09/01/2019
EVENING

Aakash
Medical | IIT-JEE | Foundations
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Time : 3 hrs. M.M. : 360

Answers & Solutions

for

JEE (MAIN)-2019

(Online CBT Mode)

(Physics, Chemistry and Mathematics)

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 **(four)** marks for each correct response.
4. *Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for a question in the answer sheet.*
5. There is only one correct response for each question.

PHYSICS

1. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to

(1) 500 Hz (2) 753 Hz

(3) 333 Hz (4) 666 Hz

Answer (4)

Sol. λ = 0.50

$$\lambda = \frac{1}{2}\text{ m}$$

$$\nu = 330 \times 2 = 660 \text{ Hz}$$

$$\nu' = \frac{\left(330 + \frac{50}{18}\right)\nu}{330} \approx 666 \text{ Hz}$$

2. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C), i.e. $\frac{B_L}{B_C}$ will be

(1) $\frac{1}{N}$ (2) N^2

(3) N (4) $\frac{1}{N^2}$

Answer (4)

Sol. $r_1 = \frac{L}{2\pi}$

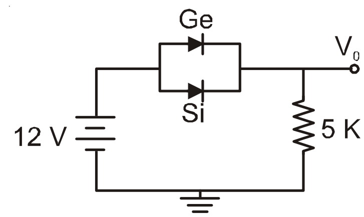
$$B_L = \frac{\mu_0 I}{2r_1}$$

$$r_2 = \frac{L}{2N\pi}$$

$$B_C = \frac{N\mu_0 I}{2r_2}$$

$$\frac{B_L}{B_C} = \frac{1}{N^2}$$

3. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_0 changes by : (assume that the Ge diode has large breakdown voltage)



(1) 0.2 V (2) 0.4 V

(3) 0.6 V (4) 0.8 V

Answer (2)

Sol. Voltage drop across diode will change from 0.3 to 0.7 V.

Value of V_0 changes by 0.4 V.

4. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength λ = 500 nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$ is

(1) 640 (2) 320

(3) 321 (4) 641

Answer (4)

Sol. $N = \frac{d\sin\theta}{\lambda} = \frac{0.320 \times 10^{-3}}{500 \times 10^{-9}} \times \frac{1}{2}$

= 320

Number of Bright fringes = 2 × 320 + 1

= 641

5. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is

(1) 5.65×10^2 J (2) 5.17×10^2 J

(3) 2.26×10^3 J (4) 3.39×10^3 J

Answer (2)

Sol. $X_L = \omega L = 20 \times 10^{-3} \times 2\pi \times 50 = 2\pi\ \Omega = 6.28\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1 \times 10^6}{2\pi \times 50 \times 120} = \frac{1000}{2\pi \times 6}\ \Omega = 26.53\ \Omega$$

$\therefore \quad z = \sqrt{R^2 + (X_C - X_L)^2}$

$\therefore \quad |Z| = \sqrt{(60)^2 + (20.25)^2} = 63.32\ \Omega$

$\therefore \quad I_{rms} = \dfrac{24}{63.32}, \quad \cos\theta = \dfrac{60}{63.32}$

$\therefore \quad \Delta E = \dfrac{24}{63.32} \times 24 \times \dfrac{60}{63.32} \times 60 = 5.17 \times 10^2 J$

6. Two point charges $q_1\left(\sqrt{10}\ \mu C\right)$ and $q_2(-25\ \mu C)$ are placed on the *x*-axis at *x* = 1 m and *x* = 4 m respectively. The electric field (in V/m) at a point *y* = 3 m on *y*-axis is,

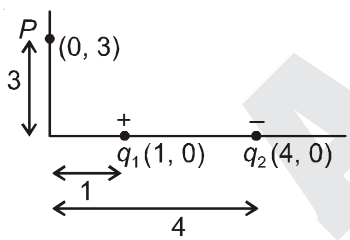
$\left[\text{take } \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{Nm}^2\ \text{C}^{-2}\right]$

(1) $\left(63\hat{i} - 27\hat{j}\right) \times 10^2$ (2) $\left(81\hat{i} - 81\hat{j}\right) \times 10^2$

(3) $\left(-81\hat{i} + 81\hat{j}\right) \times 10^2$ (4) $\left(-63\hat{i} + 27\hat{j}\right) \times 10^2$

Answer (1)

Sol.



$\vec{E} = \vec{E}_1 + \vec{E}_2$

$\vec{E}_1 = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{\sqrt{10} \times 10^{-6}}{10} \times \left[\dfrac{-\hat{i} + 3\hat{j}}{\sqrt{10}}\right]$

$\vec{E}_2 = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{(-25 \times 10^{-6})}{25} \times \left[\dfrac{-4\hat{i} + 3\hat{j}}{5}\right]$

$\vec{E} = 9 \times 10^9 \times 10^{-6}\left[\dfrac{-\hat{i} + 3\hat{j}}{10} - \left(\dfrac{-4\hat{i} + 3\hat{j}}{5}\right)\right]$

$= \dfrac{9 \times 10^3}{10}\left[-\hat{i} + 3\hat{j} + 8\hat{i} - 6\hat{j}\right]$

$= 9 \times 10^2\left[+7\hat{i} - 3\hat{j}\right]$

$= \left(63\hat{i} - 27\hat{j}\right) \times 10^2$

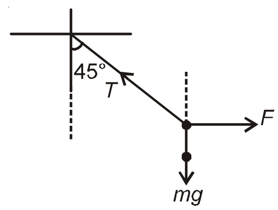
7. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the roof at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is (g = 10 ms^{-2})

(1) 100 N (2) 200 N

(3) 70 N (4) 140 N

Answer (1)

Sol.



$T\cos45° = mg$

$T\sin45° = F$

$\Rightarrow \quad F = mg$

= 10 × 10 = 100 N

8. Two Carnot engines *A* and *B* are operated in series. The first one, *A*, receives heat at T_1(=600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at T_3 (= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal

(1) 300 K (2) 400 K

(3) 600 K (4) 500 K

Answer (4)

Sol. $\dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2} = \dfrac{Q_3}{T_3}$

$W_1 = Q_1 - Q_2$

$W_2 = Q_2 - Q_3$

Given $W_1 = W_2$

$\Rightarrow \quad 2\ T_2 = T_1 + T_3$

$T_2 = \dfrac{T_1 + T_3}{2}$

T_2 = 500 K

9. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about:

[Take R = 83 J/K mole]

(1) 3 kJ (2) 14 kJ

(3) 10 kJ (4) 0.9 kJ

Answer (3)

Sol. $\dfrac{V_1}{V_2} = \sqrt{\dfrac{T_1}{T_2}}$

$T_2 = 1200$ K

$Q = n\,C_V\,(T_2 - T_1)$

$= \dfrac{15}{28} \times 5 \times \dfrac{R}{2} \times 900$

= 10 kJ

10. At a given instant, say $t = 0$, two radioactive substances A and B have equal activities. The ratio $\dfrac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. If the half-life of A is ln2, the half-life of B is

(1) 4 ln2 (2) $\dfrac{\ln 2}{2}$

(3) $\dfrac{\ln 2}{4}$ (4) 2 ln2

Answer (3)

Sol. $R_A = R_0 e^{-\lambda_A t}$

$R_B = R_0 e^{-\lambda_B t}$

$\dfrac{R_B}{R_A} = e^{-(\lambda_B - \lambda_A)t}$

$\lambda_B - \lambda_A = 3$

$\dfrac{\ln 2}{T_2} - \dfrac{\ln 2}{\ln 2} = 3$

$T_2 = \dfrac{\ln 2}{4}$

11. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about $x = 0$. When its potential energy (PE) equals kinetic energy (KE), the position of the particle will be

(1) $\dfrac{A}{\sqrt{2}}$ (2) A

(3) $\dfrac{A}{2\sqrt{2}}$ (4) $\dfrac{A}{2}$

Answer (1)

Sol. KE = potential energy

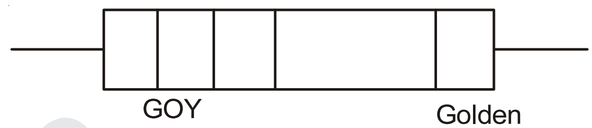
$\Rightarrow \dfrac{1}{2}mv^2 = \dfrac{1}{2}K\,x^2$

$\dfrac{1}{2}m\omega^2(A^2 - x^2) = \dfrac{1}{2}K\,x^2$

$A^2 - x^2 = x^2$

$x = \dfrac{A}{\sqrt{2}}$

12. A carbon resistance has a following colour code. What is the value of the resistance?



(1) 6.4 MΩ ± 5 % (2) 5.3 MΩ ± 5 %

(3) 64 kΩ ± 10 % (4) 530 kΩ ± 5 %

Answer (4)

Sol. G O Y Golden

↓ ↓ ↓

$R = 5 \quad 3 \times 10^4 \pm 5\ \%$

= (530 KΩ ± 5 %)

13. In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accomodated for transmitting TV signals of band width 6 MHz are (take velocity of light $c = 3 \times 10^8$ m/s, h = 6.6×10^{-34} J-s)

(1) 3.75×10^6 (2) 3.86×10^6

(3) 6.25×10^5 (4) 4.87×10^5

Answer (3)

Sol. $\nu = \dfrac{3 \times 10^8}{800} \times 10^9 = \dfrac{3 \times 10^{15}}{8}$

Signal bandwidth = $\dfrac{3 \times 10^{15}}{8} \times 0.01$

No. of channels = $\dfrac{3 \times 10^{13}}{8 \times 6 \times 10^6}$

$= 6.25 \times 10^5$

14. The energy associated with electric field is (U_E) and with magnetic field is (U_B) for an electromagnetic wave in free space, Then

(1) $U_E < U_B$ (2) $U_E = \dfrac{U_B}{2}$

(3) $U_E = U_B$ (4) $U_E > U_B$

Answer (3)

Sol. $U_E = \frac{1}{2}\varepsilon_0 E^2$

$$U_B = \frac{1}{2}\times\frac{B^2}{\mu_0}$$

$$\frac{U_E}{U_B} = \frac{E^2}{B^2}\varepsilon_0\mu_0$$

$$\frac{U_E}{U_B} = c^2\,\varepsilon_0\mu_0$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

15. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is

(1) 3.2×10^3 km (2) 1.6×10^3 km

(3) 1.28×10^4 km (4) 6.4×10^3 km

Answer (1)

Sol. $E_1 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}$

$$E_1 = \frac{GMm}{R} - \frac{GMm}{(R+h)}$$

$$E_1 = \frac{GMmh}{R(R+h)}$$

$$E_2 = \frac{1}{2}\frac{GMm}{(R+h)}$$

Given $E_1 = E_2$

$$\frac{h}{R} = \frac{1}{2}, h = \frac{R}{2}$$

16. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?

(1) 950 J (2) 900 J

(3) 850 J (4) 875 J

Answer (2)

Sol. $V = \frac{dx}{dt} = 6t$

$V(t = 0) = 0$

$V(t = 5\text{ s}) = 30$ m/s

$$\Delta KE = \frac{1}{2}2\times 30^2 = 900\text{ J}$$

17. The magnetic field associated with a light wave is given, at the origin, by $B = B_0\,[\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$. If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons?

($c = 3 \times 10^8$ ms^{-1}, $h = 6.6 \times 10^{-34}$ J-s)

(1) 8.52 eV

(2) 7.72 eV

(3) 12.5 eV

(4) 6.82 eV

Answer (2)

Sol. Maximum Angular Frequency = $6.28 \times 10^7 \times 3 \times 10^8$ rad/s

$\Rightarrow$ $f_{max} = 3 \times 10^{15}$ Hz

$$E_{max} = h\,f_{max} = \frac{6.6\times10^{-34}\times3\times10^{15}}{1.6\times10^{-19}}\text{ eV}$$

$= 12.375$ eV $\simeq$ 12.38 eV

$\Rightarrow$ $KE_{max} = 12.38 - 4.7 \simeq 7.7$ eV

18. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to:

(1) $\frac{a_1+a_2}{2}t$

(2) $\frac{2a_1a_2}{a_1+a_2}t$

(3) $\sqrt{2a_1a_2}\,t$

(4) $\sqrt{a_1a_2}\,t$

Answer (4)

Sol. $t_A = t_B - t$

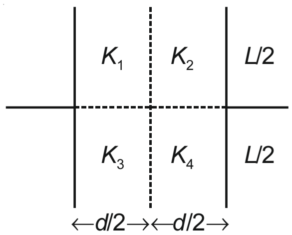
$v_A = a_1(t_B - t) = a_2t_B + v$ ...(i)

$$S = \frac{1}{2}a_1(t_B - t)^2 = \frac{1}{2}a_2\,t_B^2$$

$$\Rightarrow t_B\left[1-\sqrt{\frac{a_2}{a_1}}\right] = t \qquad \text{...(ii)}$$

Solving (i) and (ii) $v = t\sqrt{a_1a_2}$

19. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be:



(1) $K = \dfrac{(K_1+K_2)(K_3+K_4)}{2(K_1+K_2+K_3+K_4)}$

(2) $K = \dfrac{(K_1+K_2)(K_3+K_4)}{K_1+K_2+K_3+K_4}$

(3) $K = \dfrac{(K_1+K_3)(K_2+K_4)}{K_1+K_2+K_3+K_4}$

(4) $K = \dfrac{(K_1+K_4)(K_2+K_3)}{2(K_1+K_2+K_3+K_4)}$

Answer (No option is correct) [Bonus]

Sol. $C_{eq} = \dfrac{C_1C_2}{C_1+C_2} + \dfrac{C_3C_4}{C_3+C_4}$, $C_1 = K_1C$, $C = \dfrac{\epsilon_0 A/2}{d/2}$

Similarly

$C_2 = K_2C$

$C_3 = K_3C$

$C_4 = K_4C$

$$K_{eq.}\left(\frac{\epsilon_0 A}{d}\right) = \left(\frac{K_1K_2}{K_1+K_2} + \frac{K_3K_4}{K_3+K_4}\right)\frac{\epsilon_0 A/2}{d/2}$$

$$\Rightarrow K_{eq.} = \frac{K_1K_2}{K_1+K_2} + \frac{K_3K_4}{K_3+K_4}$$

No option is correct.

20. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.

The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is:

(1) 5.725 mm (2) 5.740 mm

(3) 5.755 mm (4) 5.950 mm

Answer (3)

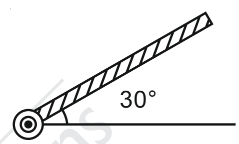
Sol. $LC = \dfrac{0.5}{100} = 0.005$ mm

Zero error, $e = -3 \times 0.005 = -0.015$ mm

Thickness = $(5.5 + 48 \times 0.005 + 0.015)$ mm

= 5.755 mm

21. A rod of length 50 cm is pivoted at one end. It is raised such that it makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be (g = 10 ms^{-2})



(1) $\sqrt{\dfrac{20}{3}}$ (2) $\sqrt{30}$

(3) $\sqrt{\dfrac{30}{2}}$ (4) $\dfrac{\sqrt{30}}{2}$

Answer (2)

Sol. Conservation of mechanical energy

$$\Rightarrow mg\frac{l}{2}\sin 30^\circ = \frac{1}{2}\frac{ml^2}{3}\cdot\omega^2$$

$$\Rightarrow \omega^2 = \frac{3g}{2l} = \frac{30}{1}$$

$$\Rightarrow \omega = \sqrt{30}\text{ rad/s}$$

22. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary winding of the transformer is 5 A and its efficiency is 90%, the output current would be:

(1) 25 A (2) 50 A

(3) 45 A (4) 35 A

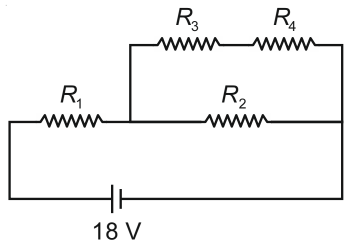
Answer (3)

Sol. $P_{input} = V_p \cdot I_p = 2300 \times 5$ W

$P_{output} = 0.9\, P_{input} = V_sI_s$

$$\Rightarrow I_s = \frac{0.9 \times 2300 \times 5}{230} = 45\text{ A}$$

23. In the given circuit the internal resistance of the 18 V cells is negligible. If R_1 = 400 Ω, R_3 = 100 Ω and R_4 = 500 Ω and the reading of an ideal voltmeter across R_4 is 5 V, then the value of R_2 will be:

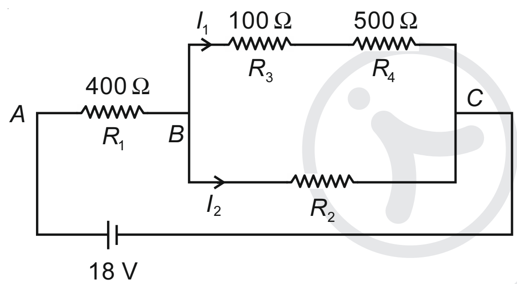


(1) 230 Ω (2) 450 Ω

(3) 550 Ω (4) 300 Ω

Answer (4)

Sol.



$$I_1 = \frac{5}{500} = 0.01\text{ A}$$

$$V_B - V_C = 600\, I_1 = 6\text{ V}$$

$$\Rightarrow \quad V_A - V_B = 12\text{ V}$$

$$\Rightarrow \quad I_1 + I_2 = \frac{12}{400} = 0.03\text{ A}$$

$$\Rightarrow \quad I_2 = 0.03 - 0.01 = 0.02\text{ A}$$

$$\Rightarrow \quad R_2 = \frac{6}{.02} = 300\ \Omega$$

24. Expression for time in terms of G(universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to:

(1) $\sqrt{\frac{Gh}{c^5}}$ (2) $\sqrt{\frac{c^3}{Gh}}$

(3) $\sqrt{\frac{Gh}{c^3}}$ (4) $\sqrt{\frac{hc^5}{G}}$

Answer (1)

Sol. $[T] = [G]^a . [h]^b . [c]^c$

$= [M^{-1}L^3T^{-2}]^a\ [ML^2T^{-1}]^b\ [LT^{-1}]^c$

$-a + b = 0$

$3a + 2b + c = 0 \quad \Rightarrow 5a + c = 0$

$-2a - b - c = 1 \quad \Rightarrow 3a + c = -1$

$$\Rightarrow a = \frac{1}{2}$$

$$b = \frac{1}{2}$$

$$c = \frac{-5}{2}$$

$$\Rightarrow [T] = \sqrt{\frac{Gh}{c^5}}$$

25. A rod of mass 'M' and length '$2L$' is suspended at its middle by a wire. It exhibits torsional oscillations; if two masses each of 'm' are attached at distance '$L/2$' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to:

(1) 0.77 (2) 0.17

(3) 0.37 (4) 0.57

Answer (3)

Sol. $I_1 = \dfrac{M(2L)^2}{12} = \dfrac{ML^2}{3}$

$$I_2 = I_1 + 2\frac{mL^2}{4} = \frac{ML^2}{3} + \frac{mL^2}{2}$$

$$\omega \propto \frac{1}{\sqrt{I}}$$

$$\frac{\omega_1}{\omega_2} = \frac{1}{0.8} = \sqrt{\frac{\frac{M}{3} + \frac{m}{2}}{\frac{M}{3}}}$$

$$\Rightarrow \frac{m}{M} = 0.375$$

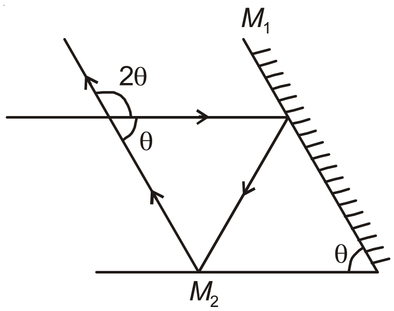
26. Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirrors willl be:

(1) 75° (2) 45°

(3) 90° (4) 60°

Answer (4)

Sol.



$3\theta = 180°$

$\Rightarrow \theta = 60°$

27. Charge is distributed within a sphere of radius R with volume charge density $\rho(r) = \frac{A}{r^2}e^{\frac{-2r}{a}}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is:

(1) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

(2) $\frac{a}{2}\log\left(1-\frac{Q}{2\pi aA}\right)$

(3) $a\log\left(1-\frac{Q}{2\pi aA}\right)$

(4) $a\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

Answer (1)

Sol. $Q = \int_0^R 4\pi r^2 \cdot \frac{A}{r^2}e^{-2r/a}\,dr$

$= \frac{4\pi Aa}{-2}e^{-2r/a}\Big|_0^R = 2\pi aA\left[1-e^{\frac{-2R}{a}}\right]$

$= e^{-2R/a} = 1-\frac{Q}{2\pi aA}$

$\Rightarrow e^{\frac{2R}{a}} = \frac{1}{\left(1-\frac{Q}{2\pi aA}\right)}$

$\Rightarrow R = \frac{a}{2}\ln\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

28. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron = 1.6×10^{-19} C)

(1) 9.1×10^{-31} kg
(2) 1.6×10^{-27} kg
(3) 1.6×10^{-19} kg
(4) 2.0×10^{-24} kg

Answer (4)

Sol. $eE = eVB$

$R = \frac{mV}{eB} \Rightarrow V = \frac{ReB}{m}$

$\Rightarrow E = \frac{ReB}{m}\cdot B \Rightarrow m = \frac{eB^2R}{E}$

$m = \frac{1.6\times10^{-19}\times(0.5)^2\times0.5\times10^{-2}}{100}$

$= 2.0 \times 10^{-24}$ kg

29. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to:

(1) 9.6 m
(2) 2.9 m
(3) 4.8 m
(4) 6.0 m

Answer (3)

Sol. Volume Flow Rate = $\frac{0.74}{60}\,m^3/s$

Speed of efflux = $\frac{0.74\times10^4}{60\times\pi\times4}$ m/s $= \sqrt{2gh}$

$\Rightarrow 9.82 = \sqrt{2\times10\times h}$

$\Rightarrow h = 4.8$ m

30. The position co-ordinates of a particle moving in a 3-D coordinate system is given by $x = a\cos\omega t$
$y = a\sin\omega t$
and $z = a\omega t$

The speed of the particle is:

(1) $2a\omega$
(2) $\sqrt{2}a\omega$
(3) $\sqrt{3}a\omega$
(4) $a\omega$

Answer (2)

Sol. $v_x = -a\omega\sin\omega t$

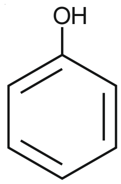
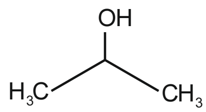
$v_y = a\omega\cos\omega t$

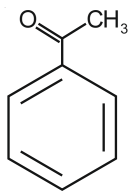
$v_z = a\omega$

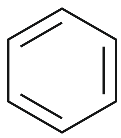
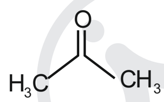
$v = \sqrt{v_x^2+v_y^2+v_z^2} = \sqrt{2}\,a\omega$

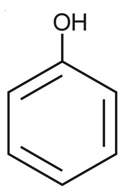
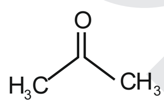
CHEMISTRY

1. The products formed in the reaction of cumene with O_2 followed by treatment with dil. HCl are :

(1)  and 

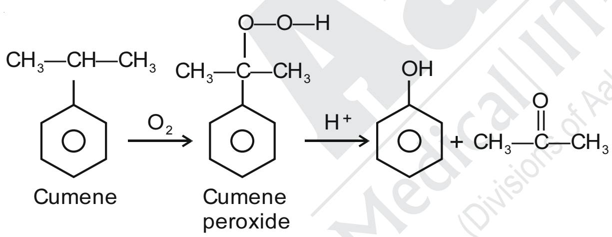
(2)  and CH_3—OH

(3)  and 

(4)  and 

Answer (4)

Sol.



2. The correct match between Item I and Item II is :

Item I	Item II
(A) Benzaldehyde	(P) Mobile phase
(B) Alumina	(Q) Adsorbent
(C) Acetonitrile	(R) Adsorbate

(1) (A) → (Q), (B) → (R), (C) → (P)

(2) (A) → (Q), (B) → (P), (C) → (R)

(3) (A) → (P), (B) → (R), (C) → (Q)

(4) (A) → (R), (B) → (Q), (C) → (P)

Answer (4)

Sol. Alumina is an adsorbent (stationary phase)

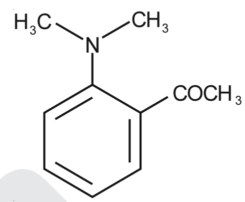
Benzaldehyde is adsorbate.

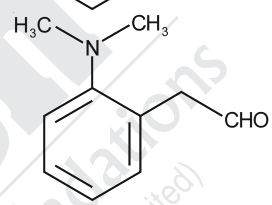
Acetonitrile is mobile phase.

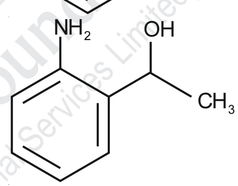
3. The tests performed on compound X and their inferences are :

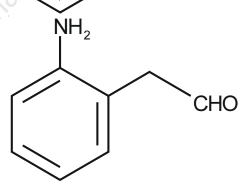
Test	Inference
(a) 2,4-DNP test	Coloured precipitate
(b) Iodoform test	Yellow precipitate
(c) Azo-dye test	No dye formation

Compound 'X' is :

(1) 

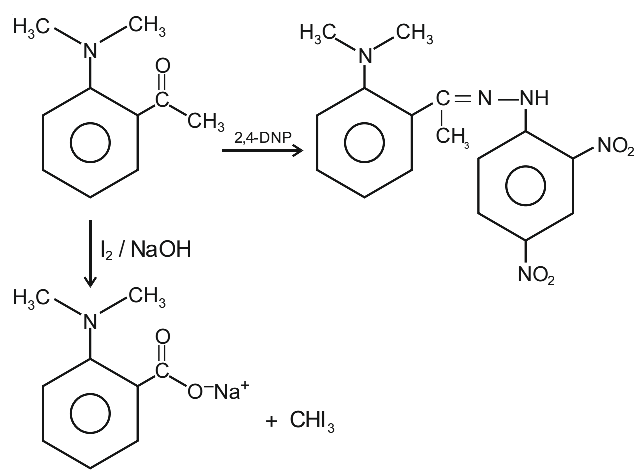
(2) 

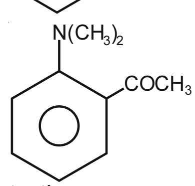
(3) 

(4) 

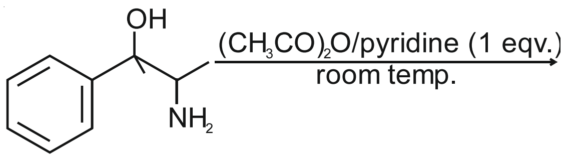
Answer (1)

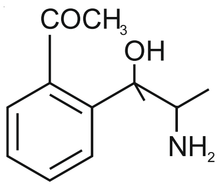
Sol.

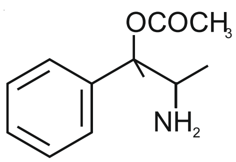


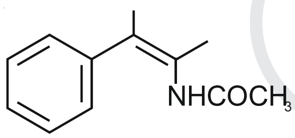
 will not give positive dye test, due to the presence of an electron withdrawing group.

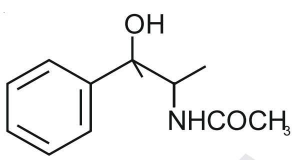
4. The major product obtained in the following reaction is



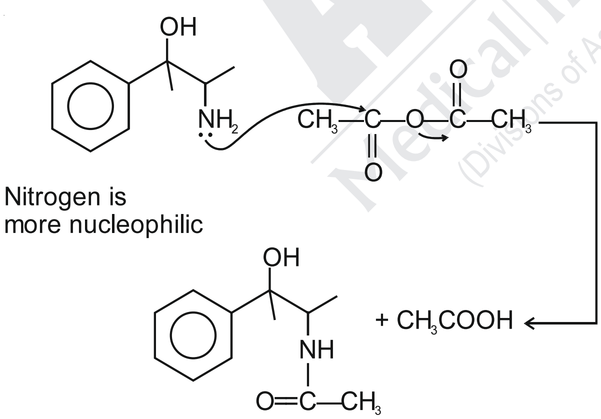
(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. 

5. Consider the following reversible chemical reactions

$A_2(g)+B_2(g) \xrightleftharpoons{K_1} 2AB(g)$ ...(1)

$6AB(g) \xrightleftharpoons{K_2} 3A_2(g)+3B_2(g)$ ...(2)

The relation between K_1 and K_2 is

(1) $K_2 = K_1^3$ (2) $K_1K_2 = \frac{1}{3}$

(3) $K_2 = K_1^{-3}$ (4) $K_1K_2 = 3$

Answer (3)

Sol. $A_2 + B_2 \xrightleftharpoons{K_1} 2AB$

$2AB \rightleftharpoons A_2 + B_2 \quad ; \quad K = \frac{1}{K_1}$

$6AB \xrightleftharpoons{K_2} 3A_2 + 3B_2 \quad ; \quad K_2 = \left(\frac{1}{K_1}\right)^3$

6. Which of the following conditions in drinking water causes methemoglobinemia?

(1) > 50 ppm of nitrate (2) > 50 ppm of lead

(3) > 50 ppm of chloride (4) > 100 ppm of sulphate

Answer (1)

Sol. Methemoglobinemia is caused by drinking water which is contaminated with nitrate.

7. The metal that forms nitride by reacting directly with N_2 of air, is

(1) Li (2) Rb

(3) Cs (4) K

Answer (1)

Sol. Only lithium react with N_2 among alkali metals

8. The entropy change associated with the conversion of 1 kg of ice at 273 K to water vapours at 383 K is (Specific heat of water liquid and water vapour are 4.2 kJ K^{-1} kg^{-1} and 2.0 kJ K^{-1} kg^{-1}; heat of liquid fusion and vapourisation of water are 334 kJ kg^{-1} and 2491 kJ kg^{-1}, respectively). (log 273 = 2.436, log 373 = 2.572, log 383 = 2.583)

(1) 8.49 kJ kg^{-1} K^{-1} (2) 7.90 kJ kg^{-1} K^{-1}

(3) 9.26 kJ kg^{-1} K^{-1} (4) 2.64 kJ kg^{-1} K^{-1}

Answer (3)

Sol. $\Delta S_{fus} = \frac{\Delta H_{fus}}{273} = \frac{334}{273} = 1.22$

$\Delta S_{vap} = \frac{\Delta H_{vap}}{373} = \frac{2491}{373} = 6.67$

$\Delta S_{water} = \frac{mCdT}{T} = mC\ln\left(\frac{T_2}{T_1}\right)$

$= 4.2 \times \ln\left(\frac{373}{273}\right) = 1.31$

$\Delta S_{vap} = mC\ln\left(\frac{T_2}{T_1}\right)$

$= 2 \times \ln\left(\frac{383}{373}\right) = 0.05$

Total entropy change

ΔS = 9.26 kJ kg^{-1} K^{-1}

9. The transition element that has lowest enthalpy of atomisation, is

(1) V (2) Cu

(3) Fe (4) Zn

Answer (4)

Sol. Zinc has least enthalpy of atomisation in 3d-transition series.

10. If the standard electrode potential for a cell is 2 V at 300 K, the equilibrium constant (K) for the reaction $Zn(s) + Cu^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Cu(s)$ at 300 K is approximately

(R = 8 $JK^{-1}mol^{-1}$, F = 96000 $Cmol^{-1}$)

(1) e^{320} (2) e^{160}

(3) e^{-160} (4) e^{-80}

Answer (2)

Sol. $\ln K = \dfrac{nFE^{\circ}_{Cell}}{RT}$

$\ln K = \dfrac{2 \times 96000 \times 2}{8 \times 300}$

$= 160$

$K = e^{160}$

11. For coagulation of arsenious sulphide sol, which one of the following salt solution will be most effective?

(1) Na_3PO_4 (2) $AlCl_3$

(3) NaCl (4) $BaCl_2$

Answer (2)

Sol. Arsenious sulphide sol is negatively charged, so according to Hardy-Schulze rule the cation which is in more oxidation state will be most effective

12. In which of the following processes, the bond order has increased and paramagnetic character has changed to diamagnetic?

(1) $N_2 \to N_2^+$ (2) $O_2 \to O_2^+$

(3) $O_2 \to O_2^{2-}$ (4) $NO \to NO^+$

Answer (4)

Sol. N_2 Diamagnetic $\to$ N_2^+ (Paramagnetic)

O_2 Paramagnetic $\to$ O_2^+ (Paramagnetic)

O_2 Paramagnetic $\to$ O_2^{2-} (Diamagenetics but bond order decreases from 2 to 1)

NO (Paramagnetic $\to$ NO^+ (Diamagnetic, bond order bond order 2.5) 3)

13. When the first electron gain enthalpy ($\Delta_{eg}H$) of oxygen is –141 kJ/mol, its second electron gain enthalpy is

(1) Almost the same as that of the first

(2) A more negative value than the first

(3) Negative, but less negative than the first

(4) A positive value

Answer (4)

Sol. Second electron gain enthalpy of oxygen is positive.

14. The pH of rain water, is approximately

(1) 7.0 (2) 7.5

(3) 5.6 (4) 6.5

Answer (3)

Sol. pH of rain water is around 5.6.

15. A solution containing 62 g ethylene glycol in 250 g water is cooled to –10°C. If K_f for water is 1.86 K kg mol^{-1}, the amount of water (in g) separated as ice is

(1) 64 (2) 32

(3) 16 (4) 48

Answer (1)

Sol. $\Delta T_f = K_f \times m$

$\Rightarrow 10 = 1.86 \times \dfrac{62 \times 1000}{62 \times w_A}$

$\Rightarrow w_A = \dfrac{1.86 \times 1000}{10}$

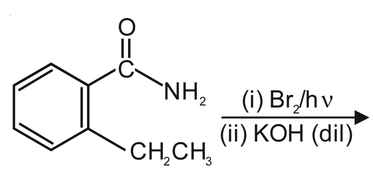
$= 186$ g

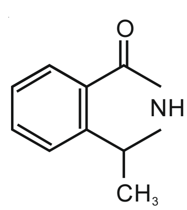
Given amount of H_2O is 250 g

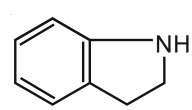
$\therefore$ The amount of water separated as ice

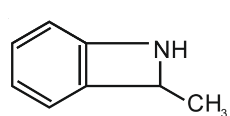
= 250 – 186 = 64 g

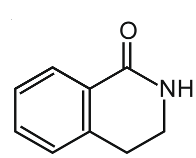
16. The major product of the following reaction



(1) 

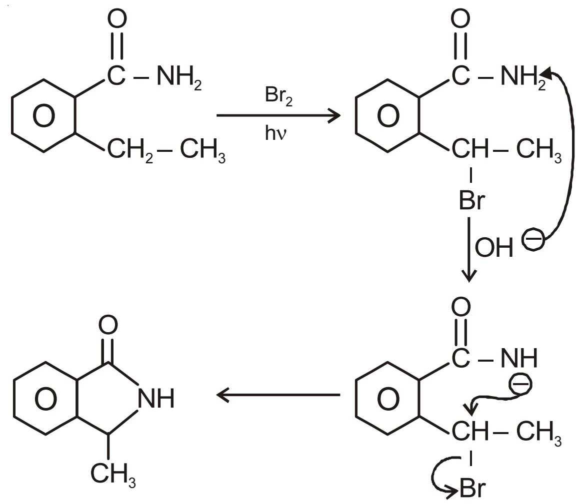
(2) 

(3) 

(4) 

Answer (1)

Sol.



17. The temporary hardness of water is due to

(1) $CaCl_2$

(2) NaCl

(3) Na_2SO_4

(4) $Ca(HCO_3)_2$

Answer (4)

Sol. Bicarbonates cause temporary hardness. Chlorides and sulphates cause permanent hardness.

18. At 100°C, copper (Cu) has FCC unit cell structure with cell edge length of x Å. What is the approximate density of Cu (in g cm^{-3}) at this temperature?

[Atomic Mass of Cu = 63.55 u]

(1) $\frac{422}{x^3}$ (2) $\frac{205}{x^3}$

(3) $\frac{105}{x^3}$ (4) $\frac{211}{x^3}$

Answer (1)

Sol. Density = $\frac{Z(M_0)}{N_A \times a^3}$

Z = 4 (FCC)

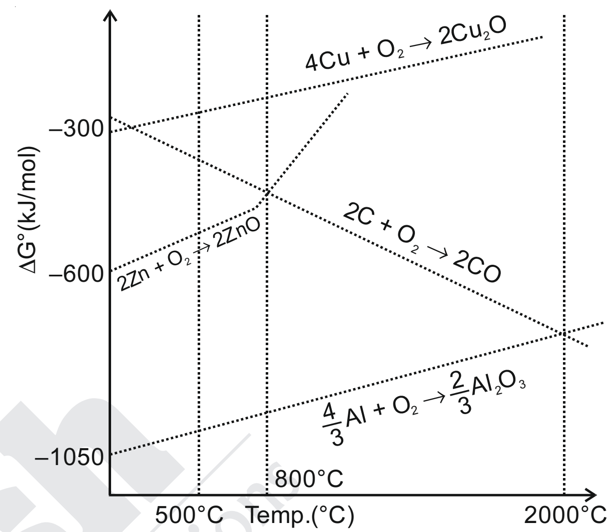
M_0= 63.5 g

$N_A = 6 \times 10^{23}$

$a = x \times 10^{-8}$ cm.

$\therefore \quad d = \frac{4 \times 63.5}{6 \times 10^{23} \times x^3 \times 10^{-24}}; \frac{422}{x^3} \frac{g}{cm^3}$

19. The correct statement regarding the given Ellingham diagram is



(1) At 800°C, Cu can be used for the extraction of Zn from ZnO

(2) At 500ºC, coke can be used for the extraction of Zn from ZnO

(3) At 1400°C, Al can be used for the extraction of Zn from ZnO

(4) Coke cannot be used for the extraction of Cu from Cu_2O

Answer (3)

Sol. In the Ellingham diagram, the metal which has a lower value of ΔG° (more negative) can reduce a metal oxide whose curve lies above it

so, Al can reduce ZnO at 1400°C

20. Homoleptic octahedral complexes of a metal ion 'M^{3+}' with three monodentate ligands L_1, L_2 and L_3 absorb wavelengths in the region of green, blue and red respectively. The increasing order of the ligand strength is:

(1) $L_1 < L_2 < L_3$ (2) $L_3 < L_2 < L_1$

(3) $L_3 < L_1 < L_2$ (4) $L_2 < L_1 < L_3$

Answer (3)

Sol. Greater the energy or lesser the wavelength of light absorbed, greater is the ligand strength

Energy : Blue > Green > Red
L_2 L_1 L_3

So, ligand strength : $L_2 > L_1 > L_3$

21. The complex that has highest crystal field splitting energy (Δ), is

(1) $[Co(NH_3)_5Cl]Cl_2$

(2) $K_2[CoCl_4]$

(3) $K_3[Co(CN)_6]$

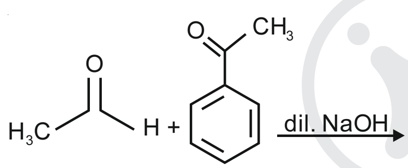
(4) $[Co(NH_3)_5(H_2O)]Cl_3$

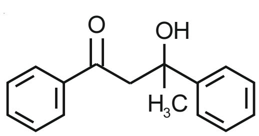
Answer (3)

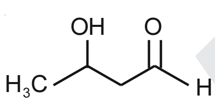
Sol. For the same metal ion, greater the co-ordination number and greater the strength of the ligands, greater is the value of crystal field splitting energy

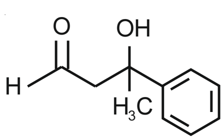
$^{-}CN > NH_3 > H_2O > Cl^{-}$

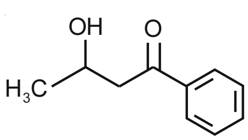
22. The major product formed in the following reaction is:



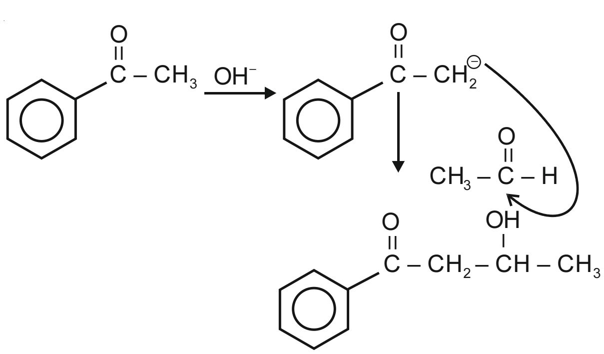
(1) 

(2) 

(3) 

(4) 

Answer (4)

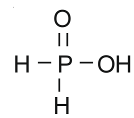
Sol. 

23. Good reducing nature of H_3PO_2 is attributed to the presence of:

(1) Two P – OH bonds (2) One P – H bond

(3) One P – OH bond (4) Two P – H bonds

Answer (4)

Sol. 

Greater the number of P–H bonds in acids of phosphorous, greater is the reducing property.

24. For the reaction, 2A + B $\rightarrow$ products, when the concentration of A and B both were doubled, the rate of the reaction increased from 0.3 mol $L^{-1}s^{-1}$ to 2.4 mol $L^{-1}s^{-1}$. When the concentration of A alone is doubled, the rate increased from 0.3 mol $L^{-1}s^{-1}$ to 0.6 mol $L^{-1}s^{-1}$. Which one of the following statements is correct

(1) Order of the reaction with respect to A is 2

(2) Order of the reaction with respect to B is 1

(3) Order of the reaction with respect to B is 2

(4) Total order of the reaction is 4

Answer (3)

Sol. $r = K[A]^x [B]^y$

$$\frac{r_2}{r_1} = 2^x . 2^y = 8 \Rightarrow x + y = 3$$

$$\frac{r_3}{r_1} = 2^x = 2 \Rightarrow x = 1$$

$\therefore$ y = 2

25. The increasing basicity order of the following compounds is:

(A) $CH_3CH_2NH_2$

(B) $CH_3CH_2NH(CH_2CH_3)$

(C) $H_3C - N(CH_3) - CH_3$

(D) $Ph - N(CH_3) - H$

(1) (D) < (C) < (B) < (A)

(2) (A) < (B) < (C) < (D)

(3) (A) < (B) < (D) < (C)

(4) (D) < (C) < (A) < (B)

Answer (4)

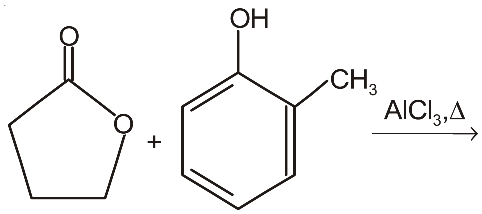
Sol. pK_b values from NCERT

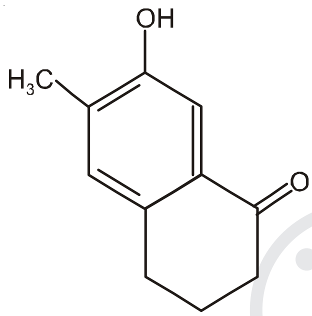
(A)	$EtNH_2$	3.29
(B)	$(Et_2)NH$	3.00
(C)	Me_3N	4.22
(D)	Ph – NH – Me	4.7

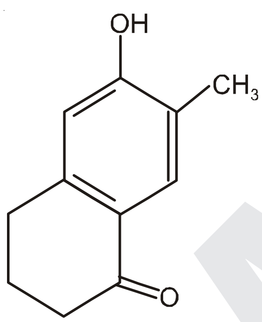
So, order of basic strength

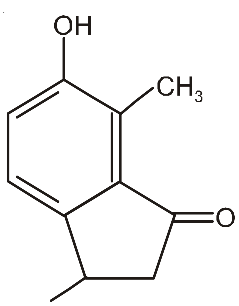
(B) > (A) > (C) > (D)

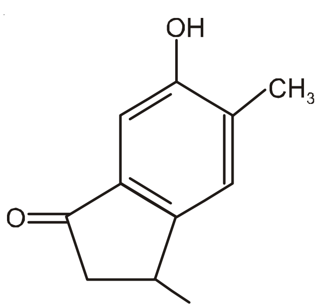
26. The major product of the following reaction is:



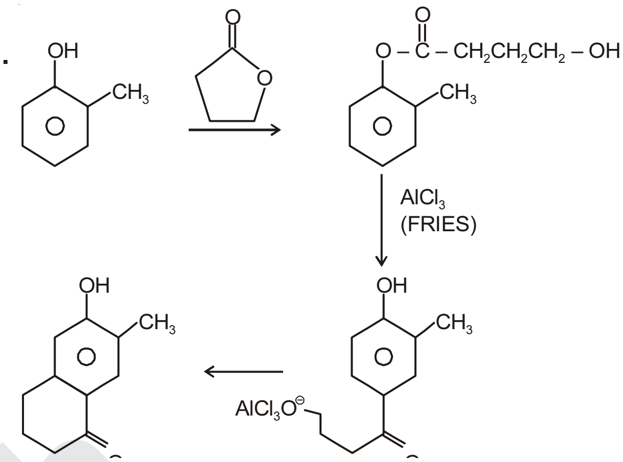
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

27. For the following reaction, the mass of water produced from 445 g of $C_{57}H_{110}O_6$ is:

$$2C_{57}H_{110}O_6(s) + 163O_2(g) \rightarrow 114CO_2(g) + 110H_2O(l)$$

(1) 890 g (2) 490 g

(3) 445 g (4) 495 g

Answer (4)

Sol. $2C_{57}H_{110}O_6(s) + 163O_2(g) \rightarrow 114CO_2(g) + 110H_2O(l)$

$$n = \frac{445}{890}$$

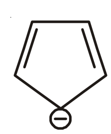
$$= 0.5$$

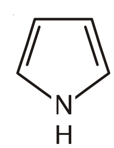
$\therefore$ Moles of water $= \frac{110}{2} \times 0.5 = 27.5$

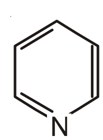
$\therefore$ Mass of water = 27.5 × 18

= 495 g

28. Which of the following compounds is not aromatic?

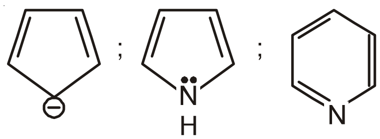
(1) 

(2) 

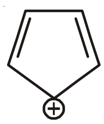
(3) 

(4) 

Answer (4)

Sol. 

Contain $6\pi e^-$ in complete conjugation and are aromatic.

 is anti-aromatic as it has $4\pi e^-$ in complete conjugation.

29. Which of the following combination of statements is true regarding the interpretation of the atomic orbitals?

(a) An electron in an orbital of high angular momentum stays away from the nucleus than an electron in the orbital of lower angular momentum

(b) For a given value of the principal quantum number, the size of the orbit is inversely proportional to the azimuthal quantum number.

(c) According to wave mechanics, the ground state angular momentum is equal to $\frac{h}{2\pi}$

(d) The plot of ψ Vs r for various azimuthal quantum numbers, shows peak shifting towards higher r value.

(1) (a), (d)

(2) (b), (c)

(3) (a), (c)

(4) (a), (b)

Answer (1)

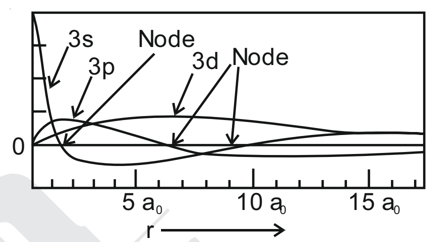
Sol. (a) Angular momentum (L) = $\frac{nh}{2\pi}$

So, as n increases, L increases.

(b) $r \propto \frac{n^2}{z}$

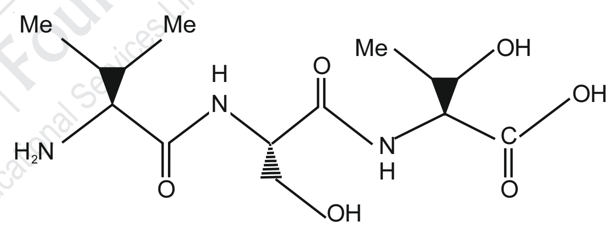
(c) For n = 1, L = $\frac{h}{2\pi}$

(d) As l increases , the peak of ψ vs r shifts towards higher 'r' value.



30. The correct sequnce of amino acids present in the tripeptide given below is :

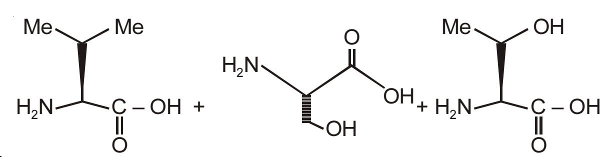
The given tripeptide contains.



(1) Leu - Ser - Thr (2) Thr - Ser - Val

(3) Val - Ser - Thr (4) Thr - Ser - Leu

Answer (3)

Sol. 

VALINE (Val) SERINE (Ser) THREONINE (Thr)

❑ ❑ ❑

MATHEMATICS

1. If $f(x) = \int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx, (x \geq 0)$, and $f(0) = 0$, then the value of $f(1)$ is

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $-\frac{1}{2}$ (4) $-\frac{1}{4}$

Answer (2)

Sol. $f(x) = \int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx,\ x \geq 0$

$= \int \frac{5x^8+7x^6}{x^{14}(x^{-5}+x^{-7}+2)^2}dx$

$= \int \frac{5x^{-6}+7x^{-8}}{(2+x^{-7}+x^{-5})^2}dx$

Let $2+x^{-7}+x^{-5} = t$

$(-7x^{-8} - 5x^{-6})dx = dt$

$f(x) = \int \frac{-dt}{t^2} = \int -t^{-2}dt = t^{-1}+c$

$f(x) = \frac{1}{2+x^{-7}+x^{-5}}+c$, $f(0) = 0 \Rightarrow c = 0$

$\therefore\ f(1) = \frac{1}{4}$

2. Let $f : [0, 1] \to R$ be such that $f(xy) = f(x).f(y)$, for all $x, y \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to

(1) 4 (2) 3

(3) 2 (4) 5

Answer (2)

Sol. $f(xy) = f(x).f(y)$ ...(1)

Put $x = y = 0$ in (1) to get $f(0) = 1$

Put $x = y = 1$ in (1) to get $f(1) = 0$ or $f(1) = 1$

$f(1) = 0$ is rejected else $y = 1$ in (1) gives $f(x) = 0$ imply $f(0) = 0$.

Hence, $f(0) = 1$ and $f(1) = 1$

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} f(x)\left(\frac{f\left(1+\frac{h}{x}\right)-f(1)}{h}\right)$

$= \frac{f(x)}{x}f'(1)$

$\Rightarrow \frac{f'(x)}{f(x)} = \frac{k}{x} \Rightarrow \ln f(x) = k \ln x + c$

$f(1) = 1 \Rightarrow \ln 1 = k \ln 1 + c \Rightarrow c = 0$

$\Rightarrow \ln f(x) = k \ln x \Rightarrow f(x) = x^k$ but $f(0) = 1$

$\Rightarrow k = 0$

$\therefore\ \boxed{f(x) = 1}$

$\frac{dy}{dx} = f(x) = 1 \Rightarrow y = x + c$, $y(0) = 1 \Rightarrow c = 1$

$\Rightarrow y = x + 1$

$\therefore\ y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right) = \frac{1}{4}+1+\frac{3}{4}+1 = 3$

3. If the lines $x = ay + b$, $z = cy + d$ and $x = a'z + b'$, $y = c'z + d'$ are perpendicular, then

(1) $ab' + bc' + 1 = 0$ (2) $cc' + a + a' = 0$

(3) $aa' + c + c' = 0$ (4) $bb' + cc' + 1 = 0$

Answer (3)

Sol. First line is : $x = ay + b$, $z = cy + d$

$\Rightarrow \frac{x-b}{a} = y = \frac{z-d}{c}$

and another line is: $x = a'z + b'$, $y = c'z + d'$

$\Rightarrow \frac{x-b'}{a'} = \frac{y-d'}{c'} = z$

$\because$ both lines are perpendicular to each other

$\therefore\ aa' + c' + c = 0$

4. If $0 \leq x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is

(1) 2 (2) 3

(3) 1 (4) 4

Answer (1)

Sol. $\sin x - \sin 2x + \sin 3x = 0$

$\sin x - 2 \sin x.\cos x + 3 \sin x - 4 \sin^3 x = 0$

$4 \sin x - 4 \sin^3 x - 2 \sin x.\cos x = 0$

$2 \sin x(1 - \sin^2 x) - \sin x.\cos x = 0$

$2 \sin x.\cos^2 x - \sin x.\cos x = 0$

$\sin x.\cos x(2 \cos x - 1) = 0$

$\therefore \quad \sin x = 0, \cos x = 0, \cos x = \frac{1}{2}$

$\therefore \quad x = 0, \frac{\pi}{3} \qquad \because \quad x \in \left[0, \frac{\pi}{2}\right)$

5. If the system of linear equations

$x - 4y + 7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then

(1) $g + h + k = 0$

(2) $g + 2h + k = 0$

(3) $g + h + 2k = 0$

(4) $2g + h + k = 0$

Answer (4)

Sol. $\because \quad x - 4y + 7z = g$ ...(i)

$3y - 5z = h$ ...(ii)

$-2x + 5y - 9z = k$ ...(iii)

from 2 (equation (i)) + equation (ii) + equation (iii):

$0 = 2g + h + k.$

$\therefore \quad 2g + h + k = 0$

then system of equation is consistent.

6. The logical statement

$[\sim (\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$ is equivalent to

(1) $(p \wedge r) \wedge \sim q$

(2) $(p \wedge \sim q) \vee r$

(3) $(\sim p \wedge \sim q) \wedge r$

(4) $\sim p \vee r$

Answer (1)

Sol. $[\sim (\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

$= [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

$= [(p \wedge \sim q) \wedge (\sim q \wedge r)] \vee [(p \wedge r) \wedge (\sim q \wedge r)]$

$= [p \wedge \sim q \wedge r] \vee [p \wedge r \wedge \sim q]$

$= (p \wedge \sim q) \wedge r$

$= (p \wedge r) \wedge \sim q$

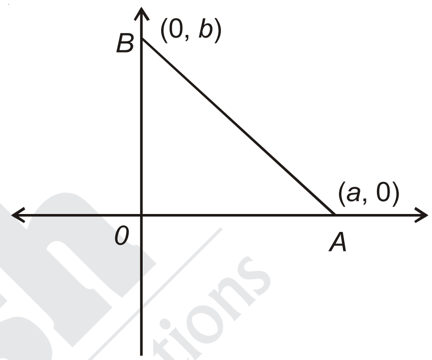
7. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is:

(1) 9 (2) 32

(3) 36 (4) 18

Answer (3)

Sol. One of the possible ΔOAB is $A(a, 0)$ and $B(0, b)$.



Area of $\Delta OAB = \frac{1}{2}|ab|$.

$\therefore \quad |ab| = 100$

$|a|\,|b| = 100$

but $100 = 1 \times 100, 2 \times 50, 4 \times 25, 5 \times 20$ or 10×10

$\therefore \quad$ For 1×100, $a = 1$ or -1 and $b = 100$ or -100

$\therefore \quad$ Total possible pairs are 8.

and for 10×10 total possible pairs are 4.

$\therefore \quad$ Total number of possible triangles with integral coordinates are $4 \times 8 + 4 = 36$.

8. Let f be a differentiable function from **R** to **R** such that $|f(x) - f(y)| \le 2|x - y|^{\frac{3}{2}}$, for all $x, y \in$ **R**. If $f(0) = 1$ then $\int_0^1 f^2(x)dx$ equal to

(1) 1 (2) 0

(3) $\frac{1}{2}$ (4) 2

Answer (1)

Sol. $\because \quad f : R \to R$

and $|f(x) - f(y)| \le 2 \cdot |x - y|^{3/2}$

$\Rightarrow \quad \left|\frac{f(x) - f(y)}{x - y}\right| \le 2\sqrt{x - y}$

$\Rightarrow \quad \lim_{x \to y}\left|\frac{f(x) - f(y)}{x - y}\right| \le \lim_{x \to y} 2\sqrt{x - y}$

$\Rightarrow \quad |f'(x)| = 0$

$\therefore \quad f(x)$ is a constant function.

$\because \quad f(0) = 1 \qquad \Rightarrow f(x) = 1$

$$\therefore \quad \int_0^1 f^2(x)dx = \int_0^1 1dx = [x]_0^1 = 1$$

9. If $x = \sin^{-1}(\sin 10)$ and $y = \cos^{-1}(\cos 10)$, then $y - x$ is equal to

(1) 7π (2) 10

(3) 0 (4) π

Answer (4)

Sol. $x = \sin^{-1}(\sin 10)$

$$x = 3\pi - 10 \qquad \begin{cases} 3\pi - \frac{\pi}{2} < 10 < 3\pi + \frac{\pi}{2} \\ \Rightarrow 3\pi - x \end{cases}$$

$$\text{and } y = \cos^{-1}(\cos 10) \qquad \begin{cases} 3\pi < 10 < 4\pi \\ \Rightarrow 4\pi - x \end{cases}$$

$y = 4\pi - 10$

$\therefore \quad y - x = (4\pi - 10) - (3\pi - 10) = \pi$

10. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5] then m lies in the interval:

(1) (–5, –4) (2) (3, 4)

(3) (4, 5) (4) (5, 6)

Answer (3)

Sol. Given quadratic equation is : $x^2 - mx + 4 = 0$

Both the roots are real and distinct.

$\therefore \quad m^2 - 4 \cdot 1 \cdot 4 > 0$

$\therefore \quad (m - 4)(m + 4) > 0$

$\therefore \quad m \in (-\infty, -4) \cup (4, \infty)$ ...(i)

$\because$ both roots lies in [1, 5]

$$\therefore \quad -\frac{-m}{2} \in (1, 5)$$

$\Rightarrow \quad m \in (2, 10)$ ...(ii)

and $1 \cdot (1 - m + 4) > 0 \quad \Rightarrow m < 5$

$\therefore \quad m \in (-\infty, 5)$ ...(iii)

and $1 \cdot (25 - 5m + 4) > 0 \quad \Rightarrow m < \frac{29}{5}$

$$\therefore \quad m \in \left(-\infty, \frac{29}{5}\right) \qquad \text{...(iv)}$$

From (i), (ii), (iii) and (iv), $m \in (4, 5)$

11. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to

(1) 0 (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

Answer (3)

Sol. $\because \quad z_0$ is a root of quadratic equation

$x^2 + x + 1 = 0$

$\therefore \quad z_0 = \omega$ or $\omega^2 \Rightarrow z_0^3 = 1$

$\therefore \quad z = 3 + 6i\, z_0^{81} - 3i\, z_0^{93}$

$= 3 + 6i - 3i$

$= 3 + 3i$

$$\therefore \quad \arg(z) = \tan^{-1}\left(\frac{3}{3}\right) = \frac{\pi}{4}$$

12. If

$$A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix},$$

then A is

(1) Invertible only if $t = \pi$

(2) Invertible for all $t \in \mathbf{R}$.

(3) Invertible only if $t = \frac{\pi}{2}$

(4) Not invertible for any $t \in \mathbf{R}$.

Answer (2)

Sol. det(A) = |A|

$$= \begin{vmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{vmatrix}$$

$$= e^t \cdot e^{-t} \cdot e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$$= e^{-t} \begin{vmatrix} 0 & 2\cos t + \sin t & 2\sin t - \cos t \\ 0 & -\cos t - 3\sin t & -\sin t + 3\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 + R_3 \end{matrix}$$

$$= e^{-t} \begin{vmatrix} 0 & -5\sin t & 5\cos t \\ 0 & -\cos t - 3\sin t & -\sin t + 3\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix} R_1 \to R_1 + 2R_2$$

$= 5e^{-t} \neq 0, \forall t \in R$

$\therefore \quad A$ is invertible

13. Let $A = \{x \in R : x$ is not a positive integer$\}$. Define a function $f : A \to R$ as $f(x) = \frac{2x}{x-1}$, then f is

(1) Injective but not surjective

(2) Neither injective nor surjective

(3) Surjective but not injective

(4) Not injective

Answer (1)

Sol. As $A = \{x \in R : x$ is not a positive integer$\}$

$f : A \to R$ given by $f(x) = \frac{2x}{x-1}$

$f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$

So, f is one-one.

As $f(x) \neq 2$ for any $x \in A \Rightarrow f$ is not onto.

$\therefore$ f is injective but not surjective.

14. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is

(1) 15 (2) 14

(3) 12 (4) 10

Answer (1)

Sol. $\left(\frac{1-t^6}{1-t}\right)^3 = (1-t^6)^3(1-t)^{-3}$

$$= (1-3t^6+3t^{12}-t^{18})\left(1+3t+\frac{3\cdot 4}{2!}t^2 + \frac{3\cdot 4\cdot 5}{3!}t^3 + \frac{3\cdot 4\cdot 5\cdot 6}{4!}t^4 + \ldots\infty\right)$$

$\therefore$ Coefficient of $t^4 = 1\cdot\frac{3\cdot 4\cdot 5\cdot 6}{4!}$

$$= \frac{3\times 4\times 5\times 6}{4\times 3\times 2\times 1}$$

$= 15$

15. For each $x \in R$, let $[x]$ be the greatest integer less than or equal to x. Then $\lim\limits_{x\to 0^-}\frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to

(1) $-\sin 1$

(2) 1

(3) $\sin 1$

(4) 0

Answer (1)

Sol. $\lim\limits_{x\to 0^-}\frac{x([x]+|x|)\cdot\sin[x]}{|x|}$

$$= \lim_{h\to 0}\frac{(0-h)([0-h]+|0-h|)\cdot\sin[0-h]}{|0-h|}$$

$$= \lim_{h\to 0}\frac{(-h)(-1+h)\sin(-1)}{h}$$

$$= \lim_{h\to 0}(1-h)\sin(-1)$$

$= -\sin 1$

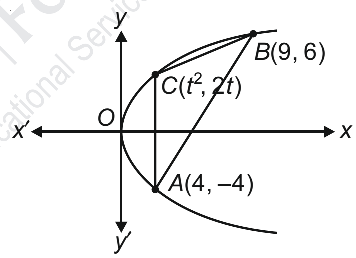
16. Let $A(4, -4)$ and $B(9, 6)$ be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is

(1) 32 (2) $31\frac{3}{4}$

(3) $31\frac{1}{4}$ (4) $30\frac{1}{2}$

Answer (3)

Sol.



Let the coordinates of C is $(t^2, 2t)$.

$\therefore$ Area of ΔACB

$$= \frac{1}{2}\left\|\begin{matrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{matrix}\right\|$$

$$= \frac{1}{2}|t^2(6+4)-2t(9-4)+1(-36-24)|$$

$$= \frac{1}{2}|10t^2-10t-60|$$

$= 5|t^2 - t - 6|$

$= 5\left|\left(t-\frac{1}{2}\right)^2-\frac{25}{4}\right|$ [Here, $t \in (0, 3)$]

For maximum area, $t = \frac{1}{2}$

$\therefore$ Maximum area $= \frac{125}{4} = 31\frac{1}{4}$ sq. units

17. If $\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{2k\sec\theta}}\,d\theta = 1-\frac{1}{\sqrt{2}}, (k>0)$, then the value of k is

(1) 4 (2) 2

(3) 1 (4) $\frac{1}{2}$

Answer (2)

Sol. $I = \int_0^{\pi/3} \frac{\tan\theta}{\sqrt{2k\sec\theta}}\,d\theta$

$= \frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\sin\theta}{\sqrt{\cos\theta}}\,d\theta$

Let $\cos\theta = t^2$

$\therefore$ $\sin\theta\, d\theta = -2t\, dt$

$= \frac{1}{\sqrt{2k}}\int_1^{\frac{1}{\sqrt{2}}}\frac{-2t\,dt}{t}$

$= \sqrt{\frac{2}{k}}\int_{\frac{1}{\sqrt{2}}}^{1} dt$

$= \sqrt{\frac{2}{k}}\left(1-\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{2}-1}{\sqrt{k}}$

$= 1-\frac{1}{\sqrt{2}}$ (Given)

$\therefore$ $k = 2$

18. The sum of the following series

$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+...+5^2)}{11}+...$$

up to 15 terms, is

(1) 7830

(2) 7820

(3) 7520

(4) 7510

Answer (2)

Sol. $S = 1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+3^2+4^2+5^2)}{11}+...$

$S = \frac{3\cdot(1)^2}{3}+\frac{6\cdot(1^2+2^2)}{5}+\frac{9\cdot(1^2+2^2+3^2)}{7}+\frac{12\cdot(1^2+2^2+3^2+4^2)}{9}+...$

n^{th} term of the series

$= t_n = \frac{3n\cdot(1^2+2^2+...+n^2)}{(2n+1)}$

$t_n = \frac{3n\cdot n(n+1)(2n+1)}{6(2n+1)} = \frac{n^3+n^2}{2}$

$\therefore$ $S_n = \Sigma t_n = \frac{1}{2}\left\{\left(\frac{n(n+1)}{2}\right)^2+\frac{n(n+1)(2n+1)}{6}\right\}$

$= \frac{n(n+1)}{4}\left(\frac{n(n+1)}{2}+\frac{2n+1}{3}\right)$

$\therefore$ $S_{15} = \frac{15\times16}{4}\left\{\frac{15\cdot16}{2}+\frac{31}{3}\right\}$

$= 60\times120+60\times\frac{31}{3}$

$= 7200 + 620$

$= 7820$

19. If $x = 3\tan t$ and $y = 3\sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$, is

(1) $\frac{1}{6\sqrt{2}}$

(2) $\frac{1}{3\sqrt{2}}$

(3) $\frac{3}{2\sqrt{2}}$

(4) $\frac{1}{6}$

Answer (1)

Sol. $\because$ $x = 3\tan t \Rightarrow \frac{dx}{dt} = 3\sec^2 t$

and $y = 3\sec t \Rightarrow \frac{dy}{dt} = 3\sec t\cdot\tan t$

$\therefore$ $\frac{dy}{dx} = \frac{\tan t}{\sec t} = \sin t$

$$\therefore \quad \frac{d^2y}{dx^2} = \frac{d}{dx}(\sin t)\cdot\frac{dt}{dx}$$

$$= \cos t \cdot \frac{1}{3\sec^2 t}$$

$$= \frac{1}{3}\cos^3 t$$

$$\therefore \quad \frac{d^2y}{dx^2}\left(\text{at } t = \frac{\pi}{4}\right) = \frac{1}{3}\cdot\left(\frac{1}{\sqrt{2}}\right)^3$$

$$= \frac{1}{6\sqrt{2}}$$

20. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to :

(1) $\sqrt{22}$ (2) $\sqrt{32}$

(3) 4 (4) 6

Answer (4)

Sol. Projection of $\vec{b}$ on $\vec{a}$ = $\dfrac{\vec{b}.\vec{a}}{|\vec{a}|} = \dfrac{b_1 + b_2 + 2}{4}$

According to question $\dfrac{b_1 + b_2 + 2}{2} = \sqrt{1+1+2} = 2$

$\Rightarrow \; b_1 + b_2 = 2 \quad$ ...(1)

Also $\vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c} = 0$

$\Rightarrow \; 8 + 5b_1 + b_2 + 2 = 0 \quad$ ...(2)

From (1) and (2),

$b_1 = -3, \; b_2 = 5$

$\Rightarrow \; \vec{b} = -3.\hat{i} + 5\hat{j} + \sqrt{2}\hat{k}$

$|\vec{b}| = \sqrt{9+25+2} = 6$

21. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is

(1) 4

(2) 5

(3) 2

(4) 3

Answer (4)

Sol. The roots of $6x^2 - 11x + \alpha = 0$ are rational numbers.

$\therefore$ Discriminant D must be perfect square number.

$D = (-11)^2 - 4\cdot6\cdot\alpha$

$= 121 - 24\alpha$ must be a perfect square

$\therefore$ $\alpha = 3, 4, 5$.

$\therefore$ 3 positive integral values are possible.

22. Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\dfrac{a}{c}$ is equal to

(1) $\dfrac{1}{2}$ (2) 4

(3) $\dfrac{7}{13}$ (4) 2

Answer (2)

Sol. Let first term and common difference be A and D respectively.

$\therefore$ $a = A + 6D$, $b = A + 10D$

and $c = A + 12D$

$\because$ a, b, c are in G.P.

$\therefore$ $b^2 = a.c.$

$\therefore$ $(A + 10D)^2 = (A + 6D)(A + 12D)$

$\therefore$ $14D + A = 0$

$\therefore$ $A = -14D$

$\therefore$ $a = -8D$, $b = -4D$ and $c = -2D$

$\therefore$ $\dfrac{a}{c} = \dfrac{-8D}{-2D} = 4$

23. If the circles $x^2 + y^2 - 16x - 20y + 164 = r^2$ and $(x - 4)^2 + (y - 7)^2 = 36$ intersect at two distinct points, then

(1) $1 < r < 11$ (2) $r > 11$

(3) $r = 11$ (4) $0 < r < 1$

Answer (1)

Sol. $x^2 + y^2 - 16x - 20y + 164 = r^2$

i.e. $(x - 8)^2 + (y - 10)^2 = r^2 \quad$ ...(1)

and $(x - 4)^2 + (y - 7)^2 = 36 \quad$ ...(2)

Both the circles intersect each other at two distinct points.

Distance between centres

$= \sqrt{(8-4)^2 + (10-7)^2} = 5$

$\therefore$ $|r-6| < 5 < |r+6|$

$\therefore$ If $|r-6| < 5 \Rightarrow r \in (1, 11)$ ...(3)

and $|r+6| > 5 \Rightarrow r \in (-\infty, -11) \cup (-1, \infty)$ ...(4)

From (3) and (4),

$r \in (1, 11)$

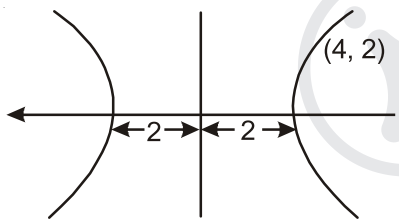
24. A hyperbola has its centre at the origin, passes through the point (4, 2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is

(1) $\frac{3}{2}$ (2) $\sqrt{3}$

(3) $\frac{2}{\sqrt{3}}$ (4) 2

Answer (3)

Sol.



Let equation of hyperbola

$$\frac{x^2}{2^2} - \frac{y^2}{b^2} = 1$$

$\because$ (4, 2) lies on hyperbola

$$\therefore \quad \frac{16}{4} - \frac{4}{b^2} = 1$$

$$\therefore \quad b^2 = \frac{4}{3}$$

$$\therefore \quad \text{Eccentricity} = \sqrt{1+\frac{\frac{4}{3}}{4}} = \sqrt{1+\frac{1}{3}} = \frac{2}{\sqrt{3}}$$

25. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(1) $x - 2y + z = 0$ (2) $x + 2y - 2z = 0$

(3) $5x + 2y - 4z = 0$ (4) $3x + 2y - 3z = 0$

Answer (1)

Sol. Let the direction ratios of the plane containing lines

$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is $< a, b, c >$

$\therefore$ $3a + 4b + 2c = 0$

$4a + 2b + 3c = 0$

$$\therefore \quad \frac{a}{12-4} = \frac{b}{8-9} = \frac{c}{6-16}$$

$$\frac{a}{8} = \frac{b}{-1} = \frac{c}{-10}$$

$\therefore$ Direction ratio of plane = $< -8, 1, 10 >$.

The direction ratio of required plane is $< l, m, n >$

Then $-8l + m + 10n = 0$ ...(3)

and $2l + 3m + 4n = 0$ ...(4)

From (3) and (4),

$$\frac{l}{-26} = \frac{m}{52} = \frac{n}{-26}$$

$\therefore$ D.R.s are $<1, -2, 1>$

$\therefore$ Equation of plane : $x - 2y + z = 0$

26. A data consists of n observations $x_1, x_2, ..., x_n$. If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$, then the standard deviation of this data is

(1) $\sqrt{7}$ (2) 5

(3) $\sqrt{5}$ (4) 2

Answer (3)

Sol. $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$

$$\sigma^2 = \frac{1}{n}A - \frac{1}{n^2}B^2 \quad \text{...(i)}$$

$$\because \quad \sum_{i=1}^{n}(x_i+1)^2 = 9n$$

$\Rightarrow$ $A + n + 2B = 9n \Rightarrow A + 2B = 8n$ ...(ii)

$$\because \quad \sum_{i=1}^{n}(x_i-1)^2 = 5n$$

$\Rightarrow$ $A + n - 2B = 5n \Rightarrow A - 2B = 4n$ ...(iii)

From (ii) and (iii),

$A = 6n, B = n$

$$\Rightarrow \sigma^2 = \frac{1}{n}\times 6n - \frac{1}{n^2}\times n^2 = 6 - 1 = 5$$

$$\Rightarrow \sigma = \sqrt{5}$$

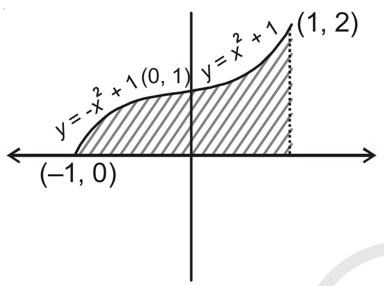
27. The area of the region $A = \{(x, y) : 0 \le y \le x|x| + 1$ and $-1 \le x \le 1\}$ in sq. units, is

(1) 2 (2) $\frac{4}{3}$

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

Answer (1)

Sol. $A = \{(x, y) : 0 \le y \le x|x| + 1$ and $-1 \le x \le 1\}$



$\therefore$ Area of shaded region

$$= \int_{-1}^{0}(-x^2+1)\,dx + \int_{0}^{1}(x^2+1)\,dx$$

$$= \left(-\frac{x^3}{3}+x\right)_{-1}^{0} + \left(\frac{x^3}{3}+x\right)_{0}^{1}$$

$$= 0 - \left(\frac{1}{3}-1\right) + \left(\frac{1}{3}+1\right) - (0+0)$$

$$= \frac{2}{3}+\frac{4}{3} = \frac{6}{3} = 2 \text{ square units}$$

28. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is

(1) $\frac{26}{49}$ (2) $\frac{21}{49}$

(3) $\frac{32}{49}$ (4) $\frac{27}{49}$

Answer (3)

Sol. Let drawing a green ball is G and a red ball is R

$\therefore$ The probability that second drawn ball is red

$$= P(G)\cdot P\left(\frac{R}{G}\right) + P(R)P\left(\frac{R}{R}\right)$$

$$= \frac{2}{7}\times\frac{6}{7}+\frac{5}{7}\times\frac{4}{7}$$

$$= \frac{12+20}{49}$$

$$= \frac{32}{49}$$

29. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1, 1), then the equation of its third side is

(1) $26x - 122y - 1675 = 0$

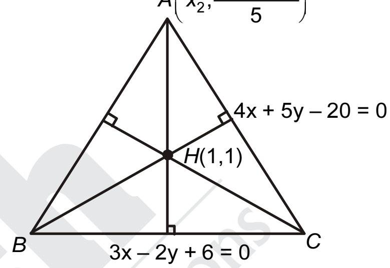
(2) $122y - 26x - 1675 = 0$

(3) $122y + 26x + 1675 = 0$

(4) $26x + 61y + 1675 = 0$

Answer (1)

Sol.



$A\left(x_2, \frac{20-4x_2}{5}\right)$

$\left(x_1, \frac{3x_1+6}{2}\right)$

$\because m_{AH}.m_{BC} = -1$

$$\left(\frac{\frac{20-4x_2}{5}-1}{x_2-1}\right)\times\frac{3}{2} = -1$$

$$\frac{15-4x_2}{5(x_2-1)} = -\frac{2}{3}$$

$$45-12x_2 = -10x_2+10$$

$$2x_2 = 35 \Rightarrow x_2 = \frac{35}{2}$$

$$\Rightarrow A\left(\frac{35}{2}, -10\right)$$

$\because m_{BH}.m_{CA} = -1$

$$\left(\frac{\frac{3x_1}{2}+3-1}{x_1-1}\right)\left(-\frac{4}{5}\right) = -1$$

$$\frac{(3x_1+4)}{2(x_1-1)}\times 4 = 5$$

$\Rightarrow$ $6x_1 + 8 = 5x_1 - 5 \Rightarrow x_1 = -13 \Rightarrow \left(-13, \frac{-33}{2}\right)$

$\Rightarrow$ Equation of line AB is

$$y+10 = \left(\frac{-\frac{33}{2}+10}{-13-35}\right)\left(x-\frac{35}{2}\right)$$

$$\Rightarrow -61y - 610 = -13x + \frac{455}{2}$$

$$\Rightarrow -122y - 1220 = -26x + 455$$

$$\Rightarrow 26x - 122y - 1675 = 0$$

30. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repitition of digits allowed) is equal to

(1) 374 (2) 375

(3) 250 (4) 372

Answer (1)

Sol. Number of numbers with '1' digit = 4 = 4

Number of numbers with '2' digits = 4 × 5 = 20

Number of numbers with '3' digits = 4 × 5 × 5

= 100

Number of numbers with '4' digits = 2 × 5 × 5 × 5

= 250

Total number of numbers = 4 + 20 + 100 + 250

= 374

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