) NH_4Cl

PART - III : MATHEMATICS

SECTION - 1

Only One Option Correct Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

37. The number of permutations of 4 letters taken from the letters of the word ''SANITATION'' are λ. If $\lambda = p_1^{\alpha} \cdot p_2^{\beta} \cdot p_3^{\gamma} \cdots$, where p_1, p_2, p_3,... are prime numbers and α, β, γ,...$\in N$ then the value of $p_1 + p_2 + p_3 + ...$ is

(A) 58 (B) 78

(C) 98 (D) 108

38. ${}^{n}C_0 \; {}^{3n}C_{2n} - {}^{n}C_1 \; {}^{3n-3}C_{2n-3} + {}^{n}C_2 \; {}^{3n-6}C_{2n-6} - ...$

(A) ${}^{3n}C_n \cdot 3^n$

(B) $(-1)^n \, {}^{3n}C_n$

(C) $(-1)^n \cdot 3^n$

(D) 3^n

39. If equation $\sin x + \cos(x + k) + \cos(x - k) = 2$ has real solutions, then range of $\sin k$ is $[a, b]$. Then $b - a$ is

(A) 0 (B) 1

(C) 2 (D) 3

40. If real roots of equation $x^2 + \frac{x^2}{(x+1)^2} = 3$ are α and β then $\alpha^2 + \beta^2$ =

(A) 3 (B) 2

(C) 1 (D) 0

41. The number of straight lines that can be drawn from $P(1, 2)$ and intersecting the two lines $x + y = 5$ and $2x - y = 7$ at A and B respectively such that the harmonic mean of PA and PB be 2, is

(A) Zero (B) Exactly one

(C) Exactly two (D) Infinite

SECTION - 2

One or More Options Correct Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONE OR MORE** is/are correct.

42. The two adjacent sides of a cyclic quadrilateral are 2 and 3 and the angle between them is 120°. If the area of quadrilateral is $4\sqrt{3}$ and remaining two sides are x and y then

(A) $x^2 + y^2 = 29$ (B) $x + y = 17$

(C) $|x - y| = 3$ (D) $xy = 10$

43. The locus of the middle points of the chords of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, which pass through a fixed point (4, 2), is a curve C. Then

(A) Centre of curve C is (2, 1) (B) Foci is curve C are $\left(\frac{43}{9}, 1\right)$ and $\left(-\frac{7}{9}, 1\right)$

(C) Eccentricity of curve C is $\frac{5}{4}$ (D) Eccentricity of curve C is $\frac{4}{3}$

44. Let $A = [a_{ij}]$ be 3 × 3 matrix and let $B = [b_{ij}]$ where $b_{ij} = 5^{i-j} a_{ij}$ $\forall$ $i, j \in \{1, 2, 3\}$. If $|A| = 600$ and $|B| = \lambda$, then

(A) Number of odd divisors of $\lambda = 6$

(B) Number of divisors of λ which are perfect square = 4

(C) Number of divisors of λ which are even = 20

(D) Number of divisors of λ which are divisible by 24 = 3

45. A ray of light is coming from the point $P(3, 5, 7)$ and it get reflected by a plane mirror $2x + y - z = 10$ and reflected ray passes through the point $R(1, 4, 9)$. Then

(A) Equation of reflected ray is $\frac{x-1}{3} = \frac{y-4}{6} = \frac{z-9}{-4}$

(B) Equation of reflected ray is $\frac{x-1}{6} = \frac{y-4}{3} = \frac{z-9}{-4}$

(C) Point of incident ray on the plane mirror is $\left(\frac{97}{19}, \frac{115}{19}, \frac{119}{19}\right)$

(D) Point of incident ray on the plane mirror is $\left(\frac{97}{17}, \frac{95}{17}, \frac{105}{17}\right)$

46. The tangent at a point $P(x, y)$ of a curve $y = f(x)$ meets the axis of y in N, a perpendicular drawn from P to x-axis which meets the axis of x in M; O is the origin and curve $y = f(x)$ passes through (1, 1). If the area of the triangle MON is 2 units. Then,

(A) Eccentricity of curve $y = f(x)$ is less than 1

(B) Eccentricity of curve $y = f(x)$ is greater than 1

(C) $y = f(x)$ is passes through (2, 5)

(D) $y = f(x)$ is passes through (–1, –1)

47. A function $f(x)$ defined on [–1, 4] as follows

$$f(x) = \begin{cases} \max\{-x, x^2\}; & -1 \le x < 0 \\ \min\{x^2, (x-2)^2\}; & 0 \le x \le 2 \\ \max\{4-x, 6x-x^2-8\}; & 2 < x \le 4 \end{cases}$$

Then

(A) $f(x)$ is discontinuous at exactly one point in [–1, 4]

(B) $f(x)$ is non-differentiable of four points in (–1, 4)

(C) Number of local maxima of $f(x)$ are two in (–1, 4)

(D) Number of local minima of $f(x)$ are two in (–1, 4)

48. If $z^3 + (3+2i)z + (-1+i\lambda) = 0$; where $i = \sqrt{-1}$, has one real root, then the value of λ does not belongs to; ($\lambda \in R$)

(A) (0, 1) (B) (–2, –1)

(C) (2, 3) (D) (–5, –1)

49. If $E_1, E_2, E_3, \ldots E_{1008}$ be 1008 independent events such that $P(E_i) = \frac{i}{2i+1}$; (i = 1, 2,..., 1008) and probability that none of the events occurs be $\frac{2^b(b!)(c!)}{(d!)}$; (where $b, c, d \in N$ and $b < c < d$) then

(A) $b + c = d$ (B) $2b + 1 = d$

(C) $2c + 1 = d$ (D) $2c - 1 = d$

SECTION - 3

Integer Value Correct Type

This section contains 5 questions. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9, both inclusive. The answer will have to be appropriately bubbled in the ORS as per the instructions as follows.
Examples- If the correct answer to question numbers X, Y and Z (say) are 6, 0 and 9 respectively, then mark 6, 0 and 9 in ORS respectively.

X	⓪①②③④⑤●⑦⑧⑨
Y	●①②③④⑤⑥⑦⑧⑨
Z	⓪①②③④⑤⑥⑦⑧●

50. $\lim_{x\to\infty}\left(\frac{x^3+4x^2+3x+5}{x^2+2x+3}+lx+m\right)=10$, where $l, m \in R$, then value of $l^2 + m$ is

51. Let $f(x)$ be a differentiable function such that $f(x)=1+\frac{x^3}{3}+\int_0^x e^{-t}f(x-t)dt$, if $\int_0^1 f(x)dx = p$.

Then the value of $5p$ is

52. Let $S = (a_6 - a_5)\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\cdots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$

where $a_1, a_2,...a_n$ are terms of an AP and $a_i > 0\ \forall_i$. If $a_1 = 100$ and $a_n = 900$, then the value of $S - 20$ is

53. If vectors $\lambda x^2\hat{i}+\hat{j}+5\hat{k}$ and $\hat{i}+\lambda x\hat{j}+5\hat{k}$ make an acute angle with each other for all real x, then number of values of λ which are perfect cube satisfies are

54. If x and y are satisfying $x^2 + y^2 - 6x - 8y = 0$ and $a = \max\{x + y\}$ and $b = \min\{x - y\}$ then value of $a + b$ is

❑ ❑ ❑

Test Booklet Code **B**

MOCK TEST

for

JEE (Advanced) 2017

Paper-II

INSTRUCTIONS FOR CANDIDATE

1. Read each question carefully.
2. It is mandatory to use blue/black ball point pen to darken the appropriate circle in the answer sheet.
3. Mark should be dark and should completely fill the circle.
4. Rough work must not be done on the answer sheet.
5. Do not use white-fluid or any other rubbing material on answer sheet.
6. Student cannot use log table and calculator or any other material in the examination hall.
7. Before attempting the question paper, student should ensure that the test paper contains all pages and no page is missing.
8. Before handing over the answer sheet to the invigilator, candidate should check that Roll No., Centre Code and Date of Birth have been filled and marked correctly.
9. Immediately after the prescribed examination time is over, the answer sheet to be returned to the invigilator.
10. Pattern of the questions are as under:
    (i) The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part has **5 sections**.
    (ii) **Section-1:** This section contains 6 multiple choice questions which have only one correct answer. Each question carries **+3 marks** each for correct answer and **–1 mark** for wrong answer.
    (iii) **Section-2:** This section contains 8 multiple choice questions which have one or more answers are correct. Each question carries **+4 marks** each for correct answer and **–2 mark** for wrong answer. **Partial +1 mark** given for darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.
    (iv) **Section-3:** This section contains two paragraphs. Based upon each paragraph, 2 multiple choice questions have to be answered. Each question has only one correct answer and carries **+3 marks** for correct answer.

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Time : 3 Hrs. **MOCK TEST** MM : 186

Paper - II (Code-B)

PART - I : PHYSICS

SECTION - 1

Only One Option Correct Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. A rod of length l and cross section area A has a variable thermal conductivity given by $k = \alpha T$, where α is a positive constant and T is temperature in kelvin. Two ends of the rod are maintained at temperature T_1 and T_2 ($T_1 > T_2$). Heat current flowing through the rod will be

(A) $\frac{A\alpha(T_1^2 - T_2^2)}{l}$ (B) $\frac{A\alpha(T_1^2 + T_2^2)}{l}$

(C) $\frac{A\alpha(T_1^2 + T_2^2)}{3l}$ (D) $\frac{A\alpha(T_1^2 - T_2^2)}{2l}$

2. The surface of lithium is irradiated with electromagnetic radiation whose magnetic component varies with time as $B = \alpha(1+\sin\omega t)\sin\omega_0 t$ where α is a constant and $\omega = 12\pi \times 10^{14}$ s^{-1} and $\omega_0 = 8\pi \times 10^{14}$ s^{-1}. Then the maximum KE of photoelectron ejected is (Given work function of lithium ϕ_{Li} = 2.39 eV)

(A) 1.75 eV (B) 3.5 eV

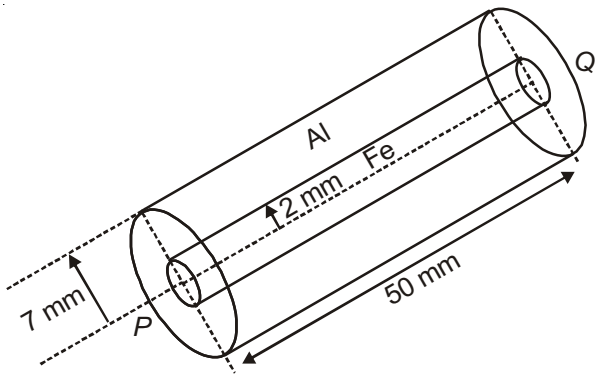
(C) 7 eV (D) 5.25 eV

3. The diameter of a cylinder is measured using a vernier calliper with no zero error. It is found that the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 divisions equivalent to 2.45 cm. The 24th division of the vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

(A) 5.112 cm (B) 5.124 cm

(C) 5.136 cm (D) 5.148 cm

4. In an aluminium (Al) bar of circular cross-section, a circular hole is drilled and is filled with iron (Fe) as shown in the figure.



The electrical resistivities of Al and Fe are $2.7\pi \times 10^{-8}$ Ωm and $1.0\pi \times 10^{-7}$ Ωm respectively. The electrical resistance between the two faces *P* and *Q* of composite bar is

(A) $\frac{2475}{64}\mu\Omega$ (B) $\frac{1875}{64}\mu\Omega$

(C) $\frac{1875}{49}\mu\Omega$ (D) $\frac{2475}{132}\mu\Omega$

5. The figures below depict two situations in which two infinitely long static line charges of constant negative line charge density $-\lambda$ are kept parallel to each other. In their resulting electric field, point charges *q* and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are)

(A) Both charges execute simple harmonic motion

(B) Both charges will continue moving in the direction of their displacement

(C) Charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement

(D) Charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement

6. An interference is observed due to two coherent sources S_1 placed at origin and S_2 placed at (0, 2λ, 0). Here λ is the wavelength of source. Then *x* coordinates on the positive *x*-axis (excluding $x = 0$ and $x = \infty$) where maximum intensity is observed is

(A) 1.5 λ (B) 3 λ

(C) 4.5 λ (D) 6 λ

SECTION - 2

One or More Options Correct Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONE OR MORE** is/are correct.

7. Suppose the potential energy between electron and proton at a distance *r* is given by $-\frac{ke^2}{4r^4}$ where $k = \frac{1}{4\pi\varepsilon_0}$.

Then according to, Bohr's theory for this hypothetical atom, which of the following statement(s) is/are true for n^{th} state of electron?

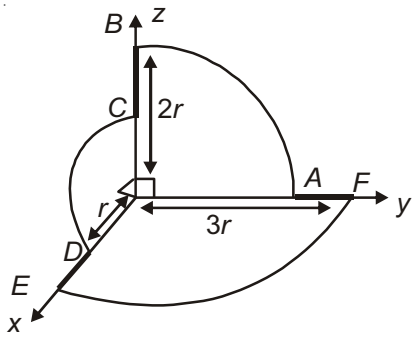
(A) The radius of orbit for revolving electron is given by $r = \frac{e}{nh}\sqrt{\frac{\pi m}{\varepsilon_0}}$

(B) The kinetic energy of electron is given by $\text{KE} = \frac{n^4h^4\varepsilon_0}{8\pi^3m^2e^2}$

(C) The potential energy of electron is given by $\text{PE} = \frac{n^4h^4\varepsilon_0}{16\pi^3m^2e^2}$

(D) The total energy of electron is $\text{TE} = n^4h^4\left(\frac{\pi\varepsilon_0}{m^2e^2}\right)$

8. A wire *ABCDEFA* is bent into three circular segments as shown



Where r = 5 cm

If a uniform magnetic field $\vec{B}$ points in the positive x direction which is increased at the rate of 5 mT/s, Then select the correct option(s).

(A) Direction of current in segment *AB* is from *B* to *A*

(B) The EMF developed in the wire is approx 4×10^{-5} V

(C) If the resistance per unit length of wire is 2 Ω/cm then induced current is 0.2 μA

(D) The EMF developed in the wire is 2×10^{-5} V

9. A certain radioactive nuclide is being manufactured in a cyclotron at a constant rate *R*. It is also decaying with disintegration constant λ. Assume that the production process has been going on for a time that is much longer than the half life of the radioactive nuclide. Which of the following statement(s) is/are correct?

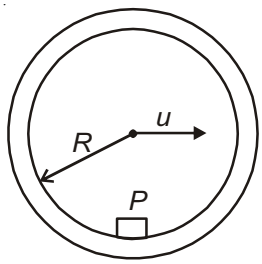
(A) The number of radioactive nuclei present after such time remains constant and is given by $N = \frac{R}{\lambda}$

(B) The number of radioactive nuclei present after such time (t) is given by $N = N_0 e^{-\lambda t}$, where N_0 = Number of radioactive nuclide present initially

(C) The number of radioactive nuclei present after such time is independent of radioactive nuclei present initially

(D) The number of radioactive nuclei present after such time is dependent of radioactive nuclei present initially

10. A small body *P*(of mass *m*) is fixed to the inside of a rigid thin hollow cylinder of radius *R* and mass equal to that of the body *P*. The cylinder rolls without slipping over the horizontal plane. When the body *P* gets into the lower position, the centre of cylinder moves with velocity *u*. Which of the following statement(s) is/are correct?



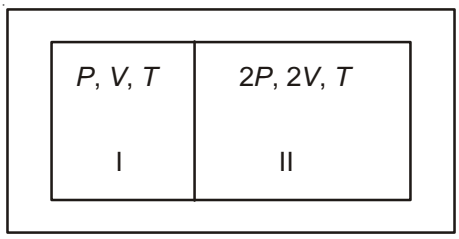
(A) The velocity of center of cylinder when body *P* is at the highest point should be less than or equal to $\sqrt{2gR}$ for the cylinder to move without bouncing

(B) The velocity of center of cylinder when body *P* is at the lowest point should be less than or equal to $\sqrt{8gR}$ for the cylinder to move without bouncing

(C) It is not possible for cylinder to move without bouncing

(D) The velocity of center of cylinder when body *P* is at the highest point should be greater than or equal to $\sqrt{4gR}$ for the cylinder to move without bouncing

11. A partition divides a container having insulated walls into two compartments I and II. The same gas fills the two compartments whose initial parameters are given. The partition is a conducting wall which can move freely without friction. Which of the following statement(s) is/are correct with reference to the final equilibrium position?



(A) The final pressure in the two compartments are equal

(B) Final volume of compartment I is $\frac{3V}{5}$

(C) Final volume of compartment II is $\frac{12V}{5}$

(D) Final temperature in the two compartments are equal

12. During an experiment an ideal gas is found to obey a condition $\frac{P^2}{\rho}$ = constant (ρ = density of gas). The gas initially at temperature T, pressure P and density ρ. The gas expands such that density changes to $\frac{\rho}{2}$. Which of the following statement(s) is/are correct?

(A) The pressure of gas changes to $\sqrt{2}P$

(B) The temperature of gas changes to $\sqrt{2}T$

(C) The graph of the above process on the P-T diagram is parabola

(D) The graph of the above process on the P-T diagram is rectangular hyperbola

13. An equiconvex lens is placed on a plane mirror. An object coincides with its image when it is at height of 24 cm from the lens. The gap between the lens (μ_g = 1.5) and mirror is filled with water $\left(\mu_w = \frac{4}{3}\right)$. Select the correct option(s).

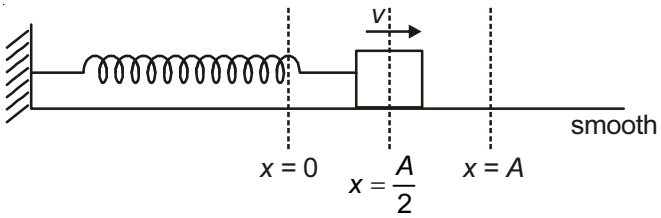
(A) The focal length of equiconvex lens is 30 cm

(B) The equivalent focal length of combination of glass lens, water lens and plane mirror is equal to –18 cm

(C) The equivalent focal length of lens-mirror combination without water is 10 cm

(D) The distance by which the object must be shifted after filling water so that it again coincides with its image is 12 cm, upward

14. A block is executing SHM on a horizontal smooth surface having angular frequency (ω) and amplitude (A). When the block is at $x = \frac{A}{2}$ and moving away from mean position, it receives an impulse in the direction of its velocity by which its kinetic energy becomes 4 times. Which of the following statement(s) is/are correct?



(A) Time period of SHM is independent of impulse

(B) The amplitude of SHM after impulse is equal to $\frac{\sqrt{13}\,A}{2}$

(C) Minimum time taken by the block to cross the same position is equal to $\left(\frac{2\cos^{-1}\frac{1}{\sqrt{13}}}{\omega}\right)$

(D) Minimum time taken by the block to cross the same position is equal to $\left(\frac{\cos^{-1}\frac{1}{\sqrt{13}}}{\omega}\right)$

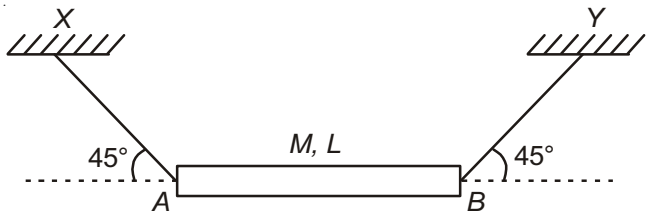
SECTION - 3

Paragraph Type

This section contains 2 paragraphs. Based on each paragraph, there are 2 questions. Each question has 4 choices (A), (B), (C) and (D). Out of which **ONLY ONE** is correct.

Paragraph - 1 (Q. Nos. 15 & 16)

A uniform rod is hanging from the ceiling by the two threads as shown.



Now the string at *B* breaks.

15. The horizontal acceleration of centre of mass a_x, vertical acceleration of centre of mass a_y and angular acceleration of rod about centre of mass α are related as

(A) $(a_x - a_y) + \alpha l = 0$ (B) $2a_x - a_y + \alpha l = 0$

(C) $2(a_x - a_y) + \alpha l = 0$ (D) $a_x + a_y + 2\alpha l = 0$

16. The tension in the string *XA* immediately after string *BY* breaks is

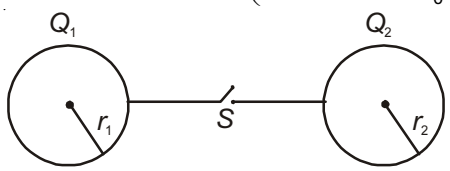
(A) $\frac{2\,mg}{5}$ (B) $\frac{mg}{5}$

(C) $\frac{\sqrt{3}}{2}mg$ (D) $\frac{mg}{\sqrt{2}}$

Paragraph - 2 (Q. Nos. 17 & 18)

Two conducting spheres of radii r_1 and r_2 having charge Q_1 and Q_2 are placed at a large distance being connected by a conducting wire of resistance *R*.

17. The variation of charge flown *q*, as function of time *t* is $\left(\text{where } k = \frac{1}{4\pi\varepsilon_0}\right)$



(A) $q = \dfrac{\dfrac{Q_1}{r_1} - \dfrac{Q_2}{r_2}}{\dfrac{1}{r_1} - \dfrac{1}{r_2}}\left(1 - e^{\dfrac{-kt\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{R}}\right)$

(B) $q = \dfrac{\dfrac{Q_1}{r_1} + \dfrac{Q_2}{r_2}}{\dfrac{1}{r_1} + \dfrac{1}{r_2}}\left(1 - e^{\dfrac{-kt\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{R}}\right)$

(C) $q = \dfrac{\dfrac{Q_1}{r_1} - \dfrac{Q_2}{r_2}}{\dfrac{1}{r_1} + \dfrac{1}{r_2}}\left(1 - e^{\dfrac{-kt\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{R}}\right)$

(D) $q = \dfrac{\dfrac{Q_1}{r_1} - \dfrac{Q_2}{r_2}}{\dfrac{1}{r_1} + \dfrac{1}{r_2}}\left(1 - e^{\dfrac{-kt\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{R}}\right)$

18. The heat dissipated *H*, in the circuit is given by

(A) $H = \dfrac{r_1 r_2}{8\pi\varepsilon_0(r_1+r_2)}\left(\dfrac{Q_1}{r_1} - \dfrac{Q_2}{r_2}\right)^2$

(B) $H = \dfrac{r_1 r_2}{8\pi\varepsilon_0(r_1+r_2)}\left(\dfrac{Q_1}{r_1} + \dfrac{Q_2}{r_2}\right)^2$

(C) $H = \dfrac{r_1 r_2}{\pi\varepsilon_0(r_1+r_2)}\left(\dfrac{Q_1}{r_1} - \dfrac{Q_2}{r_2}\right)^2$

(D) $H = \dfrac{r_1 r_2}{\pi\varepsilon_0(r_1+r_2)}\left(\dfrac{Q_1}{r_1} + \dfrac{Q_2}{r_2}\right)^2$

PART - II : CHEMISTRY

SECTION - 1

Only One Option Correct Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

19. A solid consisting of only X atoms has a close packing structure with X-X distance of 160 pm. Assuming it to be a closed packed structure of hard sphere with radius equal to half of the X-X bond length, the number of atoms in 1 cm^3 would be

(A) 6.023×10^{27}

(B) 3.45×10^{23}

(C) 6.02×10^{21}

(D) 3.8×10^{21}

20. Structure of $Sb(Ph)_5$ and $P(Ph)_5$ respectively is

(A) Trigonal bipyramidal, square pyramidal

(B) Square pyramidal, trigonal bipyramidal

(C) Trigonal bipyramidal, trigonal bipyramidal

(D) Square pyramidal, square pyramidal

21. At a given temperature consider

$Fe_2O_{3(s)} + 3CO_{(g)} \rightleftharpoons 2Fe_{(s)} + 3CO_{2(g)}$; $K_1 = 0.05$

$2CO_{2\,(g)} \rightleftharpoons 2CO_{(g)} + O_{2(g)}$; $K_2 = 2\times10^{-12}$

The equilibrium constant for the reaction

$2Fe_2O_{3(s)} \rightleftharpoons 4Fe_{(s)} + 3O_{2(g)}$ is

(A) 1×10^{-13}

(B) 2×10^{-38}

(C) 4×10^{-15}

(D) 2×10^{-24}

22. In a certain polluted area of Delhi in month of December atmosphere contain O_3 at a steady concentration of 4.0×10^{-8} mol/L, the hourly production of O_3 by all sources was estimated as 8.2×10^{-15} mol/L.

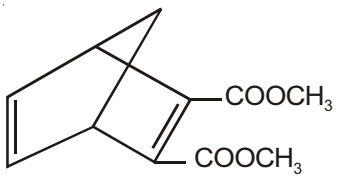
The rate constant for the destruction reaction of ozone.

$2O_3 \rightarrow 3O_2$ is

(A) 1.4×10^{-3} L $mol^{-1}s^{-1}$ (B) 14.0×10^{-3} L $mol^{-1}s^{-1}$

(C) 140×10^{-3} L $mol^{-1}s^{-1}$ (D) 0.14×10^{-3} L $mol^{-1}s^{-1}$

23. The Diels-Alder adduct can be obtained heating the appropriate diene and dienophile



The adduct shown above can be obtain from

(A) Cyclopenta-1, 3-diene and dimethyl maleate

(B) Cyclopenta-1, 3-diene and dimethyl fumarate

(C) Cyclohexa-1, 2 diene and dimethyl maleate

(D) Cyclopenta-1,3-diene and dimethyl acetylene dicarboxylate.

24. The equilibrium constants K_{P1} and K_{P2} for reaction $X \rightleftharpoons 2Y$ and $Z \rightleftharpoons P + Q$ respectively are in the ratio of 1 : 9. If the degree of dissociation of X and Z be equal then ratio of total pressure at these equilibria is

(A) 1 : 9 (B) 1 : 36

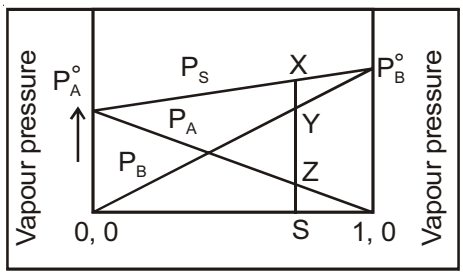
(C) 1 : 1 (D) 1 : 3

SECTION - 2

One or More Options Correct Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONE OR MORE** is/are correct.

25. Consider the following vapour pressure composition graph



Hence, which is correct?

(A) Vapour pressure of A = SZ

(B) Vapour pressure of B = ZY

(C) Vapour pressure of B = SY

(D) Vapour pressure of solution at X = SZ + SY

26. Which of the following statements is/are correct about Cu^{2+} and Cu^{+}?

(A) Cu^{2+} is more stable than Cu^{+}

(B) Cu^{2+} is paramagnetic, Cu^{+} is diamagnetic

(C) Cu^{2+} is mostly blue, whereas Cu^{+} is colourless

(D) Cu^{+} is more stable than Cu^{2+}

27. $K_4[Fe(CN)_6]$ is used to detect

(A) Cu^{2+} (B) As^{3+}

(C) Zn^{2+} (D) Fe^{3+}

28. In a lead storage battery, which option are correct?

(A) The reaction at anode is $Pb \rightarrow Pb^{2+} + 2e^{-}$

(B) The reaction taking place at cathode is $2PbSO_4 + 2H_2O \rightarrow Pb + PbO_2 + 2H_2SO_4$

(C) The overall reaction is $Pb + PbO_2 + 4H^{+} + 2SO_4^{2-} \rightarrow 2PbSO_4 + 2H_2O$

(D) $PbO_2 + 4H^{+} + 2e^{-} \rightarrow Pb^{2+} + 2H_2O$ is the reaction taking place at cathode

29. Which of the following statements is correct for group metals?

(A) On descending down the group, the lattice energy as well as hydration energy decrease

(B) Only BeF_2 is soluble whereas MgF_2, CaF_2, SrF_2 and BaF_2 are insoluble

(C) $BeCl_2$ is insoluble whereas $MgCl_2$, $CaCl_2$, $SnCl_2$, and $BaCl_2$ are soluble

(D) $BeSO_4$ is soluble whereas $BaSO_4$ is insoluble

30. Lactose is found in milk and is also known as milk sugar. Which of the following correct about lactose? (Molecular formula of lactose is $C_{12}H_{22}O_{11}$)

(A) Reducing sugar

(B) Hydrolysis product of lactose is galactose + glucose

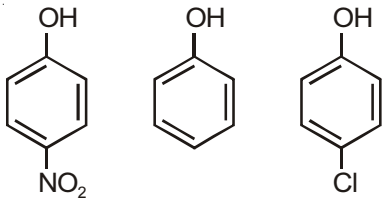
(C) Hydrolysis product of lactose is glucose + fructose

(D) β–1$\rightarrow$4 glycosidic linkage

31. The reactivity order of substituted phenols towards $R-\overset{\overset{O}{\|}}{C}-Cl$ for

(i) Electrophilic substitution reaction

(ii) Phenyl ester formation reactivity order are



I II III

(A) I > II > III for (ii) (B) I > III > II for (ii)

(C) II > III > I for (i) (D) II > I > III for (i)

32. Glycerol changes to acrolein in presence of

(A) Conc. H_2SO_4 (B) HNO_3

(C) $KHSO_4$ (D) Anhy. $CaCl_2$

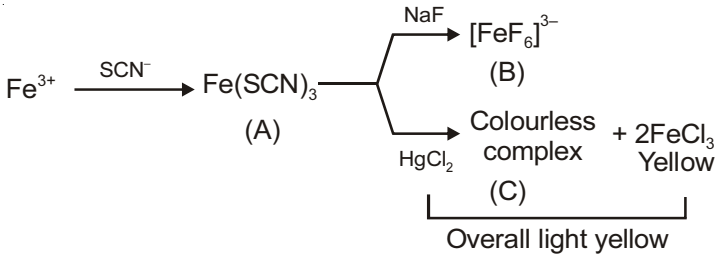
SECTION - 3

Paragraph Type

This section contains 2 paragraphs. Based on each paragraph, there are 2 questions. Each question has 4 choices (A), (B), (C) and (D). Out of which **ONLY ONE** is correct.

Paragraph - 1 (Q. Nos. 33 & 34)

The specific test for Fe^{3+} even in presence of Fe^{2+}



33. Colour of complex (A) in reaction

(A) Black ppt. (B) Yellow ppt.

(C) Blood red ppt. (D) Colourless complex

34. Complex C is

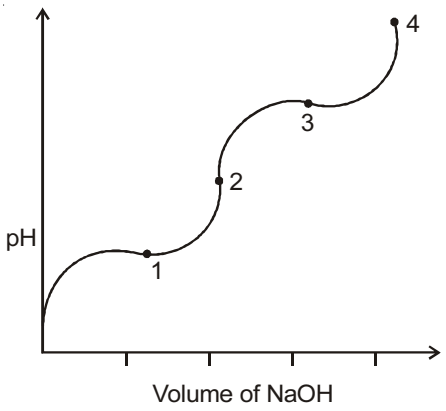
(A) $Hg[Cl_2]$ (B) $Fe[F_4]^{3-}$

(C) $Hg(SCN)_2$ (D) Both (B) & (C)

Paragraph - 2 (Q. Nos. 35 & 36)

A solution of alanine hydrochloride $[H_3N-CH(CH_3)-COOH]^+ Cl^-$ is titrated with a solution of NaOH to produce a curve as follows :

K_1 and K_2 of alanine hydrochloride are 4.6×10^{-3} and 2.0×10^{-10} respectively,



35. The pH at point 3 is

(A) 2.34 (B) 6.02

(C) 9.7 (D) 11.2

36. In the curve, at point 4 which of the molecule is found?

(A) $H_3N^+ - CH(CH_3) - COOH$ (B) $H_3N^+ - CH(CH_3) - COO^-$

(C) $H_2N - CH(CH_3) - COOH$ (D) $H_2N - CH(CH_3) - COO^-$

PART - III : MATHEMATICS

SECTION - 1

Only One Option Correct Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

37. If $\sigma(n) = \sum_{r=1}^{n}\left(r^2\left({}^nC_r - {}^nC_{r-1}\right) + (2r+1)^nC_r\right)$ then $\sigma(50)$ is equal to

(A) 2400 (B) 2500

(C) 2600 (D) 2700

38. $px + qy = 40$ is a chord of minimum length of the circle $(x - 10)^2 + (y - 20)^2 = 729$. If the chord passes through (5, 15) then $(p^{2017} + q^{2017})$ is equal to

(A) Zero (B) 1

(C) 2^{2017} (D) 2^{2018}

39. The shortest distance between the lines $2x + y + z - 1 = 0 = 3x + y + 2z - 2$ and $x = y = z$ is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

40. $\int\frac{x^7 - x^5 + x^3 - x}{x^{10} + 1}dx$ is equal to

(A) $\frac{1}{10}\ln\left(\frac{x^{10}+1}{(x^2+1)^5}\right) + C$ (B) $\frac{1}{5}\ln\left(\frac{x^{10}+1}{(x^3+1)^4}\right) + C$

(C) $\frac{1}{5}\ln\left(\frac{x^8+1}{(x^2+1)^4}\right) + C$ (D) $\frac{1}{10}\ln\left(\frac{x^8+1}{(x^2+1)^4}\right) + C$

41. The function $f(x) = [x]\,(x^2 - 25)^n\,((x^2 + 3)(x^2 + 3x + 4)^3)^m$, $m, n \in N$ attains a local minima at $x = 5$.

([.] denote the greatest integer function), then

(A) $m = 10$, $n = 5$ (B) $m = 5$, $n = 10$

(C) $m = 20$, $n = 5$ (D) $m = 10$, $n = 3$

42. The number of integers belonging to [–15, 15] but not belonging to the range of the function $f(x) = x\{x\} - x[-x]$.

(Where [ ] and { } denote the greatest integer function and fractional part respectively.)

(A) 18 (B) 17

(C) 16 (D) 15

SECTION - 2

One or More Options Correct Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONE OR MORE** is/are correct.

43. A continuous, even periodic function f with period 8 is such that $f(0) = 0$, $f(1) = -2$, $f(2) = 1$, $f(3) = 2$ and $f(4) = 3$. If $k = (f(-5) + f(20) + f(17))$, then a function, $g : R \to R$ given by $g(x) = x^k + |\sin x|$ is

(A) One-one (B) Onto

(C) Many one (D) Into

44. If ABC is a triangle in which $\sqrt{3}\sin C + \tan A = 2\sec A$ and the longest side is of length 4 units, then

(A) $S = 2 + \sqrt{3}$ (B) $r = \sqrt{3} - 1$

(C) $R = 2$ (D) $B = \frac{\pi}{3}$

45. Let f be a differentiable function in (m, n).

If $f(x)\, f'(x) \geq x\sqrt{(f(x)^8 - (f(x))^4}$,

and $\lim\limits_{x \to m+} (f(x))^2 = 1$ and $\lim\limits_{x \to n-} (f(x))^2 = 2$. Then $|m^2 - n^2|$ cannot be equal to

(A) $\frac{2}{3}$ (B) 1

(C) $\frac{4}{3}$ (D) $\frac{5}{3}$

46. Let $\langle a_i \rangle$, $i = 1, 2, ...100$ be a given sequence. It is given that a_i is i less than the sum of other 99 numbers for $i = 1, 2, ..., 100$. If $S = \sum\limits_{i=1}^{100} a_i$, then which of the following assume integral values?

(A) $7S$ (B) $21S$

(C) $35S$ (D) $49S$

47. The maximum and minimum value of $x_1^2 + x_2^2 - 12x_1 - 4x_2 + 36$ if $4x_1^2 + 9x_2^2 + 4 = 8x_1 + 36x_2\ \forall x_1, x_2 \in R$ are m and n respectively, then

(A) $\sqrt{m} + \sqrt{n} = 15$ (B) $\sqrt{m} - \sqrt{n} = 4$

(C) $m + n = 60$ (D) $m - n = 60$

48. Virat tosses a coin. There is one point for throwing a tail and 2 points for throwing a head. He repeatedly tosses the coin till he gets a sum of points equal to 7. If p be the probability of getting a sum equal to 7, then

(A) $p > \frac{1}{2}$ (B) $p > \frac{3}{5}$

(C) $p > 0.65$ (D) $p > 0.7$

49. If the equation $|x^2 - 7x + 10| - mx + 10m = 0$ has k distinct solutions, then

(A) If $k = 3$, then $m \in \{-13 - 4\sqrt{10}\}$ (B) If $k = 4$, then $m \in (-13 + 4\sqrt{10},\ 0)$

(C) If $k = 2$, then $m \in (-13 + 4\sqrt{10},\ 0)$ (D) If $k = 3$, then $m \in \{-13 + 4\sqrt{10}\}$

50. A line given by $y = \alpha$, intersects the curve $y = x - 4x^3$ at $A(x_1, \alpha)$ and $B(x_2, \alpha)$ (where $x_1 < x_2$) in first quadrant. If the area bounded by y-axis, the curve and the line $y = \alpha$ is equal to the area bounded by the line $y = \alpha$ and the curve in first quadrant, then

(A) $3\alpha - x_2 = 1$ (B) $3\alpha - x_2 = 0$

(C) $\alpha^2 + x_2^2 = \frac{5}{27}$ (D) $\alpha x_2 = \frac{1}{18}$

SECTION - 3

Paragraph Type

This section contains 2 paragraphs. Based on each paragraph, there are 2 questions. Each question has 4 choices (A), (B), (C) and (D). Out of which **ONLY ONE** is correct.

Paragraph - 1 (Q. Nos. 51 & 52)

A hyperbola $H \equiv 2x^2 - 2y^2 = 1$ intersects an ellipse $E \equiv \frac{x^2}{p^2} + \frac{y^2}{q^2} = 1$ orthogonally. The eccentricity of the ellipse E is half that of the hyperbola H.

51. The equation of E is given by

(A) $\frac{x^2}{4} + \frac{y^2}{2} = 1$ (B) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

(C) $\frac{x^2}{2} + \frac{y^2}{4} = 1$ (D) $\frac{x^2}{2} + \frac{y^2}{1} = 1$

52. The coordinates of the point from where two mutually perpendicular tangents to E can be drawn are

(A) (1, 2) (B) (2, 1)

(C) $(\sqrt{2}, 1)$ (D) (–1, 2)

Paragraph - 2 (Q. Nos. 53 & 54)

z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = 1$ and $\frac{z_1^2}{z_2 z_3} + \frac{z_2^2}{z_3 z_1} + \frac{z_3^2}{z_1 z_2} + 1 = 0$

53. Sum of all the integral values of $|z_1 + z_2 + z_3|$ is equal to

(A) 0 (B) 1

(C) 2 (D) 3

54. If $|z_1 + z_2 + z_3| = k$ where k is the greatest integral value of $|z_1 + z_2 + z_3|$, then $\text{Re}(z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1)$ equals

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$

(C) 1 (D) 2

❑ ❑ ❑

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Mock Test

Paper - I (Code-A)

MM : 186

ANSWERS

PHYSICS

1. (C)
2. (A)
3. (D)
4. (D)
5. (D)
6. (A, D)
7. (B, C)
8. (A, D)
9. (A, C, D)
10. (B)
11. (A, C)
12. (A, B, C)
13. (A, B, D)
14. (5)
15. (2)
16. (3)
17. (3)
18. (8)

CHEMISTRY

19. (A)
20. (D)
21. (B)
22. (D)
23. (C)
24. (B, C, D)
25. (B, C, D)
26. (B, C, D)
27. (A, B, C, D)
28. (B, C, D)
29. (B, C)
30. (A, C)
31. (A, B, C)
32. (4)
33. (5)
34. (8)
35. (3)
36. (3)

MATHEMATICS

37. (B)
38. (D)
39. (B)
40. (A)
41. (A)
42. (A, C, D)
43. (A, C)
44. (A, B, D)
45. (B, C)
46. (B, D)
47. (A, B, D)
48. (A, B, C, D)
49. (A, B, D)
50. (9)
51. (8)
52. (0)
53. (4)
54. (6)

ANSWERS & HINTS

PART - I (PHYSICS)

1. Answer (C)

$\lambda_{max} = \dfrac{1240 \text{ eV-nm}}{\phi(\text{in eV})}$

Plate A : $\lambda_{max} = \dfrac{1240}{2} = 620$ nm

$\Rightarrow$ For plate A, all three wavelengths can cause electron ejection

Plate B : $\lambda_{max} = \dfrac{1240}{3} = 413.3$ nm

$\Rightarrow$ For plate B, only 350 nm and 250 nm wavelength can cause electron ejection

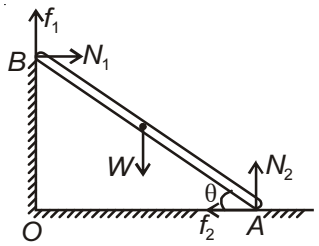
Plate C : $\lambda_{max} = \dfrac{1240}{4} = 310$ nm

$\Rightarrow$ For plate C, only 250 nm wavelength can cause electron ejection

Thus, photoelectric current $A > B > C$.

Stopping potential must be highest for plate A, as it is most photosensitive (low ϕ) [KE of photoelectrons will be highest]

2. Answer (A)



For equilibrium of rod considering limiting friction, we use.

$N_1 = f_2$

$f_1 + N_2 = W$

$(f_1)_{max} = \mu N_1$

$(f_2)_{max} = \mu N_2$

$\Rightarrow N_1 = \mu N_2$ & $\left(\mu N_1 + \dfrac{N_1}{\mu}\right) = W$

$\Rightarrow N_1\left[\dfrac{\mu^2+1}{\mu}\right] = W$

Balancing all torques about point A, we use.

$W\left[\dfrac{5}{2}\cdot\dfrac{4}{5}\right] - N_1(3) - f_1(4) = 0$

$\Rightarrow 2W - 3N_1 - 4f_1 = 0$

$\Rightarrow 2\cdot\left[\dfrac{(\mu^2+1)}{\mu}N_1\right] - 3N_1 - 4[\mu N_1] = 0$

$\Rightarrow (\mu + 2)(2\mu - 1) = 0$

$\therefore \mu = \dfrac{1}{2}$ [as $\mu \neq -2$]

3. Answer (D)

$W = nRT\ln\left(\dfrac{V_2}{V_1}\right)$ as temperature remains constant of bubble.

Ideal-gas law gives $nRT = P_1V_1 = P_2V_2$

$\Rightarrow \dfrac{V_2}{V_1} = \dfrac{P_1}{P_2} = 8$

$\therefore W = (8 \times 10^5 \text{ N/m}^2)\left[\dfrac{4}{3}\pi(8\times10^{-3})^3\right]\ln 8$

$= 3.56$ joule.

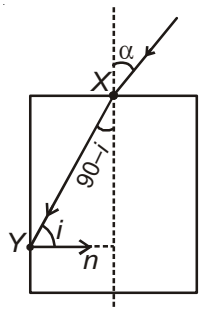
4. Answer (D)

Applying Snell's law at point X.

$1 \times \sin\alpha = \mu \times \sin(90 - i)$

$\Rightarrow \cos i = \dfrac{\sin\alpha}{\mu}$

$\therefore \sin i = \dfrac{\sqrt{\mu^2 - \sin^2\alpha}}{\mu}$



For TIR to occur at vertical surface AB, at point Y.

$i > i_C$

$\sin i > \sin i_C$

$\dfrac{\sqrt{\mu^2 - \sin^2\alpha}}{\mu} > \dfrac{1}{\mu}$

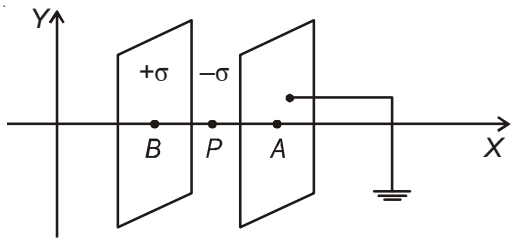
$\Rightarrow \quad \sqrt{\mu^2 - \sin^2\alpha} > 1$

$\mu > \sqrt{1+\sin^2\alpha}$

If $\alpha < 60°$, then i is always greater than critical angle.

$\therefore \quad \mu_{min} = \sqrt{1+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{1+\frac{3}{4}} = \frac{\sqrt{7}}{2}$

5. Answer (D)

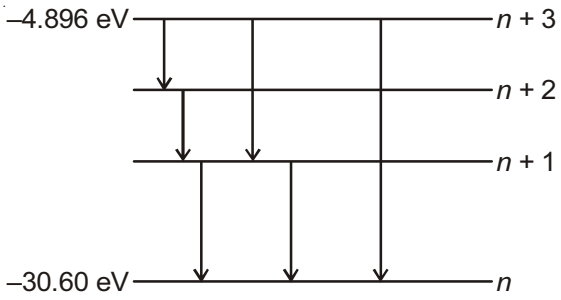


$V_P - V_A = -\int_{r=2a}^{x}\left(\frac{\sigma}{\varepsilon_0}\right)dr$

$V_P - 0 = -\frac{\sigma}{\varepsilon_0}\int_{2a}^{x} dr$

$V_P = \frac{\sigma}{\varepsilon_0}[2a - x]$

6. Answer (A, D)



For the emission of six spectral lines, there must be four energy levels.

Thus two more levels are in between –30.6 eV and –4.896 eV.

If n represents the energy level of energy –30.60 eV, then $(n + 3)$ represent the energy level of energy –4.896 eV.

$\therefore \quad -13.6\frac{z^2}{n^2} = -30.6 \text{ eV} \qquad ...(i)$

$-13.6\frac{z^2}{(n+3)^2} = -4.896 \text{ eV} \qquad ...(ii)$

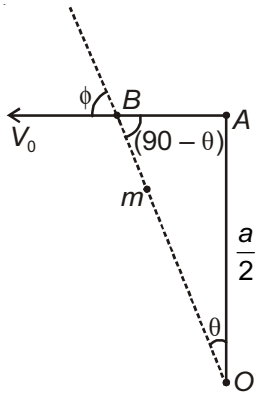
Solving equation (i) & (ii)

$z = 3$, $n = 2$

The smallest wavelength corresponding to maximum energy difference transition.

$\therefore \quad \lambda_{min} = \frac{1240 \text{ eV} - \text{nm}}{[-4.896 - (-30.60)]\text{eV}} = 482 \text{ Å}$

7. Answer (B, C)



Let the line joining running man, car and centre makes an angle θ with vertical OA, at any time t.

$AB = \frac{a}{2}\tan\theta$

Velocity of observer (man) in the car at time t.

$(V_0)_{car} = \frac{d}{dt}(AB)$

$= \frac{a}{2}\sec^2\theta \cdot \frac{d\theta}{dt}$ but $\frac{d\theta}{dt} = \omega = 22 \text{ rad/s}$

$\therefore \quad (V_0)_{car} = 11a \sec^2\theta$

Now, apparent frequency heard by observer(man) in the car at time t.

$n' = \left[\frac{V - V_0\cos\phi}{V}\right]n$

where V = velocity of sound in air

and ϕ = angle between velocity vector of observer car and the line joining man in car and centre.

$\therefore \quad n' = \left[\frac{V - 11a\sec^2\theta\sin\theta}{V}\right]n$

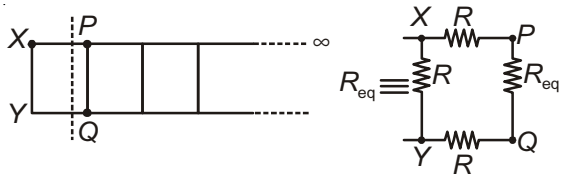
But $\sec^2\theta \sin\theta$ is a increasing function in the range $0 \le \theta \le \frac{\pi}{2}$.

n' will be minimum when $\sec^2\theta \sin\theta$ is maximum, which occurs at $\theta = 45°$, because after that direction of velocity of car will alter.

$$\therefore \quad (n')_{min} = \left(\frac{V - 11a\sqrt{2}}{V}\right) n = \frac{330 - 110}{330} \times 300$$

$$= 200 \text{ Hz}$$

8. Answer (A, D)



Equivalent thermal resistance between X & Y.

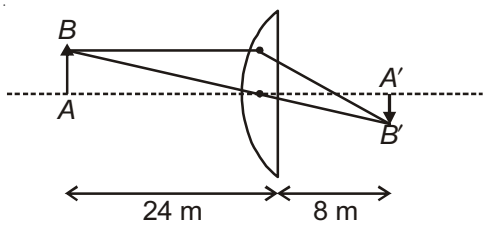
$$\frac{1}{R_{eq}} = \frac{1}{(R + R_{eq} + R)} + \frac{1}{R} \text{ where } R = \frac{l}{kA}$$

On solving $R_{eq} = R(\sqrt{3} - 1) = \frac{l(\sqrt{3} - 1)}{kA}$

$\therefore$ Rate of heat flow between X & $Y = \frac{\theta_1 - \theta_2}{R_{eq}}$

$$= \frac{kA(\theta_1 - \theta_2)}{l(\sqrt{3} - 1)}$$

9. Answer (A, C, D)



$$\mu = \frac{\lambda_a}{\lambda_m} = \frac{3}{2}$$

$$\frac{1}{f} = \frac{(\mu - 1)}{R} = \frac{\left(\frac{3}{2} - 1\right)}{R}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{2R}$$

From thin lens formula

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{2R} = \frac{1}{8} - \frac{1}{(-24)}$$

$$\Rightarrow \quad R = 3 \text{ m}$$

$$f = 6 \text{ m}$$

10. Answer (B)

$T \propto S^x r^y \rho^z$

$T = kS^x r^y \rho^z$

By substituting the dimension of each quantity in both sides

$[M^0L^0T^1] = K[MT^{-2}]^x [L]^y [ML^{-3}]^z = [M^{x+y}L^{y-3z}T^{-2x}]$

$$\therefore \quad x = -\frac{1}{2}, \; z = \frac{1}{2}, \; y = \frac{3}{2}$$

$$\therefore \quad T = K\sqrt{\frac{\rho r^3}{S}}$$

11. Answer (A, C)

$\phi_i = BA\cos 0 = BA$

$\phi_f = 0$

$|\Delta\phi| = |\phi_f - \phi_i| = BA$

Average induced emf $= \frac{BA}{t}$

$$= \frac{1.2 \times \pi[0.05]^2}{0.2}$$

$$= 0.047 \text{ volts}$$

$\Rightarrow$ Charge $= \frac{|\Delta\phi|}{R} = 9.42 \times 10^{-4}$ C

$\therefore$ Direction of current in R is clockwise as the flux decreases, induced field must support the existing field

12. Answer (A, B, C)

If $\vec{B}$ is uniform then L_{eq} for whole wire

$$\left|\overrightarrow{L_{eq}}\right| = 2(L + R)$$

Net force on wire will be $\vec{F} = I\left(\overrightarrow{L_{eq}} \times \vec{B}\right)$

If $\overrightarrow{L_{eq}} \perp \vec{B}$ then $|\vec{F}| = I\,|\overrightarrow{L_{eq}}|\,|\vec{B}|$

$$= 2I\,(L + R)\,B$$

If $\overrightarrow{L_{eq}} \parallel \vec{B}$ then $\vec{F} = 0$

13. Answer (A, B, D)

$$E_{mix} = \frac{1\times\frac{3RT}{2}+1\times\frac{5RT}{2}}{1+1} = 2RT$$

$$\gamma_{mix} = \frac{3}{2}$$

$$M_{mix} = \frac{1\times 4+1\times 32}{1+1} = 18$$

$$V_S = \sqrt{\frac{\gamma RT}{M}}$$

$$\frac{V_{mix}}{V_{He}} = \sqrt{\frac{\gamma_{mix}\cdot M_{He}}{\gamma_{He}\cdot M_{mix}}} = \frac{1}{\sqrt{5}}$$

$$V_{rms} \propto \frac{1}{\sqrt{M}}$$

$$\frac{V_{He}}{V_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{He}}} = 2\sqrt{2}$$

14. Answer (5)

$$E = \frac{1242 \text{ eV-nm}}{\lambda(\text{nm})}$$

$$E[\lambda = 108.72 \text{ nm}] = \frac{1242}{108.72} = 11.424 \text{ eV}$$

$$E[\lambda = 30.44 \text{ nm}] = \frac{1242}{30.44} = 40.80 \text{ eV}$$

$\therefore$ Total energy emitted (ΔE) = (11.424 + 40.80) eV

= 52.224 eV

$$\therefore \quad \Delta E = 13.6z^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$$52.224 = 13.6\times 4\left[\frac{1}{1^2}-\frac{1}{n^2}\right]$$

$$\Rightarrow \quad 1-\frac{1}{n^2} = 0.96$$

$$\Rightarrow \quad \frac{1}{n^2} = 0.04 = \frac{4}{100}$$

$\therefore$ n = 5 (*i.e.*, 4th excited state)

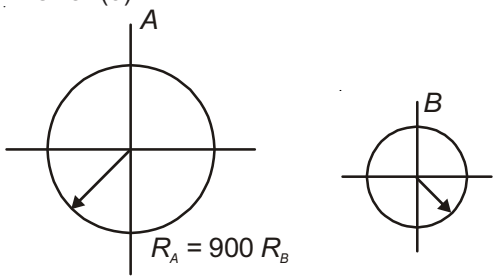
15. Answer (2)

When B is maximum then rate of disintegration of A = Rate of disintegration of B

$\therefore$ $\lambda N_1 = (2\lambda).\ N_2$

$$\therefore \quad \frac{N_1}{N_2} = 2$$

16. Answer (3)



From Wien's displacement law

$$\frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A}$$

From Stefan's Law

$P = \sigma e A T^4$

$\therefore$ $P_A = \sigma e\ 4\pi\ [900\ R]^2 \times T_A^4$

$P_B = \sigma e\ 4\pi\ R^2 \times T_B^4$

Given $P_A = 10^4\ P_B$

$\Rightarrow$ $(900)^2 \times T_A^4 = 10^4 \times T_B^4$

$$\Rightarrow \quad \frac{T_B}{T_A} = 3$$

17. Answer (3)

$$V_T = \frac{2}{9}r^2\frac{(\sigma-\rho)\,g}{\eta}$$

$$\therefore \quad \frac{V_A}{V_B} = \left(\frac{r_A}{r_B}\right)^2\cdot\left(\frac{\sigma_A-\rho_1}{\sigma_B-\rho_2}\right)\left(\frac{\eta_2}{\eta_1}\right)$$

$$= \left(\frac{2}{1}\right)^2\left(\frac{9-1.8}{9-2.6}\right)\left(\frac{2}{3}\right) = 4\times\frac{7.2}{6.4}\times\frac{2}{3} = 3$$

18. Answer (8)

$$I_{max} = \frac{10}{\left(\frac{3}{2}\right)} \text{ A}$$

$$I_{min} = \frac{10}{12} \text{ A}$$

$$\frac{I_{max}}{I_{min}} = \frac{12\times 2}{3} = 8$$

PART - II (CHEMISTRY)

19. Answer (A)

Energy of photon = ΔE

$$\Delta E = 2.18\times10^{-18}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]Z^2$$

$$= 2.18\times10^{-18}\left[\frac{1}{(1)^2}-\frac{1}{(2)^2}\right](3)^2$$

$$= 2.18\times9\times\frac{3}{4}\times10^{-18}$$

$= 14.715 \times 10^{-18}$ J

Ionisation energy of hydrogen = 2.18×10^{-18} J

$E = E_0$ + kinetic energy of electron

K.E. = $E - E_0$

K.E. = $(14.715-2.18)10^{-18}$

K.E. = 12.535×10^{-18} J

20. Answer (D)

Availability of electrons is more.

21. Answer (B)

$$M_{(s)} \longrightarrow M_{(g)} \qquad \Delta H_{sub} = 80 \text{ kJ/mole}$$

Now

$$\begin{array}{ll} M_{(g)} \longrightarrow M^+_{(g)} + e & \Delta H = 300 \text{ kJ/mole} \\ M^+_{(g)} \longrightarrow M^+_{(aq)} & \Delta H = -400 \text{ kJ/mole} \\ \hline M_{(g)} \longrightarrow M^+_{(aq)} & \Delta H = -100 \text{ kJ/mole} \end{array}$$

Similarly

$$\begin{array}{ll} M_{(g)} \longrightarrow M^+_{(g)} & \Delta H_1 = 300 \text{ kJ/mole} \\ M^+_{(g)} \longrightarrow M^{+2}_{(g)} & \Delta H_2 = 700 \text{ kJ/mole} \\ M^{+2}_{(g)} \longrightarrow M^{+2}_{(aq)} & \Delta H = -1200 \text{ kJ/mole} \\ \hline M_{(g)} \longrightarrow M^{+2}_{(aq)} & \Delta H = -200 \text{ kJ/mole} \end{array}$$

$M^{+2}_{(aq)}$ is more stable than $M^+_{(aq)}$ since ΔH is more negative hence more stability of ion.

22. Answer (D)

23. Answer (C)

Partial pressure of gases in mixture after opening stopcock when no reaction occur.

$P_1 \times V_1 = P_2V_2$

$$P_{(NO)} = \frac{1.5\times200}{350}$$

$= 0.875$ atm

$$P(O_2) = \frac{0.5\times150}{350}$$

$= 0.214$ atm

Now, gases react, here O_2 act as limiting reagent

	$2NO$	$+ \ O_2$	$\longrightarrow 2NO_2$	$\longrightarrow N_2O_4$
Initial	0.875 atm	0.214 atm	—	
Final	0.447 atm	zero	—	0.214 atm

[Since N_2O_4 is solidified so only NO exert pressure]

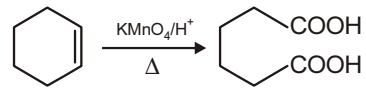
$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

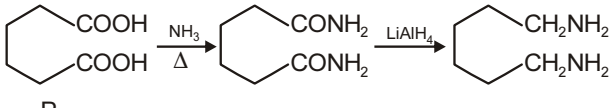
$$\frac{0.447}{P_2} = \frac{300}{220}$$

$$P_2 = \frac{0.447\times220}{300}$$

$= 0.327$ atm

24. Answer (B, C, D)





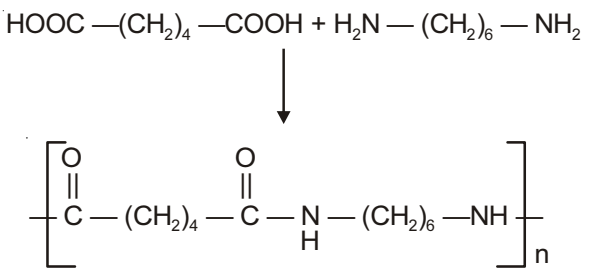
P
Adipic acid

Q

R

Now, P and R react

$P + R \rightarrow$ Nylon-66



Nylon-66

25. Answer (B, C, D)

$$6XeF_4 + 12H_2O \longrightarrow 2XeO_3 + 3O_2 + 12H_2F_2 + 4Xe$$

26. Answer (B, C, D)

Mass of CaO = 0.56 g

Moles of CaO = $\frac{0.56}{56} = 0.01$

Moles of $CaCl_2$ = 0.01

Mass of $CaCl_2$ = 0.01 × 111 gm = 1.11 g

Mass of NaCl = 4.44 – 1.11 = 3.33 g

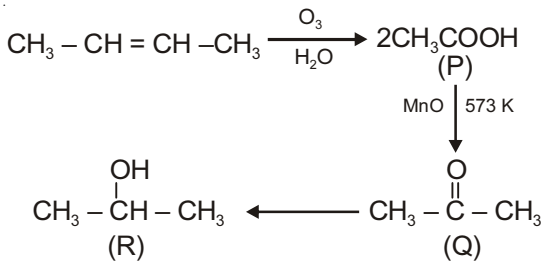
Percentage of $CaCl_2$ = $\frac{1.11}{4.44} \times 100$ = 25%

27. Answer (A, B, C, D)

Corundum Al_2O_3

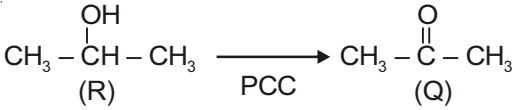
Feldspar $KAlSi_3O_8$

28. Answer (B, C, D)



Q – do not give Fehling solution test.

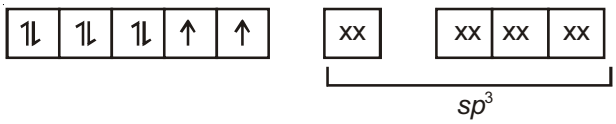
R – will form iodoform.



29. Answer (B, C)

$Ni^{+2} = [Ar]3d^8$

(B) $[NiCl_4]^{2-}$

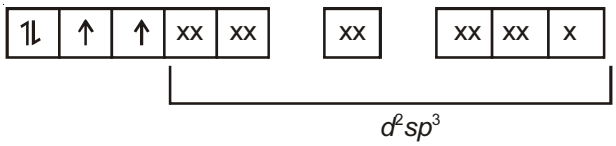


Number of unpaired electrons = 2

n = 2

(C) $Mn^{+3} = 3d^4$

$[Mn(CN)_6]^{3-}$



Number of unpaired electron = 2

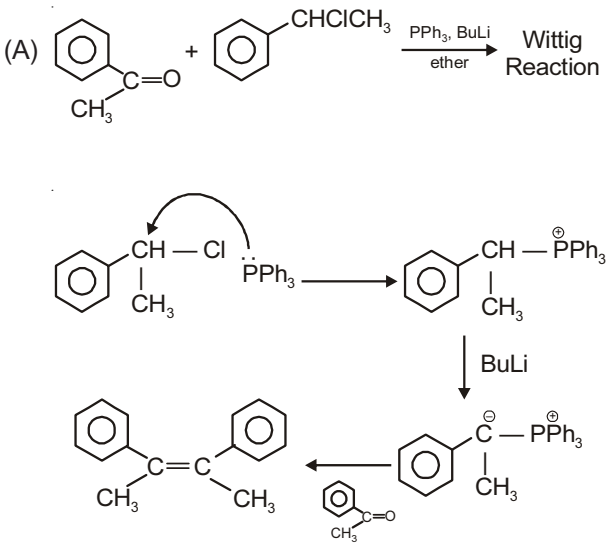
Magnetic moment = $\sqrt{n(n+2)}$ BM

= $\sqrt{2(2+2)}$ BM

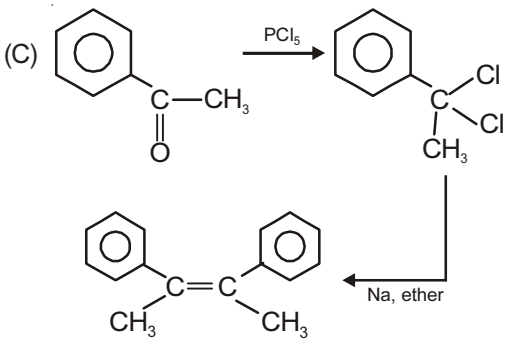
= $\sqrt{8}$

Magnetic moment = 2.82 BM

30. Answer (A, C)



(C)



31. Answer (A, B, C)

These ions are precipitated as sulphide in alkaline medium. Their K_{sp} is high they are group-IV cation.

32. Answer (4)

Glucose, Fructose, HCOOH, CH_3CH_2CHO

33. Answer (5)

(a), (c), (e), (f) and (g) are coloured

$K_3[VF_6]$ $V^{+3} = 3d^2$

$[Cr(NH_3)_6]Cl_3$ $Cr^{+3} = 3d^3$

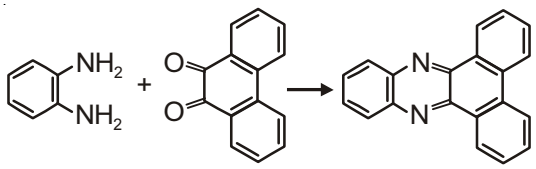
Partially filled *d*-orbitals

$Cu^{+1} = 3d^{10}$

$Ti^{+4} = 3d^0$

Colourless

34. Answer (8)



Phenanthraquinone

Degree of unsaturation = 16

$2x = 16$

$x = 8$

35. Answer (3)

Gram equivalent of $KMnO_4$ = gram equivalent of FeC_2O_4

$N_1V_1 = \frac{w}{Eq.wt.}$

$0.1 \times V_1 = \frac{14.4}{144} \times 3$

V_1 = 3 litres

n factor of $KMnO_4$ = 5

n factor of FeC_2O_4 = 3

36. Answer (3)

Na_2O, KNO_2, CH_3CH_2COONa

Salt of strong base and weak acid

PART - III (MATHEMATICS)

37. Answer (B)

There are 10 letters

A, A, I, I, N, N, T, T, O, S

Then the number of permutations

$$N = \text{Coefficient of } x^4 \text{ in } 4!\left[\left(1+\frac{x}{1!}+\frac{x^2}{2!}\right)^4(1+x)^2\right]$$

$$= \text{Coefficient of } x^4 \text{ in } 4!\left[(1+x)^4 + 4(1+x)^3\cdot\frac{x^2}{2} + 6(1+x)^2\frac{x^4}{4} + 4(1+x)\frac{x^6}{8} + \frac{x^8}{16}\right](1+x)^2$$

$$= \text{Coefficient of } x^4 \text{ in } 4!\left[(1+x)^6 + 2(1+x)^5x^2 + \frac{3}{2}(1+x)^4\cdot x^4 + (1+x)^3\cdot\frac{x^6}{2} + \frac{x^8}{16}(1+x)^2\right]$$

$$N = 4!\left[{}^6C_4 + 2\cdot{}^5C_2 + \frac{3}{2}{}^4C_0\right]$$

$$= 4!\left[15 + 20 + \frac{3}{2}\right] = 24\left[\frac{60+40+3}{2}\right]$$

$$= 12 \times 73 = 2^2 \cdot 3^1 \cdot 73^1$$

$\Rightarrow$ $p_1 = 2$, $p_2 = 3$ and $p_3 = 73$

$\therefore$ $p_1 + p_2 + p_3 = 78$.

38. Answer (D)

$${}^nC_0\ {}^{3n}C_{2n} - {}^nC_1\ {}^{3n-3}C_{2n-3} + {}^nC_2\ {}^{3n-6}C_{2n-6} - \cdots$$

$$= {}^nC_0\ {}^{3n}C_n - {}^nC_1\ {}^{3n-3}C_n + {}^nC_2\ {}^{3n-6}C_n \cdots$$

= Coefficient of x^n in the expansion of $((1 + x)^3 - 1)^n$

= Coefficient of x^n in the expansion of $(3x + 3x^2 + x^3)^n$

= Coefficient of x^n in the expansion of $x^n(3 + 3x + x^2)^n$

$= 3^n$

39. Answer (B)

Given equation can be written as

$2\cos k \cdot \cos x + \sin x = 2$

Since for real solutions of $a\cos x + b\sin x = C$

$|C| \le \sqrt{a^2 + b^2}$

$\therefore \quad |2| \le \sqrt{1+4\cos^2 k} \quad \Rightarrow \quad 2 \le \sqrt{1+4\cos^2 k}$

$\Rightarrow \quad \sin^2 k \le \frac{1}{4} \quad \Rightarrow \quad -\frac{1}{2} \le \sin k \le \frac{1}{2}$

$\therefore \quad a = -\frac{1}{2}, b = \frac{1}{2} \quad \Rightarrow \quad b - a = 1$

40. Answer (A)

$x^2 + \frac{x^2}{(x+1)^2} = 3$ can be written as

$\Rightarrow \quad \left(x - \frac{x}{(x+1)}\right)^2 + 2 \cdot x \cdot \frac{x}{x+1} = 3$

$\Rightarrow \quad \left(\frac{x^2}{x+1}\right)^2 + 2\left(\frac{x^2}{x+1}\right) - 3 = 0$

$t^2 + 2t - 3 = 0; \left(t = \frac{x^2}{x+1}\right)$

$t = -3$ and $t = 1$

$\therefore \quad \frac{x^2}{x+1} = 1 \quad \Rightarrow \quad x^2 - x - 1 = 0$ ...(1)

which gives real roots

and $\frac{x^2}{x+1} = -3 \quad \Rightarrow \quad x^2 + 3x + 3 = 0$ ...(2)

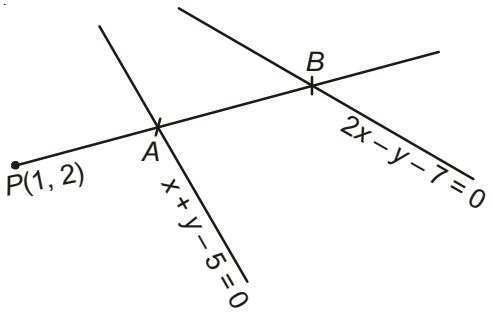
which gives non-real roots

$\therefore$ From equation (1)

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 1 - 2(-1) = 1 + 2 = 3$

41. Answer (A)



Let any straight line passes through $P(1, 2)$ is

$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = r_1, r_2$ where $r_1 = PA$, $r_2 = PB$

$\therefore$ Coordinate of $A(1 + r_1\cos\theta, 2 + r_1\sin\theta)$ which satisfies $x + y = 5$ *i.e.*,

$r_1 = \frac{2}{\cos\theta + \sin\theta}$ ...(I)

Similarly coordinate of $B(1 + r_2\cos\theta, 2 + r_2\sin\theta)$ which satisfies $2x - y = 7$ *i.e.*,

$r_2 = \frac{7}{2\cos\theta - \sin\theta}$ ...(II)

Now given that r_1 2, r_2 are in H.P.

$\therefore \quad \frac{2}{2} = \frac{1}{r_1} + \frac{1}{r_2} \quad \Rightarrow \quad \frac{\cos\theta + \sin\theta}{2} + \frac{2\cos\theta - \sin\theta}{7} = 1$

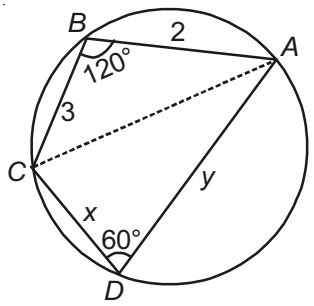
$\Rightarrow$ $11\cos\theta + 5\sin\theta = 14$

$\because$ L.H.S. is always less than R.H.S.

So there is no real value of θ.

$\therefore$ We cannot draw any real straight line.

42. Answer (A, C, D)



Let $AB = 2$ and $BC = 5$ and $\angle ABC = 120°$

$\angle CDA = 180° - 120° = 60°$

Now area of quadrilateral $ABCD = 4\sqrt{3}$

$\Rightarrow$ Area of ΔABC + area of $\Delta ACD = 4\sqrt{3}$

$\Rightarrow \quad \frac{1}{2}AB \cdot BC\sin 120° + \frac{1}{2}CD \cdot DA\sin 60° = 4\sqrt{3}$

$\Rightarrow \quad \frac{1}{2} \times 2 \times 3\sin 120° + \frac{1}{2}xy\sin 60° = 4\sqrt{3}$

$\Rightarrow \quad \boxed{xy = 10}$ ...(1)

Now again

$AC^2 = x^2 + y^2 - 2xy\cos 60°$

$= AB^2 + BC^2 - 2AB \times BC\cos 120°$

$\Rightarrow x^2 + y^2 - xy = 4 + 9 - 2\cdot 2\cdot 3\cdot\left(-\frac{1}{2}\right)$

$\Rightarrow x^2 + y^2 - 10 = 19$

$\Rightarrow \boxed{x^2 + y^2 = 29}$ ...(2)

On solving (1) and (2), we have

Either $x = 5$, $y = 2$ or $x = 2$, $y = 5$

43. Answer (A, C)

Equation of chord of hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Whose middle point be (h, k) is

$\frac{xh}{16} - \frac{ky}{9} = \frac{h^2}{16} - \frac{k^2}{9}$; (Using $T = S_1$)

It passes through (4, 2)

$\therefore \frac{4h}{16} - \frac{2k}{9} = \frac{h^2}{16} - \frac{k^2}{9}$

$\Rightarrow 36h - 32k = 9h^2 - 16k^2$

$\Rightarrow 9h^2 - 36h - 16k^2 + 32k = 0$

$\Rightarrow 9(h^2 - 4h) - 16(k^2 - 2k) = 0$

$\Rightarrow 9(h^2 - 4h + 4) - 16(k^2 - 2k + 1) = 36 - 16$

$\Rightarrow \frac{(h-2)^2}{\frac{20}{9}} - \frac{(k-1)^2}{\frac{20}{16}} = 1$

$\boxed{\frac{(x-2)^2}{\frac{20}{9}} - \frac{(y-1)^2}{\frac{20}{16}} = 1}$

Clearly centre be (2, 1) and eccentricity be $\frac{5}{4}$.

44. Answer (A, B, D)

Let $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, Then $B = \begin{vmatrix} a_{11} & \frac{a_{12}}{5} & \frac{a_{13}}{5^2} \\ 5a_{21} & a_{22} & \frac{a_{23}}{5} \\ 5^2 a_{31} & 5a_{32} & a_{33} \end{vmatrix}$,

$\Rightarrow |B| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = |A|$.

$\Rightarrow |B| = 600 = 2^3 \times 3^1 \times 5^2$

$\Rightarrow \lambda = 2^3 \cdot 3^1 \cdot 5^2$

For number of odd division of λ = (1 + 1) (2 + 1)

= 2 × 3 = 6

Number of divisors, of λ which are perfect square we consider 2^0, 2^2 and 5^0, 5^2

$\therefore$ The number of such divisors = 2 × 2 = 4

Number of divisors of λ which are even

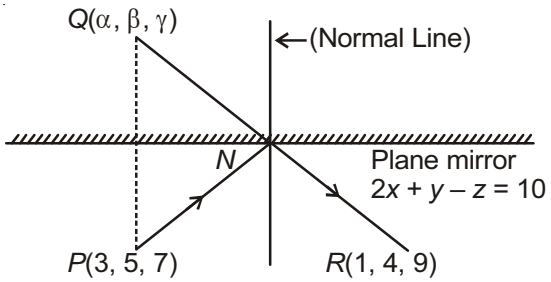
3 × 2 × 3 = 18

Number of divisors of λ which are divisible by 24;

$(2^3 \cdot 3)5^2 = 2 + 1 = 3$

45. Answer (B, C)

Let image of $P(3, 5, 7)$ w.r.t the plane mirror $2x + y - z = 10$, is $Q(\alpha, \beta, \gamma)$



$\therefore \frac{\alpha - 3}{2} = \frac{\beta - 5}{1} = \frac{\gamma - 7}{-1}$

$= \frac{-2[(2)(3) + (1)(5) + (-1)(7) - 10]}{2^2 + 1^2 + (-1)^2}$

$\Rightarrow \frac{\alpha - 3}{2} = \frac{\beta - 5}{1} = \frac{\gamma - 7}{-1} = \frac{12}{6} = 2$

$\Rightarrow \alpha = 7, \beta = 7, \gamma = 5$

$\Rightarrow Q = (7, 7, 5)$

Now equation of QR is $\frac{x-1}{6} = \frac{y-4}{3} = \frac{z-9}{-4}$

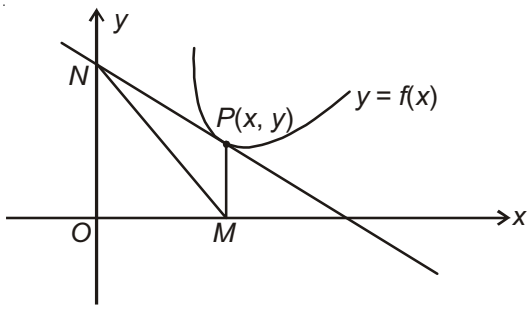
which intersect the plane at $N = \left(\frac{97}{19}, \frac{115}{19}, \frac{119}{19}\right)$.

46. Answer (B, D)

Equation of tangent

$Y - y = \frac{dy}{dx}(X - x)$

We get, coordinate of $N = \left(0, y - x\frac{dy}{dx}\right)$



$\Rightarrow$ $ON = y - x\frac{dy}{dx}$

and $OM = x$.

Given area of $\Delta MON = \frac{1}{2}OM \times ON = 2$

$\Rightarrow$ $\frac{1}{2}x\left(y - x\frac{dy}{dx}\right) = 4$

$\Rightarrow$ $\frac{dy}{dx} - \frac{1}{x}y = \frac{-4}{x^2}$

$\therefore$ I.F. $e^{-\int \frac{dx}{x}} = e^{-\log x} = \frac{1}{x}$

$\therefore$ Solution is $y \cdot \frac{1}{x} = -\int \frac{4}{x^3}dx + C$

$\frac{y}{x} = \frac{2}{x^2} + C$, which passes through (1, 1)

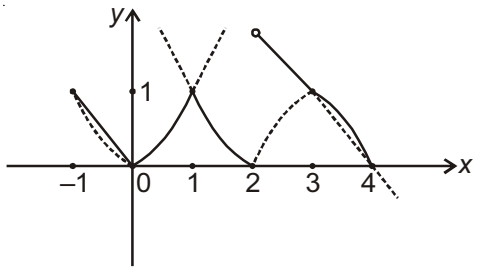
$\therefore$ $C = -1$

$\therefore$ Equation of curve is $x^2 + xy - 2 = 0$

which is hyperbola.

47. Answer (A, B, D)

$$f(x) = \begin{cases} \max\{-x, x^2\}; & -1 \le x < 0 \\ \min\{x^2, (x-2)^2\}; & 0 \le x \le 2 \\ \max\{4-x, 6x-x^2-8\}; & 2 < x \le 4 \end{cases}$$



From the graph we clearly see $f(x)$ is discontinuous at $x = 2$ and non-differentiable at $x = 0$, 1, 2 and 3.

$f(x)$ has local maxima at $x = 1$, *i.e.,* at one point

$f(x)$ has local minima at $x = 0$ & $x = 2$ *i.e.,* at two points

48. Answer (A, B, C, D)

Let 'α' be the real root of given equation then

$\alpha^3 + (3 + 2i)\alpha + (-1 + i\lambda) = 0$ ...(1)

$\overline{\alpha^3} + \overline{(3+2i)\alpha} + \overline{(-1+i\lambda)} = 0$

$\Rightarrow$ $\alpha^3 + (3 - 2i)\alpha + (-1 - i\lambda) = 0$ ...(2)

(1) – (2), $\Rightarrow 4i\alpha + 2i\lambda = 0$

$\Rightarrow$ $\alpha = -\frac{\lambda}{2}$ substitute in (1)

$-\frac{\lambda^3}{8} + (3+2i)\left(-\frac{\lambda}{2}\right) + (-1+i\lambda) = 0$

$-\lambda^3 + 4(3 + 2i)(-\lambda) + 8(-1 + i\lambda) = 0$

$\Rightarrow$ $-\lambda^3 - 12\lambda - 8i\lambda - 8 + 8i\lambda = 0$

$\Rightarrow$ $\lambda^3 + 12\lambda + 8 = 0$

Now we discuss location of roots of $f(\lambda) = 0$;

when $f(\lambda) = \lambda^3 + 12\lambda + 8$

$f'(\lambda) = 3\lambda^2 + 12 > 0$, $\forall \lambda \in R$

Now $f(-1)$ = –ve, $f(0)$ = +ve

$\therefore$ $\lambda \in (-1, 0)$

49. Answer (A, B, D)

Given that $E_1, E_2, ..E_{1008}$ are independent events. Therefore $E_1', E_2', ..., E_{1008}'$ are also independent events.

Given that $P(E_i) = \frac{i}{2i+1}$; (i = 1, 2,.., 1008)

Then the probability that none of the events occurs

$= (1 - P(E_1))(1 - P(E_2)) ... (1 - P(E_{1008}))$

$= \left(1-\frac{1}{3}\right)\left(1-\frac{2}{5}\right)\left(1-\frac{3}{7}\right)\left(1-\frac{4}{9}\right)...\left(1-\frac{1008}{2017}\right)$

$= \frac{2}{3}\cdot\frac{3}{5}\cdot\frac{4}{7}\cdot\frac{5}{9}\cdots\frac{1009}{2017}$

$= \frac{(1009!)2.4.6...2016}{2.3.4.5.6.7...2016.2017}$

$= \frac{(1009!)2^{1008}.(1.2.3....1008)}{2017!}$

$= \frac{(1008!)(1009!).2^{1008}}{2017!}$

$\therefore$ $b = 1008$, $c = 1009$ and $d = 2017$

50. Answer (9)

$$\lim_{x\to\infty}\left(\frac{x^3+4x^2+3x+5}{x^2+2x+3}+lx+m\right) = 10$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{x^3+4x^2+3x+5+lx^3+(2l+m)x^2+(3l+2m)x+3m}{x^2+2x+3}\right)=10$$

$$\Rightarrow \lim_{x\to\infty}\frac{(l+1)x^3+(2l+m+4)x^2+(3l+2m+3)x+3m+5}{(x^2+2x+3)}=10$$

$\Rightarrow$ $l + 1 = 0$, $2l + m + 4 = 10$

$\therefore$ $l = -1$, $m = 8$

Then value of $l^2 + m = 9$

51. Answer (8)

Given that $f(x) = 1+\frac{x^3}{3}+\int_0^x e^{-t}f(x-t)dt$,

Replace t by $x - t$, we get

$$f(x) = 1+\frac{x^3}{3}+\int_0^x e^{-x+t}f(t)dt$$

$$f(x) = 1+\frac{x^3}{3}+e^{-x}\int_0^x e^{t}f(t)dt \quad ...(1)$$

Now differentiate both sides w.r.t. x, we get

$$f'(x) = x^2 + e^{-x}e^{x}f(x) - e^{-x}\int_0^x e^{t}f(t)dt \quad ...(2)$$

Now (1) + (2), we get

$$f'(x)+f(x) = 1+x^2+\frac{x^3}{3}+f(x)$$

$$\Rightarrow f'(x) = 1+x^2+\frac{x^3}{3}$$

$$\Rightarrow f(x) = x+\frac{x^3}{3}+\frac{x^4}{12}+C$$

But from (1) we have $f(0) = 1$

$\Rightarrow$ $C = 1$

$$\therefore f(x) = x+\frac{x^3}{3}+\frac{x^4}{12}+1$$

Now,

$$\int_0^1 f(x)dx = x+\frac{x^2}{2}+\frac{x^4}{12}+\frac{x^5}{60}\Big|_0^1 = 1+\frac{1}{2}+\frac{1}{12}+\frac{1}{60}$$

$$= \frac{60+30+5+1}{60} = \frac{96}{60}=\frac{8}{5}$$

$$\Rightarrow p = \frac{8}{5} \Rightarrow 5p = 8$$

52. Answer (0)

$\because$ $a_1, a_2 ... a_n$ are in AP

$\Rightarrow$ $a_2 - a_1 = a_3 - a_2 = ... = a_n - a_{n-1} =$ 'd'

$$\therefore S = d\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+...+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$$

$$= -\left(\frac{a_1-a_2}{\sqrt{a_1}+\sqrt{a_2}}+\frac{a_2-a_3}{\sqrt{a_2}+\sqrt{a_3}}+...+\frac{a_{n-1}-a_n}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$$

$$= -\left(\sqrt{a_1}-\sqrt{a_2}+\sqrt{a_2}-\sqrt{a_3}+...\sqrt{a_{n-1}}+\sqrt{a_n}\right)$$

$$= -\left(\sqrt{a_1}-\sqrt{a_n}\right) = -\left(\sqrt{100}-\sqrt{900}\right)$$

$= -(10 - 30)$

$S = 20$

$\boxed{S - 20 = 0}$

53. Answer (4)

Let $\vec{a} = \lambda x^2\hat{i}+\hat{j}+5\hat{k}$ and $\vec{b} = \hat{i}+\lambda x\hat{j}+5\hat{k}$

According to given condition $\vec{a}\cdot\vec{b} > 0$

$\Rightarrow$ $\lambda x^2 + \lambda x + 25 > 0$, $\forall x \in R$

i.e., $\lambda^2 - 100\lambda < 0$ & $\lambda > 0$

$\Rightarrow$ $\lambda(\lambda - 10) < 0$ & $\lambda > 0$

$\Rightarrow$ $\lambda \in (0, 100)$

λ = 1, 8, 27 & 64 are perfect cube

54. Answer (6)

Given that $x^2 + y^2 - 6x - 8y = 0$

$(x - 2)^2 + (y - 4)^2 = 5^2$

i.e., $x = 3 + 5\cos\theta$, $y = 4 + 5\sin\theta$

$\therefore$ $a = \max\{x + y\} = \max\{7 + 5\cos\theta + 5\sin\theta\}$

$= 7+5\sqrt{2}$

$b = \min\{x - y\} = \min\{-1 + 5\cos\theta - 5\sin\theta\}$

$= -1-5\sqrt{2}$

$\therefore$ $a + b = 6$

❑ ❑ ❑

Time : 3 Hrs.

MOCK TEST

MM : 186

Paper - 2 (Code-C)

ANSWERS

PHYSICS

1. (D)
2. (A)
3. (B)
4. (B)
5. (D)
6. (A)
7. (A, B)
8. (A, B)
9. (A, C)
10. (A, B)
11. (A, B, C, D)
12. (B, D)
13. (B, D)
14. (A, B, C)
15. (C)
16. (B)
17. (C)
18. (A)

CHEMISTRY

19. (B)
20. (B)
21. (B)
22. (A)
23. (D)
24. (B)
25. (A, C, D)
26. (A, B, C)
27. (A, C, D)
28. (A, C, D)
29. (A, B, C, D)
30. (A, B, D)
31. (C)
32. (A, C)
33. (C)
34. (C)
35. (C)
36. (D)

MATHEMATICS

37. (C)
38. (D)
39. (A)
40. (A)
41. (B)
42. (A)
43. (B, C)
44. (B, C, D)
45. (C, D)
46. (D)
47. (C, D)
48. (A, B, C)
49. (B, D)
50. (B, C, D)
51. (D)
52. (C)
53. (D)
54. (B)

ANSWERS & HINTS

PART - I (PHYSICS)

1. Answer (D)

Heat Current $i = -kA\frac{dT}{dx}$

$idx = -kAdT$

$\Rightarrow \quad i\int_0^l dx = -A\alpha\int_{T_1}^{T_2} TdT$

$\Rightarrow \quad il = -\frac{A\alpha}{2}(T_2^2 - T_1^2) = \frac{A\alpha}{2}(T_1^2 - T_2^2)$

2. Answer (A)

$B = \alpha(1+\sin\omega t)\sin\omega_0 t$

$B = \alpha\{\sin\omega_0 t + \sin\omega t \sin\omega_0 t\}$

$= \alpha\left\{\sin\omega_0 t + \frac{1}{2}\cos(\omega-\omega_0)t - \frac{1}{2}\cos(\omega+\omega_0)t\right\}$

This light has 3 frequencies and maximum frequency is $\omega_0 + \omega$

Thus maximum KE will be

$\hbar(\omega + \omega_0) - \phi_{Li}$

$= 4.14 \times 10^{-15} \times 10 \times 10^{14}$ eV – 2.39 eV

= 4.14 eV – 2.39 eV = 1.75 eV

3. Answer (B)

MSD = 0.05 cm

VSD = $\frac{49}{1000} = 0.049$ cm

LC = 0.05 – 0.049 = 0.001 cm

Diameter = $5.10 + 24 \times 0.001 = 5.124$ cm

4. Answer (B)

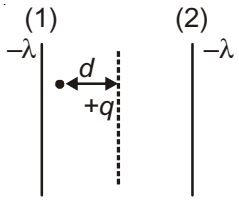
$R_{Al} = \frac{\rho l}{A} = 3\times10^{-5}\,\Omega = 30\ \mu\Omega$

$R_{Fe} = \frac{\rho' l'}{A'} = \frac{25}{2}\times10^{-4}\ \Omega = 1250\ \mu\Omega$

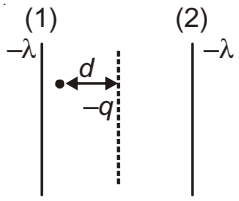
Given connection is in parallel

$R = \frac{R_{Al}R_{Fe}}{R_{Al}+R_{Fe}} = \frac{1875}{64}\ \mu\Omega$

5. Answer (D)



As q is displaced towards wire (1), attractive force due to (1) increases and due to (2) decreases. Thus F_{net} is leftward. ∴ no SHM



As $-q$ is displaced towards wire (1), repulsive force due to (1) increases and due to (2) decreases. Thus F_{net} is rightward. ∴ SHM

6. Answer (A)

x = 0, path difference 2λ

Hence 2nd order maxima obtained

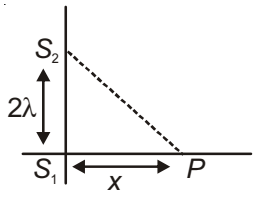
First order

$S_2P - S_1P = \lambda$

$\sqrt{x^2+4\lambda^2} - x = \lambda$

So $x^2 + 4h^2 = \lambda^2 + x^2 + 2\lambda x$

$x = \frac{3}{2}\lambda$



7. Answer (A, B)

$U = -\frac{ke^2}{4r^4}$

Force $F = -\frac{dU}{dr} = +\frac{ke^2}{4r^5}(-4) = \frac{-ke^2}{r^5}$

According to Bohr theory

Centripetal force $\frac{ke^2}{r^5} = \frac{mv^2}{r}$ ...(i)

Also $mvr = \frac{nh}{2\pi}$

$\Rightarrow \quad v = \frac{nh}{2\pi rm}$

Substituting v in (i) we get

$$\frac{ke^2}{r^5} = \frac{m}{r}\frac{n^2h^2}{4\pi^2r^2m^2}$$

$$r^2 = \frac{ke^2 4\pi^2 m}{n^2h^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2 4\pi^2 m}{n^2h^2}$$

$$r = \left(\frac{\pi e^2 m}{\varepsilon_0 n^2h^2}\right)^{1/2} = \frac{e}{nh}\sqrt{\frac{\pi m}{\varepsilon_0}}$$

Now $v = \frac{nh}{2\pi rm}$

$$= \frac{nh}{2\pi m\frac{e}{nh}\sqrt{\frac{\pi m}{\varepsilon_0}}}$$

$$= \frac{n^2h^2}{2\pi me}\sqrt{\frac{\varepsilon_0}{\pi m}}$$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{n^4h^4}{4\pi^2m^2e^2}\frac{\varepsilon_0}{\pi m}$$

$$= \frac{n^4h^4\varepsilon_0}{8\pi^3m^2e^2}$$

$$\text{PE} = -\frac{ke^2}{4r^4} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{4\frac{e^4}{n^4h^4}\frac{\pi^2m^2}{\varepsilon_0^2}}$$

$$= -\frac{n^4h^4\varepsilon_0}{16\pi^3m^2e^2}$$

TE = KE + PE

$$= \frac{n^4h^4\varepsilon_0}{\pi^3m^2e^2}\left\{\frac{1}{8}-\frac{1}{16}\right\}$$

$$= n^4\pi\left(\frac{h^4}{16\pi^4}\right)\frac{\varepsilon_0}{m^2e^2}$$

$$= n^4\hbar^4\frac{\pi\varepsilon_0}{m^2e^2}$$

8. Answer (A, B)

For segment *CD*, *EF* ; $\vec{B} \perp \vec{A}$

$$|e| = \left|\frac{d\phi}{dt}\right| = \frac{\pi}{4}(2r)^2\frac{dB}{dt}$$

$$= \frac{\pi}{4}\times10^2\times10^{-4}\times5\times10^{-3}\ \text{V}$$

$$= 3.925 \times 10^{-5} \quad \approx 4\times10^{-5}\ \text{V}$$

Total resistance of wire $= \lambda \times l$

$$= 2\times\left\{\frac{\pi}{2}(3r+2r+r)+4r\right\}$$

$$= (\pi \times 6r + 8r)\ \Omega = 134.2\ \Omega$$

Current $I = \frac{4\times10^{-5}}{134.2} = 0.0298\times10^{-5} \approx 0.3\ \mu\text{A}$

Direction is from *B* to *A* due to Lenz law

9. Answer (A, C)

$dN = -N\lambda dt + R\,dt$

$$\int_{N_0}^{N} dN = -\int_0^t\left(N-\frac{R}{\lambda}\right)\lambda\,dt$$

$$\frac{\left(N-\frac{R}{\lambda}\right)}{N_0-\frac{R}{\lambda}} = e^{-\lambda t}$$

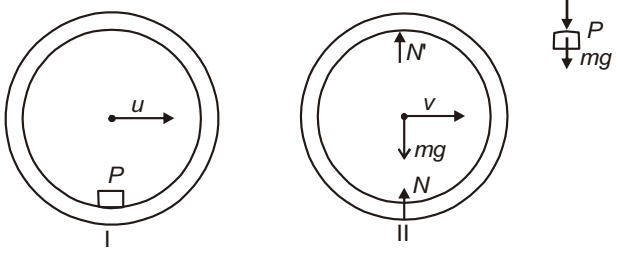
$$N-\frac{R}{\lambda} = \left(N_0-\frac{R}{\lambda}\right)e^{-\lambda t}$$

$t \to \infty$

$N = \frac{R}{\lambda}$ irrespective of value of N_0

10. Answer (A, B)

The tendency of bouncing of cylinder will be maximum when *P* is at the highest point of cylinder during rolling motion.



Law of conservation of energy

$$\frac{1}{2}mu^2 + \frac{1}{2}(mR^2)\left(\frac{u}{R}\right)^2 - mgR$$

$$= \frac{1}{2}m(2v)^2 + \frac{1}{2}mv^2 + \frac{1}{2}(mR^2)\left(\frac{v}{R}\right)^2 + mgR$$

$mu^2 = 3mv^2 + 2mgR$

$\Rightarrow \quad 3v^2 = u^2 - 2gR$

Now at final position

$$N' + mg = \frac{mv^2}{R} \text{ (for body } P\text{)}$$

Also $N + N' = mg$ (for cylinder)

$$\Rightarrow \quad 2mg - \frac{mv^2}{R} = N$$

If cylinder does not bounce

$$N \geq 0 \Rightarrow 2g \geq \frac{v^2}{R}$$

$$v \leq \sqrt{2gR} \qquad \frac{u^2 - 2gR}{3} \leq 2gR$$

or $\quad u \leq \sqrt{8gR}$

11. Answer (A, B, C, D)

In equilibrium position net force on partition will be zero. Hence pressure on both side are same

$PV = nRT$

$$n_1 = \frac{PV}{RT}$$

$$n_2 = \frac{(2P)(2V)}{RT} = \frac{4PV}{RT} \Rightarrow n_2 = 4n_1$$

moles remain conserved.

at equilibrium, $P_1V_1 = n_1RT_1$ & $P_2V_2 = n_2RT_2$

$P_1 = P_2$ and $T_1 = T_2$

$$\frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{1}{4} \qquad V_2 = 4V_1 \text{ and } V_1 + V_2 = 3V$$

$$\Rightarrow V_1 = \frac{3}{5}V$$

$$V_2 = \frac{12}{5}V$$

Hence B, C are correct.

12. Answer (B, D)

$$\frac{P^2}{\rho} = \text{constant} \qquad \frac{P^2RT}{PM} = \text{constant}$$

PT = constant

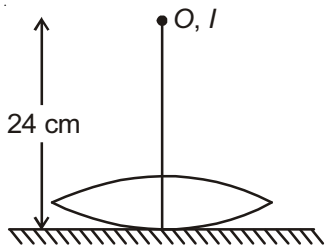
$$\frac{P^2}{\rho} = \frac{P'^2}{\rho/2} \Rightarrow P' = \frac{P}{\sqrt{2}}$$

also $T' = T\sqrt{2}$

PT = constant.

$\therefore$ P-T curve is a rectangular hyperbola.

13. Answer (B, D)



Let F_1 be equivalent focal length of the lens mirror combination

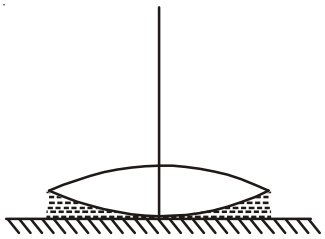
$\therefore \quad 2F_1 = 24$ cm

$F_1 = 12$ cm

$$\Rightarrow \quad \frac{1}{F_1} = \frac{2}{f_l} + \frac{1}{\infty}$$

$f_l = 24$ cm = focal length of lens.

When filled with water combination of glass lens, water lens and a plane mirror



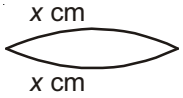
F_2 = equivalent focal length of new combination

$$\frac{1}{F_2} = \frac{-2}{f_l} + \frac{-2}{f_w} + \frac{1}{\infty}$$

$$\frac{1}{F_2} = \frac{-2}{f_l} + \frac{-2}{f_w} \qquad \text{...(i)}$$

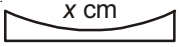
Applying lens maker formula on glass lens and water lens

$$\frac{1}{f_l} = (15 - 1)\left(\frac{1}{x} - \frac{1}{-x}\right)$$



$x = 24$ cm

$$\frac{1}{f_w} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{-x} - \frac{1}{\infty}\right)$$



$f_w = -72$ cm

Substituting in (i) we get

$$\frac{1}{F_2} = \frac{-2}{24} + \frac{-2}{-72}$$

$F_2 = -18$ cm

$2F_2 = -36$ cm

Shift = (36 – 24) = 12 cm upward.

14. Answer (A, B, C)

$KE_f = 4\ KE_i$

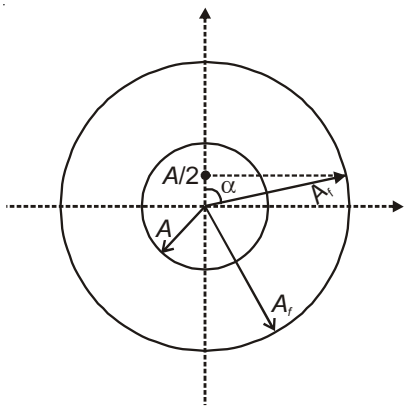
$\Rightarrow \quad v_f = 2\ v_i$

$$v_i = \omega\sqrt{A^2 - \left(\frac{A}{2}\right)^2}$$

$$v_i = \omega\sqrt{A^2 - \left(\frac{A}{2}\right)^2} \Rightarrow v_i = \frac{\omega\sqrt{3}\ A}{2}$$

By applying conservation of energy after impulse

$$\frac{1}{2}m\,v_f^2 + \frac{1}{2}k\left[\frac{A}{2}\right]^2 = \frac{1}{2}kA_f^2 \quad \Rightarrow A_f = \frac{\sqrt{13}\ A}{2}$$



Minimum time taken by particle to cross the same position again

$$\cos\alpha = \frac{\frac{A}{2}}{A_f} = \frac{1}{\sqrt{13}}$$

$$\alpha = \cos^{-1}\frac{1}{\sqrt{13}}$$

$$2\alpha = \omega t_{min} \Rightarrow t_{min} = \frac{2\cos^{-1}\frac{1}{\sqrt{13}}}{\omega}$$

Solution for Q. No. 15 and 16

15. Answer (C)

16. Answer (B)

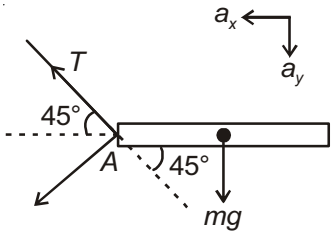
Point *A* will undergo circular motion about *X*

Torque about COM = $(T \sin 45°)\ \frac{l}{2} = \frac{ml^2}{12}\alpha$

COM acceleration

$$a_x = \frac{T\cos 45°}{m}$$

$$a_y = \frac{mg - T\sin 45°}{m}$$



Immediately after cutting of string, the speed of point *A* is zero, hence its centripetal acceleration towards point *X* must be zero.

$$a_x\cos 45° - a_y\cos 45° + \alpha\frac{l}{2}\cos 45° = 0$$

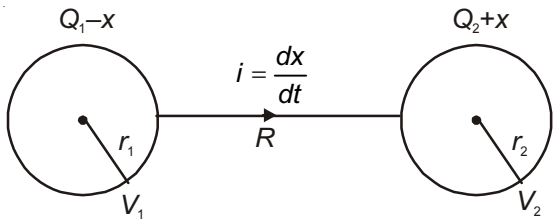
$$2(a_x - a_y) + \alpha l = 0$$

$$2\left\{\frac{T}{\sqrt{2}m} - g + \frac{T}{\sqrt{2}m}\right\} + \frac{6T}{\sqrt{2}m} = 0$$

$$\left(\frac{2T}{m} - g + \frac{3T}{m}\right) = 0$$

$$T = \frac{mg}{5}$$

17. Answer (C)



Applying KVL

$$V_1 - \left(\frac{dx}{dt}\right)R = V_2$$

$$\frac{k(Q_1 - x)}{r_1} - \frac{dx}{dt}R = k\frac{(Q_2 + x)}{r_2} \quad \text{where } k = \frac{1}{4\pi\in_0}$$

$$R\frac{dx}{dt} = \frac{kQ_1}{r_1} - \frac{kQ_2}{r_2} - k\left(\frac{1}{r_1} + \frac{1}{r_2}\right)x$$

$$R\int_0^x \frac{dx}{k\left(\frac{Q_1}{r_1} - \frac{Q_2}{r_2}\right) - k\left(\frac{1}{r_1} + \frac{1}{r_2}\right)x} = \int_0^t dt$$

$$\frac{R}{-k\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}\ln\frac{\left[k\left(\frac{Q_1}{r_1}-\frac{Q_2}{r_2}\right)-k\left(\frac{1}{r_1}+\frac{1}{r_2}\right)x\right]}{k\left(\frac{Q_1}{r_1}-\frac{Q_2}{r_2}\right)}=t$$

$$1-\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)x}{\frac{Q_1}{r_1}-\frac{Q_2}{r_2}}=e^{\frac{-tk\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{R}}$$

or $$x=\frac{\frac{Q_1}{r_1}-\frac{Q_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}}\left\{1-e^{\frac{-tk\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{R}}\right\}$$

18. Answer (A)

$$H=\frac{C_1C_2}{2(C_1+C_2)}|V_1-V_2|^2 \qquad C_1=4\pi\varepsilon_0 r_1$$

$$C_2=4\pi\varepsilon_0 r_2$$

$$V_1=\frac{Q_1}{4\pi\varepsilon_0 r_1}$$

$$V_2=\frac{Q_2}{4\pi\varepsilon_0 r_2}$$

$$\therefore \quad H=\frac{r_1r_2}{8\pi\varepsilon_0(r_1+r_2)}\left(\frac{Q_1}{r_1}-\frac{Q_2}{r_2}\right)^2$$

PART - II (CHEMISTRY)

19. Answer (B)

CCP refers to FCC crystal structure.

X-X bond length = 160 pm

For FCC $a\sqrt{2}=4r$

a(edge length) = $\frac{4r}{\sqrt{2}}=160\sqrt{2}=226$

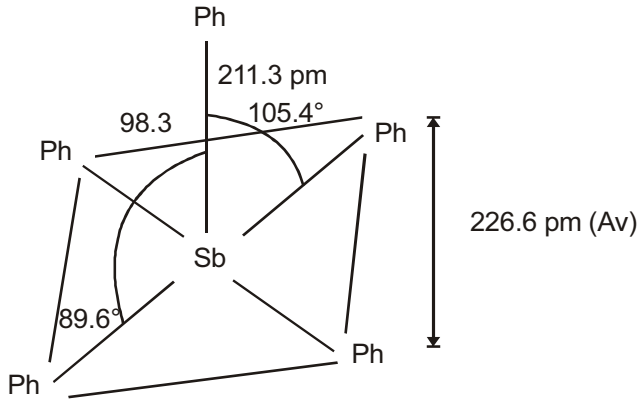
Volume of unit cell = $a^3=(226\times10^{-10})^3$

Number of atoms in FCC = 4

Number of atoms in 1 $cm^3=\frac{4}{(226\times10^{-10})^3}$

$=3.45\times10^{23}$

20. Answer (B)



→ $Sb(Ph)_5$ - Slightly distorted square pyramidal

→ $P(Ph)_5$ Trigonal bipyramidal

21. Answer (B)

$K=(k_1)^2.(k_2)^3$

$=(0.05)^2.(2\times10^{-12})^3$

$=2\times10^{-38}$

22. Answer (A)

Rate of destruction of O_3 must equal to rate of formation

$2O_3\longrightarrow 3O_2$

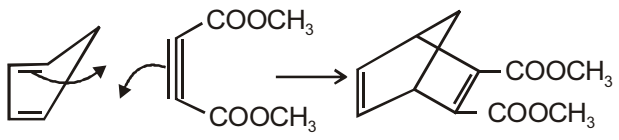
Rate of destruction of $O_3=\frac{-\Delta[O_3]}{\Delta t}=K[O_3]^2$

$$K=\frac{-\Delta[O_3]}{\Delta t}\times\frac{1}{[O_3]^2}$$

$$=\frac{8.2\times10^{-15}\ mol\ L^{-1}h^{-1}}{3600\ s\text{-}h^{-1}}\times\frac{1}{(4.0\times10^{-8}\ mol\ L^{-1})^2}$$

$=1.42\times10^{-3}$

23. Answer (D)



Cyclopenta-1,3 diene — Dimethyl acetylene dicarboxylate

24. Answer (B)

$$X \rightleftharpoons 2Y \qquad Z \rightleftharpoons P + Q$$

1	0	1	0	0
$1-\alpha$	2α	$1-\alpha$	α	α

$$K_{P1} = \frac{P_y^2}{P_x} = \frac{\left(\frac{2\alpha}{1+\alpha}.P_1\right)^2}{\left(\frac{1-\alpha}{1+\alpha}.P_1\right)} = \frac{4\alpha^2 P_1}{1-\alpha^2}$$

Also

$$KP_2 = \frac{P_P.P_Q}{P_z} = \frac{\left(\frac{\alpha}{1+\alpha}.P_2\right)\left(\frac{\alpha}{1+\alpha}.P_2\right)}{\left(\frac{1-\alpha}{1+\alpha}.P_2\right)} = \frac{\alpha^2 P_2}{1-\alpha^2}$$

$$\frac{K_{P1}}{K_{P2}} = \frac{1}{9} \Rightarrow \frac{4\alpha^2 P_1}{\alpha^2 P_2} = \frac{1}{9}$$

$$\therefore \frac{P_1}{P_2} = \frac{1}{36}$$

$$\boxed{P_1 : P_2 = 1 : 36}$$

25. Answer (A, C, D)

26. Answer (A, B, C)

$Cu^{2+} > Cu^{+}$ stable due to higher hydration energy.

Cu^{2+} paramagnetic –$4s^0 3d^9$ (unpaired electron) and coloured.

27. Answer (A, C, D)

(A) $2Cu^{2+} + K_4[Fe(CN)_6] \rightarrow \underset{\text{Chocolate Brown ppt}}{Cu_2[Fe(CN)_6]} + 4K^+$

(B) As^{3+} does not react with $K_4[Fe(CN)_6]$

(C) $2Zn^{2+} + K_4[Fe(CN)_6] \rightarrow \underset{\text{Bluish white ppt}}{Zn_2[Fe(CN)_6]} + 4K^+$

(D) $4Fe^{3+} + 3K_4[Fe(CN)_6] \rightarrow \underset{\text{Prussian blue}}{Fe_4[Fe(CN)_6]_3} + 12K^+$

28. Answer (A, C, D)

29. Answer (A, B, C, D)

(A) Is correct—lattice and hydration energy decreases down the group

(B) Is correct — BeF_2 having high hydration energy

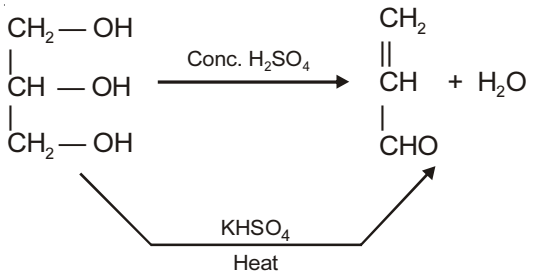
(C) Is correct — $BeCl_2$ is insoluble due to covalent in nature.

(D) Also is correct — $BeSO_4$ is soluble due to high hydration energy.

30. Answer (A, B, D)

31. Answer (C)

32. Answer (A, C)



(D) Not possible

33. Answer (C)

34. Answer (C)

35. Answer (C)

At point 3, both species will remain.

$$H_3N^+ - \underset{\underset{CH_3}{|}}{CH} - COO^- \longrightarrow H_2N - \underset{\underset{CH_3}{|}}{CH} - COO^- + H^+$$

$[H^+]$ At point 3 = $K_2 = 2 \times 10^{-10}$

$pH = pK_a$

$pH = 10 - \log 2$ $(\because \log 2 = 0.3010)$

$pH = 9.7$

36. Answer (D)

As pH increases, neutralisation proceeds very fast most acidic proton O–H then proton from H_3–N^+ — also goes into solution

PART - III (MATHEMATICS)

37. Answer (C)

$$\sigma(n) = \sum_{r=1}^{n}\left\{r^2\left({}^nC_r - {}^nC_{r-1}\right) + 2r\,{}^nC_r + {}^nC_r\right\}$$

$$= \sum_{r=1}^{n}\left\{(r^2 + 2r + 1){}^nC_r - r^2 \cdot {}^nC_{r-1}\right\}$$

$$= \sum_{r=1}^{n}\left\{(r+1)^2\,{}^nC_r - r^2 \cdot {}^nC_{r-1}\right\}$$

$$= \sum_{r=1}^{n}\left(V_{r+1} - V_r\right)$$

$= (V_2 - V_1) + (V_3 - V_2) + .. + (V_{n+1} - V_n)$

$= V_{n+1} - V_1$

$= (n+1)^2\,{}^nC_n - 1 = n^2 + 2n$

so, $\sigma(50) = 2600$

38. Answer (D)

(5, 15) lies inside the circle and (5, 15) should be the mid-point of the chord.

$$\therefore \quad \frac{-p}{q} = -1 \qquad \Rightarrow \quad p = q$$

Now, $5p + 15q = 40 \Rightarrow p + 3q = 8 \Rightarrow p = q = 2$

39. Answer (A)

Equation of plane through first two planes is

$2x + y + z - 1 + \lambda(3x + y + 2z - 2) = 0$

If it is parallel to second line, then

$(2 + 3\lambda)1 + (1 + \lambda)1 + (1 + 2\lambda) \cdot 1 = 0$

$$\Rightarrow \quad \lambda = \frac{-2}{3}$$

$\therefore$ Equation of the plane is $y - z + 1 = 0$

Distance from (0, 0, 0) = $\frac{1}{\sqrt{2}}$.

40. Answer (A)

$$I = \int \frac{x^6 - x^4 + x^2 - 1}{x^{10} + 1}\,x\,dx$$

Let $x^2 = t$

$\therefore \quad 2x\,dx = dt$

$$I = \frac{1}{2}\int \frac{t^3 - t^2 + t - 1}{t^5 + 1}\,dt$$

$$= \frac{1}{2}\int \frac{(t^2+1)(t-1)}{t^5+1}\,dt$$

$$= \frac{1}{2}\int \frac{t^4 - 1}{(t^5+1)(t+1)}\,dt$$

$$= \frac{1}{2}\int \left(\frac{t^4}{t^5+1} - \frac{1}{t+1}\right)dt$$

$$= \frac{1}{2}\left[\frac{1}{5}\ln(t^5+1) - \ln(t+1)\right] + C$$

$$= \frac{1}{2}\left[\frac{1}{5}\ln(t^5+1) - \frac{1}{5}\ln(t+1)^5\right] + C$$

$$= \frac{1}{10}\ln\left(\frac{x^{10}+1}{(x^2+1)^{10}}\right) + C$$

41. Answer (B)

$f(x) = [x]\,(x^2 - 25)^n\,(x^2 + 3)^m\,(x^2 + 3x + 4)^{3m}$

$f(5) = 0$, $f(5\_) = 4$ (negative)^n (+) (+)

For a local minima at $x = 5$, $f(5^-) > 0$

For this n must be even, $m \in R$

42. Answer (A)

$f(x) = x\{x\} - x\{-x\}$

$$= \begin{cases} x^2, & x \in I \\ x^2 + x, & x \notin I \end{cases}$$

The integers not belonging to the range of $f(x)$ are 1.2, 2.3, 3.4 *i.e.,* 2, 6, 12.

Further $x^2 + x \geq -\frac{1}{4}$ so $\{-15, -14, ..., -1\} \notin R_f$.

So required number of elements = 3 + 15 = 18.

43. Answer (B, C)

$f(-5) = 2$, $f(20) = f(4) = 3$, $f(17) = -2$

$\therefore \quad k = 2 + 3 - 2 = 3$

Now $g(x) = x^3 + |\sin x|$

$\lim\limits_{x\to-\infty} g(x) = -\infty$ and $\lim\limits_{x\to+\infty} g(x) = \infty$

$g(0) = 0$, $g(x)$ is neither increasing nor decreasing on R

$\therefore \quad g(x)$ is many-one and onto.

44. Answer (B, C, D)

$$\sqrt{3}\sin C = \frac{2}{\cos A} - \frac{\sin A}{\cos A}$$

$$\Rightarrow \quad \sqrt{3}\sin C\cos A = 2 - \sin A$$

$$\Rightarrow \quad \sqrt{3}\sin C \cdot \cos A > 0, \text{ as } 2 - \sin A > 0$$

$\Rightarrow$ A is an acute angle.

Now, $\sin C \leq 1$

$$\therefore \quad \sqrt{3}\sin C \cdot \cos A \leq \sqrt{3}\cos A$$

$$\Rightarrow \quad 2 - \sin A \leq \sqrt{3}\cos A$$

$$\Rightarrow \quad \sqrt{3}\cos A + \sin A \geq 2$$

But, $\sqrt{3}\cos A + \sin A \leq 2$

$$\Rightarrow \quad \sqrt{3}\cos A + \sin A = 2$$

$$\Rightarrow \quad 2\sin\left(\frac{\pi}{3} + A\right) = 2$$

$$\Rightarrow \quad \sin\left(\frac{\pi}{3} + A\right) = \sin\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\pi}{3} + A = \frac{\pi}{2}$$

$$\Rightarrow \quad \boxed{A = \frac{\pi}{6}}$$

$$\therefore \quad \sqrt{3}\sin C\cos\frac{\pi}{6} = 2 - \sin\frac{\pi}{6}$$

$$\Rightarrow \quad \frac{3}{2}\sin C = \frac{3}{2}$$

$$\Rightarrow \quad \sin C = 1 \Rightarrow \boxed{C = \frac{\pi}{2}}$$

$$\therefore \quad \boxed{B = \frac{\pi}{3}}$$

Consequently $c = 4$

Now, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$\Rightarrow \quad \frac{a}{\frac{1}{2}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{c}{1} = \frac{a+b+c}{\frac{1}{2} + \frac{\sqrt{3}}{2} + 1}$$

$$\Rightarrow \quad 4 = \frac{2S}{\left(\frac{1+\sqrt{3}+2}{2}\right)} \quad \Rightarrow \quad 2S = 4\left(\frac{3+\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad S = 3 + \sqrt{3}$$

Also, $a = 4 \cdot \frac{1}{2} = 2$, $b = 4\frac{\sqrt{3}}{2} = 2\sqrt{3}$

$$\therefore \quad \Delta = \frac{1}{2}ab = \frac{1}{2} \cdot 2 \cdot 2\sqrt{3} = 2\sqrt{3}$$

$$r = \frac{\Delta}{S} = \frac{2\sqrt{3}}{3+\sqrt{3}} = \frac{2}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{2(\sqrt{3}-1)}{2}$$

$$R = \frac{c}{2\sin C} = \frac{4}{2} = 2$$

45. Answer (C, D)

$$f(x)f'(x) \geq x(f(x))^2\sqrt{(f(x))^4 - 1}$$

$$\Rightarrow \quad \frac{2f(x)f'(x)}{(f(x))^2\sqrt{(f(x))^4 - 1}} \geq 2x$$

$\Rightarrow \quad \frac{d}{dx}\left(\sec^{-1}(f(x))^2 - x^2\right) \geq 0$

$\therefore \quad g(x) = \sec^{-1}(f(x))^2 - x^2$ is a non-decreasing function.

$\Rightarrow \quad \lim\limits_{x \to m+} g(x) \leq \lim\limits_{x \to n-} g(x)$

$\Rightarrow \quad 0 - m^2 \leq \frac{\pi}{3} - n^2$

$\Rightarrow \quad n^2 - m^2 \leq \frac{\pi}{3}$

46. Answer (D)

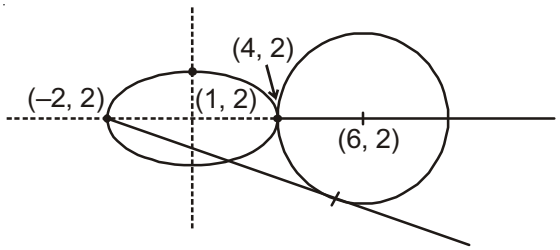
It is given that $a_i + i = S - a_i$

Putting $i = 1, 2, 3, ..100$ and adding

$2S + \frac{100 \times 101}{2} = 100S$

$\Rightarrow \quad S = \frac{25 \times 101}{49} \quad \Rightarrow \quad 49S = 25 \times 101$

47. Answer (C, D)



m and n are the powers of the points lying on the ellipse

$\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$ w.r.t. the circle $x^2 + y^2 - 12x - 4y + 36 = 0$

Clearly, $m = 60$, $n = 0$

48. Answer (A, B, C)

If $p(k)$ denote the probability of getting a score k, then

$p(1) = P(T) = \frac{1}{2}$, $p(2) = P(TT \text{ or } H) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

A score of k can be obtained either by

(i) Throwing a tail, when the score is $k - 1$ or

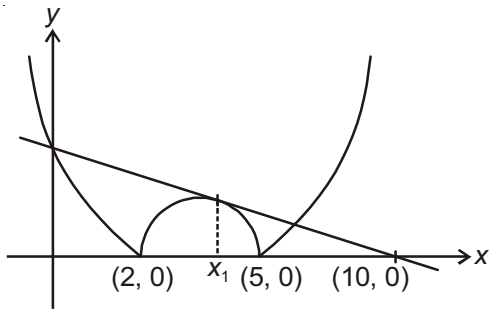
(ii) By throwing a head when the score is $k - 2$

Now, $P(k) = \frac{1}{2}P(k-1) + \frac{1}{2}P(k-2)$

$= \frac{1}{2}[P(k-1) + P(k-2)]$

So, $p = P(7) = \frac{85}{128} \sim 0.66406$

49. Answer (B, D)



For 3 distinct real solutions, $y = m(x - 10)$ will be a tangent to $y = -x^2 + 7x - 10$ and $2 < x_1 < 5$

$\Rightarrow$ Discriminant of $\{x^2 - (m-7)x + (10 - 10m) = 0\} = 0$

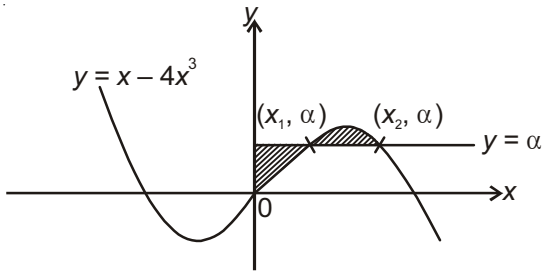
$\Rightarrow \quad (m-7)^2 - 4(10 - 10m) = 0$

$\Rightarrow \quad m^2 + 26m + 9 = 0$

$\Rightarrow \quad m = -13 \pm 4\sqrt{10}$

and $2 < \frac{7-m}{2} < 5 \quad \Rightarrow \quad m \in (-3, 3)$

50. Answer (B, C, D)



Given that $\int_0^{x_2} (x - 4x^3)dx = \int_0^{x_2} \alpha \cdot dx$

$\Rightarrow \quad \left(\frac{x^2}{2} - x^4\right)_0^{x_2} = \alpha x_2$

$\Rightarrow \quad \frac{x_2^2}{2} - x_2^4 = \alpha x_2$

$\Rightarrow \quad \frac{x_2}{2} - x_2^3 = \alpha$, also $\boxed{\alpha - x_2 - 4x_2^3}$

$\Rightarrow \quad \frac{x_2}{2} - x_2^3 = x_2 - 4x_2^3$

$\Rightarrow \quad 3x_2^3 = \frac{x_2}{2}$

$\Rightarrow \quad 6x_2^3 - x_2 = 0$

$\Rightarrow \quad x_2\left(6x_2^2 - 1\right) = 0 \quad \Rightarrow \quad \boxed{x_2 = \frac{1}{\sqrt{6}}}$

$\therefore \quad \alpha = \frac{1}{\sqrt{6}} - \frac{4}{6\sqrt{6}}$

$= \frac{1}{\sqrt{6}}\left(1 - \frac{2}{3}\right)$

$\boxed{\alpha = \frac{1}{3\sqrt{6}}}$

Solutions of Questions 51 & 52

51. Answer (D)

As E and H are orthogonal.

$\therefore$ They are confocal.

$\therefore$ Coordinates of the foci are $(\pm ae, 0) = (\pm 1, 0)$

Eccentricity e of $E = \frac{1}{\sqrt{2}}$; $p = \sqrt{2}, q = 1$

$\therefore$ Equation of ellipse E is $\frac{x^2}{2} + \frac{y^2}{1} = 1$

52. Answer (C)

The point must lie on the director circle of E *i.e.,* $x^2 + y^2 = 3$

Solutions of Questions 53 & 54

As $z_1^3 + z_2^3 + z_3^3 + z_1 z_2 z_3 = 0$

$\Rightarrow (z_1 + z_2 + z_3)\{(z_1 + z_2 + z_3)^2 - 3z_1z_2 - 3z_2z_3 - 3z_3z_1\} = -4z_1z_2z_3$

$\Rightarrow (z_1 + z_2 + z_3)^3 - 3((z_1 + z_2 + z_3)(z_1z_2 + z_2z_3 + z_3z_1) + 4z_1z_2z_3 = 0$

$\Rightarrow (z_1 + z_2 + z_3)^3 - z_1z_2z_3$

$\left\{3(z_1 + z_2 + z_3)\left(\frac{1}{z_3} + \frac{1}{z_2} + \frac{1}{z_1}\right) - 4\right\} = 0$

$\Rightarrow (z_1 + z_2 + z_3)^3 = z_1z_2z_3$

$\left\{3(z_1 + z_2 + z_3)\left(\bar{z}_3 + \bar{z}_2 + \bar{z}_1\right) - 4\right\}$

$\Rightarrow \left|z_1 + z_2 + z_3\right|^3 = 1\left|3(z_1 + z_2 + z_3)(\overline{z_1 + z_2 + z_3}) - 4\right|$

Let $\left|z_1 + z_2 + z_3\right| = m$, then

$m^3 = |3m^2 - 4| \quad \Rightarrow \quad m = 1, 2$

53. Answer (D)

Sum of all the integral values of $|z_1 + z_2 + z_3| = 1+2=3$

54. Answer (B)

Given that, $|z_1 + z_2 + z_3| = 2$

Now, $|z_1 + z_2 + z_3|^2 = 4$

$\Rightarrow (z_1 + z_2 + z_3)(\bar{z}_1 + \bar{z}_2 + \bar{z}_3) = 4$

$\Rightarrow 3 + z_1\bar{z}_2 + z_1\bar{z}_3 + z_2\bar{z}_1 + z_2\bar{z}_3 + z_3\bar{z}_1 + z_3\bar{z}_2 = 4$

$\Rightarrow (z_1\bar{z}_2 + z_2\bar{z}_1) + (z_1\bar{z}_3 + z_3\bar{z}_1) + (z_2\bar{z}_3 + z_3\bar{z}_2) = 1$

$\Rightarrow 2\left\{\text{Re}(z_1\bar{z}_2) + \text{Re}(z_3\bar{z}_1) + \text{Re}(z_2\bar{z}_3)\right\} = 1$

$\Rightarrow \text{Re}(z_1\bar{z}_2 + z_1\bar{z}_3 + z_2\bar{z}_3) = \frac{1}{2}$

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