11/01/2019
Evening

Aakash
Medical | IIT-JEE | Foundations
(Divisions of Aakash Educational Services Limited)
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Time : 3 hrs. **M.M. : 360**

Answers & Solutions

for

JEE (MAIN)-2019

(Online CBT Mode)

(Physics, Chemistry and Mathematics)

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test consists of **90** questions. The maximum marks are **360.**
3. There are ***three*** parts consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 **(four)** marks for each correct response.
4. *Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for a question in the answer sheet.*
5. There is only one correct response for each question.

PHYSICS

1. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^{3} A/m is applied. Its magnetic susceptibility is
   (1) 3.3×10^{-2}
   (2) 2.3×10^{-2}
   (3) 3.3×10^{-4}
   (4) 4.3×10^{-2}

Answer (3)

Sol. $M = \chi H$

$$\Rightarrow \quad \chi = \frac{20\times10^{-6}}{60\times10^{3}\times10^{-6}}$$

$= 3.3 \times 10^{-4}$

2. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} Cm. What is the potential energy of the electric dipole?
   (1) -9×10^{-20} J (2) -10×10^{-29} J
   (3) -7×10^{-27} J (4) -20×10^{-18} J

Answer (3)

Sol. $U = -\vec{p}\cdot\vec{E}$

$= -pE\cos45°$

$$= -10^{-29}\times10^{3}\times\frac{1}{\sqrt{2}}$$

$U = -7 \times 10^{-27}$ J

3. A particle of mass m is moving in a straight line with momentum p. Starting at time $t = 0$, a force $F = kt$ acts in the same direction on the moving particle during time interval T so that its momentum changes from p to $3p$. Here k is a constant. The value of T is
   (1) $\sqrt{\frac{2k}{p}}$
   (2) $2\sqrt{\frac{p}{k}}$
   (3) $\sqrt{\frac{2p}{k}}$
   (4) $2\sqrt{\frac{k}{p}}$

Answer (2)

Sol. $F = kt$

$$\frac{dp}{dt} = kt$$

$$\int_{p}^{3p} dp = k\int_{0}^{t} t\,dt$$

$$2p = \frac{kt^2}{2}$$

$$t = 2\sqrt{\frac{p}{k}}$$

4. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water? [Specific Heat Capacities of water and metal are, respectively, 4200 $Jkg^{-1}K^{-1}$ and 400 $Jkg^{-1}K^{-1}$]
   (1) 25% (2) 20%
   (3) 30% (4) 15%

Answer (2)

Sol. Heat lost = Heat gained

Let final temperature be T

$0.1(500 - T) \times 400 = (800 + 0.5 \times 4200)[T - 30]$

$$(500 - T) = \frac{2900}{40}[T - 30]$$

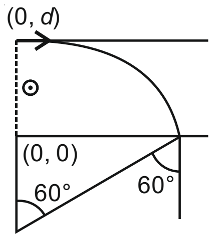
$T = 36.39$

$$\% \text{ increase} = \frac{36.39 - 30}{30} \simeq 20\%$$

5. The region between $y = 0$ and $y = d$ contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v} = v\hat{i}$. if $d = \frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is
   (1) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$ (2) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$
   (3) $\frac{qvB}{m}\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$ (4) $\frac{qvB}{m}\left(\frac{-\hat{j} + \hat{i}}{\sqrt{2}}\right)$

Answer (No option is correct) [BONUS]

Sol. Assuming particle enters from (0, *d*)



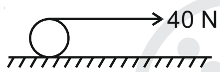
$$F = qvB(-\sin 60°\hat{i} - \cos 60°\hat{j})$$

$$F = -\frac{qvB}{2}\left(\sqrt{3}\hat{i} + \hat{j}\right)$$

$$a = -\frac{qvB}{2m}\left(\sqrt{3}\hat{i} + \hat{j}\right)$$

None of the option is correct.

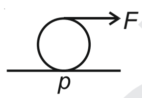
6. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) :



(1) 16 rad/s^2
(2) 20 rad/s^2
(3) 12 rad/s^2
(4) 10 rad/s^2

Answer (1)

Sol. $\tau_p = I_p\alpha$



$F(2R) = 2MR^2\alpha$

$$\alpha = \frac{F}{MR} = \frac{40}{0.5\times 5}$$

$= 16$ rad/s^2

7. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by

(1) 10^{-3} rad/s
(2) 10^{-1} rad/s
(3) 10^{-5} rad/s
(4) 1 rad/s

Answer (1)

Sol. $T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

$$\omega = \sqrt{\frac{g_{eff}}{l}}$$

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{eff}}{g_{eff}}$$

$$= \frac{1}{2}\frac{(2\omega^2 A)}{g}$$

$$\frac{\Delta\omega}{\omega} = 10^{-3}\text{ rad/s}$$

8. If speed (*V*), acceleration (*A*) and force (*F*) are considered as fundamental units, the dimension of Young's modulus will be

(1) $V^{-2}A^2F^{-2}$
(2) $V^{-2}A^2F^2$
(3) $V^{-4}A^2F$
(4) $V^{-4}A^{-2}F$

Answer (3)

Sol. $V = L^1T^{-1}$

$A = L^1T^{-2}$

$F = M^1L^1T^{-2}$

$$Y = \frac{\text{Force}}{\text{Area}}$$

$Y = M^1L^{-1}T^{-2}$

$[M^1L^{-1}T^{-2}] = [F]^\alpha [A]^\beta [V]^\gamma$

$\alpha = 1$, $\beta = 2$, $\gamma = -4$

9. When 100 g of a liquid *A* at 100°C is added to 50 g of a liquid *B* at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid *A* at 100°C is added to 50 g of liquid *B* at 50°C, will be

(1) 85°C
(2) 80°C
(3) 70°C
(4) 60°C

Answer (2)

Sol. $100\ S_1\ (100 - 90) = 50\ S_2\ (90 - 75)$

$20\ S_1 = 15\ S_2$

$4\ S_1 = 3\ S_2$

Let final temperature be T

$100\ S_1\ (100 - T) = 50\ S_2\ (T - 50)$

$75\ S_2\ (100 - T) = 50\ S_2\ (T - 50)$

$3(100 - T) = 2T - 100$

$T = 80$°C

10. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by:

[Given permittivity of space $\epsilon_0 = 9\times 10^{-12}$ SI units, Speed of light $c = 3\times 10^8$ m/s]

(1) 1.4 kV/m
(2) 1 kV/m
(3) 2 kV/m
(4) 0.7 kV/m

Answer (1)

Sol. $\frac{1}{2}\epsilon_0 E_0^2 = I$

$$I = \frac{P}{Ac}$$

$$E_0^2 \times \frac{1}{2}\times 9\times 10^{-12} = \frac{27\times 10^{-3}}{A\times 3\times 10^8}$$

$$E_0^2 = \frac{9\times10^{-8}\times2\times10^{-3}}{9\times10^{-12}\times10^{-5}}$$

$$E_0^2 = 2\times10^6$$

E_0 = 1.4 × 10^3 V/m

= 1.4 kV/m

11. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2 s. The period of oscillation of the same pendulum on the planet would be:

(1) $\frac{2}{\sqrt{3}}$ s (2) $\frac{3}{2}$ s

(3) $\frac{\sqrt{3}}{2}$ s (4) $2\sqrt{3}$ s

Answer (4)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$$

$$T_2 = 2\sqrt{\frac{g_1}{g_2}}$$

$$g_2 = \frac{3GM}{(3R)^2} = \frac{g}{3}$$

$$T_2 = 2\sqrt{3} \text{ s}$$

12. In a hydrogen like atom, when an electron jumps from the *M*-shell to the *L*-shell, the wavelength of emitted radiation is λ. If an electron jumps from *N*-shell to the *L*-shell, the wavelength of emitted radiation will be:

(1) $\frac{25}{16}\lambda$ (2) $\frac{16}{25}\lambda$

(3) $\frac{20}{27}\lambda$ (4) $\frac{27}{20}\lambda$

Answer (3)

Sol. $\frac{1}{\lambda} = RZ^2\left[\frac{1}{4}-\frac{1}{9}\right] = \frac{5RZ^2}{36}$

$$\frac{1}{\lambda_1} = RZ^2\left[\frac{1}{4}-\frac{1}{16}\right] = \frac{3RZ^2}{16}$$

$$\lambda_1 = \frac{16}{3RZ^2},\ \lambda = \frac{36}{5RZ^2}$$

$$\frac{\lambda_1}{\lambda} = \frac{16\times5}{3\times36} = \frac{20}{27}$$

$$\lambda_1 = \frac{20}{27}\lambda$$

13. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to: $\left(\frac{hc}{e} = 1240 \text{ nm-V}\right)$

(1) 1.0 V (2) 2.0 V

(3) 1.5 V (4) 0.5 V

Answer (1)

Sol. $h\nu = \phi + eV_0$

$$\frac{hc}{\lambda_1} = \phi + eV_1$$

$$\frac{hc}{\lambda_2} = \phi + eV_2$$

$$1240\left[\frac{1}{300}-\frac{1}{400}\right] = e(V_1 - V_2)$$

$(V_1 - V_2) \approx 1.0$ V

14. Two rods *A* and *B* of identical dimensions are at temperature 30°C. If *A* is heated upto 180°C and *B* upto *T*°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of *A* and *B* is 4 : 3, then the value of *T* is:

(1) 270°C

(2) 230°C

(3) 250°C

(4) 200°C

Answer (2)

Sol. $\Delta\ell = \ell_0\,\alpha\,(\Delta T)$

$\alpha_A(180 - 30) = \alpha_B(T - 30)$

$4(180 - 30) = 3(T - 30)$

T = 230°C

15. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation $VT = K$, where K is a constant. In this process, the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (R is gas constant):

(1) $\frac{3}{2}R\Delta T$

(2) $\frac{1}{2}R\Delta T$

(3) $\frac{2K}{3}\Delta T$

(4) $\frac{1}{2}KR\Delta T$

Answer (2)

Sol. $VT = K$

$PV = RT$

$\Rightarrow V \cdot PV = K'$

$PV^2 = K'$

$$\Rightarrow C = \frac{3R}{2} - \frac{R}{2-1} = \frac{R}{2}$$

$$\Delta Q = nC\Delta T = \frac{1}{2}R\Delta T$$

16. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is:

(1) 100 Ω (2) 125 Ω

(3) 80 Ω (4) 120 Ω

Answer (3)

Sol. Full scale deflection current, $I_g = 30 \times 0.005 = 0.15$ A

$15 = (20 + R) \times 0.15$

$\Rightarrow R = 80\ \Omega$

17. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $x_0/2$?

(1) 40 (2) 60

(3) 35 (4) 25

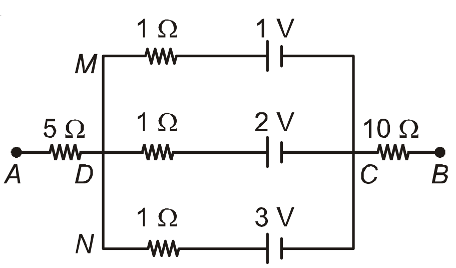
Answer (4)

Sol. $\frac{2x_0}{3} = 100°\text{C}$

$$\frac{x_0}{2} = \frac{x_0}{3} + \frac{x_0}{6}$$

$$= 0° + \frac{1}{6} \times \frac{3 \times 100}{2} = 25°\text{C}$$

18. In the circuit shown, the potential difference between A and B is:



(1) 6 V (2) 3 V

(3) 2 V (4) 1 V

Answer (3)

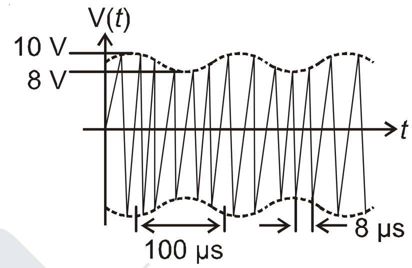
Sol. $\frac{1}{r_{eq}} = 3$

$$r_{eq} = \frac{1}{3}\ \Omega$$

$$\frac{E_{eq} \times 3}{1} = \frac{1}{1} + \frac{2}{1} + \frac{3}{1} = 6$$

$\Rightarrow E_{eq} = 2$ V

19. An amplitude modulated signal is plotted below:



Which one of the following best describes the above signal?

(1) $(9 + \sin(2\pi \times 10^4 t))\sin(2.5\pi \times 10^5 t)$ V

(2) $(9 + \sin(4\pi \times 10^4 t))\sin(5\pi \times 10^5 t)$ V

(3) $(1 + 9\sin(2\pi \times 10^4 t))\sin(2.5\pi \times 10^5 t)$ V

(4) $(9 + \sin(2.5\pi \times 10^5 t))\sin(2\pi \times 10^4 t)$ V

Answer (1)

Sol. $C_m = (A_C + A_m \sin\omega_m t)\sin\omega_C t$

From the graph $A_C + A_m = 10$

$A_C - A_m = 8$

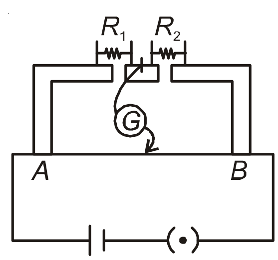
$\Rightarrow A_C = 9$ V, $A_m = 1$ V

$$\omega_m = \frac{2\pi}{100 \times 10^{-6}} = 2\pi \times 10^4\ \text{s}^{-1}$$

$$\omega_C = \frac{2\pi}{8 \times 10^{-6}} = 2.5\pi \times 10^5\ \text{s}^{-1}$$

$\Rightarrow C_m = (9 + \sin 2\pi \times 10^4 t)\sin(2.5\pi \times 10^5 t)$ V

20. In the experimental set up of metre bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10 Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)$ Ω such that the null point shifts back to its initial position is:



(1) 60 Ω (2) 30 Ω

(3) 40 Ω (4) 20 Ω

Answer (1)

Sol. $\dfrac{R_1}{R_2} = \dfrac{40}{60} = \dfrac{2}{3}$

$$\frac{R_1 + 10}{R_2} = 1$$

$$\Rightarrow \quad \frac{R_1}{R_1 + 10} = \frac{2}{3}$$

$\Rightarrow \quad R_1 = 20\ \Omega$

Now $\dfrac{30 \times R}{30 + R} = 20$

$\Rightarrow \quad 30R = 600 + 20R$

$\Rightarrow \quad R = 60\ \Omega$

21. A particle of mass m and charge q is in an electric and magnetic field given by

$$\vec{E} = 2\hat{i} + 3\hat{j}\ ;\ \vec{B} = 4\hat{j} + 6\hat{k}$$

The charged particle is shifted from the origin to the point $P(x = 1\ ;\ y = 1)$ along a straight path. The magnitude of the total work done is:

(1) $(0.15)q$

(2) $5q$

(3) $(0.35)q$

(4) $(2.5)q$

Answer (2)

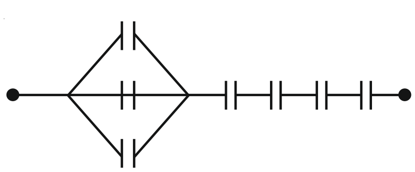
Sol. The straight path from origin to $P(x = 1, y = 1)$ is $y = x$

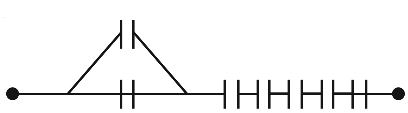
Work is done by electric force only

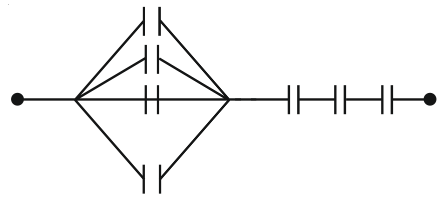
$$W = q\int \vec{E} \cdot \overrightarrow{dr} = q\int_0^1 2dx + q3\int_0^1 dy$$

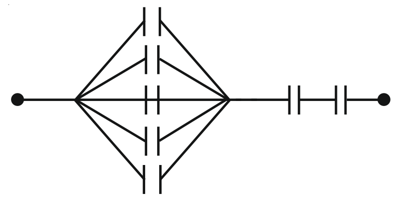
$= 2q + 3q = 5q$

22. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\dfrac{6}{13}\right)$ μF. Which of the combinations, shown in figures below, will achieve the desired value?

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. $C = \dfrac{C_1C_2}{C_1 + C_2}$, where $C_1 = 6$ μF

$$\frac{1}{C_2} = \frac{4}{2} \Rightarrow C_2 = \frac{1}{2}\ \mu\text{F}$$

$$C = \frac{6 \times \frac{1}{2}}{6 + \frac{1}{2}} = \frac{6}{13}\ \mu\text{F}$$

23. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then

(1) $K_2 = 2K_1$

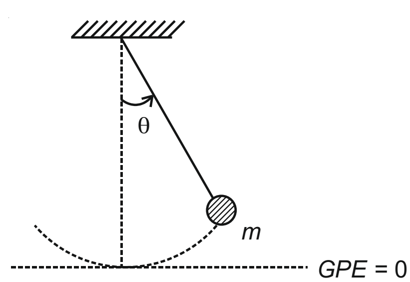
(2) $K_2 = \dfrac{K_1}{4}$

(3) $K_2 = K_1$

(4) $K_2 = \dfrac{K_1}{2}$

Answer (1)

Sol.



$U = mgL(1 - \cos\theta)$

$GPE = 0$

$K_1 = mgL(1 - \cos\theta)$

$K_2 = mg(2L)(1 - \cos\theta)$

$K_2 = 2K_1$

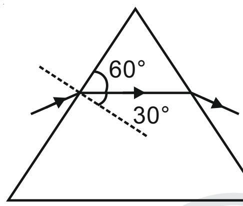
24. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is:

(1) 90°   (2) 30°

(3) 45°   (4) 60°

Answer 4)

Sol. For minimum deviation the ray passes symmetrically



$\Rightarrow$ $r = 30°$

$$\sin i = \sqrt{3}\sin 30° = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ $i = 60°$

25. In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is

(1) 05   (2) 09

(3) 10   (4) 04

Answer (4)

Sol. Angular width between second and first diffraction minima $= \dfrac{\lambda}{a}$

Angular width of a fringe $= \dfrac{\lambda}{d}$

$\therefore \quad n = \dfrac{d}{a} = \dfrac{19.44}{4.05}$

$\Rightarrow$ No. of bright fringes = 04

26. A particle moves from the point $(2.0\hat{i} + 4.0\hat{j})$ m, at $t = 0$, with an initial velocity $(5.0\hat{i} + 4.0\hat{j})$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i} + 4.0\hat{j})$ ms^{-2}. What is the distance of the particle from the origin at time 2 s?

(1) $20\sqrt{2}$ m   (2) 15 m

(3) $10\sqrt{2}$ m   (4) 5 m

Answer (1)

Sol. $\vec{r} = \vec{r}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2$

$$= (2\hat{i} + 4\hat{j}) + (5\hat{i} + 4\hat{j}) \times 2 + \frac{1}{2}[4\hat{i} + 4\hat{j}] \times 2^2$$

$$= (20\hat{i} + 20\hat{j}) \text{ m}$$

$$|\vec{r}| = 20\sqrt{2} \text{ m}$$

27. The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5 m, the angle between the force and the position vector is (in radians):

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

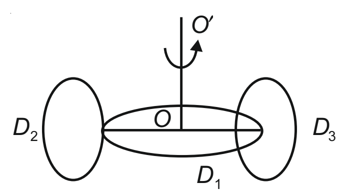
Answer (2)

Sol. $\tau = Fr\sin\theta$

$2.5 = 1 \times 5 \sin\theta$

$\Rightarrow \theta = \dfrac{\pi}{6}$

28. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1 as shown in the figure, will be:



(1) $3MR^2$

(2) $\frac{4}{5}MR^2$

(3) MR^2

(4) $\frac{2}{3}MR^2$

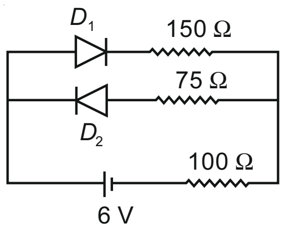
Answer (1)

Sol. $I = I_1 + I_2 + I_3$

$$= \frac{MR^2}{2} + 2\left[\frac{MR^2}{4} + MR^2\right]$$

$$= \frac{MR^2}{2} + 2 \times \frac{5MR^2}{4} = 3MR^2$$

29. The circuit shown below contains two ideal diodes, each with a forward resistance of 50 Ω. If the battery voltage is 6 V, the current through the 100 Ω resistance (in amperes) is:



(1) 0.036

(2) 0.020

(3) 0.030

(4) 0.027

Answer (2)

Sol. $I = \frac{6}{50+150+100} = \frac{6}{300}\ \text{A} = 0.02\text{A}$

30. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil:

(1) Increases by a factor of 3

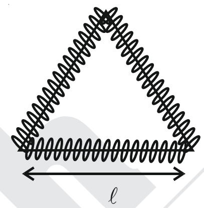
(2) Decreases by a factor of $9\sqrt{3}$

(3) Decreases by a factor of 9

(4) Increases by a factor of 27

Answer (1)

Sol.



$LI = (\mu_0 nI)\ A\ (n.3\ell)$

Where n = No. of turns/length

$L \propto \ell$

❑ ❑ ❑

CHEMISTRY

1. Given the equilibrium constant :
   K_C of the reaction :
   $Cu(s) + 2Ag^{+}(aq) \rightarrow Cu^{2+}(aq) + 2Ag(s)$ is
   10×10^{15}, calculate the E^0_{cell} of this reaction at 298 K

   $$\left[2.303\frac{RT}{F} \text{at 298 K} = 0.059 \text{ V}\right]$$

   (1) 0.4736 mV (2) 0.4736 V
   (3) 0.04736 V (4) 0.04736 mV

Answer (2)

Sol. $E^0_{cell} = \frac{0.059}{n} \log K_C$

$$= \frac{0.059}{2} \log 10^{16}$$

$= 0.472$ V

2. Among the colloids cheese (C), milk (M) and smoke (S), the correct combination of the dispersed phase and dispersion medium, respectively is :
   (1) C : solid in liquid; M : liquid in liquid; S : gas in solid
   (2) C : liquid in solid; M : liquid in solid; S : solid in gas
   (3) C : liquid in solid; M : liquid in liquid; S : solid in gas
   (4) C : solid in liquid; M : solid in liquid; S : solid in gas

Answer (3)

Sol.

		Dispersed phase	Dispersion medium
(C)	Cheese	liquid	solid
(M)	Milk	liquid	liquid
(S)	Smoke	solid	gas

3. The radius of the largest sphere which fits properly at the centre of the edge of a body centred cubic unit cell is : (Edge length is represented by 'a')
   (1) 0.027 a (2) 0.047 a
   (3) 0.067 a (4) 0.134 a

Answer (3)

Sol. For BCC

$$\sqrt{3}\, a = 4R$$

$$\Rightarrow R = \frac{\sqrt{3}\, a}{4}$$

$\therefore$ Empty space at edge = a – 2R

$$= a - \frac{\sqrt{3}\, a}{2}$$

= diameter of sphere.

$$\therefore \quad r_{sphere} = \frac{a - \frac{\sqrt{3}}{2}a}{2} = \left(\frac{2-\sqrt{3}}{4}\right)a$$

= 0.067 a

4. The reaction $2X \rightarrow B$ is a zeroth order reaction. If the initial concentration of X is 0.2 M, the half-life is 6 h. When the initial concentration of X is 0.5 M, the time required to reach its final concentration of 0.2 M will be :
   (1) 12.0 h (2) 7.2 h
   (3) 9.0 h (4) 18.0 h

Answer (4)

Sol. For the reaction $2X \rightarrow B$, follow zeroth order

Rate equation is

$2\,Kt = [A]_0 - [A]$

For the half-life

$$2\,Kt = \frac{[A]_0}{2}$$

$$K = \frac{0.2}{2 \times 2 \times 6}$$

$$K = \frac{1}{120} \text{ M hr}^{-1}$$

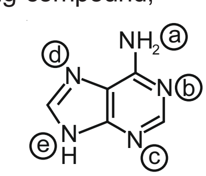
$\therefore$ time required to reach from 0.5 M to 0.2 M

$2\,Kt = [A]_0 - [A]$

$t = (0.5 - 0.2) \times 60$

= 18 hour

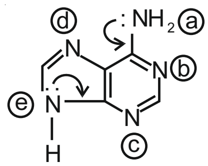
5. In the following compound,



   the favourable site/s for protonation is/are :
   (1) (a)
   (2) (b), (c) and (d)
   (3) (a) and (d)
   (4) (a) and (e)

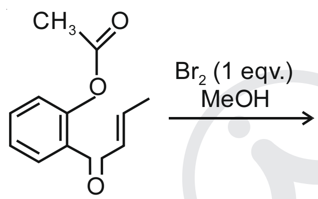
Answer (2)

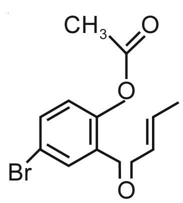
Sol. The lone pair which is participating in resonance and aromaticity will not be a favourable site for protonation.

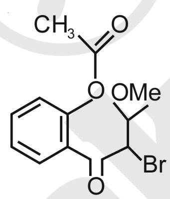


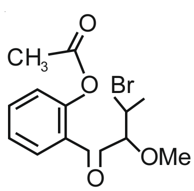
lone pair of N atom b, c and d is not the part of aromaticity.

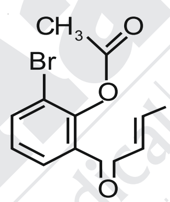
6. The major product obtained in the following conversion is :



(1) 

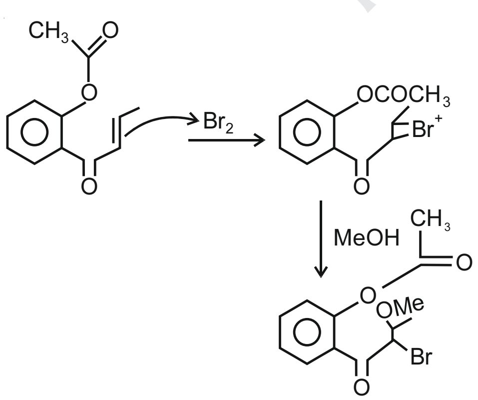
(2) 

(3) 

(4) 

Answer (2)

Sol.



7. K_2HgI_4 is 40% ionised in aqueous solution. The value of its van't Hoff factor (i) is :

(1) 1.6 (2) 2.0

(3) 2.2 (4) 1.8

Answer (4)

Sol. $K_2HgI_4 \rightleftharpoons 2K^+ + [HgI_4]^{2-}$

n = 3

$\because \quad \alpha = \dfrac{i-1}{n-1}$

$0.4 = \dfrac{i-1}{3-1}$

i = 1.8

8. Match the following items in column I with the corresponding items in column II.

Column-I		Column-II	
(i)	$Na_2CO_3.10H_2O$	(A)	Portland cement ingredient
(ii)	$Mg(HCO_3)_2$	(B)	Castner-Kellner process
(iii)	NaOH	(C)	Solvay process
(iv)	$Ca_3Al_2O_6$	(D)	Temporary hardness

(1) (i)(B), (ii)(C), (iii)(A), (iv)(D)

(2) (i)(C), (ii)(D), (iii)(B), (iv)(A)

(3) (i)(D), (ii)(A), (iii)(B), (iv)(C)

(4) (i)(C), (ii)(B), (iii)(D), (iv)(A)

Answer (2)

Sol. (i) $Na_2CO_3.10H_2O$ is prepared by Solvay process

(ii) $Mg(HCO_3)_2$ is the reason of temporary hardness

(iii) NaOH is prepared by Castner-Kellener process

(iv) $Ca_3Al_2O_6$ is the ingredient of Portland cement

9. The de Broglie wavelength (λ) associated with a photoelectron varies with the frequency (ν) of the incident radiation as, [ν_0 is threshold frequency]:

(1) $\lambda \propto \dfrac{1}{(\nu-\nu_0)}$ (2) $\lambda \propto \dfrac{1}{(\nu-\nu_0)^{\frac{1}{4}}}$

(3) $\lambda \propto \dfrac{1}{(\nu-\nu_0)^{\frac{1}{2}}}$ (4) $\lambda \propto \dfrac{1}{(\nu-\nu_0)^{\frac{3}{2}}}$

Answer (3)

Sol. According to de-Broglie wavelength equation

$$\lambda = \frac{h}{mv} \Rightarrow \lambda \propto \frac{1}{v}$$

From photoelectric effect.

$$h\nu - h\nu_0 = \frac{1}{2}mv^2$$

$$v \propto (\nu - \nu_0)^{1/2}$$

$$\therefore \quad \lambda \propto \frac{1}{(\nu-\nu_0)^{1/2}}$$

10. The correct option with respect to the Pauling electronegativity values of the elements is:

(1) Si < Al (2) P > S

(3) Te > Se (4) Ga < Ge

Answer (4)

Sol. Correct order of electronegativity is

Si > Al

S > P

Se > Te

Ge > Ga.

11. The correct match between **Item I** and **Item II** is:

Item I	**Item II**
(A) Ester test	(P) Tyr
(B) Carbylamine test	(Q) AsP
(C) Phthalein dye test	(R) Ser
	(S) Lys

(1) (A) → (Q); (B) → (S); (C) → (P)

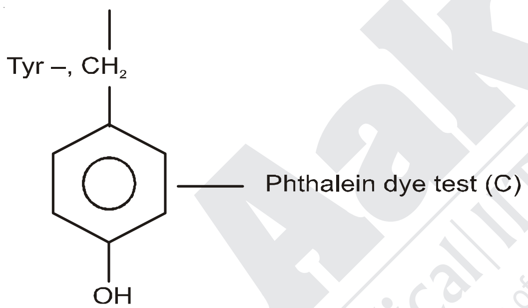
(2) (A) → (R); (B) → (Q); (C) → (P)

(3) (A) → (Q); (B) → (S); (C) → (R)

(4) (A) → (R); (B) → (S); (C) → (Q)

Answer (1)

Sol. (P)



(Q) AsP – $-CH_2-COOH$ – Ester test (A)

(R) Ser – $-CH_2-OH$ – Ester test (A)

(S) Lys – $-(CH_2)_4-NH_2$ – Carbylamine test (B)

12. The correct match between **Item I** and **Item II** is:

Item I	**Item II**
(A) Allosteric effect	(P) Molecule binding to the active site of enzyme
(B) Competitive inhibitor	(Q) Molecule crucial for communication in the body
(C) Receptor	(R) Molecule binding to a site other than the active site of enzyme
(D) Poison	(S) Molecule binding to the enzyme covalently

(1) (A) → (P); (B) → (R); (C) → (S); (D) → (Q)

(2) (A) → (R); (B) → (P); (C) → (Q); (D) → (S)

(3) (A) → (P); (B) → (R); (C) → (Q); (D) → (S)

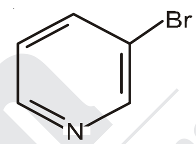
(4) (A) → (R); (B) → (P); (C) → (S); (D) → (Q)

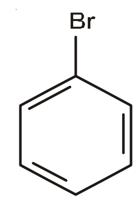
Answer (2)

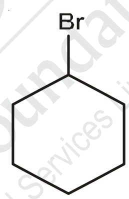
Sol.

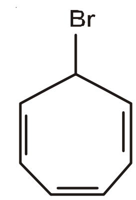
Allosteric effect:	Molecule bind to a site other than the active site of enzyme
Competitive inhibitor:	Molecule bind to the acitve site
Receptor:	Molecule crucial for communication in the body.
Poison:	Molecule binding to the enzyme covalently

13. Which of the following compounds will produce a precipitate with $AgNO_3$?

(1)


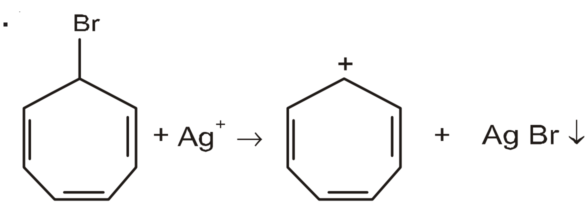
(2)


(3)


(4)


Answer (4)

Sol.



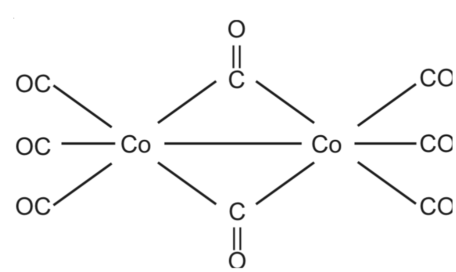
14. The number of bridging CO ligand(s) and Co-Co bond(s) in $Co_2(CO)_8$, respectively are:

(1) 2 and 1 (2) 0 and 2

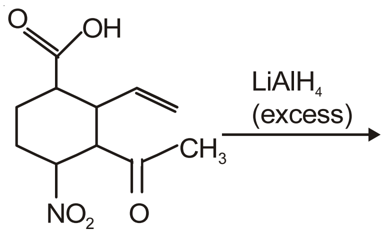
(3) 2 and 0 (4) 4 and 0

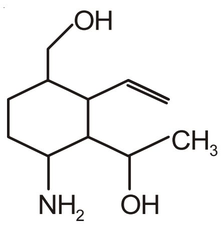
Answer (1)

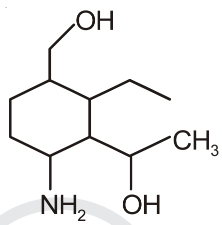
Sol. Structure of $Co_2(CO)_8$

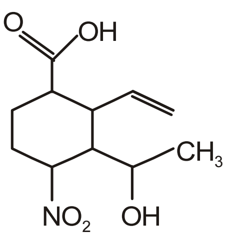


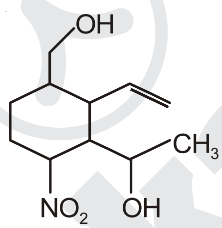
15. The major product obtained in the following reaction is:



(1) 

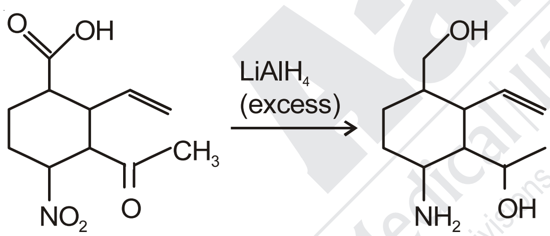
(2) 

(3) 

(4) 

Answer (1)

Sol.



16. $\underline{A} \xrightarrow{4\,KOH, O_2} \underset{(Green)}{2\underline{B}} + 2H_2O$

$3\underline{B} \xrightarrow{4\,HCl} \underset{(Purple)}{2\underline{C}} + MnO_2 + 2H_2O$

$2\underline{C} \xrightarrow{H_2O,\,KI} 2\underline{A} + 2KOH + \underline{D}$

In the above sequence of reactions, $\underline{A}$ and $\underline{D}$, respectively, are :

(1) KI and K_2MnO_4

(2) KIO_3 and MnO_2

(3) MnO_2 and KIO_3

(4) KI and $KMnO_4$

Answer (3)

Sol. $\underset{(A)}{2MnO_2} + 4KOH + O_2 \longrightarrow \underset{\substack{(Green)\\(B)}}{2K_2MnO_4} + 2H_2O$

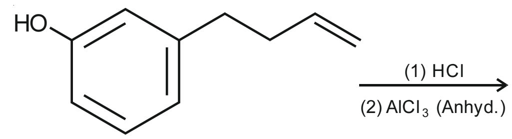
$K_2MnO_4 + 4HCl \longrightarrow \underset{\substack{(Purple)\\(C)}}{2KMnO_4} + MnO_2 + 2H_2O$

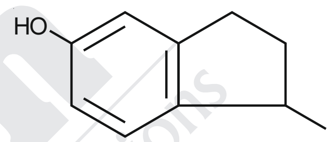
$2KMnO_4 + H_2O + KI \longrightarrow 2MnO_2 + 2KOH + \underset{(D)}{KIO_3}$

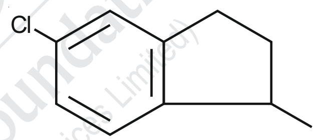
A – MnO_2

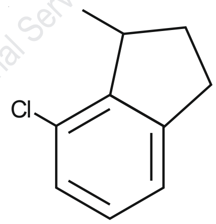
D – KIO_3

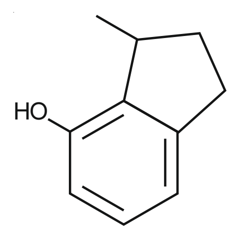
17. The major product of the following reaction is :



(1) 

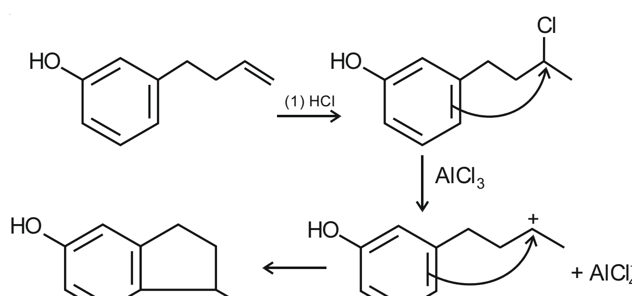
(2) 

(3) 

(4) 

Answer (1)

Sol.



(Major product)

Para attack will form major product because at ortho position steric crowding is applicable.

18. Taj Mahal is being slowly disfigured and discoloured. This is primarily due to :

(1) Acid rain (2) Water pollution

(3) Global warming (4) Soil pollution

Answer (1)

Sol. Taj Mahal is being slowly disfigured and discoloured due to **Acid Rain.**

19. The relative stability of +1 oxidation state of group 13 elements follows the order :

(1) Tl < In < Ga < Al

(2) Al < Ga < Tl < In

(3) Al < Ga < In < Tl

(4) Ga < Al < In < Tl

Answer (3)

Sol. Due to inert pair effect, as we move down the group-13 elements, stability of +1 oxidation state increases.

$\therefore$ Correct order of stability is

Al < Ga < In < Tl

20. The homopolymer formed from 4-hydroxy-butanoic acids is :

(1) $\left[\overset{O}{\overset{\|}{C}}(CH_2)_3-O\right]_n$ (2) $\left[\overset{O}{\overset{\|}{C}}(CH_2)_2\overset{O}{\overset{\|}{C}}-O\right]_n$

(3) $\left[O\overset{O}{\overset{\|}{C}}(CH_2)_3-O\right]_n$ (4) $\left[\overset{O}{\overset{\|}{C}}(CH_2)_2\overset{O}{\overset{\|}{C}}\right]_n$

Answer (1)

Sol.

$$\underset{\underset{OH}{|}}{CH_2}-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-OH \xrightarrow{\text{Polymerisation}} \left[O(CH_2)_3-\overset{O}{\overset{\|}{C}}\right]_n$$

P4HB Polymer

21. The reaction that does NOT define calcination is:

(1) $Fe_2O_3.XH_2O \xrightarrow{\Delta} Fe_2O_3 + XH_2O$

(2) $CaCO_3.MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2CO_2$

(3) $ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2$

(4) $2Cu_2S + 3O_2 \xrightarrow{\Delta} 2Cu_2O + 2SO_2$

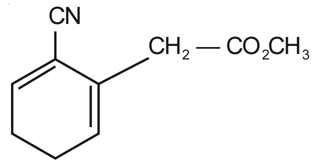
Answer (4)

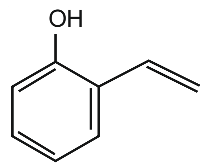
Sol. Calcination involves heating in absence of air.

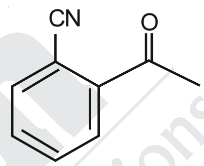
$\therefore$ $2Cu_2S + 3O_2 \xrightarrow{\Delta} 2Cu_2O + 2SO_2$

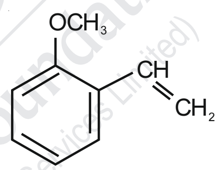
This is not calcination rather it is roasting (heating in a regular supply of air)

22. Which of the following compounds reacts with ethylmagnesium bromide and also decolourizes bromine water solution?

(1) 

(2) 

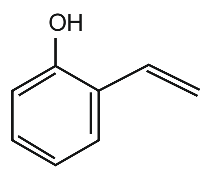
(3) 

(4) 

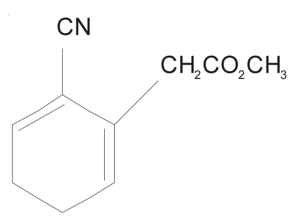
Answer should be both (1) and (2)

Sol. Phenol or unsaturated hydrocarbon (alkene or alkyne) decolourised bromine water solution.

C_2H_5MgBr will react with carbonyl carbon or acidic hydrogen.



reacts with C_2H_5MgBr and also decolourized bromine water solution



will also give postive bromine water test and give substitution product with EtMgBr

23. The hydride that is NOT electron deficient is

(1) SiH_4 (2) GaH_3

(3) B_2H_6 (4) AlH_3

Answer (1)

Sol. SiH_4 – Electron precise hydride

GaH_3 – Electron deficient hydride

B_2H_6 – Electron deficient hydride

AlH_3 – Electron deficient hydride

24. For the equilibrium

$2H_2O \rightleftharpoons H_3O^+ + OH^-$, the value of ΔG° at 298 K is approximately

(1) –80 kJ mol^{-1} (2) –100 kJ mol^{-1}

(3) 80 kJ mol^{-1} (4) 100 kJ mol^{-1}

Answer (3)

Sol. $\Delta G = \Delta G^\circ + RT \ln Q$

At equilibrium

$\Delta G = 0$ and $Q = K_{eq}$

$\Rightarrow \Delta G^\circ = -2.303\ RT \log K_w$

$= -2.303 \times 8.314 \times 298 \log 10^{-14}$

$\simeq 80$ kJ/mol

25. The coordination number of Th in $K_4[Th(C_2O_4)_4(OH_2)_2]$ is

$\left(C_2O_4^{2-} = \text{Oxalato}\right)$

(1) 10 (2) 6

(3) 14 (4) 8

Answer (1)

Sol. $K_4[Th(C_2O_4)_4(OH_2)_2]$

$C_2O_4^{2-}$ is bidentate ligand and H_2O is monodentate ligand.

∴ Co-ordination no. of Th = 2 × 4 + 2 = 10

26. A compound 'X' on treatment with Br_2/NaOH, provided C_3H_9N, which gives positive carbylamine test. Compound 'X' is :

(1) $CH_3CH_2CH_2CONH_2$ (2) $CH_3COCH_2NHCH_3$

(3) $CH_3CH_2COCH_2NH_2$ (4) $CH_3CON(CH_3)_2$

Answer (1)

Sol. C_3H_9N gives carbylamine test.

∴ C_3H_9N is primary aliphatic amine.

$X + Br_2 + NaOH \longrightarrow \underset{\text{(Primary amine)}}{C_3H_9N}$

∴ X is acid amide having formula

$CH_3-CH_2-CH_2-\overset{\overset{\displaystyle O}{||}}{C}-NH_2$

27. The higher concentration of which gas in air can cause stiffness of flower buds?

(1) SO_2 (2) CO

(3) CO_2 (4) NO_2

Answer (1)

Sol. High concentration of SO_2 leads to stiffness of flower buds.

28. The reaction

$MgO(s) + C(s) \rightarrow Mg(s) + CO(g)$, for which $\Delta_rH^\circ = +491.1$ kJ mol^{-1} and $\Delta_rS^\circ = 198.0$ $JK^{-1}mol^{-1}$, is not feasible at 298 K. Temperature above which reaction will be feasible is

(1) 2040.5 K (2) 1890.0 K

(3) 2480.3 K (4) 2380.5 K

Answer (3)

Sol. $MgO(s) + C(s) \longrightarrow Mg(s) + CO(g)$

For reaction to be spontaneous

$\Delta_rH^\circ - T\Delta_rS^\circ < O$

$\Rightarrow T > \dfrac{\Delta_rH^\circ}{\Delta_rS^\circ}$

$T > \dfrac{491.1 \times 1000}{198}$

$T > 2480.3$ K

29. 25 ml of the given HCl solution requires 30 mL of 0.1 M sodium carbonate solution. What is the volume of this HCl solution required to titrate 30 mL of 0.2 M aqueous NaOH solution

(1) 25 mL (2) 12.5 mL

(3) 50 mL (4) 75 mL

Answer (1)

Sol. 25 mL of HCl solution required 30 mL of 0.1 M Na_2CO_3 solution

∴ 25 × M × 1 = 30 × 0.1 × 2

$\Rightarrow M = \dfrac{6}{25} = 0.24$ M

Now, HCl solution is titrated with NaOH solution.

∴ V × 0.24 × 1 = 30 × 0.2 × 1

⇒ V = 25 mL

30. The standard reaction Gibbs energy for a chemical reaction at an absolute temperature T is given by

$\Delta_rG^\circ = A - BT$

Where A and B are non-zero constants. Which of the following is true about this reaction?

(1) Exothermic if B < 0

(2) Endothermic if A > 0

(3) Endothermic if A < 0 and B > 0

(4) Exothermic if A > 0 and B < 0

Answer (2)

Sol. $\Delta_rG^\circ = A - BT$

A and B are non-zero constants

∵ $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = A - BT$

∴ Reaction will be endothermic if A > 0.

MATHEMATICS

1. Let A and B be two invertible matrices of order 3 × 3. If $det(ABA^T) = 8$ and $det(AB^{-1}) = 8$, then $det(BA^{-1}B^T)$ is equal to :

(1) 1

(2) 16

(3) $\frac{1}{16}$

(4) $\frac{1}{4}$

Answer (3)

Sol. Let $|A| = x$, $|B| = y$

$$\Rightarrow \quad |A^T| = x,\ |A^{-1}| = \frac{1}{x},\ |B^T| = y,\ |B^{-1}| = \frac{1}{y}$$

$\because \quad |ABA^T| = 8 \quad \Rightarrow \quad |A|\,|B|\,|A^T| = 8$

$\Rightarrow \quad x \cdot y \cdot x = 8 \quad \Rightarrow \quad x^2y = 8$ ...(i)

$\because \quad |AB^{-1}| = 8 \Rightarrow |A|\,|B^{-1}| = 8 \Rightarrow x \cdot \frac{1}{y} = 8$ ...(ii)

From (i) & (ii)

$$x = 4,\ y = \frac{1}{2}$$

$$\Rightarrow \quad |BA^{-1}B^T| = |B||A^{-1}||B^T| = y.\frac{1}{x}.y = \frac{y^2}{x} = \frac{1}{16}$$

Option (3) is correct.

2. A circle cuts a chord of length $4a$ on the x-axis and passes through a point on the y-axis, distant $2b$ from the origin. Then the locus of the centre of this circle, is :

(1) A hyperbola

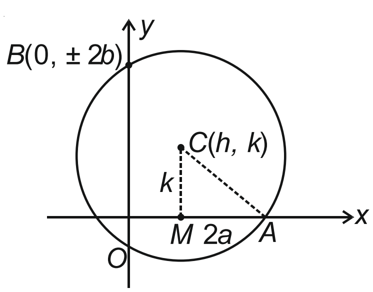
(2) A parabola

(3) An ellipse

(4) A straight line

Answer (2)

Sol.



Let centre is $C(h, k)$

$CB = CA = r$

$\Rightarrow \quad CB^2 = CA^2$

$(h - 0)^2 + (k \pm 2b)^2 = CM^2 + MA^2$

$h^2 + (k \pm 2b)^2 = k^2 + 4a^2$

$h^2 + k^2 + 4b^2 \pm 4bk = k^2 + 4a^2$

Locus of $C(h, k)$

$x^2 + 4b^2 \pm 4by = 4a^2$

It is a parabola

Option (2) is correct.

3. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is

(1) $\frac{1}{2}$

(2) $\frac{m+n}{6mn}$

(3) 1

(4) $\frac{1}{4}$

Answer (4)

Sol. $E = \frac{x^m y^n}{(1+x^{2m})(1+y^{2n})} = \frac{1}{(x^{-m}+x^m)(y^{-n}+y^n)}$

$$\frac{x^m + y^{-m}}{2} \ge (x^m.x^{-m})^{\frac{1}{2}} \Rightarrow x^m + x^{-m} \ge 2$$

Similarly $y^{-n} + y^n \ge 2$

$\Rightarrow \quad (x^m + x^{-m})(y^{-n} + y^n) \ge 4$

$$\Rightarrow \quad \frac{1}{(x^m + x^{-m})(y^{-n} + y^n)} \le \frac{1}{4}$$

Option (4) is correct

4. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a + b + c)(x + a + b + c)^2$, $x \ne 0$ and $a + b + c \ne 0$, then x is equal to :

(1) $2(a + b + c)$

(2) $-(a + b + c)$

(3) abc

(4) $-2(a + b + c)$

Answer (4)

Sol. $\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_1 \to C_1 - C_3,\ C_2 \to C_2 - C_3$

$$\Delta = (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & -b-c-a & 2b \\ c+a+b & c+a+b & c-a-b \end{vmatrix}$$

$= (a + b + c)(a + b + c)^2$

Option (4) is correct

5. Let a function $f : (0, \infty) \to (0, \infty)$ be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then f is :

(1) Injective only

(2) Both injective as well as surjective

(3) Not injective but it is surjective

(4) Neither injective nor surjective

Answer (Question is wrong)

Sol.

$f : (0, \infty) \to (0, \infty)$

$f(x) = \left|1 - \frac{1}{x}\right|$ is not a function

$\because f(1) = 0$ and $1 \in$ domain but $0 \notin$ co-domain

6. If $\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then $f(x)$ is equal to :

(1) $\frac{1}{3}(x+1)$ (2) $\frac{1}{3}(x+4)$

(3) $\frac{2}{3}(x-4)$ (4) $\frac{2}{3}(x+2)$

Answer (2)

Sol. Let $I = \int \frac{x+1}{\sqrt{2x-1}}dx$

Let $\sqrt{2x-1} = t$

$\therefore\ 2x - 1 = t^2$

$\Rightarrow\ dx = t\,dt$

$$I \int \frac{(t^2+3)}{2}dt = \frac{t^3}{6} + \frac{3t}{2} + C$$

$$= \frac{(2x-1)^{\frac{3}{2}}}{6} + \frac{3}{2}(2x-1)^{\frac{1}{2}} + C$$

$$= \sqrt{2x-1}\left(\frac{x+4}{3}\right) + C$$

$$= f(x).\sqrt{2x-1} + C$$

where $f(x) = \frac{x+4}{3}$

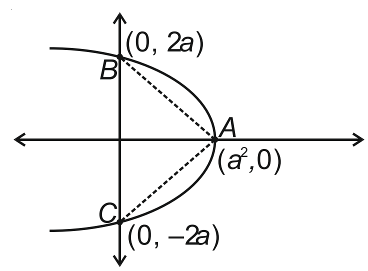
7. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is :

(1) $5\sqrt{5}$ (2) $(10)^{2/3}$

(3) $5(2^{1/3})$ (4) 5

Answer (4)

Sol. $y^2 = -4(x - a^2)$



Area $= \frac{1}{2}(4a)(a^2)$

$= 2a^3$

As $2a^2 = 250$

$\Rightarrow\ a = 5$

8. Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$).

Then $|z|$ is equal to :

(1) $\frac{\sqrt{41}}{4}$

(2) $\frac{5}{4}$

(3) $\frac{5}{3}$

(4) $\frac{\sqrt{34}}{3}$

Answer (3)

Sol. Given, $|z| + z = 3 + i$

Let $z = a + ib$

$$\Rightarrow \sqrt{a^2+b^2} + a + ib = 3 + i$$

$$\Rightarrow b = 1, \sqrt{a^2+b^2} + a = 3$$

$$\sqrt{a^2+1} = 3 - a$$

$$a^2 + 1 = a^2 + 9 - 6a$$

$$6a = 8$$

$$a = \frac{4}{3}$$

$$|z| = \sqrt{\left(\frac{4}{3}\right)^2 + 1} = \sqrt{\frac{16}{9}+1} = \frac{5}{3}$$

9. Let $f(x) = \dfrac{x}{\sqrt{a^2+x^2}} - \dfrac{(d-x)}{\sqrt{b^2+(d-x)^2}}, x \in R$ where a, b and d are non-zero real constants. Then :

(1) f is an increasing function of x

(2) f is a decreasing function of x

(3) f is neither increasing nor decreasing function of x

(4) f' is not a continuous function of x

Answer (1)

Sol. $f(x) = \dfrac{x}{\sqrt{a^2+x^2}} - \dfrac{(d-x)}{\sqrt{b^2+(d-x)^2}}$

$$= \frac{x}{\sqrt{a^2+x^2}} + \frac{(x-d)}{\sqrt{b^2+(x-d)^2}}$$

$$f'(x) = \frac{\sqrt{a^2+x^2} - \dfrac{x(2x)}{2\sqrt{a^2+x^2}}}{(a^2+x^2)} + \frac{\sqrt{b^2+(x-d)^2} - \dfrac{(x-d)2(x-d)}{2\sqrt{b^2+(x-d)^2}}}{\left(b^2+(x-d)^2\right)}$$

$$= \frac{a^2+x^2-x^2}{(a^2+x^2)^{3/2}} + \frac{b^2+(x-d)^2-(x-d)^2}{\left(b^2+(x-d)^2\right)^{3/2}}$$

$$\frac{a^2}{(a^2+x^2)^{3/2}} + \frac{b^2}{\left(b^2+(x-d)^2\right)^{3/2}} > 0$$

Hence $f(x)$ is increasing function.

10. Contrapositive of the statement

"If two numbers are not equal, then their squares are not equal." is :

(1) If the squares of two numbers are equal, then the numbers are equal

(2) If the squares of two numbers are not equal, then the numbers are equal

(3) If the squares of two numbers are equal, then the numbers are not equal

(4) If the squares of two numbers are not equal, then the numbers are not equal

Answer (1)

Sol. Contrapositive of "If A then B" is "If ~B then ~A"

Hence contrapositive of "If two numbers are not equal, then their squares are not equal" is "If squares of two numbers are equal, then the two numbers are equal".

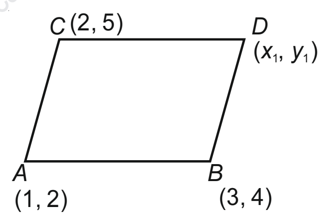
11. If in a parallelogram $ABDC$, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is :

(1) $5x + 3y - 11 = 0$ (2) $3x + 5y - 13 = 0$

(3) $3x - 5y + 7 = 0$ (4) $5x - 3y + 1 = 0$

Answer (4)

Sol. Mid-point of AD = mid-point of BC



$$\left(\frac{x_1+1}{2}, \frac{y_1+2}{2}\right) = \left(\frac{3+2}{2}, \frac{4+5}{2}\right)$$

$\therefore (x_1, y_1) = (4, 7)$

$\therefore$ Equation of AD : $y - 7 = \dfrac{2-7}{1-4}(x-4)$

$$y - 7 = \frac{5}{3}(x-4)$$

$$3y - 21 = 5x - 20$$

$$5x - 3y + 1 = 0$$

12. $\lim\limits_{x\to 0} \dfrac{x\cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to :

(1) 2 (2) 4

(3) 1 (4) 0

Answer (3)

Sol. $\lim_{x\to 0} \dfrac{x\cot 4x}{\sin^2 x \cdot \cot^2 2x}$

$= \lim_{x\to 0} \dfrac{x \cdot \tan^2 2x}{\sin^2 x \cdot \tan 4x}$

$= \lim_{x\to 0} \left(\dfrac{x}{\sin x}\right)^2 \cdot \left(\dfrac{\tan 2x}{2x}\right)^2 \cdot \left(\dfrac{4x}{\tan 4x}\right) \cdot \dfrac{4}{2^2} = 1$

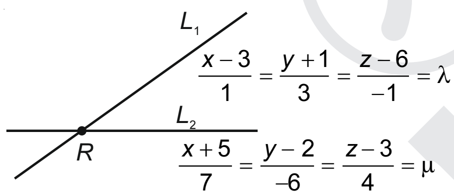
13. Two lines $\dfrac{x-3}{1} = \dfrac{y+1}{3} = \dfrac{z-6}{-1}$ and $\dfrac{x+5}{7} = \dfrac{y-2}{-6} = \dfrac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates :

(1) (–2, 4, 7) (2) (2, 4, 7)

(3) (2, –4, –7) (4) (2, –4, 7)

Answer (3)

Sol. Let the coordinate of A with respect to line



$L_1 = (\lambda + 3, 3\lambda - 1, -\lambda + 6)$

and coordinate of A w.r.t.

line $L_2 = (7\mu - 5, -6\mu + 2, 4\mu + 3)$.

$\therefore \lambda - 7\mu = -8,\ 3\lambda + 6\mu = 3,\ \lambda + 4\mu = 3$

from above equations : $\lambda = -1, \mu = 1$

$\therefore$ Coordinate of point of intersection $R = (2, -4, 7)$.

Image of R w.r.t. xy plane = (2, –4, –7).

14. Let $(x + 10)^{50} + (x - 10)^{50}$

$= a_0 + a_1x + a_2x^2 + ... + a_{50}x^{50}$, for all

$x \in R$; then $\dfrac{a_2}{a_0}$ is equal to :

(1) 12.25 (2) 12.75

(3) 12.00 (4) 12.50

Answer (1)

Sol. $(x + 10)^{50} + (x - 10)^{50}$

$= a_0 + a_1x + a_2x^2 + ... + a_{50}x^{50}$

$\therefore a_0 + a_1x + a_2x^2 + ... + a_{50}x^{50}$

$= 2({}^{50}C_0x^{50} + {}^{50}C_2x^{48}\cdot 10^2 + {}^{50}C_4x^{46}\cdot 10^4 + ...)$

$\therefore \quad a_0 = 2\cdot{}^{50}C_{50}10^{50}$

$a_2 = 2\cdot{}^{50}C_2\cdot 10^{48}$

$\therefore \quad \dfrac{a_2}{a_0} = \dfrac{{}^{50}C_2 \times 10^{48}}{{}^{50}C_{50}10^{50}}$

$= \dfrac{50\times 49}{2\times 100} = \dfrac{49}{4}$

= 12.25

15. Given $\dfrac{b+c}{11} = \dfrac{c+a}{12} = \dfrac{a+b}{13}$ for ΔABC with usual natation. If $\dfrac{\cos A}{\alpha} = \dfrac{\cos B}{\beta} = \dfrac{\cos C}{\gamma}$, then the ordered triplet (α, β, γ) has a value :

(1) (3, 4, 5) (2) (7, 19, 25)

(3) (19, 7, 25) (4) (5, 12, 13)

Answer (2)

Sol. $\because \dfrac{b+c}{11} = \dfrac{c+a}{12} = \dfrac{a+b}{13} = k\ (\text{Say})$.

$\therefore \quad b + c = 11k,\ c + a = 12k,\ a + b = 13k$

$\therefore \quad a + b + c = 18k$

$\therefore \quad a = 7k,\ b = 6k$ and $c = 5k$

$\therefore \quad \cos A = \dfrac{36k^2 + 25k^2 - 49k^2}{2.30k^2} = \dfrac{1}{5}$

and $\cos B = \dfrac{49k^2 + 25k^2 - 36k^2}{2.35k^2} = \dfrac{19}{35}$

and $\cos C = \dfrac{49k^2 + 36k^2 - 25k^2}{2.42k^2} = \dfrac{5}{7}$

$\therefore \quad \cos A : \cos B : \cos C = 7 : 19 : 25$

$\therefore \quad \dfrac{\cos A}{7} = \dfrac{\cos B}{19} = \dfrac{\cos C}{25}$

16. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is :

(1) 2 : 1 (2) 1 : 3

(3) 4 : 1 (4) 3 : 1

Answer (4)

Sol. Let first term and common difference of AP be a and d respectively.

$\therefore \quad t_{19} = a + 18d = 0$

$\therefore \quad a = -18d$ ...(i)

$\therefore \quad \dfrac{t_{49}}{t_{29}} = \dfrac{a + 48d}{a + 28d}$

$= \dfrac{-18d + 48d}{-18d + 28d} = \dfrac{30d}{10d} = 3$

$t_{49} : t_{29} = 3 : 1$

17. The number of function *f* from {1, 2, 3, ...,20} onto {1, 2, 3, ..., 20} such that *f*(*k*) is a multiple of 3, whenever *k* is a multiple of 4, is :

(1) $5^6 \times 15$ (2) $6^5 \times (15)!$

(3) $5! \times 6!$ (4) $(15)! \times 6!$

Answer (4)

Sol. Domain and codomain = {1, 2, 3, ..., 20}.

There are five multiple of 4 as 4, 8, 12, 16 and 20.

and there are 6 multiple of 3 as 3, 6, 9, 12, 15, 18.

when ever *k* is multiple of 4 then *f*(*k*) is multiple of 3 then total number of arrangement

$= {}^6c_5 \times 5! = 6!$

Remaining 15 are arrange is 15! ways.

∴ given function in onto

∴ Total number of arrangement = 15! · 6!

18. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x\left(\tan^5 x + \cot^5 x\right)}$ equals:

(1) $\frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(2) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

(3) $\frac{\pi}{40}$

(4) $\frac{1}{5}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

Answer (1)

Sol. $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{dx}{\sin 2x\left(\tan^5 x + \cot^5 x\right)}$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan^5 x \cdot \sec^2 x}{2\frac{\sin x}{\cos x}\left(\left(\tan^5 x\right)^2 + 1\right)}$$

$$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan^4 x \cdot \sec^2 x}{\left(\tan^5 x\right)^2 + 1}\, dx.$$

Let $\tan^5 x = t$.

$5\tan^4 x \cdot \sec^2 x\, dx = dt$.

$$= \frac{1}{10}\int_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1} \frac{dt}{t^2 + 1}$$

$$= \frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$$

19. Let α and β the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$ $(0 < \theta < 45°)$, and $\alpha < \beta$. Then

$\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to :

(1) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$ (2) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

(3) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$ (4) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

Answer (2)

Sol. $x^2\sin\theta - x(\sin\theta \cdot \cos\theta + 1) + \cos\theta = 0.$

$x^2\sin\theta - x\sin\theta \cdot \cos\theta - x + \cos\theta = 0.$

$x\sin\theta(x - \cos\theta) - 1(x - \cos\theta) = 0.$

$(x - \cos\theta)(x\sin\theta - 1) = 0.$

∴ $x = \cos\theta, \operatorname{cosec}\theta, \theta \in = (0, 45°)$

∴ $\alpha = \cos\theta, \beta = \operatorname{cosec}\theta$

$$\sum_{n=0}^{\infty}\alpha^n = 1 + \cos\theta + \cos^2\theta + \ldots\ldots \infty = \frac{1}{1-\cos\theta}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{\beta^n} = 1 - \frac{1}{\operatorname{cosec}\theta} + \frac{1}{\operatorname{cosec}^2\theta} - \frac{1}{\operatorname{cosec}^3\theta} + \ldots\ldots \infty$$

$= 1 - \sin\theta + \sin^2\theta - \sin^3\theta + \ldots\ldots \infty.$

$$= \frac{1}{1+\sin\theta}$$

$$\therefore \sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$$

$$= \sum_{n=0}^{\infty}\alpha^n + \sum_{n=0}^{\infty}\frac{(-1)^n}{\beta^n}$$

$$= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}.$$

20. A bag contains 30 white ball and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If *X* be the number of white balls drawn, then $\left(\frac{\text{mean of } X}{\text{Standard deviation of } X}\right)$ is equal to :

(1) $3\sqrt{2}$ (2) $4\sqrt{3}$

(3) $\frac{4\sqrt{3}}{3}$ (4) 4

Answer (2)

Sol. $p = \frac{30}{40} = \frac{3}{4}, \quad q = \frac{10}{40} = \frac{1}{4}$

$n = 16$

$$\frac{\text{Mean of } X}{\text{standard deviation of } X} = \frac{np}{\sqrt{npq}} = \frac{\sqrt{np}}{\sqrt{q}}$$

$$= \sqrt{\frac{16 \times \frac{3}{4}}{\frac{1}{4}}} = \sqrt{48} = 4\sqrt{3}$$

21. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is :

(1) 2

(2) $\frac{13}{8}$

(3) $\frac{13}{6}$

(4) $\frac{13}{12}$

Answer (4)

Sol. $2b = 5$

$2ae = 13$

Now, $b^2 = a^2(e^2 - 1)$

$\Rightarrow a^2 = 36$

$\therefore a = 6$

$ae = \frac{13}{2} \Rightarrow e = \frac{13}{12}$

22. Let $S_n = 1 + q + q^2 + \ldots + q^n$ and

$$T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \ldots + \left(\frac{q+1}{2}\right)^n$$

where q is a real number and $q \neq 1$.

If ${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + \ldots + {}^{101}C_{101} \cdot S_{100} = \alpha\, T_{100}$.

(1) 200
(2) 202
(3) 2^{99}
(4) 2^{100}

Answer (4)

Sol. $S_n = \left(\frac{1-q^{n+1}}{1-q}\right), \quad T_n = \frac{1-\left(\frac{q+1}{2}\right)^{n+1}}{1-\left(\frac{q+1}{2}\right)}$

$$\Rightarrow T_{100} = \frac{1-\left(\frac{q+1}{2}\right)^{101}}{1-\left(\frac{q+1}{2}\right)}$$

$$S_n = \frac{1}{1-q} - \frac{q^{n+1}}{1-q} \qquad T_{100} = \frac{2^{101} - (q+1)^{101}}{2^{100}(1-q)}$$

$\Rightarrow {}^{101}C_1 + {}^{101}C_2\, S_1 + {}^{101}C_3\, S_2 + \ldots + {}^{101}C_{101}\, S_{100}$

$$= \left(\frac{1}{1-q}\right)\left({}^{101}C_2 + \ldots + {}^{101}C_{101}\right) - \frac{1}{1-q}\left({}^{101}C_2\, q^2 + {}^{101}C_3\, q^3 + \ldots + {}^{101}C_{101}\, q^{101}\right) + 101$$

$$= \frac{1}{1-q}\left(2^{101} - 1 - 101\right) - \left(\frac{1}{1-q}\right)\left((1+q)^{101} - 1 - {}^{101}C_1\, q\right) + 101$$

$$= \frac{1}{1-q}\left[2^{101} - 102 - (1+q)^{101} + 1 + 101q\right] + 101$$

$$= \frac{1}{1-q}\left[2^{101} - 101 + 101q - (1+q)^{101}\right] + 101$$

$$= \left(\frac{1}{1-q}\right)\left[2^{101} - (1+q)^{101}\right]$$

$= 2^{100}\, T_{100}$

23. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it?

(1) $\left(4\sqrt{3}, 2\sqrt{3}\right)$
(2) $\left(4\sqrt{3}, 2\sqrt{2}\right)$
(3) $\left(4\sqrt{2}, 2\sqrt{2}\right)$
(4) $\left(4\sqrt{2}, 2\sqrt{3}\right)$

Answer (2)

Sol. Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Now, $\frac{2b^2}{a} = 8, \quad 2ae = b^2$ and $b^2 = a^2(1-e^2)$

gives $a = 8$, $b^2 = 32$

$\therefore$ The equation of the ellipse is

$$\frac{x^2}{64} + \frac{y^2}{32} = 1$$

Clearly $\left(4\sqrt{3}, 2\sqrt{2}\right)$ lies on it

24. Let S = {1, 2, ..., 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is

(1) $\frac{7}{2^{20}}$ (2) $\frac{6}{2^{20}}$

(3) $\frac{4}{2^{20}}$ (4) $\frac{5}{2^{20}}$

Answer (4)

Sol. Total number of subset = 2^{20}

Now sum of all number from 1 to 20 = $\frac{20\times 21}{2} = 210$

Now we have to find the sets which has sum 7.

(1) {7}

(2) {1, 6}

(3) {2, 5}

(4) {3, 4}

(5) {1, 2, 4}

So, there is only 5 sets which has sum 203

So required probability $= \frac{5}{2^{20}}$

25. If the point (2, α, β) lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to

(1) 5 (2) 12

(3) 17 (4) 7

Answer (4)

Sol. Let the normal to the required plane is $\vec{n}$

$$\Rightarrow \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -4 & 4 \\ 2 & -5 & 0 \end{vmatrix} = 20\hat{i} + 8\hat{j} - 12\hat{j}$$

$\Rightarrow$ Equation of the plane is

$(x-3) \times 20 + (y-4) \times 8 + (z-2) \times (-12) = 0$

$5x - 15 + 2y - 8 - 3z + 6 = 0$

$5x + 2y - 3z - 17 = 0$ passes through $(2, \alpha, \beta)$

$\Rightarrow 10 + 2\alpha - 3\beta - 17 = 0 \quad \Rightarrow \quad 2\alpha - 3\beta = 7$

So, option (4) is correct

26. All x satisfying the inequality $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$, lie in the interval

(1) $(\cot 2, \infty)$

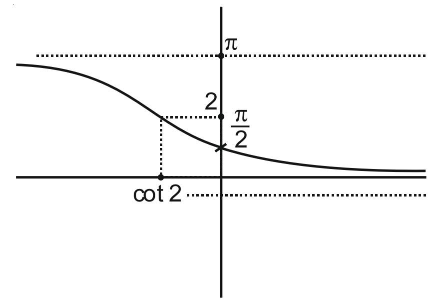
(2) $(\cot 5, \cot 4)$

(3) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$

(4) $(-\infty, \cot 5) \cup (\cot 2, \infty)$

Answer (1)

Sol.



$(\cot^{-1}x - 5)(\cot^{-1} - 2) > 0$

$\cot^{-1}x \in (-\infty, 2) \cup (5, \infty)$ ...(i)

But $\cot^{-1}x$ lies in $(0, \pi)$

From equation (i)

So, $\cot^{-1}x \in (0, 2)$

By graph,

$x \in (\cot 2, \infty)$

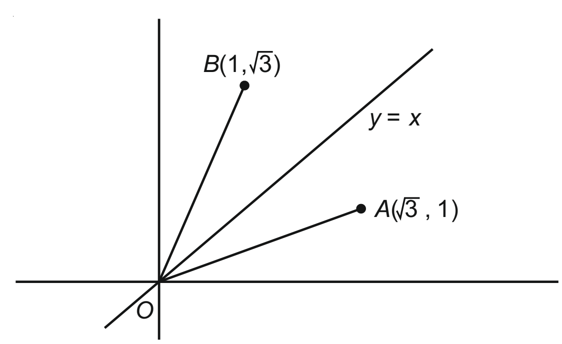
27. Let $\sqrt{3}\hat{i} + \hat{j}$, $\hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is

(1) 3 (2) 1

(3) 4 (4) 2

Answer (2)

Sol. By observing point A, B angle bisector of acute angle, OA and OB would be $\boxed{y = x}$



Now, according to question

$$\left|\frac{\beta - (1-\beta)}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$$

$\Rightarrow 2\beta = \pm 3 + 1$

$\beta = 2$ or $\beta = -1$

28. The solution of the differential equation, $\frac{dy}{dx} = (x-y)^2$, when $y(1) = 1$, is

(1) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

(2) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x+y-2$

(3) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

(4) $\log_e\left|\frac{2-x}{2-y}\right| = x-y$

Answer (3)

Sol. $\frac{dy}{dx} = (x-y)^2$ ...(i)

Let $x - y = t$

$1 - \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$

From equation (i)

$\left(1 - \frac{dt}{dx}\right) = (t)^2 \Rightarrow 1 - t^2 = \frac{dt}{dx} \Rightarrow \int dx = \int \frac{dt}{1-t^2}$

$\Rightarrow -x = \frac{1}{2\times1}\ln\left|\frac{t-1}{t+1}\right| + c$

$\Rightarrow -x = \frac{1}{2}\ln\left|\frac{x-y-1}{x-y+1}\right| + c \quad \because$ given $y(1) = 1$

$-1 = \frac{1}{2}\ln\left|\frac{1-1-1}{1-1+1}\right| + c \Rightarrow \boxed{c = -1}$

So, $2(x-1) = \ln\left|\frac{1-x+y}{1-y+x}\right|$

29. Let K be the set of all real values of x where the function

$f(x) = \sin|x| - |x| + 2(x - \pi)$ is not differentiable. Then the set K is equal to

(1) $\{\pi\}$

(2) ϕ (an empty set)

(3) $\{0\}$

(4) $\{0, \pi\}$

Answer (2)

Sol. $f(x) = \sin|x| - |x| + 2(x-\pi)\cos|x|$

For $x > 0$

$f(x) = \sin x - x + 2(x-\pi)\cos x$

$f'(x) = \cos x - 1 + 2(1-0)\cos x - 2\sin(x-\pi)$

$f'(x) = 3\cos x - 2(x-\pi)\sin x - 1$

By observing this, it is differentiable for $x > 0$

Now for $x < 0$

$f(x) = -\sin x + x + 2(x-\pi)\cos x$

$f'(x) = -\cos x + 1 - 2(x-\pi)\sin x + 2\cos x$

$f'(x) = \cos x + 1 - 2(x-\pi)\sin x$

By observing this, it is differentiable for all $x < 0$

Now check for $x = 0$

$f'(0^+)$ R.H.D. = 3 – 1 = 2

$f'(0^-)$ L.H.D. = 1 + 1 = 2

L.H.D. = R.H.D.

It is differentiable for $x = 0$, So it is differentiable everywhere

30. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is

(1) $\frac{187}{24}$

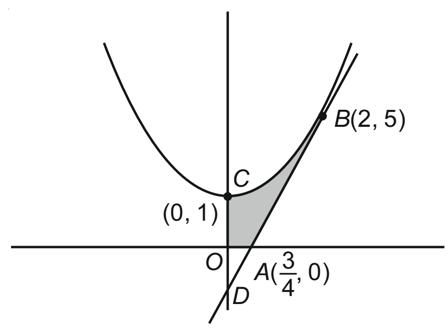
(2) $\frac{8}{3}$

(3) $\frac{14}{3}$

(4) $\frac{37}{24}$

Answer (4)

Sol.



Given $x^2 = y - 1$

Equation of tangent at (2, 5) to parabola is

$\boxed{4x - y = 3}$

Now required area

$= \int_O^2 \{(x^2+1) - (4x-3)\}dx - \text{Area of } \Delta AOD$

$= \int_O^2 (x^2 - 4x + 4)dx - \frac{1}{2}\times\frac{3}{4}\times3$

$= \left[\frac{(x-2)^3}{3}\right]_0^2 - \frac{9}{8} = \frac{37}{24}$