All India Aakash Test Series for JEE (Advanced)-2021

TEST - 1A (Paper-1) - Code-E

Test Date : 17/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(A, C)	19.	(B, C, D)	37.	(A, C)
2.	(B, D)	20.	(A, B, D)	38.	(B, C, D)
3.	(A, C, D)	21.	(B)	39.	(B, D)
4.	(A, C, D)	22.	(A, B, C, D)	40.	(A, D)
5.	(A, B, C, D)	23.	(B, C)	41.	(B, C)
6.	(A, B)	24.	(A, B, C)	42.	(B, D)
7.	(33)	25.	(10)	43.	(15)
8.	(24)	26.	(51)	44.	(63)
9.	(17)	27.	(68)	45.	(11)
10.	(12)	28.	(15)	46.	(12)
11.	(72)	29.	(12)	47.	(24)
12.	(16)	30.	(18)	48.	(04)
13.	(18)	31.	(20)	49.	(01)
14.	(15)	32.	(50)	50.	(07)
15.	(C)	33.	(A)	51.	(C)
16.	(B)	34.	(B)	52.	(C)
17.	(C)	35.	(A)	53.	(A)
18.	(A)	36.	(C)	54.	(D)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (A, C)

Hint : $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

Sol. :

Displacement $\vec{r} = \vec{r}_f - \vec{r}_i$

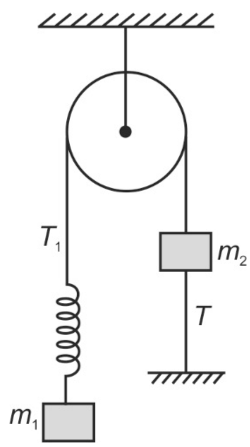
$\Rightarrow \vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) - (2\hat{i} + 3\hat{j} + 5\hat{k}) = \hat{i} + \hat{j}$

$\vec{r}$ makes 45° with +ve x-axis in anticlockwise sense.

2. Answer (B, D)

Hint. : Sudden impulsive force by spring is zero.

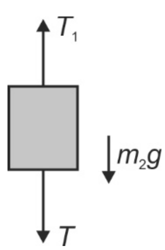
Sol. : Let the tension in string BC is T at equilibrium.



Then for m_1 to be in equilibrium $K\Delta x = m_1g$

That means spring will be in extended condition and it will transmit $T_1 = K\Delta x = m_1g$ force on string attached with spring.

So, for (m_2)



$\Rightarrow T = m_1g - m_2g$

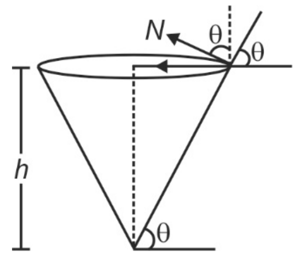
When string BC is burnt suddenly then spring still transmit the same force so acceleration of mass m_1 is zero. And acceleration of mass m_2 is

$$a_2 = \frac{(m_1 - m_2)g}{m_2}$$

3. Answer (A, C, D)

Hint : $v^2 = gr\tan\theta$

Sol. :



$$\tan\theta = \frac{h}{r}$$

$$\Rightarrow \quad r = \frac{h}{\tan\theta}$$

Along the plane, with respect to cone the particle is in state of equilibrium.

$$\therefore \; mg\sin\theta = \frac{mv^2}{r}\cdot\cos\theta$$

$$\Rightarrow \quad gr\cdot\frac{\sin\theta}{\cos\theta} = v^2 \quad \Rightarrow \quad v^2 = \frac{gh}{\tan\theta}\tan\theta$$

$\therefore \quad v^2 = gh$

Also, $N\cos\theta = mg$

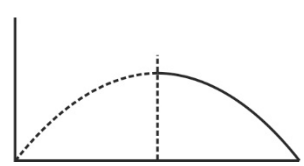
And $N\sin\theta = \dfrac{mv^2}{r}$

$$\therefore \quad N\sin\theta = \frac{mv^2}{h}\frac{\sin\theta}{\cos\theta} \quad \Rightarrow \quad N\cos\theta = \frac{mv^2}{h}$$

4. Answer (A, C, D)

Hint : For velocity to become perpendicular to initial direction $\theta > \dfrac{\pi}{4}$.

Sol. :



For velocity to become perpendicular to initial direction $\theta > \dfrac{\pi}{4}$.

For same case, $m_1 = \tan\theta_1 = \tan\theta$

And $m_2 = \tan\theta_2 = \dfrac{(u\sin\theta - gt)}{u\cos\theta}$

$\therefore \quad (m_1m_2 = -1) \Rightarrow \dfrac{(u\sin\theta - gt)}{u\cos\theta}\cdot\dfrac{\sin\theta}{\cos\theta} = -1$

$\Rightarrow \quad u\sin^2\theta - gt\sin\theta = -u\cos^2\theta$

$\Rightarrow \quad u = gt\sin\theta \quad \therefore \quad t = \dfrac{u}{g\sin\theta}$

And 1 s before, $\theta_1 = \tan^{-1}\left(\dfrac{g}{u\cos\theta}\right)$

So just after 1 s and before 1 s, $\Delta\theta = 2\theta$

$$\Delta\theta = 2\tan^{-1}\left(\frac{g}{u\cos\theta}\right)$$

5. Answer (A, B, C, D)

Hint : Tangential force will change the speed and perpendicular force will change the direction.

Sol. : If $\vec{v}$ is opposite to $\vec{F}$ the particle may retrace its path.

If $\vec{F}$ is perpendicular to $\vec{v}$ and so $\vec{F}$ will provide the centripetal force and if $|\vec{F}|$ is constant, then radius of curvature will be constant. And if at a particular time instant $\vec{v}$ and $\vec{F}$ are some angle other than 0° or 180° and $\vec{F}$ is constant, then it's analogues of projectile motion. Particle will trace the parabolic path.

6. Answer (A, B)

Hint : $v = \dfrac{dx}{dt}; \quad a = \dfrac{d^2x}{dt^2}$

Sol. : $x = \alpha t^3 + \beta t^2 + \gamma t + \delta$

$\therefore \quad \dfrac{dx}{dt} = v = 3\alpha t^2 + 2\beta t + \gamma$

$\dfrac{d^2x}{dt^2} = a = 6\alpha t + 2\beta$

$\therefore \quad 6\alpha t + 2\beta = 3\alpha t^2 + 2\beta t + \gamma$

$\Rightarrow \quad 3\alpha t^2 + (2\beta - 6\alpha)t + \gamma - 2\beta = 0$

Here, $4(3\alpha - \beta)^2 - 4 \times 3\alpha(\gamma - 2\beta) = 0$ for unique t

$\Rightarrow \quad 9\alpha^2 + \beta^2 - 3\alpha\gamma = 0$

And for that time instant $t > 0$

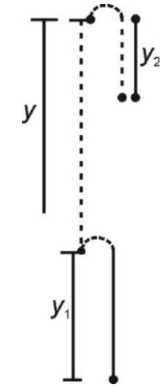
$\therefore \quad t = \dfrac{2(3\alpha - \beta)}{2\times 3\alpha} = \dfrac{3\alpha - \beta}{3\alpha}$

$\therefore \quad 3\alpha - \beta > 0$

7. Answer (33)

Hint : For vertical upward motion, $y = u_y t - \dfrac{1}{2}gt^2$

Sol. : Let v_0 was the velocity of dropping of 1st stone, then



$\vec{y}_1 = v_0 2 - \dfrac{1}{2}\times 10\times 4 = 2v_0 - 20$

$\Rightarrow \quad |\vec{y}_1| = (20 - 2v_0)$ is the distance from dropping point.

After 1 sec balloon shall have velocity $v_2 = (v_0 + 1)$

And it must have travelled $|y| = \left(v_0 + \dfrac{1}{2}\right)\cdot$

Then 1 sec after 2nd particle will be at

$y_2 = (v_0 + 1) - \dfrac{1}{2}\times 10 \times 1$

$\Rightarrow \quad |\vec{y}_2| = 5 - v_0 - 1 = 4 - v_0$

Distance from dropping point

$\therefore \quad$ Separation $\Rightarrow s = |\vec{y}_1| + |\vec{y}| - |\vec{y}_2|$

$\Rightarrow \quad s = 20 - 2v_0 + v_0 + \dfrac{1}{2} - 4 + v_0$

$\Rightarrow \quad s = 16 + \dfrac{1}{2} = \dfrac{33}{2}$

$\therefore \quad 2s = 33$

8. Answer (24)

Hint : $v = \dfrac{dr}{dt}, \quad a = \dfrac{d^2r}{dt^2}$

Sol. : $\vec{r} = 2t\hat{i} + 4t^2\hat{j} + \hat{k}$

$\therefore \quad \vec{v} = \dfrac{dr}{dt} = 2\hat{i} + 8t\hat{j}$

And $\vec{a} = \dfrac{d^2r}{dt^2} = 8\hat{j}$

Acceleration is always along y direction.

So, velocity in y direction at $t = 3$ s is

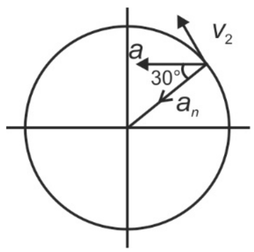
$|\vec{v}_y| = |8t| \Rightarrow (8\times 3) = 24$

9. Answer (17)

Hint : $\tan\theta = \frac{|a_n|}{|a_t|}$

Sol. : Angle with velocity vector is 30°.

$\therefore\ \tan 30° = \frac{|a_n|}{|a_t|} = \frac{V^2}{Ra}$



$\Rightarrow\ \frac{1}{\sqrt{3}} = \frac{a^2t^2}{Ra}$

$\Rightarrow\ \frac{1}{\sqrt{3}} = \frac{a^2t^2}{Ra} \Rightarrow \frac{at^2}{R} = \frac{1}{\sqrt{3}}$

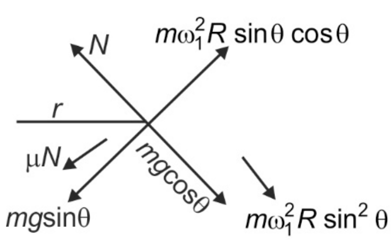
$\therefore\ t^2 = \frac{51}{\sqrt{3}\times\sqrt{3}} = 17$

10. Answer (12)

Hint : $N = mg\cos\theta + m\omega_1^2 R\sin^2\theta$

$mg\sin\theta + \mu N = m\omega_1^2 R\sin\theta\cdot\cos\theta$

Sol. : Let ω_1 be the maximum angular speed and ω_2 be the minimum angular speed, then



$N = mg\cos\theta + m\omega_1^2 R\sin^2\theta$

And $mg\sin\theta + \mu N = m\omega_1^2 R\sin\theta\cdot\cos\theta$

$\Rightarrow\ mg\sin\theta + \mu mg\cos\theta + \mu m\omega_1^2 R\sin^2\theta$

$= m\omega_1^2 R\sin\theta\cos\theta$

$\Rightarrow\ \omega_1^2 R\sin\theta(\cos\theta - \mu\sin\theta) = g(\sin\theta + \mu\cos\theta)$

$\therefore\ \omega_1^2 = \frac{g(\sin\theta+\mu\cos\theta)}{R\sin\theta(\cos\theta-\mu\sin\theta)}$

Similarly, $\omega_2^2 = \frac{g(\sin\theta-\mu\cos\theta)}{R\sin\theta(\cos\theta+\mu\sin\theta)}$

$\therefore\ \frac{\omega_1^2}{\omega_2^2} = \frac{\left(\frac{1}{\sqrt{2}}+\frac{1}{2\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}+\frac{2}{2\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)}$

$\Rightarrow\ \frac{\omega_1^2}{\omega_2^2} = \frac{\left(\frac{3}{2\sqrt{2}}\right)^2}{\left(\frac{1}{2\sqrt{2}}\right)^2} = 9$

$\therefore\ X = \frac{\omega_1}{\omega_2} = 3 \quad \therefore\ 4X = 12$

11. Answer (72)

Hint : $\frac{mg}{120}x\mu = \frac{mg}{120}(120-x)$

Sol. : For state of impending motion, let x be the length on the table, then

$\frac{m}{120}xg\mu = \frac{mg}{120}(120-x)$

$\Rightarrow\ \frac{2}{3}x = 120 - x \Rightarrow \frac{5x}{3} = 120$

$\Rightarrow\ x = \frac{120\times 3}{5} = 72$

12. Answer (16)

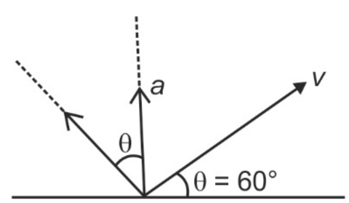
Hint : $a = \frac{dv}{dt}$

Sol. : $v(t) = \frac{8}{\sqrt{3}}\hat{i} + 8t\hat{j}$

$\therefore\ \vec{a} = 8\hat{j}$

At $t = 1$ s, $v = \frac{8}{\sqrt{3}}\hat{i} + 8\hat{j}$

$\therefore\ \tan\theta = \frac{8}{8}\cdot\sqrt{3} \ \therefore\ \theta = 60°$



$\therefore\ a_n = a\cos 60° = 8\times\frac{1}{2} = 4$

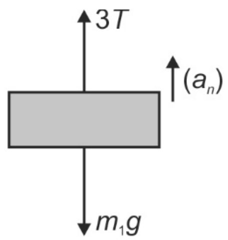
$\therefore\ 4|\vec{a}_n| = 16$

13. Answer (18)

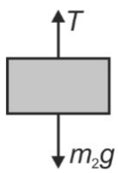
Hint : $3T - m_1g = m_1a_1$

$m_2g - T = m_2a_2$

Sol. :



$\Rightarrow \quad 3T - m_1g = m_1a_1 \quad ...(1)$



$\Rightarrow \quad m_2g - T = m_2a_2$

$\Rightarrow \quad 3m_2g - 3T = 3m_2a_2$

$\therefore \quad 3m_2g - m_1g = m_1a_1 + 3m_2a_2$

Also, $a_2 = 3a_1$

$\therefore \quad (3m_2 - m_1)g = m_1a_1 + gm_2a_1$

$\therefore \quad a_1 = \frac{4g}{20} = 2 \text{ m/s}^2$

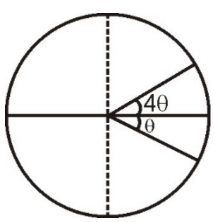
$\therefore \quad a_2 = 6 \text{ m/s}^2$

$\therefore \quad 3a_2 = 18 \text{ m/s}^2$

14. Answer (15)

Hint : $2T\Delta\theta = \Delta m\omega^2 R$

Sol. :



$\lambda = \frac{m}{2\pi R}$

$\Delta\ell = 2R\Delta Q$

$\therefore \quad 2T\sin\Delta\theta = \Delta m\omega^2 R$

$\Rightarrow \quad 2T\Delta\theta = \Delta m\omega^2 R$

$\Rightarrow \quad 2T\Delta\theta = \frac{m}{2\pi R}2R\Delta\theta \cdot \omega^2 R$

$\Rightarrow \quad T = \frac{m\omega^2 R}{2\pi} = 6.28 \times 5 \times 5 \times \frac{6}{10 \times 2 \times 3.14}$

$\Rightarrow \quad T = 15$

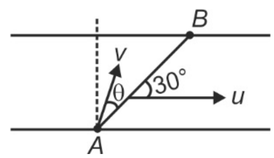
15. Answer (C)

16. Answer (B)

Hint for Q.Nos. 15 and 16 :

For forward and backward, both motion, person must maintain the same angle with line *AB*.

Solution for Q.Nos. 15 and 16 :



Clearly $\sin 30° = \frac{d}{AB}$

$\Rightarrow \quad AB = 2d$

For forward and backward, both motion, person must maintain the same angle with line *AB*.

$\therefore \quad v\sin\theta = u\sin 30° \quad \Rightarrow \quad \sin\theta = \frac{u}{4\sqrt{3}}$

From *A* to *B* $\Rightarrow \quad v\cos\theta + \frac{u\sqrt{3}}{2} = \frac{2d}{T_1}$

And from *B* to *A* $\Rightarrow \quad v\cos\theta - \frac{u\sqrt{3}}{2} = \frac{2d}{T_2}$

$\therefore \quad u\sqrt{3} = 2d\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\Rightarrow \quad u\sqrt{3} = 2d\left(\frac{T_2 - T_1}{T_1T_2}\right)$

$\therefore \quad u = \frac{2d}{\sqrt{3}}\left[\frac{T_2 - T_1}{T_1T_2}\right]$

And $\sin\theta = \frac{1}{4\sqrt{3}} \cdot \frac{2d}{\sqrt{3}}\left(\frac{T_2 - T_1}{T_1T_2}\right)$

$\therefore \quad \theta = \sin^{-1}\left(\frac{d}{6}\frac{(T_2 - T_1)}{T_1T_2}\right)$

17. Answer (C)

18. Answer (A)

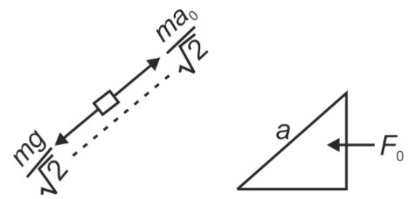
Hint for Q.Nos. 17 and 18 :

Motion is accelerated reference frame.

Solution for Q.Nos. 17 to 18 :

Let the force be F_0 when small block does not slide with respect to wedge.

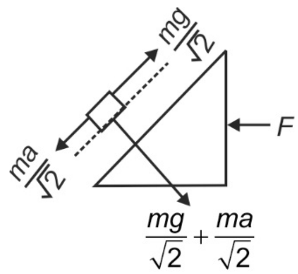
Then



$$\frac{mg}{\sqrt{2}} = \frac{ma_2}{\sqrt{2}} \quad \Rightarrow \quad a_0 = g$$

And $F_0 = 2ma_0 = 2mg$

As $F = \frac{3}{2}F_0 \ \therefore \ \frac{3}{2}\cdot 2mg \quad \Rightarrow F = 3mg$



Now as $F = 3mg$, let the acceleration of block be a_1 and wedge be a, then

$$\frac{ma}{\sqrt{2}} - \frac{mg}{\sqrt{2}} = ma_1$$

$$\therefore \ a_1 = \frac{1}{\sqrt{2}} - \frac{g}{\sqrt{2}}$$

And for wedge $3mg - \left(\frac{mg}{2} + \frac{ma}{2}\right) = ma$

$\Rightarrow$ $6mg - mg - ma = 2ma$

$\Rightarrow$ $5g = 3a \ \Rightarrow \ a = \frac{5g}{3}$

$$\therefore \ a_1 = \frac{1}{\sqrt{2}}\left(\frac{5}{3} - 1\right)g = \frac{2g}{3\sqrt{2}} = \frac{\sqrt{2}g}{3}$$

So, $\frac{L}{2} = \frac{1}{2}\frac{\sqrt{2}g}{3}\cdot t^2$

$$\Rightarrow \ t = \left[\frac{3L}{\sqrt{2}g}\right]^{\frac{1}{2}}$$

PART - II (CHEMISTRY)

19. Answer (B, C, D)

Hint : Oxygen is the limiting reagent.

Sol. : Number of moles of $Mg = \frac{1}{24}$

Number of moles of $O_2 = \frac{1}{64}$

	$2Mg$	+ O_2	$\rightarrow$ $2MgO$
Initial moles	$\frac{1}{24}$	$\frac{1}{64}$	
Moles at the end of reaction	$\left(\frac{1}{24} - \frac{1}{32}\right)$	0	$\frac{1}{32}$

Mass of Mg left unreacted

$$= \left[\frac{1}{24} - \frac{1}{32}\right] \times 24 = 0.25 \text{ g}$$

O_2 gas is consumed completely.

Mass of MgO formed = $\frac{1}{32} \times 40 = 1.25$ g

20. Answer (A, B, D)

Hint : Particles in the right zone have greater kinetic energy in distribution curve.

Sol. : Greater the kinetic energy, greater would be the tendency to get evaporate $T_2 > T_1$.

At higher temperature, vapour phase would exist.

21. Answer (B)

Hint : $P_{real} = \frac{nRT}{V - nb} - a\left(\frac{n}{V}\right)^2$

$$P_{ideal} = \frac{nRT}{V}$$

Sol. : When cylinder is full

$$P_{real} = \frac{60 \times 0.08 \times 300}{15 - (60 \times 0.05)} - 0.25\left(\frac{60}{15}\right)^2$$

= 116 atm

$$P_{ideal} = \frac{60 \times 0.08 \times 300}{15} = 96 \text{ atm}$$

After prolonged used,

$$P_{real} = \frac{0.60 \times 0.08 \times 300}{15 - (0.6 \times 0.05)} - (0.25)\left(\frac{0.6}{15}\right)^2$$

≈ 0.96 atm

$$P_{ideal} = \frac{0.6 \times 0.08 \times 300}{15} = 0.96 \text{ atm}$$

22. Answer (A, B, C, D)

Hint : All statements are correct.

Sol. : At constant V, $P = \left(\frac{R}{V - b}\right)T$

At constant P, $V = b + \left(\frac{R}{P}\right)T$

$$Z = \frac{PV}{RT} = 1 + \frac{Pb}{RT}$$

Since Z > 1, the repulsive forces dominate over attractive forces.

23. Answer (B, C)

Hint : The probability of finding electron, ψ^2 is zero at radial nodes in an orbital.

Sol. : The radial wave function for a Bohr atom is given as

$$\psi(\text{radial}) = \frac{1}{16\sqrt{4}}\left[\frac{Z}{a_0}\right]^{\frac{3}{2}}\left[(\sigma-1)(\sigma^2-8\sigma+12)\right]e^{\frac{-\sigma}{2}}$$

where $\sigma = \frac{2Zr}{a_0}$

At radial nodes, $\psi^2 = 0$

or $(\sigma-1)=0;\ \sigma=1 \Rightarrow r=\frac{a_0}{2Z}$

or $\sigma^2 - 8\sigma + 12 = 0$; $(\sigma - 6)(\sigma - 2) = 0$

$\Rightarrow \sigma = 6$ or 2; $r = \frac{3a_0}{Z}$ or $\frac{a_0}{Z}$

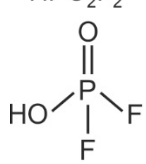
$\therefore$ Minimum position of radial node, $r = \frac{a_0}{2Z}$

Maximum position of radial node, $r = \frac{3a_0}{Z}$

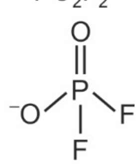
24. Answer (A, B, C)

Hint : Hybridisation of central atom in all 4 molecules is same.

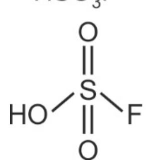
Sol. : HPO_2F_2



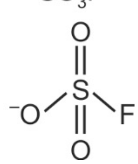
$PO_2F_2^-$



HSO_3F



SO_3F^-



25. Answer (10)

Hint : Average atomic mass = ΣX_iM_i, where X_i is the mole fraction of an isotope and M_i is its atomic mass.

Sol. : Let the mole % of ^{25}Mg be x. Therefore, mole % of ^{26}Mg is $(20 - x)$%.

$\therefore$ $0.80 \times 24 + 0.01x \times 25 + 0.01(20 - x) \times 26 = 24.3$

$\therefore$ x = 10%

26. Answer (51)

Hint : N_2 is the limiting reagent, that decides the maximum mass of NH_3.

Sol. : Number of moles of $N_2 = \frac{42}{28} = 1.5$

Number of moles of $H_2 = \frac{12}{2} = 6.0$

	N_2	+	$3H_2 \longrightarrow$	$2NH_3$
Initial moles	1.5		6.0	
Final moles	0		1.5	3.0

Maximum mass of NH_3 gas formed = 3.0 × 17 = 51 g

27. Answer (68)

Hint : Number of photons

$$= \frac{\text{Total energy absorbed}}{\text{Photon energy}}$$

Sol. : Energy of a photon of wavelength,

λ = 612 nm

$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{612\times10^{-9}} = \frac{6.6\times10^{-17}}{204}$$

Minimum energy needed to see an object

= 2.2×10^{-17} J

Number of photons required to see an object

$$= \frac{2.2\times10^{-17}\times204}{6.6\times10^{-17}} = 68$$

28. Answer (15)

Hint : Molality of solution

$$= \frac{\text{Number of moles of solute}}{\text{Mass of solvents in kg}}$$

Sol. : Molarity of the given solution = 3.9 M

Volume of solvent in 1 L solution = 1 L

Density of solvent = 0.26 g mL^{-1}

Mass of 1 L solvent = 260 gm

Molality of solution = $\frac{3.9\times1000}{260} = 15\ \text{mol kg}^{-1}$

29. Answer (12)

Hint : For n = 4, l = 0, 1, 2, and 3

For $|m_e| = 1$, $m_e = \pm1$ and

For $|m_s| = \frac{1}{2}$, $m_s = \pm\frac{1}{2}$

Sol. : For principal quantum number, $n = 4$, the possible values of azimuthal quantum number and magnetic quantum number are

$\ell = 0 \quad m_\ell = 0$

$\ell = 1 \quad m_\ell = 0, \pm 1$

$\ell = 2 \quad m_\ell = 0, \pm 1, \pm 2$

$\ell = 3 \quad m_\ell = 0, \pm 1, \pm 2, \pm 3$

Given values of magnetic and spin quantum numbers are

$|m_\ell| = 1 ; \Rightarrow m_\ell = \pm 1$

$|m_s| = \frac{1}{2}; \Rightarrow m_s = \pm\frac{1}{2}$

There are 6 orbitals which satisfy the given conditions and can accommodate 12 electrons.

30. Answer (18)

Hint : Angular momentum of electron in 3rd orbit of He^+ ion

$$mv_3r_3 = 3\left(\frac{h}{2\pi}\right)$$

Radius of electron in 3rd orbit of He^+ ion

$$r_3 = \frac{(3)^2 a_0}{2}$$

KE of electron in 3rd orbit of He^+ ion = $\frac{(mv_3)^2}{2m}$

Sol. : Angular momentum of an electron in nth orbit of a Bohr atom is given by

$$mvr = n\frac{h}{2\pi}$$

For an electron in 3rd orbit of He^+ ion,

$$mv_3r_3 = 3\frac{h}{2\pi}$$

$$mv_3 = \frac{3h}{2\pi r_3}$$

$$r_3 = \frac{(3)^2 a_0}{2} = \frac{9a_0}{2}$$

$$\therefore \quad mv_3 = \frac{3h \times 2}{2\pi \times 9a_0} = \frac{h}{3\pi a_0}$$

$$KE = \frac{(mv_3)^2}{2m} = \frac{h^2}{2m \times 9\pi^2 a_0^2} = \left(\frac{h^2}{\pi^2 m a_0^2}\right)\left(\frac{1}{18}\right)$$

$\therefore \quad x = 18$

31. Answer (20)

Hint : Molarity of stock solution × V (ml) = 0.4 × 460

Sol. : Millimoles of HCl in the final solution

= 0.4 × 460

= 184

Mass of HCl in stock solution = 29.2 gm

Number of moles of HCl in stock solution

$$= \frac{29.2}{36.5} = 0.8$$

Mass of HCl stock solution = 100 gm

Density of stock solution = 1.15 g mL^{-1}

Volume of 100 g stock solution = $\frac{100}{1.15}$ mL

Molarity of stock solution $= \frac{0.8 \times 1.15 \times 1000}{100}$

= 9.2 M

Let V ml of stock solution is required

9.2 × V = 184

$$V = \frac{184}{9.2} = 20 \text{ ml}$$

32. Answer (50)

Hint : Number of moles of C_2H_5Br

= 0.80 × Number of moles of C_2H_6 consumed

Number of moles of n-butane

= $\frac{0.56}{2}$ × Number of moles of C_2H_5Br consumed

Sol. : Let the volume of C_2H_6 required at STP be x L.

Number of moles of C_2H_6 required = $\frac{x}{22.4}$

$$C_2H_6 + Br_2 \xrightarrow[125°C]{h\nu} C_2H_5Br + HBr \text{ (80\% yield)}$$

Number of moles of C_2H_5Br produced

= 0.80 × Number of moles of C_2H_6 consumed

$$= \frac{0.80 \times x}{22.4}$$

$$2C_2H_5Br + 2Na \xrightarrow[\text{ether}]{\text{Dry}} C_4H_{10} + 2NaBr \text{ (56\% yield)}$$

Number of moles of C_4H_{10} produced

= $\frac{0.56}{2}$ × number of moles of C_2H_5Br consumed

$$= \frac{0.56 \times 0.80 \times x}{2 \times 22.4}$$

Mass of C_4H_{10} produced = $\frac{0.56 \times 0.80 \times x \times 58}{2 \times 22.4}$

= 29 g

x = 50 L

33. Answer (A)

Hint : $\text{Rate} = \frac{\text{Volume diffused}}{\text{Time}}$

Sol. : $\frac{r_X}{r_{O_2}} = \frac{V \times 5.65}{4 \times V} = \sqrt{\frac{32}{M_X}};\ M_X = 16$

34. Answer (B)

Hint : $\text{Rate} = \frac{\text{Moles diffused}}{\text{Time}}$

Sol. : $\frac{r_{H_2}}{r_{O_2}} = \frac{x \times 32 \times 30}{2 \times 60 \times 1} = \sqrt{\frac{32}{2}};\ x = 0.50\text{ g}$

35. Answer (A)

Hint & Sol. : Correct order of dipole moment

$H_2O > NH_3 > NF_3$

36. Answer (C)

Hint & Sol. : Compounds (I) and (II) have non-zero dipole moment because the resultant of all the bond dipole moments do not got cancelled.

PART - III (MATHEMATICS)

37. Answer (A, C)

Hint : Form an equation whose roots are $\frac{\alpha_i}{1+\alpha_i}$, where i = 1, 2, 3, 4.

Sol. : $x^4 - 7x + 1 = 0$ has roots α_1, α_2, α_3 and α_4.

Let $y = \frac{x}{1+x} \Rightarrow x = \frac{y}{1-y}$

$$\left(\frac{y}{1-y}\right)^4 - 7\left(\frac{y}{1-y}\right) + 1 = 0$$

$\Rightarrow$ $y^4 - 7y(1-y)^3 + (1-y)^4 = 0$

$\Rightarrow$ $9y^4 - 25y^3 + 27y^2 - 11y + 1 = 0$ …(i)

The roots of equation (i) are $\frac{\alpha_i}{1+\alpha_i}$; i = 1, 2, 3, 4

$$\sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \text{Sum of roots of } (i) = \frac{25}{9}$$

$$\prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \text{Product of roots of } (i) = \frac{1}{9}$$

38. Answer (B, C, D)

Hint : Put $x - 2 = t$

Sol. : Let $x - 2 = t$

$\Rightarrow$ $(t+1)^4 + (t-1)^4 = k$

$\Rightarrow$ $t^4 + 6t^2 + 1 = \frac{k}{2}$

$\Rightarrow$ $(t^2 + 3)^2 = 8 + \frac{k}{2}$

$\Rightarrow$ $t^2 = -3 \pm \sqrt{8 + \frac{k}{2}}$ …(i)

When $t^2 > 0 \Rightarrow$ Two distinct real values of x

$t^2 < 0 \Rightarrow$ Two imaginary values of x.

From (i) at least one value of t^2 is negative, while other value may be positive if $k > 2$.

39. Answer (B, D)

Hint : Put $z = x + iy$ and solve for x and y.

Sol. : Let $z = x + iy$

$x + iy + 1 + i = \sqrt{x^2 + y^2}$

$\Rightarrow$ $(x+1) + i(y+1) = \sqrt{x^2 + y^2}$

$\Rightarrow$ $y + 1 = 0$ and $x + 1 = \sqrt{x^2 + y^2}$

$\Rightarrow$ $y = -1$ and $x = 0$

So, $z = -i$

40. Answer (A, D)

Hint : Range of $f(x)$ is $\left[-\frac{1}{5}, \frac{1}{3}\right]$.

Sol. : Domain of $f(x)$ is R as $x^2 + x + 4 \neq 0$.

Let $y = \frac{x+1}{x^2+x+4}$ = $yx^2 + x(y-1) + (4y-1) = 0$

$\because$ $x \in R$, $(y-1)^2 - 4y(4y-1) \geq 0$

$\Rightarrow$ $15y^2 - 2y - 1 \leq 0$

$y \in \left[-\frac{1}{5}, \frac{1}{3}\right]$

41. Answer (B, C)

Hint : Use properties.

Sol. : $\because$ $1 \notin A \cup (B \cap \{1, 2, 3\})$

$\Rightarrow$ $1 \notin A$ and $1 \notin B \cap \{1, 2, 3\}$

$\Rightarrow$ $1 \notin A$ and $1 \notin B$

$\Rightarrow$ $1 \notin A \cup B$

$\Rightarrow$ $1 \in (A \cup B)'$

$\because$ $4 \notin B \cap \{1, 2, 3\}$ and $5 \notin B \cap \{1, 2, 3\}$

So, the smallest possible set $A = \{4, 5\}$

Also, smallest possible set $B = \phi$ (when $A = \{2, 3, 4, 5\}$)

42. Answer (B, D)

Hint : Use condition for common root.

Sol. : $(a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (a_1c_2 - a_2c_1)^2$

$\Rightarrow 3(-2\lambda) = (-\lambda)^2$

$\Rightarrow \lambda = 0, -6$

43. Answer (15)

Hint : Use $\tan\theta\cdot\tan(60° - \theta)\cdot\tan(60° + \theta) = \tan 3\theta$

Sol. : $\tan 4°\cdot\tan 8°\cdot\tan 12°\cdot\ldots\tan 88°$

$= (\tan 4°\cdot\tan 56°\cdot\tan 64°)(\tan 8°\cdot\tan 52°\cdot\tan 68°)\ldots(\tan 28°\cdot\tan 32°\cdot\tan 88°)\cdot\tan 60°$

$= (\tan 12°\cdot\tan 24°\cdot\tan 36°\cdot\tan 48°\cdot\tan 60°\cdot\tan 72°\cdot\tan 84°)\sqrt{3}$

$= 3[(\tan 12°\cdot\tan 48°\cdot\tan 72°)(\tan 24°\cdot\tan 36°\cdot\tan 84°)]$

$= 3\tan 36°\cdot\tan 72°$

$= 3\cdot\dfrac{\sin 36°\cdot\cos 18°}{\cos 36°\cdot\sin 18°}$

$= 3\left[\dfrac{\sqrt{10-2\sqrt{5}}\cdot\sqrt{10+2\sqrt{5}}}{(\sqrt{5}+1)(\sqrt{5}-1)}\right]$

$= 3\left[\dfrac{\sqrt{100-20}}{4}\right]$

$= 3\sqrt{5}$

44. Answer (63)

Hint : $\tan C = -\tan(A + B) = -\dfrac{2\tan\left(\frac{A+B}{2}\right)}{1-\tan^2\left(\frac{A+B}{2}\right)}$

Sol. : $\because \quad C = \pi - (A + B)$

$\Rightarrow \tan C = -\tan(A + B)$

$\Rightarrow \tan C = -\dfrac{2\tan\left(\frac{A+B}{2}\right)}{1-\tan^2\left(\frac{A+B}{2}\right)}$

Now, $\tan\left(\dfrac{A+B}{2}\right) = \dfrac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\cdot\tan\frac{B}{2}}$

$= 8$

So, $\tan C = \dfrac{16}{63}$

$\cos C = \dfrac{63}{65}$

45. Answer (11)

Hint : Find the range of both trigonometric functions.

Sol. : $\because \quad 2k+1 \in \left[-\sqrt{193}, \sqrt{193}\right]$ …(i)

Also, $2k = 4\sec^2 y + \operatorname{cosec}^2 y$

$2k = 5 + 4\tan^2 y + \dfrac{1}{\tan^2 y}$

$2k \in [9, \infty]$ …(ii)

From (i) and (ii),

$k = 5$ or 6

46. Answer (12)

Hint : $\arg\left(\dfrac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

Sol. : $\arg(z) = \arg(\sqrt{i}) - \arg(\sqrt{3}+i)$

$= \dfrac{1}{2}\arg(i) - \dfrac{\pi}{6}$

$= \dfrac{\pi}{4} - \dfrac{\pi}{6}$

$= \dfrac{\pi}{12}$

47. Answer (24)

Hint : $(A \times B \times B) \cap (A \times A \times B) = A \times (A \cap B) \times B$

Sol. : If $(x, y, z) \in (A \times B \times B) \cap (A \times A \times B)$, then $x \in A$, $y \in A$ and $y \in B$, $z \in B$

Possible number of values of $x = 3$

Possible number of values of $y = 2$

Possible number of values of $z = 4$

$\therefore \quad n((A \times B \times B) \cap (A \times A \times B)) = 24$

48. Answer (04)

Hint : Put $\log_2 3 = a$ to simplify X and use $(\sqrt{3}-1)^2 = 2(2-\sqrt{3})$ to simplify Y.

Sol. :

$X = (4 + \log_2 3)(5 + \log_2 3) - (3 + \log_2 3)(6 + \log_2 3)$

Put $\log_2 3 = a$

$\Rightarrow X = (4 + a)(5 + a) - (3 + a)(6 + a) = 2$

$Y = \dfrac{1+\log_2(2-\sqrt{3})}{\log_2(\sqrt{3}-1)} = \dfrac{\log_2(4-2\sqrt{3})}{\log_2(\sqrt{3}-1)} = 2$

49. Answer (01)

Hint : $x \in \left[2n\pi + \frac{\pi}{4}, 2n\pi + \frac{\pi}{2}\right)$

Sol. : $\log_{\frac{\sqrt{3}}{2}}(\sin x) \le \log_{\frac{\sqrt{3}}{2}}(\cos x)$

$\Rightarrow$ $\sin x \ge \cos x$ also $\sin x > 0 \cap \cos x > 0$

$\Rightarrow$ $x \in \left[2n\pi + \frac{\pi}{4}, 2n\pi + \frac{\pi}{2}\right)$

$\because$ $x \in [0, 12]$, then $x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left[\frac{9\pi}{4}, \frac{5\pi}{2}\right)$

$\because$ x is an integer, then $x = 1$ only.

50. Answer (07)

Hint : $f(x) = 3 + 2(\tan^2 x + \cot^2 x)$

Sol. : $f(x) = \sin^2 x + \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \text{cosec}^2 x$

$\Rightarrow$ $f(x) = 3 + 2(\tan^2 x + \cot^2 x)$

$\Rightarrow$ $f(x) = 7 + 2(\tan x - \cot x)^2$

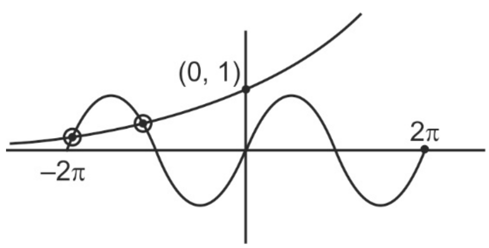
$\therefore$ Minimum value of $f(x) = 7$

51. Answer (C)

Hint : $e^x = \sin x$; draw the graphs of LHS and RHS

Sol. : $\therefore$ $e^x = \sin x$

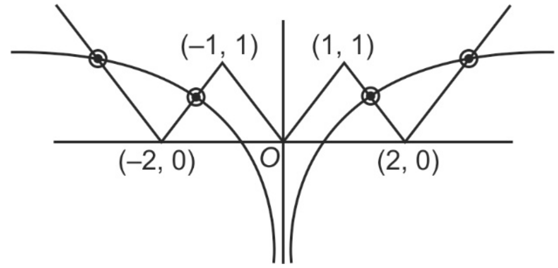
From the graph, there are two points of intersection.



52. Answer (C)

Hint : Draw graphs of LHS and RHS.

Sol. : $\log_2|x| = |||x| - 1| - 1|$



From the graph; we get 4 solutions.

53. Answer (A)

Hint : Put $x = \sin\theta \Rightarrow \sin 3\theta = \frac{1}{\sqrt{2}}$

Sol. : Let $x = \sin\theta$

$\Rightarrow$ $\sin 3\theta = \frac{1}{\sqrt{2}} = \sin 45°$

So, possible value of θ is 15°, then

$x = \sin 15° = \frac{\sqrt{3} - 1}{2\sqrt{2}}$

54. Answer (D)

Hint : Put $x = \sin\theta \Rightarrow \sin 2\theta = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

Sol. : Let $x = \sin\theta$

$\Rightarrow$ $\sin 2\theta = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

$\Rightarrow$ $\sin 2\theta = \sin 36°$

So, possible value of θ is 18°, then

$x = \sin 18° = \frac{\sqrt{5} - 1}{4}$

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All India Aakash Test Series for JEE (Advanced)-2021

TEST - 1A (Paper-1) - Code-F

Test Date : 17/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(A, B)	19.	(A, B, C)	37.	(B, D)
2.	(A, B, C, D)	20.	(B, C)	38.	(B, C)
3.	(A, C, D)	21.	(A, B, C, D)	39.	(A, D)
4.	(A, C, D)	22.	(B)	40.	(B, D)
5.	(B, D)	23.	(A, B, D)	41.	(B, C, D)
6.	(A, C)	24.	(B, C, D)	42.	(A, C)
7.	(15)	25.	(50)	43.	(07)
8.	(18)	26.	(20)	44.	(01)
9.	(16)	27.	(18)	45.	(04)
10.	(72)	28.	(12)	46.	(24)
11.	(12)	29.	(15)	47.	(12)
12.	(17)	30.	(68)	48.	(11)
13.	(24)	31.	(51)	49.	(63)
14.	(33)	32.	(10)	50.	(15)
15.	(C)	33.	(A)	51.	(C)
16.	(B)	34.	(B)	52.	(C)
17.	(C)	35.	(A)	53.	(A)
18.	(A)	36.	(C)	54.	(D)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (A, B)

Hint : $v = \frac{dx}{dt}$; $a = \frac{d^2x}{dt^2}$

Sol. : $x = \alpha t^3 + \beta t^2 + \gamma t + \delta$

$\therefore \quad \frac{dx}{dt} = v = 3\alpha t^2 + 2\beta t + \gamma$

$\frac{d^2x}{dt^2} = a = 6\alpha t + 2\beta$

$\therefore \quad 6\alpha t + 2\beta = 3\alpha t^2 + 2\beta t + \gamma$

$\Rightarrow \quad 3\alpha t^2 + (2\beta - 6\alpha)\,t + \gamma - 2\beta = 0$

Here, $4(3\alpha - \beta)^2 - 4 \times 3\alpha(\gamma - 2\beta) = 0$ for unique t

$\Rightarrow \quad 9\alpha^2 + \beta^2 - 3\alpha\gamma = 0$

And for that time instant $t > 0$

$\therefore \quad t = \frac{2(3\alpha - \beta)}{2 \times 3\alpha} = \frac{3\alpha - \beta}{3\alpha}$

$\therefore \quad 3\alpha - \beta > 0$

2. Answer (A, B, C, D)

Hint : Tangential force will change the speed and perpendicular force will change the direction.

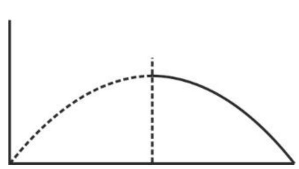
Sol. : If $\vec{v}$ is opposite to $\vec{F}$ the particle may retrace its path.

If $\vec{F}$ is perpendicular to $\vec{v}$ and so $\vec{F}$ will provide the centripetal force and if $|\vec{F}|$ is constant, then radius of curvature will be constant. And if at a particular time instant $\vec{v}$ and $\vec{F}$ are some angle other than 0° or 180° and $\vec{F}$ is constant, then it's analogues of projectile motion. Particle will trace the parabolic path.

3. Answer (A, C, D)

Hint : For velocity to become perpendicular to initial direction $\theta > \frac{\pi}{4}$.

Sol. :



For velocity to become perpendicular to initial direction $\theta > \frac{\pi}{4}$.

For same case, $m_1 = \tan\theta_1 = \tan\theta$

And $m_2 = \tan\theta_2 = \frac{(u\sin\theta - gt)}{u\cos\theta}$

$\therefore \quad (m_1m_2 = -1) \Rightarrow \frac{(u\sin\theta - gt)}{u\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} = -1$

$\Rightarrow \quad u\sin^2\theta - gt\sin\theta = -u\cos^2\theta$

$\Rightarrow \quad u = gt\sin\theta \quad \therefore \quad t = \frac{u}{g\sin\theta}$

And 1 s before, $\theta_1 = \tan^{-1}\left(\frac{g}{u\cos\theta}\right)$

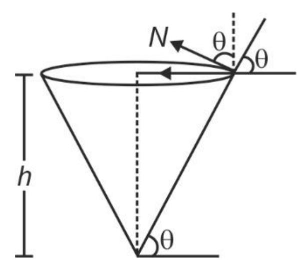
So just after 1 s and before 1 s, $\Delta\theta = 2\theta$

$\Delta\theta = 2\tan^{-1}\left(\frac{g}{u\cos\theta}\right)$

4. Answer (A, C, D)

Hint : $v^2 = gr\tan\theta$

Sol. :



$\tan\theta = \frac{h}{r}$

$\Rightarrow \quad r = \frac{h}{\tan\theta}$

Along the plane, with respect to cone the particle is in state of equilibrium.

$\therefore \; mg\sin\theta = \frac{mv^2}{r} \cdot \cos\theta$

$\Rightarrow \quad gr \cdot \frac{\sin\theta}{\cos\theta} = v^2 \quad \Rightarrow \quad v^2 = \frac{gh}{\tan\theta}\tan\theta$

$\therefore \quad v^2 = gh$

Also, $N\cos\theta = mg$

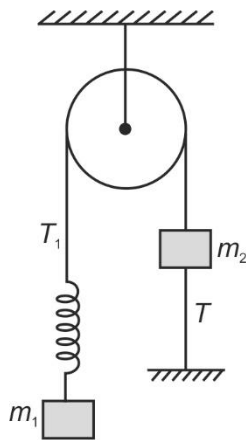
And $N\sin\theta = \frac{mv^2}{r}$

$\therefore \quad N\sin\theta = \frac{mv^2}{h}\frac{\sin\theta}{\cos\theta} \quad \Rightarrow \quad N\cos\theta = \frac{mv^2}{h}$

5. Answer (B, D)

Hint. : Sudden impulsive force by spring is zero.

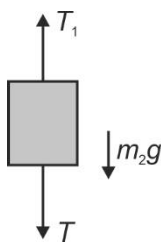
Sol. : Let the tension in string *BC* is *T* at equilibrium.



Then for m_1 to be in equilibrium $K\Delta x = m_1g$

That means spring will be in extended condition and it will transmit $T_1 = K\Delta x = m_1g$ force on string attached with spring.

So, for (m_2)



$\Rightarrow$ $T = m_1g - m_2g$

When string *BC* is burnt suddenly then spring still transmit the same force so acceleration of mass m_1 is zero. And acceleration of mass m_2 is

$$a_2 = \frac{(m_1 - m_2)g}{m_2}$$

6. Answer (A, C)

Hint : $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

Sol. :

Displacement $\vec{r} = \vec{r}_f - \vec{r}_i$

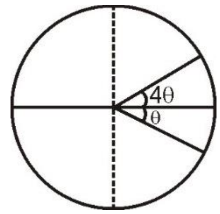
$$\Rightarrow \vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) - (2\hat{i} + 3\hat{j} + 5\hat{k}) = \hat{i} + \hat{j}$$

$\vec{r}$ makes 45° with +ve *x*-axis in anticlockwise sense.

7. Answer (15)

Hint : $2T\Delta\theta = \Delta m\omega^2 R$

Sol. :



$$\lambda = \frac{m}{2\pi R}$$

$$\Delta\ell = 2R\Delta Q$$

$\therefore$ $2T\sin\Delta\theta = \Delta m\omega^2 R$

$\Rightarrow$ $2T\Delta\theta = \Delta m\omega^2 R$

$$\Rightarrow 2T\Delta\theta = \frac{m}{2\pi R}2R\Delta\theta\cdot\omega^2 R$$

$$\Rightarrow T = \frac{m\omega^2 R}{2\pi} = 6.28\times 5\times 5\times\frac{6}{10\times 2\times 3.14}$$

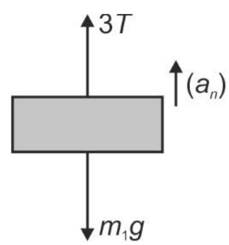
$\Rightarrow$ $T = 15$

8. Answer (18)

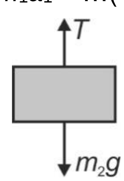
Hint : $3T - m_1g = m_1a_1$

$m_2g - T = m_2a_2$

Sol. :



$\Rightarrow$ $3T - m_1g = m_1a_1$ …(1)



$\Rightarrow$ $m_2g - T = m_2a_2$

$\Rightarrow$ $3m_2g - 3T = 3m_2a_2$

$\therefore$ $3m_2g - m_1g = m_1a_1 + 3m_2a_2$

Also, $a_2 = 3a_1$

$\therefore$ $(3m_2 - m_1)g = m_1a_1 + gm_2a_1$

$$\therefore a_1 = \frac{4g}{20} = 2\text{ m/s}^2$$

$\therefore$ $a_2 = 6\text{ m/s}^2$

$\therefore$ $3a_2 = 18\text{ m/s}^2$

9. Answer (16)

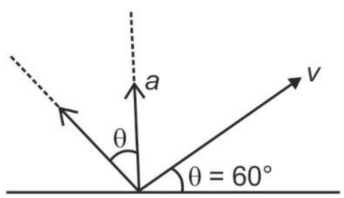
Hint : $a = \frac{dv}{dt}$

Sol. : $v(t) = \frac{8}{\sqrt{3}}\hat{i} + 8t\hat{j}$

$\therefore \quad \vec{a} = 8\hat{j}$

At $t = 1$ s, $v = \frac{8}{\sqrt{3}}\hat{i} + 8\hat{j}$

$\therefore \quad \tan\theta = \frac{8}{8}\cdot\sqrt{3} \ \therefore \ \theta = 60°$



$\therefore \quad a_n = a\cos 60° = 8 \times \frac{1}{2} = 4$

$\therefore \quad 4|\vec{a}_n| = 16$

10. Answer (72)

Hint : $\frac{mg}{120}x\mu = \frac{mg}{120}(120 - x)$

Sol. : For state of impending motion, let x be the length on the table, then

$$\frac{m}{120}xg\mu = \frac{mg}{120}(120 - x)$$

$$\Rightarrow \frac{2}{3}x = 120 - x \quad \Rightarrow \quad \frac{5x}{3} = 120$$

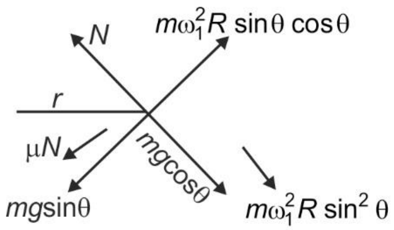
$$\Rightarrow \quad x = \frac{120 \times 3}{5} = 72$$

11. Answer (12)

Hint : $N = mg\cos\theta + m\omega_1^2 R\sin^2\theta$

$$mg\sin\theta + \mu N = m\omega_1^2 R\sin\theta\cdot\cos\theta$$

Sol. : Let ω_1 be the maximum angular speed and ω_2 be the minimum angular speed, then



$$N = mg\cos\theta + m\omega_1^2 R\sin^2\theta$$

And $mg\sin\theta + \mu N = m\omega_1^2 R\sin\theta\cdot\cos\theta$

$$\Rightarrow \quad mg\sin\theta + \mu mg\cos\theta + \mu m\omega_1^2 R\sin^2\theta$$

$$= m\omega_1^2 R\sin\theta\cos\theta$$

$$\Rightarrow \quad \omega_1^2 R\sin\theta(\cos\theta - \mu\sin\theta) = g(\sin\theta + \mu\cos\theta)$$

$$\therefore \quad \omega_1^2 = \frac{g(\sin\theta + \mu\cos\theta)}{R\sin\theta(\cos\theta - \mu\sin\theta)}$$

Similarly, $\omega_2^2 = \frac{g(\sin\theta - \mu\cos\theta)}{R\sin\theta(\cos\theta + \mu\sin\theta)}$

$$\therefore \quad \frac{\omega_1^2}{\omega_2^2} = \frac{\left(\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}} + \frac{2}{2\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)}$$

$$\Rightarrow \quad \frac{\omega_1^2}{\omega_2^2} = \frac{\left(\frac{3}{2\sqrt{2}}\right)^2}{\left(\frac{1}{2\sqrt{2}}\right)^2} = 9$$

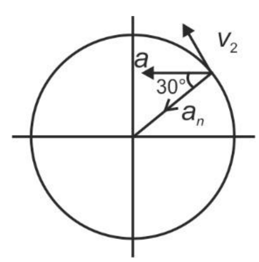
$$\therefore \quad X = \frac{\omega_1}{\omega_2} = 3 \quad \therefore \quad 4X = 12$$

12. Answer (17)

Hint : $\tan\theta = \frac{|a_n|}{|a_t|}$

Sol. : Angle with velocity vector is 30°.

$$\therefore \quad \tan 30° = \frac{|a_n|}{|a_t|} = \frac{V^2}{Ra}$$



$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{a^2t^2}{Ra}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{a^2t^2}{Ra} \quad \Rightarrow \quad \frac{at^2}{R} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad t^2 = \frac{51}{\sqrt{3}\times\sqrt{3}} = 17$$

13. Answer (24)

Hint : $v = \frac{dr}{dt}$, $a = \frac{d^2r}{dt^2}$

Sol. : $\vec{r} = 2t\hat{i} + 4t^2\hat{j} + \hat{k}$

$\therefore \vec{v} = \frac{dr}{dt} = 2\hat{i} + 8t\hat{j}$

And $\vec{a} = \frac{d^2r}{dt^2} = 8\hat{j}$

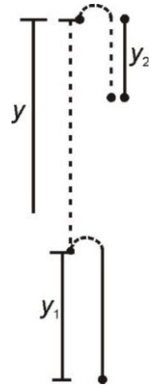
Acceleration is always along *y* direction.
So, velocity in *y* direction at *t* = 3 s is

$|\vec{v}_y| = |8t| \Rightarrow (8 \times 3) = 24$

14. Answer (33)

Hint : For vertical upward motion, $y = u_y t - \frac{1}{2}gt^2$

Sol. : Let v_0 was the velocity of dropping of 1st stone, then



$\vec{y}_1 = v_0 2 - \frac{1}{2} \times 10 \times 4 = 2v_0 - 20$

$\Rightarrow |\vec{y}_1| = (20 - 2v_0)$ is the distance from dropping point.

After 1 sec balloon shall have velocity $v_2 = (v_0 + 1)$

And it must have travelled $|y| = \left(v_0 + \frac{1}{2}\right)$.

Then 1 sec after 2nd particle will be at

$y_2 = (v_0 + 1) - \frac{1}{2} \times 10 \times 1$

$\Rightarrow |\vec{y}_2| = 5 - v_0 - 1 = 4 - v_0$

Distance from dropping point

$\therefore$ Separation $\Rightarrow s = |\vec{y}_1| + |\vec{y}| - |\vec{y}_2|$

$\Rightarrow s = 20 - 2v_0 + v_0 + \frac{1}{2} - 4 + v_0$

$\Rightarrow s = 16 + \frac{1}{2} = \frac{33}{2}$

$\therefore 2s = 33$

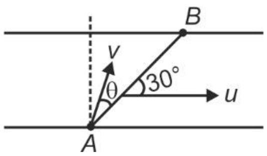
15. Answer (C)

16. Answer (B)

Hint for Q.Nos. 15 and 16 :

For forward and backward, both motion, person must maintain the same angle with line *AB*.

Solution for Q.Nos. 15 and 16 :



Clearly $\sin 30° = \frac{d}{AB}$

$\Rightarrow AB = 2d$

For forward and backward, both motion, person must maintain the same angle with line *AB*.

$\therefore v \sin\theta = u \sin 30° \Rightarrow \sin\theta = \frac{u}{4\sqrt{3}}$

From *A* to *B* $\Rightarrow v\cos\theta + \frac{u\sqrt{3}}{2} = \frac{2d}{T_1}$

And from *B* to *A* $\Rightarrow v\cos\theta - \frac{u\sqrt{3}}{2} = \frac{2d}{T_2}$

$\therefore u\sqrt{3} = 2d\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\Rightarrow u\sqrt{3} = 2d\left(\frac{T_2 - T_1}{T_1 T_2}\right)$

$\therefore u = \frac{2d}{\sqrt{3}}\left[\frac{T_2 - T_1}{T_1 T_2}\right]$

And $\sin\theta = \frac{1}{4\sqrt{3}} \cdot \frac{2d}{\sqrt{3}}\left(\frac{T_2 - T_1}{T_1 T_2}\right)$

$\therefore \theta = \sin^{-1}\left(\frac{d}{6}\frac{(T_2 - T_1)}{T_1 T_2}\right)$

17. Answer (C)

18. Answer (A)

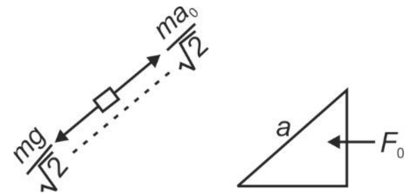
Hint for Q.Nos. 17 and 18 :

Motion is accelerated reference frame.

Solution for Q.Nos. 17 and 18 :

Let the force be F_0 when small block does not slide with respect to wedge.

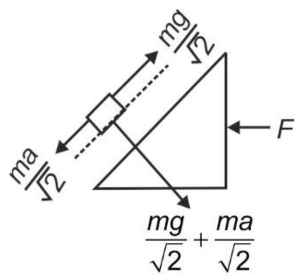
Then



$$\frac{mg}{\sqrt{2}} = \frac{ma_2}{\sqrt{2}} \Rightarrow a_0 = g$$

And $F_0 = 2ma_0 = 2mg$

As $F = \frac{3}{2}F_0 \;\therefore\; \frac{3}{2}\cdot 2mg \;\Rightarrow\; F = 3mg$



Now as $F = 3mg$, let the acceleration of block be a_1 and wedge be a, then

$$\frac{ma}{\sqrt{2}} - \frac{mg}{\sqrt{2}} = ma_1$$

$$\therefore\; a_1 = \frac{1}{\sqrt{2}} - \frac{g}{\sqrt{2}}$$

And for wedge $3mg - \left(\frac{mg}{2} + \frac{ma}{2}\right) = ma$

$\Rightarrow$ $6mg - mg - ma = 2ma$

$\Rightarrow$ $5g = 3a \;\Rightarrow\; a = \frac{5g}{3}$

$$\therefore\; a_1 = \frac{1}{\sqrt{2}}\left(\frac{5}{3} - 1\right)g = \frac{2g}{3\sqrt{2}} = \frac{\sqrt{2}g}{3}$$

So, $\frac{L}{2} = \frac{1}{2}\frac{\sqrt{2}g}{3}\cdot t^2$

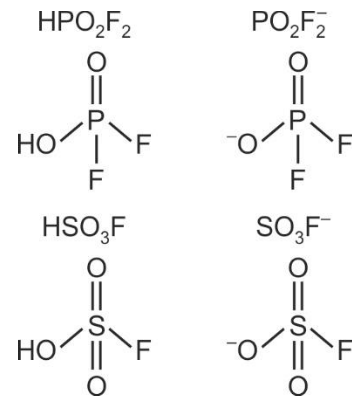
$$\Rightarrow\; t = \left[\frac{3L}{\sqrt{2}g}\right]^{\frac{1}{2}}$$

PART - II (CHEMISTRY)

19. Answer (A, B, C)

Hint : Hybridisation of central atom in all 4 molecules is same.

Sol. :



20. Answer (B, C)

Hint : The probability of finding electron, ψ^2 is zero at radial nodes in an orbital.

Sol. : The radial wave function for a Bohr atom is given as

$$\psi(\text{radial}) = \frac{1}{16\sqrt{4}}\left[\frac{Z}{a_0}\right]^{\frac{3}{2}}\left[(\sigma-1)(\sigma^2 - 8\sigma + 12)\right]e^{\frac{-\sigma}{2}}$$

where $\sigma = \frac{2Zr}{a_0}$

At radial nodes, $\psi^2 = 0$

or $(\sigma - 1) = 0;\; \sigma = 1 \Rightarrow r = \frac{a_0}{2Z}$

or $\sigma^2 - 8\sigma + 12 = 0$; $(\sigma - 6)(\sigma - 2) = 0$

$\Rightarrow\; \sigma = 6$ or 2; $r = \frac{3a_0}{Z}$ or $\frac{a_0}{Z}$

$\therefore$ Minimum position of radial node, $r = \frac{a_0}{2Z}$

Maximum position of radial node, $r = \frac{3a_0}{Z}$

21. Answer (A, B, C, D)

Hint : All statements are correct.

Sol. : At constant V, $P = \left(\frac{R}{V-b}\right)T$

At constant P, $V = b + \left(\frac{R}{P}\right)T$

$$Z = \frac{PV}{RT} = 1 + \frac{Pb}{RT}$$

Since Z > 1, the repulsive forces dominate over attractive forces.

22. Answer (B)

Hint : $P_{real} = \frac{nRT}{V - nb} - a\left(\frac{n}{V}\right)^2$

$$P_{ideal} = \frac{nRT}{V}$$

Sol. : When cylinder is full

$$P_{real} = \frac{60 \times 0.08 \times 300}{15-(60 \times 0.05)} - 0.25\left(\frac{60}{15}\right)^2$$

= 116 atm

$$P_{ideal} = \frac{60 \times 0.08 \times 300}{15} = 96 \text{ atm}$$

After prolonged used,

$$P_{real} = \frac{0.60 \times 0.08 \times 300}{15-(0.6 \times 0.05)} - (0.25)\left(\frac{0.6}{15}\right)^2$$

≈ 0.96 atm

$$P_{ideal} = \frac{0.6 \times 0.08 \times 300}{15} = 0.96 \text{ atm}$$

23. Answer (A, B, D)

Hint : Particles in the right zone have greater kinetic energy in distribution curve.

Sol. : Greater the kinetic energy, greater would be the tendency to get evaporate $T_2 > T_1$.

At higher temperature, vapour phase would exist.

24. Answer (B, C, D)

Hint : Oxygen is the limiting reagent.

Sol. : Number of moles of $Mg = \frac{1}{24}$

Number of moles of $O_2 = \frac{1}{64}$

	$2Mg$	+	O_2	$\rightarrow$	$2MgO$
Initial moles	$\frac{1}{24}$		$\frac{1}{64}$		
Moles at the end of reaction	$\left(\frac{1}{24}-\frac{1}{32}\right)$		0		$\frac{1}{32}$

Mass of Mg left unreacted

$$= \left[\frac{1}{24}-\frac{1}{32}\right] \times 24 = 0.25 \text{ g}$$

O_2 gas is consumed completely.

Mass of MgO formed = $\frac{1}{32} \times 40 = 1.25$ g

25. Answer (50)

Hint : Number of moles of C_2H_5Br

= 0.80 × Number of moles of C_2H_6 consumed

Number of moles of n-butane

= $\frac{0.56}{2}$ × Number of moles of C_2H_5Br consumed

Sol. : Let the volume of C_2H_6 required at STP be x L.

Number of moles of C_2H_6 required = $\frac{x}{22.4}$

$$C_2H_6 + Br_2 \xrightarrow[125°C]{h\nu} C_2H_5Br + HBr \ (80\% \text{ yield})$$

Number of moles of C_2H_5Br produced

= 0.80 × Number of moles of C_2H_6 consumed

$= \frac{0.80 \times x}{22.4}$

$$2C_2H_5Br + 2Na \xrightarrow[\text{ether}]{\text{Dry}} C_4H_{10} + 2NaBr \ (56\% \text{ yield})$$

Number of moles of C_4H_{10} produced

$= \frac{0.56}{2}$ × number of moles of C_2H_5Br consumed

$= \frac{0.56 \times 0.80 \times x}{2 \times 22.4}$

Mass of C_4H_{10} produced = $\frac{0.56 \times 0.80 \times x \times 58}{2 \times 22.4}$

= 29 g

x = 50 L

26. Answer (20)

Hint : Molarity of stock solution × V (ml) = 0.4 × 460

Sol. : Millimoles of HCl in the final solution

= 0.4 × 460

= 184

Mass of HCl in stock solution = 29.2 gm

Number of moles of HCl in stock solution

$= \frac{29.2}{36.5} = 0.8$

Mass of HCl stock solution = 100 gm

Density of stock solution = 1.15 g mL^{-1}

Volume of 100 g stock solution = $\frac{100}{1.15}$ mL

Molarity of stock solution $= \frac{0.8 \times 1.15 \times 1000}{100}$

= 9.2 M

Let V ml of stock solution is required

9.2 × V = 184

$$V = \frac{184}{9.2} = 20 \text{ ml}$$

27. Answer (18)

Hint : Angular momentum of electron in 3rd orbit of He^+ ion

$$mv_3r_3 = 3\left(\frac{h}{2\pi}\right)$$

Radius of electron in 3rd orbit of He^+ ion

$$r_3 = \frac{(3)^2 a_0}{2}$$

KE of electron in 3rd orbit of He^+ ion = $\frac{(mv_3)^2}{2m}$

Sol. : Angular momentum of an electron in nth orbit of a Bohr atom is given by

$$mvr = n\frac{h}{2\pi}$$

For an electron in 3rd orbit of He^+ ion,

$$mv_3r_3 = 3\frac{h}{2\pi}$$

$$mv_3 = \frac{3h}{2\pi r_3}$$

$$r_3 = \frac{(3)^2 a_0}{2} = \frac{9a_0}{2}$$

$$\therefore \ mv_3 = \frac{3h \times 2}{2\pi \times 9a_0} = \frac{h}{3\pi a_0}$$

$$KE = \frac{(mv_3)^2}{2m} = \frac{h^2}{2m \times 9\pi^2 a_0^2} = \left(\frac{h^2}{\pi^2 m a_0^2}\right)\left(\frac{1}{18}\right)$$

$\therefore$ x = 18

28. Answer (12)

Hint : For n = 4, l = 0, 1, 2, and 3

For $|m_e|$ = 1, m_e = ±1 and

For $|m_s| = \frac{1}{2}$, $m_s = \pm\frac{1}{2}$

Sol. : For principal quantum number, *n* = 4, the possible values of azimuthal quantum number and magnetic quantum number are

$\ell = 0$ $m_\ell = 0$

$\ell = 1$ $m_\ell = 0, \pm 1$

$\ell = 2$ $m_\ell = 0, \pm 1, \pm 2$

$\ell = 3$ $m_\ell = 0, \pm 1, \pm 2, \pm 3$

Given values of magnetic and spin quantum numbers are

$|m_\ell| = 1$; $\Rightarrow m_\ell = \pm 1$

$|m_s| = \frac{1}{2}; \Rightarrow m_s = \pm\frac{1}{2}$

There are 6 orbitals which satisfy the given conditions and can accommodate 12 electrons.

29. Answer (15)

Hint : Molality of solution

$$= \frac{\text{Number of moles of solute}}{\text{Mass of solvents in kg}}$$

Sol. : Molarity of the given solution = 3.9 M

Volume of solvent in 1 L solution = 1 L

Density of solvent = 0.26 g mL^{-1}

Mass of 1 L solvent = 260 gm

Molality of solution = $\frac{3.9 \times 1000}{260} = 15 \text{ mol kg}^{-1}$

30. Answer (68)

Hint : Number of photons

$$= \frac{\text{Total energy absorbed}}{\text{Photon energy}}$$

Sol. : Energy of a photon of wavelength,

λ = 612 nm

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{612 \times 10^{-9}} = \frac{6.6 \times 10^{-17}}{204}$$

Minimum energy needed to see an object

= 2.2×10^{-17} J

Number of photons required to see an object

$$= \frac{2.2 \times 10^{-17} \times 204}{6.6 \times 10^{-17}} = 68$$

31. Answer (51)

Hint : N_2 is the limiting reagent, that decides the maximum mass of NH_3.

Sol. : Number of moles of $N_2 = \frac{42}{28} = 1.5$

Number of moles of $H_2 = \frac{12}{2} = 6.0$

	N_2 +	$3H_2 \longrightarrow$	$2NH_3$
Initial moles	1.5	6.0	
Final moles	0	1.5	3.0

Maximum mass of NH_3 gas formed = 3.0 × 17

= 51 g

32. Answer (10)

Hint : Average atomic mass = $\Sigma X_i M_i$, where X_i is the mole fraction of an isotope and M_i is its atomic mass.

Sol. : Let the mole % of ^{25}Mg be x. Therefore, mole % of ^{26}Mg is (20 – x)%.

$\therefore$ 0.80 × 24 + 0.01x × 25 + 0.01 (20 – x) × 26 = 24.3

$\therefore$ x = 10%

33. Answer (A)

Hint : $\text{Rate} = \frac{\text{Volume diffused}}{\text{Time}}$

Sol. : $\frac{r_X}{r_{O_2}} = \frac{V \times 5.65}{4 \times V} = \sqrt{\frac{32}{M_X}}$; $M_X = 16$

34. Answer (B)

Hint : $\text{Rate} = \frac{\text{Moles diffused}}{\text{Time}}$

Sol. : $\frac{r_{H_2}}{r_{O_2}} = \frac{x \times 32 \times 30}{2 \times 60 \times 1} = \sqrt{\frac{32}{2}}$; x = 0.50 g

35. Answer (A)

Hint & Sol. : Correct order of dipole moment

$H_2O > NH_3 > NF_3$

36. Answer (C)

Hint & Sol. : Compounds (I) and (II) have non-zero dipole moment because the resultant of all the bond dipole moments do not got cancelled.

PART - III (MATHEMATICS)

37. Answer (B, D)

Hint : Use condition for common root.

Sol. : $(a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (a_1c_2 - a_2c_1)^2$

$\Rightarrow$ $3(-2\lambda) = (-\lambda)^2$

$\Rightarrow$ $\lambda = 0, -6$

38. Answer (B, C)

Hint : Use properties.

Sol. : $\because$ $1 \notin A \cup (B \cap \{1, 2, 3\})$

$\Rightarrow$ $1 \notin A$ and $1 \notin B \cap \{1, 2, 3\}$

$\Rightarrow$ $1 \notin A$ and $1 \notin B$

$\Rightarrow$ $1 \notin A \cup B$

$\Rightarrow$ $1 \in (A \cup B)'$

$\because$ $4 \notin B \cap \{1, 2, 3\}$ and $5 \notin B \cap \{1, 2, 3\}$

So, the smallest possible set $A = \{4, 5\}$

Also, smallest possible set $B = \phi$ (when $A = \{2, 3, 4, 5\}$)

39. Answer (A, D)

Hint : Range of $f(x)$ is $\left[-\frac{1}{5}, \frac{1}{3}\right]$.

Sol. : Domain of $f(x)$ is R as $x^2 + x + 4 \neq 0$.

Let $y = \frac{x+1}{x^2+x+4}$ $= yx^2 + x(y - 1) + (4y - 1) = 0$

$\because$ $x \in R$, $(y - 1)^2 - 4y(4y - 1) \geq 0$

$\Rightarrow$ $15y^2 - 2y - 1 \leq 0$

$y \in \left[-\frac{1}{5}, \frac{1}{3}\right]$

40. Answer (B, D)

Hint : Put $z = x + iy$ and solve for x and y.

Sol. : Let $z = x + iy$

$x + iy + 1 + i = \sqrt{x^2 + y^2}$

$\Rightarrow$ $(x + 1) + i(y + 1) = \sqrt{x^2 + y^2}$

$\Rightarrow$ $y + 1 = 0$ and $x + 1 = \sqrt{x^2 + y^2}$

$\Rightarrow$ $y = -1$ and $x = 0$

So, $z = -i$

41. Answer (B, C, D)

Hint : Put $x - 2 = t$

Sol. : Let $x - 2 = t$

$\Rightarrow$ $(t + 1)^4 + (t - 1)^4 = k$

$\Rightarrow$ $t^4 + 6t^2 + 1 = \frac{k}{2}$

$\Rightarrow$ $(t^2 + 3)^2 = 8 + \frac{k}{2}$

$\Rightarrow$ $t^2 = -3 \pm \sqrt{8 + \frac{k}{2}}$ …(i)

When $t^2 > 0 \Rightarrow$ Two distinct real values of x

$t^2 < 0 \Rightarrow$ Two imaginary values of x.

From (i) at least one value of t^2 is negative, while other value may be positive if $k > 2$.

42. Answer (A, C)

Hint : Form an equation whose roots are $\frac{\alpha_i}{1+\alpha_i}$, where $i = 1, 2, 3, 4$.

Sol. : $x^4 - 7x + 1 = 0$ has roots $\alpha_1, \alpha_2, \alpha_3$ and α_4.

Let $y = \frac{x}{1+x} \Rightarrow x = \frac{y}{1-y}$

$$\left(\frac{y}{1-y}\right)^4 - 7\left(\frac{y}{1-y}\right) + 1 = 0$$

$\Rightarrow\ y^4 - 7y(1-y)^3 + (1-y)^4 = 0$

$\Rightarrow\ 9y^4 - 25y^3 + 27y^2 - 11y + 1 = 0$ …(i)

The roots of equation (i) are $\dfrac{\alpha_i}{1+\alpha_i}$; $i = 1, 2, 3, 4$

$$\sum_{i=1}^{4}\frac{\alpha_i}{1+\alpha_i} = \text{Sum of roots of } (i) = \frac{25}{9}$$

$$\prod_{i=1}^{4}\frac{\alpha_i}{1+\alpha_i} = \text{Product of roots of } (i) = \frac{1}{9}$$

43. Answer (07)

Hint : $f(x) = 3 + 2(\tan^2 x + \cot^2 x)$

Sol. : $f(x) = \sin^2 x + \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \text{cosec}^2 x$

$\Rightarrow\ f(x) = 3 + 2(\tan^2 x + \cot^2 x)$

$\Rightarrow\ f(x) = 7 + 2(\tan x - \cot x)^2$

$\therefore$ Minimum value of $f(x) = 7$

44. Answer (01)

Hint : $x \in \left[2n\pi + \dfrac{\pi}{4}, 2n\pi + \dfrac{\pi}{2}\right)$

Sol. : $\log_{\frac{\sqrt{3}}{2}}(\sin x) \le \log_{\frac{\sqrt{3}}{2}}(\cos x)$

$\Rightarrow\ \sin x \ge \cos x$ also $\sin x > 0 \cap \cos x > 0$

$\Rightarrow\ x \in \left[2n\pi + \dfrac{\pi}{4}, 2n\pi + \dfrac{\pi}{2}\right)$

$\because\ x \in [0, 12]$, then $x \in \left[\dfrac{\pi}{4}, \dfrac{\pi}{2}\right) \cup \left[\dfrac{9\pi}{4}, \dfrac{5\pi}{2}\right)$

$\because\ x$ is an integer, then $x = 1$ only.

45. Answer (04)

Hint : Put $\log_2 3 = a$ to simplify X and use $(\sqrt{3}-1)^2 = 2(2-\sqrt{3})$ to simplify Y.

Sol. :

$X = (4 + \log_2 3)(5 + \log_2 3) - (3 + \log_2 3)(6 + \log_2 3)$

Put $\log_2 3 = a$

$\Rightarrow\ X = (4 + a)(5 + a) - (3 + a)(6 + a) = 2$

$$Y = \frac{1+\log_2(2-\sqrt{3})}{\log_2(\sqrt{3}-1)} = \frac{\log_2(4-2\sqrt{3})}{\log_2(\sqrt{3}-1)} = 2$$

46. Answer (24)

Hint : $(A \times B \times B) \cap (A \times A \times B) = A \times (A \cap B) \times B$

Sol. : If $(x, y, z) \in (A \times B \times B) \cap (A \times A \times B)$, then $x \in A$, $y \in A$ and $y \in B$, $z \in B$

Possible number of values of $x = 3$

Possible number of values of $y = 2$

Possible number of values of $z = 4$

$\therefore\ n((A \times B \times B) \cap (A \times A \times B)) = 24$

47. Answer (12)

Hint : $\arg\left(\dfrac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

Sol. : $\arg(z) = \arg(\sqrt{i}) - \arg(\sqrt{3} + i)$

$$= \frac{1}{2}\arg(i) - \frac{\pi}{6}$$

$$= \frac{\pi}{4} - \frac{\pi}{6}$$

$$= \frac{\pi}{12}$$

48. Answer (11)

Hint : Find the range of both trigonometric functions.

Sol. : $\because\ 2k + 1 \in \left[-\sqrt{193}, \sqrt{193}\right]$ …(i)

Also, $2k = 4\sec^2 y + \text{cosec}^2 y$

$2k = 5 + 4\tan^2 y + \dfrac{1}{\tan^2 y}$

$2k \in [9, \infty]$ …(ii)

From (i) and (ii),

$k = 5$ or 6

49. Answer (63)

Hint : $\tan C = -\tan(A + B) = -\dfrac{2\tan\left(\frac{A+B}{2}\right)}{1-\tan^2\left(\frac{A+B}{2}\right)}$

Sol. : $\because\ C = \pi - (A + B)$

$\Rightarrow\ \tan C = -\tan(A + B)$

$\Rightarrow\ \tan C = -\dfrac{2\tan\left(\frac{A+B}{2}\right)}{1-\tan^2\left(\frac{A+B}{2}\right)}$

Now, $\tan\left(\dfrac{A+B}{2}\right) = \dfrac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1-\tan\frac{A}{2}\cdot\tan\frac{B}{2}}$

$= 8$

So, $\tan C = \dfrac{16}{63}$

$\cos C = \dfrac{63}{65}$

50. Answer (15)

Hint : Use $\tan\theta\cdot\tan(60° - \theta)\cdot\tan(60° + \theta) = \tan3\theta$

Sol. : $\tan4°\cdot\tan8°\cdot\tan12°\cdot\ldots\tan88°$

$= (\tan4°\cdot\tan56°\cdot\tan64°)(\tan8°\cdot\tan52°\cdot\tan68°)\ldots(\tan28°\cdot\tan32°\cdot\tan88°)\cdot\tan60°$

$= (\tan12°\cdot\tan24°\cdot\tan36°\cdot\tan48°\cdot\tan60°\cdot\tan72°\cdot\tan84°)\sqrt{3}$

$= 3[(\tan12°\cdot\tan48°\cdot\tan72°)(\tan24°\cdot\tan36°\cdot\tan84°)]$

$= 3\tan36°\cdot\tan72°$

$= 3\cdot\dfrac{\sin36°\cdot\cos18°}{\cos36°\cdot\sin18°}$

$= 3\left[\dfrac{\sqrt{10-2\sqrt{5}}\cdot\sqrt{10+2\sqrt{5}}}{(\sqrt{5}+1)(\sqrt{5}-1)}\right]$

$= 3\left[\dfrac{\sqrt{100-20}}{4}\right]$

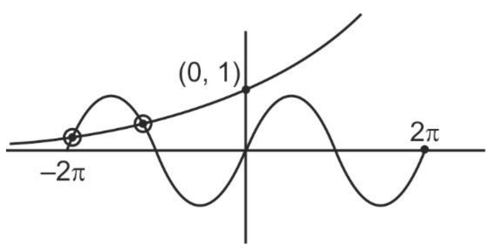
$= 3\sqrt{5}$

51. Answer (C)

Hint : $e^x = \sin x$; draw the graphs of LHS and RHS

Sol. : $\therefore\ e^x = \sin x$

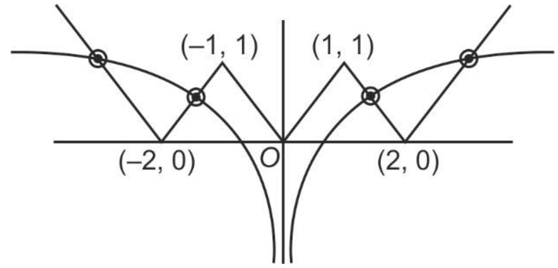
From the graph, there are two points of intersection.



52. Answer (C)

Hint : Draw graphs of LHS and RHS.

Sol. : $\log_2|x| = |||x| - 1| - 1|$



From the graph; we get 4 solutions.

53. Answer (A)

Hint : Put $x = \sin\theta \Rightarrow \sin3\theta = \dfrac{1}{\sqrt{2}}$

Sol. : Let $x = \sin\theta$

$\Rightarrow\ \sin3\theta = \dfrac{1}{\sqrt{2}} = \sin45°$

So, possible value of θ is 15°, then

$x = \sin15° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

54. Answer (D)

Hint : Put $x = \sin\theta \Rightarrow \sin2\theta = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$

Sol. : Let $x = \sin\theta$

$\Rightarrow\ \sin2\theta = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$

$\Rightarrow\ \sin2\theta = \sin36°$

So, possible value of θ is 18°, then

$x = \sin18° = \dfrac{\sqrt{5}-1}{4}$

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