All India Aakash Test Series for JEE (Advanced)-2021

TEST - 1A (Paper-2) - Code-G

Test Date : 17/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(B, C)	19.	(B, C, D)	37.	(B, D)
2.	(A, C, D)	20.	(B, C)	38.	(A, C)
3.	(B, C, D)	21.	(A, B, C, D)	39.	(A, B, D)
4.	(A, B, C)	22.	(A, D)	40.	(A, D)
5.	(A, B, C)	23.	(A, B, C)	41.	(A, B)
6.	(A, B, C, D)	24.	(B, C, D)	42.	(B, C)
7.	(12)	25.	(40)	43.	(02)
8.	(51)	26.	(28)	44.	(15)
9.	(02)	27.	(32)	45.	(20)
10.	(24)	28.	(23)	46.	(03)
11.	(15)	29.	(55)	47.	(08)
12.	(12)	30.	(06)	48.	(16)
13.	(03)	31.	(22)	49.	(20)
14.	(15)	32.	(28)	50.	(06)
15.	(B)	33.	(D)	51.	(D)
16.	(A)	34.	(B)	52.	(B)
17.	(B)	35.	(A)	53.	(C)
18.	(C)	36.	(C)	54.	(C)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (B, C)

Hint : For constant acceleration,

$$s = \frac{1}{2}at^2; \quad v = u + at$$

Sol. : $v_1 = \alpha t_1$

Also, $\alpha t_1 - \beta(t - t_1) = 0$

$$\therefore \quad t_1 = \frac{\beta t}{\alpha + \beta} \text{ and } t - t_1 = \frac{\alpha t}{(\alpha + \beta)}$$

$$\therefore \quad v_1 = \alpha t_1 = \frac{\alpha\beta \cdot t}{\alpha + \beta}$$

Also, $s = s_1 + s_2 = \frac{1}{2}\frac{\alpha \cdot \beta^2 t^2}{(\alpha+\beta)^2} + \frac{1}{2}\frac{\beta \cdot \alpha^2 t^2}{(\alpha+\beta)^2}$

$$\therefore \quad s = \frac{1}{2}\frac{\alpha\beta \cdot t^2(\alpha+\beta)}{(\alpha+\beta)^2}$$

$$\Rightarrow \quad s = \frac{1}{2}\frac{\alpha\beta \cdot t^2}{(\alpha+\beta)}$$

2. Answer (A, C, D)

Hint : If $f_r = \mu N$, from there onwards the particle will start sliding.

Sol. : For $\mu_1 = 0.8$, $\mu_2 = 0.8$, both of the body shall not have any tendency to move so there will not be any tension in the string.

And for $\mu_1 = 0.5 = \mu_2$, both of the body shall have same acceleration along the plane. So, yet again there will not be any tension in the string.

In case if $\mu_1 = 0.4$ and $\mu_2 = 0.8$, the acceleration of the system

$$a_0 = \frac{\left(150\times\frac{3}{5}\right) - \left(\frac{4}{10}\times 50\times\frac{4}{5}\right) - \left(\frac{6}{10}\times 10\times 10\times\frac{4}{5}\right)}{15}$$

$$\Rightarrow \quad a_0 = \frac{90 - 16 - 48}{15} = \frac{26}{15}$$

Now, FBD of (m_1)

$$\Rightarrow \quad 50\times\frac{3}{5} - \frac{4}{10}\times 50\times\frac{4}{5} - T = 5\times\frac{26}{15}$$

$$\Rightarrow \quad 30 - 16 - \frac{26}{3} = T$$

$$\Rightarrow \quad T = \frac{16}{3}N \quad \Rightarrow \quad \frac{42-26}{3} = T$$

3. Answer (B, C, D)

Hint : Motion will be symmetrical about the vertical line.

Sol. : Motion will be symmetrical about the vertical line. So from 16.45 m to maximum height, the particle will take 2 s.

$$\therefore \quad \text{In 2 s, } h_1 = \frac{1}{2}\times 10\times 2\times 2 = 20 \text{ m}$$

So, maximum height = 16.45 + 20 = 36.45 m

Speed of projection $\Rightarrow v^2 = 2 \times 10 \times 36.45 = 729$

$$\Rightarrow \quad v = 27 \text{ m/s and } T = \frac{2u}{g} = 5.4 \text{ s}$$

4. Answer (A, B, C)

Hint : $v = \sqrt{4a^2 + b^2}$

Sol. : Angle between acceleration vector and velocity vector is $\frac{\pi}{2}$.

Speed at any time $|\vec{v}| = \sqrt{4a^2 + b^2}$

$$\therefore \quad T = \frac{2\pi}{2} = \pi \text{ s}$$

$$\therefore \quad S = \pi\sqrt{4a^2 + b^2}$$

5. Answer (A, B, C)

Hint : If A remains stationary, then net force on A by plane is in vertically upward direction.

Sol. : If A remains stationary, then net force on A is in vertically upward direction. So, mass B will also experience the force because of A in vertically downward direction only, hence both of mass A and B will remain stationary.

For equilibrium of B, horizontal force will be zero.

$\Rightarrow \quad N \sin\theta = f_r \cos\theta$

And if B starts moving, then N force should be less than $mg \cos\theta$.

6. Answer (A, B, C, D)

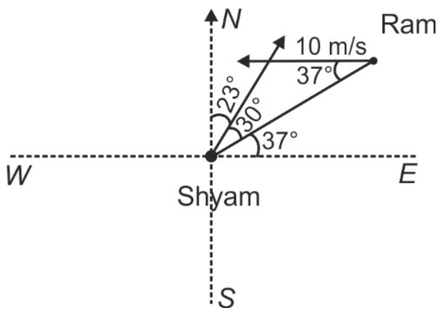
Hint : Uniform motion.

Sol. : For uniform motion in one direction values of average speed, instantaneous speed, magnitude of average velocity, magnitude of instantaneous velocity, all happens to be same.

7. Answer (12)

Hint : $v \sin 30° = 10 \sin 37°$

Sol. : Refer the figure



If $v \sin 30° = 10 \sin 37°$, then Shyam collides with Ram.

$$\therefore \ \frac{v}{2} = 10 \times \frac{3}{5} \quad \Rightarrow \quad v = 12 \text{ m/s}$$

8. Answer (51)

Hint : Displacement = Area under v-t curve

Sol. : $\therefore \ |\vec{r}\,| = (8 \times 6) + \left(\frac{1}{2} \times 2 \times 8\right) - \left(\frac{1}{2} \times 2 \times 5\right)$

$\Rightarrow \ |\vec{r}\,| = 48 + 8 - 5 = 51$

9. Answer (02)

Hint & Sol. : $a_\tau = \alpha L$

$$\mu\sqrt{\alpha^2 L^2 + g^2} = \omega^2 L$$

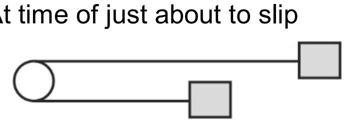
$$\omega = \alpha t$$

$\therefore \ t = 2$ s

10. Answer (24)

Hint : $m_1\omega_0^2 R_1 - \mu m_1 g - m_2 \omega_0^2 R_2 - \mu m_2 g = 0$

Sol. : At time of just about to slip

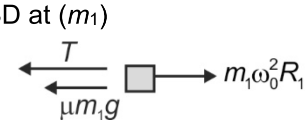


$$m_1\omega_0^2 R_1 - \mu m_1 g - m_2 \omega_0^2 R_2 - \mu m_2 g = 0$$

$$\Rightarrow \ 8\omega_0^2(1) - 2 \times \omega_0^2 \cdot \frac{1}{2} = \mu g(10)$$

$$\Rightarrow \ 7\omega_0^2 = \frac{7}{10} \times 10 \times 10 \quad \Rightarrow \quad \omega_0^2 = 10$$

Now, FBD at (m_1)



$$\Rightarrow \ 8 \times (1) \times 10 = T + \frac{7}{10} \times 8 \times 10$$

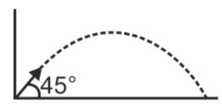
$\Rightarrow \ 80 - 56 = T$

$\Rightarrow \ T = 24$ N

11. Answer (15)

Hint : Finally, at the time of striking the ground the velocity vector will be perpendicular to initial direction.

Sol. : Its a projectile motion with initial speed $|\vec{u}\,| = 75\sqrt{2}$ m/s and angle 45°.



And acceleration is in –ve y-direction as $a = g = -10j$.

So, definitely total time of flight T will be the time instant when its velocity will be perpendicular to its 'initial velocity'.

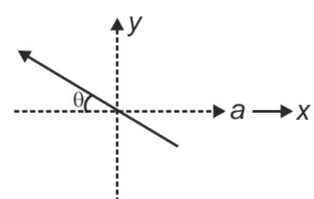
$$\therefore \ t = T = \frac{2u\sin\theta}{g} = \frac{2 \times 75\sqrt{2} \times 1}{\sqrt{2} \times 10}$$

$\therefore \ T = 15$ s

12. Answer (12)

Hint : $\vec{a}_{P,\,qr} = \vec{a}_{P,\,M} + \vec{a}_{M,\,gr}$

Sol. :



$$\vec{a}_{pgr} = \vec{a}_{P,\,M} + \vec{a}_{M,\,gr}$$

$\because$ String is fixed so load P will slide on wedge with an acceleration a_0 only.

$$\vec{a}_{pgr} = (a_0 - a_0\cos\theta)\hat{i} - a_0\sin\theta\hat{j} = a_0\frac{2}{5}\hat{i} - a_0\frac{4}{5}\hat{j}$$

$$\therefore \ |\vec{a}| = \frac{2a_0}{5}\sqrt{4+1} = \frac{2}{5} \times 6\sqrt{5} \cdot \sqrt{5} = 12$$

13. Answer (03)

Hint : $2F = mg \sin\theta + \mu mg \cos\theta$

$F = mg \sin\theta - \mu mg \cos\theta$

Sol. : $2F = mg \sin\theta + \mu mg \cos\theta$

$F = mg \sin\theta - \mu mg \cos\theta$

$\therefore$ and $\mu = \tan\phi$

Now, $2mg \sin\theta - 2\mu mg \cos\theta = mg \sin\theta + \mu mg \cos\theta$

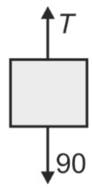
$\Rightarrow \ mg \sin\theta = 3\mu mg \cos\theta \ \Rightarrow \ \sin\theta = 3\mu \cos\theta$

$$\therefore \ \tan\theta = 3\tan\phi \ \Rightarrow \ \frac{\tan\theta}{\tan\phi} = 3$$

14. Answer (15)

Hint : Just after cutting the string, force on B is 90 N.

Sol. : Before cutting the string, force on B is



$\therefore \quad T = 90$ N

So, just after cutting the string resulting force on B is 90 N.

$$\therefore \quad a_0 = \frac{90}{6} = 15\ \text{m/s}^2$$

15. Answer (B)

Hint : Constraint motion

Sol. : Speed of $R = v_1 = 3$ m/s

Speed of $Q = v_1 + 2v_2 = 3 + 4 = 7$ m/s

Speed of $P = 3v_1 + 2v_2 = 9 + 4 = 13$ m/s

Speed of P w.r.t. $A = 3v_1 + 2v_2 - v_1 = 10$ m/s

16. Answer (A)

Hint : $\dfrac{dy}{dt} = 3$ m/s; $\quad \dfrac{dx}{dt} = y$

Sol. : $\dfrac{dy}{dt} = 3$ m/s $\quad \therefore \quad y = 3t \qquad \ldots(1)$

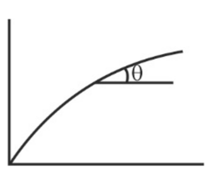
$$\frac{dx}{dt} = 3t \quad \therefore \quad x = \frac{3t^2}{2}$$

And $\dfrac{d^2x}{dt^2} = 3$

$$\therefore \quad x = \frac{3}{2}\cdot\left(\frac{y}{3}\right)^2 \quad \Rightarrow \quad x = \frac{y^2}{6}$$

$$\therefore \quad 6(1) = 2y\frac{dy}{dx} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{3}{y}$$

At $y = 3$ m, $\dfrac{dy}{dx} = \tan\theta = 1 \quad \therefore \quad \theta = 45°$



$$|\vec{a}| = \frac{d^2x}{dt^2} = 3\ \text{m/s}^2$$

$$\therefore \quad |a_n| = \frac{3}{\sqrt{2}}, \ |\vec{a}_z| = \frac{3}{\sqrt{2}}$$

$$x\ (\text{at } y = 4\text{ m}) = \frac{4\times 4}{6} = 8/3$$

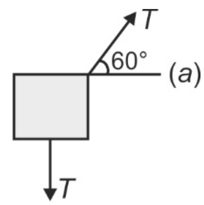
At $y = 4$ m ; $t = 4/3 \quad \therefore \quad v_x = 3 \times 4/3 = 4$ m/s

$\therefore \quad |\vec{v}| = 5$ m/s

17. Answer (B)

Hint : Constraint motion

Sol. :



For (A),

$$\Rightarrow \quad \frac{T}{2} = ma \ \ldots(1) \qquad 2mg - T = 2m\frac{a}{2} \text{ for } (B)$$

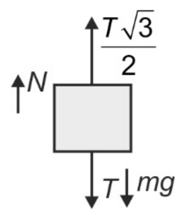
$\therefore \quad T = 2ma \quad \therefore \quad 2mg = ma + 2ma$

$$\Rightarrow \quad \frac{2mg}{3m} = a \quad \Rightarrow \quad a = \frac{2\times 10}{3} = \frac{20}{3}$$

$$\therefore \quad \text{Acceleration of } B = \frac{a}{2} = \frac{10}{3}$$

$$T = 2\times(1)\times\frac{20}{3} = \frac{40}{3}$$

And



$$\Rightarrow \quad N + \frac{T\sqrt{3}}{2} = mg + T$$

$$\Rightarrow \quad N = 10 + \frac{40}{3} - \frac{20\sqrt{3}}{3} = \frac{70 - 20\sqrt{3}}{3}$$

18. Answer (C)

Hint : If $f_r = 0$, then $mg\dfrac{3}{5} = m\omega_0^2 R\dfrac{4}{5}$

Sol. : For particular speed when no friction force is acting $mg \times \dfrac{3}{5} = m\omega_0^2 \cdot R \cdot \dfrac{4}{5}$

$$\Rightarrow \quad \frac{10\times 3}{5} = \omega_0^2 \times \frac{2\times 4}{5}$$

$$\Rightarrow \quad \omega_0^2 = \frac{30}{8} \quad \Rightarrow \quad \omega_0 \simeq 1.94\ \text{rad/s}^2$$

So, if ω is higher than ω_0, the friction force will act downward along the plane and if ω is less than ω_0, then friction force will act upward along the plane.

Now, if ω_1 is the maximum value of ω, when the mass will not skid, then

$$mg\frac{3}{5}+\mu N = m\omega^2 R\cos\theta$$

$$N = mg\frac{4}{5}+m\omega^2 R\sin\theta = \left(mg\frac{4}{5}+m\omega^2\times 2\times\frac{3}{5}\right)$$

$$\therefore\quad mg\frac{3}{5}+\frac{4}{10}mg\frac{4}{5}+\frac{4}{10}m\omega^2\times\frac{6}{5}=m\omega^2\times 2\times\frac{4}{5}$$

$$\Rightarrow\ 3g+\frac{16g}{10}=\omega^2(8-2.4)\ \Rightarrow\ \omega^2=\frac{(30+16)}{8-2.4}$$

ω_{max} = 2.86 rad/s

$$\therefore\ \text{Similarly}\ \omega_{min}=\sqrt{\frac{30-16}{8+2.4}}=1.16\ \text{rad/s}$$

So, finally if ω is less than 1.16 rad/s the mass will skid downward and if ω is greater than 2.86, then mass will skid upward along the plane. And in between the range of ω in $1.16 < \omega < 2.86$, the mass will remain stationary with respect to wedge.

PART - II (CHEMISTRY)

19. Answer (B, C, D)

Hint : B.O. of CO = 3

B.O. of CO^+ = 3.5

Sol. :

	BO
N_2	3
N_2^+	2.5

. Bond length $\propto \frac{1}{\text{Bond order}}$

NO^+	NO^-
BO → 3	BO → 2
O_2 BO = 2	O_3 BO → 1.5

20. Answer (B, C)

Hint : Mass of air displaced = V × d

Sol. : Weight of payload = 80 × 10^3

Let number of balloons required be x.

$$80\times10^3+0+100x=\frac{nRT}{P}x\times1.25$$

x = 26.8

$\therefore$ Number of balloon = 27 and 30

21. Answer (A, B, C, D)

Hint : Number of moles = Molarity × volume (L)

Sol. : Let the volumes of 0.1 M $Fe_2(SO_4)_3$ and 0.1 M $Al_2(SO_4)_3$ solutions to be mixed be V_1 and V_2 respectively.

Number of moles of cations = $0.2V_1 + 0.2V_2$

= $0.2(V_1 + V_2)$

Number of moles of anions = $0.3V_1 + 0.3V_2$

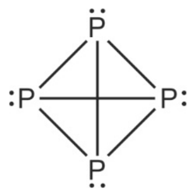
= $0.3(V_1 + V_2)$

$$\frac{\text{Moles of cations}}{\text{Moles of anions}}=\frac{0.2(V_1+V_2)}{0.3(V_1+V_2)}=\frac{2}{3}$$

So, molar ratio of cations to anions will be always 2 : 3 irrespective of the values of V_1 and V_2.

22. Answer (A, D)

Hint : Each P is sp^3 hybridised.

Sol. : 

23. Answer (A, B, C)

Hint : Time taken to complete 1 revolution = $\frac{2\pi r}{v}$

Sol. : Time taken to complete one revolution in an orbit, $T\propto\frac{n^3}{Z^2}$

$$T_2(H)\propto\frac{(2)^3}{(1)^2}\propto 8;\quad T_4(He^+)\propto\frac{(4)^3}{(2)^2}\propto 16$$

$$\therefore\ \frac{T_2(H)}{T_4(He^+)}=\frac{8}{16}=\frac{1}{2}$$

$$\frac{n_2(H)}{n_4(H)}=\frac{2}{4}=\frac{1}{2}$$

Radius of n^{th} orbit, $r_n=\left(\frac{n^2}{Z}\right)(r_1)_H$

$$r_2(H)=\frac{(2)^2}{1}r_1(H);\quad r_4(He^+)=\frac{(4)^2}{2}r_1(H)=8r_1(H)$$

$$\frac{r_2(H)}{r_4(He^+)}=\frac{4}{8}=\frac{1}{2}$$

Energy of electron in n^{th} orbit, $E_n=-13.6\frac{Z^2}{n^2}$ eV

$$E_2(H)=-13.6\frac{(1)^2}{(2)^2}=-\frac{13.6}{4}\ \text{eV}$$

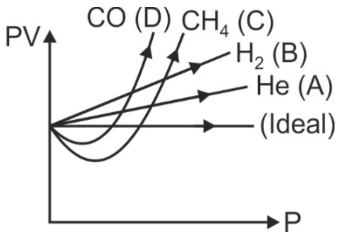
$$E_4(He^+)=-13.6\frac{(2)^2}{(4)^2}=-\frac{13.6}{4}\ \text{eV}$$

$$\therefore\ \frac{E_2(H)}{E_4(He^+)}=1$$

24. Answer (B, C, D)

Hint : A – He, B – H_2, C – CH_4 and D – CO

Sol. :



25. Answer (40)

Hint & Sol. :

N_2 ⇒ u = 10

O_2 ⇒ v = 10

CN^- ⇒ w = 10

NO^+ ⇒ x = 10

26. Answer (28)

Hint : Increase in weight is due to oxidation of FeO, present in the mixture, to Fe_2O_3.

Sol. : Let the mass of FeO in 100 gm mixture of FeO and Fe_2O_3 be x gm. On heating the entire amount of FeO is converted into Fe_2O_3.

$$4FeO + O_2 \longrightarrow 2Fe_2O_3$$

Percentage increase in weight = $\frac{32 \times x}{4 \times 72} = 8$

x = 72 gm

∴ Percentage of Fe_2O_3 in the given mixture = 28%

27. Answer (32)

Hint : Eka silicon is Germanium.

Sol. : Atomic number of eka-silicon (Germanium) = 32

28. Answer (23)

Hint : Bohr's model

Sol. : For H-atom, transition $n_2 \to n_1$, λ = 92 nm

$$\frac{1}{92} = R_H(1)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad \text{...(i)}$$

For He^+ ion,

$$\frac{1}{\lambda} = R_H(2)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad \text{...(ii)}$$

Solving (i) and (ii), we get

$$\lambda = \frac{92}{4} = 23 \text{ nm}$$

29. Answer (55)

Hint : Moles of $CaCl_2$ = Moles of $CaCO_3$ = Moles of CaO

Sol. : Mass of mixture of $CaCl_2$ and NaCl = 3.70 gm

$$CaCl_2 + Na_2CO_3 \longrightarrow CaCO_3 + 2NaCl$$

$$CaCO_3 \xrightarrow{\Delta} \underset{0.84\text{ gm}}{CaO} + CO_2$$

Moles of $CaCl_2$ = Moles of $CaCO_3$ = Moles of CaO

$$= \frac{0.84}{56} = 0.015$$

Mass of $CaCl_2$ = 0.015 × 111 gm

% of $CaCl_2$ = $\frac{0.015 \times 111 \times 100}{3.70} = 45\%$

∴ % of NaCl in the given mix = 55%

30. Answer (06)

Hint : For Be, Mg and He, EGE is positive.

Sol. : EGE is negative for Li, C, F, Cl, Na, K.

31. Answer (22)

Hint : KE of iodine atoms = $\frac{1}{2}[IE - BE]$

Sol. : Incident energy (IE) given to I_2 molecule

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4500 \times 10^{-10}} = 4.4 \times 10^{-19} \text{ J}$$

Minimum energy required to dissociate I_2 molecule

$$= \frac{240 \times 1000}{6 \times 10^{23}} = 4.0 \times 10^{-19} \text{ J}$$

KE of 2 iodine atoms = $(4.4 – 4.0)10^{-19}$

= 4×10^{-20} J

KE / Iodine atom = $2.0 \times 10^{-20} = x \times 10^y$

x + |y| = 2 + |–20| = 22

32. Answer (28)

Hint : Excitation energy = $13.6\left[\frac{1}{(1)^2} - \frac{1}{n^2}\right]$

Sol. : Energy absorbed by H-atoms = 13.3875 eV

Let the electron of H-atom be excited from ground state to n^{th} orbit.

$$\therefore \quad 13.3875 = 13.6\left[\frac{1}{(1)^2} - \frac{1}{(n)^2}\right]$$

On solving, n = 8

Total number of lines in the spectrum of hydrogen

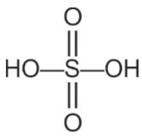
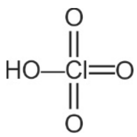
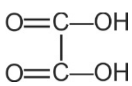
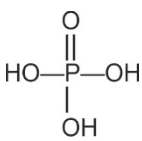
$$= \sum(8-1) = \sum(7) = 28$$

33. Answer (D)

Hint : Number of hybridised orbitals, H is given by

$H = \frac{1}{2}[V + M - C + A]$.

Sol. :

Acid	**Basicity**	**No. of π-bonds**	**% of O**	**Hybridisation of central atom**
	2	2	≃ 64%	sp^3
	1	3	≃ 64%	sp^3
	2	2	≃ 71%	sp^2
	3	1	≃ 64%	sp^3

34. Answer (B)

Hint : Molality

$$= \frac{\text{Molarity} \times 1000}{[1000d - \text{Molarity} \times \text{Molar mass of solute}]}$$

Sol. :

NaOH solution :

Molarity = $\frac{6 \times 1000}{40 \times 250} = 0.6$ M

Molality = 0.6 m $\left\{\because d_{H_2O} = \frac{1 \text{ gm}}{\text{cc}}\right\}$

Mole fraction of NaOH = $\frac{0.15}{0.15 + \frac{250}{18}}$ = 0.01

Glucose solution :

Molarity = $\frac{54 \times 1000}{180 \times 500} = 0.6$ M

Molality = 0.6 m

Mole fraction of glucose = $\frac{0.30}{0.30 + \frac{500}{18}} = 0.01$

	NaOH +	HCl ⟶	NaCl +	H_2O
m mole	70	10		
	60	0	10	10

Molarity of excess NaOH $= \frac{60}{100} = 0.6$ M

Benzene – Toluene mixture

Number of moles of benzene = $\frac{31.2}{78} = 0.4$

Number of moles of toluene = $\frac{55.2}{92} = 0.6$

Mole fraction of benzene = 0.4

Molarity of benzene in solution

$$= \frac{0.4 \times 1000}{\frac{55.2}{0.87}} = 6.3 \text{ M}$$

35. Answer (A)

Hint : d_{z^2} has no planar nodes but has two angular nodes.

Sol. : Number of radial/spherical nodes in an orbital = n – l – 1

Number of planar/angular nodes in an orbital = l

Orbital d_{z^2} has two angular nodes but no planar nodes.

36. Answer (C)

Hint : Emission spectrum of H-atom and H-like species has lines in the UV region, IR and far IR region.

Sol. :

Z = 39, n_1 = 5 and n_2 = 13

$$\frac{1}{\lambda} = R_H(39)^2\left[\frac{1}{(5)^2} - \frac{1}{(13)^2}\right]; \ \lambda = \frac{25}{1296\,R_H}$$

Z = 48, n_1 = 4 and n_2 = 5

$$\frac{1}{\lambda} = R_H(48)^2\left[\frac{1}{(4)^2} - \frac{1}{(5)^2}\right]; \ \lambda = \frac{25}{1296\,R_H}$$

Z = 2, n_1 = 2 and n_2 = 3

$$\frac{1}{\lambda} = R_H(2)^2\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right]; \ \lambda = \frac{9}{5\,R_H}$$

Z = 1, n_1 = 1 and n_2 = 3

$$\frac{1}{\lambda} = R_H(1)^2\left[\frac{1}{(1)^2} - \frac{1}{(3)^2}\right]; \ \lambda = \frac{9}{8\,R_H}$$

All transitions from higher energy orbits to 4th orbit lie in 'far IR region' for H-atom.

All transitions from higher energy orbit to 2nd orbit lie in UV region except H-atom which lie in visible region.

PART - III (MATHEMATICS)

37. Answer (B, D)

Hint : Put $z = x + iy$ and solve for x and y.

Sol. : Let $z = x + iy$

$x^2 + y^2 - 5(x + iy) = 10i$

$\Rightarrow x^2 + y^2 - 5x = 0$ and $-5y = 10$

$\Rightarrow y = -2$ and $x = 1$ or 4

$\Rightarrow z = 1 - 2i$ or $4 - 2i$

38. Answer (A, C)

Hint : $f(x)$ and $g(x)$ have a common factor.

Sol. : $f(x)$ and $g(x)$ both are factors of a cubic polynomial then $f(x) = 0$ and $g(x) = 0$ have a common root.

$x^2 + (k^2 - 29)x - k = 0$ ...(i)

$2x^2 + (2k - 43)x + k = 0$ ...(ii)

Applying condition for common roots,

$(2k - 58 - 2k + 43)(-2k^2 + 43k - k^2 + 29k)$

$= (2k + k)^2$

$\Rightarrow 4k^2 - 120k = 0 \Rightarrow k = 0$ or 30

39. Answer (A, B, D)

Hint : $-\sqrt{3}x = 2y = z$

Sol. : $(-\sqrt{3}x)^2 + (2y)^2 + (z)^2 - (-\sqrt{3}x)(2y)$

$- (2y)(z) - (z)(-\sqrt{3}x) = 0$

$\Rightarrow -\sqrt{3}x = 2y = z = k$ (where $k \in R - \{0\}$)

$\Rightarrow x = \frac{-k}{\sqrt{3}}, y = \frac{k}{2}, z = k$

So, $xy = -\frac{k^2}{2\sqrt{3}} < 0$, $yz = \frac{k^2}{2} > 0$

$x(x + y + z) = \frac{k^2}{3} - \frac{k^2}{2\sqrt{3}} - \frac{k^2}{\sqrt{3}} = k^2\left(\frac{1}{3} - \frac{\sqrt{3}}{2}\right) < 0$

and $xy + zx = -\frac{k^2}{2\sqrt{3}} - \frac{k^2}{\sqrt{3}} < 0$

40. Answer (A, D)

Hint : $[x]^2 - [x] - 20 > 0 \Rightarrow [x] \in (-\infty, -4) \cup (5, \infty)$

Sol. : $f(x)$ exists if

$[x]^2 - [x] - 20 > 0 \Rightarrow [x] \in (-\infty, -4) \cup (5, \infty)$

$\Rightarrow x \in (-\infty, -4) \cup [6, \infty)$

41. Answer (A, B)

Hint : Put $z = ki$, where $k \in R$

Sol. : If the equation has a purely imaginary root, then $z = ki$; $(k \in R)$ will satisfy the equation.

$-k^2(1 - i) + k(-2i + 1) - 3i = m$

$\Rightarrow -k^2 + k = m$ and $k^2 - 2k - 3 = 0$

$k = -1, 3$

So, $m = -2, -6$

42. Answer (B, C)

Hint : $\sqrt{3}\sin x + \cos x = 2(\cos x + \cos 5x)$

$\Rightarrow \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \cos 5x$

Sol. : $\sqrt{3}\sin x + \cos x = 2(\cos 5x + \cos x)$

$\Rightarrow \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \cos 5x$

$\Rightarrow \cos 5x = \cos\left(\frac{2\pi}{3} - x\right)$

$\Rightarrow 5x = 2n\pi \pm \left(\frac{2\pi}{3} - x\right)$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{9}$ or $x = \frac{n\pi}{2} - \frac{\pi}{6}$

43. Answer (02)

Hint : $\cot\theta - \tan\theta = 2\cot 2\theta$

Sol. : $\tan 55° - \tan 35° = \frac{\sin(55° - 35°)}{\cos 55° \cdot \cos 35°}$

$= \frac{2\sin 20°}{2\sin 35° \cdot \cos 35°}$

$= \frac{2\sin 20°}{\sin 70°} = 2\tan 20°$

And $2\cot 40° - \cot 20° + 2\tan 20°$

$= (\cot 20° - \tan 20°) - \cot 20° + 2\tan 20°$

$= \tan 20°$

So, $\frac{\tan 55° - \tan 35°}{2\cot 40° - \cot 20° + 2\tan 20°} = 2$

44. Answer (15)

Hint : $(x - 3)^2 + 2\cos^2\left(\frac{\pi x}{n}\right) = 0$

Sol. : $(x^2 - 6x + 9) + \left(1 + \cos\left(\frac{2\pi x}{n}\right)\right) = 0$

$\Rightarrow (x - 3)^2 + 2\cos^2\left(\frac{\pi x}{n}\right) = 0$

$\Rightarrow$ $x = 3$ and $\cos\left(\frac{\pi x}{n}\right) = 0$

$\Rightarrow$ $\alpha = 3$ $\Rightarrow \cos\left(\frac{3\pi}{n}\right) = 0$

$\Rightarrow n = 2$ or 6

Maximum value of $\alpha^2 + n = 15$

45. Answer (20)

Hint : $\cos^2\frac{r\pi}{7} = \frac{1}{2}\left(1+\cos\frac{2r\pi}{7}\right)$

Sol. : $\cos^2\frac{\pi}{7}+\cos^2\frac{2\pi}{7}+\cos^2\frac{3\pi}{7}$

$= \frac{1}{2}\left[\left(1+\cos\frac{2\pi}{7}\right)+\left(1+\cos\frac{4\pi}{7}\right)+\left(1+\cos\frac{6\pi}{7}\right)\right]$

$= \frac{1}{2}\left[3+\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)\right]$

$= \frac{1}{2}\left[3+\frac{\sin\frac{3\pi}{7}}{\sin\frac{\pi}{7}}\cdot\cos\frac{4\pi}{7}\right]$

$= \frac{1}{2}\left[3-\frac{2\sin\frac{3\pi}{7}\cdot\cos\frac{3\pi}{7}}{2\sin\frac{\pi}{7}}\right]$

$= \frac{1}{2}\left[3-\frac{1}{2}\right] = \frac{5}{4}$

46. Answer (03)

Hint : $f(x) = 1 - 2\left(\frac{x}{1+x^2}\right)$

Sol. : $f(x) = \frac{1+x^2-2x}{1+x^2} = 1 - 2\left(\frac{x}{1+x^2}\right)$

$\because$ $x + \frac{1}{x} \in (-\infty, -2] \cup [2, \infty)$

$\Rightarrow$ $\frac{x^2+1}{x} \in (-\infty, -2] \cup [2, \infty)$

$\Rightarrow$ $\frac{x}{x^2+1} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

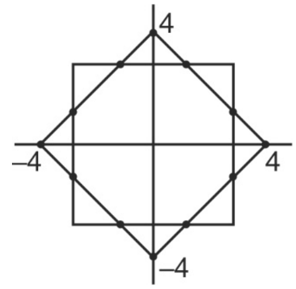
$f(x) \in [0, 2]$

Number of integers in the range of $f(x)$ is 3.

47. Answer (08)

Hint : Draw graph

Sol. : If we draw the two graph $|x| + |y| = 4$ and $|x + y| + |x - y| = 5$



There are 8 points of intersection.

48. Answer (16)

Hint : $x^3 - 2x^2 - 11x + 12 < 0 \cap x^2 - 2x - 48 < 0$

Sol. : $\because$ $x^3 - 2x^2 - 11x + 12 < 0$ and

$x^2 - 2x - 48 < 0$

$(x - 1)(x + 3)(x - 4) < 0$ and $(x - 8)(x + 6) < 0$

$x \in (-\infty, -3) \cup (1, 4)$ and $x \in (-6, 8)$

So, $x \in (-6, -3) \cup (1, 4)$

Integral value of x will be –5, –4, 2, 3.

49. Answer (20)

Hint : $(a - b)(a^2 + b^2 + ab) = a^3 - b^3$

Sol. :

$X =$ $X = \log_3(3-\sqrt[3]{2})+\log_3(3^2+(\sqrt[3]{2})^2+3\cdot\sqrt[3]{2})$

$X = \log_3(3^3 - 2) = \log_3 25$

Now, $5X\cdot\log_5 9 = 5(\log_3 25)(\log_5 9)$

$= 5(2\log_3 5)(2\log_5 3)$

$= 20$

50. Answer (06)

Hint : Product of roots is negative.

Sol. :

Case 1 : One positive and one negative root $a\cdot c < 0$

$\Rightarrow$ $m(m^2 - 6m - 7) < 0$

$\Rightarrow$ $m \in (-\infty, -1) \cup (0, 7)$

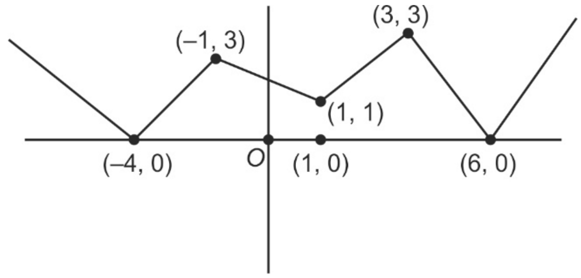
Case 2 : Two equal positive roots

$D = 0 \cap m < 0 \Rightarrow$ no integral value of m exist.

51. Answer (D)

Hint : Draw graph of $y = f(x)$

Sol. : $y = f(x)$



52. Answer (B)

Hint : $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

$\cos 2A + \cos 2B + \cos 2C$

$= -1 - 4\cos A\cos B\cos C$

$\tan A + \tan B + \tan C = \tan A\cdot\tan B\cdot\tan C$

Sol. :

(P) $\because$ $\cos 2A + \cos 2B + \cos 2C$

$= -1 - 4\cos A\cos B\cos C$

$= -\dfrac{\sqrt{3}+1}{2}$

(Q) $\sin^2 A + \sin^2 B + \sin^2 C$

$= \dfrac{3-(\cos 2A+\cos 2B+\cos 2C)}{2}$

$= \dfrac{7+\sqrt{3}}{4}$

(R) $\tan A + \tan B + \tan C = \tan A\cdot\tan B\cdot\tan C$

$= \dfrac{3+\sqrt{3}}{\sqrt{3}-1}$

(S) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

$= \dfrac{3+\sqrt{3}}{2}$

53. Answer (C)

Hint : $f(x) = -2(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}ax+1)$

Sol. : $f(x) = (a-1)(x^2+\sqrt{3}x+1)^2 - (a+1)[(x^2+1)^2-3x^2]$

$= (a-1)(x^2+\sqrt{3}x+1)^2 - (a+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)$

$= -2(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}ax+1)$

$\therefore$ $x^2+\sqrt{3}x+1>0$

(P) If $f(x) < 0 \Rightarrow x^2-\sqrt{3}ax+1>0$ for all $x\in R$

$\Rightarrow 3a^2 - 4 < 0 \Rightarrow a \in \left(-\dfrac{2}{\sqrt{3}}, \dfrac{2}{\sqrt{3}}\right)$

(Q) $f(x) > 0 \Rightarrow x^2 - \sqrt{3}ax+1<0 \Rightarrow a\in\phi$

(R) If $f(x) = 0$ has real roots, then

$x^2-\sqrt{3}ax+1 = 0$ has real roots

$3a^2 - 4 > 0$

$a\in\left(-\infty, -\dfrac{2}{\sqrt{3}}\right)\cup\left(\dfrac{2}{\sqrt{3}}, \infty\right)$

(S) If $f(x) = 0$ has all roots imaginary, then

$x^2-\sqrt{3}ax+1 = 0$ has imaginary roots

($\because$ $x^2+\sqrt{3}x+1 = 0$ already has imaginary roots)

$a\in\left(-\dfrac{2}{\sqrt{3}}, \dfrac{2}{\sqrt{3}}\right)$

Sum of all roots = $S = \sqrt{3}a-\sqrt{3}$

$S\in\left(-2-\sqrt{3}, 2-\sqrt{3}\right)$

54. Answer (C)

Hint : $\arg(z_1\cdot z_2) = \arg(z_1) + \arg(z_2)$

and $\arg\left(\dfrac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

Sol. :

(P) $z+\bar{z}+z\bar{z}$ is purely real,

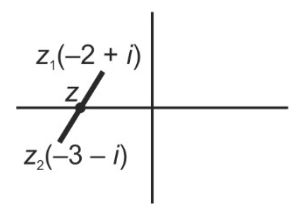
$\arg\left(z+\bar{z}+z\bar{z}\right)$ is 0 or π

(Q) $\arg\left(\dfrac{(1+\sqrt{3}i)(\sqrt{3}-i)(1+i)}{(\sqrt{3}+i)}\right)$

$= \arg(1+\sqrt{3}i)+\arg(\sqrt{3}-i)+\arg(1+i) - \arg(\sqrt{3}+i)$

$= \dfrac{\pi}{3}+\left(-\dfrac{\pi}{6}\right)+\dfrac{\pi}{4}-\dfrac{\pi}{6}$

$= \dfrac{\pi}{4}$



(R) $\arg(z)$ may be π

(S) $z_1 = iz_2$

$\Rightarrow$ $\arg(z_1) = \dfrac{\pi}{2}+\arg(z_2)$ ...(i)

also, $\arg(z_1z_2) = \dfrac{2\pi}{3}$

$\arg(z_1) + \arg(z_2) = \dfrac{2\pi}{3}$ ...(ii)

(i) + (ii),

$\arg(z_1) = \dfrac{7\pi}{12}$

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All India Aakash Test Series for JEE (Advanced)-2021

TEST - 1A (Paper-2) - Code-H

Test Date : 17/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(A, B, C, D)	19.	(B, C, D)	37.	(B, C)
2.	(A, B, C)	20.	(A, B, C)	38.	(A, B)
3.	(A, B, C)	21.	(A, D)	39.	(A, D)
4.	(B, C, D)	22.	(A, B, C, D)	40.	(A, B, D)
5.	(A, C, D)	23.	(B, C)	41.	(A, C)
6.	(B, C)	24.	(B, C, D)	42.	(B, D)
7.	(15)	25.	(28)	43.	(06)
8.	(03)	26.	(22)	44.	(20)
9.	(12)	27.	(06)	45.	(16)
10.	(15)	28.	(55)	46.	(08)
11.	(24)	29.	(23)	47.	(03)
12.	(02)	30.	(32)	48.	(20)
13.	(51)	31.	(28)	49.	(15)
14.	(12)	32.	(40)	50.	(02)
15.	(C)	33.	(C)	51.	(C)
16.	(B)	34.	(A)	52.	(C)
17.	(A)	35.	(B)	53.	(B)
18.	(B)	36.	(D)	54.	(D)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (A, B, C, D)

Hint : Uniform motion.

Sol. : For uniform motion in one direction values of average speed, instantaneous speed, magnitude of average velocity, magnitude of instantaneous velocity, all happens to be same.

2. Answer (A, B, C)

Hint : If A remains stationary, then net force on A by plane is in vertically upward direction.

Sol. : If A remains stationary, then net force on A is in vertically upward direction. So, mass B will also experience the force because of A in vertically downward direction only, hence both of mass A and B will remain stationary.

For equilibrium of B, horizontal force will be zero.

$\Rightarrow$ $N\sin\theta = f_r\cos\theta$

And if B starts moving, then N force should be less than $mg\cos\theta$.

3. Answer (A, B, C)

Hint : $v = \sqrt{4a^2 + b^2}$

Sol. : Angle between acceleration vector and velocity vector is $\frac{\pi}{2}$.

Speed at any time $|\vec{v}| = \sqrt{4a^2 + b^2}$

$\therefore$ $T = \frac{2\pi}{2} = \pi$ s

$\therefore$ $S = \pi\sqrt{4a^2 + b^2}$

4. Answer (B, C, D)

Hint : Motion will be symmetrical about the vertical line.

Sol. : Motion will be symmetrical about the vertical line. So from 16.45 m to maximum height, the particle will take 2 s.

$\therefore$ In 2 s, $h_1 = \frac{1}{2}\times 10\times 2\times 2 = 20$ m

So, maximum height = 16.45 + 20 = 36.45 m

Speed of projection $\Rightarrow$ $v^2 = 2 \times 10 \times 36.45 = 729$

$\Rightarrow$ $v = 27$ m/s and $T = \frac{2u}{g} = 5.4$ s

5. Answer (A, C, D)

Hint : If $f_r = \mu N$, from there onwards the particle will start sliding.

Sol. : For $\mu_1 = 0.8$, $\mu_2 = 0.8$, both of the body shall not have any tendency to move so there will not be any tension in the string.

And for $\mu_1 = 0.5 = \mu_2$, both of the body shall have same acceleration along the plane. So, yet again there will not be any tension in the string.

In case if $\mu_1 = 0.4$ and $\mu_2 = 0.8$, the acceleration of the system

$$a_0 = \frac{\left(150\times\frac{3}{5}\right) - \left(\frac{4}{10}\times 50\times\frac{4}{5}\right) - \left(\frac{6}{10}\times 10\times 10\times\frac{4}{5}\right)}{15}$$

$$\Rightarrow\ a_0 = \frac{90-16-48}{15} = \frac{26}{15}$$

Now, FBD of (m_1)

$$\Rightarrow\ 50\times\frac{3}{5} - \frac{4}{10}\times 50\times\frac{4}{5} - T = 5\times\frac{26}{15}$$

$$\Rightarrow\ 30 - 16 - \frac{26}{3} = T$$

$$\Rightarrow\ T = \frac{16}{3}N \ \Rightarrow\ \frac{42-26}{3} = T$$

6. Answer (B, C)

Hint : For constant acceleration,

$$s = \frac{1}{2}at^2;\quad v = u + at$$

Sol. : $v_1 = \alpha t_1$

Also, $\alpha t_1 - \beta(t - t_1) = 0$

$$\therefore\ t_1 = \frac{\beta t}{\alpha+\beta} \text{ and } t - t_1 = \frac{\alpha t}{(\alpha+\beta)}$$

$$\therefore\ v_1 = \alpha t_1 = \frac{\alpha\beta\cdot t}{\alpha+\beta}$$

Also, $s = s_1 + s_2 = \frac{1}{2}\frac{\alpha\cdot\beta^2 t^2}{(\alpha+\beta)^2} + \frac{1}{2}\frac{\beta\cdot\alpha^2 t^2}{(\alpha+\beta)^2}$

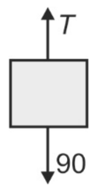
$$\therefore\ s = \frac{1}{2}\frac{\alpha\beta\cdot t^2(\alpha+\beta)}{(\alpha+\beta)^2}$$

$$\Rightarrow\ s = \frac{1}{2}\frac{\alpha\beta\cdot t^2}{(\alpha+\beta)}$$

7. Answer (15)

Hint : Just after cutting the string, force on *B* is 90 N.

Sol. : Before cutting the string, force on *B* is



$\therefore \quad T = 90$ N

So, just after cutting the string resulting force on *B* is 90 N.

$$\therefore \quad a_0 = \frac{90}{6} = 15 \text{ m/s}^2$$

8. Answer (03)

Hint : $2F = mg\sin\theta + \mu mg\cos\theta$

$F = mg\sin\theta - \mu mg\cos\theta$

Sol. : $2F = mg\sin\theta + \mu mg\cos\theta$

$F = mg\sin\theta - \mu mg\cos\theta$

$\therefore$ and $\mu = \tan\phi$

Now, $2mg\sin\theta - 2\mu mg\cos\theta = mg\sin\theta + \mu mg\cos\theta$

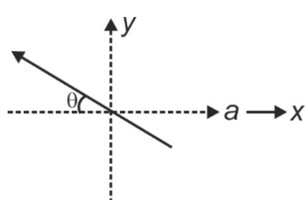
$\Rightarrow \ mg\sin\theta = 3\mu mg\cos\theta \ \Rightarrow \ \sin\theta = 3\mu\cos\theta$

$$\therefore \quad \tan\theta = 3\tan\phi \Rightarrow \frac{\tan\theta}{\tan\phi} = 3$$

9. Answer (12)

Hint : $\vec{a}_{P,\,gr} = \vec{a}_{P,\,M} + \vec{a}_{M,\,gr}$

Sol. :



$\vec{a}_{pgr} = \vec{a}_{P,\,M} + \vec{a}_{M,\,gr}$

$\because$ String is fixed so load *P* will slide on wedge with an acceleration a_0 only.

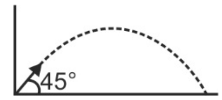
$$\vec{a}_{pgr} = (a_0 - a_0\cos\theta)\hat{i} - a_0\sin\theta\hat{j} = a_0\frac{2}{5}\hat{i} - a_0\frac{4}{5}\hat{j}$$

$$\therefore \quad |\vec{a}| = \frac{2a_0}{5}\sqrt{4+1} = \frac{2}{5}\times 6\sqrt{5}\cdot\sqrt{5} = 12$$

10. Answer (15)

Hint : Finally, at the time of striking the ground the velocity vector will be perpendicular to initial direction.

Sol. : Its a projectile motion with initial speed $|\vec{u}| = 75\sqrt{2}$ m/s and angle 45°.



And acceleration is in –ve *y*-direction as $a = g = -10j$.

So, definitely total time of flight *T* will be the time instant when its velocity will be perpendicular to its 'initial velocity'.

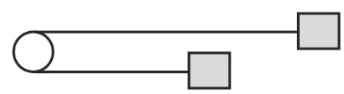
$$\therefore \quad t = T = \frac{2u\sin\theta}{g} = \frac{2\times 75\sqrt{2}\times 1}{\sqrt{2}\times 10}$$

$\therefore \quad T = 15$ s

11. Answer (24)

Hint : $m_1\omega_0^2R_1 - \mu m_1g - m_2\omega_0^2R_2 - \mu m_2g = 0$

Sol. : At time of just about to slip

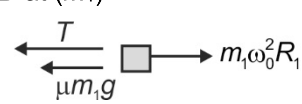


$$m_1\omega_0^2R_1 - \mu m_1g - m_2\omega_0^2R_2 - \mu m_2g = 0$$

$$\Rightarrow \quad 8\omega_0^2(1) - 2\times\omega_0^2\cdot\frac{1}{2} = \mu g(10)$$

$$\Rightarrow \quad 7\omega_0^2 = \frac{7}{10}\times 10\times 10 \quad \Rightarrow \quad \omega_0^2 = 10$$

Now, FBD at (m_1)



$$\Rightarrow \quad 8\times(1)\times 10 = T + \frac{7}{10}\times 8\times 10$$

$\Rightarrow \quad 80 - 56 = T$

$\Rightarrow \quad T = 24$ N

12. Answer (02)

Hint & Sol. : $a_\tau = \alpha L$

$$\mu\sqrt{\alpha^2L^2 + g^2} = \omega^2L$$

$\omega = \alpha t$

$\therefore \quad t = 2$ s

13. Answer (51)

Hint : Displacement = Area under *v*-*t* curve

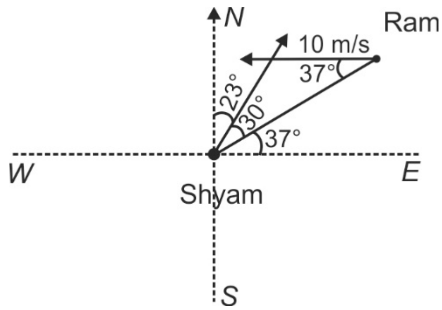
Sol. : $\therefore \quad |\vec{r}| = (8\times 6) + \left(\frac{1}{2}\times 2\times 8\right) - \left(\frac{1}{2}\times 2\times 5\right)$

$\Rightarrow \quad |\vec{r}| = 48 + 8 - 5 = 51$

14. Answer (12)

Hint : $v \sin 30° = 10 \sin 37°$

Sol. : Refer the figure



If $v \sin 30° = 10 \sin 37°$, then Shyam collides with Ram.

$$\therefore \frac{v}{2} = 10 \times \frac{3}{5} \Rightarrow v = 12 \text{ m/s}$$

15. Answer (C)

Hint : If $f_r = 0$, then $mg\frac{3}{5} = m\omega_0^2 R\frac{4}{5}$

Sol. : For particular speed when no friction force is acting $mg \times \frac{3}{5} = m\omega_0^2 \cdot R \cdot \frac{4}{5}$

$$\Rightarrow \frac{10 \times 3}{5} = \omega_0^2 \times \frac{2 \times 4}{5}$$

$$\Rightarrow \omega_0^2 = \frac{30}{8} \Rightarrow \omega_0 \simeq 1.94 \text{ rad/s}^2$$

So, if ω is higher than ω_0, the friction force will act downward along the plane and if ω is less than ω_0, then friction force will act upward along the plane.

Now, if ω_1 is the maximum value of ω, when the mass will not skid, then

$$mg\frac{3}{5} + \mu N = m\omega^2 R \cos\theta$$

$$N = mg\frac{4}{5} + m\omega^2 R \sin\theta = \left(mg\frac{4}{5} + m\omega^2 \times 2 \times \frac{3}{5}\right)$$

$$\therefore mg\frac{3}{5} + \frac{4}{10}mg\frac{4}{5} + \frac{4}{10}m\omega^2 \times \frac{6}{5} = m\omega^2 \times 2 \times \frac{4}{5}$$

$$\Rightarrow 3g + \frac{16g}{10} = \omega^2(8 - 2.4) \Rightarrow \omega^2 = \frac{(30+16)}{8-2.4}$$

ω_{max} = 2.86 rad/s

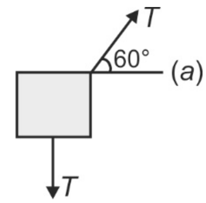
$$\therefore \text{Similarly } \omega_{\min} = \sqrt{\frac{30-16}{8+2.4}} = 1.16 \text{ rad/s}$$

So, finally if ω is less than 1.16 rad/s the mass will skid downward and if ω is greater than 2.86, then mass will skid upward along the plane. And in between the range of ω in $1.16 < \omega < 2.86$, the mass will remain stationary with respect to wedge.

16. Answer (B)

Hint : Constraint motion

Sol. :



For (*A*),

$$\Rightarrow \frac{T}{2} = ma \text{ ...(1)} \quad 2mg - T = 2m\frac{a}{2} \text{ for } (B)$$

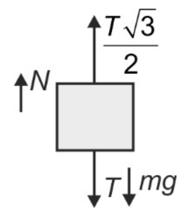
$$\therefore T = 2ma \quad \therefore 2mg = ma + 2ma$$

$$\Rightarrow \frac{2mg}{3m} = a \Rightarrow a = \frac{2 \times 10}{3} = \frac{20}{3}$$

$$\therefore \text{Acceleration of } B = \frac{a}{2} = \frac{10}{3}$$

$$T = 2 \times (1) \times \frac{20}{3} = \frac{40}{3}$$

And



$$\Rightarrow N + \frac{T\sqrt{3}}{2} = mg + T$$

$$\Rightarrow N = 10 + \frac{40}{3} - \frac{20\sqrt{3}}{3} = \frac{70 - 20\sqrt{3}}{3}$$

17. Answer (A)

Hint : $\frac{dy}{dt} = 3$ m/s; $\quad \frac{dx}{dt} = y$

Sol. : $\frac{dy}{dt} = 3$ m/s $\quad \therefore \; y = 3t \quad$ …(1)

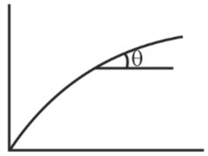
$$\frac{dx}{dt} = 3t \quad \therefore \quad x = \frac{3t^2}{2}$$

And $\frac{d^2x}{dt^2} = 3$

$$\therefore x = \frac{3}{2} \cdot \left(\frac{y}{3}\right)^2 \Rightarrow x = \frac{y^2}{6}$$

$$\therefore 6(1) = 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{3}{y}$$

At $y = 3$ m, $\frac{dy}{dx} = \tan\theta = 1 \quad \therefore \quad \theta = 45°$



$$|\vec{a}| = \frac{d^2x}{dt^2} = 3\text{ m/s}^2$$

$$\therefore\ |a_n| = \frac{3}{\sqrt{2}},\ |\vec{a}_z| = \frac{3}{\sqrt{2}}$$

x (at y = 4 m) = $\frac{4\times4}{6} = 8/3$

At y = 4 m ; t = 4/3 $\therefore$ v_x = 3 × 4/3 = 4 m/s

$\therefore$ $|\vec{v}| = 5$ m/s

18. Answer (B)

Hint : Constraint motion

Sol. : Speed of R = v_1 = 3 m/s

Speed of Q = $v_1 + 2v_2$ = 3 + 4 = 7 m/s

Speed of P = $3v_1 + 2v_2$ = 9 + 4 = 13 m/s

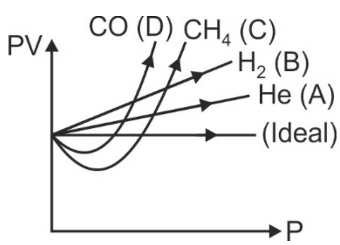
Speed of P w.r.t. A = $3v_1 + 2v_2 - v_1$ = 10 m/s

PART - II (CHEMISTRY)

19. Answer (B, C, D)

Hint : A – He, B – H_2, C – CH_4 and D – CO

Sol. :



20. Answer (A, B, C)

Hint : Time taken to complete 1 revolution = $\frac{2\pi r}{v}$

Sol. : Time taken to complete one revolution in an orbit, $T \propto \frac{n^3}{Z^2}$

$$T_2(H) \propto \frac{(2)^3}{(1)^2} \propto 8\ ; \qquad T_4(He^+) \propto \frac{(4)^3}{(2)^2} \propto 16$$

$$\therefore\ \frac{T_2(H)}{T_4(He^+)} = \frac{8}{16} = \frac{1}{2}$$

$$\frac{n_2(H)}{n_4(H)} = \frac{2}{4} = \frac{1}{2}$$

Radius of n^{th} orbit, $r_n = \left(\frac{n^2}{Z}\right)(r_1)_H$

$$r_2(H) = \frac{(2)^2}{1}r_1(H);\quad r_4(He^+) = \frac{(4)^2}{2}r_1(H) = 8r_1(H)$$

$$\frac{r_2(H)}{r_4(He^+)} = \frac{4}{8} = \frac{1}{2}$$

Energy of electron in n^{th} orbit, $E_n = -13.6\,\frac{Z^2}{n^2}$ eV

$$E_2(H) = -13.6\frac{(1)^2}{(2)^2} = -\frac{13.6}{4}\text{ eV}$$

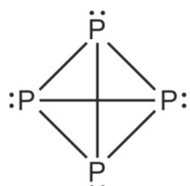
$$E_4(He^+) = -13.6\frac{(2)^2}{(4)^2} = -\frac{13.6}{4}\text{ eV}$$

$$\therefore\ \frac{E_2(H)}{E_4(He^+)} = 1$$

21. Answer (A, D)

Hint : Each P is sp^3 hybridised.

Sol. :



22. Answer (A, B, C, D)

Hint : Number of moles = Molarity × volume (L)

Sol. : Let the volumes of 0.1 M $Fe_2(SO_4)_3$ and 0.1 M $Al_2(SO_4)_3$ solutions to be mixed be V_1 and V_2 respectively.

Number of moles of cations = $0.2V_1 + 0.2V_2$

= $0.2(V_1 + V_2)$

Number of moles of anions = $0.3V_1 + 0.3V_2$

= $0.3(V_1 + V_2)$

$$\frac{\text{Moles of cations}}{\text{Moles of anions}} = \frac{0.2(V_1+V_2)}{0.3(V_1+V_2)} = \frac{2}{3}$$

So, molar ratio of cations to anions will be always 2 : 3 irrespective of the values of V_1 and V_2.

23. Answer (B, C)

Hint : Mass of air displaced = V × d

Sol. : Weight of payload = 80 × 10^3

Let number of balloons required be x.

$$80\times10^3 + 0 + 100x = \frac{nRT}{P}x\times1.25$$

x = 26.8

$\therefore$ Number of balloon = 27 and 30

24. Answer (B, C, D)

Hint : B.O. of CO = 3

B.O. of CO^+ = 3.5

Sol. :

	BO
N_2	3
N_2^+	2.5

. Bond length $\propto \frac{1}{\text{Bond order}}$

NO^+ NO^-

BO → 3 BO → 2

O_2 BO = 2 O_3 BO → 1.5

25. Answer (28)

Hint : Excitation energy = $13.6\left[\frac{1}{(1)^2}-\frac{1}{n^2}\right]$

Sol. : Energy absorbed by H-atoms = 13.3875 eV

Let the electron of H-atom be excited from ground state to n[th] orbit.

$\therefore \quad 13.3875 = 13.6\left[\frac{1}{(1)^2}-\frac{1}{(n)^2}\right]$

On solving, n = 8

Total number of lines in the spectrum of hydrogen

$= \sum(8-1) = \sum(7) = 28$

26. Answer (22)

Hint : KE of iodine atoms = $\frac{1}{2}[\text{IE}-\text{BE}]$

Sol. : Incident energy (IE) given to I_2 molecule

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{4500\times10^{-10}} = 4.4\times10^{-19}\ \text{J}$$

Minimum energy required to dissociate I_2 molecule

$$= \frac{240\times1000}{6\times10^{23}} = 4.0\times10^{-19}\ \text{J}$$

KE of 2 iodine atoms $= (4.4 - 4.0)10^{-19}$

$= 4 \times 10^{-20}$ J

KE / Iodine atom = $2.0 \times 10^{-20} = x \times 10^{y}$

$x + |y| = 2 + |-20| = 22$

27. Answer (06)

Hint : For Be, Mg and He, EGE is positive.

Sol. : EGE is negative for Li, C, F, Cl, Na, K.

28. Answer (55)

Hint : Moles of $CaCl_2$ = Moles of $CaCO_3$ = Moles of CaO

Sol. : Mass of mixture of $CaCl_2$ and NaCl = 3.70 gm

$$CaCl_2 + Na_2CO_3 \longrightarrow CaCO_3 + 2NaCl$$

$$CaCO_3 \xrightarrow{\Delta} \underset{0.84\ \text{gm}}{CaO} + CO_2$$

Moles of $CaCl_2$ = Moles of $CaCO_3$ = Moles of CaO

$= \frac{0.84}{56} = 0.015$

Mass of $CaCl_2$ = 0.015 × 111 gm

% of $CaCl_2$ = $\frac{0.015\times111\times100}{3.70} = 45\%$

$\therefore$ % of NaCl in the given mix = 55%

29. Answer (23)

Hint : Bohr's model

Sol. : For H-atom, transition $n_2 \rightarrow n_1$, λ = 92 nm

$$\frac{1}{92} = R_H(1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right] \quad \ldots(i)$$

For He^+ ion,

$$\frac{1}{\lambda} = R_H(2)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right] \quad \ldots(ii)$$

Solving (i) and (ii), we get

$\lambda = \frac{92}{4} = 23$ nm

30. Answer (32)

Hint : Eka silicon is Germanium.

Sol. : Atomic number of eka-silicon (Germanium) = 32

31. Answer (28)

Hint : Increase in weight is due to oxidation of FeO, present in the mixture, to Fe_2O_3.

Sol. : Let the mass of FeO in 100 gm mixture of FeO and Fe_2O_3 be x gm. On heating the entire amount of FeO is converted into Fe_2O_3.

$$4FeO + O_2 \longrightarrow 2Fe_2O_3$$

Percentage increase in weight = $\frac{32\times x}{4\times72} = 8$

x = 72 gm

$\therefore$ Percentage of Fe_2O_3 in the given mixture = 28%

32. Answer (40)

Hint & Sol. :

N_2 ⇒ u = 10

O_2 ⇒ v = 10

CN^- ⇒ w = 10

NO^+ ⇒ x = 10

33. Answer (C)

Hint : Emission spectrum of H-atom and H-like species has lines in the UV region, IR and far IR region.

Sol. :

Z = 39, n_1 = 5 and n_2 = 13

$$\frac{1}{\lambda} = R_H(39)^2\left[\frac{1}{(5)^2} - \frac{1}{(13)^2}\right];\ \lambda = \frac{25}{1296\ R_H}$$

Z = 48, n_1 = 4 and n_2 = 5

$$\frac{1}{\lambda} = R_H(48)^2\left[\frac{1}{(4)^2} - \frac{1}{(5)^2}\right];\ \lambda = \frac{25}{1296\ R_H}$$

Z = 2, n_1 = 2 and n_2 = 3

$$\frac{1}{\lambda} = R_H(2)^2\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right];\ \lambda = \frac{9}{5\ R_H}$$

Z = 1, n_1 = 1 and n_2 = 3

$$\frac{1}{\lambda} = R_H(1)^2\left[\frac{1}{(1)^2} - \frac{1}{(3)^2}\right];\ \lambda = \frac{9}{8\ R_H}$$

All transitions from higher energy orbits to 4^{th} orbit lie in 'far IR region' for H-atom.

All transitions from higher energy orbit to 2^{nd} orbit lie in UV region except H-atom which lie in visible region.

34. Answer (A)

Hint : d_{z^2} has no planar nodes but has two angular nodes.

Sol. : Number of radial/spherical nodes in an orbital = n – l – 1

Number of planar/angular nodes in an orbital = l

Orbital d_{z^2} has two angular nodes but no planar nodes.

35. Answer (B)

Hint : Molality

$$= \frac{\text{Molarity} \times 1000}{[1000d - \text{Molarity} \times \text{Molar mass of solute}]}$$

Sol. :

NaOH solution :

$$\text{Molarity} = \frac{6 \times 1000}{40 \times 250} = 0.6\ \text{M}$$

$$\text{Molality} = 0.6\ \text{m} \left\{\because d_{H_2O} = \frac{1\ \text{gm}}{\text{cc}}\right\}$$

$$\text{Mole fraction of NaOH} = \frac{0.15}{0.15 + \frac{250}{18}} = 0.01$$

Glucose solution :

$$\text{Molarity} = \frac{54 \times 1000}{180 \times 500} = 0.6\ \text{M}$$

Molality = 0.6 m

$$\text{Mole fraction of glucose} = \frac{0.30}{0.30 + \frac{500}{18}} = 0.01$$

$$NaOH + HCl \longrightarrow NaCl + H_2O$$

	NaOH	HCl	NaCl	H_2O
m mole	70	10		
	60	0	10	10

$$\text{Molarity of excess NaOH} = \frac{60}{100} = 0.6\ \text{M}$$

Benzene – Toluene mixture

$$\text{Number of moles of benzene} = \frac{31.2}{78} = 0.4$$

$$\text{Number of moles of toluene} = \frac{55.2}{92} = 0.6$$

Mole fraction of benzene = 0.4

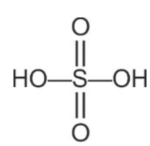
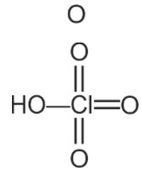
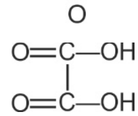
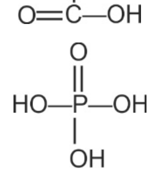
Molarity of benzene in solution

$$= \frac{0.4 \times 1000}{\frac{55.2}{0.87}} = 6.3\ \text{M}$$

36. Answer (D)

Hint : Number of hybridised orbitals, H is given by $H = \frac{1}{2}[V + M - C + A]$.

Sol. :

Acid	Basicity	No. of π-bonds	% of O	Hybridisation of central atom
	2	2	≃ 64%	sp^3
	1	3	≃ 64%	sp^3
	2	2	≃ 71%	sp^2
	3	1	≃ 64%	sp^3

PART - III (MATHEMATICS)

37. Answer (B, C)

Hint : $\sqrt{3}\sin x + \cos x$ = 2(cos$x$ + cos5x)

$\Rightarrow \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \cos 5x$

Sol. : $\sqrt{3}\sin x + \cos x$ = 2(cos5x + cos$x$)

$\Rightarrow \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \cos 5x$

$\Rightarrow \cos 5x = \cos\left(\frac{2\pi}{3} - x\right)$

$\Rightarrow 5x = 2n\pi \pm \left(\frac{2\pi}{3} - x\right)$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{9}$ or $x = \frac{n\pi}{2} - \frac{\pi}{6}$

38. Answer (A, B)

Hint : Put $z = ki$, where $k \in R$

Sol. : If the equation has a purely imaginary root, then $z = ki$; ($k \in R$) will satisfy the equation.

$-k^2(1 - i) + k(-2i + 1) - 3i = m$

$\Rightarrow -k^2 + k = m$ and $k^2 - 2k - 3 = 0$

$k = -1, 3$

So, $m = -2, -6$

39. Answer (A, D)

Hint : $[x]^2 - [x] - 20 > 0 \Rightarrow [x] \in (-\infty, -4) \cup (5, \infty)$

Sol. : $f(x)$ exists if

$[x]^2 - [x] - 20 > 0 \Rightarrow [x] \in (-\infty, -4) \cup (5, \infty)$

$\Rightarrow x \in (-\infty, -4) \cup [6, \infty)$

40. Answer (A, B, D)

Hint : $-\sqrt{3}x = 2y = z$

Sol. : $(-\sqrt{3}x)^2 + (2y)^2 + (z)^2 - (-\sqrt{3}x)(2y) - (2y)(z) - (z)(-\sqrt{3}x) = 0$

$\Rightarrow -\sqrt{3}x = 2y = z = k$ (where $k \in R - \{0\}$)

$\Rightarrow x = \frac{-k}{\sqrt{3}}, y = \frac{k}{2}, z = k$

So, $xy = -\frac{k^2}{2\sqrt{3}} < 0$, $yz = \frac{k^2}{2} > 0$

$x(x + y + z) = \frac{k^2}{3} - \frac{k^2}{2\sqrt{3}} - \frac{k^2}{\sqrt{3}} = k^2\left(\frac{1}{3} - \frac{\sqrt{3}}{2}\right) < 0$

and $xy + zx = -\frac{k^2}{2\sqrt{3}} - \frac{k^2}{\sqrt{3}} < 0$

41. Answer (A, C)

Hint : $f(x)$ and $g(x)$ have a common factor.

Sol. : $f(x)$ and $g(x)$ both are factors of a cubic polynomial then $f(x) = 0$ and $g(x) = 0$ have a common root.

$x^2 + (k^2 - 29)x - k = 0$ …(i)

$2x^2 + (2k - 43)x + k = 0$ …(ii)

Applying condition for common roots,

$(2k - 58 - 2k + 43)(-2k^2 + 43k - k^2 + 29k)$

$= (2k + k)^2$

$\Rightarrow 4k^2 - 120k = 0 \Rightarrow k = 0$ or 30

42. Answer (B, D)

Hint : Put $z = x + iy$ and solve for x and y.

Sol. : Let $z = x + iy$

$x^2 + y^2 - 5(x + iy) = 10i$

$\Rightarrow x^2 + y^2 - 5x = 0$ and $-5y = 10$

$\Rightarrow y = -2$ and $x = 1$ or 4

$\Rightarrow z = 1 - 2i$ or $4 - 2i$

43. Answer (06)

Hint : Product of roots is negative.

Sol. :

Case 1 : One positive and one negative root $a \cdot c < 0$

$\Rightarrow m(m^2 - 6m - 7) < 0$

$\Rightarrow m \in (-\infty, -1) \cup (0, 7)$

Case 2 : Two equal positive roots

$D = 0 \cap m < 0 \Rightarrow$ no integral value of m exist.

44. Answer (20)

Hint : $(a - b)(a^2 + b^2 + ab) = a^3 - b^3$

Sol. :

$X = X = \log_3(3 - \sqrt[3]{2}) + \log_3(3^2 + (\sqrt[3]{2})^2 + 3\cdot\sqrt[3]{2})$

$X = \log_3(3^3 - 2) = \log_3 25$

Now, $5X \cdot \log_5 9 = 5(\log_3 25)(\log_5 9)$

$= 5(2\log_3 5)(2\log_5 3)$

$= 20$

45. Answer (16)

Hint : $x^3 - 2x^2 - 11x + 12 < 0 \cap x^2 - 2x - 48 < 0$

Sol. : $\because$ $x^3 - 2x^2 - 11x + 12 < 0$ and

$x^2 - 2x - 48 < 0$

$(x - 1)(x + 3)(x - 4) < 0$ and $(x - 8)(x + 6) < 0$

$x \in (-\infty, -3) \cup (1, 4)$ and $x \in (-6, 8)$

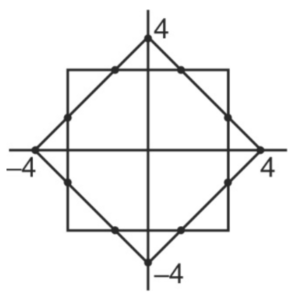
So, $x \in (-6, -3) \cup (1, 4)$

Integral value of x will be –5, –4, 2, 3.

46. Answer (08)

Hint : Draw graph

Sol. : If we draw the two graph $|x| + |y| = 4$ and $|x + y| + |x - y| = 5$



There are 8 points of intersection.

47. Answer (03)

Hint : $f(x) = 1 - 2\left(\frac{x}{1+x^2}\right)$

Sol. : $f(x) = \frac{1+x^2-2x}{1+x^2} = 1 - 2\left(\frac{x}{1+x^2}\right)$

$\because$ $x + \frac{1}{x} \in (-\infty, -2] \cup [2, \infty)$

$\Rightarrow$ $\frac{x^2+1}{x} \in (-\infty, -2] \cup [2, \infty)$

$\Rightarrow$ $\frac{x}{x^2+1} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$f(x) \in [0, 2]$

Number of integers in the range of $f(x)$ is 3.

48. Answer (20)

Hint : $\cos^2\frac{r\pi}{7} = \frac{1}{2}\left(1+\cos\frac{2r\pi}{7}\right)$

Sol. : $\cos^2\frac{\pi}{7}+\cos^2\frac{2\pi}{7}+\cos^2\frac{3\pi}{7}$

$= \frac{1}{2}\left[\left(1+\cos\frac{2\pi}{7}\right)+\left(1+\cos\frac{4\pi}{7}\right)+\left(1+\cos\frac{6\pi}{7}\right)\right]$

$= \frac{1}{2}\left[3+\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)\right]$

$= \frac{1}{2}\left[3+\frac{\sin\frac{3\pi}{7}}{\sin\frac{\pi}{7}}\cdot\cos\frac{4\pi}{7}\right]$

$= \frac{1}{2}\left[3-\frac{2\sin\frac{3\pi}{7}\cdot\cos\frac{3\pi}{7}}{2\sin\frac{\pi}{7}}\right]$

$= \frac{1}{2}\left[3-\frac{1}{2}\right] = \frac{5}{4}$

49. Answer (15)

Hint : $(x - 3)^2 + 2\cos^2\left(\frac{\pi x}{n}\right) = 0$

Sol. : $(x^2 - 6x + 9) + \left(1+\cos\left(\frac{2\pi x}{n}\right)\right) = 0$

$\Rightarrow$ $(x - 3)^2 + 2\cos^2\left(\frac{\pi x}{n}\right) = 0$

$\Rightarrow$ $x = 3$ and $\cos\left(\frac{\pi x}{n}\right) = 0$

$\Rightarrow$ $\alpha = 3$ $\Rightarrow \cos\left(\frac{3\pi}{n}\right) = 0$

$\Rightarrow n = 2$ or 6

Maximum value of $\alpha^2 + n = 15$

50. Answer (02)

Hint : $\cot\theta - \tan\theta = 2\cot2\theta$

Sol. : $\tan55° - \tan35° = \frac{\sin(55°-35°)}{\cos55°\cdot\cos35°}$

$= \frac{2\sin20°}{2\sin35°\cdot\cos35°}$

$= \frac{2\sin20°}{\sin70°} = 2\tan20°$

And $2\cot40° - \cot20° + 2\tan20°$

$= (\cot20° - \tan20°) - \cot20° + 2\tan20°$

$= \tan20°$

So, $\frac{\tan55°-\tan35°}{2\cot40°-\cot20°+2\tan20°} = 2$

51. Answer (C)

Hint : $\arg(z_1\cdot z_2) = \arg(z_1) + \arg(z_2)$

and $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

Sol. :

(P) $z+\bar{z}+z\bar{z}$ is purely real,

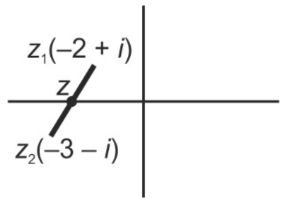
$\arg\left(z+\bar{z}+z\bar{z}\right)$ is 0 or π

(Q) $\arg\left(\dfrac{(1+\sqrt{3}i)(\sqrt{3}-i)(1+i)}{(\sqrt{3}+i)}\right)$

$= \arg(1+\sqrt{3}i)+\arg(\sqrt{3}-i)+\arg(1+i)$
$\qquad - \arg(\sqrt{3}+i)$

$= \dfrac{\pi}{3}+\left(-\dfrac{\pi}{6}\right)+\dfrac{\pi}{4}-\dfrac{\pi}{6}$

$= \dfrac{\pi}{4}$



(R) $\arg(z)$ may be π

(S) $z_1 = iz_2$

$\Rightarrow \arg(z_1) = \dfrac{\pi}{2}+\arg(z_2)$ ...(i)

also, $\arg(z_1z_2) = \dfrac{2\pi}{3}$

$\arg(z_1) + \arg(z_2) = \dfrac{2\pi}{3}$ ...(ii)

(i) + (ii),

$\arg(z_1) = \dfrac{7\pi}{12}$

52. Answer (C)

Hint : $f(x) = -2(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}ax+1)$

Sol. : $f(x) = (a-1)(x^2+\sqrt{3}x+1)^2 - (a+1)$
$\qquad [(x^2+1)^2-3x^2]$

$= (a-1)(x^2+\sqrt{3}x+1)^2 - (a+1)$
$\qquad (x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)$

$= -2(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}ax+1)$

$\therefore \quad x^2+\sqrt{3}x+1>0$

(P) If $f(x) < 0 \Rightarrow x^2-\sqrt{3}ax+1>0$ for all $x\in R$

$\Rightarrow 3a^2-4<0 \Rightarrow a \in \left(-\dfrac{2}{\sqrt{3}}, \dfrac{2}{\sqrt{3}}\right)$

(Q) $f(x) > 0 \Rightarrow x^2-\sqrt{3}ax+1<0 \Rightarrow a \in \phi$

(R) If $f(x) = 0$ has real roots, then

$x^2-\sqrt{3}ax+1 = 0$ has real roots

$3a^2-4>0$

$a \in \left(-\infty, -\dfrac{2}{\sqrt{3}}\right)\cup\left(\dfrac{2}{\sqrt{3}}, \infty\right)$

(S) If $f(x) = 0$ has all roots imaginary, then

$x^2-\sqrt{3}ax+1 = 0$ has imaginary roots

($\because$ $x^2+\sqrt{3}x+1 = 0$ already has imaginary roots)

$a \in \left(-\dfrac{2}{\sqrt{3}}, \dfrac{2}{\sqrt{3}}\right)$

Sum of all roots $= S = \sqrt{3}a-\sqrt{3}$

$S \in \left(-2-\sqrt{3}, 2-\sqrt{3}\right)$

53. Answer (B)

Hint : $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

$\cos 2A + \cos 2B + \cos 2C$
$\qquad = -1 - 4\cos A\cos B\cos C$

$\tan A + \tan B + \tan C = \tan A\cdot\tan B\cdot\tan C$

Sol. :

(P) $\because$ $\cos 2A + \cos 2B + \cos 2C$

$= -1 - 4\cos A\cos B\cos C$

$= -\dfrac{\sqrt{3}+1}{2}$

(Q) $\sin^2 A + \sin^2 B + \sin^2 C$

$= \dfrac{3-(\cos 2A+\cos 2B+\cos 2C)}{2}$

$= \dfrac{7+\sqrt{3}}{4}$

(R) $\tan A + \tan B + \tan C = \tan A\cdot\tan B\cdot\tan C$

$= \dfrac{3+\sqrt{3}}{\sqrt{3}-1}$

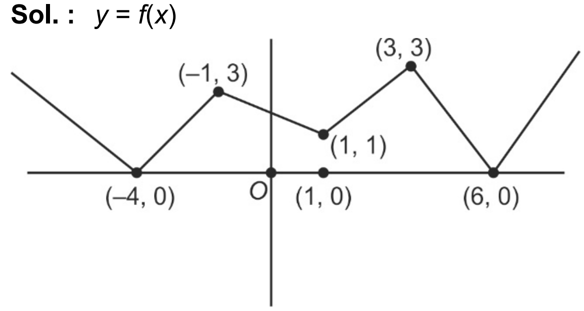
(S) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

$= \dfrac{3+\sqrt{3}}{2}$

54. Answer (D)

Hint : Draw graph of $y = f(x)$

Sol. : $y = f(x)$



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