All India Aakash Test Series for JEE (Advanced)-2020

TEST - 4A (Paper-1) - Code-A

Test Date : 24/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(2)	21.	(1)	41.	(4)
2.	(7)	22.	(5)	42.	(5)
3.	(3)	23.	(8)	43.	(2)
4.	(2)	24.	(3)	44.	(5)
5.	(6)	25.	(4)	45.	(6)
6.	(2)	26.	(3)	46.	(7)
7.	(4)	27.	(5)	47.	(9)
8.	(5)	28.	(6)	48.	(4)
9.	(A, D)	29.	(A, B)	49.	(B, C)
10.	(A, B, C)	30.	(A, B)	50.	(A, B, D)
11.	(A, B, C)	31.	(A, B, D)	51.	(A, B, C)
12.	(B, C)	32.	(B, D)	52.	(A, C, D)
13.	(A, C, D)	33.	(C)	53.	(B, C, D)
14.	(B, C, D)	34.	(A, C)	54.	(B, C, D)
15.	(A, B)	35.	(A, B, C, D)	55.	(B, C, D)
16.	(A, C)	36.	(C)	56.	(A, C, D)
17.	(A, C)	37.	(C, D)	57.	(B, D)
18.	(B, D)	38.	(A, B, C)	58.	(A, C)
19.	A $\rightarrow$ (Q)	39.	A $\rightarrow$ (P, R)	59.	A $\rightarrow$ (T)
	B $\rightarrow$ (S)		B $\rightarrow$ (Q)		B $\rightarrow$ (Q)
	C $\rightarrow$ (R, T)		C $\rightarrow$ (Q, S, T)		C $\rightarrow$ (P, R)
	D $\rightarrow$ (P)		D $\rightarrow$ (Q)		D $\rightarrow$ (S)
20.	A $\rightarrow$ (Q)	40.	A $\rightarrow$ (P, Q, S)	60.	A $\rightarrow$ (S, T)
	B $\rightarrow$ (S)		B $\rightarrow$ (P, R)		B $\rightarrow$ (P, S)
	C $\rightarrow$ (P, R)		C $\rightarrow$ (P, R, T)		C $\rightarrow$ (T)
	D $\rightarrow$ (T)		D $\rightarrow$ (R)		D $\rightarrow$ (R, T)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (2)

Hint : $\frac{hc}{X_{\min}} = \Delta E - \phi$

Solution :

$$\frac{\lambda_{H_2}}{\lambda_{gas}} = \frac{\sqrt{2m\left(\frac{3}{4}z^2\Delta E_0 - \phi\right)}}{\sqrt{2m\left(\frac{3}{4}\Delta E_0 - \phi\right)}}$$

$$\Rightarrow \frac{\frac{3}{4}\Delta E_0 z^2 - \phi}{\frac{3}{4}\Delta E_0 - \phi} = \frac{61}{10} \qquad \text{...(i)}$$

Also, $\frac{3}{4}\Delta E_0 z^2 - \frac{\Delta E_0 z^2}{4} = 2\Delta E_0$

$$\Rightarrow \frac{\Delta E_0 z^2}{2} = 2\Delta E_0 \qquad \therefore z = 2$$

Now in equation (i)

$$10(3\Delta E_0 - \phi) = 61\left(\frac{3}{4}\Delta E_0 - \phi\right)$$

$$\Rightarrow 30\Delta E_0 - 10\phi = \frac{183}{4}\Delta E_0 - 61\phi$$

$$\Rightarrow 51\phi = \frac{63}{4}\Delta E_0$$

$$\Rightarrow \phi = \frac{63 \times 13.6}{4 \times 51}\text{eV} = \frac{21 \times 2}{10}\text{eV}$$

$\therefore K = 2$

2. Answer (7)

Hint : $A_R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$

Solution :

Let the amplitude of wave through S_1 and S_2 be A. So, if $A^2 = I_0$

Then $4A^2 = I \Rightarrow A^2 = \frac{I}{4}$

After passing through P_1 amplitude would be A

and after passing through P_2 amplitude would be $\frac{A}{2}$

$$\Delta x = (\mu_1 t - \mu_2 t) = 0.5 \times 40 \times 10^{-6}$$

$$\therefore \Delta\phi = \frac{1}{2} \times 40 \times 10^{-6} \times \frac{2\pi}{4000} \times 10^{10} = 50 \times 2\pi$$

So, construction interference would occur at O

$$\therefore A_{\text{result}} = A + \frac{A}{2} = \frac{3A}{2}$$

and $I' = \frac{9}{4}A^2 = \frac{9}{4}\cdot\frac{I}{4} = \left(\frac{9}{16}\right)I$

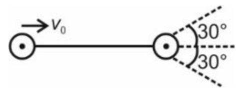
$\therefore x = 9, y = 16$

$\Rightarrow y - x = 7$

3. Answer (3)

Hint : $mv_0 = 2mv\cos 30°$; $\frac{1}{2}mv_0^2 = x\left[\frac{3}{4}\Delta E_0\right]$

Solution :



Let the final speed of (both) the H-atom and neutron is v then, $mv_0 = 2mv\cos 30°$

$$\Rightarrow v = \frac{v_0}{\sqrt{3}}$$

Also, $\frac{1}{2}mv_0^2 = \frac{1}{2}\cdot 2m\cdot\frac{v_0^2}{3} + \frac{3}{4}\Delta E_0$

$$\therefore \frac{1}{2}mv_0^2\left(1 - \frac{2}{3}\right) = \frac{3}{4}\Delta E_0$$

$$\Rightarrow \frac{1}{2}mv_0^2 = \frac{9}{4}\Delta E_0$$

$$\because \frac{1}{2}mv_0^2 = x\left(\frac{3}{4}\Delta E_0\right)$$

$$\therefore x\left[\frac{3}{4}\Delta E_0\right] = \frac{9}{4}\Delta E_0$$

$\therefore x = 3$

4. Answer (2)

Hint : $d\sin\theta = (2\mu t + t) - 2\mu t$

Solution :

$d\sin\theta = (2\mu t + t) - 2\mu t$

$$\Rightarrow d\frac{y}{D} = t \Rightarrow \qquad y = \frac{Dt}{d}$$

$$\Rightarrow y = \frac{1 \times 2 \times 10^{-5}}{1 \times 10^{-3}} = 2 \times 10^{-2}\text{m}$$

5. Answer (6)

Hint : $\Delta E_0\left|1 - \frac{1}{n^2}\right| = \Delta E$

Solution :

$\Delta E = 30$ eV

42.5% of 30 eV = 12.75 eV

$$13.6\left|1 - \frac{1}{n^2}\right| = 12.75$$

So we get $n = 4$ (is the energy level to which hydrogen gets excited)

So, number of wavelengths = 6

6. Answer (2)

Hint : $d\sin\theta = \frac{\lambda}{2}$ $\left|\text{for } 1^{st} \text{ minima } \theta = 0.75°\right|$

Solution :

For first minima $\theta = 0.75°$

$\therefore \quad d = \frac{\lambda}{2\sin(0.75°)} = 1.98\times10^{-5}$ m

$\Rightarrow \quad d \approx 2\times10^{-2}$mm

7. Answer (4)

Hint : ΔE(for reaction) = 4(7.30 MeV) –3(2.40 MeV) –2(1.00 MeV)

Solution :

ΔE(for reaction) = [4(7.30) – 3(2.40) –2(1.0)]MeV

$\Rightarrow \quad \Delta E$ = 20 MeV

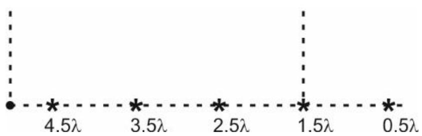
$\Rightarrow \quad \frac{1}{5}^{th}$ of this energy will be taken away by helium and rest are for neutron.

8. Answer (5)

Hint :

Minima will be at the position where path differences are 4.5λ, 3.5λ, 2.5λ, 1.5λ, 0.5λ.

Solution :



Minima will be at those points where path differences are 4.5λ, 3.5λ, 2.5λ, 1.5λ and 0.5λ

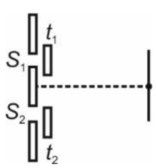
So five minima are observed.

9. Answer (A, D)

Hint : Effective optical path difference :

$\mu_2 t_2 - \mu_1 t_1 + (t_1 - t_2)$

Solution :



If $t_1 > t_2$

Then phase lead by wave from S_2

$\left[\mu_2 t_2 + (t_1 - t_2) - \mu_1 t_1\right]$

So *f* ring will shift towards S_2 to counter that much extra lead of phase.

10. Answer (A, B, C)

Hint :

The least count of the vernier caliper is the difference of the smallest unit on vernier scale and main scale.

Solution :

The least count of the vernier caliper is the difference of the smallest unit on vernier scale and main scale.

11. Answer (A, B ,C)

Hint : $\frac{hc}{\lambda} = \Delta E_0 Z^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$

Solution :

$$\frac{hc}{\lambda_B} = \Delta E_0 Z^2\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}\Delta E_0 Z^2$$

$$\therefore \quad \lambda_B = \frac{hc}{\Delta E_0 Z^2}\,\frac{36}{5}$$

$$\lambda_L = \frac{hc}{\Delta E_0 Z^2}$$

$$\because \quad \lambda_B - \lambda_L = \Delta\lambda = \frac{hc}{\Delta E_0 Z^2}\cdot\frac{31}{5}$$

$$\Rightarrow \quad \frac{5}{31}\Delta\lambda = \left(\frac{hc}{\Delta E_0 Z^2}\right)$$

$$\because \quad \Delta E_0 = Rch$$

$$\therefore \quad \frac{5}{31}\Delta\lambda\cdot Rhc = \frac{hc}{Z^2}$$

$$\Rightarrow \quad R = \frac{31}{5\Delta\lambda\cdot Z^2}$$

Shortest wavelength of Balmer series

$$\frac{hc}{\lambda'_B} = \frac{\Delta E_0 Z^2}{4}$$

$$\Rightarrow \quad \lambda'_B = 4\left[\frac{hc}{\Delta E_0 Z^2}\right]$$

$$\Rightarrow \quad \lambda'_B = 4\left[\frac{5}{31}\Delta\lambda\right] = \frac{20\Delta\lambda}{31}$$

And longest wavelength of Lyman series

$$\frac{hc}{\lambda'_L} = \Delta E_0 Z^2\left(1 - \frac{1}{4}\right) = \frac{3}{4}\Delta E_0 Z^2$$

$$\Rightarrow \quad \lambda'_L = \frac{4}{3}\left(\frac{hc}{\Delta E_0 Z^2}\right) = \frac{4}{3}\cdot\frac{5}{31}\Delta\lambda$$

$$\Rightarrow \quad \lambda'_L = \frac{20}{93}\Delta\lambda$$

12. Answer (B, C)

Hint : $\Delta w = \frac{\lambda D}{d}$.

Solution :

Fringe width $\Delta w = \frac{\lambda D}{d}$

So if λ increases then fringe width also increases.

13. Answer (A, C, D)

Hint : $\lambda = \frac{h}{P}$

Solution :

$$2m\,6(\hat{i}+2\hat{j}) = P_B$$

$$\therefore \quad \lambda = \frac{h}{12m\sqrt{5}}$$

$$\Rightarrow \quad \frac{h}{m} = 12\sqrt{5}\,\lambda$$

$$P_A = 2m(\hat{i}+2\hat{j}) = 2m\sqrt{5}$$

$$\therefore \quad \lambda_A = \frac{h}{2\sqrt{5}\,m} = \frac{12\sqrt{5}\,\lambda}{2\sqrt{5}} = 6\lambda$$

$$v_0 = \frac{m(2\hat{i}+4\hat{j})+2m(6\hat{i}+12\hat{j})}{3m} = \frac{14m\hat{i}+28m\hat{j}}{3m}$$

$$P_{cm} = 3mv_0 = 14m(\hat{i}+2\hat{j})$$

$$\Rightarrow \quad \lambda_{cm} = \frac{h}{P_{cm}} = \frac{h}{14\sqrt{5}m} = \frac{12\sqrt{5}\lambda}{14\sqrt{5}} = \frac{6}{7}\lambda$$

Now, $\vec{v}_{AC} = \vec{v}_{Aqr} - \vec{v}_{cqr} = 2\hat{i}+4\hat{j}-\frac{14}{3}\hat{i}-\frac{28}{3}\hat{j}$

$$\vec{v}_{AC} = \frac{-8}{3}\hat{i}-\frac{16}{3}\hat{j} = \frac{-8}{3}(\hat{i}+2\hat{j})$$

$$\therefore \quad |\vec{P}_{AC}| = \frac{8m}{3}\sqrt{5}$$

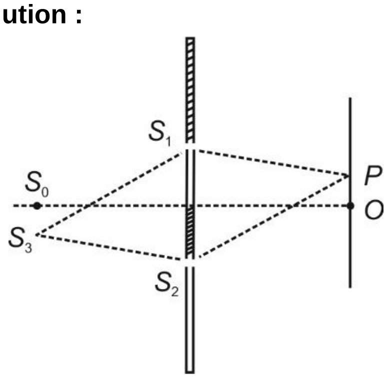
$$\therefore \quad \vec{\lambda}_{AC} = \frac{h}{(P_{AC})} = \frac{h\times 3}{8\sqrt{5}m} = \frac{3}{8\sqrt{5}}\times 12\sqrt{5}\,\lambda$$

$$\vec{\lambda}_{AC} = \frac{9}{2}\lambda$$

14. Answer (B, C, D)

Hint : Will be shifted upward by $\frac{d^2}{4\lambda D}$

Solution :



$\Delta S_3S_1S_2 \equiv \Delta PS_2S_1$

$\therefore \quad S_0S_3 = OP$

$\Rightarrow$ shifting $\Delta y = \frac{d}{4}$

$\therefore$ Fringe width will remain same as $\frac{\lambda D}{d}$

$\therefore$ Number of fringe crossing through O is

$$N = \frac{d.d}{4\lambda D} = \frac{d^2}{4\lambda D}$$

15. Answer (A, B)

Hint :

$R_n = R_0\,(n^2)$

$\therefore \quad A_n = 4\pi R_0^2 \cdot n^4$

Solution :

$R_n = R_0\, n^2$

$\therefore \quad A_n = 4\pi R_0^2 \cdot n^4$

And $A_1 = 4\pi R_0^2$

$$\therefore \quad \frac{A_n}{A_1} = n^4$$

$$\Rightarrow \quad \ln\left(\frac{A_n}{A_1}\right) = 4\ln n$$

Straight line of slope 4 and pass through origin.

16. Answer (A, C)

Hint : Least count $= \left(\frac{1}{50}\right)$mm.

Solution :

Least count $= \frac{1}{50}$mm $= 0.02$ mm

Reading = (1 mm × 18) + 0.02 × 20

= 18.4 mm

17. Answer (A, C)

Hint : $\frac{hc}{\lambda} = \Delta E_0 (Z-1)^2 \cdot \frac{3}{4}$ (for K_α lines)

Solution :

$$\frac{hc}{\lambda_z} = \Delta E_0 (Z-1)^2 \cdot \frac{3}{4}$$

$$\frac{hc}{\lambda_1} = \Delta E_0 (Z_1-1)^2 \frac{3}{4}$$

$$\frac{hc}{\lambda_2} = \Delta E_0 (Z_2-1)^2 . \frac{3}{4}$$

$$\therefore \quad \frac{\lambda_z}{\lambda_1} = 4 = \frac{(Z_1-1)^2}{(Z-1)^2}$$

$$\Rightarrow \quad \frac{Z_1-1}{Z-1} = 2$$

$\therefore \quad Z_1 = 2Z - 1$

Similarly, $\frac{\lambda_z}{\lambda_2} = \frac{(Z_2-1)^2}{(Z-1)^2} = \frac{1}{4}$

$\Rightarrow \frac{Z_2-1}{Z-1} = \frac{1}{2}$

$\Rightarrow Z_2 = \frac{Z+1}{2}$

18. Answer (B, D)

Hint : $N = N_0 e^{-\lambda t}$

Solution :

Let N_0 be the number of active nuclei at 6 : 10 AM in 1 mL of dose.

Then at 8 : 00 AM, number of active nuclei becomes $\frac{N_0}{2}$ in 1 mL. So effectively $\frac{N_0}{2}$ no. of nuclei is to be administrated.

$\Rightarrow (1\text{mL})\cdot\frac{N_0}{2} = \text{constant}$

At 7 : 05 AM let N_1 be the active nuclei then

$N_1 = \frac{N_0}{e^{\frac{\ln 2\times 55}{110}}} = \frac{N_0}{\sqrt{2}}$

So $x.\frac{N_0}{\sqrt{2}} = (1\text{mL})\frac{N_0}{2}$

$\Rightarrow x = \left(\frac{1}{\sqrt{2}}\right)\text{mL}$

At 9 : 50 $N_3 = \frac{N_0}{4}$ And at 8 : 55 AM. $N_2 = \frac{N_0}{2\sqrt{2}}$

$\therefore \frac{(1\text{mL})\frac{N}{2}}{\text{At } 8:00\text{ AM}} = \frac{(\sqrt{2}\text{ mL})\frac{N_0}{2\sqrt{2}}}{\text{At } 8:55\text{ AM}} = \frac{(2\text{ mL})\left(\frac{N_0}{4}\right)}{\text{At } 9:00\text{ AM}}$

19. Answer A(Q); B(S); C(R, T); D(P)

Hint : Apply Bohr's model.

Solution :

$\frac{mV^2}{r} = \frac{kZe^2}{r^2}$

$mvr = \frac{nn}{2\pi}$

$r_n \propto \frac{n^2}{Z}$

$V_n \propto \frac{Z}{n}$

$T = \frac{2\pi r_n}{v_n}$

$T \propto \frac{n^3}{Z^2}$

$i = \frac{e}{T} = \frac{Z^2}{n^3}$

$B \propto \frac{i}{r} = \frac{Z^2 Z}{n^3 n^2} = \frac{Z^3}{n^5}$

20. Answer A(Q); B(S); C(P, R); D(T)

Hint : Fringe width $\Delta w = \frac{\lambda D}{d}$

Position of minima $(2n+1)\frac{\lambda D}{2d}$

Solution :

(A) If at S_3 and S_4 there is destructive interference then final intensity on second screen is zero

$\therefore d_1 = d_2 = (2n+1)\frac{\lambda D}{2d}$

(B) If $d_1 = \frac{3\lambda D}{2d}$ then destructive interference at S_3 and if $d_4 = \frac{\lambda D}{3d}$ then resulting intensity at S_4 is I_0. So final intensity at screen 2, is I_0.

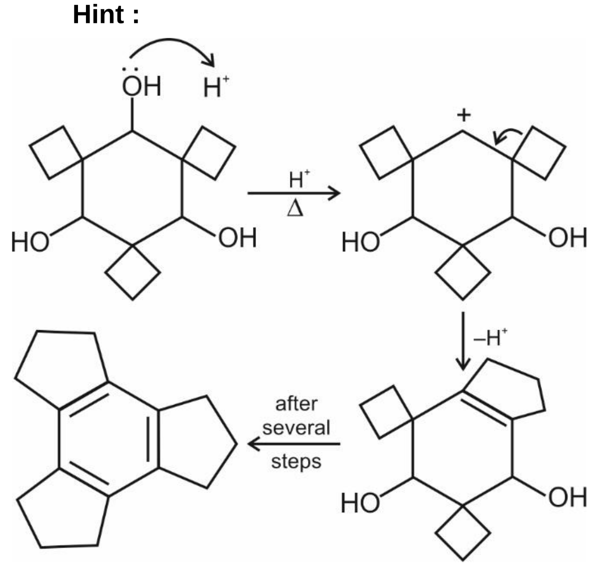
(C) If $d_1 = d_2 = \frac{\lambda D}{3d}$ then resulting intensity at S_3 and S_4 are I_0 and final intensity at screen 2 is $4I_0$. Also if $d_1 = \frac{\lambda D}{2d}$ then intensity at S_3 is zero and for $d_2 = \frac{\lambda D}{d}$ the intensity at S_4 is $4I_0$. So final intensity at screen 2 is $4I_0$.

(D) If constructive interference happens at S_3 and S_4 then find intensity at screen 2 is $16I_0$.

PART - II (CHEMISTRY)

21. Answer (1)

Hint :



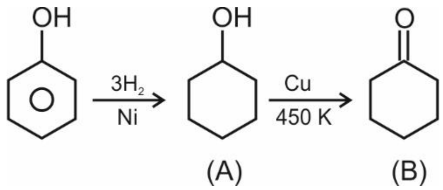
Solution :

x = 1

Grignard reagent will not interact with product.

22. Answer (5)

Hint :



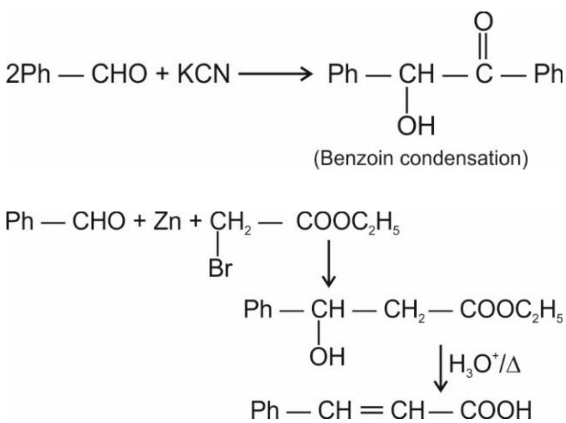
Solution :

A and B are non-aromatic compounds

Statements P, Q, R, U and V are incorrect.

23. Answer (8)

Hint :



Solution :

Difference in molar mass, M = 64; $\frac{M}{8} = 8$

24. Answer (3)

Hint : Isoelectric point is the pH when an amino acid exist in zwitterionic form and shows no net migration towards any electrode.

Solution :

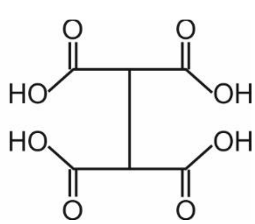
$$pI = \frac{pK_{a_1} + pK_{a_2}}{2}$$

$$= \frac{2+4}{2} = 3$$

25. Answer (4)

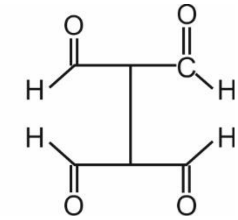
Hint :

P is



Solution :

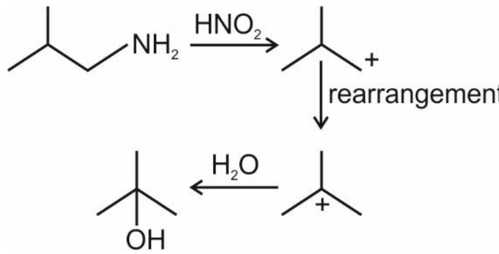
Q is



Each —CHO can produce an oxime.

26. Answer (3)

Hint :



Solution :

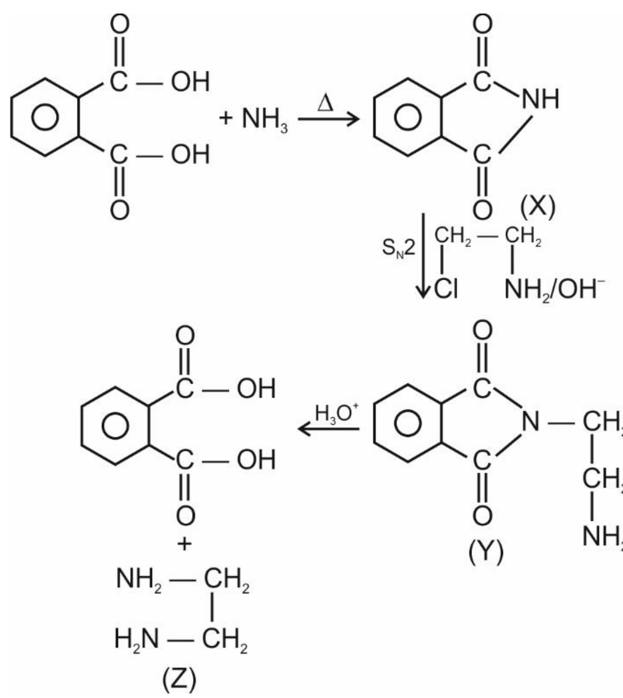
$$(C_2H_5)_4NOH \xrightarrow{\Delta} (C_2H_5)_3N + C_2H_4$$

$$>N - H \xrightarrow{HNO_2} >N \rightarrow NO$$

Reactions A, D and E are correct.

27. Answer (5)

Hint :



Solution :

x′ = 1

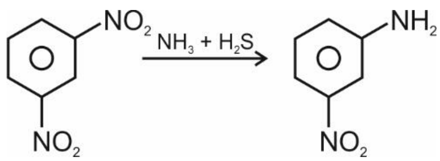
z′ = 2

n = 2

Nucleophilic substitution takes place via S_N2

28. Answer (6)

Hint :



Solution :

$$R-NO_2 \xrightarrow{+6e^-} R-NH_2$$

29. Answer (A, B)

Hint : Ethers have lower boiling point than alcohols as there is hydrogen bonding involved in between two alcohol molecule.

Solution :

Compounds with multiple hydroxy functional group are having greater boiling point than mono hydroxy compound.

30. Answer (A, B)

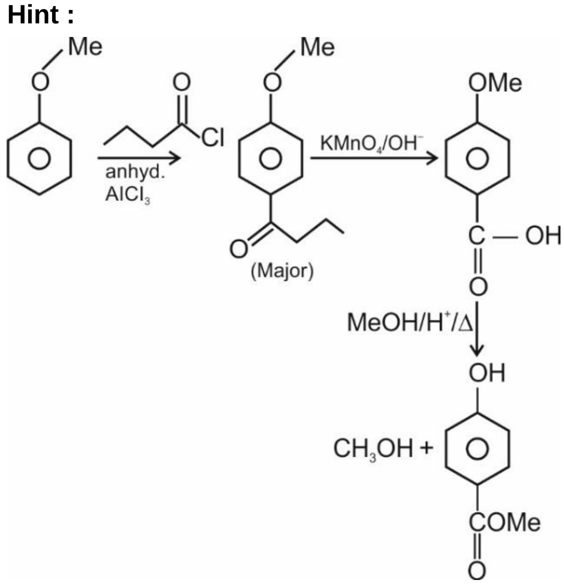
Hint : PCl_5 cause substitution of —OH by — Cl group.

Solution :

P is $CH_3-\underset{\underset{Cl}{|}}{CH}-CH_2-C(Me)_3$

31. Answer (A, B, D)

Hint :

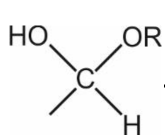


Solution :

Ether do not undergo oxidation and in alkaline medium there is no hydrolysis that can occur on ether linkage.

32. Answer (B, D)

Hint :

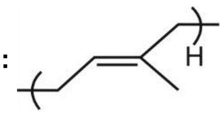
Hemiacetal forms are 

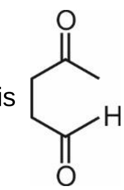
. And these are reducing in nature.

Solution :

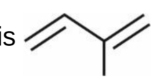
Both C_1 and C_2 are having hemiacetal that's why both are reducing.

33. Answer (C)

Hint : 

Polymer will yield upon ozonolysis 

Solution :

Monomer of the polymer is 

34. Answer (A, C)

Hint : Histamine is not an antacid. It stimulates the secretion of acid in stomach.

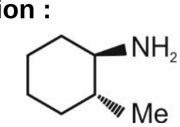
Solution :

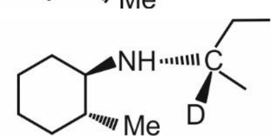
Brompheniramine is an antihistaminics

35. Answer (A, B, C, D)

Hint : During Hoffmann bromamide degradation stereochemistry of migrating group does not change.

Solution :

P is 

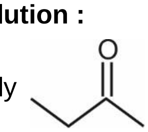
Q is 

36. Answer (C)

Hint :

All given species can give yellow ppt with 2, 4-DNP.

Solution :

Only 

can give yellow ppt with I_2 in NaOH.

37. Answer (C, D)

Hint : Correct order for basic strength is C > A > D > B

Solution :

More is the availability of lone pair of e^- on N-atom, greater would be the basic strength.

38. Answer (A, B, C)

Hint : The amines which can show optical activity, are resolvable.

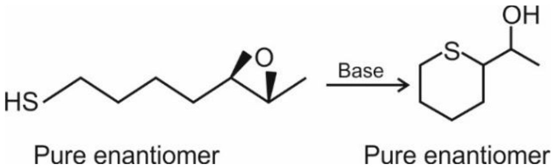
Solution :

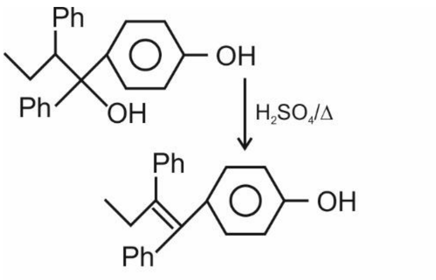
Amines which are bonded with four bulky groups and cyclic amines cannot undergo inversion.

39. Answer A(P, R); B(Q); C(Q, S, T); D(Q)

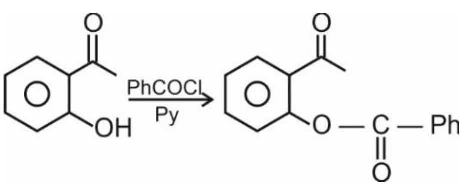
Hint : Gauche and anti-form are diastereomers of each other.

Solution :





Mixture of E and Z isomer



40. Answer A(P, Q, S); B(P, R); C(P, R, T); D(R)

Hint : Cellulose has β-links

Starch has α-links

Solution :

Sucrose upon hydrolysis gives α-D-glucose and β-D-fructose.

Maltose gives only α-D-glucose.

PART - III (MATHEMATICS)

41. Answer (4)

Hint : Fundamental principle of counting.

Solution : Total combinations of a, b, c and $d = 6^4$

$(a - 3)$, $(b - 4)$, $(c - 5)$ and $(d - 6)$ are integers.

Their product is 1 then

(i) All of them should be 1 (Not possible as $d \neq 7$)

(ii) All of them should be –1 (one case $a = 2$, $b = 3$, $c = 4$, $d = 5$)

(iii) Two of them are 1 and remaining two are –1 (three cases)

Total favourable cases = 4

Required probability = $\frac{4}{6^4}$

42. Answer (5)

Hint : Slope of normal is 2.

Solution : $xy^2 = 8$

$$\Rightarrow \frac{dy}{dx}_{(2,2)} = -\frac{1}{2}$$

Slope of normal = 2

So, unit vector along normal $= \frac{i + 2j}{\sqrt{5}} = \vec{x}$

Length of projection $= \left|\frac{3-8}{\sqrt{5}}\right| = \sqrt{5}$

43. Answer (2)

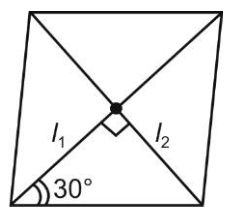
Hint : One angle of rhombus is $\frac{\pi}{3}$

Solution : Angle between two given lines;

$$\cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Area of rhombus = $2\sqrt{3}$



$$\frac{1}{2}(2l_1)(2l_2) = 2\sqrt{3}$$

$l_1 l_2 = \sqrt{3}$ …(i)

Also, $\frac{l_2}{l_1} = \frac{1}{\sqrt{3}}$ …(ii)

So $l_1 = \sqrt{3}$ and $l_2 = 1$

Side length of rhombus $= \sqrt{l_1^2 + l_2^2} = 2$

44. Answer (5)

Hint : Both the lines are parallel

Solution : $2x - 2y + z - 9 = 0 = x + 2y + 2z + 12$ is line of intersection of planes $P_1 : 2x - 2y + z - 9 = 0$ and $P_2 : x + 2y + 2z + 12 = 0$.

Another line $L : \frac{x}{2} = \frac{y}{1} = \frac{z}{-2}$ is parallel to both planes P_1 and P_2.

Distance between L and $P_1 = \left|\frac{9}{\sqrt{4+4+1}}\right| = 3$

and distance between L and $P_2 = \left|\frac{12}{\sqrt{4+4+1}}\right| = 4$

then distance between the two lines

$= \sqrt{3^2+4^2} = 5$

45. Answer (6)

Hint : $P(A\cap B) = P(A)\cdot P(B)$

Solution : $P(A\cap B) = P(A)\cdot P(B)$

$\Rightarrow\ P(A) = \frac{1}{5}$

Now, $P\left(\frac{\bar{A}}{A\cup B}\right) = \frac{P(\bar{A}\cap(A\cup B))}{P(A\cup B)}$

$\Rightarrow\ P\left(\frac{\bar{A}}{A\cup B}\right) = \frac{P(B)-P(A\cap B)}{\frac{1}{5}+\frac{1}{2}-\frac{1}{10}} = \frac{\frac{1}{2}-\frac{1}{10}}{\frac{6}{10}} = \frac{2}{3}$

So, $9P\left(\frac{\bar{A}}{A\cup B}\right) = 6$

46. Answer (7)

Hint : Count cases when number of letters between A and E are 0, 1 or 2

Solution : Total words = $\lfloor 8$

There are 6 cases when number of letters between A and E (or A and I) is zero.

There are 4 cases when number of letters between A and E (or A and I) is one.

There are 2 cases when number of letters between A and E (or A and I) is two.

So, favourable words = $(\lfloor 2 \times \lfloor 5)$

Required probability = $\frac{12\lfloor 2\lfloor 5}{\lfloor 8}$

$\Rightarrow\ P = \frac{1}{14}$

$\Rightarrow\ \frac{1}{2P} = 7$

47. Answer (9)

Hint : Use formula for division into groups.

Solution : Total number of ways of forming groups = $\frac{\lfloor 8}{(\lfloor 2)^4\cdot\lfloor 4}\cdot\frac{\lfloor 4}{(\lfloor 2)^2\cdot\lfloor 2}$. If 5th ranked player is in the final, then he played a match against a higher ranked player in second round.

So in first round 5th ranked player played a match against 6th, 7th or 8th ranked player and remaining two of these played against each other.

Number of favourable ways for round one

$= {}^3C_1\cdot 1\cdot\frac{\lfloor 4}{(\lfloor 2)^2\lfloor 2}$

And number of favourable ways for round two = 1

Required probability $= \frac{{}^3C_1\cdot\frac{\lfloor 4}{(\lfloor 2)^2\lfloor 2}\times 1}{\frac{\lfloor 8}{(\lfloor 2)^4\lfloor 4}\cdot\frac{\lfloor 4}{(\lfloor 2)^2\lfloor 2}}$

$= \frac{1}{35}$

Number of divisors of 36 will be 9.

48. Answer (4)

Hint : Angle between two lines = $\frac{\bar{b}_1\cdot\bar{b}_2}{|\bar{b}_1||\bar{b}_2|}$

(where b_1 and b_2 are the vectors along the lines)

Solution :

$$\cos\alpha = \frac{a(\sin\theta-2)+b\sqrt{5}\cos\theta+(2\sin\theta+1)}{\sqrt{(\sin\theta-2)^2+5\cos^2\theta+(2\sin\theta+1)^2}\sqrt{a^2+b^2+1^2}}$$

$$\Rightarrow\ \cos\alpha = \frac{\sin\theta(a+2)+b\sqrt{5}\cos\theta-2a+1}{\sqrt{10}\sqrt{a^2+b^2+1}}$$

$\because$ α is independent of θ, then $a + 2 = 0$ and $b = 0$

$\Rightarrow\ \cos\alpha = \frac{5}{\sqrt{10}\sqrt{5}} = \frac{1}{\sqrt{2}}$

$\Rightarrow\ \alpha = \frac{\pi}{4}$

49. Answer (B, C)

Hint : If odds in favour of an event is p then its probability is $\frac{p}{1+p}$

Solution : Let odds in favour of an event is p

then its probability is $\frac{p}{1+p}$

$p = 3q$ …(i)

Probability of 1st event = $\frac{p}{1+p}$

Probability 2nd event = $\frac{q}{1+q}$

$\frac{p}{1+p} = 2\left(\frac{q}{1+q}\right)$ …(ii)

From (i) and (ii) p = 0 or 1

Probability of 1st event = 0 or $\frac{1}{2}$

50. Answer (A, B, D)

Hint : Use binomial distribution

Solution : Probability of A winning the game

$= {}^3C_2 \cdot p^2(1-p) + {}^3C_3 p^3$

$= p^2[3-2p]$

Probability of B winning the game

$= {}^5C_3 p^3(1-p)^2 + {}^5C_4 p^4(1-p) + {}^5C_5 p^5$

$= p^3\left[6p^2 - 15p + 10\right]$

Now, $p^2(3 - 2p) = p^3(6p^2 - 15p + 10)$

$\Rightarrow \; p = 0, 1, \frac{1}{2}$

51. Answer (A, B, C)

Hint : $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{b}\cdot\vec{c}) - (\vec{a}\cdot\vec{b})\vec{c}$

Solution : $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$

$\Rightarrow \; (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$

$\Rightarrow \; (\vec{a}\cdot b)\vec{c} = (\vec{b}\cdot\vec{c})\vec{a}$

So either $\vec{a}$ and $\vec{c}$ are collinear or

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = 0$

which means either $\vec{b}$ is a null vector or $\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{c}$.

52. Answer (A, C, D)

Hint : $\left|\lambda\vec{a} + \mu\vec{b}\right|^2 = 4\lambda^2 + \mu^2 + 2\lambda\mu$

Solution : $\left|\lambda\vec{a} + \mu\vec{b}\right|^2 = \lambda^2\left|\vec{a}\right|^2 + \mu^2\left|\vec{b}\right|^2 + 2\lambda\mu\vec{a}\cdot\vec{b}$

$= 4\lambda^2 + \mu^2 + 2\lambda\mu$

(A) $\left|\vec{a} - \vec{b}\right|^2 = 4 + 1 - 2 = 3 \Rightarrow \left|\vec{a} - \vec{b}\right| = \sqrt{3}$

(B) $\left|\vec{b} - \frac{1}{2}\vec{a}\right|^2 = 1 + 1 - 1 = 1 \Rightarrow \left|\vec{b} - \frac{1}{2}\vec{a}\right| = 1$

(C) $\left|\frac{3\vec{a} - 7\vec{b}}{2}\right|^2 = \frac{36 + 49 - 42}{4} = \frac{43}{4}$

$\Rightarrow \left|3\vec{a} - 7\vec{b}\right| = \sqrt{\frac{43}{2}}$

(D) $\left|2\vec{a} - 5\vec{b}\right|^2 = 16 + 25 - 20 = 21$

$\left|2\vec{a} - 5\vec{b}\right| = \sqrt{21}$

53. Answer (B, C, D)

Hint : Equation must be inconsistent and planes should be non parallel.

Solution :

$$\begin{vmatrix} 1 & k & 1 \\ 1 & 1 & k \\ 1 & -3 & 3 \end{vmatrix} = 0$$

$\Rightarrow \; k = 1, -1$

But $k \neq 1$ (The two planes will be parallel so triangular prism can't be formed).

54. Answer (B, C, D)

Hint : Assume points $B(\vec{b})$ and $D(\vec{d})$ then use scalar product.

Solution : Consider A as origin and position vectors of B, D and C as $\vec{b}, \vec{d}, \vec{b}+\vec{d}$ respectively.

Here $\left|\vec{b}\right| = x$ and $\left|\vec{d}\right| = y$

Also $\left|\vec{b} + \vec{d}\right| = z$

$\Rightarrow \; x^2 + y^2 + 2\vec{b}\cdot\vec{d} = z^2$ …(i)

$\overrightarrow{BD}\cdot\overrightarrow{DA} = (\vec{d} - \vec{b})\cdot(-\vec{d})$

$= -y^2 + \vec{b}\cdot\vec{d}$

$= -y^2 + \frac{z^2 - x^2 - y^2}{2}$ (from (i))

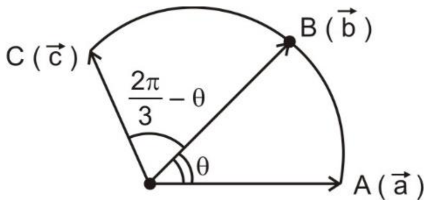
$= -\frac{1}{2}x^2 - \frac{3}{2}y^2 + \frac{1}{2}z^2$

55. Answer (B, C, D)

Hint : Consider $\vec{c} = \lambda\vec{a} + \mu\vec{b}$

Solution :

$\because$ $\vec{a}, \vec{b}, \vec{c}$ are coplanar then



Let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$

$\vec{a}\cdot\vec{c} = \lambda\vec{a}\cdot\vec{a} + \mu\vec{a}\cdot\vec{b}$

$-\frac{1}{2} = \lambda + \mu\cos\theta$ ...(i)

Similarly $\vec{b}\cdot\vec{c} = \lambda\vec{a}\cdot\vec{b} + \mu\vec{b}\cdot\vec{b}$

$\cos\left(\frac{2\pi}{3} - \theta\right) = \lambda\cos\theta + \mu$ ...(ii)

From (i) and (ii)

$\lambda = -\frac{\sqrt{3}}{2}\cot\theta - \frac{1}{2}$ and $\mu = \frac{\sqrt{3}}{2}\operatorname{cosec}\theta$

So, $\bar{c} = -\left(\frac{\sqrt{3}\cot\theta + 1}{2}\right)\bar{a} + \left(\frac{\sqrt{3}}{2}\operatorname{cosec}\theta\right)\bar{b}$

56. Answer (A, C, D)

Hint : Use Bayes theorem

Solution : Let events

E_1 = Coins show 2 heads (die X is rolled)

E_2 = Coins show 1 head (die Y is rolled)

E_3 = Coins show no head (die Z is rolled)

And A = Die shows red face.

$P(E_1) = \frac{1}{4}$ $\quad P(E_2) = \frac{2}{4}$ $\quad P(E_3) = \frac{1}{4}$

$P\left(\frac{A}{E_1}\right) = \frac{1}{6}$ $\quad P\left(\frac{A}{E_2}\right) = \frac{3}{6}$ $\quad P\left(\frac{A}{E_3}\right) = \frac{2}{6}$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)\cdot P\left(\frac{A}{E_3}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right) + P(E_3)\cdot P\left(\frac{A}{E_3}\right)}$$

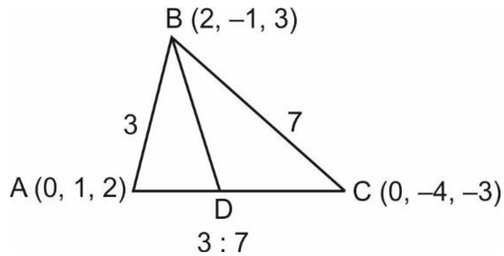
$$= \frac{\frac{1}{4}\cdot\frac{2}{6}}{\frac{1}{4}\cdot\frac{1}{6} + \frac{2}{4}\cdot\frac{3}{6} + \frac{1}{4}\cdot\frac{2}{6}}$$

$$= \frac{2}{1+6+2} = \frac{2}{9}$$

57. Answer (B, D)

Hint : $\because \frac{AD}{CD} = \frac{AB}{BC}$

Solution :



$\because \frac{AD}{CD} = \frac{AB}{BC} = \frac{3}{7}$

Using section formula;

$D\left(0, -\frac{1}{2}, \frac{1}{2}\right)$

(A) DR's of $BD \propto 2, -\frac{1}{2}, \frac{5}{2}$ or $4, -1, 5$

(B) $BD = \sqrt{4 + \frac{1}{4} + \frac{25}{4}} = \sqrt{\frac{21}{2}}$

(C) Area of

$$\Delta ABD = \frac{1}{2}\left|\overrightarrow{BA}\times\overrightarrow{BD}\right| = \frac{1}{2}\left|-\frac{9}{2}\hat{i} - 3\hat{j} + 3\hat{k}\right|$$

$$= \frac{\sqrt{153}}{4}$$

(D) Area of

$$\Delta ABC = \frac{1}{2}\left|\overrightarrow{BA}\times\overrightarrow{BC}\right| = \frac{1}{2}\left|-15\hat{i} - 10\hat{j} + 10\hat{k}\right|$$

$$= \frac{5\sqrt{17}}{2}$$

58. Answer (A, C)

Hint : Probability of a selected number to be even is $\frac{3}{7}$ and to be odd is $\frac{4}{7}$

Solution : (A and B) abc is even if at least one of a, b or c is even, so

Required probability = $1 - \left(\frac{4}{7}\right)^3$

$$= \frac{279}{7^3}$$

(C) $(ab + c)$ is even then,

Case (1) : If c is even then at least one of a or b is even.

Case (2) : If c is odd then both a and b are odd

Required probability

$$=\frac{3}{7}\left(1-\left(\frac{4}{7}\right)^2\right)+\left(\frac{4}{7}\right)^3=\frac{163}{7^3}$$

(B) $(a + b + c)$ is even if all are even or anyone is even and remaining two are odd.

Required probability = $\left(\frac{3}{7}\right)^3 + {}^3C_1\cdot\frac{3}{7}\left(\frac{4}{7}\right)^2=\frac{171}{7^3}$

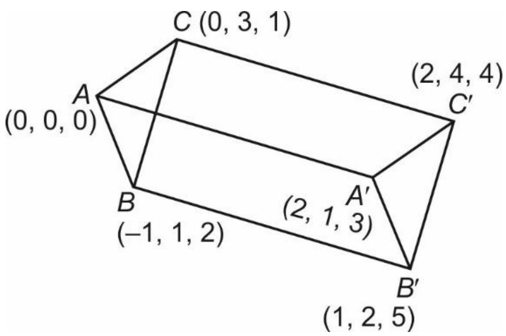
59. Answer A(T); B(Q); C(P, R); D(S)

Hint : Three faces of a triangular prism are parallelogram.

Solution :

(A) Let point $B'(x, y, z)$

$\because$ $AA'B'B$ is a parallelogram



So, $x_1 + 0 = 2 - 1 \Rightarrow x_1 = 1$

$y_1 + 0 = 1 + 1 \Rightarrow y_1 = 2$

$z_1 + 0 = 2 + 3 \Rightarrow z_1 = 5$

$B'(1, 2, 5)$

(B) Similarly $C'(2, 4, 4)$

(C) Point of intersection of diagonals of face $AA'B'B$ is $\left(\frac{1}{2}, 1, \frac{5}{2}\right)$

Point of intersection of diagonals of face $AA'C'C$ is (1, 2, 2)

Point of intersection of diagonals of face $BB'C'C$ is $\left(\frac{1}{2}, \frac{5}{2}, 3\right)$

(D) D' is midpoint of B' and C', so

$D'\left(\frac{3}{2}, 3, \frac{9}{2}\right)$

60. Answer A(S, T); B(P, S); C(T); D(R, T)

Hint : $p(E)=\frac{n(E)}{n(S)}$

Solution :

Total number in set $S = 5 \times 5 \times 4 \times 3 = 300$

(A) If number is divisible by 4, the last two digits will be 04, 12, 20, 24, 32, 40, or 52.

Total number of numbers = $12 \times 3 + 9 \times 4 = 72$

Required probability = $\frac{72}{300}=\frac{6}{25}$

(B) There should be either only one odd digit or only even digit in the number.

Total such numbers = ${}^3C_1 \cdot 3\underline{|3} + {}^2C_1 \cdot \underline{|4} + 3\underline{|3}$

= 54 + 48 + 18

= 120

Required probability = $\frac{120}{300}=\frac{2}{5}$

(C) There are only 5 combinations of 4 digits possible (1, 2, 4, 5); (0, 3, 4, 5); (0, 2, 3, 4); (0, 1, 3, 5) or (0, 1, 2, 3)

Number of numbers divisible by 6 using (1, 2, 4, 5) = 12

Number of numbers divisible by 6 using (0, 3, 4, 5) = 10

Number of numbers divisible by 6 using (0, 2, 3, 4) = 14

Number of numbers divisible by 6 using (0, 1, 3, 5) = 6

Number of numbers divisible by 6 using (0, 1, 2, 3) = 10

Total numbers divisible by 6 = 52

Required probability = $\frac{52}{300}=\frac{13}{75}$

(D) If $abcd$ is divisible by 11 then

$a + c = b + d$

Total number of numbers divisible by 11 = 48

Required probability = $\frac{48}{300}=\frac{4}{25}$

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All India Aakash Test Series for JEE (Advanced)-2020

TEST - 4A (Paper-1) - Code-B

Test Date : 24/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(5)	21.	(6)	41.	(4)
2.	(4)	22.	(5)	42.	(9)
3.	(2)	23.	(3)	43.	(7)
4.	(6)	24.	(4)	44.	(6)
5.	(2)	25.	(3)	45.	(5)
6.	(3)	26.	(8)	46.	(2)
7.	(7)	27.	(5)	47.	(5)
8.	(2)	28.	(1)	48.	(4)
9.	(B, D)	29.	(A, B, C)	49.	(A, C)
10.	(A, C)	30.	(C, D)	50.	(B, D)
11.	(A, C)	31.	(C)	51.	(A, C, D)
12.	(A, B)	32.	(A, B, C, D)	52.	(B, C, D)
13.	(B, C, D)	33.	(A, C)	53.	(B, C, D)
14.	(A, C, D)	34.	(C)	54.	(B, C, D)
15.	(B, C)	35.	(B, D)	55.	(A, C, D)
16.	(A, B, C)	36.	(A, B, D)	56.	(A, B, C)
17.	(A, B, C)	37.	(A, B)	57.	(A, B, D)
18.	(A, D)	38.	(A, B)	58.	(B, C)
19.	A $\rightarrow$ (Q)	39.	A $\rightarrow$ (P, Q, S)	59.	A $\rightarrow$ (S, T)
	B $\rightarrow$ (S)		B $\rightarrow$ (P, R)		B $\rightarrow$ (P, S)
	C $\rightarrow$ (P, R)		C $\rightarrow$ (P, R, T)		C $\rightarrow$ (T)
	D $\rightarrow$ (T)		D $\rightarrow$ (R)		D $\rightarrow$ (R, T)
20.	A $\rightarrow$ (Q)	40.	A $\rightarrow$ (P, R)	60.	A $\rightarrow$ (T)
	B $\rightarrow$ (S)		B $\rightarrow$ (Q)		B $\rightarrow$ (Q)
	C $\rightarrow$ (R, T)		C $\rightarrow$ (Q, S, T)		C $\rightarrow$ (P, R)
	D $\rightarrow$ (P)		D $\rightarrow$ (Q)		D $\rightarrow$ (S)

HINTS & SOLUTIONS

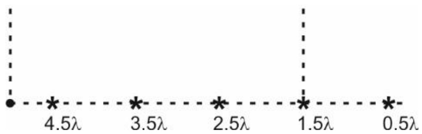
PART - I (PHYSICS)

1. Answer (5)

Hint :

Minima will be at the position where path differences are 4.5λ, 3.5λ, 2.5λ, 1.5λ, 0.5λ.

Solution :



Minima will be at those points where path differences are 4.5λ, 3.5λ, 2.5λ, 1.5λ and 0.5λ

So five minima are observed.

2. Answer (4)

Hint : ΔE(for reaction) = 4(7.30 MeV) –3(2.40 MeV) –2(1.00 MeV)

Solution :

ΔE(for reaction) = [4(7.30) – 3(2.40) –2(1.0)]MeV

$\Rightarrow \Delta E = 20$ MeV

$\Rightarrow \frac{1}{5}^{\text{th}}$ of this energy will be taken away by helium and rest are for neutron.

3. Answer (2)

Hint : $d\sin\theta = \frac{\lambda}{2}$ $\quad |\text{for } 1^{\text{st}} \text{ minima } \theta = 0.75°|$

Solution :

For first minima θ = 0.75°

$\therefore \quad d = \frac{\lambda}{2\sin(0.75°)} = 1.98\times10^{-5}$ m

$\Rightarrow \quad d \approx 2\times10^{-2}$mm

4. Answer (6)

Hint : $\Delta E_0\left|1-\frac{1}{n^2}\right| = \Delta E$

Solution :

$\Delta E = 30$ eV

42.5% of 30 eV = 12.75 eV

$13.6\left|1-\frac{1}{n^2}\right| = 12.75$

So we get $n = 4$ (is the energy level to which hydrogen gets excited)

So, number of wavelengths = 6

5. Answer (2)

Hint : $d\sin\theta = (2\mu t + t) - 2\mu t$

Solution :

$d\sin\theta = (2\mu t + t) - 2\mu t$

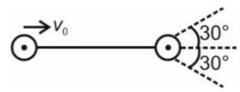
$\Rightarrow \quad d\frac{y}{D} = t \Rightarrow \qquad y = \frac{Dt}{d}$

$\Rightarrow \quad y = \frac{1\times2\times10^{-5}}{1\times10^{-3}} = 2\times10^{-2}\text{m}$

6. Answer (3)

Hint : $mv_0 = 2mv\cos30°$; $\frac{1}{2}mv_0^2 = x\left[\frac{3}{4}\Delta E_0\right]$

Solution :



Let the final speed of (both) the H-atom and neutron is v then, $mv_0 = 2mv\cos30°$

$\Rightarrow \quad v = \frac{v_0}{\sqrt{3}}$

Also, $\frac{1}{2}mv_0^2 = \frac{1}{2}\cdot 2m\cdot\frac{v_0^2}{3} + \frac{3}{4}\Delta E_0$

$\therefore \quad \frac{1}{2}mv_0^2\left(1-\frac{2}{3}\right) = \frac{3}{4}\Delta E_0$

$\Rightarrow \quad \frac{1}{2}mv_0^2 = \frac{9}{4}\Delta E_0$

$\because \quad \frac{1}{2}mv_0^2 = x\left(\frac{3}{4}\Delta E_0\right)$

$\therefore \quad x\left[\frac{3}{4}\Delta E_0\right] = \frac{9}{4}\Delta E_0$

$\therefore \quad x = 3$

7. Answer (7)

Hint : $A_R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$

Solution :

Let the amplitude of wave through S_1 and S_2 be A. So, if $A^2 = I_0$

Then $4A^2 = I \Rightarrow A^2 = \frac{I}{4}$

After passing through P_1 amplitude would be A and after passing through P_2 amplitude would be $\frac{A}{2}$

$\Delta x = (\mu_1 t - \mu_2 t) = 0.5\times40\times10^{-6}$

$\therefore \quad \Delta\phi = \frac{1}{2}\times40\times10^{-6}\times\frac{2\pi}{4000}\times10^{10} = 50\times2\pi$

So, construction interference would occur at O

$\therefore \quad A_{\text{result}} = A + \frac{A}{2} = \frac{3A}{2}$

and $I' = \frac{9}{4}A^2 = \frac{9}{4}\cdot\frac{I}{4} = \left(\frac{9}{16}\right)I$

$\therefore \quad x = 9, y = 16$

$\Rightarrow \quad y - x = 7$

8. Answer (2)

Hint : $\frac{hc}{X_{min}} = \Delta E - \phi$

Solution :

$$\frac{\lambda_{H_2}}{\lambda_{gas}} = \frac{\sqrt{2m\left(\frac{3}{4}z^2\Delta E_0 - \phi\right)}}{\sqrt{2m\left(\frac{3}{4}\Delta E_0 - \phi\right)}}$$

$$\Rightarrow \frac{\frac{3}{4}\Delta E_0 z^2 - \phi}{\frac{3}{4}\Delta E_0 - \phi} = \frac{61}{10} \quad \ldots(i)$$

Also, $\frac{3}{4}\Delta E_0 z^2 - \frac{\Delta E_0 z^2}{4} = 2\Delta E_0$

$$\Rightarrow \frac{\Delta E_0 z^2}{2} = 2\Delta E_0 \qquad \therefore\ z = 2$$

Now in equation (i)

$$10(3\Delta E_0 - \phi) = 61\left(\frac{3}{4}\Delta E_0 - \phi\right)$$

$$\Rightarrow 30\Delta E_0 - 10\phi = \frac{183}{4}\Delta E_0 - 61\phi$$

$$\Rightarrow 51\phi = \frac{63}{4}\Delta E_0$$

$$\Rightarrow \phi = \frac{63 \times 13.6}{4 \times 51}\text{eV} = \frac{21 \times 2}{10}\text{eV}$$

$\therefore$ $K = 2$

9. Answer (B, D)

Hint : $N = N_0 e^{-\lambda t}$

Solution :

Let N_0 be the number of active nuclei at 6 : 10 AM in 1 mL of dose.

Then at 8 : 00 AM, number of active nuclei becomes $\frac{N_0}{2}$ in 1 mL. So effectively $\frac{N_0}{2}$ no. of nuclei is to be administrated.

$$\Rightarrow (1\,\text{mL}) \cdot \frac{N_0}{2} = \text{constant}$$

At 7 : 05 AM let N_1 be the active nuclei then

$$N_1 = \frac{N_0}{e^{\frac{\ln 2 \times 55}{110}}} = \frac{N_0}{\sqrt{2}}$$

So $x.\frac{N_0}{\sqrt{2}} = (1\,\text{mL})\frac{N_0}{2}$

$$\Rightarrow x = \left(\frac{1}{\sqrt{2}}\right)\text{mL}$$

At 9 : 50 $N_3 = \frac{N_0}{4}$ And at 8 : 55 AM. $N_2 = \frac{N_0}{2\sqrt{2}}$

$$\therefore \frac{(1\,\text{mL})\frac{N}{2}}{\text{At } 8:00\text{ AM}} = \frac{(\sqrt{2}\,\text{mL})\frac{N_0}{2\sqrt{2}}}{\text{At } 8:55\text{ AM}} = \frac{(2\,\text{mL})\left(\frac{N_0}{4}\right)}{\text{At } 9:00\text{ AM}}$$

10. Answer (A, C)

Hint : $\frac{hc}{\lambda} = \Delta E_0 (Z-1)^2 \cdot \frac{3}{4}$ (for K_α lines)

Solution :

$$\frac{hc}{\lambda_z} = \Delta E_0 (Z-1)^2 \cdot \frac{3}{4}$$

$$\frac{hc}{\lambda_1} = \Delta E_0 (Z_1-1)^2 \frac{3}{4}$$

$$\frac{hc}{\lambda_2} = \Delta E_0 (Z_2-1)^2 . \frac{3}{4}$$

$$\therefore \frac{\lambda_z}{\lambda_1} = 4 = \frac{(Z_1-1)^2}{(Z-1)^2}$$

$$\Rightarrow \frac{Z_1-1}{Z-1} = 2$$

$\therefore$ $Z_1 = 2Z - 1$

Similarly, $\frac{\lambda_z}{\lambda_2} = \frac{(Z_2-1)^2}{(Z-1)^2} = \frac{1}{4}$

$$\Rightarrow \frac{Z_2-1}{Z-1} = \frac{1}{2}$$

$$\Rightarrow Z_2 = \frac{Z+1}{2}$$

11. Answer (A, C)

Hint : Least count $= \left(\frac{1}{50}\right)$mm.

Solution :

Least count $= \frac{1}{50}\text{mm} = 0.02\text{ mm}$

Reading = (1 mm × 18) + 0.02 × 20

= 18.4 mm

12. Answer (A, B)

Hint :

$R_n = R_0 (n^2)$

$\therefore$ $A_n = 4\pi R_0^2 \cdot n^4$

Solution :

$R_n = R_0 n^2$

$\therefore$ $A_n = 4\pi R_0^2 \cdot n^4$

And $A_1 = 4\pi R_0^2$

$$\therefore \frac{A_n}{A_1} = n^4$$

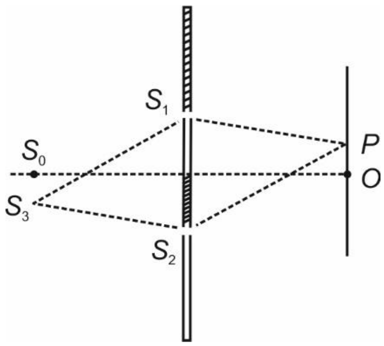
$$\Rightarrow \ln\left(\frac{A_n}{A_1}\right) = 4\ln n$$

Straight line of slope 4 and pass through origin.

13. Answer (B, C, D)

Hint : Will be shifted upward by $\frac{d^2}{4\lambda D}$

Solution :



$\Delta S_3S_1S_2 \equiv \Delta PS_2S_1$

$\therefore$ $S_0S_3 = OP$

$\Rightarrow$ shifting $\Delta y = \frac{d}{4}$

$\therefore$ Fringe width will remain same as $\frac{\lambda D}{d}$

$\therefore$ Number of fringe crossing through O is

$$N = \frac{d.d}{4\lambda D} = \frac{d^2}{4\lambda D}$$

14. Answer (A, C, D)

Hint : $\lambda = \frac{h}{P}$

Solution :

$$2m\,6(\hat{i} + 2\hat{j}) = P_B$$

$$\therefore \quad \lambda = \frac{h}{12m\sqrt{5}}$$

$$\Rightarrow \quad \frac{h}{m} = 12\sqrt{5}\,\lambda$$

$$P_A = 2m(\hat{i} + 2\hat{j}) = 2m\sqrt{5}$$

$$\therefore \quad \lambda_A = \frac{h}{2\sqrt{5}\,m} = \frac{12\sqrt{5}\,\lambda}{2\sqrt{5}} = 6\lambda$$

$$v_0 = \frac{m(2\hat{i} + 4\hat{j}) + 2m(6\hat{i} + 12\hat{j})}{3m} = \frac{14m\hat{i} + 28m\hat{j}}{3m}$$

$$P_{cm} = 3mv_0 = 14m(\hat{i} + 2\hat{j})$$

$$\Rightarrow \quad \lambda_{cm} = \frac{h}{P_{cm}} = \frac{h}{14\sqrt{5}m} = \frac{12\sqrt{5}\lambda}{14\sqrt{5}} = \frac{6}{7}\lambda$$

Now, $\vec{v}_{AC} = \vec{v}_{Aqr} - \vec{v}_{cqr} = 2\hat{i} + 4\hat{j} - \frac{14}{3}\hat{i} - \frac{28}{3}\hat{j}$

$$\vec{v}_{AC} = \frac{-8}{3}\hat{i} - \frac{16}{3}\hat{j} = \frac{-8}{3}(\hat{i} + 2\hat{j})$$

$$\therefore \quad |\vec{P}_{AC}| = \frac{8m}{3}\sqrt{5}$$

$$\therefore \quad \vec{\lambda}_{AC} = \frac{h}{(P_{AC})} = \frac{h \times 3}{8\sqrt{5}m} = \frac{3}{8\sqrt{5}} \times 12\sqrt{5}\,\lambda$$

$$\vec{\lambda}_{AC} = \frac{9}{2}\lambda$$

15. Answer (B, C)

Hint : $\Delta w = \frac{\lambda D}{d}$.

Solution :

Fringe width $\Delta w = \frac{\lambda D}{d}$

So if λ increases then fringe width also increases.

16. Answer (A, B ,C)

Hint : $\frac{hc}{\lambda} = \Delta E_0 Z^2 \left(\frac{1}{n^2} - \frac{1}{m^2}\right)$

Solution :

$$\frac{hc}{\lambda_B} = \Delta E_0 Z^2 \left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}\Delta E_0 Z^2$$

$$\therefore \quad \lambda_B = \frac{hc}{\Delta E_0 Z^2}\frac{36}{5}$$

$$\lambda_L = \frac{hc}{\Delta E_0 Z^2}$$

$$\because \quad \lambda_B - \lambda_L = \Delta\lambda = \frac{hc}{\Delta E_0 Z^2} \cdot \frac{31}{5}$$

$$\Rightarrow \quad \frac{5}{31}\Delta\lambda = \left(\frac{hc}{\Delta E_0 Z^2}\right)$$

$\because$ $\Delta E_0 = Rch$

$$\therefore \quad \frac{5}{31}\Delta\lambda \cdot Rhc = \frac{hc}{Z^2}$$

$$\Rightarrow \quad R = \frac{31}{5\Delta\lambda \cdot Z^2}$$

Shortest wavelength of Balmer series

$$\frac{hc}{\lambda'_B} = \frac{\Delta E_0 Z^2}{4}$$

$$\Rightarrow \quad \lambda'_B = 4\left[\frac{hc}{\Delta E_0 Z^2}\right]$$

$$\Rightarrow \quad \lambda'_B = 4\left[\frac{5}{31}\Delta\lambda\right] = \frac{20\Delta\lambda}{31}$$

And longest wavelength of Lyman series

$$\frac{hc}{\lambda'_L} = \Delta E_0 Z^2\left(1 - \frac{1}{4}\right) = \frac{3}{4}\Delta E_0 Z^2$$

$$\Rightarrow \quad \lambda'_L = \frac{4}{3}\left(\frac{hc}{\Delta E_0 Z^2}\right) = \frac{4}{3} \cdot \frac{5}{31}\Delta\lambda$$

$$\Rightarrow \quad \lambda'_L = \frac{20}{93}\Delta\lambda$$

17. Answer (A, B, C)

Hint :

The least count of the vernier caliper is the difference of the smallest unit on vernier scale and main scale.

Solution :

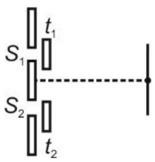
The least count of the vernier caliper is the difference of the smallest unit on vernier scale and main scale.

18. Answer (A, D)

Hint : Effective optical path difference :

$\mu_2 t_2 - \mu_1 t_1 + (t_1 - t_2)$

Solution :



If $t_1 > t_2$

Then phase lead by wave from S_2

$\left[\mu_2 t_2 + (t_1 - t_2) - \mu_1 t_1\right]$

So *f* ring will shift towards S_2 to counter that much extra lead of phase.

19. Answer A(Q); B(S); C(P, R); D(T)

Hint : Fringe width $\Delta w = \dfrac{\lambda D}{d}$

Position of minima $(2n+1)\dfrac{\lambda D}{2d}$

Solution :

(A) If at S_3 and S_4 there is destructive interference then final intensity on second screen is zero

$\therefore \quad d_1 = d_2 = (2n+1)\dfrac{\lambda D}{2d}$

(B) If $d_1 = \dfrac{3\lambda D}{2d}$ then destructive interference at S_3 and if $d_4 = \dfrac{\lambda D}{3d}$ then resulting intensity at S_4 is I_0. So final intensity at screen 2, is I_0.

(C) If $d_1 = d_2 = \dfrac{\lambda D}{3d}$ then resulting intensity at S_3 and S_4 are I_0 and final intensity at screen 2 is $4I_0$. Also if $d_1 = \dfrac{\lambda D}{2d}$ then intensity at S_3 is zero and for $d_2 = \dfrac{\lambda D}{d}$ the intensity at S_4 is $4I_0$. So final intensity at screen 2 is $4I_0$.

(D) If constructive interference happens at S_3 and S_4 then find intensity at screen 2 is $16I_0$.

20. Answer A(Q); B(S); C(R, T); D(P)

Hint : Apply Bohr's model.

Solution :

$$\frac{mV^2}{r} = \frac{kZe^2}{r^2}$$

$$mvr = \frac{nn}{2\pi}$$

$$r_n \propto \frac{n^2}{Z}$$

$$V_n \propto \frac{Z}{n}$$

$$T = \frac{2\pi r_n}{v_n}$$

$$T \propto \frac{n^3}{Z^2}$$

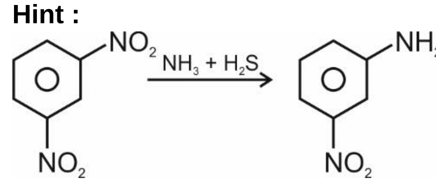
$$i = \frac{e}{T} = \frac{Z^2}{n^3}$$

$$B \propto \frac{i}{r} = \frac{Z^2 Z}{n^3 n^2} = \frac{Z^3}{n^5}$$

PART - II (CHEMISTRY)

21. Answer (6)

Hint :

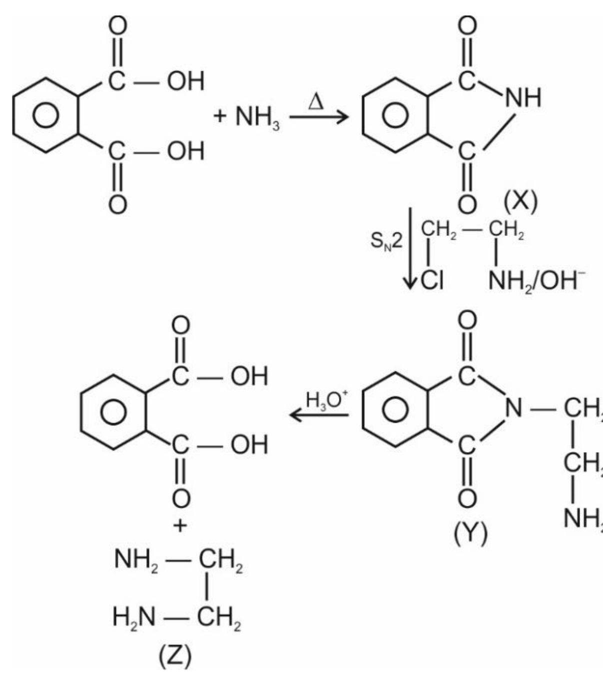


Solution :

$$R-NO_2 \xrightarrow{+6e^-} R-NH_2$$

22. Answer (5)

Hint :



Solution :

$x' = 1$

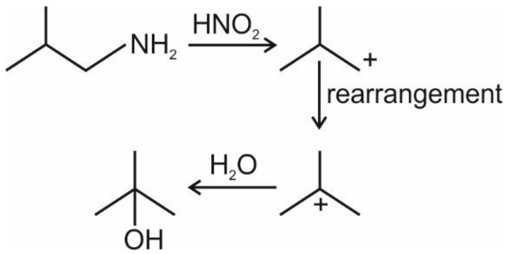
$z' = 2$

$n = 2$

Nucleophilic substitution takes place via S_N2

23. Answer (3)

Hint :



Solution :

$$(C_2H_5)_4NOH \xrightarrow{\Delta} (C_2H_5)_3N + C_2H_4$$

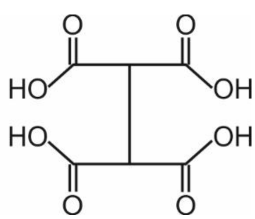
$$\rangle N-H \xrightarrow{HNO_2} \rangle N \rightarrow NO$$

Reactions A, D and E are correct.

24. Answer (4)

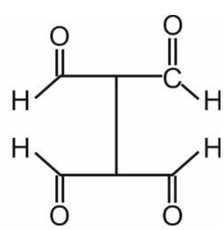
Hint :

P is



Solution :

Q is



Each —CHO can produce an oxime.

25. Answer (3)

Hint : Isoelectric point is the pH when an amino acid exist in zwitterionic form and shows no net migration towards any electrode.

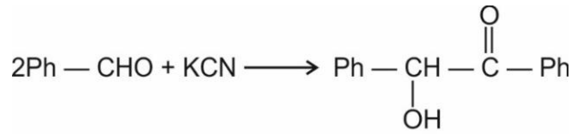
Solution :

$$pI = \frac{pK_{a_1} + pK_{a_2}}{2}$$

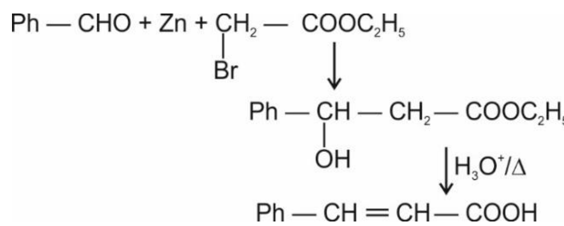
$$= \frac{2+4}{2} = 3$$

26. Answer (8)

Hint :



(Benzoin condensation)

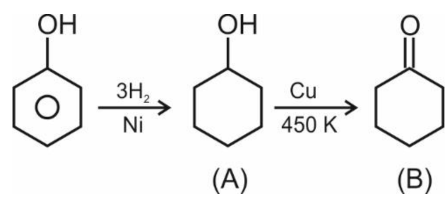


Solution :

Difference in molar mass, M = 64; $\frac{M}{8} = 8$

27. Answer (5)

Hint :



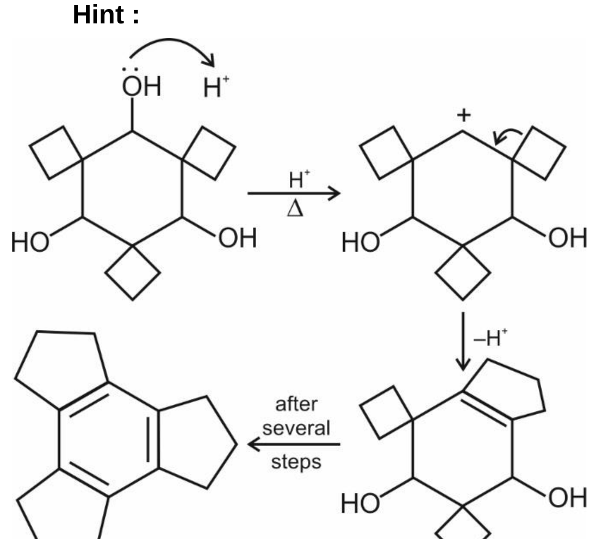
Solution :

A and B are non-aromatic compounds

Statements P, Q, R, U and V are incorrect.

28. Answer (1)

Hint :



Solution :

x = 1

Grignard reagent will not interact with product.

29. Answer (A, B, C)

Hint : The amines which can show optical activity, are resolvable.

Solution :

Amines which are bonded with four bulky groups and cyclic amines cannot undergo inversion.

30. Answer (C, D)

Hint : Correct order for basic strength is C > A > D > B

Solution :

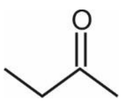
More is the availability of lone pair of e^- on N-atom, greater would be the basic strength.

31. Answer (C)

Hint :

All given species can give yellow ppt with 2, 4-DNP.

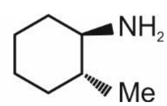
Solution :

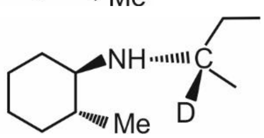
Only  can give yellow ppt with I_2 in NaOH.

32. Answer (A, B, C, D)

Hint : During Hoffmann bromamide degradation stereochemistry of migrating group does not change.

Solution :

P is 

Q is 

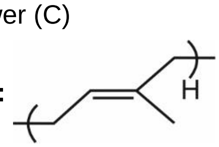
33. Answer (A, C)

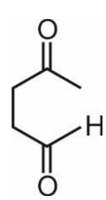
Hint : Histamine is not an antacid. It stimulates the secretion of acid in stomach.

Solution :

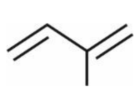
Brompheniramine is an antihistaminics

34. Answer (C)

Hint : 

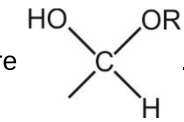
Polymer will yield upon ozonolysis 

Solution :

Monomer of the polymer is 

35. Answer (B, D)

Hint :

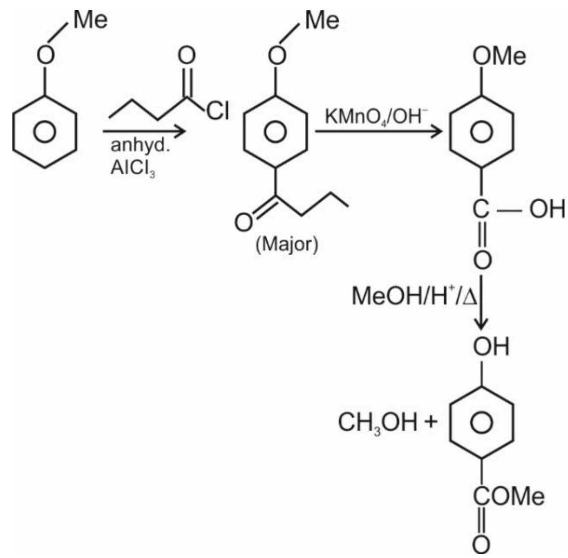
Hemiacetal forms are  . And these are reducing in nature.

Solution :

Both C_1 and C_2 are having hemiacetal that's why both are reducing.

36. Answer (A, B, D)

Hint :



Solution :

Ether do not undergo oxidation and in alkaline medium there is no hydrolysis that can occur on ether linkage.

37. Answer (A, B)

Hint : PCl_5 cause substitution of —OH by — Cl group.

Solution :

P is $CH_3 — \underset{\underset{Cl}{|}}{CH} — CH_2 — C(Me)_3$

38. Answer (A, B)

Hint : Ethers have lower boiling point than alcohols as there is hydrogen bonding involved in between two alcohol molecule.

Solution :

Compounds with multiple hydroxy functional group are having greater boiling point than mono hydroxy compound.

39. Answer A(P, Q, S); B(P, R); C(P, R, T); D(R)

Hint : Cellulose has β-links

Starch has α-links

Solution :

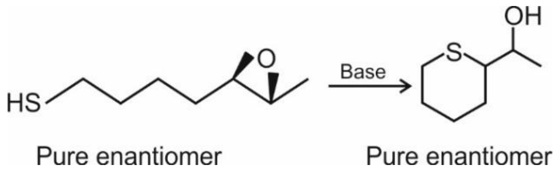
Sucrose upon hydrolysis gives α-D-glucose and β-D-fructose.

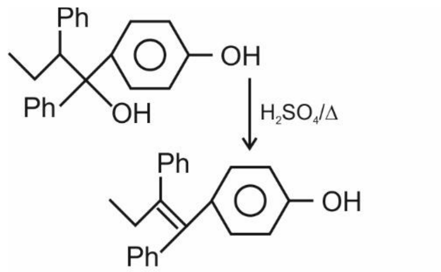
Maltose gives only α-D-glucose.

40. Answer A(P, R); B(Q); C(Q, S, T); D(Q)

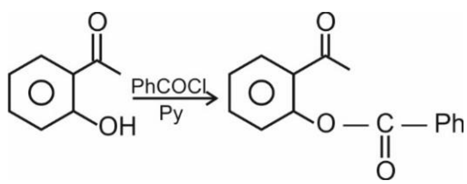
Hint : Gauche and anti-form are diastereomers of each other.

Solution :





Mixture of E and Z isomer



PART - III (MATHEMATICS)

41. Answer (4)

Hint : Angle between two lines = $\dfrac{\bar{b}_1 \cdot \bar{b}_2}{|\bar{b}_1||\bar{b}_2|}$

(where b_1 and b_2 are the vectors along the lines)

Solution :

$$\cos\alpha = \frac{a(\sin\theta - 2) + b\sqrt{5}\cos\theta + (2\sin\theta + 1)}{\sqrt{(\sin\theta-2)^2 + 5\cos^2\theta + (2\sin\theta+1)^2}\sqrt{a^2+b^2+1^2}}$$

$$\Rightarrow \cos\alpha = \frac{\sin\theta(a+2) + b\sqrt{5}\cos\theta - 2a + 1}{\sqrt{10}\sqrt{a^2+b^2+1}}$$

$\because$ α is independent of θ, then $a + 2 = 0$ and $b = 0$

$$\Rightarrow \cos\alpha = \frac{5}{\sqrt{10}\sqrt{5}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \alpha = \frac{\pi}{4}$$

42. Answer (9)

Hint : Use formula for division into groups.

Solution : Total number of ways of forming groups = $\dfrac{\lfloor 8}{(\lfloor 2)^4 \cdot \lfloor 4} \cdot \dfrac{\lfloor 4}{(\lfloor 2)^2 \cdot \lfloor 2}$. If 5^{th} ranked player is in the final, then he played a match against a higher ranked player in second round.

So in first round 5^{th} ranked player played a match against 6^{th}, 7^{th} or 8^{th} ranked player and remaining two of these played against each other.

Number of favourable ways for round one

$$= {}^3C_1 \cdot 1 \cdot \frac{\lfloor 4}{(\lfloor 2)^2 \lfloor 2}$$

And number of favourable ways for round two = 1

$$\text{Required probability} = \frac{{}^3C_1 \cdot \dfrac{\lfloor 4}{(\lfloor 2)^2 \lfloor 2} \times 1}{\dfrac{\lfloor 8}{(\lfloor 2)^4 \lfloor 4} \cdot \dfrac{\lfloor 4}{(\lfloor 2)^2 \lfloor 2}}$$

$$= \frac{1}{35}$$

Number of divisors of 36 will be 9.

43. Answer (7)

Hint : Count cases when number of letters between *A* and *E* are 0, 1 or 2

Solution : Total words = $\lfloor 8$

There are 6 cases when number of letters between *A* and *E* (or *A* and *I*) is zero.

There are 4 cases when number of letters between *A* and *E* (or *A* and *I*) is one.

There are 2 cases when number of letters between *A* and *E* (or *A* and *I*) is two.

So, favourable words = $(\lfloor 2 \times \lfloor 5)$

Required probability = $\dfrac{12 \lfloor 2 \lfloor 5}{\lfloor 8}$

$$\Rightarrow P = \frac{1}{14}$$

$$\Rightarrow \frac{1}{2P} = 7$$

44. Answer (6)

Hint : $P(A \cap B) = P(A) \cdot P(B)$

Solution : $P(A \cap B) = P(A) \cdot P(B)$

$$\Rightarrow P(A) = \frac{1}{5}$$

Now, $P\left(\dfrac{\bar{A}}{A \cup B}\right) = \dfrac{P(\bar{A} \cap (A \cup B))}{P(A \cup B)}$

$$\Rightarrow P\left(\frac{\bar{A}}{A \cup B}\right) = \frac{P(B) - P(A \cap B)}{\frac{1}{5} + \frac{1}{2} - \frac{1}{10}} = \frac{\frac{1}{2} - \frac{1}{10}}{\frac{6}{10}} = \frac{2}{3}$$

So, $9P\left(\dfrac{\bar{A}}{A \cup B}\right) = 6$

45. Answer (5)

Hint : Both the lines are parallel

Solution : $2x - 2y + z - 9 = 0 = x + 2y + 2z + 12$ is line of intersection of planes $P_1 : 2x - 2y + z - 9 = 0$ and $P_2 : x + 2y + 2z + 12 = 0$.

Another line $L : \frac{x}{2} = \frac{y}{1} = \frac{z}{-2}$ is parallel to both planes P_1 and P_2.

Distance between L and $P_1 = \left|\frac{9}{\sqrt{4+4+1}}\right| = 3$

and distance between L and $P_2 = \left|\frac{12}{\sqrt{4+4+1}}\right| = 4$

then distance between the two lines

$= \sqrt{3^2 + 4^2} = 5$

46. Answer (2)

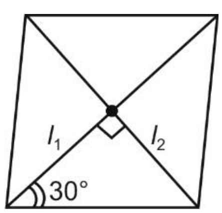
Hint : One angle of rhombus is $\frac{\pi}{3}$

Solution : Angle between two given lines;

$\cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

Area of rhombus = $2\sqrt{3}$



$\frac{1}{2}(2l_1)(2l_2) = 2\sqrt{3}$

$l_1 l_2 = \sqrt{3}$ …(i)

Also, $\frac{l_2}{l_1} = \frac{1}{\sqrt{3}}$ …(ii)

So $l_1 = \sqrt{3}$ and $l_2 = 1$

Side length of rhombus $= \sqrt{l_1^2 + l_2^2} = 2$

47. Answer (5)

Hint : Slope of normal is 2.

Solution : $xy^2 = 8$

$\Rightarrow \frac{dy}{dx}_{(2,2)} = -\frac{1}{2}$

Slope of normal = 2

So, unit vector along normal $= \frac{i + 2j}{\sqrt{5}} = \vec{x}$

Length of projection $= \left|\frac{3-8}{\sqrt{5}}\right| = \sqrt{5}$

48. Answer (4)

Hint : Fundamental principle of counting.

Solution : Total combinations of a, b, c and $d = 6^4$

$(a-3)$, $(b-4)$, $(c-5)$ and $(d-6)$ are integers.

Their product is 1 then

(i) All of them should be 1 (Not possible as $d \neq 7$)

(ii) All of them should be –1 (one case $a = 2$, $b = 3$, $c = 4$, $d = 5$)

(iii) Two of them are 1 and remaining two are –1 (three cases)

Total favourable cases = 4

Required probability = $\frac{4}{6^4}$

49. Answer (A, C)

Hint : Probability of a selected number to be even is $\frac{3}{7}$ and to be odd is $\frac{4}{7}$

Solution : (*A* and *B*) *abc* is even if at least one of *a*, *b* or *c* is even, so

Required probability = $1 - \left(\frac{4}{7}\right)^3$

$= \frac{279}{7^3}$

(C) $(ab + c)$ is even then,

Case (1) : If c is even then at least one of a or b is even.

Case (2) : If c is odd then both a and b are odd

Required probability

$= \frac{3}{7}\left(1 - \left(\frac{4}{7}\right)^2\right) + \left(\frac{4}{7}\right)^3 = \frac{163}{7^3}$

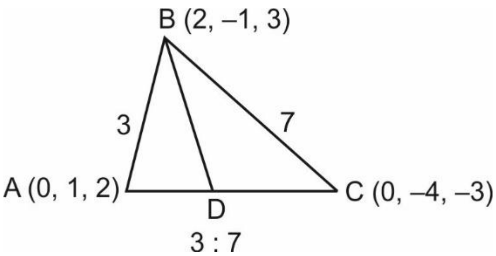
(B) $(a + b + c)$ is even if all are even or anyone is even and remaining two are odd.

Required probability = $\left(\frac{3}{7}\right)^3 + {}^3C_1 \cdot \frac{3}{7}\left(\frac{4}{7}\right)^2 = \frac{171}{7^3}$

50. Answer (B, D)

Hint : $\because \dfrac{AD}{CD} = \dfrac{AB}{BC}$

Solution :



$\because \dfrac{AD}{CD} = \dfrac{AB}{BC} = \dfrac{3}{7}$

Using section formula;

$D\left(0, -\dfrac{1}{2}, \dfrac{1}{2}\right)$

(A) DR's of $BD \propto 2, -\dfrac{1}{2}, \dfrac{5}{2}$ or $4, -1, 5$

(B) $BD = \sqrt{4 + \dfrac{1}{4} + \dfrac{25}{4}} = \sqrt{\dfrac{21}{2}}$

(C) Area of

$$\Delta ABD = \frac{1}{2}\left|\overrightarrow{BA} \times \overrightarrow{BD}\right| = \frac{1}{2}\left|-\frac{9}{2}\hat{i} - 3\hat{j} + 3\hat{k}\right|$$

$$= \frac{\sqrt{153}}{4}$$

(A) Area of

$$\Delta ABC = \frac{1}{2}\left|\overrightarrow{BA} \times \overrightarrow{BC}\right| = \frac{1}{2}\left|-15\hat{i} - 10\hat{j} + 10\hat{k}\right|$$

$$= \frac{5\sqrt{17}}{2}$$

51. Answer (A, C, D)

Hint : Use Bayes theorem

Solution : Let events

E_1 = Coins show 2 heads (die X is rolled)

E_2 = Coins show 1 head (die Y is rolled)

E_3 = Coins show no head (die Z is rolled)

And A = Die shows red face.

$P(E_1) = \dfrac{1}{4}$ $\quad P(E_2) = \dfrac{2}{4}$ $\quad P(E_3) = \dfrac{1}{4}$

$P\left(\dfrac{A}{E_1}\right) = \dfrac{1}{6}$ $\quad P\left(\dfrac{A}{E_2}\right) = \dfrac{3}{6}$ $\quad P\left(\dfrac{A}{E_3}\right) = \dfrac{2}{6}$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)\cdot P\left(\frac{A}{E_3}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right) + P(E_3)\cdot P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{4}\cdot\frac{2}{6}}{\frac{1}{4}\cdot\frac{1}{6} + \frac{2}{4}\cdot\frac{3}{6} + \frac{1}{4}\cdot\frac{2}{6}}$$

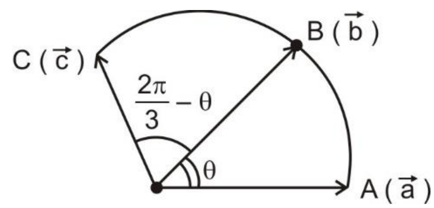
$$= \frac{2}{1+6+2} = \frac{2}{9}$$

52. Answer (B, C, D)

Hint : Consider $\vec{c} = \lambda\vec{a} + \mu\vec{b}$

Solution :

$\because$ $\vec{a}, \vec{b}, \vec{c}$ are coplanar then



Let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$

$\vec{a}\cdot\vec{c} = \lambda\vec{a}\cdot\vec{a} + \mu\vec{a}\cdot\vec{b}$

$-\dfrac{1}{2} = \lambda + \mu\cos\theta$ …(i)

Similarly $\vec{b}\cdot\vec{c} = \lambda\vec{a}\cdot\vec{b} + \mu\vec{b}\cdot\vec{b}$

$\cos\left(\dfrac{2\pi}{3} - \theta\right) = \lambda\cos\theta + \mu$ …(ii)

From (i) and (ii)

$\lambda = -\dfrac{\sqrt{3}}{2}\cot\theta - \dfrac{1}{2}$ and $\mu = \dfrac{\sqrt{3}}{2}\text{cosec}\,\theta$

So, $\vec{c} = -\left(\dfrac{\sqrt{3}\cot\theta + 1}{2}\right)\vec{a} + \left(\dfrac{\sqrt{3}}{2}\text{cosec}\,\theta\right)\vec{b}$

53. Answer (B, C, D)

Hint : Assume points $B(\vec{b})$ and $D(\vec{d})$ then use scalar product.

Solution : Consider A as origin and position vectors of B, D and C as $\vec{b}, \vec{d}, \vec{b} + \vec{d}$ respectively.

Here $|\vec{b}| = x$ and $|\vec{d}| = y$

Also $|\vec{b} + \vec{d}| = z$

$\Rightarrow$ $x^2 + y^2 + 2\vec{b}\cdot\vec{d} = z^2$ …(i)

$\overrightarrow{BD}\cdot\overrightarrow{DA} = (\vec{d} - \vec{b})\cdot(-\vec{d})$

$= -y^2 + \vec{b}\cdot\vec{d}$

$= -y^2 + \dfrac{z^2 - x^2 - y^2}{2}$ (from (i))

$= -\dfrac{1}{2}x^2 - \dfrac{3}{2}y^2 + \dfrac{1}{2}z^2$

54. Answer (B, C, D)

Hint : Equation must be inconsistent and planes should be non parallel.

Solution :

$$\begin{vmatrix} 1 & k & 1 \\ 1 & 1 & k \\ 1 & -3 & 3 \end{vmatrix} = 0$$

$\Rightarrow \; k = 1, -1$

But $k \neq 1$ (The two planes will be parallel so triangular prism can't be formed).

55. Answer (A, C, D)

Hint : $|\lambda\vec{a} + \mu\vec{b}|^2 = 4\lambda^2 + \mu^2 + 2\lambda\mu$

Solution : $|\lambda\vec{a} + \mu\vec{b}|^2 = \lambda^2|\vec{a}|^2 + \mu^2|\vec{b}|^2 + 2\lambda\mu\vec{a}\cdot\vec{b}$

$= 4\lambda^2 + \mu^2 + 2\lambda\mu$

(A) $|\vec{a} - \vec{b}|^2 = 4 + 1 - 2 = 3 \Rightarrow |\vec{a} - \vec{b}| = \sqrt{3}$

(B) $\left|\vec{b} - \dfrac{1}{2}\vec{a}\right|^2 = 1 + 1 - 1 = 1 \Rightarrow \left|\vec{b} - \dfrac{1}{2}\vec{a}\right| = 1$

(C) $\left|\dfrac{3\vec{a} - 7\vec{b}}{2}\right|^2 = \dfrac{36 + 49 - 42}{4} = \dfrac{43}{4}$

$\Rightarrow |3\vec{a} - 7\vec{b}| = \sqrt{\dfrac{43}{2}}$

(D) $|2\vec{a} - 5\vec{b}|^2 = 16 + 25 - 20 = 21$

$|2\vec{a} - 5\vec{b}| = \sqrt{21}$

56. Answer (A, B, C)

Hint : $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{b}\cdot\vec{c}) - (\vec{a}\cdot\vec{b})\vec{c}$

Solution : $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$

$\Rightarrow (\vec{a}\cdot b)\vec{c} = (\vec{b}\cdot\vec{c})\vec{a}$

So either $\vec{a}$ and $\vec{c}$ are collinear or

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = 0$

which means either $\vec{b}$ is a null vector or $\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{c}$.

57. Answer (A, B, D)

Hint : Use binomial distribution

Solution : Probability of *A* winning the game

$= {}^3C_2 \cdot p^2(1-p) + {}^3C_3 p^3$

$= p^2[3 - 2p]$

Probability of *B* winning the game

$= {}^5C_3 p^3(1-p)^2 + {}^5C_4 p^4(1-p) + {}^5C_5 p^5$

$= p^3\left[6p^2 - 15p + 10\right]$

Now, $p^2(3 - 2p) = p^3(6p^2 - 15p + 10)$

$\Rightarrow \; p = 0, 1, \dfrac{1}{2}$

58. Answer (B, C)

Hint : If odds in favour of an event is *p* then its probability is $\dfrac{p}{1+p}$

Solution : Let odds in favour of an event is *p* then its probability is $\dfrac{p}{1+p}$

$p = 3q$ …(i)

Probability of 1st event = $\dfrac{p}{1+p}$

Probability 2nd event = $\dfrac{q}{1+q}$

$\dfrac{p}{1+p} = 2\left(\dfrac{q}{1+q}\right)$ …(ii)

From (i) and (ii) $p = 0$ or 1

Probability of 1st event = 0 or $\dfrac{1}{2}$

59. Answer A(S, T); B(P, S); C(T); D(R, T)

Hint : $p(E) = \dfrac{n(E)}{n(S)}$

Solution :

Total number in set $S = 5 \times 5 \times 4 \times 3 = 300$

(A) If number is divisible by 4, the last two digits will be 04, 12, 20, 24, 32, 40, or 52.

Total number of numbers = $12 \times 3 + 9 \times 4 = 72$

Required probability = $\dfrac{72}{300} = \dfrac{6}{25}$

(B) There should be either only one odd digit or only even digit in the number.

Total such numbers = $^3C_1 \cdot 3\underline{\lvert 3} + {}^2C_1 \cdot \underline{\lvert 4} + 3\underline{\lvert 3}$

= 54 + 48 + 18

= 120

Required probability = $\frac{120}{300} = \frac{2}{5}$

(C) There are only 5 combinations of 4 digits possible (1, 2, 4, 5); (0, 3, 4, 5); (0, 2, 3, 4); (0, 1, 3, 5) or (0, 1, 2, 3)

Number of numbers divisible by 6 using

(1, 2, 4, 5) = 12

Number of numbers divisible by 6 using

(0, 3, 4, 5) = 10

Number of numbers divisible by 6 using

(0, 2, 3, 4) = 14

Number of numbers divisible by 6 using

(0, 1, 3, 5) = 6

Number of numbers divisible by 6 using

(0, 1, 2, 3) = 10

Total numbers divisible by 6 = 52

Required probability = $\frac{52}{300} = \frac{13}{75}$

(D) If *abcd* is divisible by 11 then

$a + c = b + d$

Total number of numbers divisible by

11 = 48

Required probability = $\frac{48}{300} = \frac{4}{25}$

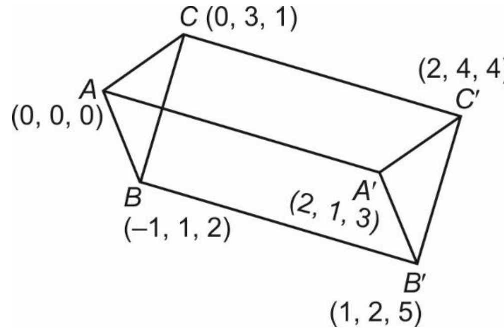
60. Answer A(T); B(Q); C(P, R); D(S)

Hint : Three faces of a triangular prism are parallelogram.

Solution :

(A) Let point $B'(x, y, z)$

$\because$ $AA'B'B$ is a parallelogram



So, $x_1 + 0 = 2 - 1 \Rightarrow x_1 = 1$

$y_1 + 0 = 1 + 1 \Rightarrow y_1 = 2$

$z_1 + 0 = 2 + 3 \Rightarrow z_1 = 5$

$B'(1, 2, 5)$

(B) Similarly $C'(2, 4, 4)$

(C) Point of intersection of diagonals of face

$AA'B'B$ is $\left(\frac{1}{2}, 1, \frac{5}{2}\right)$

Point of intersection of diagonals of face $AA'C'C$ is (1, 2, 2)

Point of intersection of diagonals of face

$BB'C'C$ is $\left(\frac{1}{2}, \frac{5}{2}, 3\right)$

(D) D' is midpoint of B' and C', so

$D'\left(\frac{3}{2}, 3, \frac{9}{2}\right)$

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