All India Aakash Test Series for JEE (Main)-2020

TEST - 4 - Code-A

Test Date : 17/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(4)	26.	(3)	51.	(4)
2.	(1)	27.	(3)	52.	(2)
3.	(2)	28.	(1)	53.	(1)
4.	(2)	29.	(3)	54.	(2)
5.	(1)	30.	(1)	55.	(4)
6.	(4)	31.	(2)	56.	(4)
7.	(2)	32.	(3)	57.	(2)
8.	(1)	33.	(2)	58.	(1)
9.	(3)	34.	(3)	59.	(2)
10.	(1)	35.	(2)	60.	(3)
11.	(3)	36.	(3)	61.	(1)
12.	(1)	37.	(3)	62.	(4)
13.	(2)	38.	(1)	63.	(2)
14.	(3)	39.	(2)	64.	(4)
15.	(4)	40.	(3)	65.	(1)
16.	(1)	41.	(4)	66.	(3)
17.	(3)	42.	(1)	67.	(1)
18.	(1)	43.	(2)	68.	(4)
19.	(2)	44.	(2)	69.	(4)
20.	(4)	45.	(1)	70.	(2)
21.	(16)	46.	(02)	71.	(24)
22.	(10)	47.	(09)	72.	(01)
23.	(14)	48.	(02)	73.	(00)
24.	(07)	49.	(12)	74.	(22)
25.	(88)	50.	(20)	75.	(04)

PART - A (PHYSICS)

1. Answer (4)

Hint : $A = \lambda N$

Sol. : $3\lambda_1 N_A = \lambda_2 N_B$

$$\frac{3\times 2}{T_1} = \frac{1}{T_2}$$

$$T_2 = \frac{T_1}{6}$$

2. Answer (1)

Hint : $hv = \phi + eV_0$

Sol. : $\phi = 5.2$ eV

$E = 6.2$ eV

$V_0 = 1$ V

$$\frac{ne}{4\pi\varepsilon_0 r} = 1$$

$$\frac{9\times 10^9 \times n \times 1.6\times 10^{-19}}{2\times 10^{-2}} = 1$$

$$n = \frac{10^{10}\times 2\times 10^{-2}}{9\times 1.6} = 1.38\times 10^7 \approx 1.4\times 10^7$$

3. Answer (2)

Hint : $\frac{1}{2}mv^2 = \frac{k\,60e^2}{d}$

Sol. : $(mv)^2 = \frac{120\,ke^2 m}{d}$

$$\lambda = h\sqrt{\frac{d}{120\,mke^2}}$$

4. Answer (2)

Hint : Fringes formed will be symmetric on screen

Sol. : Number of Fringes are $\left[\frac{d}{\lambda}\right] = 5$.

5. Answer (1)

Hint : $E \propto \frac{1}{n^2}$

Sol. : $E_n \propto \frac{1}{n^2}$

$$E_n - E_{n-1} = k\left[\frac{1}{n^2} - \frac{1}{(n-1)^2}\right]$$

$$\frac{hc}{\lambda} = \frac{1}{n^3}$$

$$\lambda \propto n^3$$

6. Answer (4)

Hint : $Y = \overline{(\overline{A+B})\cdot(A+B)}$

Sol. : $Y = (A+B) + (\overline{A+B}) = 1$

7. Answer (2)

Hint : $I = 4I_o\cos^2\frac{\phi}{2}$

Sol. : $\phi = \frac{\pi}{3}$

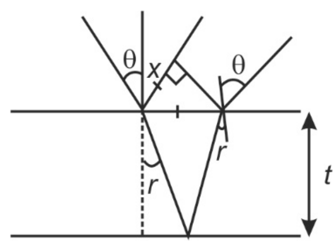
$$y_1 = \frac{\lambda D}{6d}$$

$$y_2 = -\frac{\lambda D}{6d}$$

8. Answer (1)

Hint : Effective phase difference must be $2n\pi$

Sol. : $2\mu t\sec r - x = \frac{\lambda}{2}$



$$x = 2t\tan r\sin\theta$$

$$\frac{2\mu t}{\cos r} - \frac{2t\sin r\sin\theta}{\cos r} = \frac{\lambda}{2}$$

$$\frac{2t}{\cos r}\left[\mu - \frac{\sin^2\theta}{\mu}\right] = \frac{\lambda}{2}$$

$$\Rightarrow \frac{2t\mu[\mu^2 - \sin^2\theta]}{\mu\sqrt{\mu^2-\sin^2\theta}} = \frac{\lambda}{2}$$

$$\Rightarrow t = \frac{\lambda}{4\sqrt{\mu^2-\sin^2\theta}}$$

9. Answer (3)

Hint : For polarizing angle reflected and refracted rays are mutually perpendicular.

Sol. : $\frac{\sin\theta_p}{\cos\theta_p} = \mu$

$\tan\theta_p = \mu$

$\sin\theta_c = \frac{1}{\mu}$

$\tan\theta_p \sin\theta_c = 1$

10. Answer (1)

Hint : $0.7\,N_0 = N_0 e^{-\lambda t_0}$

Sol. : $0.7 = e^{-\lambda t_0}$

$x = (1 - e^{-3\lambda t_0})$

$x = 1 - (0.7)^3$

$x = 1 - 0.49 \times 0.7$

$x = 65.7\%$

11. Answer (3)

Hint : $\frac{hc}{\lambda} = \phi + \frac{P^2}{2m}$

Sol. : $\frac{hc}{\lambda} = \frac{3hc}{5\lambda} + \frac{h^2}{2m\lambda_1^2}$

$\frac{2hc}{5\lambda} = \frac{h^2}{2m\lambda_1}$

$\lambda_1^2 = \frac{5h^2\lambda}{4hmc}$

$\lambda_1^2 = \frac{5h\lambda}{4mc}$

$\lambda_1 = \frac{1}{2}\sqrt{\frac{5h\lambda}{mc}}$

12. Answer (1)

Hint : $I = \frac{P}{4\pi d^2}$

Sol. : $I = \frac{P}{4\pi d^2}$

Pressure $= \frac{I}{L} = \frac{P}{4\pi d^2 c}$

13. Answer (2)

Hint : $\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

Sol. : $\frac{1}{\lambda} = Rz^2\left[\frac{1}{4} - \frac{1}{16}\right] \Rightarrow \lambda = \frac{16}{3Rz^2}$

$\frac{1}{\lambda_{\text{least}}} = Rz^2\left[1 - \frac{1}{16}\right]$

$\frac{1}{\lambda_{\text{least}}} = \frac{Rz^2 15}{16}$

$\lambda_{\text{least}} = \frac{16}{15Rz^2}$

14. Answer (3)

Hint : $t_{1/2} = \frac{\ln 2}{\lambda}$

Sol. : $N = N_0 e^{-\lambda t}$

$N = N_0 e^{-\frac{\lambda \ln 2}{2\lambda}}$

$N = \frac{N_0}{\sqrt{2}}$

Decayed $= \frac{N_0 - \frac{N_0}{\sqrt{2}}}{N_0} = \frac{\sqrt{2}-1}{\sqrt{2}}$

15. Answer (4)

Hint : $F = -\frac{du}{dr}$

Sol. : $\frac{mv^2}{r} = \frac{ke^2}{r^3}$

K.E. $= \frac{ke^2}{2r^2}$

$U = -\frac{ke^2}{2r^2}$

Total energy = zero

16. Answer (1)

Hint : $\lambda = \frac{h}{p}$

Sol. : $p_f = \frac{h}{\lambda_1} - \frac{h}{\lambda_2}$

$\frac{h}{\lambda} = \frac{h}{\lambda_1} - \frac{h}{\lambda_2}$

$\lambda = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}$

17. Answer (3)

Hint : $E = (B.E)_I - (B.E)_R$

Sol. : $E = E_1 - 2E_2$

18. Answer (1)

Hint : Apply KVL and KCL

Sol. : $10 - 4 - 1.2I_2 = 0$

$I_2 = 5\text{ mA}$

$4 - I_1 = 0$

$I_1 = 4\text{ mA}$

$I_z = I_2 - I_1 = 1\text{ mA}$

19. Answer (2)

Hint and Sol. : $m = \frac{v_{max} - v_{min}}{v_{max} + v_{min}} = \frac{1}{3}$

20. Answer (4)

Hint : Reading = MSR + n × LC

Sol. : $\text{LC} = \frac{0.5}{100} = 0.005\text{ mm}$

$\text{Reading} = 3 + 40 \times 0.005$

= 3.200 mm

21. Answer (16)

Hint : $\beta = \frac{I_C}{I_B}$

Sol. : $\frac{I_C R_C}{I_b R_b} = \frac{V_0}{V_i}$

$V_i = \frac{2 \times R_b}{R_C \times 50}$

V_i = 16 mV

22. Answer (10)

Hint : It will vary from $f_c - f_m$ to $f_c + f_m$

Sol. : $\Delta f = 2f_m = 10\text{ kHz}$

23. Answer (14)

Hint : For intensity to be $\frac{1}{4}$th

$\phi = 2n\pi \pm \frac{2\pi}{3}$

Sol. : $5\lambda\cos\theta = \left(5\lambda - \frac{\lambda}{3}\right)$

$5\lambda\cos\theta = \frac{14\lambda}{3}$

$\theta = \cos^{-1}\left(\frac{14}{15}\right)$

24. Answer (07)

Hint : Path difference will be 3λ corresponding to that point.

Sol. : $4\lambda\cos\theta = 3\lambda$

$\theta = \cos^{-1}\left(\frac{3}{4}\right)$

$\tan\theta = \frac{y}{D}$

$\frac{\sqrt{7}}{3} = y$

25. Answer (88)

Hint : Analyse the circuit for half of the cycle

Sol. : $P = \frac{1}{2} \times \frac{V_0^2}{2R}\left[\frac{1}{2} + \frac{4}{11}\right] = \frac{19V_0^2}{88R}$

PART - B (CHEMISTRY)

26. Answer (3)

Hint : A strong acid can produce weak acid.

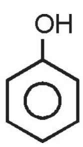
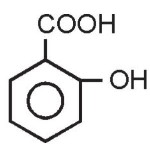
Sol. : Acidic strength of carbolic acid (Phenol) is less than H_2CO_3 so, it not produce CO_2.

27. Answer (3)

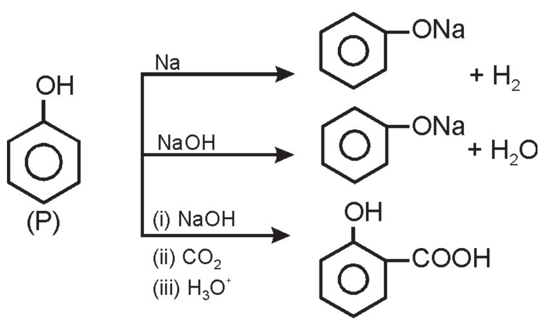
Hint : Hemiacetal can reduce Tollen's reagent

Sol. : If A give positive iodoform test so it must contain $CH_3-\overset{\overset{\Large O}{\|}}{C}$ group and A does not reduce Tollen's reagent it means A not contain $-\overset{\overset{\Large O}{\|}}{C}-H$ or hemiacetal

28. Answer (1)

Hint : P =  Q = 

Sol. :



29. Answer (3)

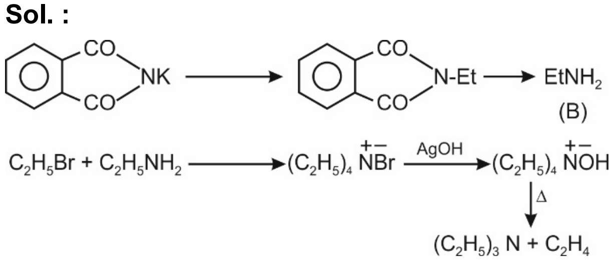
Hint : Carbylamine test is given by primary amines.

Sol. : Carbylamine test is not given by secondary amines.

30. Answer (1)

Hint : Gabriel phthalimide synthesis

Sol. :



31. Answer (2)

Hint : A and D → α

Sol. : CH_2OH at C–5 and OH at C–1 decide the α or β nomenclature.

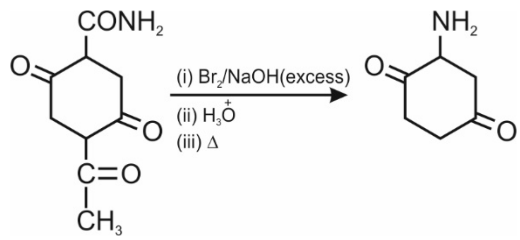
CH_2OH and OH same side → β-form

CH_2OH and OH opposite side → α-form

32. Answer (3)

Hint : β–Keto acid show decarboxylation on heating.

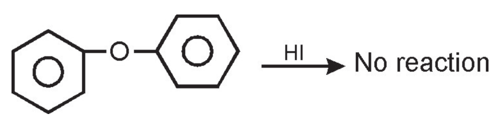
Sol. :



33. Answer (2)

Hint : B not cleaved with HI

Sol. :



34. Answer (3)

Hint : 4-methylpent-3-en-2-one

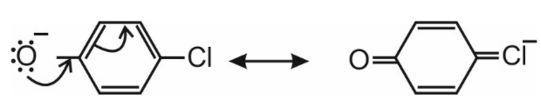
Sol. :

$\overset{5}{(CH_3)_2}\overset{4}{C}=\overset{3}{C}H\overset{2}{C}O\overset{1}{C}H_3$ 4-methylpent-3-en-2-one

35. Answer (2)

Hint : p-chlorophenol is most acidic

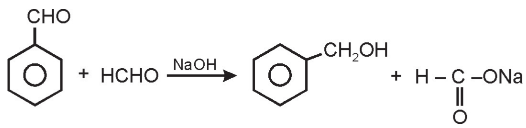
Sol. :



36. Answer (3)

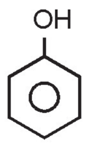
Hint : Reaction is intermolecular, it is a Cannizzaro reaction.

Sol . :



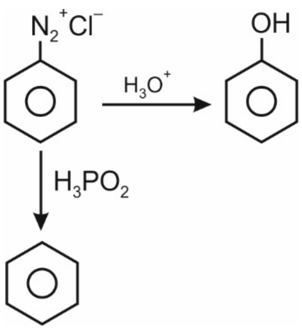
37. Answer (3)

Hint :

Q = 

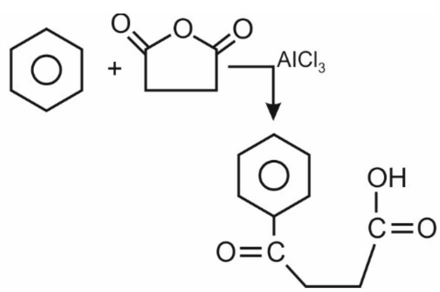
R = 

Sol. :

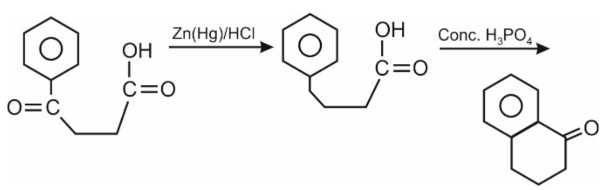


38. Answer (1)

Hint :

Q = 

Sol. :



39. Answer (2)

Hint : B.P of isomeric amine 1°> 2° > 3°

Sol. : A $\rightarrow$ 1° amine

B $\rightarrow$ 2° amine

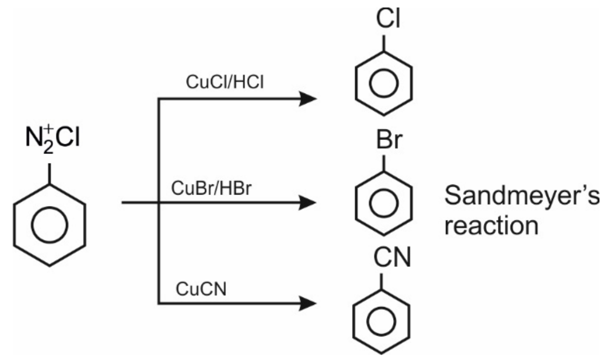
C $\rightarrow$ 3° amine

Due to less extent of H–Bonding in 3° amine, B.P. of 3° amine is minimum.

40. Answer (3)

Hint : (iv), (v), (vi) are not Sandmeyer's reaction

Sol. :



41. Answer (4)

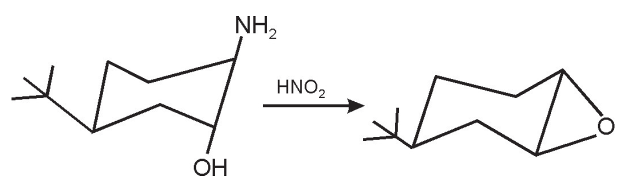
Hint : As positive charge on carbonyl carbon decrease, rate of reaction decrease.

Sol. : *sp*^{2} 'N' in 3 membered ring and bridge head is not possible.

42. Answer (1)

Hint : When NH_2 and OH are at axial position then form compound P.

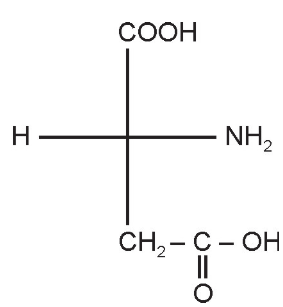
Sol. :



43. Answer (2)

Hint : $\text{Glucose} \xrightarrow{HNO_3} \text{Saccharic acid}$

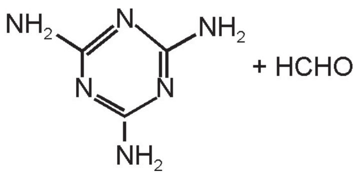
Sol. : Aspartic acid



44. Answer (2)

Hint : Melamine and formaldehyde

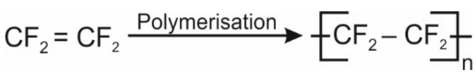
Sol. :



45. Answer (1)

Hint : Teflon

Sol. :



46. Answer (02)

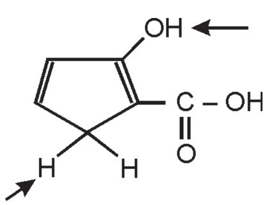
Hint : PCC oxidises alcohol to carbonyl group.

Sol. : MnO_2 oxidises allylic alcohol.

47. Answer (09)

Hint : CH_3MgBr react with acidic hydrogen

Sol. :



48. Answer (02)

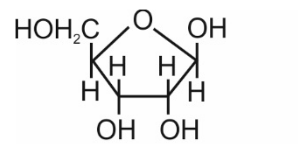
Hint : Glyptal is used in manufacture of paints and lacquers.

Sol. : Polyester is a condensation polymer. Two statements are incorrect (ii) & (vii)

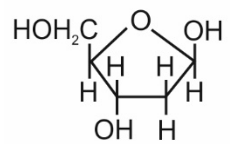
49. Answer (12)

Hint : β - D ribose has 4 chiral centre.

Sol. :



β - D - ribose

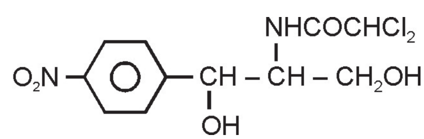


β - D - 2 deoxyribose

xy = 4 × 3 = 12

50. Answer (20)

Hint :



Sol. : Chloramphenicol has two –OH groups, two chiral carbon and zero –COOH.

5(2 + 0 + 2) = 20

PART - C (MATHEMATICS)

51. Answer (4)

Hint : Find the d.r. of normal to the plane

Sol. : $\begin{vmatrix} x-1 & y+2 & z-1 \\ 1 & 2 & 3 \\ 2 & -1 & 5 \end{vmatrix} = 0$

$\Rightarrow 13(x-1)+(y+2)-5(z-1)=0$

$\Rightarrow 13x+y-5z=6$

52. Answer (2)

Hint : $(\vec{a}+2\vec{b})\cdot\vec{c}=0$

Sol. : $\vec{a}=3\vec{b}$

$\Rightarrow p\hat{i}+2\hat{j}+3\hat{k}=3\hat{i}-3q\hat{j}+3\hat{k}$

$\Rightarrow p=3,\ q=-\dfrac{2}{3}$

$(\vec{a}+2\vec{b})=(p+2)\hat{i}+(2-2q)\hat{j}+(3+2)\hat{k}$

$=5\hat{i}+\dfrac{2}{3}\hat{j}+5\hat{k}$

$(\vec{a}+2\vec{b})\cdot\vec{c}=0$

$\Rightarrow r = \frac{-13}{9}$

$(p, q, r) = \left(3, \frac{-2}{3}, \frac{-13}{9}\right)$

53. Answer (1)

Hint : Use section formula

Sol. : *A* = (2, 4, 5), *B* = (–2, 0, 1)

$P\left(\frac{-2\lambda+2}{\lambda+1}, \frac{4}{\lambda+1}, \frac{\lambda+5}{\lambda+1}\right)$

$\Rightarrow \lambda + 33 = 2\lambda + 2$

$\Rightarrow \lambda = 31$

$\Rightarrow 31 : 1$

54. Answer (2)

Hint : Find the probability of complementary event

Sol. : Required probability

$= 1 - \left(\frac{2}{3} \times \frac{2}{3} \times \frac{3}{4} \times \frac{3}{4}\right) = \frac{3}{4}$

55. Answer (4)

Hint : d.r. of line segment is same as that of normal to the plane

Sol. : Equation of plane

$(x-5)2 + (x-0)6 + (y-2)10 = 0$

$\Rightarrow x + 3y + 5z = 0$

56. Answer (4)

Hint : Proceed with parametric form of point

Sol. : Point *Q* on the line = (–5*k* – 9, *k* + 2, 2*k* + 5)

Q lies on plane

$\Rightarrow -5k - 9 + k + 2 + 2k + 5 = 2$

$\Rightarrow k = -2$

$\therefore\ Q = (1, 0, 1)$

Distance, $PQ = \sqrt{5^2 + 1^2 + 2^2} = \sqrt{30}$

57. Answer (2)

Hint : variance = $\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$

Sol. : $\sum_{i=1}^{5} x_i = 20 \times 5 = 100$

$\Rightarrow \sum_{i=1}^{6} x_i = 100 - 100 = 0$

$\frac{\sum_{i=1}^{5} x_i^2}{5} - (20)^2 = 16$

$\Rightarrow \sum_{i=1}^{5} x_i^2 = 2080$

$\therefore\ \sum_{i=1}^{6} x_i^2 = 2080 + (-100)^2 = 12080$

Variance $= \frac{\sum_{i=1}^{6} x_i^2}{6} - \left(\frac{\sum_{i=1}^{6} x_i}{6}\right)^2$

$= \frac{12080}{6} = \frac{6040}{3}$

58. Answer (1)

Hint : Make cases

Sol. : Required probability

$= \frac{{}^5C_2({}^5C_2 + {}^5C_2 + {}^5C_3)}{{}^5C_2 \times {}^7C_3}$

$= \frac{10 + 10 + 10}{35} = \frac{6}{7}$

59. Answer (2)

Hint : $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Sol. : $|\vec{a}| = a,\ \ |\vec{b}| = b$

$ab(1 + \cos\theta) = 12$

$ab(1 - \cos\theta) = 6$

$\Rightarrow \frac{1+\cos\theta}{1-\cos\theta} = 2$

$\Rightarrow \cos\theta = \frac{1}{3}$ and $ab = 9$

$\therefore\ |\vec{a} \times \vec{b}| = ab|\sin\theta|$

$\therefore$ option (2) can be $\vec{a} \times \vec{b}$

60. Answer (3)

Hint : Use the concept of family of planes

Sol. : Required plane is

$(x+y+z-3)+\lambda(3x-2y+z+5)=0$

Parallel to *xz* plane

$\Rightarrow 0(1+3\lambda)+1(1-2\lambda)+0(1+\lambda)=0$

$\Rightarrow \lambda=\frac{1}{2}$

$\therefore$ Plane:

$(x+y+z-3)+\frac{1}{2}(3x-2y+z+5)=0$

$\Rightarrow 5x+3z-1=0$

61. Answer (1)

Hint : Find the d.r. of line.

Sol. : Any point on line is $(-\lambda + 2, -3\lambda - 1, 2\lambda + 3)$ which also passes through (1, 1, –2)

$\therefore$ d.r. of line = $(-\lambda + 1, -3\lambda - 2, 2\lambda + 5)$

This line is parallel to the given plane.

$\Rightarrow (-\lambda + 1)(-2) + (-3\lambda - 2)(1) + (2\lambda + 5)2 = 0$

$\Rightarrow 3\lambda + 6 = 0 \Rightarrow \lambda = -2$

$\therefore$ d.r. = (3, 4, 1)

$\therefore$ Equation of line: $\frac{x-1}{3}=\frac{y-1}{4}=\frac{z+2}{1}$

62. Answer (4)

Hint : Find the d.r. of lines

Sol. : First line is $\frac{x-10}{2}=y=\frac{z-3}{-1}$ and second line is $\frac{x-5}{-3}=\frac{y-2}{7}=z$ as

$2\times(-3)+1\times7+(-1)\times1=0$

$\Rightarrow L_1$ is perpendicular to L_2

63. Answer (2)

Hint : Break 5 into 3 non-zero parts

Sol. : $n(E)=3^5-{}^3C_1(2^5-2)-{}^3C_2$

$n(S)=3^5$

$P=\frac{n(E)}{n(S)}=\frac{50}{81}$

64. Answer (4)

Hint : Use total probability theorem

Sol. : Required probability

$=P(W)\cdot P\left(\frac{W}{W}\right)+P(B)\cdot P\left(\frac{W}{B}\right)$

$=\frac{3}{5}\times\frac{4}{6}+\frac{2}{5}\times\frac{3}{6}=\frac{18}{30}=\frac{3}{5}$

65. Answer (1)

Hint : Find the angle between $(\overrightarrow{OA}\times\overrightarrow{OB})$ and $(\overrightarrow{AB}\times\overrightarrow{AC})$.

Sol. : $\vec{n}_1=\overrightarrow{OA}\times\overrightarrow{OB}$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 1\\ 1 & 3 & 2\end{vmatrix}=-\hat{i}-3\hat{j}+5\hat{k}$$

$\vec{n}_2=\overrightarrow{AB}\times\overrightarrow{AC}$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 2 & 1\\ -3 & 0 & 1\end{vmatrix}=2\hat{i}-2\hat{j}+6\hat{k}$$

$$\cos\theta=\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

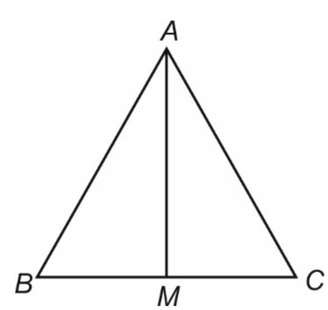
$$=\frac{-2+6+30}{\sqrt{1+9+25}\sqrt{4+4+36}}$$

$$=\frac{34}{\sqrt{35}\sqrt{44}}=\frac{17}{\sqrt{385}}$$

66. Answer (3)

Hint : Use triangle rule of vector addition

Sol. :



$\overrightarrow{AB}+\overrightarrow{BM}=\overrightarrow{AM}$ ...(1)

$\overrightarrow{AC}+\overrightarrow{CM}=\overrightarrow{AM}$ ...(2)

(1) + (2)

$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}=2\overrightarrow{AM}$

67. Answer (1)

Hint : $\vec{a},\vec{b},\vec{c}$ do not make a triangle.

Sol. : $1\le|\vec{a}+\vec{b}+\vec{c}|\le 7$

68. Answer (4)

Hint : $P_1 + \lambda P_2 = 0$

Sol. : Intersection point of

$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and xy plane

is $P(\lambda + 1, 2\lambda + 2, 3\lambda + 3)$ where $3\lambda + 3 = 0$

$\Rightarrow P(0, 0, 0)$

Plane passing through intersection of

$P_1 = 0$ and $P_2 = 0$ is

$P_1 + tP_2 = 0$

$\Rightarrow (1 + t)x + (1 - t)y + (1 + t)z + t - 1 = 0$

Passes through $P(0, 0, 0) \Rightarrow \boxed{t=1}$

$\Rightarrow$ required plane is $2x + 2z = 0$

$\Rightarrow \boxed{x+z=0}$

69. Answer (4)

Hint : $\vec{n}=\vec{b}_1\times\vec{b}_2$

Sol. : $\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 4 & 5\\ 1 & -4 & 5\end{vmatrix}=40\hat{i}-8\hat{k}$

70. Answer (2)

Hint : Use Bayes theorem

Sol. : Let B_i be the number of black balls transferred (i = 0, 1, 2, 3). B is the event of drawing a black ball. Therefore,

$P(B_0)=\frac{5}{70},\ P(B_1)=\frac{30}{70},$

$P(B_2)=\frac{30}{70},\ P(B_3)=\frac{5}{70},$

Also $P\left(\frac{B}{B_0}\right)=0, P\left(\frac{B}{B_1}\right)=\frac{1}{4},$

$P\left(\frac{B}{B_2}\right)=\frac{2}{4}, P\left(\frac{B}{B_3}\right)=\frac{3}{4},$

$\therefore$ By Bayes theorem

$$P\left(\frac{B_3}{B}\right)=\frac{\frac{5}{70}\times\frac{3}{4}}{\frac{5}{70}\times 0+\frac{30}{70}\times\frac{1}{4}+\frac{30}{70}\times\frac{2}{4}+\frac{5}{70}\times\frac{3}{4}}$$

$$=\frac{1}{7}$$

71. Answer (24)

Hint : Find $P(x = 1) + P(x = 2)$

Sol. : $P(X\le 2)=P(X=1)+P(X=2)$

$$=\frac{{}^{12}C_1\times{}^{40}C_1}{{}^{52}C_2}+\frac{{}^{12}C_2}{{}^{52}C_2}$$

$$=\frac{6\times 91}{51\times 26}=\frac{7}{17}$$

72. Answer (01)

Hint : $\sigma^2=\frac{\sum x_i^2}{N}-\left(\frac{\sum x_i}{N}\right)^2$

Sol. : $\sum(2x_i+1)^2=21n$

$\Rightarrow 4A+n+4B=29n$

$\Rightarrow A+B=7n$

Where $A=\sum_{i=1}^{n}x_i^2,\ B=\sum_{i=1}^{n}x_i$

$\sum(3x_i-1)^2=34n$

$\Rightarrow\ 9A+n-6B=34n$

$\Rightarrow\ 3A-2B=11n$

$\therefore\ A=5n,\ B=2n$

$$\sigma^2=\frac{1}{n}\sum x_i^2-\left(\frac{1}{n}\sum x_i\right)^2$$

$$=\frac{A}{n}-\frac{B^2}{n^2}=5-4=1$$

73. Answer (00)

Hint : $\vec{a}\cdot\vec{b} > 0$ (acute), $\vec{a}\cdot\vec{b} < 0$ (obtuse)

Sol. : $x^2 - x - x > 0$ …(1)

$-x < 0$ …(2)

$\cos\theta = \frac{-x}{\sqrt{1+1+x^2}}$ …(3)

$x(x-2) > 0, \because x > 0 \Rightarrow \boxed{x > 2}$ …(a)

$\frac{\pi}{2} < \theta < \frac{5\pi}{6}$

$0 > \cos\theta > \cos\frac{5\pi}{6}$

$\Rightarrow \frac{-\sqrt{3}}{2} < \frac{-x}{\sqrt{2+x^2}} < 0$

$\Rightarrow \frac{-\sqrt{3}}{2} < \frac{-x}{\sqrt{2+x^2}}$

$\Rightarrow \frac{\sqrt{3}}{2} > \frac{x}{\sqrt{2+x^2}} \Rightarrow -\sqrt{6} < x < \sqrt{6}$ …(b)

and $\frac{-x}{\sqrt{2+x^2}} < 0 \Rightarrow \boxed{x \in R - \{0\}}$ …(c)

From (a), (b) and (c), $\boxed{2 < x < \sqrt{6}}$

74. Answer (22)

Hint : Make cases

Sol. : $P(E) = \frac{{}^5C_1 \cdot {}^6C_1 + {}^5C_1 \cdot {}^3C_1 + {}^6C_1 \cdot {}^3C_1}{{}^{14}C_2}$

$= \frac{(30+15+18)}{\frac{13\cdot 14}{2}} = \frac{63}{13\cdot 7} = \frac{9}{13}$

75. Answer (04)

Hint : $\sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$

Sol. : $\bar{x}_1 = \frac{x_1 + x_2 + ... + x_n}{n}$

$\sigma^2 = \frac{(\bar{x}_1 - x_1)^2 + (\bar{x}_1 - x_2)^2 + ... + (\bar{x}_1 - x_n)^2}{n}$

$\bar{x}_2 = \frac{y_1 + y_2 + ... + y_n}{n} = 4\bar{x}_1$

$(\sigma')^2 = \frac{(\bar{x}_2 - y_1)^2 + (\bar{x}_2 - y_2)^2 + ... + (\bar{x}_2 - y_n)^2}{n}$

$= 16\frac{(\bar{x}_1 - x_1)^2 + (\bar{x}_1 - x_2)^2 + ... + (\bar{x}_1 - x_n)^2}{n}$

$\sigma'^2 = 16\sigma^2 \Rightarrow \boxed{\sigma' = 4\sigma}$

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All India Aakash Test Series for JEE (Main)-2020

TEST - 4 - Code-B

Test Date : 17/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(4)	26.	(1)	51.	(2)
2.	(2)	27.	(2)	52.	(4)
3.	(1)	28.	(2)	53.	(4)
4.	(3)	29.	(1)	54.	(1)
5.	(1)	30.	(4)	55.	(3)
6.	(4)	31.	(3)	56.	(1)
7.	(3)	32.	(2)	57.	(4)
8.	(2)	33.	(1)	58.	(2)
9.	(1)	34.	(3)	59.	(4)
10.	(3)	35.	(3)	60.	(1)
11.	(1)	36.	(2)	61.	(3)
12.	(3)	37.	(3)	62.	(2)
13.	(1)	38.	(2)	63.	(1)
14.	(2)	39.	(3)	64.	(2)
15.	(4)	40.	(2)	65.	(4)
16.	(1)	41.	(1)	66.	(4)
17.	(2)	42.	(3)	67.	(2)
18.	(2)	43.	(1)	68.	(1)
19.	(1)	44.	(3)	69.	(2)
20.	(4)	45.	(3)	70.	(4)
21.	(88)	46.	(20)	71.	(04)
22.	(07)	47.	(12)	72.	(22)
23.	(14)	48.	(02)	73.	(00)
24.	(10)	49.	(09)	74.	(01)
25.	(16)	50.	(02)	75.	(24)

PART - A (PHYSICS)

1. Answer (4)

Hint : Reading = MSR + n × LC

Sol. : $LC = \frac{0.5}{100} = 0.005\text{ mm}$

$\text{Reading} = 3 + 40 \times 0.005$

= 3.200 mm

2. Answer (2)

Hint and Sol. : $m = \frac{v_{max} - v_{min}}{v_{max} + v_{min}} = \frac{1}{3}$

3. Answer (1)

Hint : Apply KVL and KCL

Sol. : $10 - 4 - 1.2I_2 = 0$

$I_2 = 5\text{ mA}$

$4 - I_1 = 0$

$I_1 = 4\text{ mA}$

$I_z = I_2 - I_1 = 1\text{ mA}$

4. Answer (3)

Hint : $E = (B.E)_I - (B.E)_R$

Sol. : $E = E_1 - 2E_2$

5. Answer (1)

Hint : $\lambda = \frac{h}{p}$

Sol. : $p_f = \frac{h}{\lambda_1} - \frac{h}{\lambda_2}$

$\frac{h}{\lambda} = \frac{h}{\lambda_1} - \frac{h}{\lambda_2}$

$\lambda = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}$

6. Answer (4)

Hint : $F = -\frac{du}{dr}$

Sol. : $\frac{mv^2}{r} = \frac{ke^2}{r^3}$

$\text{K.E.} = \frac{ke^2}{2r^2}$

$U = -\frac{ke^2}{2r^2}$

Total energy = zero

7. Answer (3)

Hint : $t_{1/2} = \frac{\ln 2}{\lambda}$

Sol. : $N = N_0 e^{-\lambda t}$

$N = N_0 e^{-\frac{\lambda \ln 2}{2\lambda}}$

$N = \frac{N_0}{\sqrt{2}}$

$\text{Decayed} = \frac{N_0 - \frac{N_0}{\sqrt{2}}}{N_0} = \frac{\sqrt{2}-1}{\sqrt{2}}$

8. Answer (2)

Hint : $\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

Sol. : $\frac{1}{\lambda} = Rz^2\left[\frac{1}{4} - \frac{1}{16}\right] \Rightarrow \lambda = \frac{16}{3Rz^2}$

$\frac{1}{\lambda_{least}} = Rz^2\left[1 - \frac{1}{16}\right]$

$\frac{1}{\lambda_{least}} = \frac{Rz^2 15}{16}$

$\lambda_{least} = \frac{16}{15Rz^2}$

9. Answer (1)

Hint : $I = \frac{P}{4\pi d^2}$

Sol. : $I = \frac{P}{4\pi d^2}$

$\text{Pressure} = \frac{I}{L} = \frac{P}{4\pi d^2 c}$

10. Answer (3)

Hint : $\frac{hc}{\lambda} = \phi + \frac{P^2}{2m}$

Sol. : $\frac{hc}{\lambda} = \frac{3hc}{5\lambda} + \frac{h^2}{2m\lambda_1^2}$

$$\frac{2hc}{5\lambda} = \frac{h^2}{2m\lambda_1}$$

$$\lambda_1^2 = \frac{5h^2\lambda}{4hmc}$$

$$\lambda_1^2 = \frac{5h\lambda}{4mc}$$

$$\lambda_1 = \frac{1}{2}\sqrt{\frac{5h\lambda}{mc}}$$

11. Answer (1)

Hint : $0.7\,N_0 = N_0 e^{-\lambda t_0}$

Sol. : $0.7 = e^{-\lambda t_0}$

$$x = (1 - e^{-3\lambda t_0})$$

$$x = 1 - (0.7)^3$$

$$x = 1 - 0.49 \times 0.7$$

$$x = 65.7\%$$

12. Answer (3)

Hint : For polarizing angle reflected and refracted rays are mutually perpendicular.

Sol. : $\frac{\sin\theta_p}{\cos\theta_p} = \mu$

$$\tan\theta_p = \mu$$

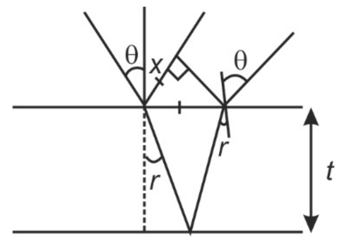
$$\sin\theta_c = \frac{1}{\mu}$$

$$\tan\theta_p \sin\theta_c = 1$$

13. Answer (1)

Hint : Effective phase difference must be $2n\pi$

Sol. : $2\mu t \sec r - x = \frac{\lambda}{2}$



$$x = 2t\tan r \sin\theta$$

$$\frac{2\mu t}{\cos r} - \frac{2t\sin r\sin\theta}{\cos r} = \frac{\lambda}{2}$$

$$\frac{2t}{\cos r}\left[\mu - \frac{\sin^2\theta}{\mu}\right] = \frac{\lambda}{2}$$

$$\Rightarrow \frac{2t\mu[\mu^2 - \sin^2\theta]}{\mu\sqrt{\mu^2 - \sin^2\theta}} = \frac{\lambda}{2}$$

$$\Rightarrow t = \frac{\lambda}{4\sqrt{\mu^2 - \sin^2\theta}}$$

14. Answer (2)

Hint : $I = 4I_o \cos^2\frac{\phi}{2}$

Sol. : $\phi = \frac{\pi}{3}$

$$y_1 = \frac{\lambda D}{6d}$$

$$y_2 = -\frac{\lambda D}{6d}$$

15. Answer (4)

Hint : $Y = \overline{(\overline{A+B})\cdot(A+B)}$

Sol. : $Y = (A+B) + (\overline{A+B}) = 1$

16. Answer (1)

Hint : $E \propto \frac{1}{n^2}$

Sol. : $E_n \propto \frac{1}{n^2}$

$$E_n - E_{n-1} = k\left[\frac{1}{n^2} - \frac{1}{(n-1)^2}\right]$$

$$\frac{hc}{\lambda} = \frac{1}{n^3}$$

$$\lambda \propto n^3$$

17. Answer (2)

Hint : Fringes formed will be symmetric on screen

Sol. : Number of Fringes are $\left[\frac{d}{\lambda}\right] = 5$.

18. Answer (2)

Hint : $\frac{1}{2}mv^2 = \frac{k\,60e^2}{d}$

Sol. : $(mv)^2 = \frac{120\,ke^2m}{d}$

$\lambda = h\sqrt{\frac{d}{120\,mke^2}}$

19. Answer (1)

Hint : $hv = \phi + eV_0$

Sol. : $\phi = 5.2$ eV

$E = 6.2$ eV

$V_0 = 1$ V

$\frac{ne}{4\pi\varepsilon_0 r} = 1$

$\frac{9\times10^9\times n\times1.6\times10^{-19}}{2\times10^{-2}} = 1$

$n = \frac{10^{10}\times2\times10^{-2}}{9\times1.6} = 1.38\times10^7 \approx 1.4\times10^7$

20. Answer (4)

Hint : $A = \lambda N$

Sol. : $3\lambda_1 N_A = \lambda_2 N_B$

$\frac{3\times2}{T_1} = \frac{1}{T_2}$

$T_2 = \frac{T_1}{6}$

21. Answer (88)

Hint : Analyse the circuit for half of the cycle

Sol. : $P = \frac{1}{2}\times\frac{V_0^2}{2R}\left[\frac{1}{2}+\frac{4}{11}\right] = \frac{19V_0^2}{88R}$

22. Answer (07)

Hint : Path difference will be 3λ corresponding to that point.

Sol. : $4\lambda\cos\theta = 3\lambda$

$\theta = \cos^{-1}\left(\frac{3}{4}\right)$

$\tan\theta = \frac{y}{D}$

$\frac{\sqrt{7}}{3} = y$

23. Answer (14)

Hint : For intensity to be $\frac{1}{4}$th

$\phi = 2n\pi \pm \frac{2\pi}{3}$

Sol. : $5\lambda\cos\theta = \left(5\lambda - \frac{\lambda}{3}\right)$

$5\lambda\cos\theta = \frac{14\lambda}{3}$

$\theta = \cos^{-1}\left(\frac{14}{15}\right)$

24. Answer (10)

Hint : It will vary from $f_c - f_m$ to $f_c + f_m$

Sol. : $\Delta f = 2f_m = 10$ kHz

25. Answer (16)

Hint : $\beta = \frac{I_C}{I_B}$

Sol. : $\frac{I_C R_C}{I_b R_b} = \frac{V_0}{V_i}$

$V_i = \frac{2\times R_b}{R_C\times50}$

$V_i = 16$ mV

PART - B (CHEMISTRY)

26. Answer (1)

Hint : Teflon

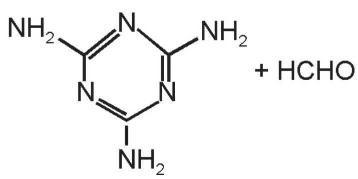
Sol. :

$$CF_2 = CF_2 \xrightarrow{\text{Polymerisation}} \left[CF_2 - CF_2 \right]_n$$

27. Answer (2)

Hint : Melamine and formaldehyde

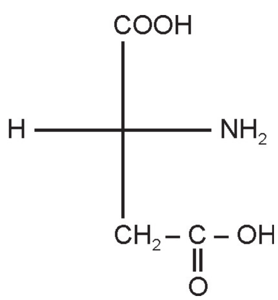
Sol. :



28. Answer (2)

Hint : $\text{Glucose} \xrightarrow{HNO_3} \text{Saccharic acid}$

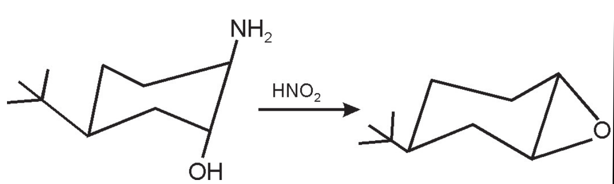
Sol. : Aspartic acid



29. Answer (1)

Hint : When NH_2 and OH are at axial position then form compound P.

Sol. :



30. Answer (4)

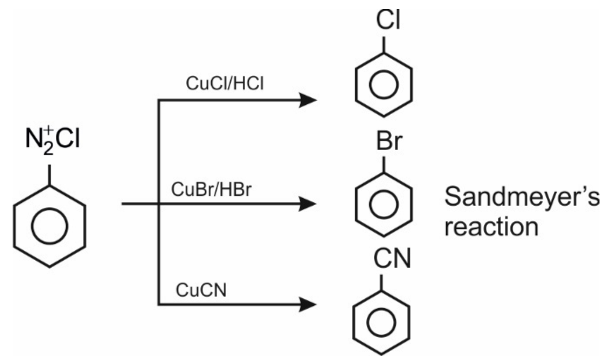
Hint : As positive charge on carbonyl carbon decrease, rate of reaction decrease.

Sol. : sp^2 'N' in 3 membered ring and bridge head is not possible.

31. Answer (3)

Hint : (iv), (v), (vi) are not Sandmeyer's reaction

Sol. :



32. Answer (2)

Hint : B.P of isomeric amine 1°> 2° > 3°

Sol. : A $\rightarrow$ 1° amine

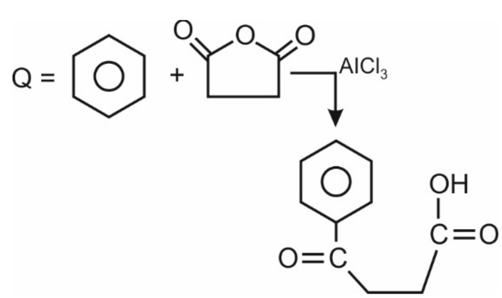
B $\rightarrow$ 2° amine

C $\rightarrow$ 3° amine

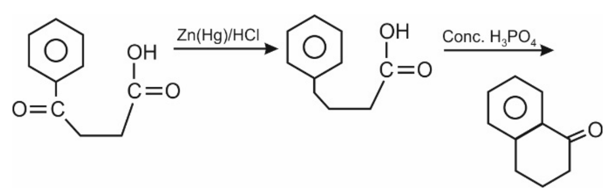
Due to less extent of H–Bonding in 3° amine, B.P. of 3° amine is minimum.

33. Answer (1)

Hint :

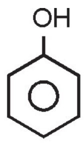


Sol. :



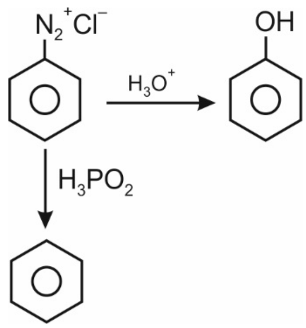
34. Answer (3)

Hint :

Q = 

R = 

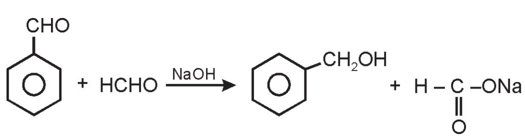
Sol. :



35. Answer (3)

Hint : Reaction is intermolecular, it is a Cannizzaro reaction.

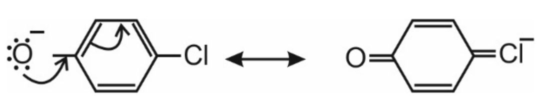
Sol . :



36. Answer (2)

Hint : p-chlorophenol is most acidic

Sol. :



37. Answer (3)

Hint : 4-methylpent-3-en-2-one

Sol. :

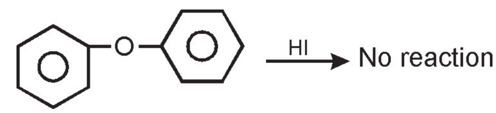
5 4 3 2 1

$(CH_3)_2C = CHCOCH_3$ 4-methylpent-3-en-2-one

38. Answer (2)

Hint : B not cleaved with HI

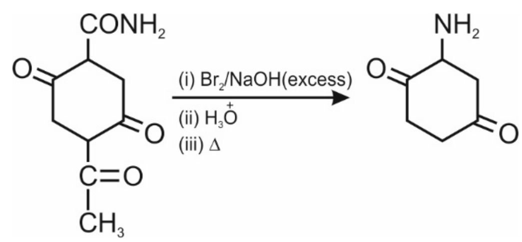
Sol. :



39. Answer (3)

Hint : β–Keto acid show decarboxylation on heating.

Sol. :



40. Answer (2)

Hint : A and D → α

Sol. : CH_2OH at C–5 and OH at C–1 decide the α or β nomenclature.

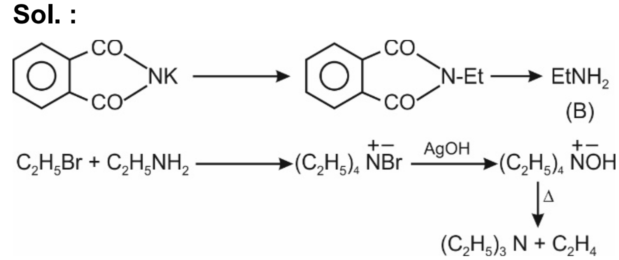
CH_2OH and OH same side → β-form

CH_2OH and OH opposite side → α-form

41. Answer (1)

Hint : Gabriel phthalimide synthesis

Sol. :

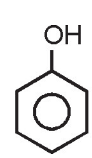


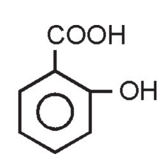
42. Answer (3)

Hint : Carbylamine test is given by primary amines.

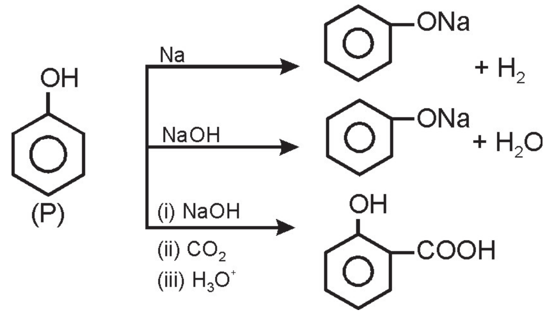
Sol. : Carbylamine test is not given by secondary amines.

43. Answer (1)

Hint : P = 

Q = 

Sol. :



44. Answer (3)

Hint : Hemiacetal can reduce Tollen's reagent

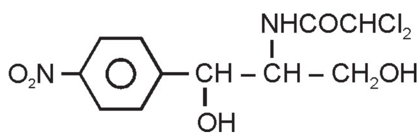
Sol. : If A give positive iodoform test so it must contain $CH_3-\underset{\underset{O}{\|}}{C}$ group and A does not reduce Tollen's reagent it means A not contain $-\underset{\underset{O}{\|}}{C}-H$ or hemiacetal

45. Answer (3)

Hint : A strong acid can produce weak acid.

Sol. : Acidic strength of carbolic acid (Phenol) is less than H_2CO_3 so, it not produce CO_2.

46. Answer (20)

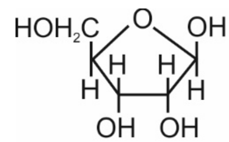
Hint : 

Sol. : Chloramphenicol has two –OH groups, two chiral carbon and zero –COOH.

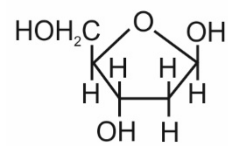
5(2 + 0 + 2) = 20

47. Answer (12)

Hint : β - D ribose has 4 chiral centre.

Sol. : 

β - D - ribose



β - D - 2 deoxyribose

xy = 4 × 3 = 12

48. Answer (02)

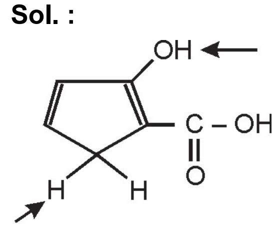
Hint : Glyptal is used in manufacture of paints and lacquers.

Sol. : Polyester is a condensation polymer. Two statements are incorrect (ii) & (vii)

49. Answer (09)

Hint : CH_3MgBr react with acidic hydrogen

Sol. :



50. Answer (02)

Hint : PCC oxidises alcohol to carbonyl group.

Sol. : MnO_2 oxidises allylic alcohol.

PART - C (MATHEMATICS)

51. Answer (2)

Hint : Use Bayes theorem

Sol. : Let B_i be the number of black balls transferred (i = 0, 1, 2, 3). B is the event of drawing a black ball. Therefore,

$$P(B_0)=\frac{5}{70},\ P(B_1)=\frac{30}{70},$$

$$P(B_2)=\frac{30}{70},\ P(B_3)=\frac{5}{70},$$

Also $P\left(\frac{B}{B_0}\right)=0, P\left(\frac{B}{B_1}\right)=\frac{1}{4},$

$$P\left(\frac{B}{B_2}\right)=\frac{2}{4}, P\left(\frac{B}{B_3}\right)=\frac{3}{4},$$

∴ By Bayes theorem

$$P\left(\frac{B_3}{B}\right) = \frac{\frac{5}{70}\times\frac{3}{4}}{\frac{5}{70}\times 0+\frac{30}{70}\times\frac{1}{4}+\frac{30}{70}\times\frac{2}{4}+\frac{5}{70}\times\frac{3}{4}}$$

$$=\frac{1}{7}$$

52. Answer (4)

Hint : $\vec{n} = \vec{b}_1 \times \vec{b}_2$

Sol. : $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 5 \\ 1 & -4 & 5 \end{vmatrix} = 40\hat{i} - 8\hat{k}$

53. Answer (4)

Hint : $P_1 + \lambda P_2 = 0$

Sol. : Intersection point of

$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and xy plane

is $P(\lambda + 1, 2\lambda + 2, 3\lambda + 3)$ where $3\lambda + 3 = 0$

$\Rightarrow P(0, 0, 0)$

Plane passing through intersection of

$P_1 = 0$ and $P_2 = 0$ is

$P_1 + tP_2 = 0$

$\Rightarrow (1 + t)x + (1 - t)y + (1 + t)z + t - 1 = 0$

Passes through $P(0, 0, 0) \Rightarrow \boxed{t=1}$

$\Rightarrow$ required plane is $2x + 2z = 0$

$\Rightarrow \boxed{x+z=0}$

54. Answer (1)

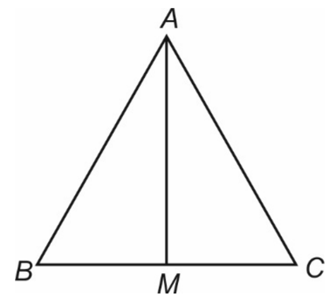
Hint : $\vec{a}, \vec{b}, \vec{c}$ do not make a triangle.

Sol. : $1 \le |\vec{a}+\vec{b}+\vec{c}| \le 7$

55. Answer (3)

Hint : Use triangle rule of vector addition

Sol. :



$\overrightarrow{AB} + \overrightarrow{BM} = \overrightarrow{AM}$ ...(1)

$\overrightarrow{AC} + \overrightarrow{CM} = \overrightarrow{AM}$ ...(2)

(1) + (2)

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = 2\overrightarrow{AM}$

56. Answer (1)

Hint : Find the angle between $(\overrightarrow{OA} \times \overrightarrow{OB})$ and $(\overrightarrow{AB} \times \overrightarrow{AC})$.

Sol. : $\vec{n}_1 = \overrightarrow{OA} \times \overrightarrow{OB}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 3 & 2 \end{vmatrix} = -\hat{i} - 3\hat{j} + 5\hat{k}$$

$\vec{n}_2 = \overrightarrow{AB} \times \overrightarrow{AC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ -3 & 0 & 1 \end{vmatrix} = 2\hat{i} - 2\hat{j} + 6\hat{k}$$

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{-2+6+30}{\sqrt{1+9+25}\sqrt{4+4+36}}$$

$$= \frac{34}{\sqrt{35}\sqrt{44}} = \frac{17}{\sqrt{385}}$$

57. Answer (4)

Hint : Use total probability theorem

Sol. : Required probability

$$= P(W)\cdot P\left(\frac{W}{W}\right) + P(B)\cdot P\left(\frac{W}{B}\right)$$

$$= \frac{3}{5}\times\frac{4}{6} + \frac{2}{5}\times\frac{3}{6} = \frac{18}{30} = \frac{3}{5}$$

58. Answer (2)

Hint : Break 5 into 3 non-zero parts

Sol. : $n(E) = 3^5 - {}^3C_1(2^5 - 2) - {}^3C_2$

$n(S) = 3^5$

$P = \frac{n(E)}{n(S)} = \frac{50}{81}$

59. Answer (4)

Hint : Find the d.r. of lines

Sol. : First line is $\frac{x-10}{2} = y = \frac{z-3}{-1}$ and second line is $\frac{x-5}{-3} = \frac{y-2}{7} = z$ as

$2 \times (-3) + 1 \times 7 + (-1) \times 1 = 0$

$\Rightarrow L_1$ is perpendicular to L_2

60. Answer (1)

Hint : Find the d.r. of line.

Sol. : Any point on line is $(-\lambda + 2, -3\lambda - 1, 2\lambda + 3)$ which also passes through $(1, 1, -2)$

$\therefore$ d.r. of line = $(-\lambda + 1, -3\lambda - 2, 2\lambda + 5)$

This line is parallel to the given plane.

$\Rightarrow (-\lambda + 1)(-2) + (-3\lambda - 2)(1) + (2\lambda + 5)2 = 0$

$\Rightarrow 3\lambda + 6 = 0 \Rightarrow \lambda = -2$

$\therefore$ d.r. = (3, 4, 1)

$\therefore$ Equation of line: $\frac{x-1}{3} = \frac{y-1}{4} = \frac{z+2}{1}$

61. Answer (3)

Hint : Use the concept of family of planes

Sol. : Required plane is

$(x + y + z - 3) + \lambda(3x - 2y + z + 5) = 0$

Parallel to xz plane

$\Rightarrow 0(1 + 3\lambda) + 1(1 - 2\lambda) + 0(1 + \lambda) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

$\therefore$ Plane:

$(x + y + z - 3) + \frac{1}{2}(3x - 2y + z + 5) = 0$

$\Rightarrow 5x + 3z - 1 = 0$

62. Answer (2)

Hint : $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Sol. : $|\vec{a}| = a, \quad |\vec{b}| = b$

$ab(1 + \cos\theta) = 12$

$ab(1 - \cos\theta) = 6$

$\Rightarrow \frac{1+\cos\theta}{1-\cos\theta} = 2$

$\Rightarrow \cos\theta = \frac{1}{3}$ and $ab = 9$

$\therefore |\vec{a} \times \vec{b}| = ab|\sin\theta|$

$\therefore$ option (2) can be $\vec{a} \times \vec{b}$

63. Answer (1)

Hint : Make cases

Sol. : Required probability

$= \frac{{}^5C_2({}^5C_2 + {}^5C_2 + {}^5C_3)}{{}^5C_2 \times {}^7C_3}$

$= \frac{10 + 10 + 10}{35} = \frac{6}{7}$

64. Answer (2)

Hint : variance = $\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$

Sol. : $\sum_{i=1}^{5} x_i = 20 \times 5 = 100$

$\Rightarrow \sum_{i=1}^{6} x_i = 100 - 100 = 0$

$\frac{\sum_{i=1}^{5} x_i^2}{5} - (20)^2 = 16$

$\Rightarrow \sum_{i=1}^{5} x_i^2 = 2080$

$\therefore \sum_{i=1}^{6} x_i^2 = 2080 + (-100)^2 = 12080$

Variance $= \dfrac{\sum_{i=1}^{6} x_i^2}{6} - \left(\dfrac{\sum_{i=1}^{6} x_i}{6}\right)^2$

$= \dfrac{12080}{6} = \dfrac{6040}{3}$

65. Answer (4)

Hint : Proceed with parametric form of point

Sol. : Point Q on the line = $(-5k - 9, k + 2, 2k + 5)$

Q lies on plane

$\Rightarrow -5k - 9 + k + 2 + 2k + 5 = 2$

$\Rightarrow k = -2$

$\therefore\ Q = (1, 0, 1)$

Distance, $PQ = \sqrt{5^2 + 1^2 + 2^2} = \sqrt{30}$

66. Answer (4)

Hint : d.r. of line segment is same as that of normal to the plane

Sol. : Equation of plane

$(x-5)2 + (x-0)6 + (y-2)10 = 0$

$\Rightarrow x + 3y + 5z = 0$

67. Answer (2)

Hint : Find the probability of complementary event

Sol. : Required probability

$= 1 - \left(\dfrac{2}{3} \times \dfrac{2}{3} \times \dfrac{3}{4} \times \dfrac{3}{4}\right) = \dfrac{3}{4}$

68. Answer (1)

Hint : Use section formula

Sol. : $A = (2, 4, 5)$, $B = (-2, 0, 1)$

$P\left(\dfrac{-2\lambda + 2}{\lambda + 1}, \dfrac{4}{\lambda + 1}, \dfrac{\lambda + 5}{\lambda + 1}\right)$

$\Rightarrow \lambda + 33 = 2\lambda + 2$

$\Rightarrow \lambda = 31$

$\Rightarrow 31 : 1$

69. Answer (2)

Hint : $(\vec{a} + 2\vec{b}) \cdot \vec{c} = 0$

Sol. : $\vec{a} = 3\vec{b}$

$\Rightarrow p\hat{i} + 2\hat{j} + 3\hat{k} = 3\hat{i} - 3q\hat{j} + 3\hat{k}$

$\Rightarrow p = 3,\ q = -\dfrac{2}{3}$

$(\vec{a} + 2\vec{b}) = (p+2)\hat{i} + (2-2q)\hat{j} + (3+2)\hat{k}$

$= 5\hat{i} + \dfrac{2}{3}\hat{j} + 5\hat{k}$

$(\vec{a} + 2\vec{b}) \cdot \vec{c} = 0$

$\Rightarrow r = \dfrac{-13}{9}$

$(p, q, r) = \left(3, \dfrac{-2}{3}, \dfrac{-13}{9}\right)$

70. Answer (4)

Hint : Find the d.r. of normal to the plane

Sol. : $\begin{vmatrix} x-1 & y+2 & z-1 \\ 1 & 2 & 3 \\ 2 & -1 & 5 \end{vmatrix} = 0$

$\Rightarrow 13(x-1) + (y+2) - 5(z-1) = 0$

$\Rightarrow 13x + y - 5z = 6$

71. Answer (04)

Hint : $\sigma^2 = \dfrac{1}{n}\sum (x_i - \bar{x})^2$

Sol. : $\bar{x}_1 = \dfrac{x_1 + x_2 + \ldots + x_n}{n}$

$\sigma^2 = \dfrac{(\bar{x}_1 - x_1)^2 + (\bar{x}_1 - x_2)^2 + \ldots + (\bar{x}_1 - x_n)^2}{n}$

$\bar{x}_2 = \dfrac{y_1 + y_2 + \ldots + y_n}{n} = 4\bar{x}_1$

$(\sigma')^2 = \dfrac{(\bar{x}_2 - y_1)^2 + (\bar{x}_2 - y_2)^2 + \ldots + (\bar{x}_2 - y_n)^2}{n}$

$= 16\dfrac{(\bar{x}_1 - x_1)^2 + (\bar{x}_1 - x_2)^2 + \ldots + (\bar{x}_1 - x_n)^2}{n}$

$\sigma'^2 = 16\sigma^2 \Rightarrow \boxed{\sigma' = 4\sigma}$

72. Answer (22)

Hint : Make cases

Sol. : $P(E) = \dfrac{{}^5C_1 \cdot {}^6C_1 + {}^5C_1 \cdot {}^3C_1 + {}^6C_1 \cdot {}^3C_1}{{}^{14}C_2}$

$$= \frac{(30+15+18)}{\frac{13\cdot 14}{2}} = \frac{63}{13\cdot 7} = \frac{9}{13}$$

73. Answer (00)

Hint : $\vec{a}\cdot\vec{b} > 0$ (acute), $\vec{a}\cdot\vec{b} < 0$ (obtuse)

Sol. : $x^2 - x - x > 0$ …(1)

$-x < 0$ …(2)

$\cos\theta = \dfrac{-x}{\sqrt{1+1+x^2}}$ …(3)

$x(x-2) > 0, \;\because\; x > 0 \Rightarrow \boxed{x > 2}$ …(a)

$\dfrac{\pi}{2} < \theta < \dfrac{5\pi}{6}$

$0 > \cos\theta > \cos\dfrac{5\pi}{6}$

$\Rightarrow \dfrac{-\sqrt{3}}{2} < \dfrac{-x}{\sqrt{2+x^2}} < 0$

$\Rightarrow \dfrac{-\sqrt{3}}{2} < \dfrac{-x}{\sqrt{2+x^2}}$

$\Rightarrow \dfrac{\sqrt{3}}{2} > \dfrac{x}{\sqrt{2+x^2}} \Rightarrow -\sqrt{6} < x < \sqrt{6}$ …(b)

and $\dfrac{-x}{\sqrt{2+x^2}} < 0 \Rightarrow \boxed{x \in R - \{0\}}$ …(c)

From (a), (b) and (c), $\boxed{2 < x < \sqrt{6}}$

74. Answer (01)

Hint : $\sigma^2 = \dfrac{\sum x_i^2}{N} - \left(\dfrac{\sum x_i}{N}\right)^2$

Sol. : $\sum (2x_i + 1)^2 = 21n$

$\Rightarrow 4A + n + 4B = 29n$

$\Rightarrow A + B = 7n$

Where $A = \sum_{i=1}^{n} x_i^2,\; B = \sum_{i=1}^{n} x_i$

$\sum (3x_i - 1)^2 = 34n$

$\Rightarrow 9A + n - 6B = 34n$

$\Rightarrow 3A - 2B = 11n$

$\therefore A = 5n,\; B = 2n$

$\sigma^2 = \dfrac{1}{n}\sum x_i^2 - \left(\dfrac{1}{n}\sum x_i\right)^2$

$= \dfrac{A}{n} - \dfrac{B^2}{n^2} = 5 - 4 = 1$

75. Answer (24)

Hint : Find $P(x = 1) + P(x = 2)$

Sol. : $P(X \le 2) = P(X = 1) + P(X = 2)$

$$= \frac{{}^{12}C_1 \times {}^{40}C_1}{{}^{52}C_2} + \frac{{}^{12}C_2}{{}^{52}C_2}$$

$$= \frac{6\times 91}{51\times 26} = \frac{7}{17}$$

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