All India Aakash Test Series for JEE (Advanced)-2020

TEST - 3A (Paper-2) - Code-C

Test Date : 06/10/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(C)	21.	(D)	41.	(B)
2.	(A)	22.	(A)	42.	(A)
3.	(A)	23.	(C)	43.	(B)
4.	(C)	24.	(D)	44.	(A)
5.	(A)	25.	(B)	45.	(A)
6.	(A, D)	26.	(A, C, D)	46.	(C, D)
7.	(A, C)	27.	(A, C, D)	47.	(A, B, C)
8.	(A, B)	28.	(B, C, D)	48.	(A, D)
9.	(A, C)	29.	(A, B, D)	49.	(C, D)
10.	(C)	30.	(A, B, C)	50.	(A, B, D)
11.	(A, D)	31.	(D)	51.	(A, C)
12.	(A, C)	32.	(D)	52.	(C, D)
13.	(A, C, D)	33.	(A)	53.	(A, B, C)
14.	(A, C)	34.	(A, B, D)	54.	(A, B, C)
15.	(A, C)	35.	(A, B, C)	55.	(C, D)
16.	A $\rightarrow$ (Q)	36.	A $\rightarrow$ (Q, R, S, T)	56.	A $\rightarrow$ (P, S)
	B $\rightarrow$ (P, T)		B $\rightarrow$ (P, Q, R)		B $\rightarrow$ (S)
	C $\rightarrow$ (P, S)		C $\rightarrow$ (P, Q, R)		C $\rightarrow$ (Q, S, T)
	D $\rightarrow$ (P, R, T)		D $\rightarrow$ (P, R, T)		D $\rightarrow$ (Q, S, T)
17.	A $\rightarrow$ (Q, T)	37.	A $\rightarrow$ (P, R, T)	57.	A $\rightarrow$ (P, Q)
	B $\rightarrow$ (R, S, T)		B $\rightarrow$ (Q, T)		B $\rightarrow$ (P, R, S, T)
	C $\rightarrow$ (P, R, S)		C $\rightarrow$ (P, S, T)		C $\rightarrow$ (R, T)
	D $\rightarrow$ (P, Q)		D $\rightarrow$ (P, Q, T)		D $\rightarrow$ (R, S, T)
18.	(25)	38.	(07)	58.	(19)
19.	(20)	39.	(03)	59.	(30)
20.	(22)	40.	(13)	60.	(32)

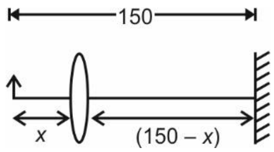
HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (C)

Hint : Displacement method.

Solution :



$$m_1 = \frac{150-x}{x}$$

$$m_2 = \frac{x}{150-x}$$

$$\because \quad \frac{m_1}{m_2} = \frac{16}{1} = \frac{(150-x)^2}{x^2}$$

$$\therefore \quad \frac{150-x}{x} = 4$$

$\Rightarrow$ x = 30 cm

$$\therefore \quad \frac{1}{120} + \frac{1}{30} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{5}{120}$$

$$\Rightarrow \quad f = 24 \text{ cm}$$

2. Answer (A)

Hint : At the instant of sharp change, the flux would remain same.

Solution : Just after changing flux would remain same

$$\therefore \quad L\frac{\varepsilon}{R} = \frac{L}{3}i'$$

$$\Rightarrow \quad i' = \frac{3\varepsilon}{R} = 3I_0$$

Now, $\varepsilon - Ri - \frac{L}{3}\frac{di}{dt} = 0$

$$\Rightarrow \quad (\varepsilon - Ri) = \frac{L}{3}\frac{di}{dt}$$

$$\Rightarrow \quad \int_0^t \frac{3dt}{L} = \int_{3i_0}^{i} \frac{di}{\varepsilon - Ri}$$

$$\Rightarrow \quad \frac{-3Rt}{L} = \ln\left[\frac{\varepsilon - Ri}{-2\varepsilon}\right]$$

$$\Rightarrow \quad Ri - \varepsilon = 2\varepsilon e^{-\frac{3Rt}{L}}$$

$$\Rightarrow \quad i = \frac{\varepsilon}{R}\left[1 + 2e^{-\frac{2Rt}{L}}\right]$$

3. Answer (A)

Hint : $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$

Solution :

$$\frac{1}{L_{eff}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$\Rightarrow \quad L_{eff} = \frac{2}{3}H$$

$$i_{total} = 2 + 3 + 5 = 10 \text{ A}$$

$$\frac{1}{2}Li^2 = \frac{q^2}{2C}$$

$$\Rightarrow \quad q^2 = \frac{2}{3} \times 10 \times 10 \times 2 \times 10^{-6}$$

$$\Rightarrow \quad q = \frac{20}{\sqrt{3}} \times 10^{-3} \text{C} = \frac{20}{\sqrt{3}} \text{mC}$$

4. Answer (C)

Hint : AC circuit Analysis.

Solution : $V_A = \frac{V}{2}$

and $V_B = \frac{VR\angle -\theta}{\sqrt{R^2 + (\omega L)^2}}$ $\quad\quad \tan\theta = \frac{\omega L}{R}$

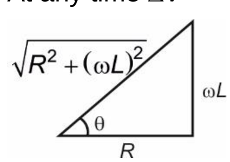
For $R \to 0$ $V_B = V$

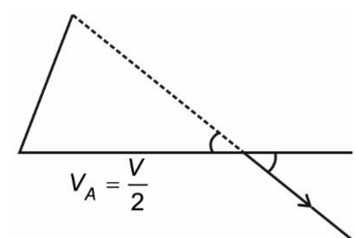
$$\Rightarrow \quad \Delta v = \frac{V}{2}$$

For $R \to \infty$ $V_B = V$

$$\Rightarrow \quad \Delta v = \frac{V}{2}$$

At any time ΔV





$$\therefore \quad \Delta V = |V_A - V_B| = \sqrt{V_A^2 + |V_B|^2 - 2(V_A)(V_B)\cdot\cos\theta}$$

$$\therefore \quad \Delta V^2 = \frac{V^2}{4} + \frac{V^2R^2}{R^2+(\omega L)^2} - \frac{2\cdot V}{2}\frac{V\,R}{\sqrt{R^2+(\omega L)^2}}\cdot\frac{R}{\sqrt{R^2+(\omega L)^2}}$$

$\Rightarrow \quad \Delta V^2 = \frac{V^2}{4}$

$\Rightarrow \quad \Delta V = \frac{V}{2}$

5. Answer (A)

Hint : Induced electric field is non-conservative in nature.

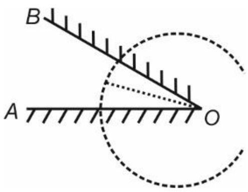
Solution : Induced electric field are produced by changing magnetic field and they form a closed loop,

So $\oint E \cdot dl \neq 0$ and potential can't be defined.

6. Answer (A, D)

Hint : Image by reflection.

Solution : As the object is not on the bisector, the polygon will be irregular.



7. Answer (A, C)

Hint : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}; m = \frac{v}{u}$

Solution : Let x be the object distance from lens and d be the distance between lens and mirror.

Then for image by lens $\frac{1}{v_1} + \frac{1}{x} = \frac{1}{f}$

$\Rightarrow \quad \frac{1}{v_1} = \frac{x-f}{xf}$

$\therefore \quad m_1 = \frac{v_1}{x} = \frac{f}{x-f} \qquad v_1 = \frac{x \cdot f}{x-f}$

For 2nd time object distance is $2d - \frac{xf}{x-f}$

$$\frac{1}{v_2} + \frac{1}{2d - \frac{xf}{x-f}} = \frac{1}{f}$$

$$m_2 = \frac{f}{2d - \frac{xf}{x-f} - f}$$

$$\therefore \quad m_1 . m_2 = \left(\frac{f}{x-f}\right) . \frac{f}{2d - \frac{xf}{x-f} - f}$$

$$= \frac{f^2(x-f)}{(x-f)(2xd - 2df - xf - xf + f^2)}$$

$$m = \frac{f^2}{f^2 - 2df + 2x(d-f)}$$

For m to be independent of x

$df = 0 \Rightarrow \quad d = f$

So, $|m| = \left|\frac{f^2}{f^2 - 2f^2}\right| = 1$

8. Answer (A, B)

Hint : It could be real image or virtual image.

Solution : $\frac{-f}{-f+u} = 2$ and $\frac{-f}{-f+u} = -2$

$\therefore \quad u = \frac{f}{2}$ and $u = \frac{3f}{2}$

9. Answer (A, C)

Hint : $|Z| = \frac{V}{I}; \; P = i_{rms} \cdot V_{rms} \cos\theta$

Solution : $\omega = 100\pi$

$V = 400 \sin\left(\omega t + \frac{\pi}{6}\right)$ volt

$I = 600 \sin(\omega t)$ mA

$\therefore \quad |Z| = \frac{V}{I} = \frac{400 \times 10^3}{600} = \frac{2000}{3} \, \Omega$

Power factor = $\frac{\sqrt{3}}{2}$

Average power dissipation:

$$\frac{400}{\sqrt{2}} \cdot \frac{600 \times 10^{-3}}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = i_{rms} \cdot V_{rms} \cos\theta$$

$\Rightarrow \quad P\langle avg \rangle = 60\sqrt{3}$ watt.

10. Answer (C)

Hint : L-R circuit.

Solution : $\tau = \frac{5L}{R}$

$i_L(t = 0) = 0$

$i_{L(t=\infty)} = \frac{4\varepsilon}{R}$

$i_L = \frac{4\varepsilon}{R}\left[1 - e^{\frac{-tR}{5L}}\right)$

11. Answer (A, D)

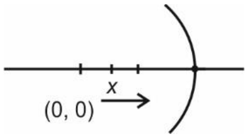
12. Answer (A, C)

13. Answer (A, C, D)

Hint for Q.Nos. 11 to 13 :

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Solution for Q.Nos. 11 to 13 :



At any time t, $x = \frac{f}{2}\cos\omega t$

$\therefore$ object distance from mirror $u = (2f - x)$ (towards left)

Let v is the position of image w.r.t. mirror then

$$\frac{1}{v} + \frac{1}{u} = -\frac{2}{R}$$

$$\Rightarrow \frac{1}{v} - \frac{1}{2f - x} = \frac{-1}{f}$$

$$\Rightarrow \frac{1}{v} = \frac{f - 2f + x}{f(2f - x)}$$

$$\Rightarrow v = \frac{f(2f - x)}{x - f}$$

$\therefore$ Distance of image from mirror

$$\Rightarrow |v| = \frac{f\left(2f - \frac{f}{2}\cos\omega t\right)}{\frac{f}{2}\cos\omega t - f}$$

$$\Rightarrow |v_{IM}| = \left|\frac{f^2(4 - \cos\omega t)\times 2}{2\cdot f(\cos\omega t - 2)}\right| = \frac{f(4 - \cos\omega t)}{2 - \cos\omega t}$$

Now position of image w.r.t. origin is

$$\vec{r}_{I_0} = \vec{r}_{IM} + \vec{r}_{M_0} = \frac{f(2f - x)}{x - f} + 2f$$

$$\Rightarrow \vec{r}_{I_0} = \frac{2f^2 - fx + 2fx - 2f^2}{x - f} = \frac{f\cdot x}{x - f}$$

$$\Rightarrow \vec{r}_{I_0} = \frac{f\cdot\frac{f}{2}\cos\omega t}{\frac{f}{2}\cos\omega t - f} = \frac{f^2\cos\omega t}{f(\cos\omega t - 2)}$$

$$\Rightarrow \vec{r}_{I_0} = -\frac{f\cos\omega t}{2 - \cos\omega t} \text{ (oscillating but not SHM)}$$

Now velocity of image is

$$d\frac{(\vec{r}_{I_0})}{dt} = v_I = \frac{(2 - \cos\omega t)f\omega\sin\omega t + f\cos\omega t\cdot\omega\sin\omega t}{(2 - \cos\omega t)^2}$$

$$\vec{v}_I = \frac{\omega f\sin\omega t(2 - \cos\omega t + \cos\omega t)}{(2 - \cos\omega t)^2} = \frac{2\omega f\sin t}{(2 - \cos\omega t)^2}$$

At $t = \frac{\pi}{3\omega}$ $\quad \vec{v}_I = \frac{2\omega f\sqrt{3}}{2\left(2 - \frac{1}{2}\right)^2} = \frac{\omega f\sqrt{3}\times 4}{9}$

Velocity of object $\frac{dx_0}{dt} = -\frac{f}{2}\omega\sin\omega t = -\frac{\omega f}{4}\sqrt{3}$

$$\therefore \quad V_{rel} = (v_2 - v_1) = \frac{4\omega f\sqrt{3}}{9} + \frac{\omega f\sqrt{3}}{4} = \sqrt{3}\omega f\left[\frac{4}{9} + \frac{1}{4}\right]$$

$$\Rightarrow v_{rel} = \frac{25}{36}\sqrt{3}\,\omega f$$

Magnification is always negative so always real image will be formed. For the time when object lies between $2f$ and f from mirror it will produce magnified image. For rest of the time it will be diminished image.

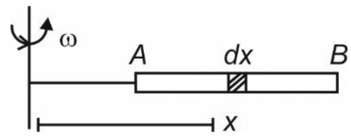
14. Answer (A, C)

15. Answer (A, C)

Hint for Q.Nos. 14 and 15 :

$eE = m\omega^2 \cdot x$

Solution for Q.Nos. 14 and 15 :



Because of pseudo force, free charge would try to shift outward since free charges are electrons. So $F_{out} = m\omega^2 x$. Because of that electric field would induce in a way that $e\vec{E} = m\omega^2 \cdot x$

$\Rightarrow \vec{E} = \frac{m\omega^2}{e}\cdot x$ (from A to B as electron has shifted towards B)

$$V_{AB} = \int_{l}^{l+L} E\,dx = \frac{m\omega^2}{e}\int x\,dx$$

$$\Rightarrow V_{AB} = \frac{m\omega^2}{2e}\left|x^2\right|_{l}^{l+L} = \frac{m\omega^2}{2e}L(L + 2l)$$

As we see $E = \frac{m\omega^2}{e}\cdot x$ implies that the field region (in rod and outside of it) would exist in x direction as per the equation then there must be uniform charge distribution.

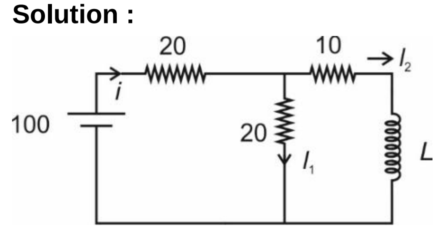
16. Answer A(Q); B(P, T); C(P, S); D(P, R, T)

Hint : $R_{eq}(t = 0) = 40\ \Omega$

$R_{eq}(t = \infty) = \frac{80}{3}\ \Omega$

transient LR circuit.

Solution :



R_{eq} $(t = 0) = 40\ \Omega$

$R_{eq}(t=\infty) = \frac{80}{3}\Omega$

Clearly at t = 0 current $i_1 = \frac{100}{40} = 2.5$ A

and at $t = \infty$ $I_1 = \frac{5}{4} = 1.25$ A

Current i_2 at t = 0 is zero at inductor and it will oppose the sudden change of current.

And i_2 at $t = \infty$ is $\frac{10}{4} = 2.5$ A

Power delivered by battery $p = \varepsilon i$

As i increases from 2.5 A to 3.75 A

So power delivered by battery increases from 250 watt to 375 watt.

17. Answer A(Q, T); B(R, S, T); C(P, R, S); D(P, Q)

Hint : For lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For mirror $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

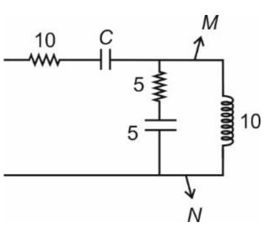
Solution : For lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For mirror $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

18. Answer (25)

Hint : $\frac{1}{Z'} = \frac{1}{5-5j} + \frac{1}{10j}$

Solution :



Let Z' be the impedance across MN.

Then $\frac{1}{z'} = \frac{1}{5-5j} + \frac{1}{10j}$

$\Rightarrow \frac{1}{Z'} = \frac{10j+5-5j}{50j+50} = \frac{5+5j}{50+50j}$

$\Rightarrow Z' = 10\ \Omega$

$\therefore \frac{1}{\omega C} = 20\ \Omega$

$\Rightarrow f = \frac{1}{2\pi \times 20 \times C} = 25$ Hz

19. Answer (20)

Hint : After first refraction from the plane surface the image seems to be at $\frac{3}{2}L$ from plane surface.

Solution : After first refraction from the plane surface p seems to be at $\frac{3}{2}L$ from plane surface.

Now for 2nd surface $\frac{1}{v} + \frac{\mu}{\left(R + \frac{3}{2}L\right)} = \frac{(1-\mu)}{-R}$

$\therefore$ $v = \infty$

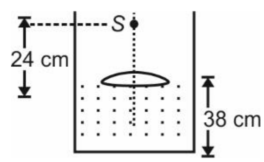
$\therefore \frac{\mu}{R + \frac{3}{2}L} = \frac{\mu - 1}{R}$

$\Rightarrow$ L = 20 cm

20. Answer (22)

Hint : $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Solution :



$$\left.\begin{aligned} \frac{7}{4v_1} + \frac{1}{24} &= \frac{\left(\frac{7}{4} - 1\right)}{6} \\ \frac{4}{3v_2} - \frac{7}{4v_1} &= \frac{\left(\frac{4}{3} - \frac{7}{4}\right)}{\infty} \end{aligned}\right| \text{from water surface}$$

$\therefore \frac{4}{3v_2} + \frac{1}{24} = \frac{3}{24}$

$\Rightarrow \frac{4}{3v_2} = \frac{1}{12}$

$\Rightarrow$ v_2 = 16 cm

$\therefore$ Distance from bottom of tank is 38 – 16 = 22 cm.

PART - II (CHEMISTRY)

21. Answer (D)

Hint : Spectator ligand will affect the C — O bond length. Order of ligand field strength of the given ligand is

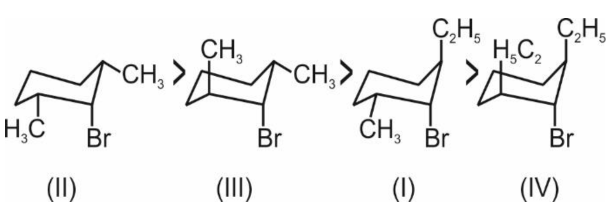
$CO \approx PF_3 > PCl_3 > PAr_3 > PMe_3$

Solution : The ligand PPh_3 is a weaker π acceptor than CO. As a result $d\pi$ electrons of metal in such mixed carbonyl will be drawn more towards CO than in pure metal carbonyl. If the Ph groups of PPh_3 are replaced by more electron attracting Cl or F groups, the tendency of $d\pi$ electrons of metal to move towards P increases. And if Ph groups of PPh_3 are replaced by electron releasing Me groups, the tendency of $d\pi$ electrons of metal to move towards P further decreases. As $d\pi$ electrons of metal move towards P more and less towards CO, the CO bond Order will be more or CO bond length will be less and vice versa.

22. Answer (A)

Hint : For the fast rate, back side of LG Should be less hindered.

Solution : In an S_N2 reaction, the leaving group must be in an axial position in order to allow backside attack to occur without steric hinderance from the cyclohexane ring. When the Br-atom is in axial position in the all cis-isomer, both the methyl groups are in equatorial positions, the structure (II) is most reactive as the approaching nucleophile experiences least crowding. In the all trans isomer (IV) both the ethyl groups are in axial positions providing the maximum crowding to the approaching nucleophile. Thus structure (IV) is least reactive. Structure (I) and (III) are cis-trans type. Their reactivity lies between those of (II) and (IV), structure (I) is less reactive than (III)as it has bulky ethyl group at the axial position



23. Answer (C)

Hint : $Fe(OH)_3$ and Na_2CrO_4 will be formed.

Solution : $Na_2O_2 + 2H_2O \rightarrow 2NaOH + H_2O_2$

$$2FeSO_4 + 4NaOH + H_2O_2 \rightarrow \underset{(Brown)}{2Fe(OH)_3}\downarrow + 2Na_2SO_4$$

$$Al_2(SO_4)_3 + NaOH \rightarrow 2NaAlO_2 + 3Na_2SO_4 + 4H_2O$$

$$Cr_2(SO_4)_3 + 10NaOH + 3H_2O_2 \rightarrow 2Na_2CrO_4 + 3Na_2SO_4 + 8H_2O$$

$Fe(OH)_3$ is a brown residue, $NaAlO_2$ is a colourless solution and Na_2CrO_4 is a yellow solution.

24. Answer (D)

Hint : Cation present is Na^+.

Solution : $Fe(CH_3COO)_3$ is red in colour.

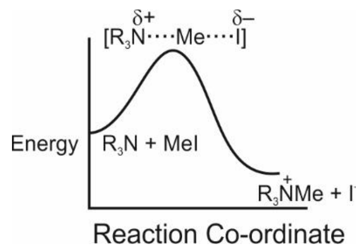
25. Answer (B)

Hint : Dispersion of charge decreases the energy.

Solution :

$$R_3N: + Me - I \rightarrow \underset{\text{Transition state}}{[R_3\overset{\delta+}{N}\cdots Me\cdots \overset{\delta-}{I}]} \rightarrow R_3\overset{+}{N}Me + I^-$$

The general energy diagram of the above reaction is



On increasing polarity of the solvent, all charged species will get solvated. Thus their energies will be lowered. In the above reaction the reactants do not carry any charge and hence their energy remains unaffected by the increase in polarity of the solvent. But the energy of transition state is lowered due to its solvation. This results in decrease of energy barrier and hence increase in the rate of reaction.

26. Answer (A, C, D)

Hint : CFSE $\propto$ stability.

Solution :

1. In a transition group, stability increases down the group due to increase in effective nuclear charge.
2. NO_2^- is stronger ligand than NH_3.
3. Chelate complexes are more stable and as the number of cyclic rings increases, the stability of the complex increases.
4. For the same metal ion, stability of the complex increases with increase in oxidation state of the metal ion.

27. Answer (A, C, D)

Hint : Fact based.

Solution :

$$2Zn^{2+} + K_4[Fe(CN)_6] \rightarrow \underset{\text{White ppt.}}{Zn_2[Fe(CN)_6]} + 4K^+$$

$$2Cd^{2+} + K_4[Fe(CN)_6] \rightarrow \underset{\text{White ppt.}}{Cd_2[Fe(CN)_6]} + 4K^+$$

$$2Cu^{2+} + K_4[Fe(CN)_6] \rightarrow \underset{\text{Brown ppt.}}{Cu_2[Fe(CN)_6]} + 4K^+$$

Al^{3+} ion does not form any precipitate with $K_4[Fe(CN)_6]$.

28. Answer (B, C, D)

Hint : With Fe, NO exist in +1 O.S.

Solution :

(I) $FeSO_4 + 5H_2O + NO \rightarrow \underset{(X)}{[Fe(H_2O)_5NO]^{2+}}$

Oxidation state of Fe changes from +2 to +1 due to transfer of an electron from NO to Fe^{+2}. Electronic configuration of Fe^+ is $3d^7$. It has three unpaired electrons and hence magnetic moment of (X) is $\sqrt{15}$ BM.

(II) $Na_2[Fe(CN)_5NO] + Na_2S \rightarrow Na_4\underset{(Y)}{[Fe(CN)_5NOS]}$

Oxidation state and hybridisation of Fe in the reactant and product (Y) of reaction (II) remains unchanged i.e. +2 and d^2sp^3 respectively.

29. Answer (A, B, D)

Hint : Less hindered more rate.

Solution :

$$\underset{\text{Rate of } S_N2 \text{ reaction}}{\underline{(CH_3)_3C-Br < CH_3CH_2CH_2CH_2-Br < CH_3-CH_2-Br < CH_3-Br}}$$

30. Answer (A, B, C)

Hint :

Less hindered double bond will be more reactive.

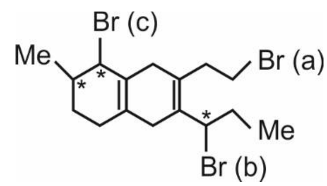
Solution :

(A) Loss of Br (a) atom in dehydrobromination reaction results in the formation of least stable alkene and hence most reactive towards hydrogenation.

(B) Dehydrohalogenation occurs.

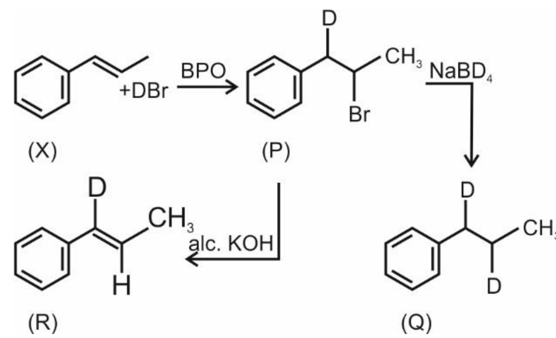
(C) It has 3 stereocentres which means 8 optically active isomers

(D) The given compound has 3 chiral centres and disubstituted cyclic ring that show geometrical isomerism.



31. Answer (D)

Hint :



Solution :

Compound (P) has two dissimilar chiral C-atoms. It has two pair of enantiomers or four pair of diastereomers.

32. Answer (D)

Hint : Compound (Q) has two chiral centres.

Solution :

No. of optical Isomers = 2^n

(for unsymmetrical molecule) = $2^n = 4$

33. Answer (A)

Hint : Saytzeff elimination.

Solution : R is Ph — CD = CH — CH_3

34. Answer (A, B, D)

35. Answer (A, B, C)

Hint : Sodium salt gives violet colour solution with sodium nitroprusside.

Solution : P, Q and R are Na_2SO_3, $Na_2S_2O_3$ and Na_2S Respectively

$$\underset{(P)}{Na_2SO_3} + 2HCl \rightarrow 2NaCl + H_2O + \underset{(X)}{SO_2(g)}$$

$$\underset{(Q)}{Na_2S_2O_3} + 2HCl \rightarrow 2NaCl + \underset{(X)}{SO_2(g)} + S\downarrow + H_2O$$

$$\underset{(R)}{Na_2S} + 2HCl \rightarrow 2NaCl + \underset{(Y)}{H_2S(g)}$$

$$SO_2 + 2H_2S \rightarrow 3S + 2H_2O$$

$$2H_2S + O_2 \rightarrow 2H_2O + 2S$$

$$H_2S + 2FeCl_3 \rightarrow 2FeCl_2 + 2HCl + S\downarrow$$

$$H_2S + (CH_3COO)_2Pb \rightarrow \underset{Black}{PbS}\downarrow + 2CH_3COOH$$

$$Na_2S + Na_2[Fe(CN)_5NO] \rightarrow \underset{\text{Violet colour}}{Na_4[Fe(CN)_5NOS]}$$

36. Answer A(Q, R, S, T); B(P, Q, R); C(P, Q, R); D(P, R, T)

Hint : Fact based

Solution :

$$Co^{2+} + 4KSCN \rightarrow \underset{\text{Blue colour}}{[Co(SCN)_4]^{2-}} + 4K^+$$

$$Co(NO_3)_2 + NaOH \rightarrow \underset{Blue}{Co(OH)(NO_3)}\downarrow + NaNO_3$$

$$Co(OH)(NO_3) + \underset{Excess}{NaOH} \xrightarrow{Warm} \underset{Pink}{Co(OH)_2}\downarrow + NaNO_3$$

$$Co^{2+} + 7NO_2^- + 2H^+ + 3K^+ \rightarrow \underset{Yellow}{K_3[Co(NO_2)_6]}\downarrow + NO\uparrow + H_2O$$

$$2Cu^{2+} + K_4[Fe(CN)_6] \rightarrow \underset{\text{Chocolate brown}}{Cu_2[Fe(CN)_6]}\downarrow + 4K^+$$

$$Cu^{2+} + 2NaOH \rightarrow \underset{\text{Bluish white}}{Cu(OH)_2}\downarrow + 2Na^+$$

$$2Cu^{2+} + 4KSCN \rightarrow \underset{Black}{2Cu(SCN)_2}\downarrow \rightarrow \underset{White}{2CuSCN}\downarrow + (SCN)_2$$

$$4Fe^{3+} + 3K_4[Fe(CN)_6] \rightarrow \underset{\text{Prussian Blue}}{Fe_4[Fe(CN)_6]_3} + 12K^+$$

$$Fe^{3+} + 3SCN^- \longrightarrow \underset{\text{Deep red colour}}{Fe(SCN)_3}$$

$$Fe^{3+} + 3NaOH \longrightarrow \underset{Brown}{Fe(OH)_3}\downarrow + 3Na^+$$

$$3Zn^{2+} + 2K_4[Fe(CN)_6] \rightarrow \underset{White}{K_2Zn_3[Fe(CN)_6]_2}\downarrow + 6K^+$$

$$Zn^{2+} + 2NaOH \rightarrow \underset{White}{Zn(OH)_2}\downarrow + 2Na^+$$

$$Zn(OH)_2 + \underset{(Excess)}{2NaOH} \rightarrow \underset{\text{Water soluble}}{Na[Zn(OH)_4]}$$

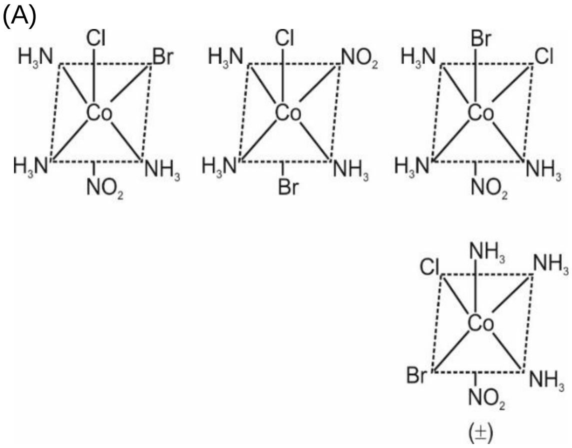
$$Zn^{2+} + (NH_4)_2[Hg(SCN)_4] \rightarrow Zn[Hg(SCN)_4]\downarrow + 2NH_4^+$$

37. Answer A(P, R, T); B(Q, T); C(P, S, T); D(P, Q, T)

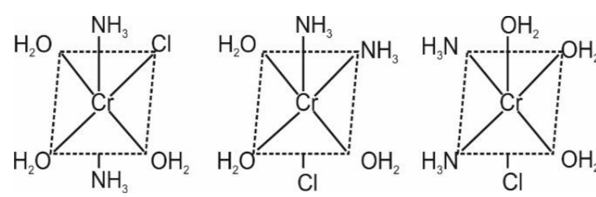
Hint :

The possible stereoisomer of the given complexes are

(A)

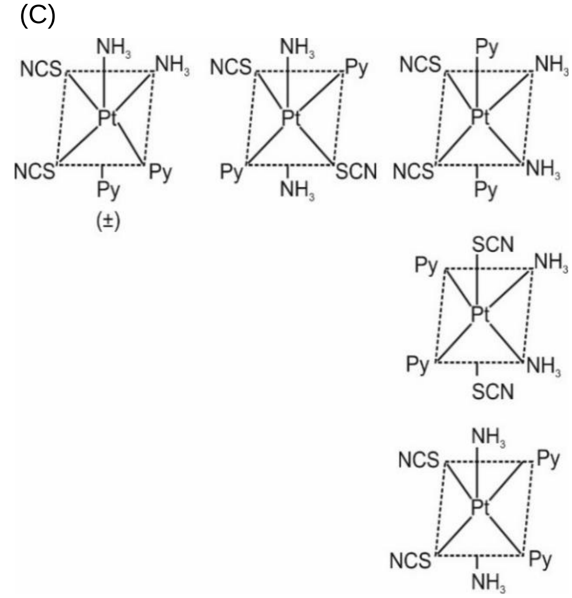


(B)

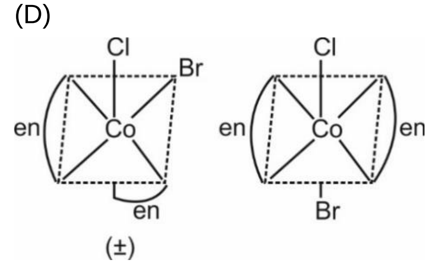


Solution :

(C)



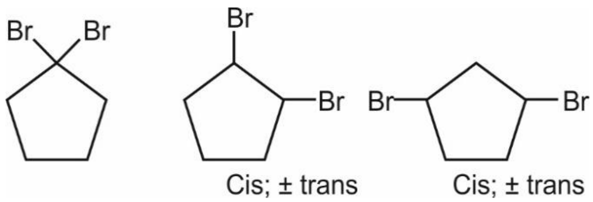
(D)



38. Answer (07)

Hint : D. B. E = 1.

Solution :



39. Answer (03)

Hint : $PbBr_2 \rightarrow$ Colourless

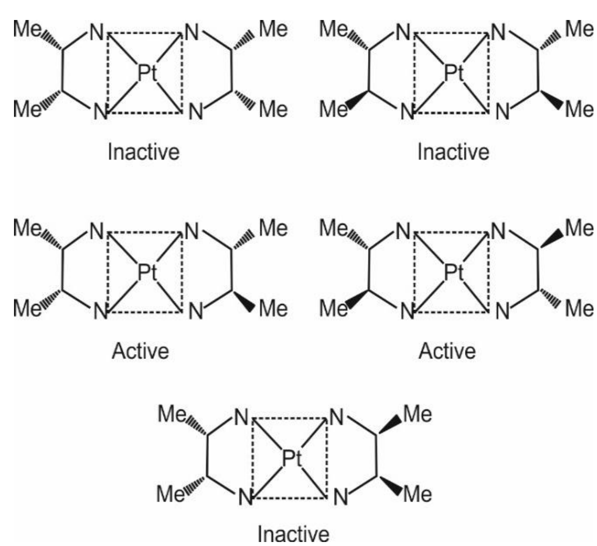
$MnS \rightarrow$ Buff

Solution : As_2S_5, PbI_2 and AgI are yellow coloured compound.

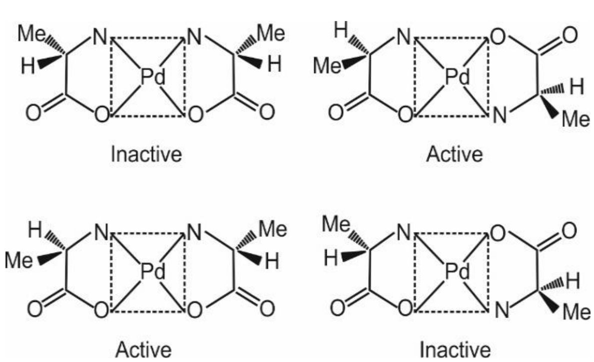
40. Answer (13)

Hint : Pt form square planar complex.

Solution :



$\therefore$ x = 7



$\therefore$ y = 6

x + y = 13

PART - III (MATHEMATICS)

41. Answer (B)

Hint : Put sec$x$ + tan$x$ = t

Solution :

$$\Rightarrow \quad \sec x\,dx = \frac{dt}{t} \text{ and } \sec x - \tan x = \frac{1}{t}$$

$$I = \int_0^{\pi/4} (2\sec x)(2\sec x)^{12}\,dx = \int_1^{1+\sqrt{2}} \frac{2}{t}\left(t + t^{-1}\right)^{12} dt$$

42. Answer (A)

Hint : $\alpha + \beta = -t^2$ and $\alpha\beta = -2t$

Solution :

$$f(t) = \int_{-1}^{2}\left(x^2 + \left(\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}\right)x + \frac{1}{\alpha^2\beta^2} + \frac{1}{\alpha\beta}\right)dx$$

$$= \frac{x^3}{3} + \left(\frac{t^4 - 2(-2t)}{4t^2}\right)\frac{x^2}{2} + \left(\frac{1}{4t^2} - \frac{1}{2t}\right)x\Bigg|_{-1}^{2}$$

$$= \left(\frac{8}{3} + \frac{1}{3}\right) + \left(\frac{t^2}{4} + \frac{1}{t}\right)\cdot\frac{3}{2} + \left(\frac{1}{4t^2} - \frac{1}{2t}\right)3$$

$$= 3 + \frac{3t^2}{8} + \frac{3}{4t^2}$$

$$f'(t) = \frac{6t}{8} - \frac{6}{4t^3} = 0 \Rightarrow t = \pm 2^{1/4}$$

$$f(t)_{\min} = 3 + \frac{3\sqrt{2}}{8} + \frac{3}{4\sqrt{2}} = 3 + \frac{3\sqrt{2}}{4}$$

43. Answer (B)

Hint : $(9 - 16x^2)^{3/2} = x^3\left(\frac{9}{x^2} - 16\right)^{3/2}$

Solution :

$$\int \frac{dx}{x^3\left(\frac{9}{x^2} - 16\right)^{3/2}}$$

Put $\frac{9}{x^2} - 16 = t^2$

$$-\frac{18}{x^3}dx = 2t\,dt$$

$$\Rightarrow -\frac{1}{18}\int \frac{2t\ dt}{t^3}$$

$$= \frac{1}{9t} + c = \frac{x}{9(9 - 16x^2)^{1/2}} + c$$

44. Answer (A)

Hint : Degree is power of highest differential coefficient when expressed as polynomial in differential coefficients

Solution :

$$y' = \frac{2\sin^{-1}x}{\sqrt{1-x^2}} - \frac{A}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad y'\sqrt{1-x^2} = 2\sin^{-1}x - A$$

$$\Rightarrow \quad y''\sqrt{1-x^2} + \frac{y'(-2x)}{\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$$

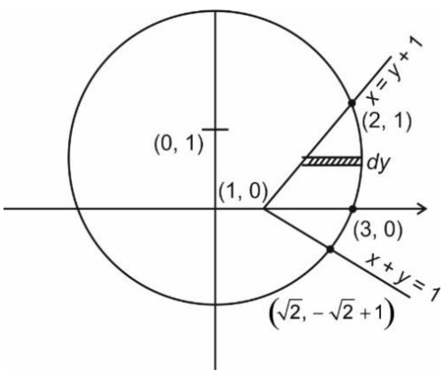
$$\Rightarrow \quad y''(1-x^2) - 2xy' = 2$$

Hence degree = 1

45. Answer (A)

Hint : Area between $f(x)$ and $g(x)$ is $\int_\alpha^\beta (f(x) - g(x))dx$, where α and β are point of intersection of the curves

Solution :



$$A = \int_{-\sqrt{2}+1}^{1} x\,dy = \int_{-\sqrt{2}+1}^{0} \left(\sqrt{-y^2+2y+3} - (1-y)\right)dy$$

$$+\int_0^1 \left(\sqrt{-y^2+2y+3} - (y+1)\right)dy$$

$$\Rightarrow \quad \int_{1-\sqrt{2}}^{1} \sqrt{4-(y-1)^2}\,dy + \int_{1-\sqrt{2}}^{0} (y-1)dy - \int_0^1 (y+1)dy$$

$$\Rightarrow \quad \left(\frac{y-1}{2}\sqrt{4-(y-1)^2} + \frac{4}{2}\sin^{-1}\left(\frac{y-1}{2}\right)\right)\Bigg|_{1-\sqrt{2}}^{1}$$

$$+\left(\frac{y^2}{2} - y\right)\Bigg|_{1-\sqrt{2}}^{0} - \left(\frac{y^2}{2} + y\right)\Bigg|_0^1$$

$$\Rightarrow \quad 0 + 0 - \left(-\frac{1}{\sqrt{2}}(0) + 2\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right) + 0 +$$

$$\left(\frac{1+2-2\sqrt{2}-2+2\sqrt{2}}{2}\right) - \left(\frac{1}{2} + 1 - 0\right)$$

$$\Rightarrow \quad -2\left(-\frac{\pi}{4}\right) + \left(\frac{1}{2}\right) - \frac{3}{2} \Rightarrow \left(\frac{\pi}{2} - 1\right) \text{ sq. units}$$

46. Answer (C, D)

Hint : Substitute $x = \sin t$

Solution :

$$\int_0^1 f(x)dx = \int_0^{\pi/2} f(\sin t)\cos t\,dt = I$$

$$= \int_0^{\pi/2} f(\cos t)\sin t\,dt = I$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \left(f(\sin t)\cos t + f(\cos t)\sin t\right)dt$$

$$\Rightarrow \quad 2I \le \int_0^{\pi/2} 1\,dx$$

$$I \le \frac{\pi}{4}$$

47. Answer (A, B, C)

Hint : Form linear differential equation

Solution :

$$\left(e^{\tan^{-1}y} - x\right)\frac{dy}{dx} = 1 + y^2$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{e^{\tan^{-1}y} - x}{1+y^2}$$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1}y}}{1+y^2}$$

$$\text{I.F.} = e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1}y}$$

$$\Rightarrow \quad \int d\left(x\,e^{\tan^{-1}y}\right) = \int \frac{e^{2\tan^{-1}y}}{1+y^2}dy$$

$$\Rightarrow \quad x\,e^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + c$$

$$\downarrow \left(\frac{1}{2}, 0\right)$$

$$\Rightarrow \quad 2x = e^{\tan^{-1}y}$$

$$\Rightarrow \quad x_0 = \frac{e^{\pi/3}}{2}$$

48. Answer (A, D)

Hint : Multiply and divide by $\sec^2 x$

Solution :

$$I = \int \left(\sin^{-11/3}x \cdot \cos^{-1/3}x\right) \cdot \frac{\sec^4 x}{\sec^4 x}dx$$

$$\Rightarrow \quad I = \int \frac{(1+\tan^2 x)\sec^2 x\,dx}{\tan^{11/3} x}$$

$\Rightarrow$ Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

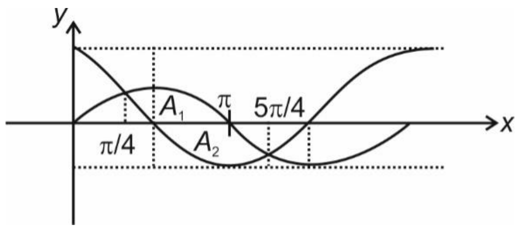
$$\Rightarrow \quad I = \int \left(\frac{1}{t^{11/3}} + \frac{t^2}{t^{11/3}}\right)dt$$

$$= -\frac{3}{8}(\tan x)^{-8/3} - \frac{3}{2}(\tan x)^{-2/3} + c$$

49. Answer (C, D)

Hint : Area between f(x) and $g(x)$ is $\int_{\alpha}^{\beta}(f(x)-g(x))dx$, where α and β are point of intersection of $f(x)$ and $g(x)$

Solution :



$$\Rightarrow \quad A_1=\int_{\pi/4}^{\pi/2}(\sin x-\cos x)dx+\int_{\pi/2}^{\pi}\sin x\,dx=-\cos x-\sin x\Big|_{\pi/4}^{\pi/2}$$

$$+(-\cos x)\Big|_{\pi/2}^{\pi}=-1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+1-0=\sqrt{2}$$

$$\text{And } A_1+A_2=\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)dx=-\cos x-\sin x\Big|_{\pi/4}^{5\pi/4}$$

$$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=2\sqrt{2}$$

$$\Rightarrow \quad A_2=\sqrt{2} \text{ and } A_1=\sqrt{2}\Rightarrow A_1A_2=2$$

50. Answer (A, B, D)

Hint : If $x \in (0, 1) \Rightarrow x^2 > x^3$ and if $x \in (1, 2)$ $x^3 > x^2$

Solution :

If $x\in(0,1)$ $\quad (2020)^{x^2}>(2020)^{x^3}$

and if $x\in(1,2)$ $\quad (2020)^{x^3}>(2020)^{x^2}$

$\Rightarrow \quad I_4 > I_3 > I_1 > I_2$

51. Answer (A, C)

52. Answer (C, D)

53. Answer (A, B, C)

Hint for Q.Nos. 51 to 53 :

$xdy + ydx = d(xy)$ and $\dfrac{xdy-ydx}{x^2}=d\left(\dfrac{y}{x}\right)$

Solution for Q.Nos. 51 to 53 :

$$\frac{xy\,dx+x^2dy}{x^2}=\frac{y^3(x\,dy-y\,dx)}{x^2}$$

$$\Rightarrow \quad \int d(xy)=\int x^2y^2\frac{y}{x}d\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \int\frac{d(xy)}{(xy)^2}=\int\frac{y}{x}d\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad -\frac{1}{xy}=\frac{1}{2}\left(\frac{y}{x}\right)^2+c$$

$$\downarrow(4,-2)$$

$\Rightarrow \quad c = 0$

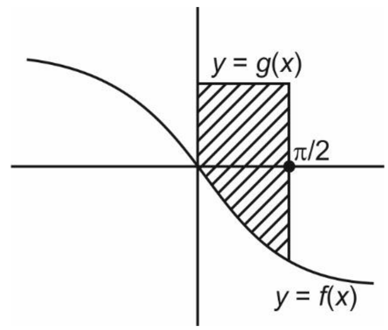
$$f(x)\equiv y=(-2x)^{1/3}$$

Also $g'(x) = \sin^{-1}\sin x\cdot 2\sin x\cos x$

$+ \cos^{-1}\cos x\, 2\cos x(-\sin x) = 0$

$\Rightarrow \quad g(x)$ = constant

Put $x = \dfrac{\pi}{4}$ we get $\quad g(x)\equiv y=\dfrac{3\pi}{16}\ \forall\ x\in\left[0,\ \dfrac{\pi}{2}\right]$



$$A=\int_0^{\pi/2}(g(x)-f(x))dx$$

$$A=\frac{3\pi^{4/3}}{8}\left[1+\frac{\pi^{2/3}}{4}\right]$$

54. Answer (A, B, C)

55. Answer (C, D)

Hint for Q.Nos. 54 and 55 :

Substitute expression in $\tan\theta$.

Solution for Q.Nos. 54 and Q.55 :

$$\int\frac{\left(\sin^{3/2}\theta+\cos^{3/2}\theta\right)d\theta}{\sqrt{\sin^3\theta\cos^3\theta\sin(\theta+\alpha)}}$$

$$\Rightarrow \quad \int\frac{\sin^{3/2}\theta\,d\theta}{\sin^{3/2}\theta\sqrt{\cos^3\theta\cdot(\sin\theta\cos\alpha+\cos\theta\sin\alpha)}}$$

$$+\int\frac{\cos^{3/2}\theta\,d\theta}{\cos^{3/2}\theta\sqrt{\sin^3\theta(\sin\theta\cos\alpha+\cos\theta\sin\alpha)}}$$

$$\Rightarrow \quad \underbrace{\int\frac{d\theta}{\cos^2\theta\sqrt{(\tan\theta\cos\alpha+\sin\alpha)}}}_{I_1}$$

$$+\underbrace{\int\frac{d\theta}{\sin^2\theta\sqrt{(\cos\alpha+\cos\theta\sin\alpha)}}}_{I_2}$$

For I_1 put $\tan\theta\cos\alpha + \sin\alpha = t^2$

$\Rightarrow$ $\cos\alpha\sec^2\theta\, d\theta = 2t\, dt$

For I_2 put $\cos\alpha + \cot\theta\sin\alpha = \mu^2$

$\Rightarrow$ $-\text{cosec}^2\theta\cdot\sin\alpha\, d\theta = 2\mu\, d\mu$

$\Rightarrow$ $\int\frac{2t\,dt}{\cos\alpha\cdot t} - \int\frac{2\mu\, d\mu}{\sin\alpha\,\mu}$

$\Rightarrow$ $2\sec\alpha\sqrt{\sin\alpha+\tan\theta\cos\alpha} - 2\,\text{cosec}\,\alpha$

$\sqrt{\cos\alpha+\cot\theta\sin\alpha}+c$

$\therefore$ $f(\theta) = \sin\alpha + \cos\alpha\tan\theta$

and $g(\theta) = \cos\alpha + \cot\theta\sin\alpha$

$f(\alpha) = 2\sin\alpha \in (0, 2)$

$g(\alpha) = 2\cos\alpha$

$\Rightarrow$ $f(\alpha)+g(\alpha)\in\left(2, 2\sqrt{2}\right]$

56. Answer A(P, S); B(S); C(Q, S, T); D(Q, S, T)

(A) **Hint :** Convert all terms into $\sin x$ and $\cos x$

$$I = \int\frac{(\sin^2 x+\cos^2 x)dx}{\sin^4 x\cos^2 x}$$

$$= \int\left(\sec^2 x + 2\,\text{cosec}^2 x + \frac{\cos^2 x}{\sin^4 x}\right)dx$$

$$= \tan x - 2\cot x - \frac{\cot^3 x}{3} + k$$

$$A = 1, B = -2 \text{ and } c = -\frac{1}{3}$$

$\Rightarrow$ $4A + B + 3C = 1$

(B) **Hint :** Distribute x^{27} in both brackets

$$g(x) = \int(x^4+x^5+x^6)^6(6x^5+5x^4+4x^3)dx$$

Put $x^6 + x^5 + x^4 = t$

$\Rightarrow$ $(6x^5 + 5x^4 + 4x^3)dx = dt$

$$= \frac{1}{7}(x^6+x^4+x^5)^7 + c$$

As $g(0) = 0$ $\quad g(x) = \frac{(x^6+x^4+x^5)^7}{7}$

$\Rightarrow$ $g(1) = \frac{3^7}{7}$

(C) $\int_0^{\pi/2} 12\sin^3 x\, G x\, d x = 12\cdot\frac{1}{4} = 3$

(D) $I = \int_{-4}^{-5} e^{(x+5)^2}dx + k\int_{1/3}^{3/2} e^{(3x-2)^2}dx$

$I_1 \quad + \quad I_2$

In I_1 put $x + 5 = y$ and in I_2 put $3x - 2 = -t$

$$I = \int_1^0 e^{y^2}dy + \frac{k}{3}\int_1^0 e^{y^2}(-dt) = 0$$

$\therefore$ $k = 3$

57. Answer A(P, Q); B(P, R, S, T); C(R, T); D(R, S, T)

Hint : Applying properties of integrals

Solution :

(A) $5\int_0^1 \sin\{x\}dx = 5\int_0^1\sin x\,dx = 5(-\cos x)\Big|_6^1$

$= 5(1-\cos 1)$

(B) $\left|\int_{-5}^{5}(-1)dx\right| = 10$

(C) $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^4}{n^5} = \int_0^1 x^4 dx = \frac{x^5}{5} = \frac{1}{5}$

(D) $\lim_{x\to 0}\frac{\left(e^{2x}-1\right)}{x^n}\cdot 2x = \lim_{n\to\infty}\frac{e^{2x}-1}{2x}\times\frac{4x^2}{x^n}$

$\Rightarrow$ $n \le 3$ for limit to be finite

58. Answer (19)

Hint : Integration by parts

Solution :

$$I_n = \int_0^1 1\cdot\frac{1}{\left(1+x^2\right)^n}dx$$

$$= \frac{x}{\left(1+x^2\right)^n}\Bigg|_0^1 + n\int_0^1\frac{2x^2dx}{\left(1+x^2\right)^{n+1}}$$

$$= \frac{1}{2^n} + 2n\left[\int_0^1\frac{dx}{\left(1+x^2\right)^n} - \int_0^1\frac{dx}{\left(1+x^2\right)^{n+1}}\right]$$

$$= \frac{1}{2^n} + 2n\ I_n - 2n\ I_{n+1}$$

$2n\, I_{n+1} = 2^{-n} + (2n - 1)I_n$

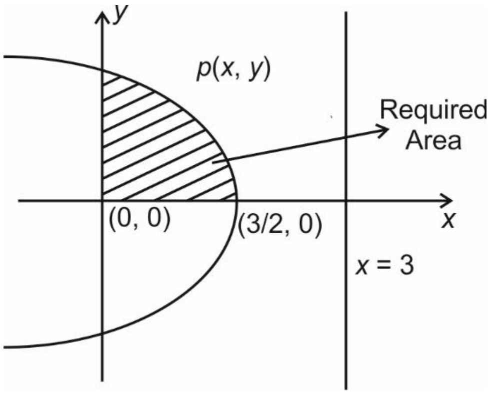
Put $n = 10$, we get

$20\, I_{11} = 2^{-10} + 19\, I_{10}$

59. Answer (30)

Hint : Distance from origin ≤ distance from line $x = 3$

Solution :



Given $\sqrt{x^2+y^2} \le |x-3|$

$\Rightarrow$ $x^2 + y^2 \le x^2 - 6x + 9$

$\Rightarrow$ $y^2 \le -6x + 9$

$\Rightarrow$ $y^2 \le -6\left(x-\frac{3}{2}\right)$

$\Rightarrow$ $A = \int_0^{3/2} \sqrt{9-6x}\, dx$

$= \left.\frac{(9-6x)^{3/2}}{\frac{3}{2}\cdot(-6)}\right|_0^{3/2}$

$= 0 + \frac{(9)^{3/2}}{9} = \frac{27}{9} = 3$ sq. units

$\Rightarrow$ $10A = 30$

60. Answer (32)

Hint : $\int_{-a}^{a} f(x)dx = 0$ if $f(x)$ is odd.

Solution :

$I = \int_{-\pi/4}^{\pi/4} \sec^2 x\, dx$

$= 2\int_0^{\pi/4} \sec^2 x\, dx$

$= 2\tan x\Big|_0^{\pi/4} = 2$

$\Rightarrow$ $I^5 = 2^5 = 32$

□ □ □

All India Aakash Test Series for JEE (Advanced)-2020

TEST - 3A (Paper-2) - Code-D

Test Date : 06/10/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(A)	21.	(B)	41.	(A)
2.	(C)	22.	(D)	42.	(A)
3.	(A)	23.	(C)	43.	(B)
4.	(A)	24.	(A)	44.	(A)
5.	(C)	25.	(D)	45.	(B)
6.	(C)	26.	(A, B, C)	46.	(A, B, D)
7.	(A, C)	27.	(A, B, D)	47.	(C, D)
8.	(A, B)	28.	(B, C, D)	48.	(A, D)
9.	(A, C)	29.	(A, C, D)	49.	(A, B, C)
10.	(A, D)	30.	(A, C, D)	50.	(C, D)
11.	(A, D)	31.	(D)	51.	(A, C)
12.	(A, C)	32.	(D)	52.	(C, D)
13.	(A, C, D)	33.	(A)	53.	(A, B, C)
14.	(A, C)	34.	(A, B, D)	54.	(A, B, C)
15.	(A, C)	35.	(A, B, C)	55.	(C, D)
16.	A $\to$ (Q, T)	36.	A $\to$ (P, R, T)	56.	A $\to$ (P, Q)
	B $\to$ (R, S, T)		B $\to$ (Q, T)		B $\to$ (P, R, S, T)
	C $\to$ (P, R, S)		C $\to$ (P, S, T)		C $\to$ (R, T)
	D $\to$ (P, Q)		D $\to$ (P, Q, T)		D $\to$ (R, S, T)
17.	A $\to$ (Q)	37.	A $\to$ (Q, R, S, T)	57.	A $\to$ (P, S)
	B $\to$ (P, T)		B $\to$ (P, Q, R)		B $\to$ (S)
	C $\to$ (P, S)		C $\to$ (P, Q, R)		C $\to$ (Q, S, T)
	D $\to$ (P, R, T)		D $\to$ (P, R, T)		D $\to$ (Q, S, T)
18.	(22)	38.	(13)	58.	(32)
19.	(20)	39.	(03)	59.	(30)
20.	(25)	40.	(07)	60.	(19)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (A)

Hint : Induced electric field is non-conservative in nature.

Solution : Induced electric field are produced by changing magnetic field and they form a closed loop,

So $\oint E \cdot dl \neq 0$ and potential can't be defined.

2. Answer (C)

Hint : AC circuit Analysis.

Solution : $V_A = \frac{V}{2}$

and $V_B = \frac{VR\angle -\theta}{\sqrt{R^2+(\omega L)^2}}$ $\tan\theta = \frac{\omega L}{R}$

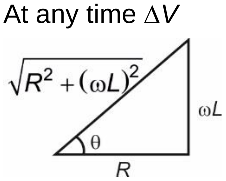
For $R \to 0$ $V_B = V$

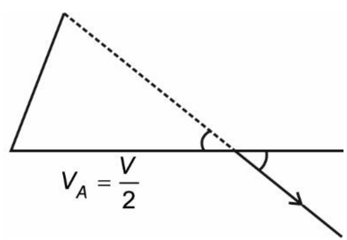
$\Rightarrow \Delta v = \frac{V}{2}$

For $R \to \infty$ $V_B = V$

$\Rightarrow \Delta v = \frac{V}{2}$

At any time ΔV





$\therefore \Delta V = |V_A - V_B| = \sqrt{V_A^2 + |V_B|^2 - 2(V_A)(V_B)\cdot\cos\theta}$

$$\therefore \Delta V^2 = \frac{V^2}{4} + \frac{V^2R^2}{R^2+(\omega L)^2} - \frac{2\cdot V}{2} \cdot \frac{V\,R}{\sqrt{R^2+(\omega L)^2}} \cdot \frac{R}{\sqrt{R^2+(\omega L)^2}}$$

$\Rightarrow \Delta V^2 = \frac{V^2}{4}$

$\Rightarrow \Delta V = \frac{V}{2}$

3. Answer (A)

Hint : $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$

Solution :

$\frac{1}{L_{eff}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$

$\Rightarrow L_{eff} = \frac{2}{3}$ H

$i_{total} = 2 + 3 + 5 = 10$ amp

$\frac{1}{2}Li^2 = \frac{q^2}{2C}$

$\Rightarrow q^2 = \frac{2}{3} \times 10 \times 10 \times 2 \times 10^{-6}$

$\Rightarrow q = \frac{20}{\sqrt{3}} \times 10^{-3}\text{C} = \frac{20}{\sqrt{3}}\text{mC}$

4. Answer (A)

Hint : At the instant of sharp change, the flux would remain same.

Solution : Just after changing flux would remain same

$\therefore L\frac{\varepsilon}{R} = \frac{L}{3}i'$

$\Rightarrow i' = \frac{3\varepsilon}{R} = 3I_0$

Now, $\varepsilon - Ri - \frac{L}{3}\frac{di}{dt} = 0$

$\Rightarrow (\varepsilon - Ri) = \frac{L}{3}\frac{di}{dt}$

$\Rightarrow \int_0^t \frac{3dt}{L} = \int_{3i_0}^{i} \frac{di}{\varepsilon - Ri}$

$\Rightarrow \frac{-3Rt}{L} = \ln\left[\frac{\varepsilon - Ri}{-2\varepsilon}\right]$

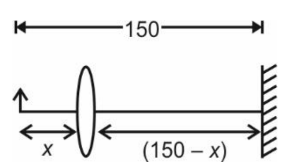
$\Rightarrow Ri - \varepsilon = 2\varepsilon e^{-\frac{3Rt}{L}}$

$\Rightarrow i = \frac{\varepsilon}{R}\left[1 + 2e^{-\frac{2Rt}{L}}\right]$

5. Answer (C)

Hint : Displacement method.

Solution :



$m_1 = \frac{150 - x}{x}$

$m_2 = \frac{x}{150 - x}$

$\because \frac{m_1}{m_2} = \frac{16}{1} = \frac{(150-x)^2}{x^2}$

$\therefore \quad \frac{150-x}{x}=4$

$\Rightarrow \quad x = 30$ cm

$\therefore \quad \frac{1}{120}+\frac{1}{30}=\frac{1}{f}$

$\Rightarrow \quad \frac{1}{f}=\frac{5}{120}$

$\Rightarrow \quad f = 24$ cm

6. Answer (C)

Hint : L-R circuit.

Solution : $\tau=\frac{5L}{R}$

$i_L(t=0)=0$

$i_{L(t=\infty)}=\frac{4\varepsilon}{R}$

$i_L=\frac{4\varepsilon}{R}\left[1-e^{\frac{-tR}{5L}}\right)$

7. Answer (A, C)

Hint : $|Z|=\frac{V}{I}$; $P=i_{\text{rms}}\cdot V_{\text{rms}}\cos\theta$

Solution : $\omega = 100\pi$

$V=400\sin\left(\omega t+\frac{\pi}{6}\right)$ volt

$I = 600\sin(\omega t)$ mA

$\therefore \quad |Z|=\frac{V}{I}=\frac{400\times10^3}{600}=\frac{2000}{3}\,\Omega$

Power factor = $\frac{\sqrt{3}}{2}$

Average power dissipation:

$$\frac{400}{\sqrt{2}}\cdot\frac{600\times10^{-3}}{\sqrt{2}}\times\frac{\sqrt{3}}{2}=i_{\text{rms}}\cdot V_{\text{rms}}\cos\theta$$

$\Rightarrow \quad P\langle\text{avg}\rangle=60\sqrt{3}$ watt.

8. Answer (A, B)

Hint : It could be real image or virtual image.

Solution : $\frac{-f}{-f+u}=2$ and $\frac{-f}{-f+u}=-2$

$\therefore \quad u=\frac{f}{2}$ and $u=\frac{3f}{2}$

9. Answer (A, C)

Hint : $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}; m=\frac{v}{u}$

Solution : Let x be the object distance from lens and d be the distance between lens and mirror.

Then for image by lens $\frac{1}{v_1}+\frac{1}{x}=\frac{1}{f}$

$\Rightarrow \quad \frac{1}{v_1}=\frac{x-f}{xf}$

$\therefore \quad m_1=\frac{v_1}{x}=\frac{f}{x-f} \qquad v_1=\frac{x\cdot f}{x-f}$

For 2nd time object distance is $2d-\frac{xf}{x-f}$

$$\frac{1}{v_2}+\frac{1}{2d-\frac{xf}{x-f}}=\frac{1}{f}$$

$$m_2=\frac{f}{2d-\frac{xf}{x-f}-f}$$

$$\therefore \quad m_1.m_2=\left(\frac{f}{x-f}\right).\frac{f}{2d-\frac{xf}{x-f}-f}$$

$$=\frac{f^2(x-f)}{(x-f)(2xd-2df-xf-xf+f^2)}$$

$$m=\frac{f^2}{f^2-2df+2x(d-f)}$$

For m to be independent of x

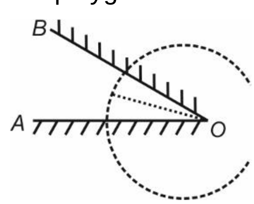
$df=0 \quad \Rightarrow \quad d=f$

So, $|m|=\left|\frac{f^2}{f^2-2f^2}\right|=1$

10. Answer (A, D)

Hint : Image by reflection.

Solution : As the object is not on the bisector, the polygon will be irregular.



11. Answer (A, D)

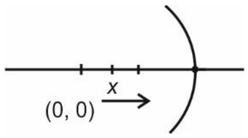
12. Answer (A, C)

13. Answer (A, C, D)

Hint for Q.Nos. 11 to 13 :

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Solution for Q.Nos. 11 to 13 :



At any time t, $x = \frac{f}{2}\cos\omega t$

$\therefore$ object distance from mirror $u = (2f - x)$ (towards left)

Let v is the position of image w.r.t. mirror then

$$\frac{1}{v} + \frac{1}{u} = -\frac{2}{R}$$

$$\Rightarrow \frac{1}{v} - \frac{1}{2f - x} = \frac{-1}{f}$$

$$\Rightarrow \frac{1}{v} = \frac{f - 2f + x}{f(2f - x)}$$

$$\Rightarrow v = \frac{f(2f - x)}{x - f}$$

$\therefore$ Distance of image from mirror

$$\Rightarrow |v| = \frac{f\left(2f - \frac{f}{2}\cos\omega t\right)}{\frac{f}{2}\cos\omega t - f}$$

$$\Rightarrow |v_{IM}| = \left|\frac{f^2(4 - \cos\omega t)\times 2}{2\cdot f(\cos\omega t - 2)}\right| = \frac{f(4 - \cos\omega t)}{2 - \cos\omega t}$$

Now position of image w.r.t. origin is

$$\vec{r}_{I_0} = \vec{r}_{IM} + \vec{r}_{M_0} = \frac{f(2f - x)}{x - f} + 2f$$

$$\Rightarrow \vec{r}_{I_0} = \frac{2f^2 - fx + 2fx - 2f^2}{x - f} = \frac{f\cdot x}{x - f}$$

$$\Rightarrow \vec{r}_{I_0} = \frac{f\cdot\frac{f}{2}\cos\omega t}{\frac{f}{2}\cos\omega t - f} = \frac{f^2\cos\omega t}{f(\cos\omega t - 2)}$$

$$\Rightarrow \vec{r}_{I_0} = -\frac{f\cos\omega t}{2 - \cos\omega t}$$ (oscillating but not SHM)

Now velocity of image is

$$d\frac{\left(\vec{r}_{I_0}\right)}{dt} = v_I = \frac{(2 - \cos\omega t)f\omega\sin\omega t + f\cos\omega t\cdot\omega\sin\omega t}{(2 - \cos\omega t)^2}$$

$$\vec{v}_I = \frac{\omega f\sin\omega t(2 - \cos\omega t + \cos\omega t)}{(2 - \cos\omega t)^2} = \frac{2\omega f\sin t}{(2 - \cos\omega t)^2}$$

At $t = \frac{\pi}{3\omega}$ $\quad \vec{v}_I = \frac{2\omega f\sqrt{3}}{2\left(2 - \frac{1}{2}\right)^2} = \frac{\omega f\sqrt{3}\times 4}{9}$

Velocity of object $\frac{dx_0}{dt} = -\frac{f}{2}\omega\sin\omega t = -\frac{\omega f}{4}\sqrt{3}$

$$\therefore \quad V_{rel} = (v_2 - v_1) = \frac{4\omega f\sqrt{3}}{9} + \frac{\omega f\sqrt{3}}{4} = \sqrt{3}\omega f\left[\frac{4}{9} + \frac{1}{4}\right]$$

$$\Rightarrow v_{rel} = \frac{25}{36}\sqrt{3}\,\omega f$$

Magnification is always negative so always real image will be formed. For the time when object lies between $2f$ and f from mirror it will produce magnified image. For rest of the time it will be diminished image.

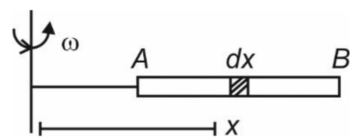
14. Answer (A, C)

15. Answer (A, C)

Hint for Q.Nos. 14 and 15 :

$eE = m\omega^2\cdot x$

Solution for Q.Nos. 14 and 15 :



Because of pseudo force, free charge would try to shift outward since free charges are electrons. So $F_{out} = m\omega^2 x$. Because of that electric field would induce in a way that $e\vec{E} = m\omega^2\cdot x$

$\Rightarrow \vec{E} = \frac{m\omega^2}{e}\cdot x$ (from A to B as electron has shifted towards B)

$$V_{AB} = \int_{l}^{l+L} Edx = \frac{m\omega^2}{e}\int xdx$$

$$\Rightarrow V_{AB} = \frac{m\omega^2}{2e}\left|x^2\right|_{l}^{l+L} = \frac{m\omega^2}{2e}L(L + 2l)$$

As we see $E = \frac{m\omega^2}{e}\cdot x$ implies that the field region (in rod and outside of it) would exist in x direction as per the equation then there must be uniform charge distribution.

16. Answer A(Q, T); B(R, S, T); C(P, R, S); D(P, Q)

Hint : For lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For mirror $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Solution : For lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For mirror $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

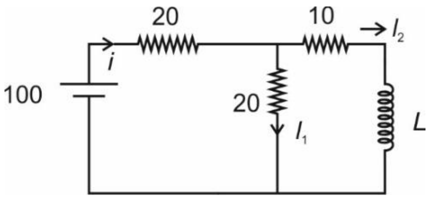
17. Answer A(Q); B(P, T); C(P, S); D(P, R, T)

Hint : $R_{eq}(t = 0) = 40\ \Omega$

$R_{eq}(t = \infty) = \frac{80}{3}\ \Omega$

transient *LR* circuit.

Solution :



$R_{eq}\ (t = 0) = 40\ \Omega$

$$R_{eq}(t=\infty) = \frac{80}{3}\Omega$$

Clearly at $t = 0$ current $i_1 = \frac{100}{40} = 2.5$ A

and at $t = \infty$ $I_1 = \frac{5}{4} = 1.25$ A

Current i_2 at $t = 0$ is zero at inductor and it will oppose the sudden change of current.

And i_2 at $t = \infty$ is $\frac{10}{4} = 2.5$ A

Power delivered by battery $p = \varepsilon i$

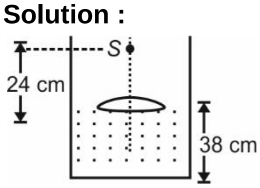
As i increases from 2.5 A to 3.75 A

So power delivered by battery increases from 250 watt to 375 watt.

18. Answer (22)

Hint : $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Solution :



$$\left.\begin{aligned}\frac{7}{4v_1} + \frac{1}{24} &= \frac{\left(\frac{7}{4} - 1\right)}{6} \\ \frac{4}{3v_2} - \frac{7}{4v_1} &= \frac{\left(\frac{4}{3} - \frac{7}{4}\right)}{\infty}\end{aligned}\right|\text{from water surface}$$

$$\therefore \quad \frac{4}{3v_2} + \frac{1}{24} = \frac{3}{24}$$

$$\Rightarrow \quad \frac{4}{3v_2} = \frac{1}{12}$$

$\Rightarrow$ $v_2 = 16$ cm

$\therefore$ Distance from bottom of tank is 38 – 16 = 22 cm.

19. Answer (20)

Hint : After first refraction from the plane surface the image seems to be at $\frac{3}{2}L$ from plane surface.

Solution : After first refraction from the plane surface p seems to be at $\frac{3}{2}L$ from plane surface.

Now for 2nd surface $\frac{1}{v} + \frac{\mu}{\left(R + \frac{3}{2}L\right)} = \frac{(1-\mu)}{-R}$

$\therefore$ $v = \infty$

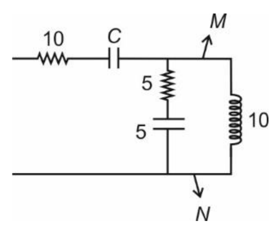
$$\therefore \quad \frac{\mu}{R + \frac{3}{2}L} = \frac{\mu - 1}{R}$$

$\Rightarrow$ $L = 20$ cm

20. Answer (25)

Hint : $\frac{1}{z'} = \frac{1}{5 - 5j} + \frac{1}{10j}$

Solution :



Let Z′ be the impedance across *MN*.

Then $\frac{1}{Z'} = \frac{1}{5 - 5j} + \frac{1}{10j}$

$$\Rightarrow \quad \frac{1}{Z'} = \frac{10j + 5 - 5j}{50j + 50} = \frac{5 + 5j}{50 + 50j}$$

$\Rightarrow$ $Z' = 10\ \Omega$

$$\therefore \quad \frac{1}{\omega C} = 20\ \Omega$$

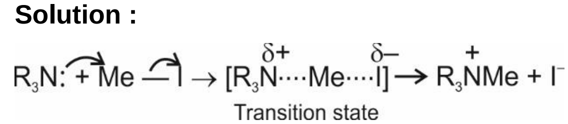
$$\Rightarrow \quad f = \frac{1}{2\pi \times 20 \times C} = 25\ \text{Hz}$$

PART - II (CHEMISTRY)

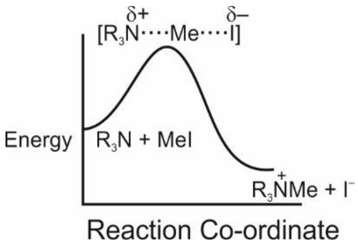
21. Answer (B)

Hint : Dispersion of charge decreases the energy.

Solution :



The general energy diagram of the above reaction is



On increasing polarity of the solvent, all charged species will get solvated. Thus their energies will be lowered. In the above reaction the reactants do not carry any charge and hence their energy remains unaffected by the increase in polarity of the solvent. But the energy of transition state is lowered due to its solvation. This results in decrease of energy barrier and hence increase in the rate of reaction.

22. Answer (D)

Hint : Cation present is Na^+.

Solution : $Fe(CH_3COO)_3$ is red in colour.

23. Answer (C)

Hint : $Fe(OH)_3$ and Na_2CrO_4 will be formed.

Solution : $Na_2O_2 + 2H_2O \rightarrow 2NaOH + H_2O_2$

$$2FeSO_4 + 4NaOH + H_2O_2 \rightarrow \underset{(Brown)}{2Fe(OH)_3\downarrow} + 2Na_2SO_4$$

$$Al_2(SO_4)_3 + NaOH \rightarrow 2NaAlO_2 + 3Na_2SO_4 + 4H_2O$$

$$Cr_2(SO_4)_3 + 10NaOH + 3H_2O_2 \rightarrow$$

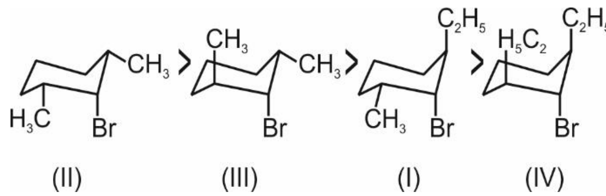
$$2Na_2CrO_4 + 3Na_2SO_4 + 8H_2O$$

$Fe(OH)_3$ is a brown residue, $NaAlO_2$ is a colourless solution and Na_2CrO_4 is a yellow solution.

24. Answer (A)

Hint : For the fast rate, back side of LG Should be less hindered.

Solution : In an S_N2 reaction, the leaving group must be in an axial position in order to allow backside attack to occur without steric hinderance from the cyclohexane ring. When the Br-atom is in axial position in the all cis-isomer, both the methyl groups are in equatorial positions, the structure (II) is most reactive as the approaching nucleophile experiences least crowding. In the all trans isomer (IV) both the ethyl groups are in axial positions providing the maximum crowding to the approaching nucleophile. Thus structure (IV) is least reactive. Structure (I) and (III) are cis-trans type. Their reactivity lies between those of (II) and (IV), structure (I) is less reactive than (III)as it has bulky ethyl group at the axial position



25. Answer (D)

Hint : Spectator ligand will affect the C — O bond length. Order of ligand field strength of the given ligand is

$CO \approx PF_3 > PCl_3 > PAr_3 > PMe_3$

Solution : The ligand PPh_3 is a weaker π acceptor than CO. As a result $d\pi$ electrons of metal in such mixed carbonyl will be drawn more towards CO than in pure metal carbonyl. If the Ph groups of PPh_3 are replaced by more electron attracting Cl or F groups, the tendency of $d\pi$ electrons of metal to move towards P increases. And if Ph groups of PPh_3 are replaced by electron releasing Me groups, the tendency of $d\pi$ electrons of metal to move towards P further decreases. As $d\pi$ electrons of metal move towards P more and less towards CO, the CO bond Order will be more or CO bond length will be less and vice versa.

26. Answer (A, B, C)

Hint :

Less hindered double bond will be more reactive.

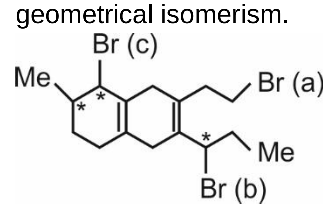
Solution :

(A) Loss of Br (a) atom in dehydrobromination reaction results in the formation of least stable alkene and hence most reactive towards hydrogenation.

(B) Dehydrohalogenation occurs.

(C) It has 3 stereocentres which means 8 optically active isomers

(D) The given compound has 3 chiral centres and disubstituted cyclic ring that show geometrical isomerism.



27. Answer (A, B, D)

Hint : Less hindered more rate.

Solution :

$$\underset{\text{Rate of } S_N2 \text{ reaction}}{\underline{(CH_3)_3C-Br < CH_3CH_2CH_2CH_2-Br < CH_3-CH_2-Br < CH_3-Br}}$$

28. Answer (B, C, D)

Hint : With Fe, NO exist in +1 O.S.

Solution :

(I) $$FeSO_4 + 5H_2O + NO \rightarrow \underset{(X)}{\left[Fe(H_2O)_5NO\right]^{2+}}$$

Oxidation state of Fe changes from +2 to +1 due to transfer of an electron from NO to Fe^{+2}. Electronic configuration of Fe^{+} is $3d^7$. It has three unpaired electrons and hence magnetic moment of (X) is $\sqrt{15}$ BM .

(II) $$Na_2[Fe(CN)_5NO] + Na_2S \rightarrow Na_4\underset{(Y)}{\left[Fe(CN)_5NOS\right]}$$

Oxidation state and hybridisation of Fe in the reactant and product (Y) of reaction (II) remains unchanged i.e. +2 and d^2sp^3 respectively.

29. Answer (A, C, D)

Hint : Fact based.

Solution :

$$2\overset{2+}{Zn} + K_4[Fe(CN)_6] \rightarrow \underset{\text{White ppt.}}{Zn_2[Fe(CN)_6]} + 4K^+$$

$$2\overset{2+}{Cd} + K_4[Fe(CN)_6] \rightarrow \underset{\text{White ppt.}}{Cd_2[Fe(CN)_6]} + 4K^+$$

$$2\overset{2+}{Cu} + K_4[Fe(CN)_6] \rightarrow \underset{\text{Brown ppt.}}{Cu_2[Fe(CN)_6]} + 4K^+$$

Al^{3+} ion does not form any precipitate with $K_4[Fe(CN)_6]$.

30. Answer (A, C, D)

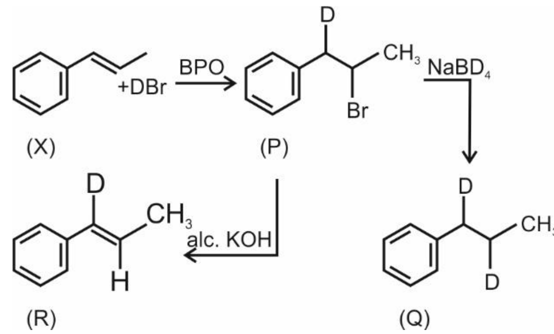
Hint : CFSE $\propto$ stability.

Solution :

1. In a transition group, stability increases down the group due to increase in effective nuclear charge.
2. NO_2^- is stronger ligand than NH_3.
3. Chelate complexes are more stable and as the number of cyclic rings increases, the stability of the complex increases.
4. For the same metal ion, stability of the complex increases with increase in oxidation state of the metal ion.

31. Answer (D)

Hint :



Solution :

Compound (P) has two dissimilar chiral C-atoms. It has two pair of enantiomers or four pair of diastereomers.

32. Answer (D)

Hint : Compound (Q) has two chiral centres.

Solution :

No. of optical Isomers = 2^n

(for unsymmetrical molecule) = $2^n = 4$

33. Answer (A)

Hint : Saytzeff elimination.

Solution : R is Ph — CD = CH — CH_3

34. Answer (A, B, D)

35. Answer (A, B, C)

Hint : Sodium salt gives violet colour solution with sodium nitroprusside.

Solution : P, Q and R are Na_2SO_3, $Na_2S_2O_3$ and Na_2S Respectively

$$\underset{(P)}{Na_2SO_3} + 2HCl \rightarrow 2NaCl + H_2O + \underset{(X)}{SO_2}(g)$$

$$\underset{(Q)}{Na_2S_2O_3} + 2HCl \rightarrow 2NaCl + \underset{(X)}{SO_2}(g) + S\downarrow + H_2O$$

$$\underset{(R)}{Na_2S} + 2HCl \rightarrow 2NaCl + \underset{(Y)}{H_2S}(g)$$

$SO_2 + 2H_2S \rightarrow 3S + 2H_2O$

$2H_2S + O_2 \rightarrow 2H_2O + 2S$

$H_2S + 2FeCl_3 \rightarrow 2FeCl_2 + 2HCl + S\downarrow$

$H_2S + (CH_3COO)_2Pb \rightarrow \underset{\text{Black}}{PbS}\downarrow + 2CH_3COOH$

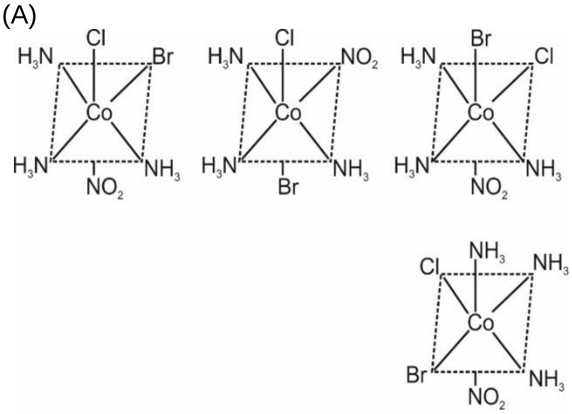
$Na_2S + Na_2[Fe(CN)_5NO] \rightarrow \underset{\text{Violet colour}}{Na_4[Fe(CN)_5NOS]}$

36. Answer A(P, R, T); B(Q, T); C(P, S, T); D(P, Q, T)

Hint :

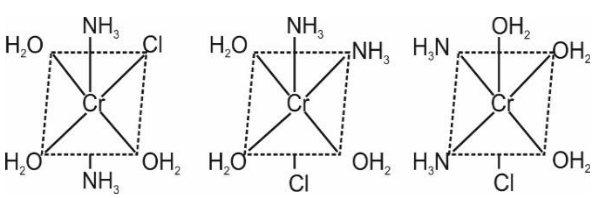
The possible stereoisomer of the given complexes are

(A)



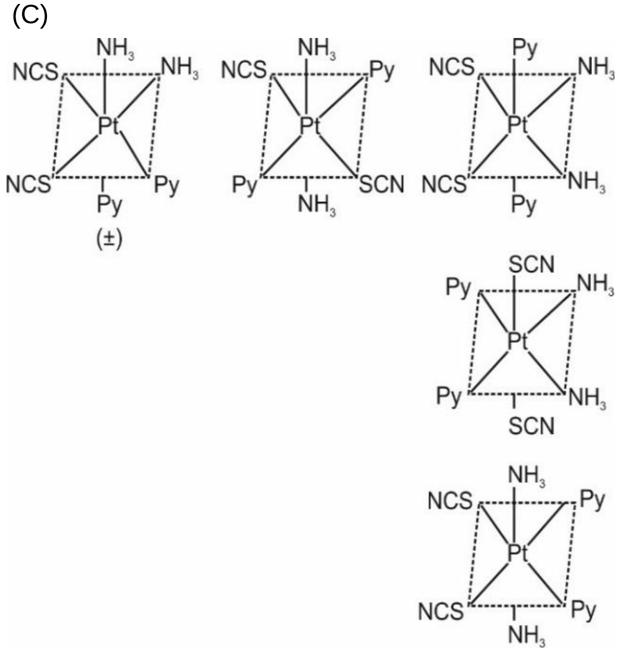
(±)

(B)

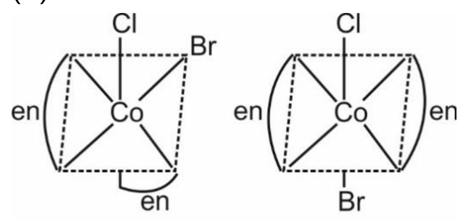


Solution :

(C)



(D)



(±)

37. Answer A(Q, R, S, T); B(P, Q, R); C(P, Q, R); D(P, R, T)

Hint : Fact based

Solution :

$Co^{2+} + 4KSCN \rightarrow \underset{\text{Blue colour}}{[Co(SCN)_4]^{2-}} + 4K^+$

$Co(NO_3)_2 + NaOH \rightarrow \underset{\text{Blue}}{Co(OH)(NO_3)}\downarrow + NaNO_3$

$Co(OH)(NO_3) + \underset{\text{Excess}}{NaOH} \xrightarrow{\text{Warm}} \underset{\text{Pink}}{Co(OH)_2}\downarrow + NaNO_3$

$Co^{2+} + 7NO_2^- + 2H^+ + 3K^+ \rightarrow \underset{\text{Yellow}}{K_3[Co(NO_2)_6]}\downarrow + NO\uparrow + H_2O$

$2Cu^{2+} + K_4[Fe(CN)_6] \rightarrow \underset{\text{Chocolate brown}}{Cu_2[Fe(CN)_6]}\downarrow + 4K^+$

$Cu^{2+} + 2NaOH \rightarrow \underset{\text{Bluish white}}{Cu(OH)_2}\downarrow + 2Na^+$

$2Cu^{2+} + 4KSCN \rightarrow \underset{\text{Black}}{2Cu(SCN)_2}\downarrow \rightarrow \underset{\text{White}}{2CuSCN}\downarrow + (SCN)_2$

$4Fe^{3+} + 3K_4[Fe(CN)_6] \rightarrow \underset{\text{Prussian Blue}}{Fe_4[Fe(CN)_6]_3} + 12K^+$

$Fe^{3+} + 3SCN^- \longrightarrow \underset{\text{Deep red colour}}{Fe(SCN)_3}$

$Fe^{3+} + 3NaOH \longrightarrow \underset{\text{Brown}}{Fe(OH)_3}\downarrow + 3Na^+$

$3Zn^{2+} + 2K_4[Fe(CN)_6] \rightarrow \underset{\text{White}}{K_2Zn_3[Fe(CN)_6]_2}\downarrow + 6K^+$

$Zn^{2+} + 2NaOH \rightarrow \underset{\text{White}}{Zn(OH)_2}\downarrow + 2Na^+$

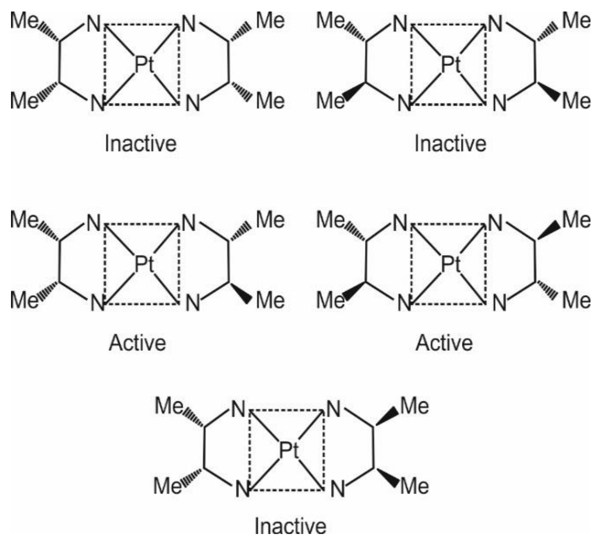
$Zn(OH)_2 + \underset{\text{(Excess)}}{2NaOH} \rightarrow \underset{\text{Water soluble}}{Na[Zn(OH)_4]}$

$Zn^{2+} + (NH_4)_2[Hg(SCN)_4] \rightarrow Zn[Hg(SCN)_4]\downarrow + 2NH_4^+$

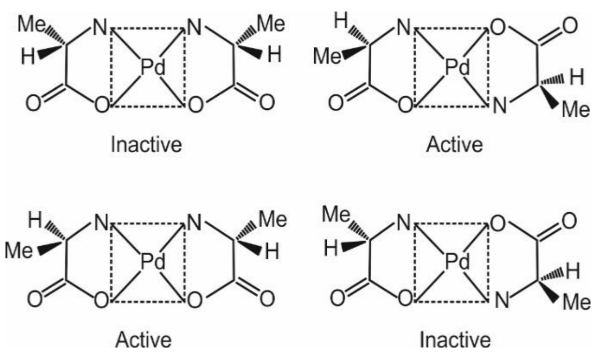
38. Answer (13)

Hint : Pt form square planar complex.

Solution :



$\therefore$ x = 7



$\therefore$ y = 6

x + y = 13

39. Answer (03)

Hint : $PbBr_2 \rightarrow$ Colourless

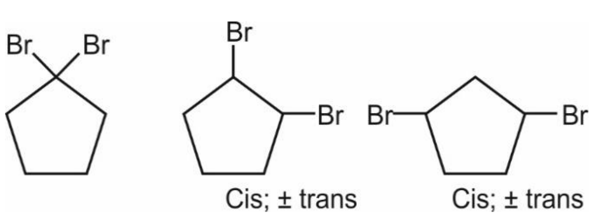
$MnS \rightarrow$ Buff

Solution : As_2S_5, PbI_2 and AgI are yellow coloured compound.

40. Answer (07)

Hint : D. B. E = 1.

Solution :

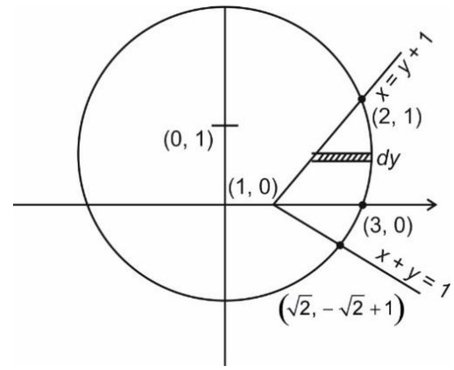


PART - III (MATHEMATICS)

41. Answer (A)

Hint : Area between $f(x)$ and $g(x)$ is $\int_{\alpha}^{\beta}(f(x)-g(x))dx$, where α and β are point of intersection of the curves

Solution :



$$A=\int_{-\sqrt{2}+1}^{1} x\,dy=\int_{-\sqrt{2}+1}^{0}\left(\sqrt{-y^2+2y+3}-(1-y)\right)dy+\int_{0}^{1}\left(\sqrt{-y^2+2y+3}-(y+1)\right)dy$$

$$\Rightarrow \int_{1-\sqrt{2}}^{1}\sqrt{4-(y-1)^2}\,dy+\int_{1-\sqrt{2}}^{0}(y-1)dy-\int_{0}^{1}(y+1)dy$$

$$\Rightarrow \left(\frac{y-1}{2}\sqrt{4-(y-1)^2}+\frac{4}{2}\sin^{-1}\left(\frac{y-1}{2}\right)\right)\Bigg|_{1-\sqrt{2}}^{1}+\left(\frac{y^2}{2}-y\right)\Bigg|_{1-\sqrt{2}}^{0}-\left(\frac{y^2}{2}+y\right)\Bigg|_{0}^{1}$$

$$\Rightarrow 0+0-\left(-\frac{1}{\sqrt{2}}(0)+2\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)+0+\left(\frac{1+2-2\sqrt{2}-2+2\sqrt{2}}{2}\right)-\left(\frac{1}{2}+1-0\right)$$

$$\Rightarrow -2\left(-\frac{\pi}{4}\right)+\left(\frac{1}{2}\right)-\frac{3}{2}\Rightarrow\left(\frac{\pi}{2}-1\right)\text{ sq. units}$$

42. Answer (A)

Hint : Degree is power of highest differential coefficient when expressed as polynomial in differential coefficients

Solution :

$$y'=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}-\frac{A}{\sqrt{1-x^2}}$$

$$\Rightarrow y'\sqrt{1-x^2}=2\sin^{-1}x-A$$

$$\Rightarrow y''\sqrt{1-x^2}+\frac{y'(-2x)}{\sqrt{1-x^2}}=\frac{2}{\sqrt{1-x^2}}$$

$$\Rightarrow y''\left(1-x^2\right)-2xy'=2$$

Hence degree = 1

43. Answer (B)

Hint : $(9-16x^2)^{3/2} = x^3\left(\frac{9}{x^2}-16\right)^{3/2}$

Solution :

$$\int \frac{dx}{x^3\left(\frac{9}{x^2}-16\right)^{3/2}}$$

Put $\frac{9}{x^2}-16=t^2$

$$-\frac{18}{x^3}dx = 2t\,dt$$

$$\Rightarrow -\frac{1}{18}\int\frac{2t\ dt}{t^3}$$

$$=\frac{1}{9t}+c=\frac{x}{9(9-16x^2)^{1/2}}+c$$

44. Answer (A)

Hint : $\alpha+\beta=-t^2$ and $\alpha\beta=-2t$

Solution :

$$f(t)=\int_{-1}^{2}\left(x^2+\left(\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}\right)x+\frac{1}{\alpha^2\beta^2}+\frac{1}{\alpha\beta}\right)dx$$

$$=\frac{x^3}{3}+\left(\frac{t^4-2(-2t)}{4t^2}\right)\frac{x^2}{2}+\left(\frac{1}{4t^2}-\frac{1}{2t}\right)x\Bigg|_{-1}^{2}$$

$$=\left(\frac{8}{3}+\frac{1}{3}\right)+\left(\frac{t^2}{4}+\frac{1}{t}\right)\cdot\frac{3}{2}+\left(\frac{1}{4t^2}-\frac{1}{2t}\right)3$$

$$=3+\frac{3t^2}{8}+\frac{3}{4t^2}$$

$$f'(t)=\frac{6t}{8}-\frac{6}{4t^3}=0\Rightarrow t=\pm 2^{1/4}$$

$$f(t)_{\min}=3+\frac{3\sqrt2}{8}+\frac{3}{4\sqrt2}=3+\frac{3\sqrt2}{4}$$

45. Answer (B)

Hint : Put $\sec x+\tan x=t$

Solution :

$$\Rightarrow\ \sec x\,dx=\frac{dt}{t}\ \text{ and }\ \sec x-\tan x=\frac{1}{t}$$

$$I=\int_0^{\pi/4}(2\sec x)(2\sec x)^{12}dx=\int_1^{1+\sqrt2}\frac{2}{t}\left(t+t^{-1}\right)^{12}dt$$

46. Answer (A, B, D)

Hint : If $x\in(0,1)\Rightarrow x^2>x^3$ and if $x\in(1,2)$ $x^3>x^2$

Solution :

If $x\in(0,1)$ $\quad(2020)^{x^2}>(2020)^{x^3}$

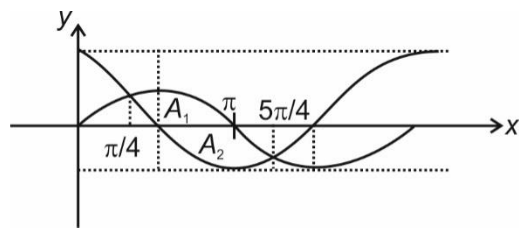
and if $x\in(1,2)$ $\quad(2020)^{x^3}>(2020)^{x^2}$

$\Rightarrow$ $I_4>I_3>I_1>I_2$

47. Answer (C, D)

Hint : Area between f(x) and $g(x)$ is $\int_\alpha^\beta (f(x)-g(x))dx$, where α and β are point of intersection of $f(x)$ and $g(x)$

Solution :



$$\Rightarrow\quad A_1=\int_{\pi/4}^{\pi/2}(\sin x-\cos x)dx+\int_{\pi/2}^{\pi}\sin x\,dx=-\cos x-\sin x\Big|_{\pi/4}^{\pi/2}$$

$$+(-\cos x)\Big|_{\pi/2}^{\pi}=-1+\frac{1}{\sqrt2}+\frac{1}{\sqrt2}+1-0=\sqrt2$$

$$\text{And } A_1+A_2=\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)dx=-\cos x-\sin x\Big|_{\pi/4}^{5\pi/4}$$

$$=\frac{1}{\sqrt2}+\frac{1}{\sqrt2}+\frac{1}{\sqrt2}+\frac{1}{\sqrt2}=2\sqrt2$$

$$\Rightarrow\quad A_2=\sqrt2 \text{ and } A_1=\sqrt2\Rightarrow A_1A_2=2$$

48. Answer (A, D)

Hint : Multiply and divide by $\sec^2 x$

Solution :

$$I=\int\left(\sin^{-11/3}x\cdot\cos^{-1/3}x\right)\cdot\frac{\sec^4 x}{\sec^4 x}dx$$

$$\Rightarrow\quad I=\int\frac{(1+\tan^2 x)\sec^2 x\,dx}{\tan^{11/3}x}$$

$\Rightarrow$ Put $\tan x=t\Rightarrow\sec^2 x\,dx=dt$

$$\Rightarrow\quad I=\int\left(\frac{1}{t^{11/3}}+\frac{t^2}{t^{11/3}}\right)dt$$

$$=-\frac{3}{8}(\tan x)^{-8/3}-\frac{3}{2}(\tan x)^{-2/3}+c$$

49. Answer (A, B, C)

Hint : Form linear differential equation

Solution :

$$\left(e^{\tan^{-1}y}-x\right)\frac{dy}{dx}=1+y^2$$

$$\Rightarrow \quad \frac{dx}{dy}=\frac{e^{\tan^{-1}y}-x}{1+y^2}$$

$$\Rightarrow \quad \frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$$

$$\text{I.F.}=e^{\int\frac{dy}{1+y^2}}=e^{\tan^{-1}y}$$

$$\Rightarrow \quad \int d\left(x\,e^{\tan^{-1}y}\right)=\int\frac{e^{2\tan^{-1}y}}{1+y^2}dy$$

$$\Rightarrow \quad x\,e^{\tan^{-1}y}=\frac{1}{2}e^{2\tan^{-1}y}+c$$

$$\downarrow\left(\frac{1}{2},0\right)$$

$$\Rightarrow \quad 2x=e^{\tan^{-1}y}$$

$$\Rightarrow \quad x_0=\frac{e^{\pi/3}}{2}$$

50. Answer (C, D)

Hint : Substitute $x = \sin t$

Solution :

$$\int_0^1 f(x)dx=\int_0^{\pi/2} f(\sin t)\cos t\,dt=I$$

$$=\int_0^{\pi/2} f(\cos t)\sin t\,dt=I$$

$$\Rightarrow \quad 2I=\int_0^{\pi/2}\left(f(\sin t)\cos t+f(\cos t)\sin t\right)dt$$

$$\Rightarrow \quad 2I\le\int_0^{\pi/2}1\,dx$$

$$I\le\frac{\pi}{4}$$

51. Answer (A, C)

52. Answer (C, D)

53. Answer (A, B, C)

Hint for Q.Nos. 51 to 53 :

$xdy + ydx = d(xy)$ and $\dfrac{xdy-ydx}{x^2}=d\left(\dfrac{y}{x}\right)$

Solution for Q.Nos. 51 to 53 :

$$\frac{xy\,dx+x^2dy}{x^2}=\frac{y^3\left(x\,dy-y\,dx\right)}{x^2}$$

$$\Rightarrow \quad \int d(xy)=\int x^2y^2\frac{y}{x}d\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \int\frac{d(xy)}{(xy)^2}=\int\frac{y}{x}d\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad -\frac{1}{xy}=\frac{1}{2}\left(\frac{y}{x}\right)^2+c$$

$$\downarrow(4,-2)$$

$$\Rightarrow \quad c=0$$

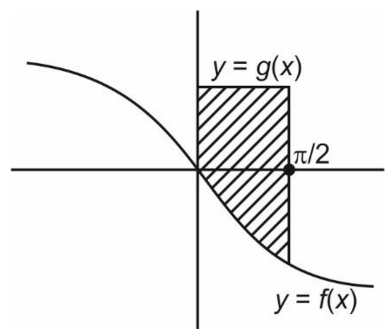
$$f(x)\equiv y=(-2x)^{1/3}$$

Also $g'(x) = \sin^{-1}\sin x\cdot 2\sin x\cos x$

$+ \cos^{-1}\cos x\ 2\cos x(-\sin x) = 0$

$\Rightarrow$ $g(x)$ = constant

Put $x=\dfrac{\pi}{4}$ we get $g(x)\equiv y=\dfrac{3\pi}{16}\ \forall\ x\in\left[0,\dfrac{\pi}{2}\right]$



$$A=\int_0^{\pi/2}\left(g(x)-f(x)\right)dx$$

$$A=\frac{3\pi^{4/3}}{8}\left[1+\frac{\pi^{2/3}}{4}\right]$$

54. Answer (A, B, C)

55. Answer (C, D)

Hint for Q.Nos. 54 and 55 :

Substitute expression in $\tan\theta$.

Solution for Q.Nos. 54 and Q.55 :

$$\int\frac{\left(\sin^{3/2}\theta+\cos^{3/2}\theta\right)d\theta}{\sqrt{\sin^3\theta\cos^3\theta\sin(\theta+\alpha)}}$$

$$\Rightarrow \int \frac{\sin^{3/2}\theta\, d\theta}{\sin^{3/2}\theta\sqrt{\cos^3\theta\cdot(\sin\theta\cos\alpha+\cos\theta\sin\alpha)}}$$

$$+\int \frac{\cos^{3/2}\theta\, d\theta}{\cos^{3/2}\theta\sqrt{\sin^3\theta(\sin\theta\cos\alpha+\cos\theta\sin\alpha)}}$$

$$\Rightarrow \underbrace{\int \frac{d\theta}{\cos^2\theta\sqrt{(\tan\theta\cos\alpha+\sin\alpha)}}}_{I_1}$$

$$+\underbrace{\int \frac{d\theta}{\sin^2\theta\sqrt{(\cos\alpha+\cos\theta\sin\alpha)}}}_{I_2}$$

For I_1 put $\tan\theta \cos\alpha + \sin\alpha = t^2$

$\Rightarrow$ $\cos\alpha \sec^2\theta\, d\theta = 2t\, dt$

For I_2 put $\cos\alpha + \cot\theta \sin\alpha = \mu^2$

$\Rightarrow$ $-\text{cosec}^2\theta\cdot\sin\alpha\, d\theta = 2\mu\, d\mu$

$$\Rightarrow \int\frac{2t\,dt}{\cos\alpha\cdot t}-\int\frac{2\mu\, d\mu}{\sin\alpha\,\mu}$$

$$\Rightarrow 2\sec\alpha\sqrt{\sin\alpha+\tan\theta\cos\alpha}-2\,\text{cosec}\,\alpha$$

$$\sqrt{\cos\alpha+\cot\theta\sin\alpha}+c$$

$\therefore$ $f(\theta) = \sin\alpha + \cos\alpha \tan\theta$

and $g(\theta) = \cos\alpha + \cot\theta \sin\alpha$

$f(\alpha) = 2\sin\alpha \in (0, 2)$

$g(\alpha) = 2\cos\alpha$

$$\Rightarrow f(\alpha)+g(\alpha)\in\left(2, 2\sqrt{2}\right]$$

56. Answer A(P, Q); B(P, R, S, T); C(R, T); D(R, S, T)

Hint : Applying properties of integrals

Solution :

(A) $5\int_0^1 \sin\{x\}dx = 5\int_0^1 \sin x\, dx = 5(-\cos x)\Big|_6^1$

$$=5(1-\cos 1)$$

(B) $\left|\int_{-5}^{5}(-1)dx\right| = 10$

(C) $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^4}{n^5}=\int_0^1 x^4dx=\frac{x^5}{5}=\frac{1}{5}$

(D) $\lim_{x\to 0}\frac{(e^{2x}-1)}{x^n}\cdot 2x = \lim_{n\to\infty}\frac{e^{2x}-1}{2x}\times\frac{4x^2}{x^n}$

$\Rightarrow$ $n \le 3$ for limit to be finite

57. Answer A(P, S); B(S); C(Q, S, T); D(Q, S, T)

(A) **Hint :** Convert all terms into sin$x$ and cos$x$

$$I=\int\frac{(\sin^2 x+\cos^2 x)dx}{\sin^4 x\cos^2 x}$$

$$=\int\left(\sec^2 x+2\,\text{cosec}^2 x+\frac{\cos^2 x}{\sin^4 x}\right)dx$$

$$=\tan x-2\cot x-\frac{\cot^3 x}{3}+k$$

$$A=1, B=-2 \text{ and } c=-\frac{1}{3}$$

$\Rightarrow$ $4A + B + 3C = 1$

(B) **Hint :** Distribute x^{27} in both brackets

$$g(x)=\int(x^4+x^5+x^6)^6(6x^5+5x^4+4x^3)dx$$

Put $x^6 + x^5 + x^4 = t$

$\Rightarrow$ $(6x^5 + 5x^4 + 4x^3)dx = dt$

$$=\frac{1}{7}(x^6+x^4+x^5)^7+c$$

As $g(0) = 0$ $\quad g(x)=\frac{(x^6+x^4+x^5)^7}{7}$

$\Rightarrow$ $g(1)=\frac{3^7}{7}$

(C) $\int_0^{\pi/2} 12\sin^3 x\, G x\, d x = 12\cdot\frac{1}{4}=3$

(D) $I=\underbrace{\int_{-4}^{-5}e^{(x+5)^2}dx}_{I_1}+\underbrace{k\int_{1/3}^{3/2}e^{(3x-2)^2}dx}_{I_2}$

In I_1 put $x + 5 = y$ and in I_2 put $3x - 2 = -t$

$$I=\int_1^0 e^{y^2}dy+\frac{k}{3}\int_1^0 e^{y^2}(-dt)=0$$

$\therefore$ $k = 3$

58. Answer (32)

Hint : $\int_{-a}^{a} f(x)dx = 0$ if $f(x)$ is odd.

Solution :

$$I = \int_{-\pi/4}^{\pi/4} \sec^2 x\, dx$$

$$= 2\int_{0}^{\pi/4} \sec^2 x\, dx$$

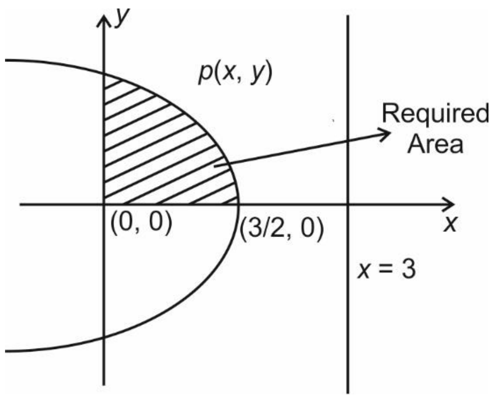
$$= 2\tan x\Big|_0^{\pi/4} = 2$$

$\Rightarrow$ $I^5 = 2^5 = 32$

59. Answer (30)

Hint : Distance from origin ≤ distance from line $x = 3$

Solution :



Given $\sqrt{x^2+y^2} \le |x-3|$

$\Rightarrow$ $x^2 + y^2 \le x^2 - 6x + 9$

$\Rightarrow$ $y^2 \le -6x + 9$

$\Rightarrow$ $y^2 \le -6\left(x - \frac{3}{2}\right)$

$\Rightarrow$ $A = \int_0^{3/2} \sqrt{9-6x}\, dx$

$$= \left.\frac{(9-6x)^{3/2}}{\frac{3}{2}\cdot(-6)}\right|_0^{3/2}$$

$$= 0 + \frac{(9)^{3/2}}{9} = \frac{27}{9} = 3 \text{ sq. units}$$

$\Rightarrow$ $10A = 30$

60. Answer (19)

Hint : Integration by parts

Solution :

$$I_n = \int_0^1 1\cdot\frac{1}{(1+x^2)^n}dx$$

$$= \left.\frac{x}{(1+x^2)^n}\right|_0^1 + n\int_0^1 \frac{2x^2dx}{(1+x^2)^{n+1}}$$

$$= \frac{1}{2^n} + 2n\left[\int_0^1 \frac{dx}{(1+x^2)^n} - \int_0^1 \frac{dx}{(1+x^2)^{n+1}}\right]$$

$$= \frac{1}{2^n} + 2n\ I_n - 2n\ I_{n+1}$$

$2n\, I_{n+1} = 2^{-n} + (2n - 1)I_n$

Put $n = 10$, we get

$20\, I_{11} = 2^{-10} + 19\, I_{10}$

□ □ □