# All India Aakash Test Series for JEE (Advanced)-2020

## TEST - 2A (Paper-1) - Code-C

Click here to access Code-D

**Test Date : 24/11/2019**

## ANSWERS

| PHYSICS | | CHEMISTRY | | MATHEMATICS | |
|---|---|---|---|---|---|
| 1. | (C) | 21. | (B) | 41. | (B) |
| 2. | (B) | 22. | (A) | 42. | (C) |
| 3. | (B) | 23. | (B) | 43. | (B) |
| 4. | (D) | 24. | (B) | 44. | (D) |
| 5. | (D) | 25. | (B) | 45. | (A) |
| 6. | (C) | 26. | (D) | 46. | (B) |
| 7. | (A, B, D) | 27. | (B, D) | 47. | (B, C, D) |
| 8. | (B) | 28. | (A, B, D) | 48. | (A, B) |
| 9. | (B, D) | 29. | (A, B, C) | 49. | (A, B, C) |
| 10. | (B, D) | 30. | (C) | 50. | (C, D) |
| 11. | (B, C, D) | 31. | (A) | 51. | (A, D) |
| 12. | (A) | 32. | (C) | 52. | (A) |
| 13. | (C) | 33. | (C) | 53. | (D) |
| 14. | (C) | 34. | (B) | 54. | (B) |
| 15. | (D) | 35. | (C) | 55. | (C) |
| 16. | A → (P, S) | 36. | A → (Q, S, T) | 56. | A → (Q, R, S) |
| | B → (Q, R) | | B → (P, R, S) | | B → (Q) |
| | C → (P, S) | | C → (P, R, S, T) | | C → (R, S) |
| | D → (P, R) | | D → (Q, R, S) | | D → (P, T) |
| 17. | A → (P, T) | 37. | A → (Q, R, T) | 57. | A → (S) |
| | B → (Q, R) | | B → (P) | | B → (Q, R, S, T) |
| | C → (R, S, T) | | C → (R, S, T) | | C → (R) |
| | D→ (Q, R) | | D → (T) | | D → (P, Q, R, S, T) |
| 18. | (16) | 38. | (25) | 58. | (36) |
| 19. | (50) | 39. | (04) | 59. | (16) |
| 20. | (29) | 40. | (12) | 60. | (45) |

# HINTS & SOLUTIONS

## PART - I (PHYSICS)

1. Answer (C)

**Hint:** Second overtone contains 3 loops

**Solution:**

$$\frac{\lambda}{2}=\frac{2}{3}\Rightarrow\lambda=\frac{4}{3}\text{m}$$

Amp. = 2$A$ sin $kx$ = $A_{max}$ sin(kx)

$$\therefore A=(A_{\max})\sin\left(\frac{2\pi\times3}{4}\times\frac{1}{6}\right)$$

$$\Rightarrow A=(2)\times\left(\frac{1}{\sqrt{2}}\right)\text{mm}$$

$$=\sqrt{2}\text{ mm}$$

2. Answer (B)

**Hint :** At maximum temperature $\frac{dT}{dV}=0$

**Solution :**

$$\left[P_0+(1-\alpha)V^2\right]V=nRT$$

$$\Rightarrow T=\frac{P_0V+(1-\alpha)V^3}{nR}$$

$$\therefore\frac{dT}{dV}=0\text{ at }V^2=\frac{P_0}{3(\alpha-1)}$$

$$\therefore P=P_0+(1-\alpha)\times\frac{P_0}{3(\alpha-1)}$$

$$\Rightarrow P=\frac{2P_0}{3}$$

3. Answer (B)

**Hint :** $Q = Q_0\, e^{-t/\tau}$ during discharging

**Solution :**

$$Q_{.0}=CV_0, C_2\left(\frac{C}{K}\right)$$

$$\therefore V_2=\frac{CV_0}{\left(\frac{C}{K}\right)}=KV_0$$

$$\tau=R\times C_2=\frac{RC}{K}$$

$$\therefore V=V_2e^{-t/\tau}$$

$$\Rightarrow\frac{V_0}{2}=KV_0\times e^{-\frac{t}{\tau}}$$

$$\Rightarrow\frac{1}{2K}=e^{-t/\tau}$$

$$\Rightarrow\ln(2K)=\frac{t}{\tau}$$

$$\Rightarrow t=\tau\ln(2K)$$

$$t=\frac{RC}{K}\ln(2K)$$

4. Answer (D)

**Hint:** Use reverse symmetry concept

**Solution:**

Using KVL and KCL, we get

$$R_{eq}=\frac{2R_1R_2+R_2R_3+R_3R_1}{(R_1+R_2+2R_3)}$$

$$=\frac{2\times(2\times3)+(3\times1)+(1\times2)}{(2+3+1\times2)}$$

$$=\frac{12+3+2}{7}=\frac{17}{7}\Omega$$

5. Answer (D)

**Hint:** Heat current remains constant

**Solution :**

$$\frac{\frac{(T_1-T)}{L}}{k2\pi a\times\left(\frac{a+b}{2}\right)}=\frac{\frac{T_1-T_2}{L}}{k\times\pi a\times b}$$

$$\Rightarrow T=\frac{T_1a+T_2b}{(a+b)}$$

6. Answer (C)

**Hint :** $E_{\text{axis}}=\frac{qx}{4\pi\varepsilon_0\left(R^2+x^2\right)^{3/2}}$

**Solution:**

$$E=\frac{q}{4\pi\varepsilon_0d^2}-\frac{q\times d}{4\pi\varepsilon_0\left(d^2+R^2\right)^{3/2}}$$

$$=\frac{3qR^2}{8\pi\varepsilon_0d^4}$$

7. Answer (A, B, D)

**Hint:** Use KVL and KCL.

**Solution :**

$$R_{eq}=1+\frac{20}{9}=\frac{29}{9}\ \Omega$$

$$\therefore I_0=\frac{58}{(29/9)}=18\text{ A}$$

$\therefore 1_{(2\Omega)} = \frac{18}{2} = 9$ A

$1_{(3\Omega)} = \frac{6}{6+3} \times 9 = 6$ A

$I_{(5\Omega)} = \frac{4}{9} \times (18) = 8$ A

$I_{(4\Omega)} = \frac{5}{9} \times (18) = 10$ A

$\therefore V_{(4\Omega)} = 4 \times 10 = 40\,\text{V}$

$P_{(5\Omega)} = 8^2 \times 5 = 320\,\text{W}$

8. Answer (B)

**Hint :** Use Gauss's law

**Solution :**

σ on outer surface becomes uniform. Potential at outside points is only due to charge on outer surface of shell.

$\therefore V_A = V_B$

9. Answer (B, D)

**Hint :** Apparent wavelength changes when source moves.

**Solution :**

$$f' = \frac{(340-10)}{(340-20)} \times (200) = 206\,\text{Hz}$$

$$\lambda' = \lambda_0 = V_s \times T = \frac{340}{200} - 20 \times \frac{1}{200} = 1.6\text{ m}$$

10. Answer (B, D)

**Hint :** $V_{rms}^2 = \frac{\int u^2 dN}{N}$

**Solution :**

$$N = \text{Area} = \frac{1}{2} \times 10 \times 10 = 50$$

$$\frac{dN}{du} = u + 10$$

$$\therefore V_{rms}^2 = \frac{\int u^2 \times (10-u)\,du}{N} = \frac{\int_0^{10} (10u^2 - u^3)\,du}{50}$$

$$V_{rms}^2 = \frac{1000 \times (4-3)}{12 \times 50} = \frac{2500}{3 \times 50}$$

$$\Rightarrow V_{rms} = \sqrt{\frac{50}{3}}\text{ m/s}$$

11. Answer (B, C, D)

**Hint :** Use KVL and KCL.

**Solution :**

$Q_{total}$ = 180 – 70 = 110 μC

$$q_A = \frac{2}{2+6+3} \times (110) = 20\ \mu\text{C}$$

$$q_B = \frac{6}{2+3+6} \times (110) = 60\ \mu\text{C}$$

$$q_C = \frac{3}{2+6+3} \times (110) = 30\ \mu\text{C}$$

$$\Delta q_S = (20+30) - (-70) = 120\ \mu\text{C}$$

12. Answer (A)

**Hint:** Flux is proportional to charge

**Solution :**

$$\frac{2\pi(1-\cos\alpha)}{4\pi} \times \left(\frac{q_1}{\varepsilon_0}\right)$$

$$= \frac{2\pi(1-\cos\beta)}{4\pi} \times \left(\frac{q_2}{\varepsilon_0}\right)$$

$$\Rightarrow \frac{q_1}{q_2} = \frac{1-\cos\beta}{1-\cos\alpha} = \frac{1-0}{1-\frac{1}{2}} = 2$$

13. Answer (C)

**Hint :** Flux is proportional to charge

**Solution:**

$q_1 = 3q_2$

⇒ one third of total flux of $q_1$ will terminate at $q_2$

$$\therefore \frac{4\pi}{3} = 2\pi(1 - \cos\alpha_{max})$$

$$\Rightarrow \cos(\alpha_{max}) = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\Rightarrow \tan(\alpha_{max}) = 2\sqrt{2} \quad \Rightarrow \alpha_{max} = \tan^{-1}(2\sqrt{2})$$

14. Answer (C)

**Hint :** $A \to B$ Isochoric $C \to D$ Isochoric

$B \to C$ Isothermal $D \to A$ Isothermal

15. Answer (D)

**Hint :** $W$ isothermal $= nRT_0 \ln\left(\frac{V_2}{V_1}\right)$

**Solution of Q.Nos. 14 and 15**

$$W_{BC} = 2P_0V_0 \ln\left(\frac{V_C}{V_B}\right) = -P_0V_0 \ln(2)$$

$$\Rightarrow V_C = \frac{V_0}{\sqrt{2}}$$

$$\therefore P_C = \frac{2P_0V_0}{V_C} = 2\sqrt{2}P_0$$

$$\therefore W_{DA} = \left(\sqrt{2}P_0\right)\left(\frac{V_0}{\sqrt{2}}\right)\ln\left(\sqrt{2}\right) = \frac{P_0V_0}{2}\ln(2)$$

$$\therefore W_{ABCDA} = 0 + -P_0V_0\ln(2) + 0 + \frac{1}{2}P_0V_0\ln(2)$$

$$= -\frac{P_0V_0}{2}\ln(2)$$

16. Answer A(P, S); B(Q, R); C(P, S); D(P, R)

**Hint :** Capacitance increases due to slab.

**Solution :**

Total capacitance increases, so charge on $A$ increases as well as voltage increases.

∴ Voltage on $B$ decreases. So, charge on it decreases

∴ Charge on $C$ and $D$ increases

17. Answer A(P, T); B(Q, R); C(R, S, T); D(Q, R)

**Hint :** Use Gauss's law.

**Solution :**

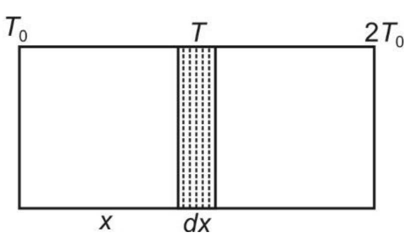
Electric field is uniform in spherical cavity in sphere and in cylindrical cavity in cylinder.

18. Answer (16)

**Hint :** Use $PV = nRT$

**Solution :**

$$T = T_0 + \frac{T_0}{L}x$$




$$\therefore \int dn = \int_{x=0}^{L} \frac{P \times Adx}{R\left(T_0 + \frac{T_0}{L}x\right)}$$

$$\Rightarrow n = \frac{PAL}{RT_0}\ln(2)$$

$$\Rightarrow n = \frac{PAL}{4RT_0}\ln(16)$$

∴ 16

19. Answer (50)

**Hint :** Voltmeter are not ideal

**Solution :**

Let resistance of each voltmeter be $R_0$

∴ $Ri' = 20\, R_0\, (I - i)$ ……………….(i)

and $2Ri' = 30$ ……………….(ii)

$$\Rightarrow i' = \frac{3}{4}i, \quad \therefore\ i_{(V_2)} = I - i' = I - \frac{3i}{4}$$

$$\therefore 2Ri' = 30 = R_0(I - i) = R_0\left(I - \frac{3i}{4}\right)$$

$$\Rightarrow i = 400\ \mu\text{A}$$

$$\therefore R = \frac{20}{400 \times 10^{-6}} = 50 \times 10^3\,\Omega$$

$= 50\ \text{k}\Omega$

20. Answer (29)

**Hint:** $BC$ is isothermal

**Solution :**

$$(3P_0) \times V_C = P_0 \times V_0$$

$$\Rightarrow V_C = \frac{V_0}{3}$$

∵ $CA$ is a adiabatic.

$$\therefore\ (3P_0) \times \left(\frac{V_0}{3}\right)^{r} = \left(\frac{P_0}{2}\right)(V_0)^{r}$$

$$\Rightarrow\ \gamma = \frac{\ln 6}{\ln 3} = \frac{\ln 2 + \ln 3}{\ln 3} = \frac{18}{11}$$

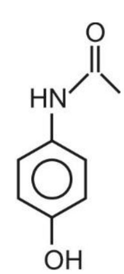
∴ $p + q = 18 + 11 = 29$

## PART - II (CHEMISTRY)

21. Answer (B)

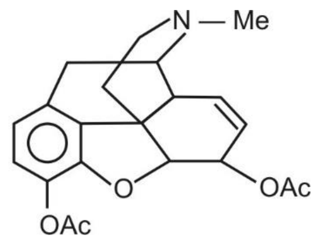
**Hint :**

Paracetamol

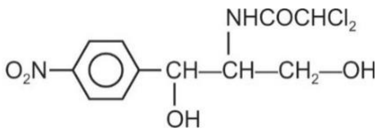


**Solution :**

Heroin




Chloramphenicol




22. Answer (A)

**Hint :** A paired with T (A = T)

**Solution :**

G paired with C (G ≡ C)

23. Answer (B)

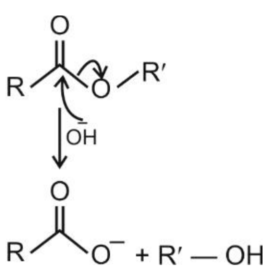
**Hint :** In DMF $S_N2$ mechanism is favoured during nucleophilic substitution reaction.

**Solution :**

Electron withdrawing group increases the tendency of $S_N2$.

24. Answer (B)

**Hint :** Hydrolysis of ester under alkaline condition occurs as



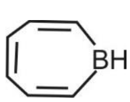
**Solution :**

Greater the extent of electron withdrawing strength of R, greater will be the rate of reaction

25. Answer (B)

**Hint :** Compound which are planar, has (4n + 2) $\pi e^-$ are aromatic

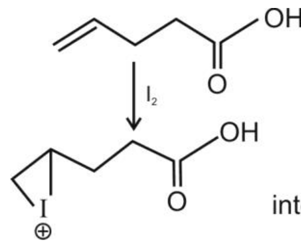
**Solution :**




Boron has vacant 2*p* orbital hence planar ($sp^2$) and has $6\pi e^-$

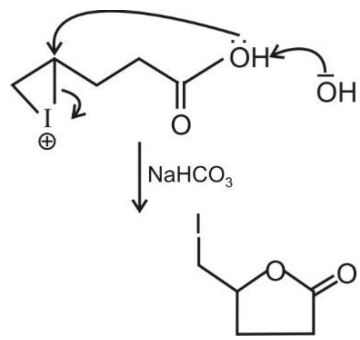
26. Answer (D)

**Hint :**



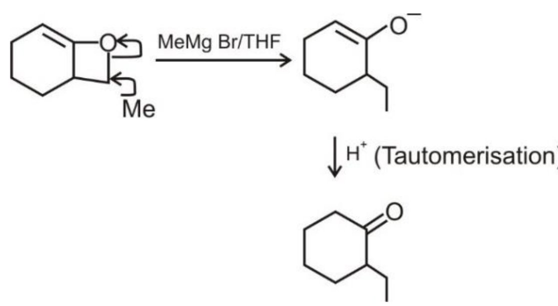
intermediate

**Solution :**

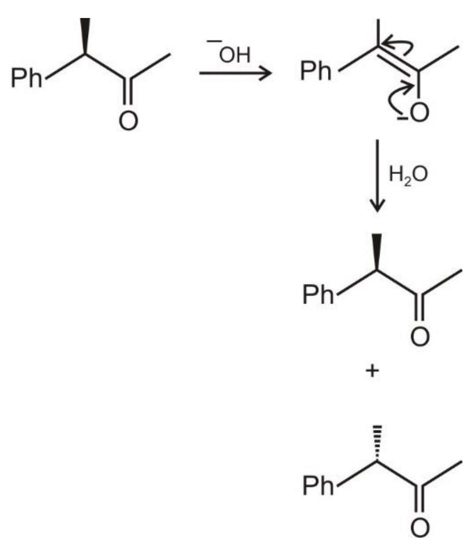


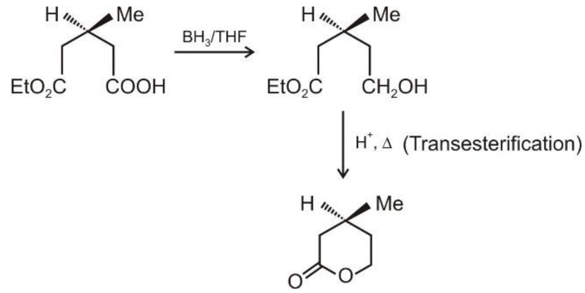

27. Answer (B, D)

**Hint :**



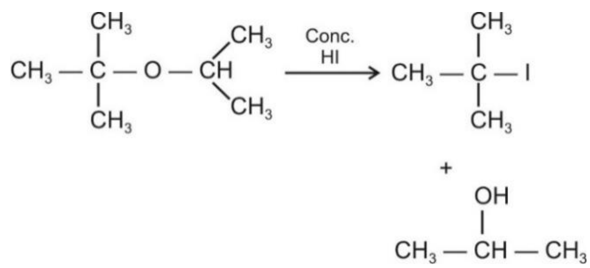

**Solution :**






28. Answer (A, B, D)

**Hint :**



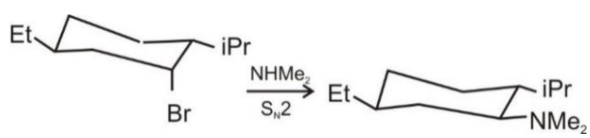

**Solution :**

Formed alcohol is 2°

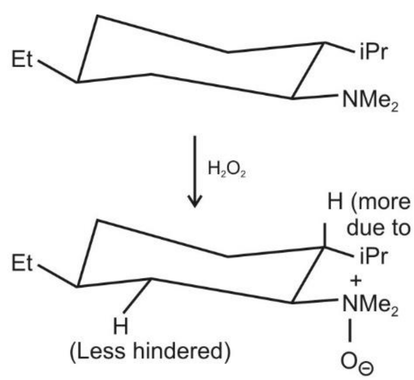
$$CH_3-\underset{\underset{CH_3}{|}}{CH}-OH \xrightarrow{Br_2/NaOH} CH_3-COO^- + CHBr_3 + NaBr$$

29. Answer (A, B, C)

**Hint :**

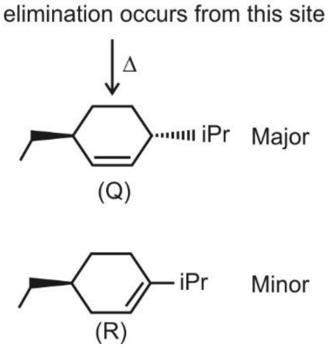



**Solution :**




∴ Major elimination occurs from this site

[This is cope elimination, which occurs through syn elimination mechanism from the less hindered site]




30. Answer (C)

**Hint :** Hoffmann bromamide reaction.

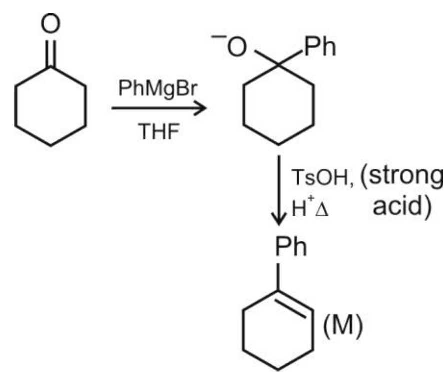
**Solution :**

$$Ph-CH_2-\overset{\overset{O}{||}}{C}-NH_2 \xrightarrow[NaOH]{Br_2} Ph-CH_2-NH_2$$

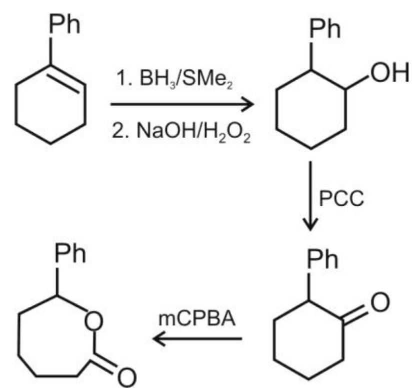
T

31. Answer (A)

**Hint :**




**Solution :**




32. Answer (C)

**Hint :** $SO_2 + O_3 \longrightarrow SO_3 + O_2$

$O_3$ is consumed by $SO_2$ only

**Solution :**

$$NO + \dot{O} \longrightarrow NO_2$$

So more of $O_3$ is consumed

33. Answer (C)

**Hint :** $CCl_2F_2$ is Freon-12

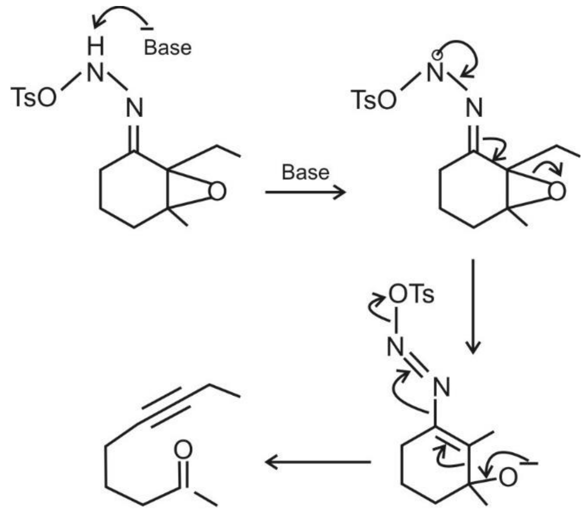
**Solution :**

Freons initiate radical chain reactions.

34. Answer (B)

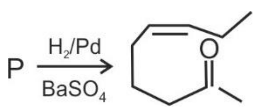
35. Answer (C)

**Hint and Solution for Q. No. 34 and 35**




(P)

**Solution :**




36. Answer A(Q, S, T); B(P, R, S); C(P, R, S, T); D(Q, R, S)

**Hint :**

| **Reducing sugars** | **Non-reducing Sugars** |
|---|---|
| Maltose | |
| Lactose | Cellulose |
| Glucose | Sucrose |
| Fructose | |

**Solution :**

Sucrose $\longrightarrow$ $\alpha$-glucose + $\beta$-fructose

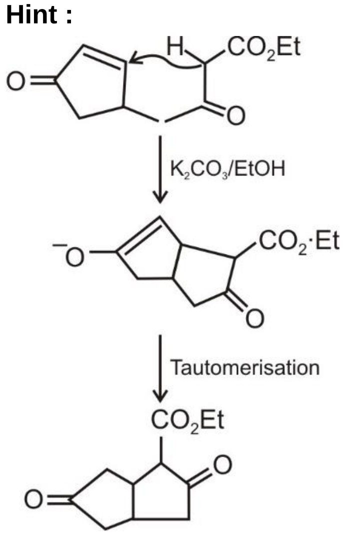
Maltose $\longrightarrow$ 2 $\alpha$-glucose

Lactose $\longrightarrow$ $\beta$-galactose + $\beta$-glucose

Cellulose $\longrightarrow$ $\beta$-glucose

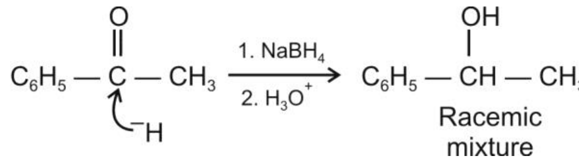
37. Answer A(Q, R, T); B(P); C(R, S, T); D(T)

**Hint :**




**Solution :**

In aldol condensation $H_2O$ elimination through E1cB mechanism.




38. Answer (25)

**Hint :** Since, the sample has [$\alpha$] to be +4.25 it means (+) alanine is present in excess.

**Solution :**

Optical purity = $\frac{4.25}{8.5} \times 100 = 50\%$. This means that 50% of the sample is pure (+) alanine and the other 50% is racemic. In which equal amount (i.e. 25% each) of (+) and (–) alanine is present.

39. Answer (04)

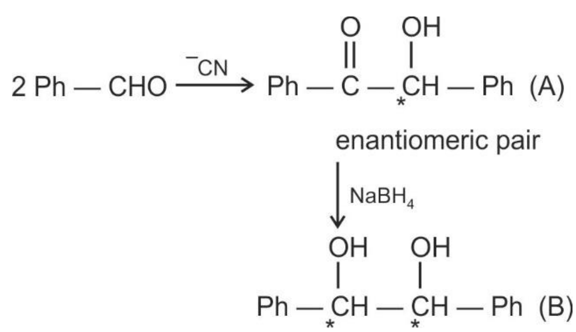
**Hint :** Since, six 1° H′s contribute to the 42% yield of 1-chloro propane, we can say that one 1° H leads to 7% (42/6) of this product. Similarly each 2° hydrogen contributes 28% (56/2) yield to the 2-chloro propane product.

**Solution :**

So the relative rate of the reaction of each 2°H compared to 1° H is $\frac{28}{7} = 4$

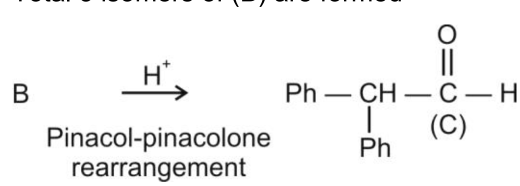
40. Answer (12)

**Hint :**




**Solution :**

Total 3 isomers of (B) are formed



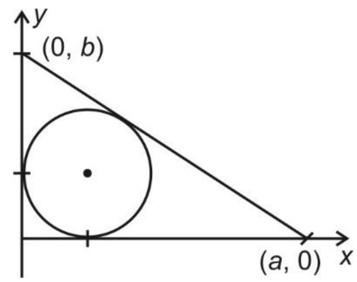

Degree of unsaturation of (C) is 9

## PART - III (MATHEMATICS)

41. Answer (B)

**Hints :** Circumcentre is mid point of hypotenuse.

**Solution :**




Clearly $a > 2$, $b > 2$

$\Rightarrow \quad \frac{1}{a} < \frac{1}{2}, \ \frac{1}{b} < 2$

$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} < 1$

Also, $rS = \Delta$

$\Rightarrow \quad 1\left(\frac{a+b+\sqrt{a^2+b^2}}{2}\right) = \frac{1}{2}ab$

$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} + \sqrt{\frac{a^2+b^2}{a^2b^2}} = 1$

42. Answer (C)

**Hint :** First find point of intersection of lines.

**Solution :**

The vertices of the triangle are

$O(0, 0),\ A\left(\frac{1}{\ell+m}, \frac{1}{\ell+m}\right), B\left(\frac{1}{\ell-m}, \frac{-1}{\ell-m}\right)$

Let circumcenter is $(h, k)$

$\therefore \quad h = \frac{\ell}{\ell^2 - m^2},\ k = \frac{-m}{\ell^2 - m^2}$

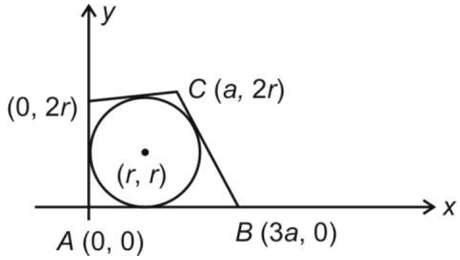
$\Rightarrow \quad h^2 + k^2 = \frac{1}{\left(\ell^2 - m^2\right)^2}$ and $h^2 - k^2 = \frac{1}{\ell^2 - m^2}$

$\Rightarrow$ Required locus $x^2 + y^2 = \left(x^2 - y^2\right)^2$

43. Answer (B)

**Hint :** Use condition for tangency.

**Solution :**




Area of trapezium = $\frac{1}{2}(a+3a)(2r) = 4$

$\Rightarrow \quad ar = 1$

Equation of $BC$ is $y = -r^2\left(x - \frac{3}{r}\right)$

$\Rightarrow \quad y + r^2x - 3r = 0$

As $BC$ is a tangent

$\Rightarrow \quad \frac{\left|r + r^3 - 3r\right|}{\sqrt{1+r^4}} = r$

$\Rightarrow \quad r = \frac{\sqrt{3}}{2}$

44. Answer (D)

**Hint :** Find chord of contact equation.

**Solution :**

Equation of tangent at (1, 2) to $C_1$ is

$x + 2y - 5 = 0$ …(1)

Let point $T$ is $(h, k)$

$\therefore$ Equation of C.O.C. w.r.t. $C_2$ is

$xh + yk - 9 = 0$ …(2)

$\Rightarrow \quad \frac{h}{1} = \frac{k}{2} = \frac{9}{5}$

$\Rightarrow \quad h = \frac{9}{5},\ k = \frac{18}{5}$

45. Answer (A)

**Hint :** Think of quadratic equation to solve.

**Solution :**

Let equation of circle is

$(x - r)^2 + y^2 = r^2$ …(1)

$\Rightarrow \quad (at^2 - r)^2 + 4a^2t^2 \geq r^2$

$\Rightarrow \quad a^2t^4 + r^2 - 2rat^2 + 4a^2t^2 \geq r^2$

$\Rightarrow \quad a^2t^4 - 2art^2 + 4a^2t^2 \geq 0$

$\Rightarrow \quad at^2 - 2r + 4a \geq 0$

$\Rightarrow \quad r \leq \frac{a}{2}(t^2 + 4) \leq 2a$

$\therefore$ Maximum value of $r = 2a$

46. Answer (B)

**Hint :** Tangency condition.

**Solution :**

Let the line is $y = mx + 5$

$\because m > 0$ and is least $\therefore$ the line should touch the ellipse

$\Rightarrow \quad 25 = 16m^2 + 9$

$\Rightarrow \quad 16m^2 = 16$

$\Rightarrow \quad m = \pm 1 \qquad \Rightarrow m = 1$

47. Answer (B, C, D)

**Hint :** A.M $\geq$ G.M

**Solution :**

$\therefore uv < 0 \Rightarrow u + \frac{1}{u} \geq 2, \quad v + \frac{1}{v} \leq -2$

or $\quad u + \frac{1}{u} \leq -2 \quad$ or $\quad v + \frac{1}{v} \geq 2$

$\Rightarrow \quad \sec^{-1}\left(u + \frac{1}{u}\right) \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

$\sec^{-1}\left(v + \frac{1}{v}\right) \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$

$\therefore \quad t \in \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$

48. Answer (A, B)

**Hint :** Conversion into trigonometric function values.

**Solution :**

$\because \tan\alpha = \frac{36}{77}, \qquad \tan\beta = \frac{3}{4}, \qquad \tan\gamma = \frac{8}{15}$

$\tan(\alpha + \beta + \gamma) = \frac{\Sigma(\tan\alpha) - \pi(\tan\alpha)}{1 - \Sigma\tan\alpha\tan\beta} = \infty$

$\Rightarrow \quad \alpha + \beta + \gamma = \frac{\pi}{2}$

$\therefore$ Option (A) and (B) are correct.

49. Answer (A, B, C)

**Hint :** Concept of orthogonality of two curves.

**Solution :**

Due to orthogonal intersection of ellipse and hyperbola

$a^2 + b^2 = 16$

$\Rightarrow \quad a^2e^2 = 16$

$\Rightarrow \quad a^2 = 4 \qquad \Rightarrow \quad b^2 = 12$

$\therefore$ No director circle of hyperbola is possible.

50. Answer (C, D)

**Hint :** Property of normal.

**Solution :**

$\because$ Normal intersects the parabola $y^2 = 4ax$ again

$\therefore x_1 x_2 = 4a^2$ and $y_1 y_2 = 8a^2$

$\because a = 2 \qquad \Rightarrow x_1 x_2 = 16$ and $y_1 y_2 = 32$

51. Answer (A, D)

**Hint :** Form family of circles.

**Solution :**

Circle with points $\left(2t_1, \frac{2}{t_1}\right)$ and $\left(2t_2, \frac{2}{t_2}\right)$ as diameter is

$(x - 2t_1)(x - 2t_2) + \left(y - \frac{2}{t_1}\right)\left(y - \frac{2}{t_2}\right) = 0$

Also $t_1t_2 = -1$

Hence the equation of circle is $(x^2 + y^2 - 8) - 2(t_1 + t_2)(x - y) = 0$

The point of intersection of $x^2 + y^2 = 8$ and $x - y = 0$ are (2, 2) and (–2, –2)

52. Answer (A)

53. Answer (D)

**Hint for Q. No. 52 and 53**

**Hint :** Family of circles.

**Solution for Q. No. 52 and 53**

Let $\Sigma$ is $x^2 + y^2 - 9x - 12y + 53 + \lambda(2x + 3y - 27) = 0$

Given circle $x^2 + y^2 - 4x - 6y - 3 = 0$

$\therefore$ Equation of common chord

$-5x + 6y + 56 + \lambda(2x + 3y - 27) = 0$

$\therefore$ Chord passes through the point of intersection of $5x + 6y - 56 = 0$ and $2x + 3y - 27 = 0$

*i.e.* $\left(2, \frac{23}{3}\right)$

$\because$ $\Sigma$ intersects $x^2 + y^2 = 29$ orthogonally.

$53 - 27\lambda - 29 = 0$

$\lambda = \frac{24}{27} = \frac{8}{9}$

$\therefore$ Circle is

$x^2 + y^2 + \left(\frac{16}{9} - 9\right)x + \left(\frac{29}{9} - 12\right)y + 29 = 0$

$\therefore$ Center is $\left(\frac{65}{18}, \frac{14}{3}\right)$

54. Answer (B)

55. Answer (C)

**Hint for Q. No. 54 and 55**

Mathematical induction approach.

**Solution for Q. No. 54 and 55**

Put $n = 2$

$\Rightarrow \quad f(1) + 2f(2) = 6f(2)$

$\Rightarrow \quad 4f(2) = f(1)$

$\Rightarrow \quad f(2) = \frac{1}{8}$

Similarly $f(3) = \frac{1}{12}$, $f(4) = \frac{1}{16}$ … and so on

$\therefore \quad f(n) = \frac{1}{4n} \qquad \therefore \quad f(1010) = \frac{1}{4040}$

56. Answer A(Q, R, S); B(Q); C(R, S); D(P, T)

**Hint :** Equality hold conditions for I.T.F.

**Solution :**

(A) $(\sin^{-1}x)^2 = (\sin^{-1}y)^2 = \frac{\pi^2}{4}$

$\Rightarrow \quad x = \pm1$ and $y = \pm1$

$\therefore \quad x^3 + y^3 = -2, 0, 2$

(B) $(\cos^{-1}x)^2 = (\cos^{-1}y)^2 = \pi^2$

$\Rightarrow$ $x = y = -1$

$\therefore$ $x^5 + y^5 = -2$

(C) $(\sin^{-1}x)^2 = \dfrac{\pi^2}{4}$ and $(\cos^{-1}y)^2 = \pi^2$

$\Rightarrow$ $\sin^{-1}x = \pm\dfrac{\pi}{2}$ and $\cos^{-1}y = \pi$

$\Rightarrow$ $x = \pm 1$ and $y = -1$

(D) $|\sin^{-1}x - \sin^{-1}y| = \pi$

$\Rightarrow$ either $\sin^{-1}x = -\dfrac{\pi}{2}$ and $\sin^{-1}y = \dfrac{\pi}{2}$

or $\sin^{-1}x = \dfrac{\pi}{2}$ and $\sin^{-1}y = -\dfrac{\pi}{2}$

$x = -1$ and $y = 1$ or $x = 1$ and $y = -1$

$\therefore$ $x^y = (-1)^1$ or $(1)^{-1}$

$= -1$ or $1$

57. Answer A(S); B(Q, R, S, T); C(R); D(P, Q, R, S, T)

**Hint :** Eccentricity formula for conic.

**Solution :**

(A) $\because \sqrt{c^2 + d^2} = a;\quad \sqrt{a^2 - b^2} = c$

$\Rightarrow$ $c^2 + d^2 = a^2$ and $a^2 - b^2 = c^2$

$\Rightarrow$ $d = b \qquad \Rightarrow \dfrac{d}{b} = 1$

(B) Now $e_1 = 1 - \dfrac{b^2}{a^2} \quad e_2 = 1 + \dfrac{d^2}{c^2}$

$\Rightarrow$ $e_1^2 + e_2^2 = 2 + b^2\left(\dfrac{a^2 - c^2}{a^2c^2}\right)$

$e_1 + e_2 = e_1^2 + \dfrac{1}{e_1^2} > 2$

(C) $2\tan^{-1}\left(\dfrac{d}{c}\right) = \dfrac{2\pi}{3} \Rightarrow d = \sqrt{3}c \Rightarrow d^2 = 3c^2$

$\Rightarrow$ $a^2 = 4c^2 \qquad \Rightarrow a = 2c$

$\therefore$ $4e_1 = 4\sqrt{1 - \dfrac{b^2}{a^2}}$

$= 4\sqrt{1 - \dfrac{3c^2}{4c^2}} = 2$

(D) $b^2 = a^2(1 - e_1^2)$

$\Rightarrow$ $a^2 = 2b^2 \qquad \Rightarrow c^2 = b^2$

For P.O.I. $\dfrac{h^2}{b^2} - \dfrac{k^2}{b^2} = \dfrac{h^2}{a^2} + \dfrac{k^2}{b^2}$

$\Rightarrow h^2\left(\dfrac{a^2 - b^2}{a^2b^2}\right) = \dfrac{2k^2}{b^2}$

$\Rightarrow \dfrac{h^2}{k^2} = \dfrac{2a^2}{a^2e_1^2} = 4$

58. Answer (36)

**Hint :** Point of intersection of two normals.

**Solution :**

Let $P(t_1)$ and $Q(t_2)$ are points

$\therefore$ $t_2 = 2t_1$

$\because$ P.O.I of normals

$R\left(2a + a\left(t_1^2 + t_1t_2 + t_2^2\right), -t_1t_2\left(t_1 + t_2\right)\right)$

$R\left(2 + t_1^2 + t_1t_2 + t_2^2,\ -t_1t_2\left(t_1 + t_2\right)\right)$

$\therefore$ $x = 2 + 7t_1^2, \quad y = -6t_1^3$

$\left(\dfrac{x-2}{7}\right)^3 = t_1^6 = \left(\dfrac{-y}{6}\right)^2 = \dfrac{y^2}{36}$

$\therefore$ Locus is $y^2 = \dfrac{36}{343}(x-2)^3$

$\therefore$ $k = 36$

59. Answer (16)

**Hint :** Monotonicity of function.

**Solution :**

$\because$ $x \in [-1, 1]$

Also $f(x)$ is an increasing function in domain

$\therefore$ $p = f(-1)$ and $q = f(1)$

$\Rightarrow$ $p = -\dfrac{\pi}{2} - \dfrac{\pi}{2} + (-2) = -\pi - 2$

and $q = \dfrac{\pi}{2} + \dfrac{\pi}{2} + 6 = \pi + 6$

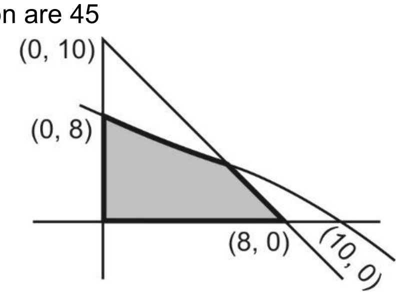
$\therefore$ $p + q = 4 \qquad \Rightarrow (p + q)^2 = 16$

60. Answer (45)

**Hint :** Linear inequalities of two variables.

**Solution :**

Total number of integral co-ordinates in shaded region are 45

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# All India Aakash Test Series for JEE (Advanced)-2020

## TEST - 2A (Paper-1) - Code-D

**Test Date : 24/11/2019**

## ANSWERS

| PHYSICS | | CHEMISTRY | | MATHEMATICS | |
|---|---|---|---|---|---|
| 1. | (C) | 21. | (D) | 41. | (B) |
| 2. | (D) | 22. | (B) | 42. | (A) |
| 3. | (D) | 23. | (B) | 43. | (D) |
| 4. | (B) | 24. | (B) | 44. | (B) |
| 5. | (B) | 25. | (A) | 45. | (C) |
| 6. | (C) | 26. | (B) | 46. | (B) |
| 7. | (B, C, D) | 27. | (A) | 47. | (A, D) |
| 8. | (B, D) | 28. | (C) | 48. | (C, D) |
| 9. | (B, D) | 29. | (A, B, C) | 49. | (A, B, C) |
| 10. | (B) | 30. | (A, B, D) | 50. | (A, B) |
| 11. | (A, B, D) | 31. | (B, D) | 51. | (B, C, D) |
| 12. | (A) | 32. | (C) | 52. | (A) |
| 13. | (C) | 33. | (C) | 53. | (D) |
| 14. | (C) | 34. | (B) | 54. | (B) |
| 15. | (D) | 35. | (C) | 55. | (C) |
| 16. | A → (P, T) | 36. | A → (Q, R, T) | 56. | A → (S) |
| | B → (Q, R) | | B → (P) | | B → (Q, R, S, T) |
| | C → (R, S, T) | | C → (R, S, T) | | C → (R) |
| | D → (Q, R) | | D → (T) | | D → (P, Q, R, S, T) |
| 17. | A → (P, S) | 37. | A → (Q, S, T) | 57. | A → (Q, R, S) |
| | B → (Q, R) | | B → (P, R, S) | | B → (Q) |
| | C → (P, S) | | C → (P, R, S, T) | | C → (R, S) |
| | D → (P, R) | | D → (Q, R, S) | | D → (P, T) |
| 18. | (29) | 38. | (12) | 58. | (45) |
| 19. | (50) | 39. | (04) | 59. | (16) |
| 20. | (16) | 40. | (25) | 60. | (36) |

# HINTS & SOLUTIONS

## PART - I (PHYSICS)

1. Answer (C)

**Hint :** $E_{\text{axis}} = \dfrac{qx}{4\pi\varepsilon_0 \left(R^2 + x^2\right)^{3/2}}$

**Solution:**

$$E = \frac{q}{4\pi\varepsilon_0 d^2} - \frac{q \times d}{4\pi\varepsilon_0 \left(d^2 + R^2\right)^{3/2}} = \frac{3qR^2}{8\pi\varepsilon_0 d^4}$$

2. Answer (D)

**Hint:** Heat current remains constant

**Solution :**

$$\frac{\dfrac{(T_1 - T)}{L}}{k2\pi a \times \left(\dfrac{a+b}{2}\right)} = \frac{\dfrac{T_1 - T_2}{L}}{k \times \pi a \times b}$$

$$\Rightarrow T = \frac{T_1 a + T_2 b}{(a+b)}$$

3. Answer (D)

**Hint:** Use reverse symmetry concept

**Solution:**

Using KVL and KCL, we get

$$R_{eq} = \frac{2R_1R_2 + R_2R_3 + R_3R_1}{(R_1 + R_2 + 2R_3)}$$

$$= \frac{2 \times (2 \times 3) + (3 \times 1) + (1 \times 2)}{(2 + 3 + 1 \times 2)}$$

$$= \frac{12 + 3 + 2}{7} = \frac{17}{7}\Omega$$

4. Answer (B)

**Hint :** $Q = Q_0 e^{-t/\tau}$ during discharging

**Solution :**

$$Q_{.0} = CV_0, C_2\left(\frac{C}{K}\right)$$

$$\therefore V_2 = \frac{CV_0}{\left(\dfrac{C}{K}\right)} = KV_0$$

$$\tau = R \times C_2 = \frac{RC}{K}$$

$$\therefore V = V_2 e^{-t/\tau}$$

$$\Rightarrow \frac{V_0}{2} = KV_0 \times e^{-\frac{t}{\tau}}$$

$$\Rightarrow \frac{1}{2K} = e^{-t/\tau}$$

$$\Rightarrow \ln(2K) = \frac{t}{\tau}$$

$$\Rightarrow t = \tau \ln(2K)$$

$$t = \frac{RC}{K}\ln(2K)$$

5. Answer (B)

**Hint :** At maximum temperature $\dfrac{dT}{dV} = 0$

**Solution :**

$$\left[P_0 + (1-\alpha)V^2\right]V = nRT$$

$$\Rightarrow T = \frac{P_0V + (1-\alpha)V^3}{nR}$$

$$\therefore \frac{dT}{dV} = 0 \text{ at } V^2 = \frac{P_0}{3(\alpha - 1)}$$

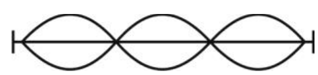
$$\therefore P = P_0 + (1-\alpha) \times \frac{P_0}{3(\alpha-1)}$$

$$\Rightarrow P = \frac{2P_0}{3}$$

6. Answer (C)

**Hint:** Second overtone contains 3 loops

**Solution:**



$$\frac{\lambda}{2} = \frac{2}{3} \Rightarrow \lambda = \frac{4}{3}\text{m}$$

Amp. = $2A \sin kx = A_{max} \sin(kx)$

$$\therefore A = (A_{max})\sin\left(\frac{2\pi \times 3}{4} \times \frac{1}{6}\right)$$

$$\Rightarrow A = (2) \times \left(\frac{1}{\sqrt{2}}\right)\text{mm} = \sqrt{2}\text{ mm}$$

7. Answer (B, C, D)

**Hint :** Use KVL and KCL.

**Solution :**

$Q_{total}$ = 180 – 70 = 110 μC

$$q_A = \frac{2}{2+6+3} \times (110) = 20\ \mu\text{C}$$

$$q_B = \frac{6}{2+3+6} \times (110) = 60\ \mu\text{C}$$

$$q_C = \frac{3}{2+6+3} \times (110) = 30\ \mu\text{C}$$

$$\Delta q_S = (20 + 30) - (-70) = 120\ \mu\text{C}$$

8. Answer (B, D)

**Hint :** $V_{rms}^2 = \frac{\int u^2 dN}{N}$

**Solution :**

$N = \text{Area} = \frac{1}{2}\times 10\times 10 = 50$

$\frac{dN}{du} = u + 10$

$\therefore V_{rms}^2 = \frac{\int u^2\times(10-u)du}{N} = \frac{\int_0^{10}(10u^2 - u^3)du}{50}$

$V_{rms}^2 = \frac{1000\times(4-3)}{12\times 50} = \frac{2500}{3\times 50}$

$\Rightarrow V_{rms} = \sqrt{\frac{50}{3}}\,\text{m/s}$

9. Answer (B, D)

**Hint :** Apparent wavelength changes when source moves.

**Solution :**

$f' = \frac{(340-10)}{(340-20)}\times(200) = 206\text{ Hz}$

$\lambda' = \lambda_0 = V_s\times T = \frac{340}{200} - 20\times\frac{1}{200} = 1.6\text{ m}$

10. Answer (B)

**Hint :** Use Gauss's law

**Solution :**

$\sigma$ on outer surface becomes uniform. Potential at outside points is only due to charge on outer surface of shell.

$\therefore V_A = V_B$

11. Answer (A, B, D)

**Hint:** Use KVL and KCL.

**Solution :**

$R_{eq} = 1 + \frac{20}{9} = \frac{29}{9}\,\Omega$

$\therefore I_0 = \frac{58}{(29/9)} = 18\text{ A}$

$\therefore 1_{(2\Omega)} = \frac{18}{2} = 9\text{ A}$

$1_{(3\Omega)} = \frac{6}{6+3}\times 9 = 6\text{ A}$

$I_{(5\Omega)} = \frac{4}{9}\times(18) = 8\text{ A}$

$I_{(4\Omega)} = \frac{5}{9}\times(18) = 10\text{ A}$

$\therefore V_{(4\Omega)} = 4\times 10 = 40\text{ V}$

$P_{(5\Omega)} = 8^2\times 5 = 320\text{ W}$

12. Answer (A)

**Hint:** Flux is proportional to charge

**Solution :**

$\frac{2\pi(1-\cos\alpha)}{4\pi}\times\left(\frac{q_1}{\varepsilon_0}\right) = \frac{2\pi(1-\cos\beta)}{4\pi}\times\left(\frac{q_2}{\varepsilon_0}\right)$

$\Rightarrow \frac{q_1}{q_2} = \frac{1-\cos\beta}{1-\cos\alpha} = \frac{1-0}{1-\frac{1}{2}} = 2$

13. Answer (C)

**Hint :** Flux is proportional to charge

**Solution:**

$q_1 = 3q_2$

$\Rightarrow$ one third of total flux of $q_1$ will terminate at $q_2$

$\therefore \frac{4\pi}{3} = 2\pi(1-\cos\alpha_{max})$

$\Rightarrow \cos(\alpha_{max}) = 1 - \frac{2}{3} = \frac{1}{3}$

$\Rightarrow \tan(\alpha_{max}) = 2\sqrt{2} \quad \Rightarrow \alpha_{max} = \tan^{-1}(2\sqrt{2})$

14. Answer (C)

**Hint :** $A\to B$ Isochoric $\quad C\to D$ Isochoric

$B\to C$ Isothermal $\quad D\to A$ Isothermal

15. Answer (D)

**Hint :** $W$ isothermal $= nRT_0\ln\left(\frac{V_2}{V_1}\right)$

**Solution of Q.Nos. 14 and 15**

$W_{BC} = 2P_0V_0\ln\left(\frac{V_C}{V_B}\right) = -P_0V_0\ln(2)$

$\Rightarrow V_C = \frac{V_0}{\sqrt{2}}$

$\therefore P_C = \frac{2P_0V_0}{V_C} = 2\sqrt{2}P_0$

$\therefore W_{DA} = (\sqrt{2}P_0)\left(\frac{V_0}{\sqrt{2}}\right)\ln(\sqrt{2}) = \frac{P_0V_0}{2}\ln(2)$

$$\therefore W_{ABCDA} = 0 + -P_0V_0\ln(2) + 0 + \frac{1}{2}P_0V_0\ln(2)$$

$$= -\frac{P_0V_0}{2}\ln(2)$$

16. Answer A(P, T); B (Q, R); C(R, S, T); D(Q, R)

**Hint :** Use Gauss's law.

**Solution :**

Electric field is uniform in spherical cavity in sphere and in cylindrical cavity in cylinder.

17. Answer A(P, S); B(Q, R); C(P, S); D(P, R)

**Hint :** Capacitance increases due to slab.

**Solution :**

Total capacitance increases, so charge on *A* increases as well as voltage increases.

$\therefore$ Voltage on *B* decreases. So, charge on it decreases

$\therefore$ Charge on *C* and *D* increases

18. Answer (29)

**Hint:** *BC* is isothermal

**Solution :**

$$(3P_0)\times V_C = P_0 \times V_0$$

$$\Rightarrow V_C = \frac{V_0}{3}$$

$\because$ *CA* is a adiabatic.

$$\therefore \quad (3P_0)\times\left(\frac{V_0}{3}\right)^{r} = \left(\frac{P_0}{2}\right)(V_0)^{r}$$

$$\Rightarrow \quad \gamma = \frac{\ln 6}{\ln 3} = \frac{\ln 2 + \ln 3}{\ln 3} = \frac{18}{11}$$

$\therefore$ $p + q = 18 + 11 = 29$

19. Answer (50)

**Hint :** Voltmeter are not ideal

**Solution :**

Let resistance of each voltmeter be $R_0$

$\therefore$ $Ri' = 20\,R_0\,(I - i)$ ..................(i)

and $2Ri' = 30$ ..................(ii)

$$\Rightarrow i' = \frac{3}{4}i, \qquad \therefore\ i_{(V_2)} = I - i' = I - \frac{3i}{4}$$

$$\therefore 2Ri' = 30 = R_0(I - i) = R_0\left(I - \frac{3i}{4}\right)$$

$$\Rightarrow i = 400\ \mu\text{A}$$

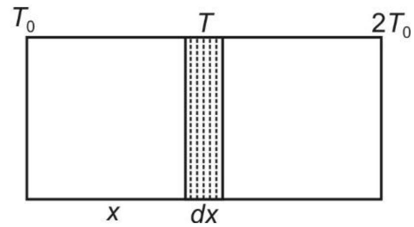
$$\therefore R = \frac{20}{400\times10^{-6}} = 50\times10^{3}\,\Omega = 50\text{ k}\Omega$$

20. Answer (16)

**Hint :** Use $PV = nRT$

**Solution :**

$$T = T_0 + \frac{T_0}{L}x$$




$$\therefore \int dn = \int_{x=0}^{L} \frac{P\times A\,dx}{R\left(T_0 + \frac{T_0}{L}x\right)}$$

$$\Rightarrow n = \frac{PAL}{RT_0}\ln(2)$$

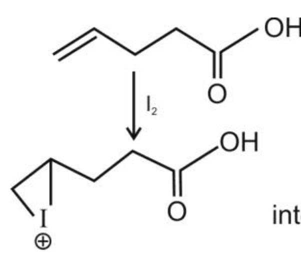
$$\Rightarrow n = \frac{PAL}{4RT_0}\ln(16)$$

$\therefore$ 16

## PART - II (CHEMISTRY)

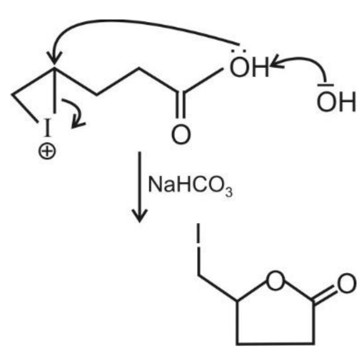
21. Answer (D)

**Hint :**




intermediate

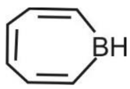
**Solution :**



22. Answer (B)

**Hint :** Compound which are planar, has $(4n + 2)\ \pi e^-$ are aromatic

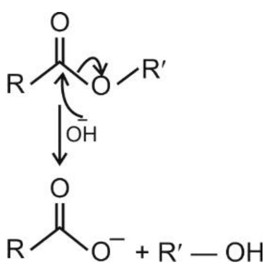
**Solution :**




Boron has vacant 2*p* orbital hence planar ($sp^2$) and has $6\pi e^-$

23. Answer (B)

**Hint :** Hydrolysis of ester under alkaline condition occurs as




**Solution :**
Greater the extent of electron withdrawing strength of R, greater will be the rate of reaction

24. Answer (B)

**Hint :** In DMF $S_N2$ mechanism is favoured during nucleophilic substitution reaction.

**Solution :**
Electron withdrawing group increases the tendency of $S_N2$.

25. Answer (A)

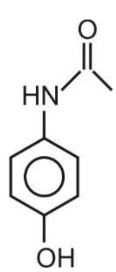
**Hint :** A paired with T (A = T)

**Solution :**
G paired with C (G ≡ C)

26. Answer (B)

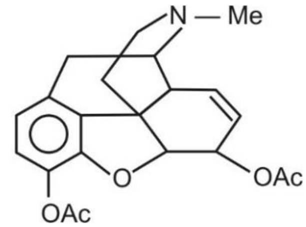
**Hint :**

Paracetamol

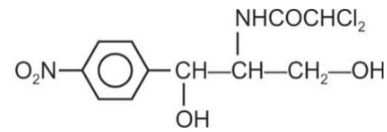



**Solution :**

Heroin

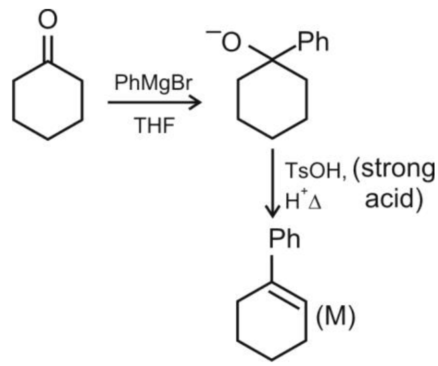



Chloramphenicol

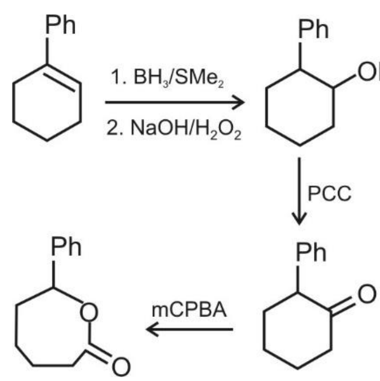



27. Answer (A)

**Hint :**




**Solution :**




28. Answer (C)

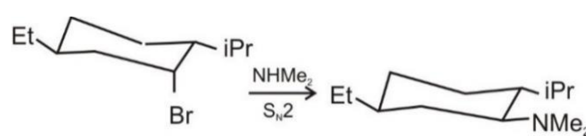
**Hint :** Hoffmann bromamide reaction.

**Solution :**

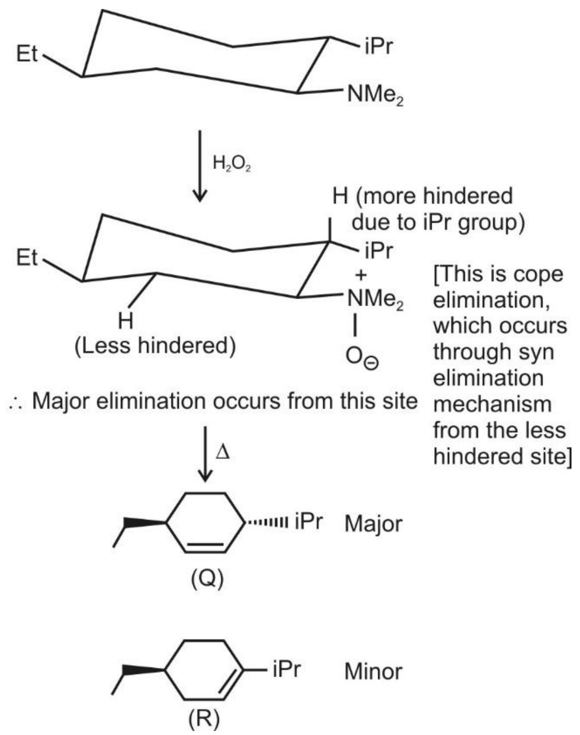
$$\underset{T}{Ph-CH_2-\overset{O}{\overset{||}{C}}-NH_2} \xrightarrow[NaOH]{Br_2} Ph-CH_2-NH_2$$

29. Answer (A, B, C)

**Hint :**

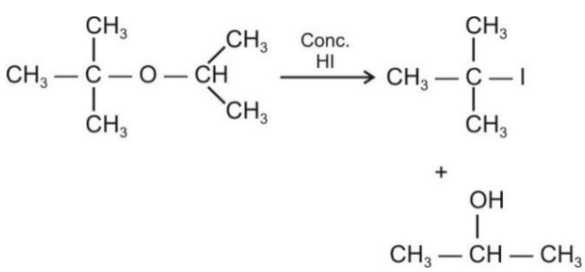


**Solution :**




30. Answer (A, B, D)

**Hint :**



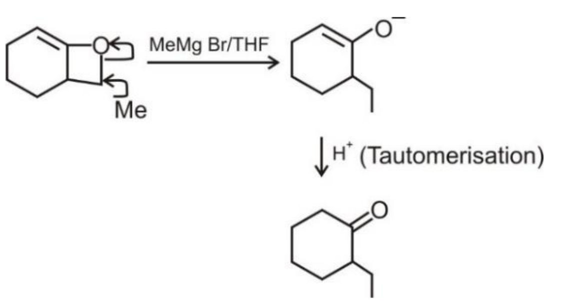

**Solution :**

Formed alcohol is 2°

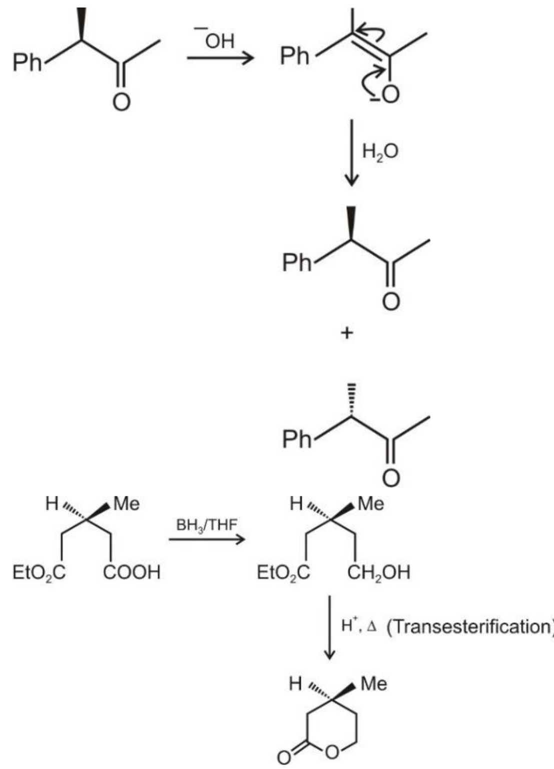
$$CH_3-\underset{\underset{CH_3}{|}}{CH}-OH \xrightarrow{Br_2/NaOH} CH_3-COO^- + CHBr_3 + NaBr$$

31. Answer (B, D)

**Hint :**




**Solution :**




32. Answer (C)

**Hint :** $SO_2 + O_3 \longrightarrow SO_3 + O_2$

$O_3$ is consumed by $SO_2$ only

**Solution :**

$$NO + \dot{O} \longrightarrow NO_2$$

So more of $O_3$ is consumed

33. Answer (C)

**Hint :** $CCl_2F_2$ is Freon-12

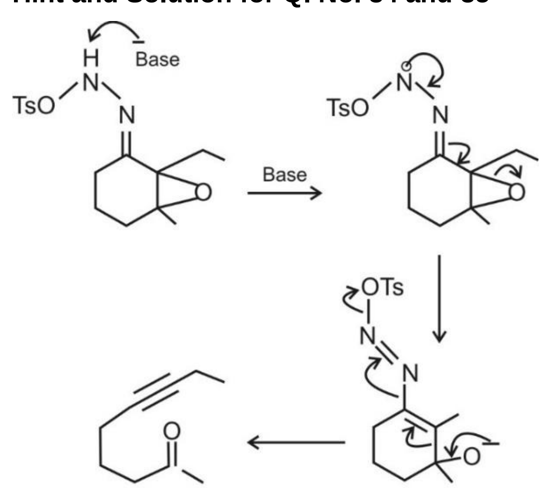
**Solution :**

Freons initiate radical chain reactions.

34. Answer (B)

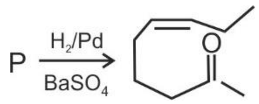
35. Answer (C)

**Hint and Solution for Q. No. 34 and 35**



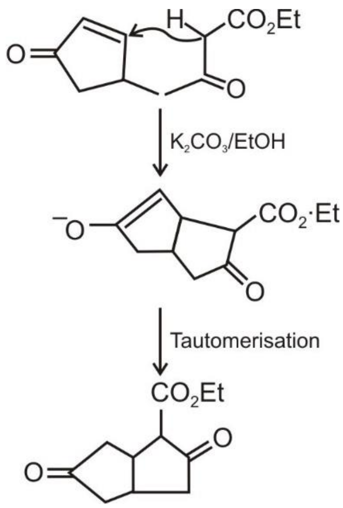

(P)

**Solution :**



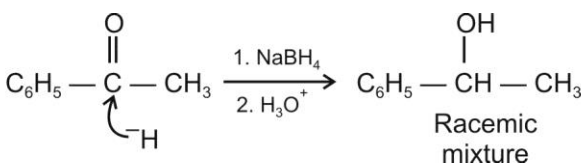

36. Answer A(Q, R, T); B(P); C(R, S, T); D(T)

**Hint :**




**Solution :**

In aldol condensation $H_2O$ elimination through E1cB mechanism.




37. Answer A(Q, S, T); B(P, R, S); C(P, R, S, T); D(Q, R, S)

**Hint :**

| **Reducing sugars** | **Non-reducing Sugars** |
|---|---|
| Maltose | |
| Lactose | Cellulose |
| Glucose | Sucrose |
| Fructose | |

**Solution :**

Sucrose $\longrightarrow$ $\alpha$-glucose + $\beta$-fructose

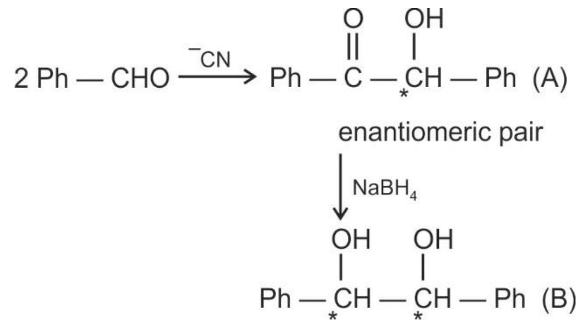
Maltose $\longrightarrow$ 2 $\alpha$-glucose

Lactose $\longrightarrow$ $\beta$-galactose + $\beta$-glucose

Cellulose $\longrightarrow$ $\beta$-glucose

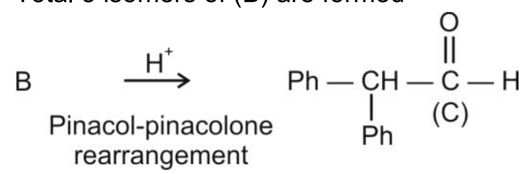
38. Answer (12)

**Hint :**




**Solution :**

Total 3 isomers of (B) are formed




Degree of unsaturation of (C) is 9

39. Answer (04)

**Hint :** Since, six 1° H′s contribute to the 42% yield of 1-chloro propane, we can say that one 1° H leads to 7% (42/6) of this product. Similarly each 2° hydrogen contributes 28% (56/2) yield to the 2-chloro propane product.

**Solution :**

So the relative rate of the reaction of each 2°H compared to 1° H is $\frac{28}{7} = 4$

40. Answer (25)

**Hint :** Since, the sample has [$\alpha$] to be +4.25 it means (+) alanine is present in excess.

**Solution :**

Optical purity = $\frac{4.25}{8.5} \times 100 = 50\%$. This means that 50% of the sample is pure (+) alanine and the other 50% is racemic. In which equal amount (i.e. 25% each) of (+) and (–) alanine is present.

## PART - III (MATHEMATICS)

41. Answer (B)

**Hint :** Tangency condition.

**Solution :**

Let the line is $y = mx + 5$

$\because$ $m > 0$ and is least $\therefore$ the line should touch the ellipse

$\Rightarrow$ $25 = 16m^2 + 9$

$\Rightarrow$ $16m^2 = 16$

$\Rightarrow$ $m = \pm 1$ $\Rightarrow$ $m = 1$

42. Answer (A)

**Hint :** Think of quadratic equation to solve.

**Solution :**

Let equation of circle is

$(x - r)^2 + y^2 = r^2$ ...(1)

$\Rightarrow$ $(at^2 - r)^2 + 4a^2t^2 \geq r^2$

$\Rightarrow$ $a^2t^4 + r^2 - 2rat^2 + 4a^2t^2 \geq r^2$

$\Rightarrow$ $a^2t^4 - 2art^2 + 4a^2t^2 \geq 0$

$\Rightarrow$ $at^2 - 2r + 4a \geq 0$

$\Rightarrow$ $r \leq \frac{a}{2}(t^2 + 4) \leq 2a$

$\therefore$ Maximum value of $r = 2a$

43. Answer (D)

**Hint :** Find chord of contact equation.

**Solution :**

Equation of tangent at (1, 2) to $C_1$ is

$x + 2y - 5 = 0$ ...(1)

Let point $T$ is $(h, k)$

$\therefore$ Equation of C.O.C. w.r.t. $C_2$ is

$xh + yk - 9 = 0$ ...(2)

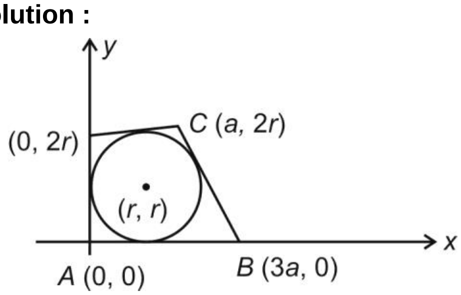
$\Rightarrow$ $\frac{h}{1} = \frac{k}{2} = \frac{9}{5}$

$\Rightarrow$ $h = \frac{9}{5},\ k = \frac{18}{5}$

44. Answer (B)

**Hint :** Use condition for tangency.

**Solution :**




Area of trapezium = $\frac{1}{2}(a + 3a)(2r) = 4$

$\Rightarrow$ $ar = 1$

Equation of $BC$ is $y = -r^2\left(x - \frac{3}{r}\right)$

$\Rightarrow$ $y + r^2x - 3r = 0$

As $BC$ is a tangent

$\Rightarrow$ $\frac{|r + r^3 - 3r|}{\sqrt{1 + r^4}} = r$

$\Rightarrow$ $r = \frac{\sqrt{3}}{2}$

45. Answer (C)

**Hint :** First find point of intersection of lines.

**Solution :**

The vertices of the triangle are

$O(0, 0),\ A\left(\frac{1}{\ell + m}, \frac{1}{\ell + m}\right), B\left(\frac{1}{\ell - m}, \frac{-1}{\ell - m}\right)$

Let circumcenter is $(h, k)$

$\therefore$ $h = \frac{\ell}{\ell^2 - m^2},\ k = \frac{-m}{\ell^2 - m^2}$

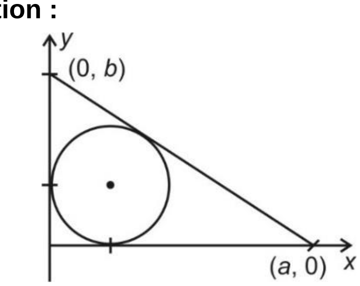
$\Rightarrow$ $h^2 + k^2 = \frac{1}{(\ell^2 - m^2)^2}$ and $h^2 - k^2 = \frac{1}{\ell^2 - m^2}$

$\Rightarrow$ Required locus $x^2 + y^2 = (x^2 - y^2)^2$

46. Answer (B)

**Hints :** Circumcentre is mid point of hypotenuse.

**Solution :**




Clearly $a > 2$, $b > 2$

$\Rightarrow$ $\frac{1}{a} < \frac{1}{2},\ \frac{1}{b} < 2$

$\Rightarrow$ $\frac{1}{a} + \frac{1}{b} < 1$

Also, $rS = \Delta$

$\Rightarrow$ $1\left(\frac{a + b + \sqrt{a^2 + b^2}}{2}\right) = \frac{1}{2}ab$

$\Rightarrow$ $\frac{1}{a} + \frac{1}{b} + \sqrt{\frac{a^2 + b^2}{a^2b^2}} = 1$

47. Answer (A, D)

**Hint :** Form family of circles.

**Solution :**

Circle with points $\left(2t_1, \frac{2}{t_1}\right)$ and $\left(2t_2, \frac{2}{t_2}\right)$ as diameter is

$$(x-2t_1)(x-2t_2)+\left(y-\frac{2}{t_1}\right)\left(y-\frac{2}{t_2}\right)=0$$

Also $t_1t_2 = -1$

Hence the equation of circle is $(x^2 + y^2 - 8) - 2(t_1 + t_2)(x - y) = 0$

The point of intersection of $x^2 + y^2 = 8$ and $x - y = 0$ are (2, 2) and (–2, –2)

48. Answer (C, D)

**Hint :** Property of normal.

**Solution :**

$\because$ Normal intersects the parabola $y^2 = 4ax$ again

$\therefore x_1 x_2 = 4a^2$ and $y_1 y_2 = 8a^2$

$\because a = 2 \quad \Rightarrow x_1 x_2 = 16$ and $y_1 y_2 = 32$

49. Answer (A, B, C)

**Hint :** Concept of orthogonality of two curves.

**Solution :**

Due to orthogonal intersection of ellipse and hyperbola

$a^2 + b^2 = 16$

$\Rightarrow \quad a^2e^2 = 16$

$\Rightarrow \quad a^2 = 4 \quad \Rightarrow \quad b^2 = 12$

$\therefore$ No director circle of hyperbola is possible.

50. Answer (A, B)

**Hint :** Conversion into trigonometric function values.

**Solution :**

$\because \tan\alpha = \frac{36}{77}, \quad \tan\beta = \frac{3}{4}, \quad \tan\gamma = \frac{8}{15}$

$$\tan(\alpha+\beta+\gamma) = \frac{\Sigma(\tan\alpha) - \pi(\tan\alpha)}{1-\Sigma\tan\alpha\tan\beta} = \infty$$

$\Rightarrow \quad \alpha + \beta + \gamma = \frac{\pi}{2}$

$\therefore$ Option (A) and (B) are correct.

51. Answer (B, C, D)

**Hint :** A.M $\ge$ G.M

**Solution :**

$\therefore uv < 0 \Rightarrow u + \frac{1}{u} \ge 2, \quad v + \frac{1}{v} \le -2$

or $\quad u + \frac{1}{u} \le -2 \quad$ or $\quad v + \frac{1}{v} \ge 2$

$\Rightarrow \quad \sec^{-1}\left(u + \frac{1}{u}\right) \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

$\sec^{-1}\left(v + \frac{1}{v}\right) \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$

$\therefore \quad t \in \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$

52. Answer (A)

53. Answer (D)

**Hint for Q. No. 52 and 53**

Family of circles.

**Solution for Q. No. 52 and 53**

Let $\Sigma$ is $x^2 + y^2 - 9x - 12y + 53 + \lambda(2x + 3y - 27) = 0$

Given circle $x^2 + y^2 - 4x - 6y - 3 = 0$

$\therefore$ Equation of common chord

$-5x + 6y + 56 + \lambda(2x + 3y - 27) = 0$

$\therefore$ Chord passes through the point of intersection of $5x + 6y - 56 = 0$ and $2x + 3y - 27 = 0$

*i.e.* $\left(2, \frac{23}{3}\right)$

$\because \quad \Sigma$ intersects $x^2 + y^2 = 29$ orthogonally.

$53 - 27\lambda - 29 = 0$

$\lambda = \frac{24}{27} = \frac{8}{9}$

$\therefore$ Circle is

$$x^2 + y^2 + \left(\frac{16}{9} - 9\right)x + \left(\frac{29}{9} - 12\right)y + 29 = 0$$

$\therefore$ Center is $\left(\frac{65}{18}, \frac{14}{3}\right)$

54. Answer (B)

55. Answer (C)

**Hind for Q. No. 54 and 55**

Mathematical induction approach.

**Solution for Q. No. 54 and 55**

Put $n = 2$

$\Rightarrow \quad f(1) + 2f(2) = 6f(2)$

$\Rightarrow \quad 4f(2) = f(1)$

$\Rightarrow \quad f(2) = \frac{1}{8}$

Similarly $f(3) = \frac{1}{12}$, $f(4) = \frac{1}{16}$ … and so on

$\therefore \quad f(n) = \frac{1}{4n} \qquad \therefore \quad f(1010) = \frac{1}{4040}$

56. Answer A(S); B(Q, R, S, T); C(R); D(P, Q, R, S, T)

**Hint :** Eccentricity formula for conic.

**Solution :**

(A) $\because \sqrt{c^2+d^2}=a; \quad \sqrt{a^2-b^2}=c$

$\Rightarrow c^2 + d^2 = a^2$ and $a^2 - b^2 = c^2$

$\Rightarrow d = b \qquad \Rightarrow \frac{d}{b}=1$

(B) Now $e_1 = 1-\frac{b^2}{a^2} \quad e_2 = 1+\frac{d^2}{c^2}$

$\Rightarrow e_1^2+e_2^2 = 2+b^2\left(\frac{a^2-c^2}{a^2c^2}\right)$

$e_1+e_2 = e_1^2+\frac{1}{e_1^2}>2$

(C) $2\tan^{-1}\left(\frac{d}{c}\right)=\frac{2\pi}{3} \Rightarrow d=\sqrt{3}c \Rightarrow d^2=3c^2$

$\Rightarrow a^2 = 4c^2 \qquad \Rightarrow a = 2c$

$\therefore 4e_1 = 4\sqrt{1-\frac{b^2}{a^2}} = 4\sqrt{1-\frac{3c^2}{4c^2}}=2$

(D) $b^2=a^2\left(1-e_1^2\right)$

$\Rightarrow a^2 = 2b^2 \qquad \Rightarrow c^2 = b^2$

For P.O.I. $\frac{h^2}{b^2}-\frac{k^2}{b^2} = \frac{h^2}{a^2}+\frac{k^2}{b^2}$

$\Rightarrow h^2\left(\frac{a^2-b^2}{a^2b^2}\right)=\frac{2k^2}{b^2}$

$\Rightarrow \frac{h^2}{k^2}=\frac{2a^2}{a^2e_1^2}=4$

57. Answer A(Q, R, S); B(Q); C(R, S); D(P, T)

**Hint :** Equality hold conditions for I.T.F.

**Solution :**

(A) $(\sin^{-1}x)^2 = (\sin^{-1}y)^2 = \frac{\pi^2}{4}$

$\Rightarrow x = \pm 1$ and $y = \pm 1$

$\therefore x^3 + y^3 = -2, 0, 2$

(B) $(\cos^{-1}x)^2 = (\cos^{-1}y)^2 = \pi^2$

$\Rightarrow x = y = -1$

$\therefore x^5 + y^5 = -2$

(C) $(\sin^{-1}x)^2 = \frac{\pi^2}{4}$ and $(\cos^{-1}y)^2 = \pi^2$

$\Rightarrow \sin^{-1}x = \pm\frac{\pi}{2}$ and $\cos^{-1}y = \pi$

$\Rightarrow x = \pm 1$ and $y = -1$

(D) $|\sin^{-1}x - \sin^{-1}y| = \pi$

$\Rightarrow$ either $\sin^{-1}x = -\frac{\pi}{2}$ and $\sin^{-1}y = \frac{\pi}{2}$

or $\sin^{-1}x = \frac{\pi}{2}$ and $\sin^{-1}y = -\frac{\pi}{2}$

$x = -1$ and $y = 1$ or $x = 1$ and $y = -1$

$\therefore x^y = (-1)^1$ or $(1)^{-1}$

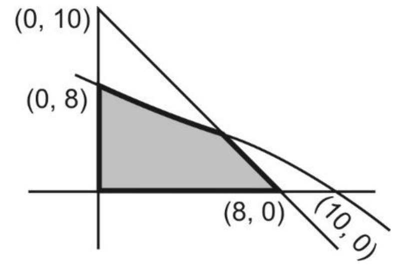
$= -1$ or $1$

58. Answer (45)

**Hint :** Linear inequalities of two variables.

**Solution :**

Total number of integral co-ordinates in shaded region are 45




59. Answer (16)

**Hint :** Monotonicity of function.

**Solution :**

$\because x \in [-1, 1]$

Also $f(x)$ is an increasing function in domain

$\therefore p = f(-1)$ and $q = f(1)$

$\Rightarrow p = -\frac{\pi}{2}-\frac{\pi}{2}+(-2) = -\pi-2$

and $q = \frac{\pi}{2}+\frac{\pi}{2}+6 = \pi+6$

$\therefore p + q = 4 \qquad \Rightarrow (p + q)^2 = 16$

60. Answer (36)

**Hint :** Point of intersection of two normals.

**Solution :**

Let $P(t_1)$ and $Q(t_2)$ are points

$\therefore t_2 = 2t_1$

$\because$ P.O.I of normals

$R\left(2a+a\left(t_1^2+t_1t_2+t_2^2\right), -t_1t_2\left(t_1+t_2\right)\right)$

$R\left(2+t_1^2+t_1t_2+t_2^2, -t_1t_2\left(t_1+t_2\right)\right)$

$\therefore x = 2+7t_1^2, \quad y = -6t_1^3$

$\left(\frac{x-2}{7}\right)^3 = t_1^6 = \left(\frac{-y}{6}\right)^2 = \frac{y^2}{36}$

$\therefore$ Locus is $y^2 = \frac{36}{343}(x-2)^3$

$\therefore k = 36$

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