WBJEE - 2019

Answer Keys by

Aakash Institute, Kolkata Centre

MATHEMATICS

Q.No.	✿	◻	◯	✚
01	B	D	C	B
02	B	A	B	A
03	D	D	C	B
04	B	D	A	A
05	A	C	A	C
06	D	D	C	C
07	D	A	C	C
08	C	D	A	A
09	C	B	*B,C	C
10	D	B	B	B
11	B	A	B	C
12	B	B	D	A
13	B	A	B	A
14	B	C	A	C
15	C	C	D	C
16	B	C	D	A
17	D	A	C	*B,C
18	A	C	C	B
19	B	B	D	B
20	D	C	B	D
21	B	A	B	B
22	A	A	B	A
23	C	C	B	D
24	A	C	C	D
25	D	A	B	C
26	A	*B,C	D	C
27	D	B	A	D
28	D	B	B	B
29	C	D	D	B
30	D	B	B	B
31	A	A	A	B
32	D	D	C	C
33	B	D	A	B
34	B	C	D	D
35	A	C	A	A
36	B	D	D	B
37	A	B	D	D
38	C	B	C	B
39	C	B	D	A
40	C	B	A	C
41	A	C	D	A
42	C	B	B	D
43	B	D	B	A
44	C	A	A	D
45	A	B	B	D
46	A	D	A	C
47	C	B	C	D
48	C	A	C	A
49	A	C	C	D
50	*B, C	A	A	B
51	C	C	A	A
52	A	C	C	B
53	D	B	C	A
54	D	A	A	B
55	B	B	D	A
56	A	A	D	C
57	C	B	B	C
58	C	A	A	A
59	B	C	C	D
60	A	C	C	D
61	B	B	B	B
62	A	D	A	A
63	B	D	B	C
64	A	B	A	C
65	C	A	B	B
66	B, D	B	A,C	A,B
67	C	A,B	B,C	A,C
68	B	A,C	B,D	A,C
69	B	A,C	C	B,C
70	A	B,C	B	B,D
71	B	B,D	B	C
72	A, B	C	A	B
73	A, C	B	B	B
74	A, C	B	A,B	A
75	B, C	A	A,C	B

*** More than one options are correct in category I.**

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Code - ✿

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ANSWERS & HINT
for
WBJEE - 2019
SUB : MATHEMATICS

CATEGORY - I (Q1 to Q50)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ¼ mark will be deducted.

1. $\lim_{x\to 0+}(x^n \ln x),\ n>0$

(A) does not exist (B) exists and is zero (C) exists and is 1 (D) exists and is e^{-1}

Ans : (B)

Hint : $\underset{x\to 0^+}{\mathrm{Lt}}\dfrac{\ell nx}{\frac{1}{x^n}}\left(\dfrac{\infty}{\infty}\right)$. Applying LH rule

$$\Rightarrow \underset{x\to 0^+}{\mathrm{Lt}}\frac{\frac{1}{x}}{\frac{-n}{x^{n+1}}}=0$$

2. If $\int \cos x \log\left(\tan\frac{x}{2}\right)dx = \sin x \log\left(\tan\frac{x}{2}\right)+f(x)$ then f(x) is equal to, (assuming c is a arbitrary real constant)

(A) c (B) c – x (C) c + x (D) 2x + c

Ans : (B)

Hint : IBP $\Rightarrow$ I = sinx log $\left(\tan\frac{x}{2}\right)-x+c$

$\therefore$ f(x) = c – x

3. $y=\int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$ is an equation of a family of

(A) straight lines (B) circles (C) ellipses (D) parabolas

Ans : (D)

Hint : Putting x = cos2θ $\therefore \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}=\cos 2\theta = x$

$\therefore y=\int x\,dx=\frac{x^2}{2}+c$

4. The value of the integration $\int_{-\pi/4}^{\pi/4}\left(\lambda|\sin x| + \frac{\mu \sin x}{1+\cos x} + \gamma\right)dx$

(A) is independent of λ only
(B) is independent of μ only
(C) is independent of γ only
(D) depends on λ, μ and γ

Ans : (B)

Hint : $I = 2\lambda\int_0^{\frac{\pi}{4}} \sin x\,dx + \mu\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan\frac{x}{2}dx + \gamma\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} dx$

$= 2\lambda\left(1-\frac{1}{\sqrt{2}}\right) - 0 + \gamma\left(\frac{\pi}{2}\right)$ ($\because \tan\frac{x}{2}$ is an odd function)

5. The value of $\lim_{x\to 0}\frac{1}{x}\left[\int_y^a e^{\sin^2 t}dt - \int_{x+y}^a e^{\sin^2 t}dt\right]$ is equal to

(A) $e^{\sin^2 y}$
(B) $e^{2\sin y}$
(C) $e^{|\sin y|}$
(D) $e^{\mathrm{cosec}^2 y}$

Ans : (A)

Hint : $= \underset{x\to 0}{\mathrm{Lt}}\frac{1}{x}\left(\int_y^a e^{\sin^2 t}dt + \int_a^{x+y} e^{\sin^2 t}dt\right)$

$= \underset{x\to 0}{\mathrm{Lt}}\frac{1}{x}\int_y^{x+y} e^{\sin^2 t}dt$ $\left(\text{form } \frac{0}{0}\right)$

Applying LH rule

$= \underset{x\to 0}{\mathrm{Lt}}\frac{e^{\sin^2(x+y)}.1-0}{1} = e^{\sin^2 y}$

6. If $\int 2^{2^x}.2^x dx = A.2^{2^x} + c$, then $A =$

(A) $\frac{1}{\log 2}$
(B) $\log 2$
(C) $(\log 2)^2$
(D) $\frac{1}{(\log 2)^2}$

Ans : (D)

Hint : $2^x = z \Rightarrow 2^x dx = \frac{dz}{\ell n\, 2}$

$\Rightarrow \frac{1}{\ell n 2}\int 2^z dz = \frac{1}{(\ell n\, 2)^2}2^{2^x} + c$

7. The value of the integeral $\int_{-1}^{1}\left\{\frac{x^{2015}}{e^{|x|}(x^2+\cos x)} + \frac{1}{e^{|x|}}\right\}dx$ is equal to

(A) 0
(B) $1 - e^{-1}$
(C) $2e^{-1}$
(D) $2(1 - e^{-1})$

Ans : (D)

Hint : $\Rightarrow \int_{-1}^{1}\left(\text{odd}+\frac{1}{e^{|x|}}\right)dx$

$= 0+2\int_{0}^{1}\frac{dx}{e^{x}}=2(1-e^{-1})$

8. $\lim_{n\to\infty}\frac{3}{n}\left\{1+\sqrt{\frac{n}{n+3}}+\sqrt{\frac{n}{n+6}}+\sqrt{\frac{n}{n+9}}+....+\sqrt{\frac{n}{n+3(n-1)}}\right\}$

(A) does not exit (B) is 1 (C) is 2 (D) is 3

Ans : (C)

Hint : $\underset{n\to\infty}{\text{Lt}}\frac{3}{n}\sum_{r=0}^{n-1}\sqrt{\frac{1}{1+3\left(\frac{r}{n}\right)}}$

$= 3\int_{0}^{1}\frac{dx}{\sqrt{1+3x}}=2\left[\sqrt{1+3x}\right]_{0}^{1} = 2$

9. The general solution of the differential equation $\left(1+e^{\frac{x}{y}}\right)dx+\left(1-\frac{x}{y}\right)e^{\frac{x}{y}}dy=0$ is (c is an arbitrary constant)

(A) $x-ye^{\frac{x}{y}}=c$ (B) $y - xe^{y} = c$ (C) $x+ye^{\frac{x}{y}}=c$ (D) $y+xe^{\frac{x}{y}}=c$

Ans : (C)

Hint : Putting $\frac{x}{y}=v\Rightarrow x=vy$

$\Rightarrow\frac{dx}{dt}=v+y\frac{dv}{dy}\ \Rightarrow y\frac{dv}{dy}=-\left(\frac{v+e^{v}}{1+e^{v}}\right)\ \Rightarrow\left(\frac{1+e^{v}}{v+e^{v}}\right)dv+\frac{dy}{y}=0$

Integrating

$\Rightarrow \ell n(v+e^{v})+\ell n\ y=c_1$

$\Rightarrow y\left(\frac{x}{y}+e^{\frac{x}{y}}\right)=c\ \Rightarrow x+ye^{\frac{x}{y}}=c$

10. General solution of $(x+y)^2\frac{dy}{dx}=a^2,\ a\neq 0$ is (c is an arbitrary constant)

(A) $\frac{x}{a}=\tan\frac{y}{a}+c$ (B) tanxy = c (C) tan (x + y) = c (D) $\tan\frac{y+c}{a}=\frac{x+y}{a}$

Ans : (D)

Hint : x + y = z, $\Rightarrow\frac{dy}{dx}=\frac{dz}{dx}-1$

$\Rightarrow\frac{z^2dz}{a^2+z^2}=dx$

Integrating

$$\Rightarrow x+y-a\tan^{-1}\frac{x+y}{a}=x+c_1 \quad \Rightarrow \tan\left(\frac{y-c_1}{a}\right)=\frac{x+y}{a}$$

$$\Rightarrow \tan\left(\frac{y+c}{a}\right)=\frac{x+y}{a} \qquad (c_1 = -c)$$

11. Let P (4, 3) be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at P intersects the X-axis at (16, 0), then the eccentricity of the hyperbola is

(A) $\frac{\sqrt{5}}{2}$ (B) 2 (C) $\sqrt{2}$ (D) $\sqrt{3}$

Ans : (B)

Hint : Normal at P(4, 3)

$\frac{a^2x}{4}+\frac{b^2y}{3}=a^2+b^2$ through (16, 0)

$\Rightarrow 4a^2 = a^2 + b^2$

$\Rightarrow \frac{b^2}{a^2}=3 \;\therefore\; e = 2$

12. If the radius of a spherical balloon increases by 0.1%, then its volume increases approximately by

(A) 0.2% (B) 0.3% (C) 0.4% (D) 0.05%

Ans : (B)

Hint : $v=\frac{4}{3}\pi r^3,\ \because \Delta r=\left(\frac{0.1}{100}\right)r\ \therefore \Delta v=\left(\frac{dv}{dr}\right)\Delta r\ =4\pi r^2\cdot\left(\frac{0.1}{100}\right)r\ =\left(\frac{0.3}{100}\right)v$

13. The three sides of a right-angled triangle are in G.P (geometric progression). If the two acute angles be α and β, then $\tan\alpha$ and $\tan\beta$ are

(A) $\frac{\sqrt{5}+1}{2}$ and $\frac{\sqrt{5}-1}{2}$ (B) $\sqrt{\frac{\sqrt{5}+1}{2}}$ and $\sqrt{\frac{\sqrt{5}-1}{2}}$ (C) $\sqrt{5}$ and $\frac{1}{\sqrt{5}}$ (D) $\frac{\sqrt{5}}{2}$ and $\frac{2}{\sqrt{5}}$

Ans : (B)

Hint : $\because \left(\frac{a}{r}\right)^2+a^2=a^2r^2$ (r > 1), $a\neq 0 \Rightarrow r^4-r^2-1=0 \Rightarrow r^2=\frac{1\pm\sqrt{5}}{2} \Rightarrow r=\pm\sqrt{\frac{\sqrt{5}+1}{2}}$

$$\Rightarrow r=\sqrt{\frac{\sqrt{5}+1}{2}}(r>1),\ \frac{1}{r}=\sqrt{\frac{\sqrt{5}-1}{2}}\ \left(\because \alpha+\beta=90^\circ \Rightarrow \tan\alpha=\cot\beta=\frac{1}{\tan\beta}\right)$$

14. If $\log_2 6+\frac{1}{2x}=\log_2\left(2^{\frac{1}{x}}+8\right)$, then the values of x are

(A) $\frac{1}{4},\frac{1}{3}$ (B) $\frac{1}{4},\frac{1}{2}$ (C) $-\frac{1}{4},\frac{1}{2}$ (D) $\frac{1}{3},\frac{1}{-2}$

Ans : (B)

Hint : $\log_2 6 + \log_2 2^{\frac{1}{2x}} = \log_2\left(2^{\frac{1}{x}} + 8\right) \Rightarrow 6\cdot 2^{\frac{1}{2x}} = 2^{\frac{1}{x}} + 8$, let $2^{\frac{1}{2x}} = a$

$\Rightarrow a^2 - 6a + 8 = 0 \Rightarrow a = 4,\ a = 2 \Rightarrow x = \frac{1}{4}, \frac{1}{2}$

15. Let z be a complex number such that the principal value of argument, arg z > 0. Then arg z – arg (–z) is

(A) $\frac{\pi}{2}$ (B) $\pm\pi$ (C) π (D) $-\pi$

Ans : (C)

Hint : $\arg(z) - \arg(-z) = \arg\left(\frac{z}{-z}\right) = \arg(-1) = \pi$

16. The general value of the real angle θ, which satisfies the equation, (cosθ + i sinθ) (cos2θ + i sin2θ)........ (cosnθ + i sinnθ) = 1 is given by, (assuming k is an integer)

(A) $\frac{2k\pi}{n+2}$ (B) $\frac{4k\pi}{n(n+1)}$ (C) $\frac{4k\pi}{n+1}$ (D) $\frac{6k\pi}{n(n+1)}$

Ans : (B)

Hint : $e^{i\theta}\cdot e^{i(2\theta)}\cdot e^{i(3\theta)}...e^{i(n\theta)} = 1 \Rightarrow e^{i\left(\theta\frac{(n(n+1))}{2}\right)} = e^{i2k\pi} \Rightarrow \theta = \frac{4k\pi}{n(n+1)}$

17. Let a, b, c be real numbers such that a + b + c < 0 and the quadratic equation $ax^2 + bx + c = 0$ has imaginary roots. Then

(A) a > 0, c > 0 (B) a > 0, c < 0 (C) a < 0, c > 0 (D) a < 0, c < 0

Ans : (D)

Hint : $f(x) = ax^2 + bx + c$, $f(1) < 0$ so $f(x) < 0\ \forall\ x \in R \Rightarrow f(0) < 0 \Rightarrow c < 0 \Rightarrow a < 0$ and $c < 0$

18. A candidate is required to answer 6 out of 12 questions which are divided into two parts A and B, each containing 6 questions and he/she is not permitted to attempt more than 4 questions from any part. In how many different ways can he/she make up his/her choice of 6 questions ?

(A) 850 (B) 800 (C) 750 (D) 700

Ans : (A)

Hint :

A	B		
4	2	$\rightarrow$	${}^6C_4 \times {}^6C_2$
2	4	$\rightarrow$	${}^6C_2 \times {}^6C_4$
3	3	$\rightarrow$	${}^6C_3 \times {}^6C_3$

Total $= 2 \times {}^6C_2 \times {}^6C_4 \times {}^6C_3 \times {}^6C_3 = 450 + 400 = 850$

19. There are 7 greetings cards, each of a different colour and 7 envelopes of same 7 colours as that of the cards. The number of ways in which the cards can be put in envelopes, so that exactly 4 of the cards go into envelopes of respective colour is,

(A) ${}^7C_3$ (B) $2.{}^7C_3$ (C) $3!\,{}^4C_4$ (D) $3!\,{}^7C_3\,{}^4C_3$

Ans : (B)

Hint : ${}^7C_4 \times 3!\left(\frac{1}{2!} - \frac{1}{3!}\right) = {}^7C_3 \times 2$

20. $7^{2n}+16n-1(n\in N)$ is divisible by

(A) 65 (B) 63 (C) 61 (D) 64

Ans : (D)

Hint : $(1+48)^n + 16n - 1 = 64n + {}^nC_2\, 48^2 + \ldots = 64k$

21. The number of irrational terms in the expansion of $\left(3^{\frac{1}{8}}+5^{\frac{1}{4}}\right)^{84}$ is

(A) 73 (B) 74 (C) 75 (D) 76

Ans : (B)

Hint : $T_{r+1} = {}^{84}C_r\left(3^{\frac{1}{8}}\right)^{84-r}\left(5^{\frac{1}{4}}\right)^{r} = {}^{84}C_r \cdot 3^{\left(\frac{84-r}{8}\right)} \cdot 5^{\frac{r}{4}}$

T_{r+1} is rational for r = 4, 12, 20, 28, 36, 44, 52, 60, 68, 76, 84, Number of Irrational terms = 85 – 11 = 74

22. Let A be a square matrix of order 3 whose all entries are 1 and let I_3 be the identity matrix of order 3. Then the matrix $A - 3I_3$ is

(A) invertible (B) orthogonal

(C) non-invertible (D) real Skew Symmetric matrix

Ans : (A)

Hint : $A-3I_3 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $|A-3I_3| \neq 0$

23. If M is any square matrix of order 3 over $\mathbb{R}$ and If M′ be the transpose of M, then adj(M′) – (adj M)′ is equal to

(A) M (B) M′ (C) null matrix (D) identity matrix

Ans : (C)

Hint : adj(M′) = (adj(M))′

24. If $A = \begin{pmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{pmatrix}$ and $|A^2| = 25$, then $|x|$ is equal to

(A) $\frac{1}{5}$ (B) 5 (C) 5^2 (D) 1

Ans : (A)

Hint : $|A^2| = |A|^2 = 25$

$\Rightarrow (25x)^2 = 25$

$\Rightarrow x = \frac{1}{5}$

25. Let A and B be two square matrices of order 3 and AB = O_3, where O_3 denotes the null matrix of order 3. Then,

(A) must be $A = O_3$, $B = O_3$
(B) if $A \neq O_3$, must be $B \neq O_3$
(C) if $A = O_3$, must be $B \neq O_3$
(D) may be $A \neq O_3$, $B \neq O_3$

Ans : (D)

Hint : $AB = O_3$

A, B may not be null matrix

26. Let P and T be the subsets of X – Y plane defined by

$P = \{(x, y) : x > 0, y > 0 \text{ and } x^2 + y^2 = 1\}$

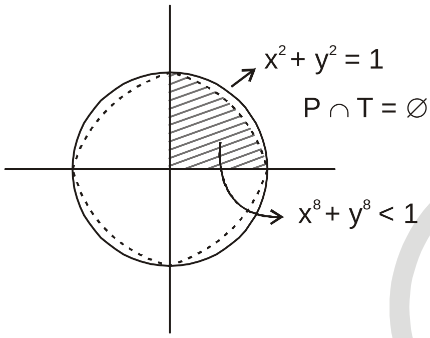
$T = \{(x, y) : x > 0, y > 0 \text{ and } x^8 + y^8 < 1\}$

Then $P \cap T$ is

(A) the void set Φ
(B) P
(C) T
(D) $P–T^C$

Ans : (A)

Hint :



27. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 - \frac{x^2}{1+x^2}$ for all $x \in \mathbb{R}$. Then

(A) f is one – one but not onto mapping
(B) f is onto but not one – one mapping
(C) f is both one – one and onto
(D) f is neither one – one nor onto

Ans : (D)

Hint : f(–x) = f(x), so many-one and into as Codomain is R

28. Let the relation ρ be defined on R as $a\rho b$ iff 1+ ab > 0. Then

(A) ρ is reflexive only
(B) ρ is equivalence relation
(C) ρ is reflexive and transitive but not symmetric
(D) ρ is reflexive and symmetric but not transitive

Ans : (D)

Hint : $1 + a^2 > 0$, so reflexive

1 + ab = 1 + ba > 0 so symmetric

1 + ab > 0 and 1 + bc > 0 does not

imply 1 + ac > 0 so not transitive

29. A problem in mathematics is given to 4 students whose chances of solving individually are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$. The probability that the problem will be solved at least by one student is

(A) $\frac{2}{3}$
(B) $\frac{3}{5}$
(C) $\frac{4}{5}$
(D) $\frac{3}{4}$

Ans : (C)

Hint : $P(A \cup B \cup C \cup D) = 1 - P(A^c \cap B^c \cap C^c \cap D^c)$

$$= 1 - P\left(A^c\right) \cdot P\left(B^c\right) \cdot P\left(C^c\right) \cdot P\left(D^c\right)$$

$$= 1 - \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{5}$$

$$= 1 - \frac{1}{5} = \frac{4}{5}$$

30. If X is a random variable such that $\sigma(X) = 2.6$, then $\sigma(1 - 4X)$ is equal to,

(A) 7.8 (B) – 10.4 (C) 13 (D) 10.4

Ans : (D)

Hint : $\sigma(a + bx) = |b|\, \sigma(x)$

31. If $e^{\sin x} - e^{-\sin x} - 4 = 0$, then the number of real values of x is

(A) 0 (B) 1 (C) 2 (D) 3

Ans : (A)

Hint : $e^{\sin x} - \frac{1}{e^{\sin x}} - 4 = 0$

$\Rightarrow e^{\sin x} = 2 \pm \sqrt{5}$

$\because -1 \le \sin x \le 1$

$\Rightarrow \frac{1}{e} \le e^{\sin x} \le e$

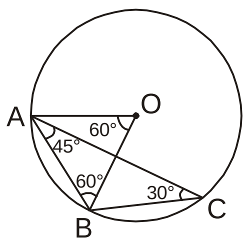
$\Rightarrow$ No solutions exist

32. The angles of a triangle are in the ratio 2 : 3 : 7 and the radius of the circumscribed circle is 10 cm. The length of the smallest side is

(A) 2 cm (B) 5 cm (C) 7 cm (D) 10 cm

Ans : (D)

Hint :



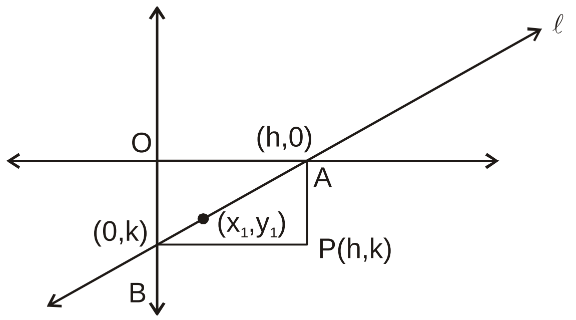
In the figure ΔABC is the respective triangle, O is the circumcentre. $\angle AOB = 2\angle ACB = 2 \times 30° = 60° \Rightarrow \angle OAB = \angle OBA = 60° \Rightarrow \Delta OAB$ is equilateral $\Rightarrow OA = AB = OB = 10$ cm.

33. A variable line passes through a fixed point (x_1, y_1) & meets the axes at A and B. If the rectangle OAPB be completed, the locus of P is, (O being the origin of the system of axes)

(A) $(y - y_1)^2 = 4(x - x_1)$ (B) $\frac{x_1}{x} + \frac{y_1}{y} = 1$ (C) $x^2 + y^2 = x_1^2 + y_1^2$ (D) $\frac{x^2}{2x_1^2} + \frac{y^2}{y_1^2} = 1$

Ans : (B)

Hint :



From the figure, ℓ passes through (h, 0), (0, k)

Thus, $\frac{x}{h}+\frac{y}{k}=1$, passes through (x_1, y_1)

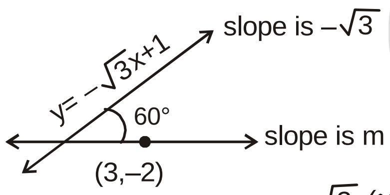
then, $\frac{x_1}{h}+\frac{y_1}{k}=1 \Rightarrow$ locus is $\frac{x_1}{x}+\frac{y_1}{y}=1$

34. A straight line through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x+y=1$. If it intersects the X-axis, then its equation will be

(A) $y+x\sqrt{3}+2+3\sqrt{3}=0$ (B) $y-x\sqrt{3}+2+3\sqrt{3}=0$ (C) $y-x\sqrt{3}-2-2\sqrt{3}=0$ (D) $x-x\sqrt{3}+2-3\sqrt{3}=0$

Ans : (B)

Hint :



$$\sqrt{3}=\left|\frac{\left(-\sqrt{3}\right)-m}{1+\left(-\sqrt{3}\right)m}\right|$$

$\Rightarrow m=\sqrt{3}$ ($\because$ it is non parallel to x axis)

$\Rightarrow \frac{y+2}{x-3}=\sqrt{3}$

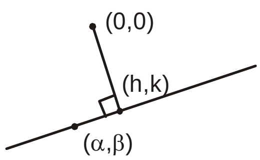
$\Rightarrow y-x\sqrt{3}+2+3\sqrt{3}=0$

35. A variable line passes through the fixed point (α, β). The locus of the foot of the perpendicular from the origin on the line is,

(A) $x^2+y^2-\alpha x-\beta y=0$ (B) $x^2-y^2+2\alpha x+2\beta y=0$ (C) $\alpha x+\beta y\pm\sqrt{\left(\alpha^2+\beta^2\right)}=0$ (D) $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$

Ans : (A)

Hint :



$$\frac{k-\beta}{h-\alpha}\times\frac{k-0}{h-0}=-1$$

$\Rightarrow$ locus is $y(y-\beta)=-x(x-\alpha)$

$\Rightarrow x^2+y^2-\alpha x-\beta y=0$

36. If the point of intersection of the lines 2ax + 4ay + c = 0 and 7bx + 3by – d = 0 lies in the 4th quadrant and is equidistant from the two axes, where a, b, c and d are non-zero numbers, then ad : bc equals to

(A) 2:3 (B) 2:1 (C) 1:1 (D) 3:2

Ans : (B)

Hint : $2ax + 4ay + c = 0$

$7bx + 3by - d = 0$

$x = \frac{4ad+3bc}{+22ab}$, $y = \frac{2ad + 7bc}{22ab}$

$x = y \Rightarrow 4ad + 3bc = 2ad+7bc$

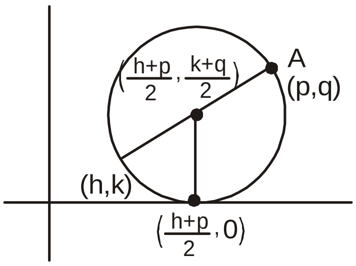
$\Rightarrow 2ad = 4bc \Rightarrow \frac{ad}{bc} = \frac{2}{1}$

37. A variable circle passes through the fixed point A(p,q) and touches x-axis. The locus of the other end of the diameter through A is

(A) $(x-p)^2 = 4qy$ (B) $(x-q)^2 = 4py$ (C) $(y-p)^2 = 4qx$ (D) $(y-q)^2 = 4px$

Ans : (A)

Hint :



$\Rightarrow (h-p)^2 + (k-q) = 4\left(\frac{k+q}{2}\right)^2$

$= k^2 + 2kq + q^2 \Rightarrow (x-p)^2 = 4qy$

38. If P(0,0), Q(1,0) and $R\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ are three given points, then the centre of the circle for which the lines PQ, QR and RP are the tangents is

(A) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (B) $\left(\frac{1}{2}, \frac{\sqrt{3}}{4}\right)$ (C) $\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$ (D) $\left(\frac{1}{2}, \frac{-1}{\sqrt{3}}\right)$

Ans : (C)

Hint : ΔPQR is equilateral, thus, the centroid is the incentre = $\left(\frac{0+1+\frac{1}{2}}{3}, \frac{0+0+\frac{\sqrt{3}}{2}}{3}\right) = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$

39. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains fixed when α varies ?

(A) directrix (B) vertices (C) foci (D) eccentricity

Ans : (C)

Hint : foci ($\pm$ ae, 0) = ($\pm \cos\alpha \,.\, \sec\alpha$, 0) = ($\pm$1,0) which is fixed

40. S and T are the foci of an ellipse and B is the end point of the minor axis. If STB is equilateral triangle, the eccentricity of the ellipse is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

Ans : (C)

Hint : $2ae = \sqrt{(ae)^2 + b^2} \Rightarrow \frac{b^2}{a^2} = 3e^2 \Rightarrow e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$

41. The equation of the directrices of the hyperbola $3x^2–3y^2–18x+12y+2 = 0$ is

(A) $x = 3\pm\sqrt{\frac{13}{6}}$ (B) $x = 3\pm\sqrt{\frac{6}{13}}$ (C) $x = 6\pm\sqrt{\frac{13}{3}}$ (D) $x = 6\pm\sqrt{\frac{3}{13}}$

Ans : (A)

Hint : $3x^2 – 3y^2 – 18x+12y+2 = 0 \Rightarrow \frac{(x-3)^2}{\left(\sqrt{\frac{13}{3}}\right)^2} - \frac{(y-2)^2}{\left(\sqrt{\frac{13}{3}}\right)^2} = 1$

Here $a = b = \sqrt{\frac{13}{3}}$ and $e = \sqrt{2}$ $\therefore$ the equations of the directrices are $x = 3\pm\sqrt{\frac{13}{6}}$

42. P is the extremity of the latusrectum of ellipse $3x^2+4y^2 = 48$ in the first quadrant. The eccentric angle of P is

(A) $\frac{\pi}{8}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Ans : (C)

Hint : $3x^2+4y^2=48 \Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1$. Here a = 4, b = $2\sqrt{3}$ and e = $\frac{1}{2}$ $\therefore$ (co-ordinates of P are (2,3)

$\therefore (4\cos\theta, 2\sqrt{3}\sin\theta) \equiv (2,3) \Rightarrow \theta = \frac{\pi}{3}$

43. The direction ratios of the normal to the plane passing through the points (1, 2, –3), (–1,–2, 1) and parallel to $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z}{4}$ is

(A) (2,3,4) (B) (14,–8,–1) (C) (–2,0,–3) (D) (1,–2,–3)

Ans : (B)

Hint : Let the direction ratios of the normal to the plane be (a,b,c), then

$$\left.\begin{aligned} 2a + 3b + 4c &= 0 \\ \text{and } -2a - 4b + 4c &= 0 \end{aligned}\right\} \Rightarrow \frac{a}{28} = \frac{b}{-16} = \frac{c}{-2} \quad \therefore (a, b, c) \equiv (14, -8, -1)$$

44. The equation of the plane, which bisects the line joining the points (1,2,3) and (3,4,5) at right angles is,

(A) x+y+z = 0 (B) x+y–z = 9 (C) x+y+z = 9 (D) x+y–z+9 = 0

Ans : (C)

Hint : Midpoint of the line segment joining (1,2,3) and (3,4,5) is (2,3,4) and the d.r.'s joining them are (2,2,2) $\equiv$ (1,1,1)
$\therefore$ the required equation of the plane is $(x–2)+(y–3)+(z–4) = 0 \Rightarrow x+y+z = 9$

45. The limit of the interior angle of a regular polygon of n sides as $n \to \infty$ is

(A) π (B) $\frac{\pi}{3}$ (C) $\frac{3\pi}{2}$ (D) $\frac{2\pi}{3}$

Ans : (A)

Hint : The limit of the interior angle of a regular polygon of n sides as $n \to \infty$ is π

46. Let f(x) > 0 for all x and f′(x) exists for all x. If f is the inverse function of h and $h'(x)=\frac{1}{1+\log x}$. Then f′(x) will be

(A) 1+log(f(x)) (B) 1+f(x) (C) 1–log(f(x)) (D) log f(x)

Ans : (A)

Hint : h(f(x)) = x $\Rightarrow$ h′(f(x)). f′(x) = 1 $\Rightarrow f'(x)=\frac{1}{h'(f(x))}$ $\Rightarrow$ f′(x) = 1+log(f(x))

47. Consider the function f(x) = cos x^2. Then

(A) f is of period 2π (B) f is of period $\sqrt{2\pi}$ (C) f is not periodic (D) f is of period π

Ans : (C)

Hint : f(x) = cosx^2, f(x+T) = cos$(x+T)^2 \neq$ f(x) for any $x \in R$ and T>0. So f(x) is not periodic

48. $\lim\limits_{x\to 0+}\left(e^x+x\right)^{1/x}$

(A) Does not exist finitely (B) is 1 (C) is e^2 (D) is 2

Ans : (C)

Hint : $\operatorname{lt}\limits_{x\to 0^+}\left(e^x+x\right)^{\frac{1}{x}} = e^{\operatorname{lt}\limits_{x\to 0^+}\frac{e^x+x-1}{x}} = e^{\operatorname{lt}\limits_{x\to 0^+}\left(\frac{e^x-1}{x}+1\right)} = e^2$

49. Let f(x) be a derivable function, f′(x) > f(x) and f(0)=0. Then

(A) f(x)>0 for all x>0 (B) f(x)<0 for all x>0

(C) no sign of f(x) can be ascertained (D) f(x) is a constant function

Ans : (A)

Hint : Let g(x) = e^{-x} f(x) $\Rightarrow$ g′(x) = e^{-x} (f′(x) – f(x)) > 0

$\Rightarrow$ g(x) is increasing function $\Rightarrow$ g(x) > g(0) for x > 0 $\Rightarrow$ e^{-x} f(x) > 0, for x > 0 $\Rightarrow$ f(x) > 0, for x > 0

50. Let f : [1,3] $\to$ R be a continuous function that is differentiable in (1, 3) an f′(x)=$|f(x)|^2$+4 for all x $\in$ (1,3). Then,

(A) f(3) – f(1) = 5 is true (B) f(3) – f(1) = 5 is false

(C) f(3) – f(1) = 7 is false (D) f(3) – f(1) < 0 only at one point of (1,3)

Ans : (B,C)*

Hint : By applying LMVT, there exist at least one $c \in$ (1,3) such that $\frac{f(3)-f(1)}{3-1}=f'(c)$

$\Rightarrow$ f(3) – f(1) = 2.$|f(c)|^2$ + 8 $\Rightarrow$ f(3) – f(1) $\geq$ 8

CATEGORY - II (Q51 to Q65)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.

51. Let a=min{$x^2+2x+3 : x \in R$} and b=$\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}$. Then $\sum_{r=0}^{n} a^r b^{n-r}$ is

(A) $\frac{2^{n+1}-1}{3.2^n}$ (B) $\frac{2^{n+1}+1}{3.2^n}$ (C) $\frac{4^{n+1}-1}{3.2^n}$ (D) $\frac{1}{2}\left(2^n-1\right)$

Ans : (C)

Hint : a=min {$(x+1)^2+2$} = 2

$$b = \mathop{lt}_{\theta\to 0}\frac{2\sin^2\frac{\theta}{2}}{4\left(\frac{\theta}{2}\right)^2} = \frac{1}{2},\ a^r b^{n-r} = \frac{2^r}{2^{n-r}} = 2^{2r-n} = \frac{4^r}{2^n},\ \sum_{r=0}^{n}a^r.b^{n-r} = \frac{1}{2^n}\sum_{r=0}^{n}4^r = \frac{1}{2^n}\left(\frac{1-4^{n+1}}{1-4}\right) = \frac{4^{n+1}-1}{3\times 2^n}$$

52. Let a>b>0 and I(n)=$a^{1/n}-b^{1/n}$, J(n)=$(a-b)^{1/n}$ for all $n\ge 2$. Then

(A) I(n) < J(n) (B) I(n) > J(n) (C) I(n) = J(n) (D) I(n) + J(n) = 0

Ans : (A)

Hint : $$\frac{I(n)}{J(n)} = \frac{a^{1/n}-b^{1/n}}{(a-b)^{1/n}} = \frac{x^{1/n}-1}{(x-1)^{1/n}} = \lim_{x\to 1^+}\frac{I(n)}{J(n)} = \lim_{x\to 1^+}\left(1-\frac{1}{x}\right)^{\left(1-\frac{1}{n}\right)} \quad \because x>0\ \&\ n\ge 2$$

53. Let $\hat{\alpha},\hat{\beta},\hat{\gamma}$ be three unit vectors such that $\hat{\alpha}\times(\hat{\beta}\times\hat{\gamma}) = \frac{1}{2}(\hat{\beta}+\hat{\gamma})$ where $\hat{\alpha}\times(\hat{\beta}\times\hat{\gamma}) = (\hat{\alpha}.\hat{\gamma})\hat{\beta}-(\hat{\alpha}.\hat{\beta})\hat{\gamma}$. If $\hat{\beta}$ is not parallel to $\hat{\gamma}$, then the angle between $\hat{\alpha}$ and $\hat{\beta}$ is

(A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Ans : (D)

Hint : $\vec{\alpha}\times(\vec{\beta}\times\vec{\gamma}) = \frac{1}{2}(\vec{\beta}+\vec{\gamma})$, $(\vec{\alpha}.\vec{\gamma})\vec{\beta}-(\vec{\alpha}.\vec{\beta})\vec{\gamma}- = \frac{1}{2}(\vec{\beta}+\vec{\gamma})$, $-(\vec{\alpha}.\vec{\beta}) = \frac{1}{2}$, $-\cos\theta = \frac{1}{2}$, $\cos\theta = \frac{-1}{2}$, $\theta = 120° = \frac{2\pi}{3}$

54. The position vectors of the points A, B, C and D are $3\hat{i}-2\hat{j}-\hat{k}$, $2\hat{i}-3\hat{j}+2\hat{k}$, $5\hat{i}-\hat{j}+2\hat{k}$ and $4\hat{i}-\hat{j}+\lambda\hat{k}$ respectively. If the points A, B, C and D lie on a plane, the value of λ is

(A) 0 (B) 1 (C) 2 (D) –4

Ans : (D)

Hint : $\vec{AB} = -\vec{i}-\vec{j}+3\vec{k}$
$$\begin{vmatrix}-1 & -1 & 3\\ 2 & 1 & 3\\ 1 & 1 & \lambda+1\end{vmatrix} = 0$$

$\vec{AC} = 2\vec{i}+\vec{j}+3\vec{k}$

$\vec{AD} = \vec{i}+\vec{j}+(\lambda+1)\vec{k}$
$$\begin{vmatrix}0 & -1 & 3\\ 1 & 1 & 3\\ 0 & 1 & \lambda+1\end{vmatrix} = 0$$

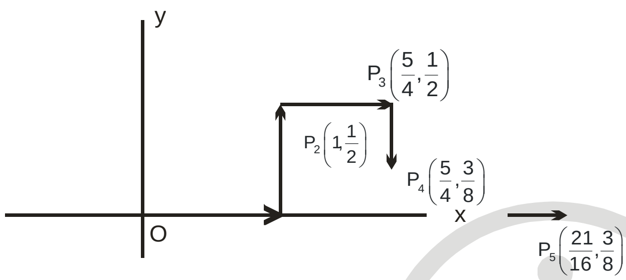
$-(-\lambda-1-3) = 0$, $(\lambda+4) = 0$, $\lambda = -4$

55. A particle starts at the origin and moves 1 unit horizontally to the right and reaches P_1, then it moves $\frac{1}{2}$ unit vertically up and reaches P_2, then it moves $\frac{1}{4}$ unit horizontally to right and reaches P_3, then it moves $\frac{1}{8}$ unit vertically down and reaches P_4, then it moves $\frac{1}{16}$ unit horizontally to right and reaches P_5 and so on. Let $P_n = (x_n, y_n)$ and $\lim_{n\to\infty} x_n = \alpha$ and $\lim_{n\to\infty} y_n = \beta$. Then (α, β) is

(A) $(2, 3)$ (B) $\left(\frac{4}{3}, \frac{2}{5}\right)$ (C) $\left(\frac{2}{5}, 1\right)$ (D) $\left(\frac{4}{3}, 3\right)$

Ans : (B)

Hint :



$x_1 = 1,\ x_2 = 1,\ x_3 = \frac{5}{4},\ x_4 = \frac{5}{4},\ x_5 = \frac{21}{16}$

$$x_n = 1 + \frac{1}{4} + \frac{1}{4^2} + \ldots\ldots\infty = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

$$y_n = 0 + \frac{1}{2} - \frac{1}{8} + \ldots\ldots\ \infty = \frac{\frac{1}{2}}{1-\left(-\frac{1}{4}\right)} = \frac{2}{5} \quad (\alpha,\beta) \equiv \left(\frac{4}{3},\frac{2}{5}\right)$$

56. For any non-zero complex number z, the minimum value of $|z| + |z - 1|$ is

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) $\frac{3}{2}$

Ans : (A)

Hint : $(|z| + |z-1|)_{min} = 1$

57. The system of equations

$\lambda x + y + 3z = 0$

$2x + \mu y - z = 0$

$5x + 7y + z = 0$

has infinitely many solutions in $\mathbb{R}$. Then,

(A) $\lambda = 2, \mu = 3$ (B) $\lambda = 1, \mu = 2$ (C) $\lambda = 1, \mu = 3$ (D) $\lambda = 3, \mu = 1$

Ans : (C)

Hint : $\begin{vmatrix} \lambda & 1 & 3 \\ 2 & \mu & -1 \\ 5 & 7 & 1 \end{vmatrix} = 0$

$\lambda(\mu+7)-1(2+5)+3(14-5\mu)=0$ or, $\lambda\mu+7\lambda-7+42-15\mu=0$ or, $\lambda\mu+7\lambda-15\mu=-35$

if $\lambda=1, \mu=3$, satisfies.

58. Lef f : $X \to Y$ and A, B are non-void subsets of Y, then (where the symbols have their usual interpretation)

(A) $f^{-1}(A)-f^{-1}(B) \supset f^{-1}(A-B)$ but the opposite does not hold

(B) $f^{-1}(A)-f^{-1}(B) \subset f^{-1}(A-B)$ but the opposite does not hold

(C) $f^{-1}(A-B)=f^{-1}(A)-f^{-1}(B)$

(D) $f^{-1}(A-B)=f^{-1}(A)\cup f^{-1}(B)$

Ans : (C)

Hint : According to definition $f^{-1}(A-B)=f^{-1}(A)-f^{-1}(B)$.

59. Let S, T, U be three non-void sets and f : $S \to T, g : T \to U$ be so that g o f : $S \to U$ is surjective. Then

(A) g and f are both surjective

(B) g is surjective, f may not be so

(C) f is surjective, g may not be so

(D) f and g both may not be surjective

Ans : (B)

Hint : g o f : $S \to U$ is onto

Let z be an arbitrary element of U $\because$ g o f : $S \to U$ onto

there exists $x \in S$

g o f(x) = z $\Rightarrow g(f(x)) = z; g(y) = z$, where y = f(x) $\in$ T for all z $\in$ U, there exists y = f(x) $\in$ T such that g(y) = z

g : $T \to U$ onto.

60. The polar coordinate of a point P is $\left(2, -\frac{\pi}{4}\right)$. The polar coordinate of the point Q, which is such that the line joining PQ is bisected perpendicularly by the initial line, is

(A) $\left(2, \frac{\pi}{4}\right)$

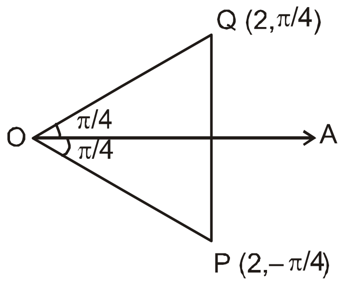
(B) $\left(2, \frac{\pi}{6}\right)$

(C) $\left(-2, \frac{\pi}{4}\right)$

(D) $\left(-2, \frac{\pi}{6}\right)$

Ans : (A)

Hint :



Point Q= (2, π/4)

61. The length of conjugate axis of a hyperbola is greater than the length of transverse axis. Then the eccentricity e is,

(A) $=\sqrt{2}$ (B) $>\sqrt{2}$ (C) $<\sqrt{2}$ (D) $<\frac{1}{\sqrt{2}}$

Ans : (B)

Hint : 2b > 2a or, b > a or, $b/a > 1$ or, $\frac{b^2}{a^2} > 1$ or, $e^2 - 1 > 1$ [$\because b^2 = a^2(e^2-1)$ or, $\frac{b^2}{a^2} = e^2 - 1$] or, $e^2 > 2$ or, e > $\sqrt{2}$

62. The value of $\lim\limits_{x\to 0+} \frac{x}{p}\left[\frac{q}{x}\right]$ is

(A) $\frac{[q]}{p}$ (B) 0 (C) 1 (D) ∞

Ans : (A)*

Hint : $= \lim\limits_{x\to 0^+} \frac{x}{p}\left(\frac{q}{x} - \left\{\frac{q}{x}\right\}\right) = \lim\limits_{x\to 0^+} \frac{q}{p} - \lim\limits_{x\to 0^+} \frac{x}{p}\left\{\frac{q}{x}\right\} = \frac{q}{p} - 0\times(\text{finite})$ $\left(0 \le \left\{\frac{q}{x}\right\} < 1\right) = \frac{q}{p}$

63. Let f(x) = $x^4 - 4x^3 + 4x^2 + c$, $c \in \mathbb{R}$. Then

(A) f(x) has infinitely many zeros in (1, 2) for all c

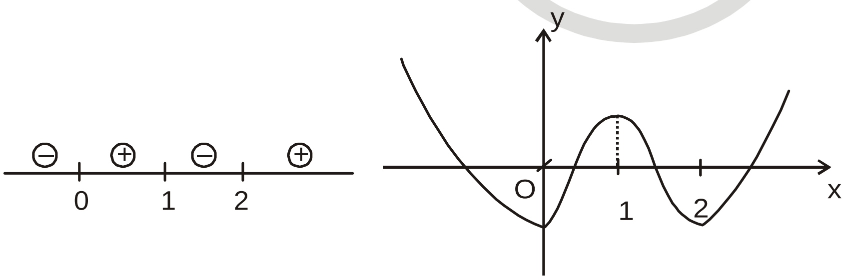
(B) f(x) has exactly one zero in (1, 2) if –1 < c < 0

(C) f(x) has double zeros in (1, 2) if –1 < c < 0

(D) Whatever be the value of c, f(x) has no zero in (1, 2)

Ans : (B)

Hint : f(x) = $x^4 - 4x^3 + 4x^2 + c$, $c \in R$ then $F'(x) = 4x^3 - 12x^2 + 8x = 4x(x^2 - 3x + 2) = 4x(x-1)(x-2)$



if –1 < c < 0

f(1) = 1 – 4 + 4 + c

= 1 + c > 0

f(2) = 16 – 32 + 16 + c

= c < 0

f(x) has exactly are zero in (1, 2) if –1 < c < 0

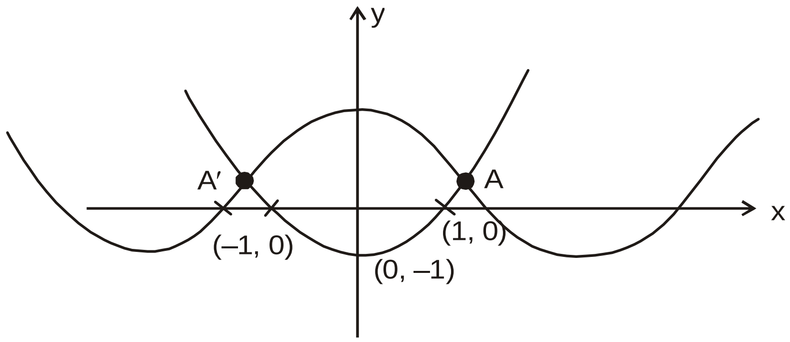
64. The graphs of the polynomial $x^2 - 1$ and cos x intersect

(A) at exactly two points (B) at exactly 3 points

(C) at least 4 but at finitely many points (D) at infinitely many points

Ans : (A)

Hint :



$y = x^2 - 1$, $y = \text{Cos } x$

Cos x and $x^2 - 1$ intersect exactly at two points at A and B.

65. A point is in motion along a hyperbola $y = \frac{10}{x}$ so that its abscissa x increases uniformly at a rate of 1 unit per second. Then, the rate of change of its ordinate, when the point passes through (5, 2)

(A) increases at the rate of $\frac{1}{2}$ unit per second
(B) decreases at the rate of $\frac{1}{2}$ unit per second
(C) decreases at the rate of $\frac{2}{5}$ unit per second
(D) increases at the rate of $\frac{2}{5}$ unit per second

Ans : (C)

Hint : $y = \frac{10}{x}$, $\frac{dx}{dt}$=1unit/second

$$\frac{dy}{dt} = -\frac{10}{x^2}.\frac{dx}{dt}$$

$$= -\frac{10}{25} \times 1 = -\frac{2}{5} \text{ unit/sec}$$

decreases at the rate of $\frac{2}{5}$ unit per second.

CATEGORY - III (Q66 to Q75)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. Let $I_n = \int_0^1 x^n \tan^{-1} x \, dx$. If $a_n I_{n+2} + b_n I_n = c_n$ for all $n \geq 1$, then

(A) a_1, a_2, a_3 are in G.P (B) b_1, b_2, b_3 are in A.P (C) c_1, c_2, c_3 are in H.P (D) a_1, a_2, a_3 are in A.P

Ans : (B,D)

Hint : $(n+1)I_n + (n+3)I_{n+2} = \int_0^1 \{(n+1)x^n + (n+3)x^{n+2}\} \tan^{-1} x \, dx$

$$=\left[\left\{(n+1)\frac{x^{n+1}}{n+1}+(n+3)\frac{x^{n+3}}{n+3}\right\}\tan^{-1}x\right]_0^1-\int_0^1 x^{n+1}(1+x^2).\frac{1}{1+x^2}dx=\left[\left(x^{n+1}+x^{n+3}\right)\tan^{-1}x\right]_0^1-\left[\frac{x^{n+2}}{n+2}\right]_0^1$$

$$=\frac{\pi}{2}-\frac{1}{n+2}$$

$$c_n=\frac{\pi}{2}-\frac{1}{n+2}$$

$\therefore$ a_n = n+3; $b_n=n+1$; $c_n=\frac{\pi}{2}-\frac{1}{n+2}$

67. Two particles A and B move from rest along a straight line with constant accelerations f and h respectively. If A takes m seconds more than B and describes n units more than that of B acquiring the same speed, then

(A) $(f+h)m^2=fhn$ (B) $(f-fh)m^2=fhn$ (C) $(h-f)n=\frac{1}{2}fhm^2$ (D) $\frac{1}{2}(f+h)n=fhm^2$

Ans : (C)

Hint : $S+n=\frac{1}{2}f(t+m)^2$ and $S=\frac{1}{2}ht^2, V=ht$

$$\therefore \frac{1}{2}ht^2+n=\frac{1}{2}f(t+m)^2 \text{ ------ (1)}$$

Also V = 0 + ht = 0 + f (t + m) $\Rightarrow t+m=\frac{ht}{f}$

From equation (1),

$$\frac{1}{2}ht^2+n=\frac{1}{2}f\left(\frac{ht}{f}\right)^2 \quad\Rightarrow\quad t^2=\frac{2nf}{h(h-f)}$$

Also,

$$ht=f(t+m) \qquad \Rightarrow t^2=\frac{m^2f^2}{(h-f)^2}$$

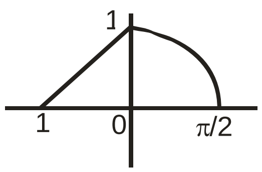
$$\therefore \frac{2nf}{h(h-f)}=\frac{m^2f^2}{(h-f)^2} \qquad \Rightarrow 2n=\frac{m^2fh}{h-f} \qquad \Rightarrow n(h-f)=\frac{1}{2}fhm^2$$

68. The area bounded by y = x + 1 and y = cos x and the x-axis, is

(A) 1 sq. unit (B) $\frac{3}{2}$ sq. unit (C) $\frac{1}{4}$ sq. unit (D) $\frac{1}{8}$ sq. unit

Ans : (B)

Hint :



$$\text{Area}=\frac{1}{2}\times1\times1+\int_0^{\pi/2}\cos x\,dx=\frac{3}{2}\text{sq.unit}$$

69. Let x_1, x_2 be the roots of $x^2 - 3x + a = 0$ and x_3, x_4 be the roots of $x^2 - 12x + b = 0$. If $x_1 < x_2 < x_3 < x_4$ and x_1, x_2, x_3, x_4 are in G.P. then ab equals

(A) $\frac{24}{5}$ (B) 64 (C) 16 (D) 8

Ans : (B)

Hint : $x_1 + x_2 = 3; x_1.x_2 = a$

$x_3 + x_4 = 12; x_3.x_4 = b$

Let r be the common ratio of G.P., then

$$\frac{x_1(1+r)}{x_1r^2(1+r)} = \frac{3}{12} \Rightarrow r = \pm 2$$

Take r = 2 ($\because$ G.P. is increasing)

$\because x_1 + x_2 = 3 \Rightarrow x_1(1 + r) = 3 \Rightarrow x_1 = 1$

$\therefore ab = x_1x_2x_3x_4 = 1.2.4.8 = 64$

70. If $\theta \in \mathbb{R}$ and $\frac{1-i\cos\theta}{1+2i\cos\theta}$ is real number, then θ will be (when I: Set of integers)

(A) $(2n+1)\frac{\pi}{2}, n \in I$ (B) $\frac{3n\pi}{2}, n \in I$ (C) $n\pi, n \in I$ (D) $2n\pi, n \in I$

Ans : (A)

Hint : $\because \frac{1-i\cos\theta}{1+2i\cos\theta}$ is real $\Rightarrow \frac{1-i\cos\theta}{1+2i\cos\theta} = \frac{1+i\cos\theta}{1-2i\cos\theta}$

$\Rightarrow 1 - 3i\cos\theta - 2\cos^2\theta = 1 + 3i\cos\theta - 2\cos^2\theta \Rightarrow \cos\theta = 0$

$\Rightarrow \theta = n\pi + \pi/2 \ (n \in I)$

71. Let $A = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}$. Then the roots of the equation $\det(A - \lambda I_3) = 0$ (where I_3 is the identity matrix of order 3) are

(A) 3, 0, 3 (B) 0, 3, 6 (C) 1, 0, –6 (D) 3, 3, 6

Ans : (B)

Hint : Let $(A - \lambda I_3) = 0 \Rightarrow \begin{vmatrix} 3-\lambda & 0 & 3 \\ 0 & 3-\lambda & 0 \\ 3 & 0 & 3-\lambda \end{vmatrix} = 0$

$\Rightarrow (3-\lambda)^3 - 9(3-\lambda) = 0 \Rightarrow (3-\lambda)[(3-\lambda)^2 - 3^2] = 0$

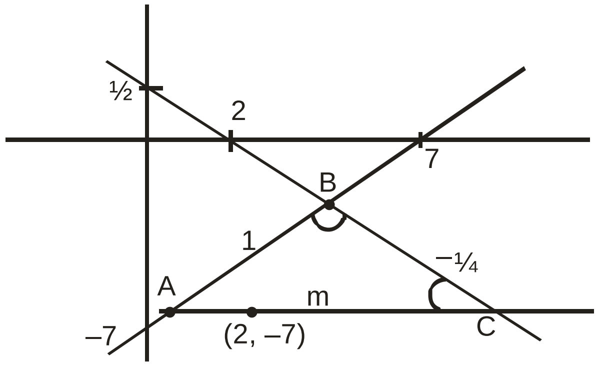
$\Rightarrow 3 - \lambda = 0$ or $3 - \lambda - 3 = 0$ or $3 - \lambda + 3 = 0 \Rightarrow \lambda = 0, 3$ or 6

72. Straight lines $x - y = 7$ and $x + 4y = 2$ intersect at B. Points A and C are so chosen on these two lines such that AB = AC. The equation of line AC passing through (2, –7) is

(A) $x - y - 9 = 0$ (B) $23x + 7y + 3 = 0$ (C) $2x - y - 11 = 0$ (D) $7x - 6y - 56 = 0$

Ans : (A, B)

Hint :



Let the slope of line AC be m,

then, AB = AC $\Rightarrow \angle ABC = \angle BCA$

$$\therefore \left|\frac{m+\frac{1}{4}}{1-\frac{m}{4}}\right| = \left|\frac{-\frac{1}{4}-1}{1-\frac{1}{4}}\right| \Rightarrow m = \frac{-23}{7}, 1$$

$\therefore$ Equation of line is: $23x + 7y + 3 = 0$, $x - y = 9$

73. Equation of a tangent to the hyperbola $5x^2 - y^2 = 5$ and which passes through an external point (2, 8) is

(A) $3x - y + 2 = 0$ (B) $3x + y - 14 = 0$ (C) $23x - 3y - 22 = 0$ (D) $3x - 23y + 178 = 0$

Ans : (A, C)

Hint : Let the tangent be $y = mx \pm \sqrt{m^2 - 5}$

Since it passes through (2, 8) $\Rightarrow (8 - 2m)^2 = m^2 - 5$

$\Rightarrow 3m^2 - 32m + 69 = 0$ $\Rightarrow 3m^2 - 9m - 23m + 69 = 0$ $\Rightarrow (3m - 23)(m - 3) = 0$ $\Rightarrow m = 3 \text{ or } \frac{23}{3}$

74. Let f and g be differentiable on the interval I and let a, b $\in$ I, a < b. Then

(A) If f (a) = 0 = f (b), the equation $f'(x) + f(x)g'(x) = 0$ is solvable in (a, b).

(B) If f (a) = 0 = f (b), the equation $f'(x) + f(x)g'(x) = 0$ may not be solvable in (a, b).

(C) If g(a) = 0 = g(b), the equation $g'(x) + kg(x) = 0$ is solvable in (a, b), $k \in \mathbb{R}$

(D) If g(a) = 0 = g(b), the equation $g'(x) + kg(x) = 0$ may not be solvable in (a, b), $k \in \mathbb{R}$

Ans : (A, C)

Hint : f(a) = 0 = f(b) $\Rightarrow f'(a).f'(b) < 0$

let $h(x) = f'(x) + f(x)g'(x)$ $h(a) = f'(a)$, $h(b) = f'(b)$

$\therefore h(a).h(b) < 0$ $\Rightarrow h(x) = 0$ has root(s) between (a, b)

Similarly, $g'(x) + kg(x) = 0$ has root(s) between (a, b) as g(a) = 0 = g(b)

75. Consider the function $f(x)=\frac{x^3}{4}-\sin\pi x+3$

(A) f(x) does not attain value within the interval [–2, 2]

(B) f(x) takes on the value $2\frac{1}{3}$ in the interval [–2, 2]

(C) f(x) takes on the value $3\frac{1}{4}$ in the interval [–2, 2]

(D) f(x) takes no value p, $1 < p < 5$ in the interval [–2, 2]

Ans : (B, C)

Hint : f (–2) = 1 and f (2) = 5 Also f is continuous.

Therefore by Intermediate value theorem, function f takes all values between 1 to 5.

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