DATE : 22/05/2016

CODE

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Aakash

Medical | IIT-JEE | Foundations

(Divisions of Aakash Educational Services Pvt. Ltd.)

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Time : 3 hrs.

Max. Marks: 186

Answers & Solutions

for

JEE (Advanced)-2016

PAPER - 1 (Code - 0)

INSTRUCTIONS

QUESTION PAPER FORMAT AND MARKING SCHEME :

1. The question paper has **three parts** (Physics, Chemistry and Mathematics).

2. Each part has three sections as detailed in the following table:

Section	Question Type	Number of Questions	Category-wise Marks for Each Questions				Maximum Marks of the section
			Full Marks	Partial Marks	Zero Marks	Negative Marks	
1	Single Correct Option	5	+3 If only the bubble corresponding to the correct option is darkened	—	0 If none of the bubbles is darkened	–1 In all other cases	15
2	One or more correct option(s)	8	+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened	+1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened	0 If none of the bubbles is darkened	–2 In all other cases	32
3	Single digit Integer (0-9)	5	+3 If only the bubble corresponding to the correct answer is darkened	—	0 In all other cases	—	15

PART–I : PHYSICS

SECTION - 1 (Maximum Marks : 15)

This section contains **FIVE questions.**

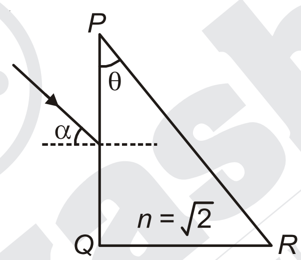
Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–1	In all other cases.

1. A parallel beam of light is incident from air at an angle α on the side PQ of a right-angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is

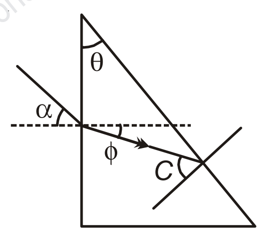


(A) 15° (B) 22.5°

(C) 30° (D) 45°

Answer (A)

Sol. : $\sin C = \frac{1}{n} = \frac{1}{\sqrt{2}}$, $C = 45°$



Also, $1 \times \sin 45 = \frac{1}{\sqrt{2}} \times \sin\phi$

$\Rightarrow \phi = 30°$

$\Rightarrow \theta = 15°$

2. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below :

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ ms^{-1} and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of J s) found from such an experiment is

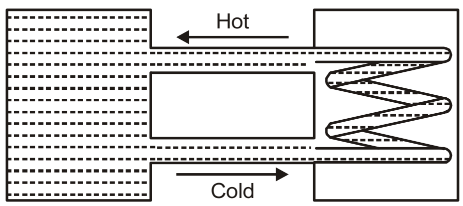
(A) 6.0×10^{-34} (B) 6.4×10^{-34}

(C) 6.6×10^{-34} (D) 6.8×10^{-34}

Answer (B)

Sol. : $\frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = e(V_1 - V_2)$

3. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600
(B) 2067
(C) 2533
(D) 3933

Answer (B)

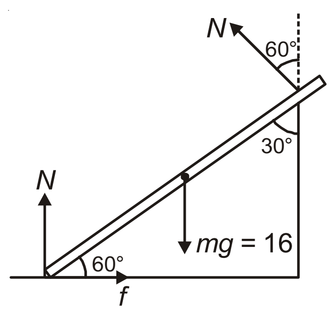
Sol. : $(3000 - P)t = mc\Delta T$

4. A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth, vertical wall of height $h(< l)$ such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/l and the frictional force f at the bottom of the stick are (g = 10 m s^{-2})

(A) $\frac{h}{l} = \frac{\sqrt{3}}{16}, \ f = \frac{16\sqrt{3}}{3}$ N
(B) $\frac{h}{l} = \frac{3}{16}, \ f = \frac{16\sqrt{3}}{3}$ N
(C) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, \ f = \frac{8\sqrt{3}}{3}$ N
(D) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, \ f = \frac{16\sqrt{3}}{3}$ N

Answer (D)

Sol. : For vertical equilibrium,



$$N + \frac{N}{2} = 16$$

$$\Rightarrow \frac{3N}{2} = 16$$

$$\Rightarrow N = \frac{32}{3} \text{ Newton}$$

For horizontal equilibrium,

$$\frac{N\sqrt{3}}{2} = f$$

$$\Rightarrow f = \frac{32}{3} \times \frac{\sqrt{3}}{2} = \frac{16\sqrt{3}}{3} \text{ Newton}$$

For rotational equilibrium about COM,

$$\left(\frac{f\sqrt{3}}{2}\right)\frac{l}{2} + N\left(\frac{2h}{\sqrt{3}} - \frac{l}{2}\right) = \frac{N}{2} \cdot \frac{l}{2}$$

$$\frac{3}{4}N\frac{l}{2}+N\left(\frac{2h}{\sqrt{3}}-\frac{l}{2}\right)=\frac{Nl}{4}$$

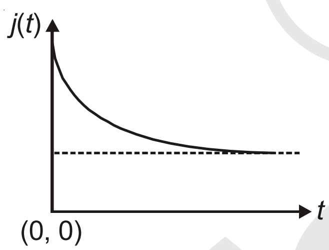
$$\Rightarrow \quad \frac{3l}{8}+\frac{2h}{\sqrt{3}}-\frac{l}{2}=\frac{l}{4}$$

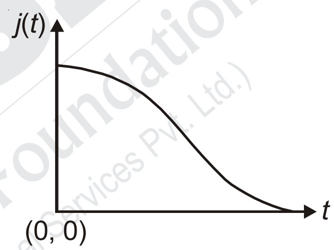
$$\Rightarrow \quad \frac{2h}{\sqrt{3}}=\frac{3l}{8}$$

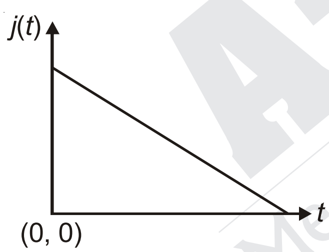
$$\Rightarrow \quad \frac{h}{l}=\frac{3\sqrt{3}}{16}$$

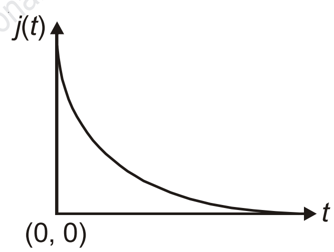
The question has been solved assuming reaction on ground to be only normal reaction (not the resultant of friction and normal reaction).

5. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density $j(t)$ at any point in the material?

(A)


(B)


(C)


(D)


Answer (D)

Sol. : It is an RC discharging circuit.

SECTION - 2 (Maximum Marks : 32)

This section contains **EIGHT** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.

For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+4	If only the bubble(s) corresponding to the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

6. A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true?

(A) The refractive index of the lens is 2.5

(B) The radius of curvature of the convex surface is 45 cm

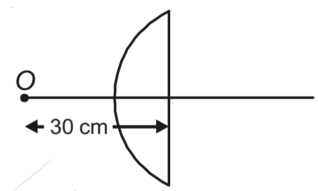
(C) The faint image is erect and real

(D) The focal length of the lens is 20 cm

Answer (A, D)

Sol. : As $m = -2 \Rightarrow v = 60$ cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{60}+\frac{1}{30}=\frac{1}{f} \Rightarrow f = 20 \text{ cm} \quad \ldots\text{(i)}$$



For reflection from convex surface,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

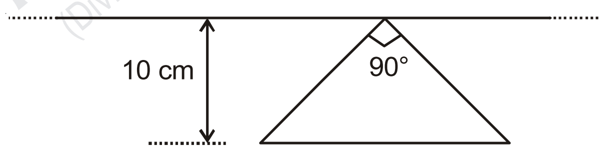
$$\frac{1}{+10}+\frac{1}{-30}=\frac{2}{R} \Rightarrow \frac{1}{10}-\frac{1}{30}=\frac{2}{R} \Rightarrow \frac{3-1}{30}=\frac{2}{30}=\frac{2}{R}$$

$$\Rightarrow R = 30 \text{ cm} \quad \ldots\text{(ii)}$$

By lens maker's formula,

$$\frac{n-1}{30}=\frac{1}{20} \Rightarrow \frac{n-1}{3}=\frac{1}{2} \Rightarrow n-1=\frac{3}{2} \Rightarrow n=2.5$$

7. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 As^{-1}. Which of the following statement(s) is(are) true?



(A) There is a repulsive force between the wire and the loop

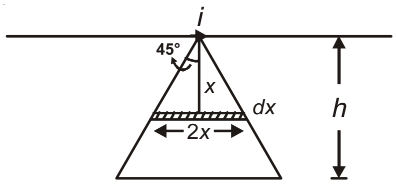
(B) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

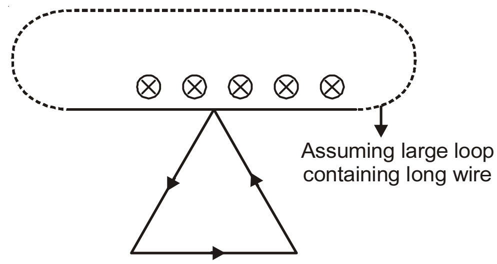
(C) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(D) The induced current in the wire is in opposite direction to the current along the hypotenuse

Answer (A, C)

Sol. : Firstly, we will find mutual inductance between loop and wire.





$$d\phi = \frac{\mu_0 i}{2\pi x} 2x\, dx$$

$$\phi = \frac{\mu_0 i}{\pi} \int dx$$

$$\phi = \frac{\mu_0 i}{\pi} h$$

$$M = \frac{\phi}{i} = \frac{\mu_0 h}{\pi}$$

Now, induced emf in wire is ε

$$\varepsilon = M\frac{di}{dt}$$

$$= \frac{\mu_0 h}{\pi}\frac{di}{dt}$$

$$= \frac{\mu_0}{\pi}\frac{10}{100} \times 10 = \left(\frac{\mu_0}{\pi}\right) \text{volts}$$

Since current in loop is anticlockwise and increasing, so magnetic field above long conducting wire will be inward and increasing. From lenz law, current in conducting long wire will be forward. Opposite currents are nearer, so there will be repulsion between them.

8. The position vector $\vec{r}$ of a particle of mass m is given by the following equation $\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$, where α = 10/3 m s^{-3}, β = 5 m s^{-2} and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle?

(A) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})\,\text{ms}^{-1}$

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -(5/3)\hat{k}$ N ms

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$ N

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -(20/3)\hat{k}$ Nm

Answer (A, B, D)

Sol. : $\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$

$\vec{v}(t) = \frac{d}{dt}\vec{r}(t) = 3\alpha t^2 \hat{i} + 2\beta t \hat{j}$ at t = 1 $\vec{v}(t) = (10\hat{i} + 10\hat{j})$ m/s

$\vec{a}(t) = \frac{d}{dt}\vec{a}(t) = 6\alpha t \hat{i} + 2\beta \hat{i}$ at t = 1 $\vec{a}(t) = (20\hat{i} + 10\hat{j})$ m/s

Now,

$$\vec{L} = m(\vec{r} \times \vec{v}) = m[2\alpha\beta t^4 \hat{k} - 3\alpha\beta t^4 \hat{k}]$$

$= -m\alpha\beta t^4 \hat{k}$ at t = 1 $\vec{L} = \frac{-5}{3}\hat{k}$

$\vec{\tau} = \frac{d\vec{L}}{dt} = -4m\alpha\beta t^3 \hat{k}$ at t = 1 $\vec{\tau} = \frac{-20}{3}\hat{k}$

9. A length-scale (l) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is(are) dimensionally correct?

(A) $l = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$

(B) $l = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$

(C) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$

(D) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

Answer (B, D)

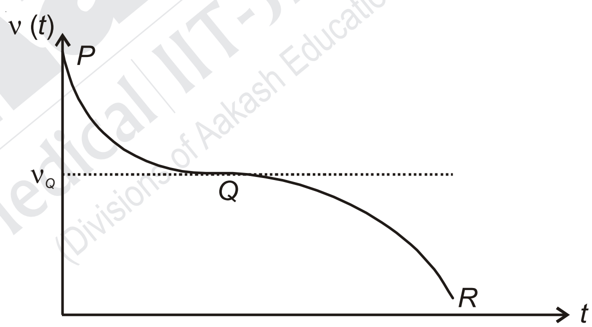
Sol. : [Energy] = $[k_B T] = \left[\frac{q^2}{\varepsilon_0 l}\right]$

$$\left[\frac{\varepsilon_0 k_B T}{q^2}\right] = \left[\frac{1}{l}\right]$$

$\Rightarrow$ $l = \sqrt{\frac{\varepsilon_0 k_B T}{nq^2}}$ and $l = \sqrt{\frac{q^2}{\varepsilon_0 n^{1/3} k_B T}}$ are dimensionally correct.

10. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $\nu(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 m s^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person?

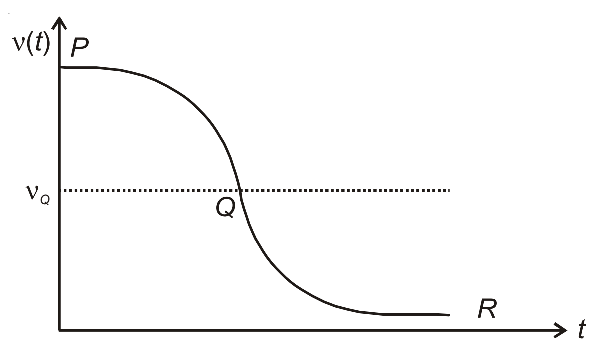
(A) The plot below represents schematically the variation of beat frequency with time



(B) The rate of change in beat frequency is maximum when the car passes through Q

(C) $\nu_P + \nu_R = 2\nu_Q$

(D) The plot below represents schematically the variation of beat frequency with time



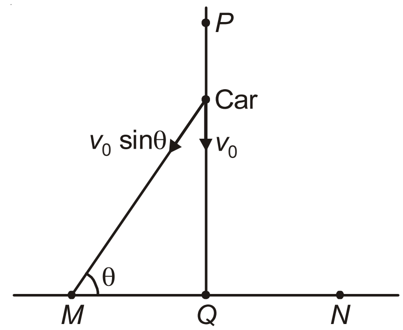
Answer (B, C, D)

Sol. : $\Delta f = 121\left(\frac{v + v_0 \sin\theta}{v}\right) - 118\left(\frac{v + v_0 \sin\theta}{v}\right)$

$= 3\left(\frac{v + v_0 \sin\theta}{v}\right)$

$\frac{d(\Delta f)}{dt} = 3\left[\frac{v_0}{v}\cos\theta\right]\frac{d\theta}{dt}$

Clearly, $\frac{d(\Delta f)}{dt}$ is maximum when θ is 0°.



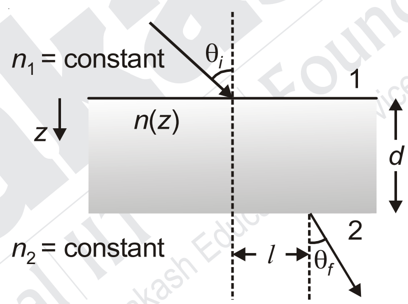
$\Rightarrow$ Rate of change in beat frequency is maximum when it crosses Q.

$\nu_P = 3\left(\frac{v + v_0 \sin\theta_0}{v}\right)$

$\nu_R = 3\left(\frac{v - v_0 \sin\theta_0}{v}\right)$

$\nu_P + \nu_R = 6 = 2\nu_Q$

11. A transparent slab of thickness d has a refractive index $n(z)$ that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 (> n_1), as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l.



Which of the following statement(s) is(are) true?

(A) l is dependent on $n(z)$

(B) $n_1 \sin\theta_i = (n_2 - n_1) \sin\theta_f$

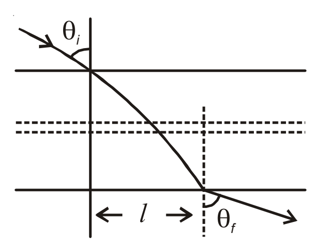
(C) $n_1 \sin\theta_i = n_2 \sin\theta_f$

(D) l is independent of n_2

Answer (A, C, D)

Sol. : Since refraction occurs at a set of parallel surfaces, $n_1 \sin\theta_i = n_2 \sin\theta_f$

The lateral displacement will depend on how the $n(z)$ varies in the medium. Clearly, l does not depend on n_2.



12. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where $n >> 1$. Which of the following statement(s) is(are) true?

(A) Relative change in the radii of two consecutive orbitals does not depend on Z

(B) Relative change in the radii of two consecutive orbitals varies as $1/n$

(C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(D) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

Answer (A, B, D)

Sol. : $r \propto \dfrac{n^2}{Z}$

$$\Rightarrow \frac{\Delta r}{r} = \frac{(n+1)^2 - n^2}{n^2} = \frac{(2n+1)}{n^2} = \frac{2n}{n^2} = \frac{2}{n} \quad (\because n >>> 1)$$

$$E \propto \frac{Z^2}{n^2}$$

$$\Rightarrow \frac{\Delta E}{E} = \frac{\frac{1}{(n+1)^2} - \frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left(\frac{n}{n+1}\right)^2 - 1 = \frac{n^2 - (n+1)^2}{(n+1)^2} \approx \frac{1}{n} \quad (\because n >> 1)$$

$$\Rightarrow L = \frac{nh}{2\pi}$$

$$\frac{\Delta L}{L} = \frac{1}{n}$$

So, correct options are (A, B, D).

13. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true?

(A) The temperature distribution over the filament is uniform

(B) The resistance over small sections of the filament decreases with time

(C) The filament emits more light at higher band of frequencies before it breaks up

(D) The filament consumes less electrical power towards the end of the life of the bulb

Answer (D)

Sol. : As the evaporation is non-uniform, temperature distribution is non-uniform.

Now, as the tungsten evaporates, its radius decreases and hence resistance increases.

Now, $\downarrow P = \dfrac{v^2}{R\uparrow}$ (as resistance increases, power decreases)

Also, $P = \dfrac{\pi r^2 v^2}{\rho l} = \sigma\varepsilon_0(2\pi r l)T^4$

$\Rightarrow T^4 \propto r$

As radius decreases, T decreases and according to Wein's distribution law

$\lambda_m T = k$ as temperature decreases, wavelength increases and hence frequency decreases.

SECTION - 3 (Maximum Marks : 15)

This section contains **FIVE** questions.

The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

For each question, darken the bubble corresponding to the correct integer in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

14. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking $hc/e = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is

Answer (6)

Sol. : Energy given by irradiation is $\frac{hc}{\lambda}$

$$= \frac{12370 \text{ eV Å}}{970 \text{ Å}}$$

$= 12.75$ eV

Energy in excited state is = – 13.6 + 12.75

= – 0.85 eV

$$-0.85 \text{ eV} = \frac{-13.6 \text{ eV}}{n^2}$$

$n^2 = 16$

$n = 4$

So number of spectral line is $\frac{n(n-1)}{2} = 6$.

15. The isotope ${}^{12}_{5}\text{B}$ having a mass 12.014 u undergoes β-decay to ${}^{12}_{6}\text{C}$. ${}^{12}_{6}\text{C}$ has an excited state of the nucleus $\left({}^{12}_{6}\text{C}^*\right)$ at 4.041 MeV above its ground state. If ${}^{12}_{5}\text{B}$ decays to ${}^{12}_{6}\text{C}^*$, the maximum kinetic energy of the β-particle in units of MeV is

(1 u = 931.5 MeV/c^2, where c is the speed of light in vacuum).

Answer (9)

Sol. : $$B_5^{12} \longrightarrow {}^*C_6^{12} + \beta_{-1}^{0} \longrightarrow C_6^{12}$$

$\Delta m = 12.014u - 12.000u$

$= .014u$

$(\Delta m)c^2 = 0.014 \times 931.5$ MeV

= 13.041 MeV

So $(\text{KE})_{max}$ of β-particle is

= 13.041 – 4.041

= 9 MeV

16. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity η = 3 poiseuilles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity η = 2 poiseuilles. The ratio of the terminal velocities of P and Q is

Answer (3)

Sol. : $V_T = \frac{2}{9}r^2\frac{(\sigma-\rho)g}{\eta}$

$$\frac{V_P}{V_Q} = \left(\frac{r_P}{r_Q}\right)^2\left(\frac{\sigma_P-\rho_1}{\sigma_Q-\rho_2}\right)\left(\frac{\eta_2}{\eta_1}\right)$$

$$= \left(\frac{1}{0.5}\right)^2\left(\frac{8-.8}{8-1.6}\right)\left(\frac{2}{3}\right)$$

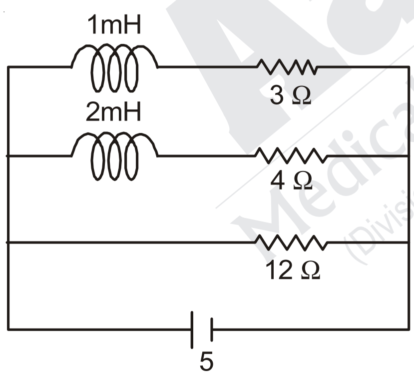
$$= 4\times\frac{7.2}{6.4}\times\frac{2}{3}$$

$$= 3$$

17. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is

Answer (8)

Sol. :



I_{min} is $\frac{5}{12}$A

I_{max} is $= 5\left(\frac{1}{12}+\frac{1}{4}+\frac{1}{3}\right)$

$$= 5\left(\frac{1+3+4}{12}\right)$$

$$= \frac{5\times8}{12}\text{A}$$

So, $\frac{I_{max}}{I_{min}} = 8$

18. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C?

Answer (9)

Sol. : Let display is $X = \log_2\left(\dfrac{P}{P_0}\right)$

At temperature T_1 = 487°C = 760 K, Power is P_1

$$X_1 = \log_2\left(\frac{P_1}{P_0}\right)$$

At T_2 = 2767°C = 3040 K, Power is P_2

$$X_2 = \log_2\left(\frac{P_2}{P_0}\right)$$

Now, $X_2 - X_1 = \log_2\left(\dfrac{P_2}{P_1}\right)$

From Stefan's law, $\dfrac{P_2}{P_1} = \left(\dfrac{T_2}{T_1}\right)^4$

$$X_2 - X_1 = 4\log_2\left(\frac{T_2}{T_1}\right)$$

$$= 4\log_2 4$$

$$= 8$$

$X_2 = 8 + X_1$

$= 9$

END OF PART I : PHYSICS

PART– II : CHEMISTRY

SECTION - 1 (Maximum Marks : 15)

This section contains **FIVE questions.**

Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Ngative Marks : –1 In all other cases.

19. The increasing order of atomic radii of the following Group 13 elements is

(A) Al < Ga < In < Tl
(B) Ga < Al < In < Tl
(C) Al < In < Ga < Tl
(D) Al < Ga < Tl < In

Answer (B)

Sol. : The increasing order of atomic radii is

	Ga	< Al	< In	< Tl
r (pm)	135	145	167	170

20. Among $[Ni(CO)_4]$, $[NiCl_4]^{2-}$, $[Co(NH_3)_4Cl_2]Cl$, $Na_3[CoF_6]$, Na_2O_2 and CsO_2, the total number of paramagnetic compounds is

(A) 2
(B) 3
(C) 4
(D) 5

Answer (B)

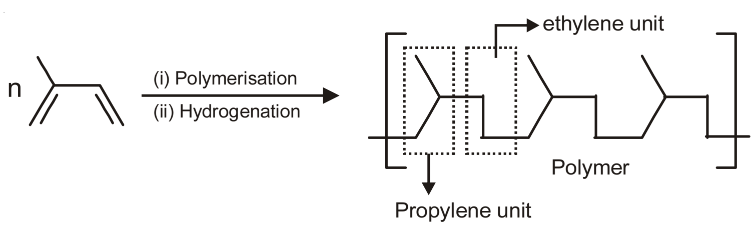
Sol. : $[NiCl_4]^{2-}$ and $Na_3[CoF_6]$ are paramagnetic since involve weak field ligand. CsO_2 is paramagnetic due to paramagnetism by O_2^-.

21. On complete hydrogenation, natural rubbber produces

(A) Ethylene-propylene copolymer
(B) Vulcanised rubber
(C) Polypropylene
(D) Polybutylene

Answer (A)

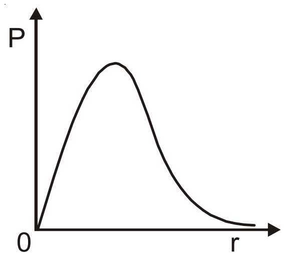
Sol. :



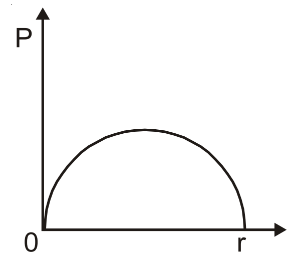
So, it is called ethylene-propylene unit.

22. P is the probability of finding the 1*s* electron of hydrogen atom in a spherical shell of infinitesimal thickness, dr at a distance r from the nucleus. The volume of this shell is $4\pi r^2 dr$. The qualitative sketch of the dependence of P on r is

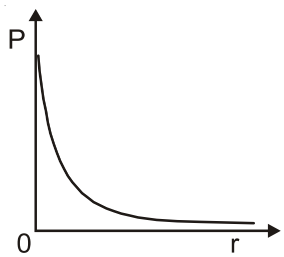
(A)



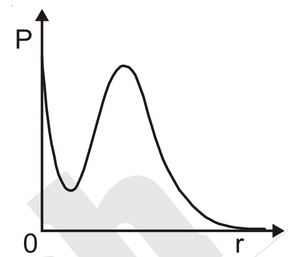
(B)



(C)

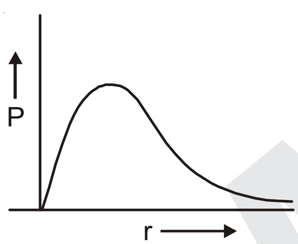


(D)



Answer (A)

Sol. : Radial probability function curve for 1s is



Here 'P' is $4\pi r^2 \psi_r^2 dr$

23. One mole of an ideal gas at 300 K in thermal contact with surroundings expands isothermally from 1.0 L to 2.0 L against a constant pressure of 3.0 atm. In this process, the change in entropy of surroundings (ΔS_{surr}) in JK^{-1} is (1 L atm = 101.3 J)

(A) 5.763

(B) 1.013

(C) –1.013

(D) –5.763

Answer (C)

Sol. : For isothermal change $\Delta U = 0$

$\therefore \quad q = -w$

$= P_{ex}\Delta U = 3 \times (2-1)$

= 3 L atm = 3 × 101.3 J

Heat released from surrounding = –3 × 101.3 J

So, $\Delta S_{surrounding} = \dfrac{-3 \times 101.3}{300} = -1.013\ JK^{-1}$

SECTION - 2 (Maximum Marks : 32)

This section contains **EIGHT** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.

For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble(s) corresponding to the correct option(s) is(are) darkened.

Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Ngative Marks : –2 In all other cases.

For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

24. The correct statement(s) about the following reaction sequence is(are)

$$\text{Cumene}(C_9H_{12}) \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } O_2} P \xrightarrow{CHCl_3/NaOH} Q(\text{major}) + R(\text{minor})$$

$$Q \xrightarrow[PhCH_2Br]{NaOH} S$$

(A) R is steam volatile

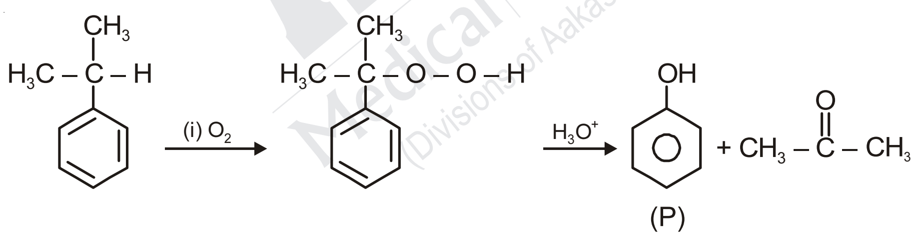
(B) Q gives dark violet coloration with 1% aqueous $FeCl_3$ solution

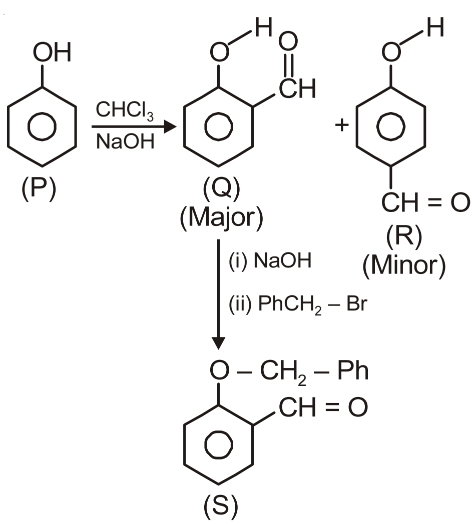
(C) S gives yellow precipitate with 2, 4-dinitrophenylhydrazine

(D) S gives dark violet coloration with 1% aqueous $FeCl_3$ solution

Answer (B, C)

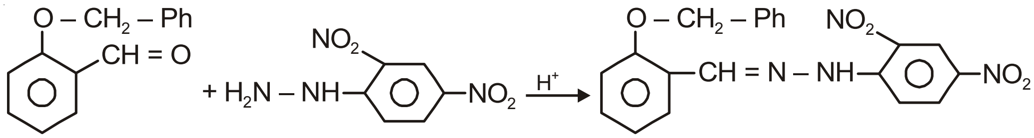
Sol. :





→ Q is steam volatile due to intramolecular hydrogen bonding but (R) is a high melting solid.

→ Since, Q contains a phenotic group, it forms dark brown solution with 1% aqueous $FeCl_3$ solution.

→ 

2, 4 DNP (yellow precipitate)

→ Since, (S) does not contain free phenolic group, therefore it will not form violet colour with $FeCl_3$ solution.

25. The compound(s) with TWO lone pairs of electrons on the central atom is(are)

(A) BrF_5 (B) ClF_3

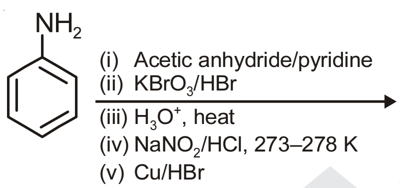
(C) XeF_4 (D) SF_4

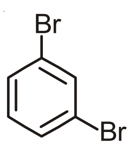
Answer (B, C)

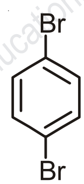
Sol. :

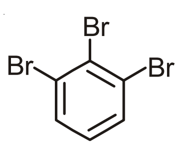
Compound		Number of lone pairs on central atom
BrF_5	→	1
ClF_3	→	2
XeF_4	→	2
SF_4	→	1

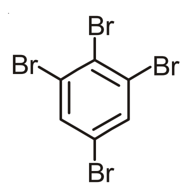
26. The product(s) of the following reaction sequence is(are)



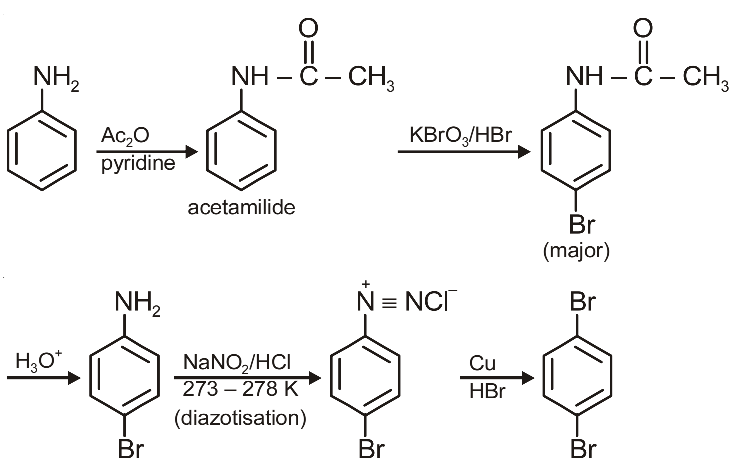
(A) 

(B) 

(C) 

(D) 

Answer (B)

Sol. : 

27. According to the Arrhenius equation,

(A) A high activation energy usually implies a fast reaction

(B) Rate constant increases with increase in temperature. This is due to a greater number of collisions whose energy exceeds the activation energy

(C) Higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant

(D) The pre-exponential factor is a measure of the rate at which collisions occur, irrespective of their energy

Answer (B, C, D)

Sol. : → A high activation energy usually implies a slow reaction.

→ Rate constant of a reaction increases with increasing temperature due to the increase in greater number of collisions whose energy exceeds the activation energy.

→ $k = P \times Z \times e^{-E_a/RT}$

So, pre-exponential factor (A) = P × Z and it is independent of activation energy or energy of molecules.

28. The crystalline form of borax has

(A) Tetranuclear $[B_4O_5(OH)_4]^{2-}$ unit

(B) All boron atoms in the same plane

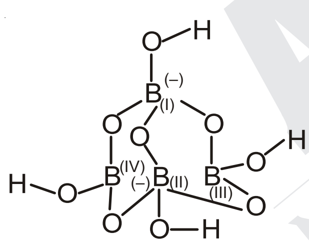
(C) Equal number of sp^2 and sp^3 hybridized boron atoms

(D) One terminal hydroxide per boron atom

Answer (A, C, D)

Sol. : Borax formula is $Na_2[B_4O_5(OH)_4].8H_2O$

$[B_4O_5(OH)_4]^{2-}$ has following structure



$B_{(I)}$ and $B_{(II)}$ are sp^3 hybridized

$B_{(III)}$ and $B_{(IV)}$ are sp^2 hybridized.

29. The reagent(s) that can selectively precipitate S^{2-} from a mixture of S^{2-} and SO_4^{2-} in aqueous solution is(are)

(A) $CuCl_2$

(B) $BaCl_2$

(C) $Pb(OOCCH_3)_2$

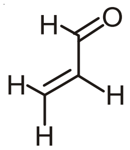
(D) $Na_2[Fe(CN)_5NO]$

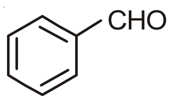
Answer (A OR A, C)

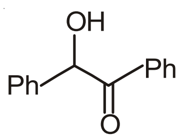
Sol. :

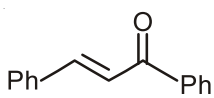
	S^{2-}	SO_4^{2-}
Cu^{2+}	CuS (ppt)	$CuSO_4$ (Soluble)
Ba^{2+}	BaS (Soluble)	$BaSO_4$ (ppt)
$Pb(OAc)_2$	PbS (ppt)	$PbSO_4$ (ppt)
$Na_2[Fe(CN)_5NO]$	$Na_4[Fe(CN)_5(NOS)]$ Colour (not a ppt)	———

30. Positive Tollen's test is observed for

(A) 

(B) 

(C) 

(D) 

Answer (A, B, C)

Sol. : All aldehydes gives Tollen's test as shown below

$$R-CH=O+[Ag(NH_3)_2]^+ \longrightarrow R-\overset{\overset{\displaystyle O}{\|}}{C}-O^- + \underset{\text{(Silver mirror)}}{Ag\downarrow} + NH_4^+$$

α-hydroxy ketones are oxidised to 1, 2 diketones and Ag^+ is reduced to Ag.

$$Ph-\overset{\overset{\displaystyle OH}{|}}{CH}-\overset{\overset{\displaystyle O}{\|}}{C}-Ph \xrightarrow{[Ag(NH_3)_2]^+} Ph-\overset{\overset{\displaystyle O}{\|}}{C}-\overset{\overset{\displaystyle O}{\|}}{C}-Ph + Ag\downarrow + NH_4^+$$

31. A plot of the number of neutrons (N) against the number of protons (P) of stable nuclei exhibits upward deviation from linearity for atomic number, Z > 20. For an unstable nucleus having N/P ratio less than 1, the possible mode(s) of decay is/are

(A) β^--decay (β emission)

(B) Orbital or K-electron capture

(C) Neutron emission

(D) β^+-decay (positron emission)

Answer (B, D)

Sol. : When N/P ratio is less than one, then proton changes into neutron.

Positron emission : ${}_{1}^{1}H \longrightarrow {}_{0}^{1}n + {}_{+1}^{0}\beta$

Capture : ${}_{1}^{1}H + {}_{-1}^{0}e \longrightarrow {}_{0}^{1}n + \text{X-rays}$

SECTION - 3 (Maximum Marks : 15)

This section contains **FIVE** questions.

The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

For each question, darken the bubble corresponding to the correct integer in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+3	If only the bubble corresponding to the correct answer is darkened.
Zero Marks	:	0	In all other cases.

32. The diffusion coefficient of an ideal gas is proportional to its mean free path and mean speed. The absolute temperature of an ideal gas is increased 4 times and its pressure is increased 2 times. As a result, the diffusion coefficient of this gas increases x times. The value of x is

Answer (4)

Sol. : Diffusion coefficient $\propto \lambda\,\mu$

Here λ = mean free path

μ = mean speed

since $\lambda \propto \frac{T}{P}$

and $\mu \propto \sqrt{T}$

$\therefore$ Diffusion coefficient $\propto \frac{T\sqrt{T}}{P}$

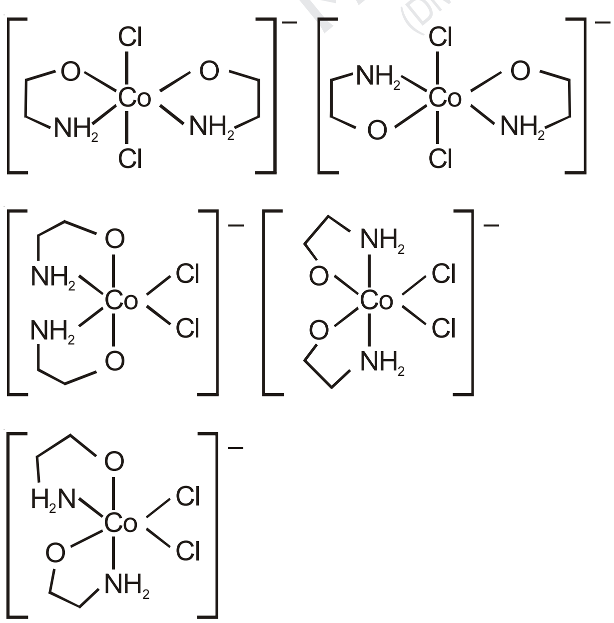
As per question 'T' is increased four times and 'P' is increased two times

Hence diffusion coefficient becomes four times

33. The number of geometric isomers possible for the complex $[CoL_2Cl_2]^-$ ($L = H_2NCH_2CH_2O^-$) is

Answer (5)

Sol. :



34. The mole fraction of a solute in a solution is 0.1. At 298 K, molarity of this solution is the same as its molality. Density of this solution at 298 K is 2.0 g cm^{-3}. The ratio of the molecular weights of the solute and solvent, $\left(\frac{MW_{solute}}{MW_{solvent}}\right)$, is

Answer (9)

Sol. : Molality (m) = $\frac{0.1\times1000}{0.9\times MW_{solvent}}$ ... (i)

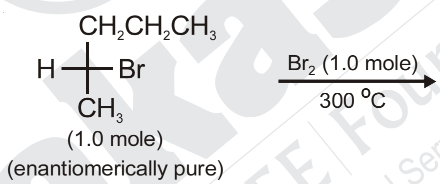
Molarity (M) = $\frac{0.1\times2\times1000}{(0.9\times MW_{solvent}+0.1\times MW_{solute})}$ ...(ii)

Equating (i) & (ii)

$$\frac{0.1\times1000}{0.9\times MW_{solvent}}=\frac{0.1\times2\times1000}{(0.9\times MW_{solvent}+0.1\times MW_{solute})}$$

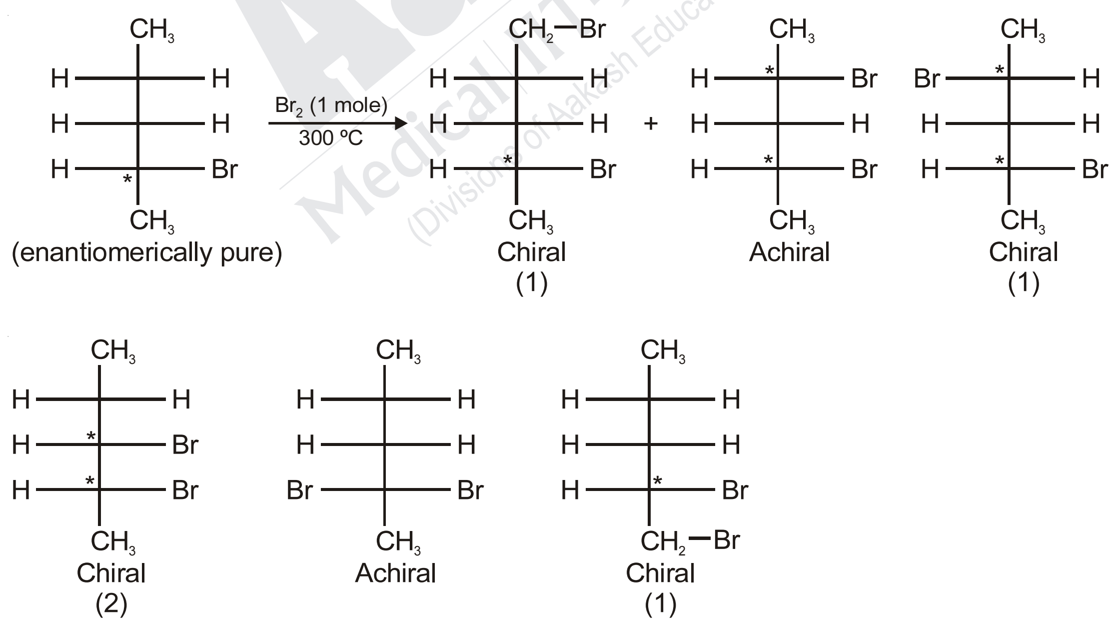
So, $\frac{MW_{solute}}{MW_{solvent}}=9$

35. In the following monobromination reaction, the number of possible chiral products is



Answer (5)

Sol. :



36. In neutral or faintly alkaline solution, 8 moles of permanganate anion quantitatively oxidize thiosulphate anions to produce X moles of a sulphur containing product. The magnitude of X is

Answer (6)

Sol. : $3S_2O_3^{-} + 8MnO_4^{-} \rightarrow 6SO_4^{2-} + 8MnO_2$

$\therefore$ 8 moles of MnO_4^{-} form 6 moles of SO_4^{2-}

PART–III : MATHEMATICS

SECTION - 1 (Maximum Marks : 15)

This section contains **FIVE questions.**

Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

For each question, darken the bubble corresponding to the correct option in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –1 In all other cases.

37. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that P(computer turns out to be defective given that it is produced in plant T_1) = 10P (computer turns out to be defective given that it is produced in plant T_2) where $P(E)$ denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$

(C) $\frac{78}{93}$ (D) $\frac{75}{83}$

Answer (C)

Sol. : $P(T_1) = \frac{1}{5}, P(T_2) = \frac{4}{5}, P(D) = \frac{7}{100}$

$$P(D) = P\left(\frac{D}{T_1}\right).P(T_1) + P\left(\frac{D}{T_2}\right).P(T_2) = 10P\left(\frac{D}{T_2}\right)P(T_1) + P\left(\frac{D}{T_2}\right)P(T_2)$$

On solving $P\left(\frac{D}{T_2}\right) = \frac{1}{40}$

$$P\left(\frac{T_2}{\bar{D}}\right) = \frac{P\left(\frac{\bar{D}}{T_2}\right).P(T_2)}{P(\bar{D})} = \frac{\frac{39}{40}\cdot\frac{4}{5}}{\frac{93}{100}} = \frac{78}{93}$$

38. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

(A) 380 (B) 320

(C) 260 (D) 95

Answer (A)

Sol. : Number of ways = ${}^6C_4 \times {}^4C_1 + {}^6C_3 \times {}^4C_1 \times {}^4C_1$

= 380

39. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

(A) $2(\sec\theta - \tan\theta)$

(B) $2\sec\theta$

(C) $-2\tan\theta$

(D) 0

Answer (C)

Sol. : $x^2 - 2x\sec\theta + 1 = 0$

$\Rightarrow$ $x = \sec\theta + \tan\theta,\ \sec\theta - \tan\theta$

as $\theta \in \left(\frac{-\pi}{6}, \frac{-\pi}{12}\right)$

So, $\alpha_1 = \sec\theta - \tan\theta,\ \beta_1 = \sec\theta + \tan\theta$

$x^2 + 2x\tan\theta - 1 = 0$

$\Rightarrow$ $x = \sec\theta - \tan\theta,\ -\sec\theta - \tan\theta$

$\alpha_2 = \sec\theta - \tan\theta,\ \beta_2 = -\sec\theta - \tan\theta$

$\alpha_1 + \beta_2 = -2\tan\theta$

40. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

(A) $-\frac{7\pi}{9}$

(B) $-\frac{2\pi}{9}$

(C) 0

(D) $\frac{5\pi}{9}$

Answer (C)

Sol. : $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$

$\Rightarrow$ $\sqrt{3}\sin x + \cos x + 2(\sin^2 x - \cos^2 x) = 0$

$\Rightarrow$ $\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \cos^2 x - \sin^2 x$

$\Rightarrow$ $\cos 2x = \cos\left(x - \frac{\pi}{3}\right)$

$\Rightarrow$ $2x = 2n\pi \pm \left(x - \frac{\pi}{3}\right), n \in I$

$\Rightarrow$ $x = 2n\pi - \frac{\pi}{3}, \frac{1}{3}\left(2n\pi + \frac{\pi}{3}\right)$

So, $x = -\frac{\pi}{3}, \frac{\pi}{9}, \frac{7\pi}{9}, \frac{-5\pi}{9}$

Sum of all the solution = 0

41. The least value of $\alpha \in \mathbb{R}$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is

(A) $\frac{1}{64}$

(B) $\frac{1}{32}$

(C) $\frac{1}{27}$

(D) $\frac{1}{25}$

Answer (C)

Sol. : Let $f(x) = 4\alpha x^2 + \frac{1}{x}$

$f'(x) = 8\alpha x - \frac{1}{x^2} = 0$

$\Rightarrow \quad x = \frac{1}{2\alpha^{1/3}}$ (point of minima)

So, $(f(x))_{min} = 4\alpha . \frac{1}{4\alpha^{2/3}} + 2\alpha^{1/3} \geq 1$

$\Rightarrow \quad \alpha^{1/3} \geq \frac{1}{3}$

$\Rightarrow \quad \alpha \geq \frac{1}{27}$

SECTION - 2 (Maximum Marks : 32)

This section contains **EIGHT** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.

For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+4	If only the bubble(s) corresponding to the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

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42. A solution curve of the differential equation $(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0$, x > 0, passes through the point (1, 3). Then the solution curve

(A) Intersects $y = x + 2$ exactly at one point

(B) Intersects $y = x + 2$ exactly at two points

(C) Intersects $y = (x + 2)^2$

(D) Does NOT intersect $y = (x + 3)^2$

Answer (A, D)

Sol. : $\left(x^2+xy+4x+2y+4\right)\frac{dy}{dx}=y^2$

$$(x+2)^2+y(x+2)=y^2\frac{dx}{dy}$$

$$\frac{dx}{dy}=\frac{(x+2)^2}{y^2}+\frac{x+2}{y}$$

$$\frac{1}{(x+2)^2}\frac{dx}{dy}-\frac{1}{y(x+2)}=\frac{1}{y^2}$$

$$t=\frac{-1}{x+2}$$

$$\frac{dt}{dy}+\frac{t}{y}=\frac{1}{y^2}$$

$$\text{IF} = e^{\int\frac{dy}{y}} = y$$

$$y\cdot t = \int\frac{y}{y^2}dy = \ln y + c$$

$$\Rightarrow \frac{-y}{x+2} = \ln y + c$$

It passes through (1, 3)

$\Rightarrow$ $c = -1 - \ln 3$

$$\Rightarrow \ln y = 1-\frac{y}{x+2}+\ln 3$$

On solving with $y = x + 2$

$\ln(x + 2) = \ln 3 \Rightarrow x = 1$ (Exactly one point)

On solving with $y = (x + 2)^2$

$\ln(x + 2)^2 = 1 - (x + 2) + \ln 3$

$= -1 + \ln 3 - x$

As graph of LHS is increasing & graph of RHS is decreasing

and $\lim\limits_{x\to 0^+}\text{LHS} > \lim\limits_{x\to 0^+}\text{RHS}$

So the curves do not intersect

Also, $(x + 3)^2 > (x + 2)^2$ for $x > 0$

So, $y = (x + 3)^2$ and $\ln y = 1 + \ln 3 - \frac{y}{x+2}$ also do not intersect

43. Consider a pyramid *OPQRS* located in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) with *O* as origin, and *OP* and *OR* along the *x*-axis and the *y*-axis, respectively. The base *OPQR* of the pyramid is a square with $OP = 3$. The point *S* is directly above the mid-point *T* of diagonal *OQ* such that $TS = 3$. Then

(A) The acute angle between *OQ* and *OS* is $\frac{\pi}{3}$

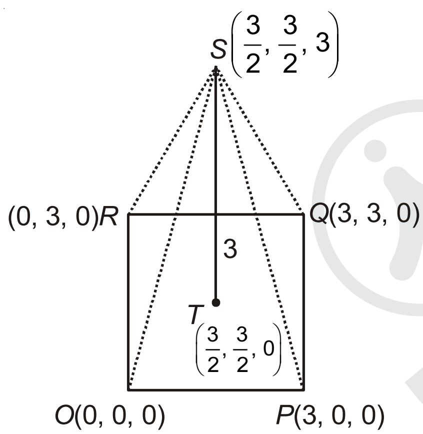
(B) The equation of the plane containing the triangle *OQS* is $x - y = 0$

(C) The length of the perpendicular from *P* to the plane containing the triangle *OQS* is $\frac{3}{\sqrt{2}}$

(D) The perpendicular distance from *O* to the straight line containing *RS* is $\sqrt{\frac{15}{2}}$

Answer (B, C, D)

Sol. :



(A) DR of *OQ*(1, 1, 0)

DR of *OS*(1, 1, 2)

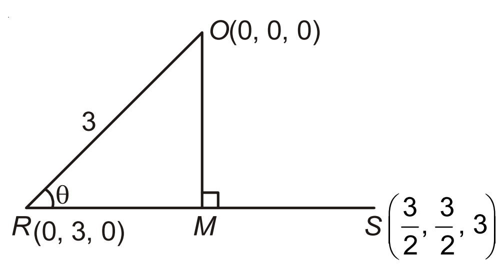
$$\text{So, } \cos\theta = \frac{1+1}{\sqrt{2}.\sqrt{6}} = \frac{1}{\sqrt{3}}$$

(B) $$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 2\hat{i} - 2\hat{j}$$

So, equation of plane is $x - y = 0$

(C) Perpendicular distance $= \frac{3-0}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

(D) DR of *OR*(0, 1, 0)



DR of *RS*(1, –1, 2)

$$\text{So, } \cos\theta = \frac{1}{\sqrt{6}}$$

$$\Rightarrow \quad OM = OR\sin\theta$$

$$= 3\frac{\sqrt{5}}{\sqrt{6}} = \sqrt{\frac{15}{2}}$$

44. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centers Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then

(A) $Q_2Q_3 = 12$

(B) $R_2R_3 = 4\sqrt{6}$

(C) Area of the triangle OR_2R_3 is $6\sqrt{2}$

(D) Area of the triangle PQ_2Q_3 is $4\sqrt{2}$

Answer (A, B, C)

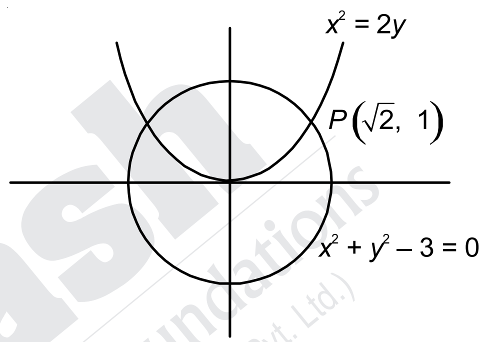
Sol. : $x^2 + y^2 = 3$

$x^2 = 2y$

$\therefore\ P\left(\sqrt{2},\ 1\right)$

Tangent at P, $\sqrt{2}x + y - 3 = 0$

Let the centre of circle be $(0, a)$



$$\therefore\ \frac{\sqrt{2}(0) + a - 3}{\sqrt{3}} = \pm 2\sqrt{3}$$

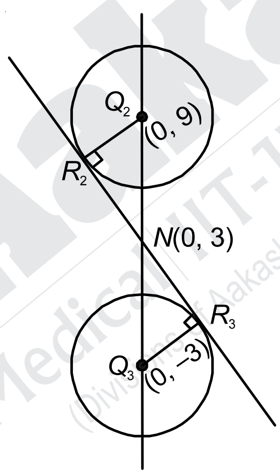
$a - 3 = \pm 6$

$a = 9, -3$

$\therefore\ Q_2\ (0, 9)$

$Q_3\ (0, -3)$

$Q_2Q_3 = 12$



$\therefore\ NR_3 = \sqrt{6^2 - 12}$

$= \sqrt{24}$

$= 2\sqrt{6}$

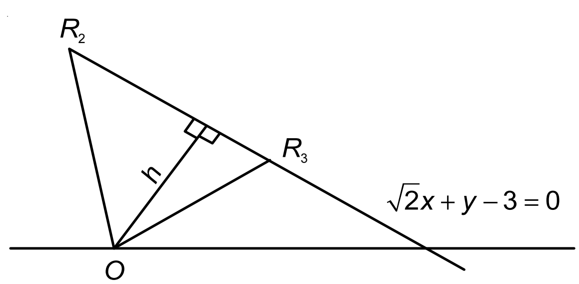
$\therefore\ R_2R_3 = 4\sqrt{6}$

Area of OR_2R_3

$$\text{Area of } \Delta = \frac{1}{2}R_2R_3 \times h$$

$$= \frac{1}{2}4\sqrt{6} \times \frac{3}{\sqrt{3}}$$

$$= \frac{4\sqrt{3} \times \sqrt{2} \times \sqrt{3}}{2}$$

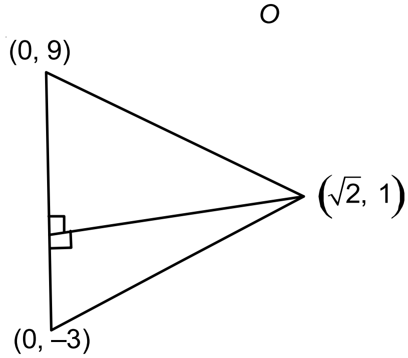


ar $(\Delta OR_2R_3) = 6\sqrt{2}$

ar (ΔPQ_2Q_3)

$$\text{Area of } \Delta PQ_2Q_3 = \frac{1}{2} \times 12 \times \sqrt{2}$$

$$= 6\sqrt{2}$$



45. Let $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \mathbb{R}$. Then

(A) $g'(2) = \frac{1}{15}$

(B) $h'(1) = 666$

(C) $h(0) = 16$

(D) $h(g(3)) = 36$

Answer (B, C)

Sol. : $g'(f(x)) \cdot f'(x) = 1$

$g'(2) \cdot f'(0) = 1$

$g'(2) = \frac{1}{f'(0)}$

$f'(x) = 3x^2 + 3$

$g'(2) = \frac{1}{3}$

$h(g(g(x))) = x$

$h(g(g(f(x)))) = f(x)$

$h(g(x)) = f(x)$

$h(g(3)) = f(3) = 38$

$h(g(f(x))) = f(f(x))$

$h(x) = f(f(x))$

$h'(x) = f'(f(x)) \cdot f'(x)$

$h'(1) = f'(f(1)) \cdot f'(1) = 111 \times 6 = 666$

$h(0) = f(f(0)) = f(2) = 16$

46. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in \mathbb{R}$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then

(A) $\alpha = 0$, $k = 8$

(B) $4\alpha - k + 8 = 0$

(C) $\det(P \operatorname{adj}(Q)) = 2^9$

(D) $\det(Q \operatorname{adj}(P)) = 2^{13}$

Answer (B, C)

Sol. : $PQ = kI$

$\Rightarrow |P||Q| = k^3$

$\Rightarrow |P|\frac{k^2}{2} = k^3$

$\Rightarrow |P| = 2k = 12\alpha + 20$ ...(i)

Also, $Q = kP^{-1}$

$= k \cdot \frac{\operatorname{adj}P}{|P|} = \frac{\operatorname{adj}P}{2}$

So, $q_{23} = -\left(\frac{3\alpha+4}{2}\right) = \frac{-k}{8}$

$\Rightarrow \quad 3\alpha + 4 = \frac{k}{4}$ ...(ii)

On solving (i) & (ii)

$k = 4,\ \alpha = -1,\ |P| = 8,\ |Q| = 8$

$\Rightarrow \quad 4\alpha - k + 8 = 0$

$|P\text{adj}Q| = |P||Q|^2 = 8^3 = 2^9$

$|Q\text{adj}P| = |Q||P|^2 = 8^3 = 2^9$

47. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s).

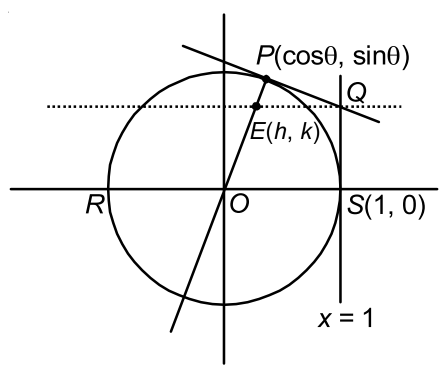
(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

Answer (A, C)

Sol. : Tangent at P : $x\cos\theta + y\sin\theta = 1$

So, $Q\left(1, \frac{1-\cos\theta}{\sin\theta}\right) \equiv \left(1, \tan\frac{\theta}{2}\right)$



Normal at P : $y = x\tan\theta$

So, $E\left(\frac{\tan\frac{\theta}{2}}{\tan\theta}, \tan\frac{\theta}{2}\right)$

$\Rightarrow \quad h = \frac{\tan\frac{\theta}{2}}{\tan\theta},\ k = \tan\frac{\theta}{2}$

On eliminating θ

$k^2 = 1 - 2h$

$\Rightarrow$ Locus of E is $y^2 = 1 - 2x$

48. Let $f : (0, \infty) \to \mathbb{R}$ be a differentiable function such that $f'(x) = 2 - \frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) \neq 1$. Then

(A) $\lim_{x\to 0+} f'\left(\frac{1}{x}\right) = 1$

(B) $\lim_{x\to 0+} xf\left(\frac{1}{x}\right) = 2$

(C) $\lim_{x\to 0+} x^2 f'(x) = 0$

(D) $|f(x)| \leq 2$ for all $x \in (0, 2)$

Answer (A)

Sol. : $f'(x) + \frac{f(x)}{x} = 2$

$$\text{I.F} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

$$f(x)\cdot x = \int 2x\,dx + c$$

$$f(x)\cdot x = x^2 + c$$

$$f(x) = x + \frac{c}{x} \qquad \because \; f(1) \neq 1$$

$$1 + \frac{c}{1} \neq 1 \qquad c \neq 0$$

$$f'(x) = 1 - \frac{c}{x^2}$$

$$\Rightarrow \quad f'\left(\frac{1}{x}\right) = 1 - cx^2$$

(A) $\lim_{x\to 0^+} f'\left(\frac{1}{x}\right) = \lim_{x\to 0^+} (1 - cx^2) = 1$

(B) $\lim_{x\to 0^+} x\cdot f\left(\frac{1}{x}\right) = \lim_{x\to 0^+} x\left(\frac{1}{x} + cx\right)$

$$= \lim_{x\to 0^+} (1 + cx^2) = 1$$

(C) $\lim_{x\to 0^+} x^2 f'(x) = \lim_{x\to 0^+} x^2\left(1 - \frac{c}{x^2}\right)$

$$= \lim_{x\to 0^+} (x^2 - c) = -c$$

(D) $f(x) = x + \frac{c}{x}$

49. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and $2s = x + y + z$. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then

(A) Area of the triangle XYZ is $6\sqrt{6}$

(B) The radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(C) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2} = \frac{4}{35}$

(D) $\sin^2\left(\frac{X+Y}{2}\right) = \frac{3}{5}$

Answer (A, C, D)

Sol. : $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2} = k$

$s - x = 4k$ ...(i)

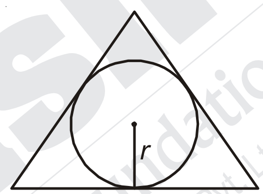
$s - y = 3k$ ...(ii)

$s - z = 2k$ ...(iii)

$3s - (x + y + z) = 9k$

$3s - 2s = 9k$

$s = 9k$



From (i), (ii), (iii)

$x = 5k$, $y = 6k$, $z = 7k$

Area of incircle $= \pi r^2 = \frac{8\pi}{3}$

$r^2 = \frac{8}{3}$

$\Rightarrow \frac{\Delta^2}{s^2} = \frac{8}{3}$

$\Rightarrow \frac{s(s-x)(s-y)(s-z)}{s^2} = \frac{8}{3}$

$\frac{(9k)(4k)(3k)(2k)}{81k^2} = \frac{8}{3}$

$k^2 = 1$

$\Rightarrow k = +1$

Now side length

$x = 5$, $y = 6$, $z = 7$ and $s = 9$

(A) Area of triangle $XYZ = \sqrt{s(s-x)(s-y)(s-z)}$

$= \sqrt{9 \cdot 4 \cdot 3 \cdot 2}$

$= 3 \cdot 2\sqrt{6}$

$= 6\sqrt{6}$

(B) Radius of circumcircle of ΔXYZ

$$R = \frac{xyz}{4\Delta} = \frac{5\cdot 6\cdot 7}{4\cdot 6\sqrt{6}} = \frac{35}{4\sqrt{6}}$$

(C) $$\sin\frac{X}{2}\cdot\sin\frac{Y}{2}\cdot\sin\frac{Z}{2} = \sqrt{\frac{(s-x)(s-z)}{xz}\frac{(s-y)(s-z)}{yz}\frac{(s-x)(s-y)}{xy}}$$

$$= \frac{(s-x)(s-y)(s-z)}{xyz}$$

$$\frac{4\cdot 3\cdot 2}{5\cdot 6\cdot 7}$$

$$= \frac{4}{35}$$

(D) $$\sin^2\left(\frac{X+Y}{2}\right) = \cos^2\frac{Z}{2} = \frac{s(s-z)}{xy} = \frac{9\cdot 2}{5\cdot 6} = \frac{3}{5}$$

SECTION - 3 (Maximum Marks : 15)

This section contains **FIVE** questions.

The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

For each question, darken the bubble corresponding to the correct integer in the ORS.

For each question, marks will be awarded in one of the following categories:

Full Marks	:	+3	If only the bubble corresponding to the correct answer is darkened.
Zero Marks	:	0	In all other cases.

50. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1 + x)^2 + (1 + x)^3 + \ldots + (1 + x)^{49} + (1 + mx)^{50}$ is $(3n + 1)\,{}^{51}C_3$ for some positive integer n. Then the value of n is

Answer (5)

Sol. : $${}^3C_3 + {}^3C_2 + {}^4C_2 + {}^5C_2 + \ldots + {}^{49}C_2 + m^2\,{}^{50}C_2 = (3n+1)\,{}^{51}C_3$$

$${}^{50}C_3 + m^2\,{}^{50}C_2 = (3n+1)\,{}^{51}C_3$$

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

$${}^{n+1}C_r - {}^nC_r = {}^nC_{r-1}$$

$$m^2\,{}^{50}C_2 = 3n\cdot{}^{51}C_3 + {}^{50}C_2$$

$$(m^2-1)\,{}^{50}C_2 = 3n\cdot{}^{51}C_3$$

$$(m^2-1)\times\frac{50\times 49}{2} = 3n\times\frac{51\times 50\times 49}{3\times 2}$$

$$\boxed{(m^2-1) = 51n}$$

$$\boxed{m^2 = 51n+1} \quad \boxed{n = 5}$$

51. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x\to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals

Answer (7)

Sol. : $$\lim_{x\to 0} \frac{x^2\left(\beta x - \frac{(\beta x)^3}{3!} + ...\right)}{\alpha x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + ...\right)}$$

$\boxed{\alpha = 1}$ $\quad \frac{\beta}{\frac{1}{3!}} = 1$

$$\beta = \frac{1}{6}$$

$$6\left(1 + \frac{1}{6}\right) = 7$$

52. Let $z = \frac{-1+\sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and $r, s \in \{1, 2, 3\}$. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is

Answer (1)

Sol. : $z = w$

$P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ $\qquad P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$

$z^{2r} + z^{4s} = -1$, $(-z)^r z^{2s} + z^{2s} z^r = 0$

$\boxed{w^{2r} + w^{4s} = -1}$, $w^{2s}((-w)^r + w^r) = 0$

$\boxed{r = 1, 3}$

$\boxed{r = 1}$ $\boxed{s = 1}$

(1, 1) only one pair

53. The total number of distinct $x \in \mathbb{R}$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

Answer (2)

Sol. : $$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$$

$$x^3\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & 1\\ 3 & 9 & 1\end{vmatrix}+x^6\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & 8\\ 3 & 9 & 27\end{vmatrix}=10$$

$R_3 \to R_3 - R_2 \quad C_3 \to C_3 - C_2$

$R_2 \to R_2 - R_1 \quad C_2 \to C_2 - C_1$

$$x^3\begin{vmatrix}1 & 1 & 1\\ 1 & 3 & 0\\ 1 & 5 & 0\end{vmatrix}+x^6\begin{vmatrix}1 & 0 & 0\\ 2 & 2 & 4\\ 3 & 6 & 18\end{vmatrix}$$

$2x^6 + 2x^3 - 10 = 0$

$6x^6 + x^3 - 5 = 0$

$(x^3 + 1)(6x^3 - 5) = 0$

54. The total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$ is

Answer (1)

Sol. : Let $f(x) = \int_0^x \frac{t^2}{1+t^4} dt - 2x + 1$

$$f'(x) = \frac{x^2}{1+x^4} - 2 < 0$$

$f(x)$ is decreasing function

$$f(1) = \int_0^1 \frac{dt}{t^2 + \frac{1}{t^2}} - 2\cdot 1 + 1 < 0$$

$f(0) > 0$

Therefore only one solution between [0, 1].

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