All India Aakash Test Series for JEE (Advanced)-2020

TEST - 2A (Paper-2) - Code-E

Test Date : 24/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(B, D)	21.	(B, C, D)	41.	(B, C)
2.	(B, D)	22.	(A, C, D)	42.	(B, D)
3.	(A, D)	23.	(D)	43.	(A)
4.	(B, C)	24.	(A, C, D)	44.	(A, B)
5.	(A)	25.	(C, D)	45.	(B, C)
6.	(D)	26.	(D)	46.	(C)
7.	(B)	27.	(B)	47.	(D)
8.	(C)	28.	(A)	48.	(A)
9.	(A)	29.	(B)	49.	(D)
10.	(C)	30.	(A)	50.	(A)
11.	(C)	31.	(B)	51.	(C)
12.	(B)	32.	(A)	52.	(D)
13.	(C)	33.	(B)	53.	(B)
14.	(B)	34.	(C)	54.	(A)
15.	(A)	35.	(A)	55.	(C)
16.	A $\rightarrow$ (P, R)	36.	A $\rightarrow$ (Q, S)	56.	A $\rightarrow$ (P)
	B $\rightarrow$ (P, S)		B $\rightarrow$ (R)		B $\rightarrow$ (Q)
	C $\rightarrow$ (Q, T)		C $\rightarrow$ (Q, R)		C $\rightarrow$ (Q, S)
	D $\rightarrow$ (Q, S)		D $\rightarrow$ (P, T)		D $\rightarrow$ (R, S)
17.	A $\rightarrow$ (R, T)	37.	A $\rightarrow$ (Q, S)	57.	A $\rightarrow$ (P, Q)
	B $\rightarrow$ (S, T)		B $\rightarrow$ (Q)		B $\rightarrow$ (P, Q)
	C $\rightarrow$ (Q, S)		C $\rightarrow$ (R, T)		C $\rightarrow$ (R, T)
	D $\rightarrow$ (Q, S)		D $\rightarrow$ (Q)		D $\rightarrow$ (S, T)
18.	(02)	38.	(07)	58.	(09)
19.	(01)	39.	(03)	59.	(00)
20.	(06)	40.	(06)	60.	(03)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (B, D)

Hint : B and C will be in series.

Solution :

$$C_{BC} = \frac{2\times3}{2+3} = \frac{6}{5}\mu\text{F}$$

$$\therefore \quad \Delta q_S = \frac{\frac{6}{5}}{\frac{6}{5}+1}\times110 = 60\ \mu\text{C}$$

$\therefore \quad q_A = 110 - 60 = 50\ \mu$C

$$V_B = \frac{60}{2} = 30\text{ V},\ V_C = \frac{60}{3} = 20\text{ V}$$

2. Answer (B, D)

Hint : Particle performs SHM.

Solution :

$$E = \frac{Qx}{4\pi\varepsilon_0(R^2+x^2)^{\frac{3}{2}}},\ x << R$$

$$\Rightarrow \quad E = \frac{Qx}{4\pi\varepsilon_0 R^3}$$

$$\therefore \quad \omega = \sqrt{\frac{Qq}{m\times4\pi\varepsilon_0 R^3}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m\times4\pi\varepsilon_0 R^3}{Qq}} = 4\pi\sqrt{\frac{\pi\varepsilon_0 mR^3}{Qq}}$$

and, $V_{max} = \omega a = \dfrac{ax}{2}\sqrt{\dfrac{Qq}{\pi\varepsilon_0 mR^3}}$

3. Answer (A, D)

Hint : Frequency as well as wavelength change.

Solution :

$$\lambda_1 = \lambda_0 - \frac{V}{5}T \quad \Rightarrow \quad \lambda_1 = \frac{4\lambda_0}{5}$$

$$\therefore \quad \lambda_2 = 2\lambda_1 = \frac{8\lambda_0}{5}$$

and, $T' = \dfrac{\lambda'}{2V+\dfrac{V}{5}} = \dfrac{5\lambda'}{11V}$

$$\therefore \quad f' = \frac{1}{T'} = \frac{11V}{5\times\left(\frac{8\lambda_0}{5}\right)} = \frac{11V}{8f_0}$$

4. Answer (B, C)

Hint : $\Delta Q = \Delta U + \Delta W$

Solution :

$$Q = \Delta U + \frac{Q}{2} \quad \Rightarrow \quad \Delta U = \frac{Q}{2}$$

$$\Rightarrow \quad n\times\left(\frac{3R}{2}\right)\cdot\Delta T = \frac{nC\Delta T}{2}$$

$\Rightarrow \quad C = 3R$

And, $\Delta U = \Delta W$

$$\Rightarrow \quad n\left(\frac{3R}{2}\right)\cdot dT = PdV$$

$\Rightarrow \quad P^3V$ = constant

$\Rightarrow \quad P^2 \times T$ = constant

$$\Rightarrow \quad P \propto \frac{1}{\sqrt{T}}$$

5. Answer (A)

Hint : A balanced wheatstone bridge is formed.

Solution :

$\therefore \quad V_D - V_C = 0$

$$\therefore \quad R_{eq} = \frac{12\times6}{12+6} = 4\ \Omega$$

$$\therefore \quad I_{battery} = \frac{20}{4} = 5\text{ A}$$

$$I_{AD} = \frac{20}{12} = \frac{5}{3}\text{A}$$

6. Answer (D)

Hint : Two spheres behave as capacitor and then become in parallel finally.

Solution :

$$Q_0 = 4\pi\varepsilon_0(2a)\times V$$

$$\therefore \quad i = \frac{V}{R}e^{-\frac{t}{\tau}},\quad \tau = R\times\left(\frac{C_1C_2}{C_1+C_2}\right)$$

$$= R\times\frac{4\pi\varepsilon_0 a\times2a}{3a}$$

$$= \frac{8\pi\varepsilon_0 Ra}{3}$$

$$\therefore \quad i = \frac{V}{R}e^{-\frac{3t}{8\pi\varepsilon_0 Ra}}$$

7. Answer (B)

Hint : Two spheres behave as capacitor and then become in parallel finally.

Solution :

Final charge on smaller sphere

$$q_2 = \frac{C_2}{C_1 + C_2} \times Q_0$$

$$= \frac{4\pi\varepsilon_0 \times a}{4\pi\varepsilon_0 (a + 2a)} \times \left[4\pi\varepsilon_0 \times (2a) \times V\right]$$

$$= \frac{1}{3} \times 8\pi\varepsilon_0 aV$$

8. Answer (C)

Hint : Two spheres behave as capacitor and then become in parallel finally.

Solution :

Total heat dissipation

$$H = \frac{1}{2} \times \left(\frac{C_1 C_2}{C_1 + C_2}\right) \times V^2$$

$$= \frac{1}{2} \times \frac{2}{3} \times (4\pi\varepsilon_0 a) V^2$$

$$= \frac{4\pi\varepsilon_0 a V^2}{3}$$

9. Answer (A)

Hint : Speed of sound, $V = \sqrt{\frac{\gamma RT}{M}}$

Solution :

$$V = \sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow \quad T = \frac{V^2 M}{\gamma R} = \frac{(300)^2 \times (29 \times 10^{-3})}{1.4 \times 8.314}$$

$$\simeq 224 \text{ K}$$

10. Answer (C)

Hint : Put the value of T_0.

Solution :

$\because \quad T = T_0 - 0.006\ h_0$

$\Rightarrow \quad 273 = 224 - 0.006 \times h_0$

$\Rightarrow \quad h_0 = 8170$ m

11. Answer (C)

Hint : Put the value of h_0.

Solution :

$$P = P_0 \left(1 - \frac{0.006 \times 8170}{273}\right)^{\frac{29 \times 10^{-3} \times 9.8}{8.31 \times 0.006}}$$

$= P_0 \times (0.82)^{5.7}$

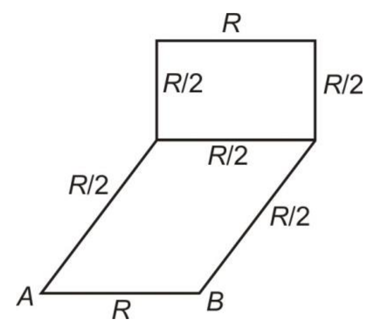
$= 0.32\ P_0$

12. Answer (B)

Hint : Use KVL and KCL

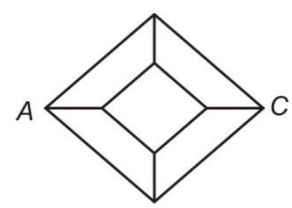
Solution :

For R_{AB}



$$\therefore \quad R_{AB} = \frac{7R}{12}$$

For R_{AC}



$$\therefore \quad R_{AC} = \frac{3R}{4}$$

$$\therefore \quad \frac{R_{AB}}{R_{AC}} = \frac{7 \times 4}{12 \times 3} = \frac{7}{9}$$

13. Answer (C)

Hint : Flux $= \frac{q}{4\pi\varepsilon_0} \times \text{Solid angle} \times 2$

Solution :

$$\phi = \frac{q}{\varepsilon_0} \times \frac{2\pi(1 - \cos\theta)}{4\pi} \times 2$$

$$= \frac{q}{\varepsilon_0}\left(1 - \frac{\ell}{\sqrt{\ell^2 + R^2}}\right)$$

$$= \frac{q}{\varepsilon_0}\left(1 - \frac{2}{\sqrt{5}}\right)$$

14. Answer (B)

Hint : Use concept of standing wave.

Solution :

$y = y_1 + y_2$

$$= a\left[\sin\left(\frac{\pi}{2}x - \omega t\right) + \sin\left(\frac{\pi}{2}x + \omega t + \frac{\pi}{3}\right)\right]$$

$$\Rightarrow \quad y = 2a\sin\left(\frac{\pi}{2}x + \frac{\pi}{6}\right)\cdot\cos\left(\text{cot} + \frac{\pi}{6}\right)$$

For nodes, $2a\sin\left(\frac{\pi}{2}x + \frac{\pi}{6}\right) = 0$

$$\Rightarrow \quad \frac{\pi}{2}x + \frac{\pi}{6} = \pi, 2\pi, 3\pi, 4\pi, ...$$

$$\Rightarrow \quad x = \frac{5}{3}, \frac{11}{3}, \frac{17}{3}, \frac{23}{3}$$

$\therefore$ For $0 \le x \le 6$,

Number of nodes = 3

15. Answer (A)

Hint : $P^{1-\gamma}\, T^{\gamma}$ = constant

Solution :

$$P_1^{1-\gamma}T_1^{\gamma} = P_2^{1-\gamma}T_2^{\gamma}$$

$$\Rightarrow \quad T_2 = 1000 \times \left(\frac{3}{2}\right)^{\left(\frac{3}{5}-1\right)} = 850 \text{ K}$$

Then, $\frac{P_3}{T_3} = \frac{P_2}{T_2} \quad \Rightarrow \quad T_3 = 425 \text{ K}$

$$\therefore \quad \Delta Q = nC_v\Delta T = 1 \times \left(\frac{3R}{2}\right) \times (850 - 425)$$

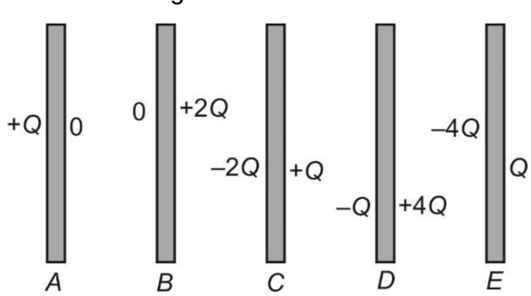
= 5300 J

16. Answer A(P, R); B(P, S); C(Q, T); D(Q, S)

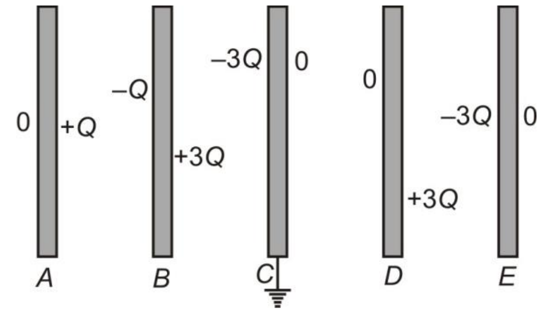
Hint : After earthing, charge on outer surface of outer most plates becomes zero.

Solution :

Before earthing



After earthing



$$V_{AB} = \frac{Qd}{\varepsilon_0 A}, V_{BC} = \frac{3Qd}{\varepsilon_0 A}, V_{CD} = 0, V_{DE} = \frac{3Qd}{\varepsilon_0 A}$$

17. Answer A(R, T); B(S, T); C(Q, S); D(Q, S)

Hint : In isothermal process $\Delta U = 0$

Solution :

For A : PV = constant

$\Rightarrow$ $\Delta U = 0$, ΔW = positive

$\Rightarrow$ ΔQ = positive

For B : $P = \frac{pRT}{m} \Rightarrow T = \text{constant}$

$\Rightarrow$ $\Delta U = 0$, ΔW = –negative, ΔQ = negative

And so on.

18. Answer (02)

Hint : Reduce it to a finite circuit.

Solution :

$R_{AB} = R_{CD}$, $V_{CD} = \frac{1}{2}V_{AB}$

$\therefore$ Current gets equally distributed

$\therefore$ $R_2 = R_{AB}$

And, $R_{AB} = R_1 + \left(\frac{R_2}{2}\right) = R_2$

$$\Rightarrow \quad \frac{R_2}{R_1} = 2$$

19. Answer (01)

Hint : Use Newton's law.

Solution :

$$\frac{-dT}{dt} = b(T - T_s)$$

$$\Rightarrow \quad \Delta T = (\Delta T)_0\, e^{-bt}$$

$\therefore$ $t_2 = 2t_0$

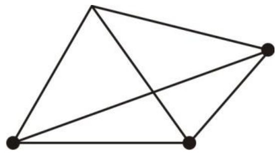
$\therefore$ $n = 1$

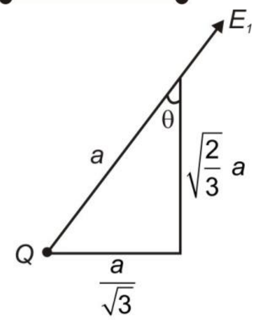
20. Answer (06)

Hint : Use superposition principle.

Solution :

$$E_1 = \frac{Q}{4\pi\varepsilon_0 a^2}$$





$$\therefore \quad E_{net} = 3\times(E_1\cos\theta) = 3\times\frac{Q}{4\pi\varepsilon_0 a^2}\times\sqrt{\frac{2}{3}}$$

$$= \frac{Q\sqrt{6}}{4\pi\varepsilon_0 a^2}$$

PART - II (CHEMISTRY)

21. Answer (B, C, D)

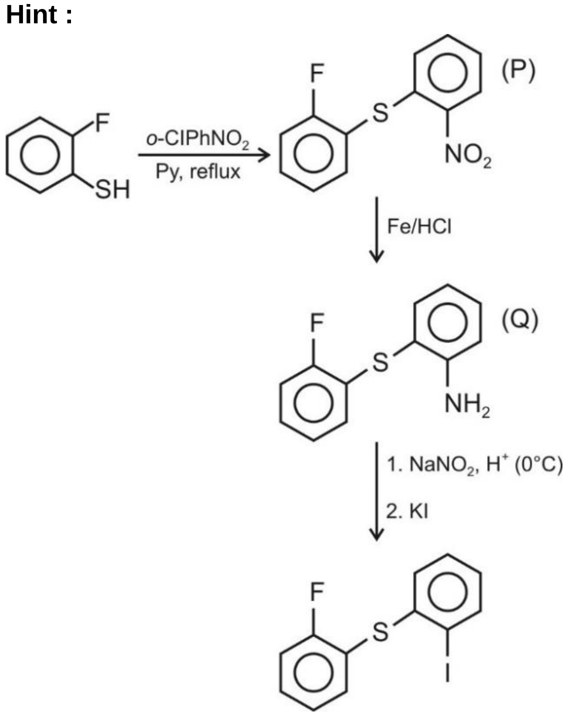
Hint : Benzaldehyde is not oxidised by Fehling's reagent.

Solution :

Acetophenone can give iodoform and bromoform.

22. Answer (A, C, D)

Hint :



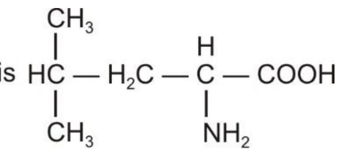
Solution :

Q is slightly basic

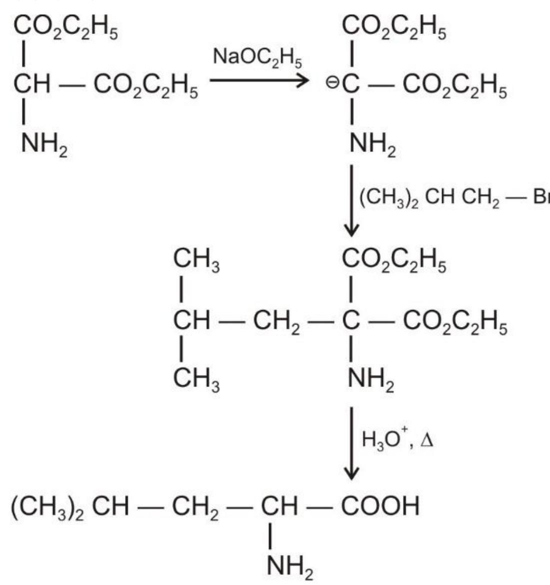
P contains fluorine atom

Because of the presence of $-NH_2$ group, Q can give coupling reaction

23. Answer (D)

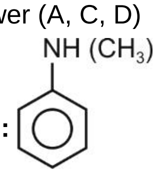
Hint : Leucine is 

Solution :

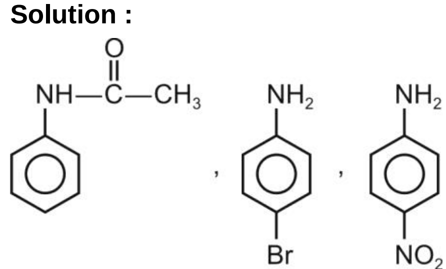


Racemic mixture

24. Answer (A, C, D)

Hint :  is more basic than aniline

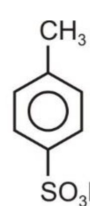
Solution :



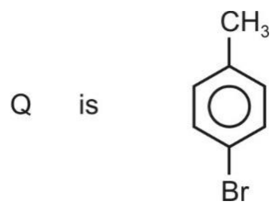
are less basic than aniline

25. Answer (C, D)

Hint :

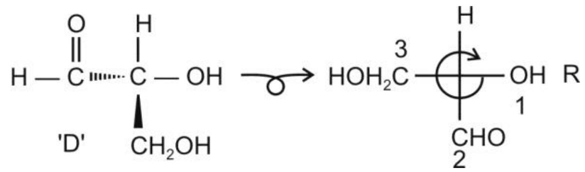
P is 

Solution :

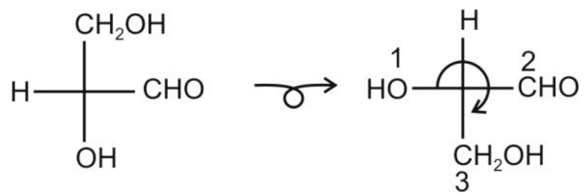


26. Answer (D)

Hint : Given D-glyceraldehyde is 'R'



Solution :



R as well as D.

27. Answer (B)

Hint : D and L convention is used for amino acids also

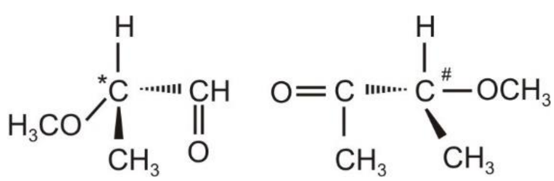
Solution :

(D) fructose is laevorotatory (l)

(D) glucose is dextrorotatory(d)

28. Answer (A)

Hint : Product obtained after ozonolysis



Solution :

C* is L

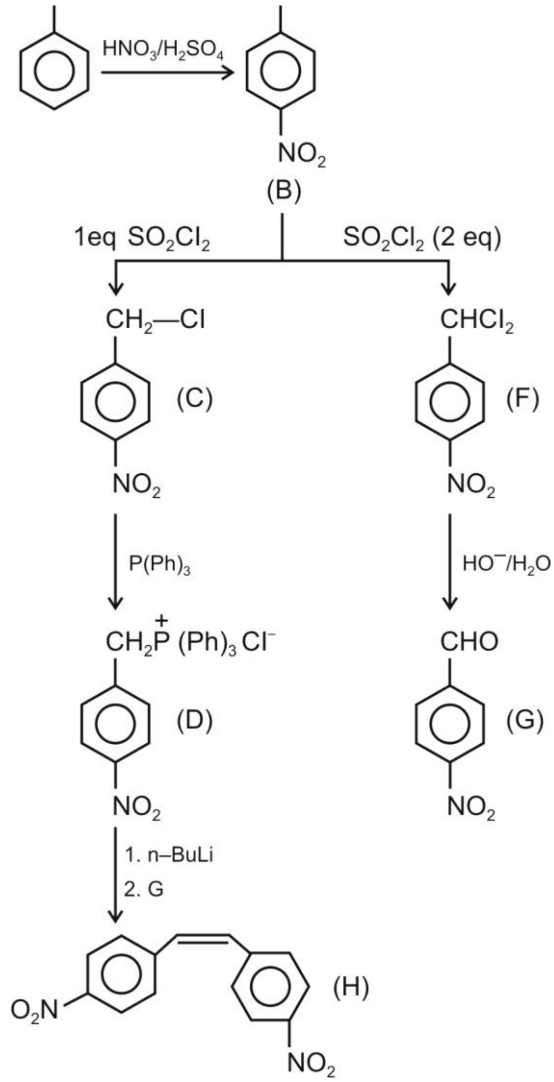
$C^{\#}$ is D.

29. Answer (B)

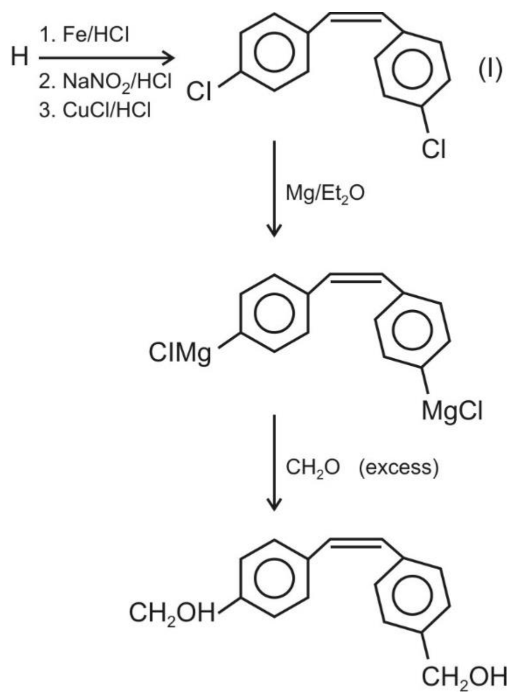
30. Answer (A)

31. Answer (B)

Hint and Solution : Q. Nos. 29 to 31



Solution :



32. Answer (A)

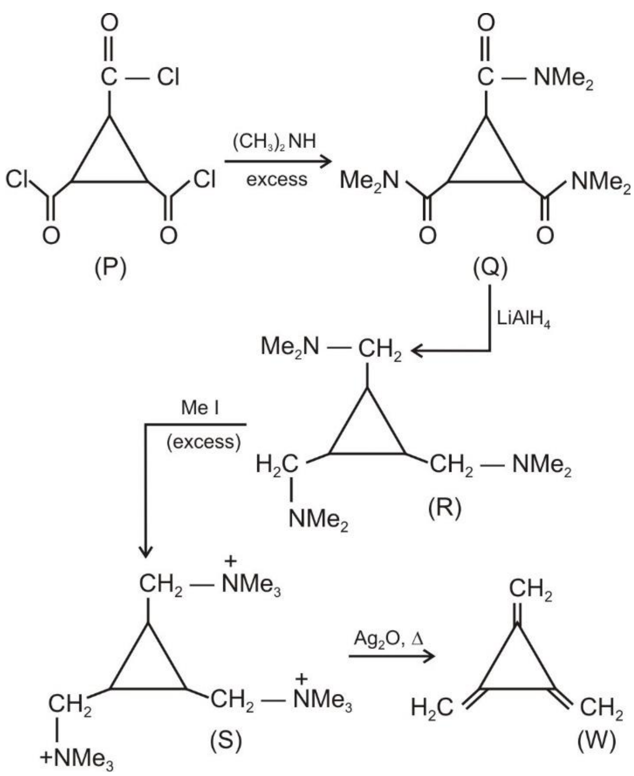
Hint : It is electrophilic substitution reaction.

Solution :

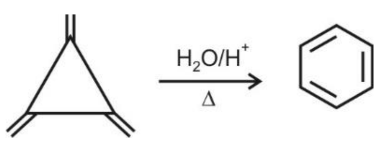
The intermediate formed when $^{+}NO_2$ attacks at position C-2 is more stable.

33. Answer (B)

Hint :

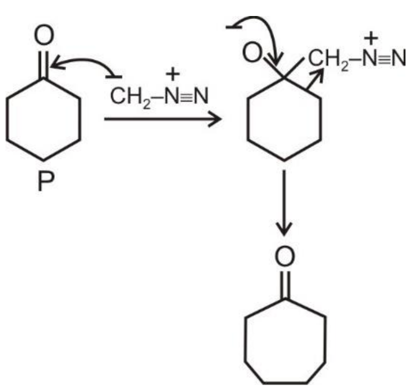


Solution :

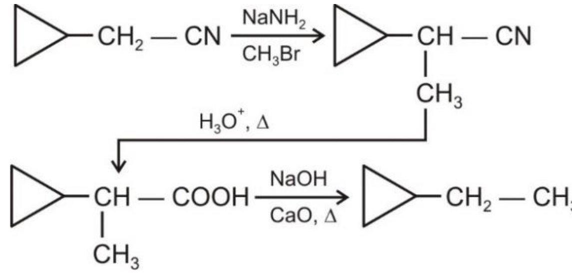


34. Answer (C)

Hint :



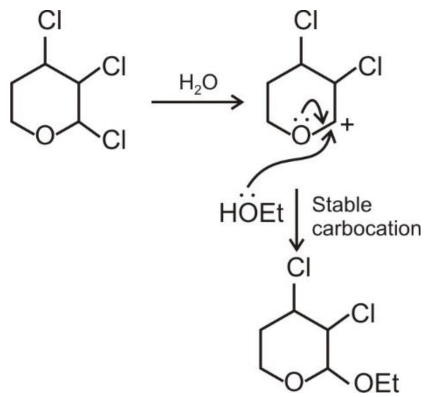
Solution :



35. Answer (A)

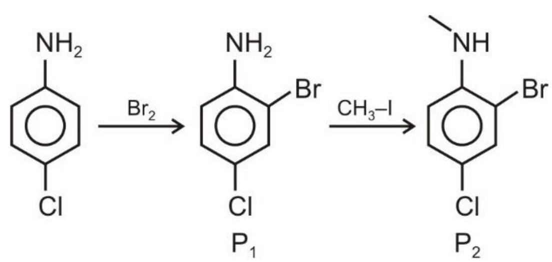
Hint : Excess of ether and water as solvent will favour S_N1 reaction.

Solution :

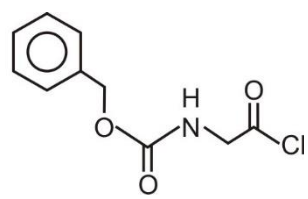


36. Answer A(Q, S); B(R); C(Q, R); D(P, T)

Hint :

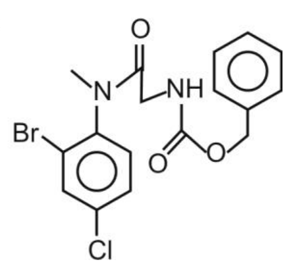


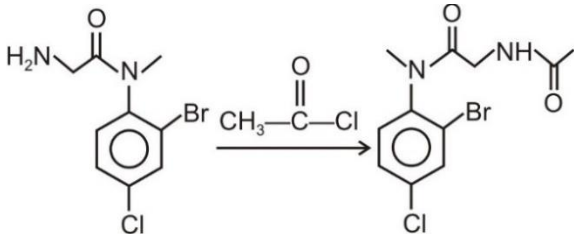
Q_1 –



Solution :

R_1 + R_2 →





37. Answer A(Q, S); B(Q); C(R, T); D(Q)

Hint : $LiAlH_4$ is a very strong reducing agent can reduce almost functional groups to lower oxidation state, except alkenes and alkynes.

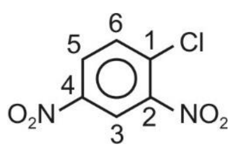
Solution :

$NaBH_4$ cannot reduce amide into amine

$NaBH_4$ can reduce only acid halide into alcohol among the given transformation.

38. Answer (07)

Hint :



Solution :

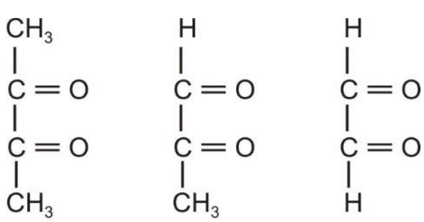
x = 1, y = 2, z = 4

39. Answer (03)

Hint :

O-xylene

Solution :



Possible Products

40. Answer (06)

Hint : O, P-substituted Bromo group are more likely to get substituted.

Solution :

The Br group at position 4 is most likely to get substitute and at position 2, is least

PART - III (MATHEMATICS)

41. Answer (B, C)

Hint : Translation and Rotation of axes.

Solution :

For $f(x,y) = 0$

new origin=

$$\left(\frac{hf-bh}{ab-h^2},\frac{gf-af}{ab-h^2}\right)\equiv\left(\frac{28}{-14},\frac{42}{-14}\right)\equiv(-2,-3)$$

For $g(x,y) = 0$

$$\theta=\frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)=\frac{1}{2}\tan^{-1}\left(\frac{2\times\sqrt{3}}{2}\right)$$

$$=\frac{1}{2}\times\frac{\pi}{3}=\frac{\pi}{6}$$

42. Answer (B, D)

Hint : Distance between two parallel lines.

Solution :

Distance between two parallel lines = $2\sqrt{5}$

$\therefore$ points on line $\dfrac{x-1}{\frac{-2}{\sqrt{5}}}=\dfrac{y+2}{\frac{1}{\sqrt{5}}}=\pm 2\sqrt{5}$

$\therefore$ points are (–3, 0) and (5, –4)

$\therefore$ Required lines are

$2x - y + 6 = 0$ and $2x - y - 14 = 0$

43. Answer (A)

Hint : Division formula between two points.

Solution :

$$A\left(\frac{ab}{a+b}\left(\frac{1-m}{-m},0\right):B\left(-0,\frac{ab}{a+b}(1-m)\right)\right)$$

Mid-point of AB is (h, k)

$$\therefore\ 2h=\frac{ab}{a+b}\frac{(m-1)}{m};\ 2k\,\frac{ab}{a+b}(1-m)$$

$$\therefore\ \frac{1}{2h}+\frac{1}{2k}=\frac{a+b}{ab}$$

$\Rightarrow$ ab $(x + y) = 2\,(a + b)\,xy$ is the locus

Let P divides AB in ratio 1 : 3

$$\therefore P\left(\frac{\frac{3ab}{a+b}\left(1-\frac{1}{m}\right)}{4},\frac{\frac{ab}{a+b}(1-m)}{4}\right)$$

$\therefore (x + 3y)ab \equiv 4(a + b)xy$ is the required locus

44. Answer (A, B)

Hint : Condition of two degree equation (to represent pair of straight line.

Solution :

$\Delta = 0 \Rightarrow abc + 2fgh = af^2 + bg^2 + ch^2$

$$\Rightarrow c = \frac{-10}{9}$$

Also

$$\cos\alpha = \left|\frac{a+b}{\sqrt{(a-b)^2+4h^2}}\right| \Rightarrow \alpha = \cos^{-1}\left(\frac{5}{\sqrt{34}}\right)$$

45. Answer (B, C)

Hint : Perpendicularity of two lines.

Solution :

L_1 and L_2 if they are $\perp$ to a common line $\Rightarrow \lambda = -1$ for two adjacent sides of a square

$L_1 \perp L_2$

$\therefore (\lambda^2 + 1)\lambda^2 = 1$

$\Rightarrow \lambda^5 + 2\lambda^3 + \lambda - 1 = 0 = f(\lambda)$

$\therefore f'(\lambda) = 5\lambda^4 + 6\lambda^2 + 1 = 0$

$\therefore f(\lambda) = 0$ has only one real root

46. Answer (C)

Hint : Family of circle with line.

Solution :

Let required circle

$x^2 + y^2 - 3x + 2y - 4 + \lambda(2x + 5y + 2) = 0$

$\therefore C\left(\frac{3-2\lambda}{2},\frac{-(5\lambda+2)}{2}\right)$ satisfy $x + y = 11$

$\therefore \lambda = -3$

$\therefore$ Required circle is $x^2 + y^2 - 9x - 13y - 10 = 0$

47. Answer (D)

Hint : Orthogonal of two circles.

Solution :

Let required circle

$(x - 1)^2 + (y + 1)^2 + \lambda(2x + 3y + 1) = 0$ (i)

Circle with diameter points (0,3) and (–2,–1) is

$x^2 + y^2 + 2x - 2y - 3 = 0$ (ii)

(i) of (ii) intersect orthogonally

$$\therefore (2\lambda-1) + -2\left(\frac{3\lambda}{2}+1\right) = \lambda - 1 \Rightarrow \lambda = \frac{-3}{2}$$

Required circle is $2x^2 + 2y^2 - 10x - 5y + 1 = 0$

48. Answer (A)

Hint : Touching concept of line with circle.

Solution :

Let required circle

$(x^2 + y^2 - 4) + \lambda(x + 2y - 4) = 0$

$$C\left(\frac{-\lambda}{2},-\lambda\right) = \lambda = \sqrt{\frac{5\lambda^2}{4}+4\lambda+4}$$

$\because$ it touches line x + 2y – 5 = 0

$$\therefore \left|\frac{\frac{-\lambda}{2}+2(-\lambda)-5}{\sqrt{5}}\right| = \sqrt{\frac{5\lambda^2}{4}+4\lambda+4}$$

$\Rightarrow 5(\lambda + 2)^2 = 5\lambda^2 + 16\lambda + 16$

$\Rightarrow \lambda = -1$

$\therefore$ Required circle $x^2 + y^2 - x - 2y = 0$

49. Answer (D)

Hint : Chord of contact of circle.

Solution :

Equation of C.O.C $hx + ky = 8$ (i)

Also $tk = h + 2t^2$ (ii) $\because (h,k)$ satisfy tangent)

$$\Rightarrow hx + y\frac{(h+2t^2)}{t} - 8 = 0$$

$$\Rightarrow 2(ty-4) + h\left(x+\frac{y}{t}\right) = 0$$

$\therefore$ Line passes through point

$y = \frac{4}{t}$ and $x = \frac{-y}{t}$

$$\Rightarrow \frac{y}{4} = -\frac{x}{y} \Rightarrow y^2 = -4x$$

50. Answer (A)

Hint : Point of intersection of two curves.

Solution :

Point of intersection of $x = -2$ and $x^2 + y^2 = 16$

$\therefore y^2 = 12 \Rightarrow y = \pm 2\sqrt{3}$

$\therefore \text{Point}\left(-2, 2\sqrt{3}\right)$

51. Answer (C)

Hint : Circumcircle of triangle *ABC*

Solution :

The equation of the circumcircle of ΔAQB is

$(x^2 + y^2 - 8) + \lambda(hx + ky - 8) = 0 \because$ (A = –1) due to (0,0) satisfy it

$\therefore$ Equation is $x^2 + y^2 - hx - ky = 0$

Now centre $\left(\frac{h}{2}, \frac{k}{2}\right)$

$\therefore$ For locus let $x = \frac{h}{2}; y = \frac{k}{2}$

$h = 2x$ and $k = 2y$

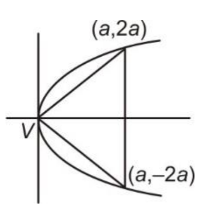
$\because tk = h + 2t^2 \therefore$ at $t = 2$

The required locus is $2y = x + 4$

52. Answer (D)

Hint : Length of latus rectum independency.

Solution :



$\theta = 2\tan^{-1} 2$

$\because \sqrt{3} < 2 < \sqrt{2} + 1$

$\frac{\pi}{3} < \tan^{-1} 2 < \frac{3\pi}{8}$

$\Rightarrow \frac{2\pi}{3} < \theta < \frac{3\pi}{-4}$

53. Answer (B)

Hint : Condition of common tangent on two curves.

Solution :

$y = \frac{x}{2} + 2$ is tangent on $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$

$\Rightarrow 4 + 4b^2 = 16 \Rightarrow b^2 + 1 = 4 \Rightarrow b = \pm\sqrt{3}$

Now tangent at other point is given by $-2y = x + 4$

$\Rightarrow x + 2y + 4 = 0$

54. Answer (A)

Hint : Locus of midpoint of Parallel chords.

Solution :

Let middle point is (h, k)

$\therefore$ Equation of chord in mid-point form is

$\frac{x}{h} + \frac{y}{k} = 2$

$\therefore -\frac{1}{h \times \frac{1}{k}} = m \Rightarrow k = -mh$

$\Rightarrow y + mx = 0$ is the required locus

55. Answer (C)

Hint : I.T.F. conversion in domain.

Solution :

Let $\sin^{-1}x = \theta \Rightarrow x = \sin\theta$

Now

$$\cos^{-1}x = \cos^{-1}(\sin\theta) = \cos^{-1}\left(-\cos\left(\frac{3\pi}{2} - \theta\right)\right)$$

$$= \pi - \cos^{-1}\left(\cos\left(\frac{3\pi}{2} - \theta\right)\right)$$

$$= \pi - \left(\frac{3\pi}{2} - \theta\right) \text{ as } \frac{3\pi}{2} - \theta \in (0, \pi)$$

$$= \theta - \frac{\pi}{2} = \sin^{-1}x - \frac{\pi}{2}$$

$$\therefore \sin^{-1}x + \cos^{-1}x = 2\sin^{-1}x - \frac{\pi}{2}$$

56. Answer A(P); B(Q); C(Q, S); D(R, S)

Hint : Type of functions concept.

Solution :

(A) $f(x)=\begin{cases}\left((1)^{1}\right)^{n} x>0=1 & ,x>0\\ \left((-1)^{-1}\right)^{n} x<0 & ,x<0\end{cases}$

$f(x)$ is an odd function. $f(x)$ is not bijective

$\because$ $f(x)$ is not one one

(B) $f(x)=\dfrac{x}{e^{x}-1}+\dfrac{x}{2}+1$

$f(x)=\dfrac{-x}{e^{-x}-1}-\dfrac{x}{2}+1=x+\dfrac{x}{e^{x}-1}-\dfrac{x}{2}+f(x)$

$\therefore$ $f(x)$ is an even function $\because$ $f(x)$ is not bijective

(C) $f(-x) = f(x)$ $\therefore$ $f(x)$ is even. $f(x)$ is periodic but time period not define

(D) $f(x) = \max\{\tan x, \cot x\}$. From graph of $f(x)$ it is clear that

$f(x)$ is neither even nor odd

$\because f(x+\pi)=\max\{\tan(x+\pi),\cot(x+\pi)\}$

$=\max\{\tan x,\cot x\}$ $f(x)$ is periodic with f(x) is periodic with π

57. Answer A(P, Q); B(P, Q); C(R, T); D(S, T)

Hint : Property of perpendicular normals.

Solution :

Equation of normal at ‘p’ is

$y = -tx + 2at + at^3$

Put $y = -2at$ $\Rightarrow$ $x = 4a + at^2$

$\therefore$G $(4a + at^2, -2at)$

Required locus $y^2 = 4a(x - 4a)$

$\because$ $a = 1$

$\therefore$ $y^2 = 4(x - 4)$

Now verify each option

58. Answer (09)

Hint : Sum of infinite G.P.

Solution :

$\dfrac{1}{1-\sin\left(\cos^{-1}x\right)}=2$

$\Rightarrow \sin\left(\cos^{-1}x\right)=\dfrac{1}{2}$

$\Rightarrow \cos^{-1}x=\dfrac{\pi}{6}\Rightarrow x=\dfrac{\sqrt{3}}{2}\Rightarrow 4x^{2}=3$

$\Rightarrow 12x^2 = 9$

59. Answer (00)

Hint : Trigonometric conversion of I.T.F.

Solution :

$\sin\left(\cos^{-1}\left(\tan\left(\tan^{-1}\left(\sqrt{x^{2}-1}\right)\right)\right)\right)$

$=\sin\left(\cos^{-1}\sqrt{x^{2}-1}\right)$

$=\sin\left(\sin^{-1}\sqrt{2-x^{2}}\right)=\sqrt{2-x^{2}}$

$\therefore$ Common domain $\left[1,\sqrt{2}\right]\Rightarrow 2-x^{2}=1+x$

$\Rightarrow x^{2}+x-1=0\Rightarrow x=\dfrac{-1\pm\sqrt{5}}{2}$

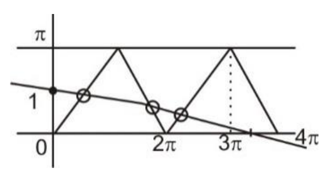
$x=\dfrac{\sqrt{5}-1}{2}\notin\left[1,\sqrt{2}\right]$

$\therefore$No solution exists

60. Answer (03)

Hint : Graphical solution.

Solution :



From graph it is clear that curves intersect at 3 points

$\therefore$ Only 3 solutions

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All India Aakash Test Series for JEE (Advanced)-2020

TEST - 2A (Paper-2) - Code-F

Test Date : 24/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(A)	21.	(C, D)	41.	(B, C)
2.	(B, C)	22.	(A, C, D)	42.	(A, B)
3.	(A, D)	23.	(D)	43.	(A)
4.	(B, D)	24.	(A, C, D)	44.	(B, D)
5.	(B, D)	25.	(B, C, D)	45.	(B, C)
6.	(D)	26.	(D)	46.	(C)
7.	(B)	27.	(B)	47.	(D)
8.	(C)	28.	(A)	48.	(A)
9.	(A)	29.	(B)	49.	(D)
10.	(C)	30.	(A)	50.	(A)
11.	(C)	31.	(B)	51.	(C)
12.	(A)	32.	(A)	52.	(C)
13.	(B)	33.	(C)	53.	(A)
14.	(C)	34.	(B)	54.	(B)
15.	(B)	35.	(A)	55.	(D)
16.	A $\rightarrow$ (R, T)	36.	A $\rightarrow$ (Q, S)	56.	A $\rightarrow$ (P, Q)
	B $\rightarrow$ (S, T)		B $\rightarrow$ (Q)		B $\rightarrow$ (P, Q)
	C $\rightarrow$ (Q, S)		C $\rightarrow$ (R, T)		C $\rightarrow$ (R, T)
	D $\rightarrow$ (Q, S)		D $\rightarrow$ (Q)		D $\rightarrow$ (S, T)
17.	A $\rightarrow$ (P, R)	37.	A $\rightarrow$ (Q, S)	57.	A $\rightarrow$ (P)
	B $\rightarrow$ (P, S)		B $\rightarrow$ (R)		B $\rightarrow$ (Q)
	C $\rightarrow$ (Q, T)		C $\rightarrow$ (Q, R)		C $\rightarrow$ (Q, S)
	D $\rightarrow$ (Q, S)		D $\rightarrow$ (P, T)		D $\rightarrow$ (R, S)
18.	(06)	38.	(06)	58.	(03)
19.	(01)	39.	(03)	59.	(00)
20.	(02)	40.	(07)	60.	(09)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (A)

Hint : A balanced wheatstone bridge is formed.

Solution :

$\therefore \quad V_D - V_C = 0$

$$\therefore \quad R_{eq} = \frac{12\times 6}{12+6} = 4\ \Omega$$

$$\therefore \quad I_{battery} = \frac{20}{4} = 5\text{ A}$$

$$I_{AD} = \frac{20}{12} = \frac{5}{3}\text{A}$$

2. Answer (B, C)

Hint : $\Delta Q = \Delta U + \Delta W$

Solution :

$$Q = \Delta U + \frac{Q}{2} \quad \Rightarrow \quad \Delta U = \frac{Q}{2}$$

$$\Rightarrow \quad n\times\left(\frac{3R}{2}\right)\cdot \Delta T = \frac{nC\,\Delta T}{2}$$

$\Rightarrow \quad C = 3R$

And, $\Delta U = \Delta W$

$$\Rightarrow \quad n\left(\frac{3R}{2}\right)\cdot dT = PdV$$

$\Rightarrow \quad P^3V$ = constant

$\Rightarrow \quad P^2 \times T$ = constant

$$\Rightarrow \quad P \propto \frac{1}{\sqrt{T}}$$

3. Answer (A, D)

Hint : Frequency as well as wavelength change.

Solution :

$$\lambda_1 = \lambda_0 - \frac{V}{5}T \quad \Rightarrow \quad \lambda_1 = \frac{4\lambda_0}{5}$$

$$\therefore \quad \lambda_2 = 2\lambda_1 = \frac{8\lambda_0}{5}$$

$$\text{and, } T' = \frac{\lambda'}{2V + \frac{V}{5}} = \frac{5\lambda'}{11V}$$

$$\therefore \quad f' = \frac{1}{T'} = \frac{11V}{5\times\left(\frac{8\lambda_0}{5}\right)} = \frac{11V}{8f_0}$$

4. Answer (B, D)

Hint : Particle performs SHM.

Solution :

$$E = \frac{Qx}{4\pi\varepsilon_0\left(R^2+x^2\right)^{\frac{3}{2}}}, \quad x << R$$

$$\Rightarrow \quad E = \frac{Qx}{4\pi\varepsilon_0 R^3}$$

$$\therefore \quad \omega = \sqrt{\frac{Qq}{m\times 4\pi\varepsilon_0 R^3}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m\times 4\pi\varepsilon_0 R^3}{Qq}} = 4\pi\sqrt{\frac{\pi\varepsilon_0 mR^3}{Qq}}$$

and, $V_{max} = \omega a = \dfrac{ax}{2}\sqrt{\dfrac{Qq}{\pi\varepsilon_0 mR^3}}$

5. Answer (B, D)

Hint : B and C will be in series.

Solution :

$$C_{BC} = \frac{2\times 3}{2+3} = \frac{6}{5}\mu\text{F}$$

$$\therefore \quad \Delta q_S = \frac{\frac{6}{5}}{\frac{6}{5}+1}\times 110 = 60\ \mu\text{C}$$

$\therefore \quad q_A = 110 - 60 = 50\ \mu$C

$$V_B = \frac{60}{2} = 30\text{ V},\ V_C = \frac{60}{3} = 20\text{ V}$$

6. Answer (D)

Hint : Two spheres behave as capacitor and then become in parallel finally.

Solution :

$$Q_0 = 4\pi\varepsilon_0(2a)\times V$$

$$\therefore \quad i = \frac{V}{R}e^{-\frac{t}{\tau}}, \quad \tau = R\times\left(\frac{C_1C_2}{C_1+C_2}\right)$$

$$= R\times\frac{4\pi\varepsilon_0 a\times 2a}{3a}$$

$$= \frac{8\pi\varepsilon_0 Ra}{3}$$

$$\therefore \quad i = \frac{V}{R}e^{-\frac{3t}{8\pi\varepsilon_0 Ra}}$$

7. Answer (B)

Hint : Two spheres behave as capacitor and then become in parallel finally.

Solution :

Final charge on smaller sphere

$$q_2 = \frac{C_2}{C_1 + C_2} \times Q_0$$

$$= \frac{4\pi\varepsilon_0 \times a}{4\pi\varepsilon_0 (a + 2a)} \times \left[4\pi\varepsilon_0 \times (2a) \times V\right]$$

$$= \frac{1}{3} \times 8\pi\varepsilon_0 aV$$

8. Answer (C)

Hint : Two spheres behave as capacitor and then become in parallel finally.

Solution :

Total heat dissipation

$$H = \frac{1}{2} \times \left(\frac{C_1 C_2}{C_1 + C_2}\right) \times V^2$$

$$= \frac{1}{2} \times \frac{2}{3} \times (4\pi\varepsilon_0 a) V^2$$

$$= \frac{4\pi\varepsilon_0 a V^2}{3}$$

9. Answer (A)

Hint : Speed of sound, $V = \sqrt{\frac{\gamma RT}{M}}$

Solution :

$$V = \sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow \quad T = \frac{V^2 M}{\gamma R} = \frac{(300)^2 \times (29 \times 10^{-3})}{1.4 \times 8.314}$$

$$\simeq 224 \text{ K}$$

10. Answer (C)

Hint : Put the value of T_0.

Solution :

$\because \quad T = T_0 - 0.006\, h_0$

$\Rightarrow \quad 273 = 224 - 0.006 \times h_0$

$\Rightarrow \quad h_0 = 8170$ m

11. Answer (C)

Hint : Put the value of h_0.

Solution :

$$P = P_0 \left(1 - \frac{0.006 \times 8170}{273}\right)^{\frac{29 \times 10^{-3} \times 9.8}{8.31 \times 0.006}}$$

$= P_0 \times (0.82)^{5.7}$

$= 0.32\, P_0$

12. Answer (A)

Hint : $P^{1-\gamma}\, T^{\gamma}$ = constant

Solution :

$$P_1^{1-\gamma} T_1^{\gamma} = P_2^{1-\gamma} T_2^{\gamma}$$

$$\Rightarrow \quad T_2 = 1000 \times \left(\frac{3}{2}\right)^{\left(\frac{3}{5} - 1\right)} = 850 \text{ K}$$

Then, $\frac{P_3}{T_3} = \frac{P_2}{T_2} \quad \Rightarrow \quad T_3 = 425$ K

$$\therefore \quad \Delta Q = nC_v \Delta T = 1 \times \left(\frac{3R}{2}\right) \times (850 - 425)$$

= 5300 J

13. Answer (B)

Hint : Use concept of standing wave.

Solution :

$y = y_1 + y_2$

$$= a\left[\sin\left(\frac{\pi}{2}x - \omega t\right) + \sin\left(\frac{\pi}{2}x + \omega t + \frac{\pi}{3}\right)\right]$$

$$\Rightarrow \quad y = 2a \sin\left(\frac{\pi}{2}x + \frac{\pi}{6}\right) \cdot \cos\left(\text{cot} + \frac{\pi}{6}\right)$$

For nodes, $2a \sin\left(\frac{\pi}{2}x + \frac{\pi}{6}\right) = 0$

$$\Rightarrow \quad \frac{\pi}{2}x + \frac{\pi}{6} = \pi, 2\pi, 3\pi, 4\pi, \ldots.$$

$$\Rightarrow \quad x = \frac{5}{3}, \frac{11}{3}, \frac{17}{3}, \frac{23}{3}$$

$\therefore \quad$ For $0 \le x \le 6$,

Number of nodes = 3

14. Answer (C)

Hint : Flux $= \frac{q}{4\pi\varepsilon_0} \times \text{Solid angle} \times 2$

Solution :

$$\phi = \frac{q}{\varepsilon_0} \times \frac{2\pi(1 - \cos\theta)}{4\pi} \times 2$$

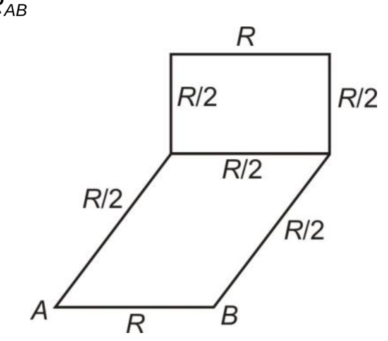
$$= \frac{q}{\varepsilon_0}\left(1 - \frac{\ell}{\sqrt{\ell^2 + R^2}}\right) = \frac{q}{\varepsilon_0}\left(1 - \frac{2}{\sqrt{5}}\right)$$

15. Answer (B)

Hint : Use KVL and KCL

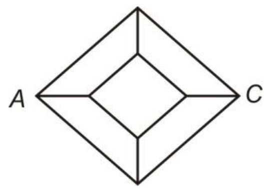
Solution :

For R_{AB}



$$\therefore \quad R_{AB} = \frac{7R}{12}$$

For R_{AC}



$\therefore \quad R_{AC} = \dfrac{3R}{4}$

$\therefore \quad \dfrac{R_{AB}}{R_{AC}} = \dfrac{7 \times 4}{12 \times 3} = \dfrac{7}{9}$

16. Answer A(R, T); B(S, T); C(Q, S); D(Q, S)

Hint : In isothermal process $\Delta U = 0$

Solution :

For A : PV = constant

$\Rightarrow \quad \Delta U = 0, \Delta W$ = positive

$\Rightarrow \quad \Delta Q$ = positive

For B : $P = \dfrac{pRT}{m} \Rightarrow T = \text{constant}$

$\Rightarrow \quad \Delta U = 0, \Delta W$ = –negative, ΔQ = negative

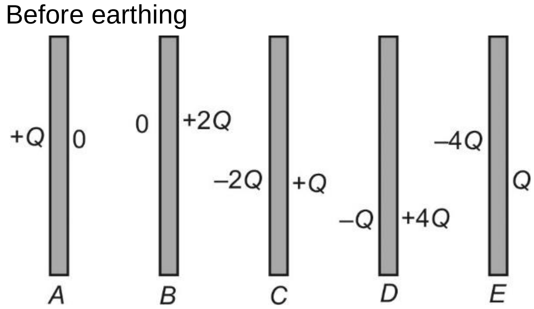
And so on.

17. Answer A(P, R); B(P, S); C(Q, T); D(Q, S)

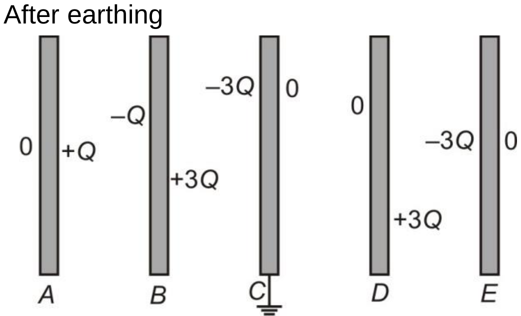
Hint : After earthing, charge on outer surface of outer most plates becomes zero.

Solution :

Before earthing



After earthing



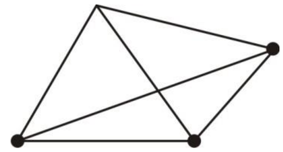
$V_{AB} = \dfrac{Qd}{\varepsilon_0 A}, V_{BC} = \dfrac{3Qd}{\varepsilon_0 A}, V_{CD} = 0, V_{DE} = \dfrac{3Qd}{\varepsilon_0 A}$

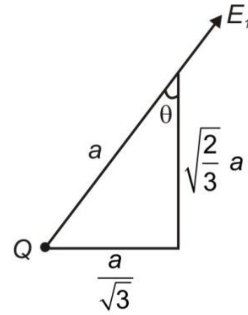
18. Answer (06)

Hint : Use superposition principle.

Solution :

$E_1 = \dfrac{Q}{4\pi\varepsilon_0 a^2}$





$\therefore \quad E_{\text{net}} = 3 \times (E_1 \cos\theta) = 3 \times \dfrac{Q}{4\pi\varepsilon_0 a^2} \times \sqrt{\dfrac{2}{3}}$

$= \dfrac{Q\sqrt{6}}{4\pi\varepsilon_0 a^2}$

19. Answer (01)

Hint : Use Newton's law.

Solution :

$\dfrac{-dT}{dt} = b(T - T_s)$

$\Rightarrow \quad \Delta T = (\Delta T)_0 e^{-bt}$

$\therefore \quad t_2 = 2t_0$

$\therefore \quad n = 1$

20. Answer (02)

Hint : Reduce it to a finite circuit.

Solution :

$R_{AB} = R_{CD}$, $V_{CD} = \dfrac{1}{2}V_{AB}$

$\therefore$ Current gets equally distributed

$\therefore \quad R_2 = R_{AB}$

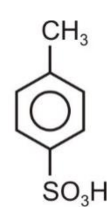
And, $R_{AB} = R_1 + \left(\dfrac{R_2}{2}\right) = R_2$

$\Rightarrow \quad \dfrac{R_2}{R_1} = 2$

PART - II (CHEMISTRY)

21. Answer (C, D)

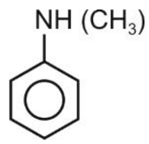
Hint :

P is 

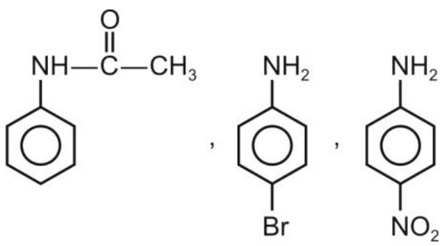
Solution :

Q is 

22. Answer (A, C, D)

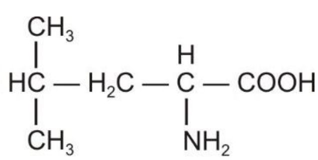
Hint :  is more basic than aniline

Solution :

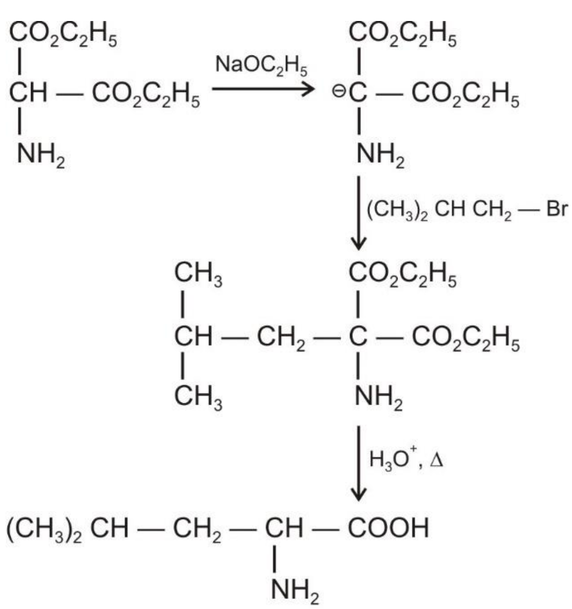


are less basic than aniline

23. Answer (D)

Hint : Leucine is 

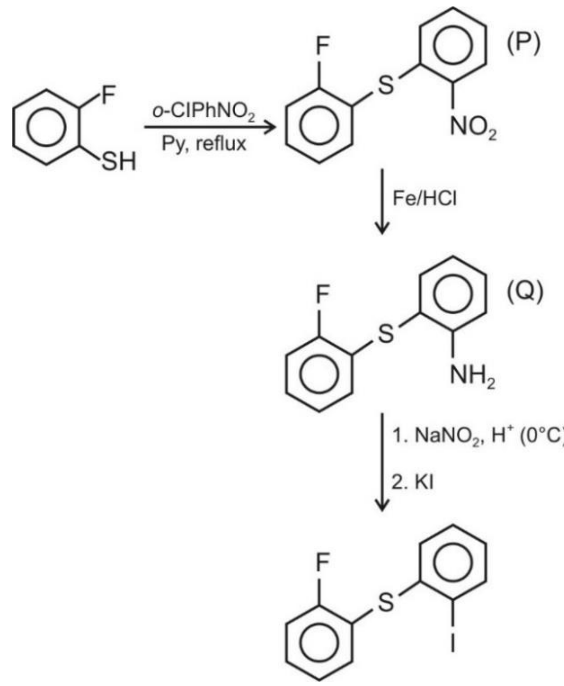
Solution :



Racemic mixture

24. Answer (A, C, D)

Hint :



Solution :

Q is slightly basic

P contains fluorine atom

Because of the presence of $-NH_2$ group, Q can give coupling reaction

25. Answer (B, C, D)

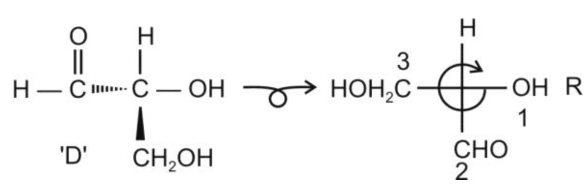
Hint : Benzaldehyde is not oxidised by Fehling's reagent.

Solution :

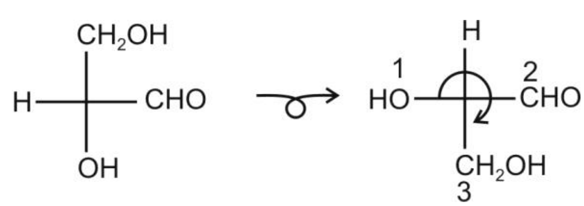
Acetophenone can give iodoform and bromoform.

26. Answer (D)

Hint : Given D-glyceraldehyde is 'R'



Solution :



R as well as D.

27. Answer (B)

Hint : D and L convention is used for amino acids also

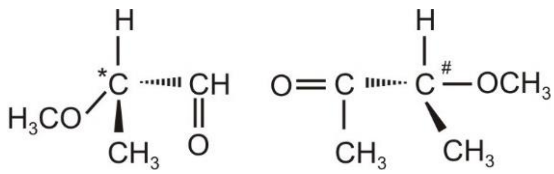
Solution :

(D) fructose is laevorotatory (l)

(D) glucose is dextrorotatory(d)

28. Answer (A)

Hint : Product obtained after ozonolysis



Solution :

C* is L

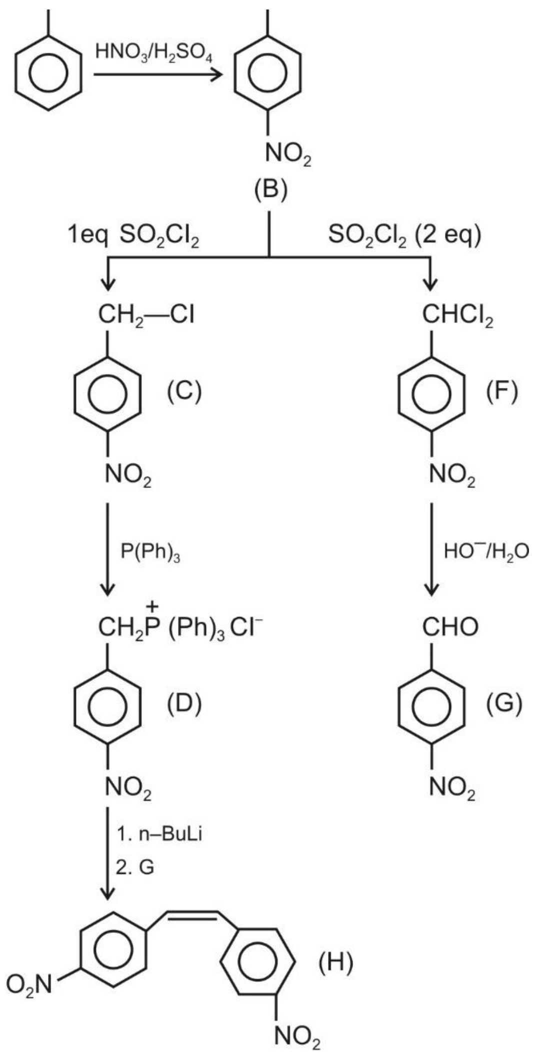
$C^{\#}$ is D.

29. Answer (B)

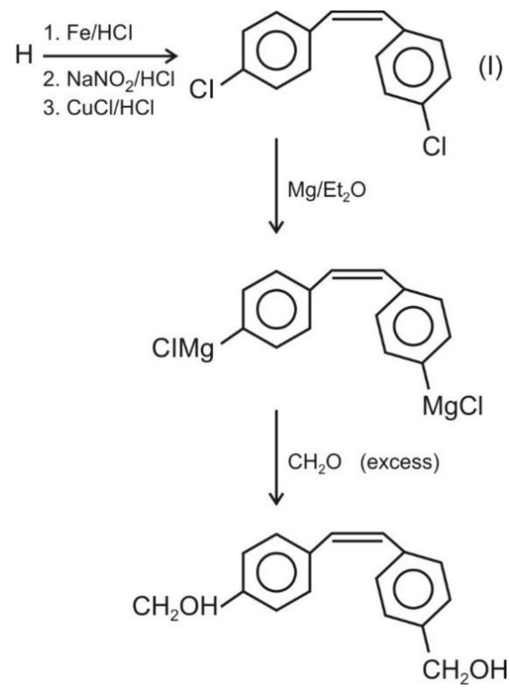
30. Answer (A)

31. Answer (B)

Hint and Solution : Q. Nos. 29 to 31



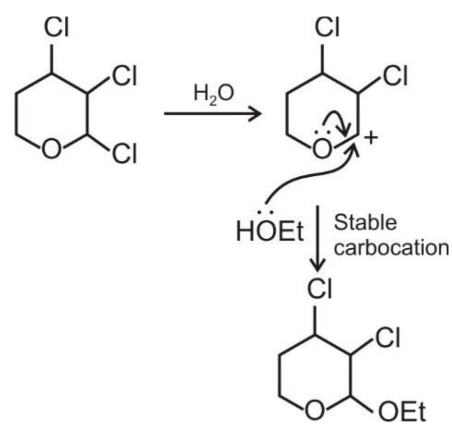
Solution :



32. Answer (A)

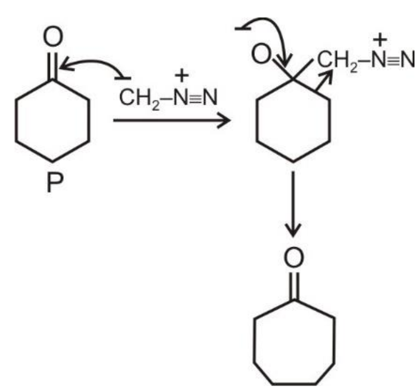
Hint : Excess of ether and water as solvent will favour S_N1 reaction.

Solution :

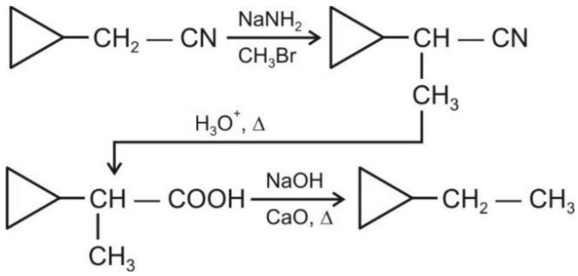


33. Answer (C)

Hint :

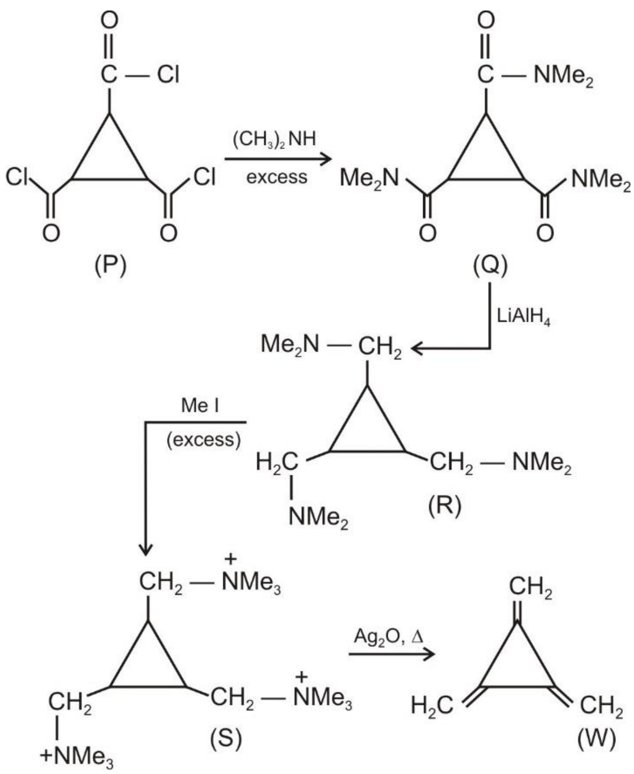


Solution :

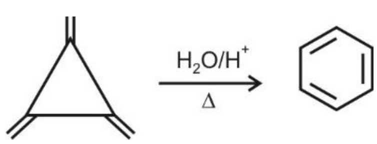


34. Answer (B)

Hint :



Solution :



35. Answer (A)

Hint : It is electrophilic substitution reaction.

Solution :

The intermediate formed when $^{+}NO_2$ attacks at position C-2 is more stable.

36. Answer A(Q, S); B(Q); C(R, T); D(Q)

Hint : $LiAlH_4$ is a very strong reducing agent can reduce almost functional groups to lower oxidation state, except alkenes and alkynes.

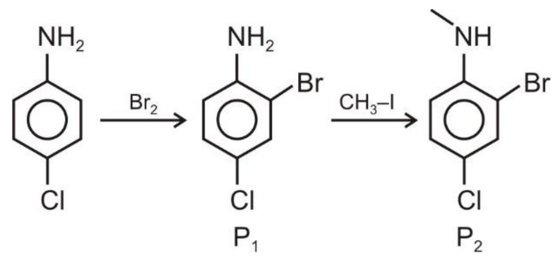
Solution :

$NaBH_4$ cannot reduce amide into amine

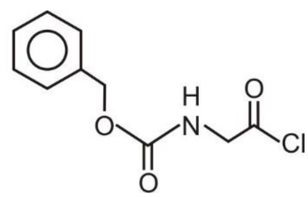
$NaBH_4$ can reduce only acid halide into alcohol among the given transformation.

37. Answer A(Q, S); B(R); C(Q, R); D(P, T)

Hint :

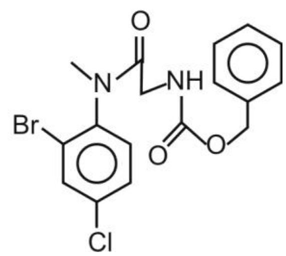


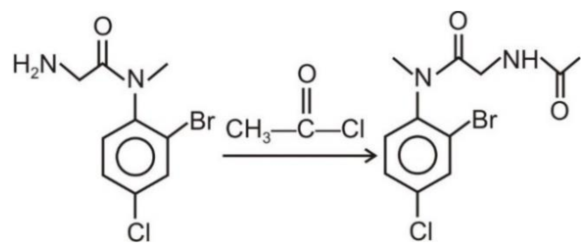
Q_1 –



Solution :

$R_1 + R_2 \rightarrow$





38. Answer (06)

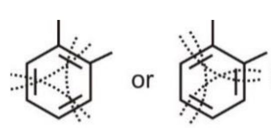
Hint : O, P-substituted Bromo group are more likely to get substituted.

Solution :

The Br group at position 4 is most likely to get substitute and at position 2, is least

39. Answer (03)

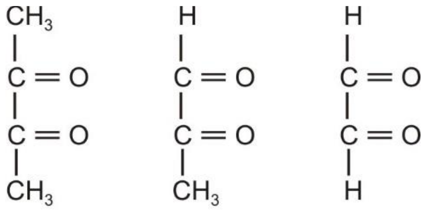
Hint :



Possible ways of ozonolysis

O-xylene

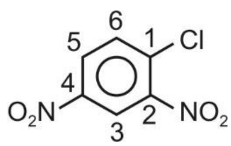
Solution :



Possible Products

40. Answer (07)

Hint :



Solution :

x = 1, y = 2, z = 4

PART - III (MATHEMATICS)

41. Answer (B, C)

Hint : Perpendicularity of two lines.

Solution :

L_1 and L_2 if they are $\perp$ to a common line $\Rightarrow \lambda = -1$ for two adjacent sides of a square

$L_1 \perp L_2$

$\therefore (\lambda^2 + 1)\lambda^2 = 1$

$\Rightarrow \lambda^5 + 2\lambda^3 + \lambda - 1 = 0 = f(\lambda)$

$\therefore f'(\lambda) = 5\lambda^4 + 6\lambda^2 + 1 = 0$

$\therefore f(\lambda) = 0$ has only one real root

42. Answer (A, B)

Hint : Condition of two degree equation (to represent pair of straight line.

Solution :

$\Delta = 0 \Rightarrow abc + 2fgh = af^2 + bg^2 + ch^2$

$$\Rightarrow c = \frac{-10}{9}$$

Also

$$\cos\alpha = \left|\frac{a+b}{\sqrt{(a-b)^2 + 4h^2}}\right| \Rightarrow \alpha = \cos^{-1}\left(\frac{5}{\sqrt{34}}\right)$$

43. Answer (A)

Hint : Division formula between two points.

Solution :

$$A\left(\frac{ab}{a+b}\left(\frac{1-m}{-m}, 0\right)\right) : B\left(-0, \frac{ab}{a+b}(1-m)\right)$$

Mid-point of AB is (h, k)

$$\therefore 2h = \frac{ab}{a+b}\frac{(m-1)}{m};\ 2k\frac{ab}{a+b}(1-m)$$

$$\therefore \frac{1}{2h} + \frac{1}{2k} = \frac{a+b}{ab}$$

$\Rightarrow$ ab $(x + y) = 2(a + b)xy$ is the locus

Let P divides AB in ratio 1 : 3

$$\therefore P\left(\frac{\frac{3ab}{a+b}\left(1-\frac{1}{m}\right)}{4}, \frac{\frac{ab}{a+b}(1-m)}{4}\right)$$

$\therefore (x + 3y)ab \equiv 4(a + b)xy$ is the required locus

44. Answer (B, D)

Hint : Distance between two parallel lines.

Solution :

Distance between two parallel lines = $2\sqrt{5}$

$\therefore$ points on line $\dfrac{x-1}{\frac{-2}{\sqrt{5}}} = \dfrac{y+2}{\frac{1}{\sqrt{5}}} = \pm 2\sqrt{5}$

$\therefore$ points are (–3, 0) and (5, –4)

$\therefore$ Required lines are

$2x - y + 6 = 0$ and $2x - y - 14 = 0$

45. Answer (B, C)

Hint : Translation and Rotation of axes.

Solution :

For $f(x,y) = 0$

new origin=

$$\left(\frac{hf-bh}{ab-h^2}, \frac{gf-af}{ab-h^2}\right) \equiv \left(\frac{28}{-14}, \frac{42}{-14}\right) \equiv (-2, -3)$$

For $g(x,y) = 0$

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2\times\sqrt{3}}{2}\right)$$

$$= \frac{1}{2}\times\frac{\pi}{3} = \frac{\pi}{6}$$

46. Answer (C)

Hint : Family of circle with line.

Solution :

Let required circle

$x^2 + y^2 - 3x + 2y - 4 + \lambda (2x + 5y + 2) = 0$

$\therefore C\left(\frac{3-2\lambda}{2}, \frac{-(5\lambda+2)}{2}\right)$ satisfy $x + y = 11$

$\therefore \lambda = -3$

$\therefore$ Required circle is $x^2 + y^2 - 9x - 13y - 10 = 0$

47. Answer (D)

Hint : Orthogonal of two circles.

Solution :

Let required circle

$(x - 1)^2 + (y + 1)^2 + \lambda(2x + 3y + 1) = 0$ (i)

Circle with diameter points (0,3) and (–2,–1) is

$x^2 + y^2 + 2x - 2y - 3 = 0$ (ii)

(i) of (ii) intersect orthogonally

$\therefore (2\lambda - 1) + -2\left(\frac{3\lambda}{2} + 1\right) = \lambda - 1 \Rightarrow \lambda = \frac{-3}{2}$

Required circle is $2x^2 + 2y^2 - 10x - 5y + 1 = 0$

48. Answer (A)

Hint : Touching concept of line with circle.

Solution :

Let required circle

$(x^2 + y^2 - 4) + \lambda(x + 2y - 4) = 0$

$C\left(\frac{-\lambda}{2}, -\lambda\right) = \lambda = \sqrt{\frac{5\lambda^2}{4} + 4\lambda + 4}$

$\because$ it touches line x + 2y – 5 = 0

$\therefore \left|\frac{\frac{-\lambda}{2} + 2(-\lambda) - 5}{\sqrt{5}}\right| = \sqrt{\frac{5\lambda^2}{4} + 4\lambda + 4}$

$\Rightarrow 5(\lambda + 2)^2 = 5\lambda^2 + 16\lambda + 16$

$\Rightarrow \lambda = -1$

$\therefore$ Required circle $x^2 + y^2 - x - 2y = 0$

49. Answer (D)

Hint : Chord of contact of circle.

Solution :

Equation of C.O.C $hx + ky = 8$ (i)

Also $tk = h + 2t^2$ (ii) $\because (h,k)$ satisfy tangent)

$\Rightarrow hx + y\frac{(h + 2t^2)}{t} - 8 = 0$

$\Rightarrow 2(ty - 4) + h\left(x + \frac{y}{t}\right) = 0$

$\therefore$ Line passes through point

$y = \frac{4}{t}$ and $x = \frac{-y}{t}$

$\Rightarrow \frac{y}{4} = -\frac{x}{y} \Rightarrow y^2 = -4x$

50. Answer (A)

Hint : Point of intersection of two curves.

Solution :

Point of intersection of $x = -2$ and $x^2 + y^2 = 16$

$\therefore y^2 = 12 \Rightarrow y = \pm 2\sqrt{3}$

$\therefore \text{Point}\left(-2, 2\sqrt{3}\right)$

51. Answer (C)

Hint : Circumcircle of triangle *ABC*

Solution :

The equation of the circumcircle of ΔAQB is

$(x^2 + y^2 - 8) + \lambda(hx + ky - 8) = 0 \because$ (A = –1) due to (0,0) satisfy it

$\therefore$ Equation is $x^2 + y^2 - hx - ky = 0$

Now centre $\left(\frac{h}{2}, \frac{k}{2}\right)$

$\therefore$ For locus let $x = \frac{h}{2}; y = \frac{k}{2}$

$h = 2x$ and $k = 2y$

$\because tk = h + 2t^2 \therefore$ at $t = 2$

The required locus is $2y = x + 4$

52. Answer (C)

Hint : I.T.F. conversion in domain.

Solution :

Let $\sin^{-1}x = \theta \Rightarrow x = \sin\theta$

Now

$$\cos^{-1}x = \cos^{-1}(\sin\theta) = \cos^{-1}\left(-\cos\left(\frac{3\pi}{2}-\theta\right)\right)$$

$$= \pi - \cos^{-1}\left(\cos\left(\frac{3\pi}{2}-\theta\right)\right)$$

$$= \pi - \left(\frac{3\pi}{2}-\theta\right) \text{ as } \frac{3\pi}{2}-\theta \in (0,\pi)$$

$$= \theta - \frac{\pi}{2} = \sin^{-1}x - \frac{\pi}{2}$$

$$\therefore \sin^{-1}x + \cos^{-1}x = 2\sin^{-1}x - \frac{\pi}{2}$$

53. Answer (A)

Hint : Locus of midpoint of Parallel chords.

Solution :

Let middle point is (h,k)

$\therefore$ Equation of chord in mid-point form is

$$\frac{x}{h} + \frac{y}{k} = 2$$

$$\therefore -\frac{1}{h \times \frac{1}{k}} = m \Rightarrow k = -mh$$

$\Rightarrow y + mx = 0$ is the required locus

54. Answer (B)

Hint : Condition of common tangent on two curves.

Solution :

$y = \frac{x}{2} + 2$ is tangent on $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$

$$\Rightarrow 4 + 4b^2 = 16 \Rightarrow b^2 + 1 = 4 \Rightarrow b = \pm\sqrt{3}$$

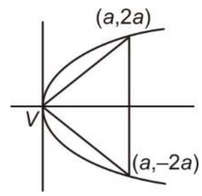
Now tangent at other point is given by $-2y = x+4$

$\Rightarrow x + 2y + 4 = 0$

55. Answer (D)

Hint : Length of latus rectum independency.

Solution :



$\theta = 2\tan^{-1} 2$

$\because \sqrt{3} < 2 < \sqrt{2} + 1$

$$\frac{\pi}{3} < \tan^{-1} 2 < \frac{3\pi}{8}$$

$$\Rightarrow \frac{2\pi}{3} < \theta < \frac{3\pi}{-4}$$

56. Answer A(P, Q); B(P, Q); C(R, T); D(S, T)

Hint : Property of perpendicular normals.

Solution :

Equation of normal at '*p*' is

$y = -tx + 2at + at^3$

Put $y = -2at \Rightarrow x = 4a + at^2$

$\therefore G\ (4a + at^2, -2at)$

Required locus $y^2 = 4a(x - 4a)$

$\because a = 1$

$\therefore y^2 = 4(x - 4)$

Now verify each option

57. Answer A(P); B(Q); C(Q, S); D(R, S)

Hint : Type of functions concept.

Solution :

(A) $f(x) = \begin{cases} \left((1)^1\right)^n x > 0 = 1 & ,x > 0 \\ \left((-1)^{-1}\right)^n x < 0 & ,x < 0 \end{cases}$

$f(x)$ is an odd function. $f(x)$ is not bijective

$\because f(x)$ is not one one

(B) $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$

$$f(x) = \frac{-x}{e^{-x} - 1} - \frac{x}{2} + 1 = x + \frac{x}{e^x - 1} - \frac{x}{2} + f(x)$$

$\therefore f(x)$ is an even function $\because f(x)$ is not bijective

(C) $f(-x) = f(x) \therefore f(x)$ is even. $f(x)$ is periodic but time period not define

(D) $f(x) = \max\{\tan x, \cot x\}$. From graph of $f(x)$ it is clear that

$f(x)$ is neither even nor odd

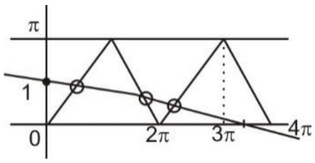
$\because f(x+\pi) = \max\{\tan(x+\pi), \cot(x+\pi)\}$

$= \max\{\tan x, \cot x\}$ $f(x)$ is periodic with f(x) is periodic with π

58. Answer (03)

Hint : Graphical solution.

Solution :



From graph it is clear that curves intersect at 3 points

$\therefore$ Only 3 solutions

59. Answer (00)

Hint : Trigonometric conversion of I.T.F.

Solution :

$\sin\left(\cos^{-1}\left(\tan\left(\tan^{-1}\left(\sqrt{x^2-1}\right)\right)\right)\right)$

$= \sin\left(\cos^{-1}\sqrt{x^2-1}\right)$

$= \sin\left(\sin^{-1}\sqrt{2-x^2}\right) = \sqrt{2-x^2}$

$\therefore$ Common domain $\left[1, \sqrt{2}\right] \Rightarrow 2 - x^2 = 1 + x$

$\Rightarrow x^2 + x - 1 = 0 \Rightarrow x = \dfrac{-1 \pm \sqrt{5}}{2}$

$x = \dfrac{\sqrt{5}-1}{2} \notin \left[1, \sqrt{2}\right]$

$\therefore$ No solution exists

60. Answer (09)

Hint : Sum of infinite G.P.

Solution :

$\dfrac{1}{1-\sin\left(\cos^{-1}x\right)} = 2$

$\Rightarrow \sin\left(\cos^{-1}x\right) = \dfrac{1}{2}$

$\Rightarrow \cos^{-1}x = \dfrac{\pi}{6} \Rightarrow x = \dfrac{\sqrt{3}}{2} \Rightarrow 4x^2 = 3$

$\Rightarrow 12x^2 = 9$

□ □ □