All India Aakash Test Series for JEE (Advanced)-2020

MOCK TEST - 6 (Paper-2) - Code-C

Test Date : 10/05/2020

ANSWERS

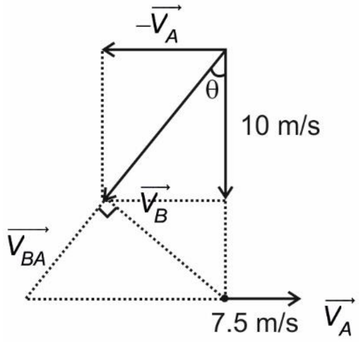
PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(C, D)	19.	(A, C)	37.	(A, C)
2.	(A, D)	20.	(B, D)	38.	(B)
3.	(A, C)	21.	(A, C)	39.	(C, D)
4.	(B, D)	22.	(B, C)	40.	(A, B)
5.	(A, C, D)	23.	(A, B, C)	41.	(A, C)
6.	(A, C, D)	24.	(A, C)	42.	(A, B, C, D)
7.	(A, C)	25.	(B)	43.	(A, C, D)
8.	(B, C)	26.	(B, C, D)	44.	(B, D)
9.	(70)	27.	(35)	45.	(04)
10.	(48)	28.	(05)	46.	(07)
11.	(60)	29.	(06)	47.	(10)
12.	(80)	30.	(11)	48.	(20)
13.	(22)	31.	(12)	49.	(09)
14.	(26)	32.	(26)	50.	(04)
15.	(D)	33.	(B)	51.	(D)
16.	(A)	34.	(A)	52.	(D)
17.	(B)	35.	(C)	53.	(B)
18.	(C)	36.	(D)	54.	(B)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (C, D)

Hint :



Solution :

$$\tan\theta = \frac{3}{4}$$

$\Rightarrow$ $d_{min} = 100 \sin\theta$

$$= 100 \times \frac{3}{5} = 60 \text{ m}$$

$$V_{BA} = \sqrt{10^2 + (7.5)^2} = \sqrt{100 + \frac{225}{4}} = 12.5 \text{ m/s}$$

$$t = \frac{100 \times \cos\theta}{12.5} = \frac{100}{25} \times \frac{4}{5} \times 2 = \frac{32}{5} = 6.4 \text{ s}$$

2. Answer (A, D)

Hint :

$$E_x = -\frac{dv}{dx}, \ \rho = \varepsilon_0 \frac{dE_x}{dx}$$

Solution :

$E = E_x = 4a_0x^3$

$$\phi = 4a_0 \cdot L^3 \cdot \frac{L^2}{4} = a_0 L^5$$

$$Q = \varepsilon_0 \phi = a_0 \varepsilon_0 L^5$$

$dE_x = 12a_0x^2dx$

$$\Rightarrow \quad A\,dE_x = \frac{\rho A dx}{\varepsilon_0} = 12\, a_0 A x^2 dx$$

$$\Rightarrow \quad \rho = 12\, \varepsilon_0 a_0 x^2$$

$$\rho\left(x = \frac{L}{2}\right) = 12\, \varepsilon_0 a_0 \cdot \frac{L^2}{4} = 3\varepsilon_0 a L^2$$

3. Answer (A, C)

Hint :

$$\lambda = \lambda_0 - VT, \ \Delta f = f_0\left(\frac{v + v_0}{v - v_5}\right) - f_0$$

Solution :

$$\lambda = \lambda_0 - VT = \frac{340}{165} - \frac{10}{165} = 2 \text{ m}$$

$$f' = f_0 \frac{340}{330} \times \frac{350}{340} = \frac{165 \times 350}{330} = 175 \text{ Hz}$$

$\Rightarrow$ Beat frequency = 10 Hz

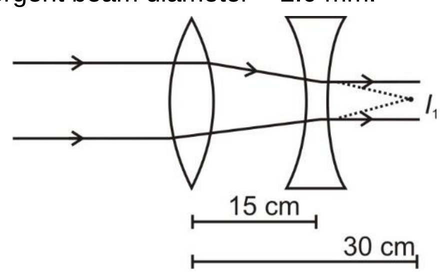
4. Answer (B, D)

Hint :

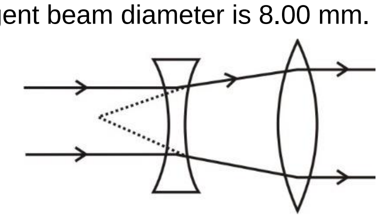
In case-I beam diameter is reduced, in case-II beam diameter is expanded.

Solution :

Emergent beam diameter = 2.0 mm.



Emergent beam diameter is 8.00 mm.

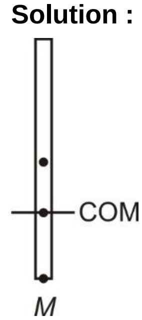


5. Answer (A, C, D)

Hint :

Apply conservation of linear and angular momentum.

Solution :



COAM $\Rightarrow$ $MV_0\frac{L}{4} = I\omega = \left(\frac{ML^2}{12}+\frac{ML^2}{16}+\frac{ML^2}{16}\right)\omega$

$$= \frac{(4+6)ML^2\omega}{48} = \frac{10}{48}ML^2\omega$$

$$\Rightarrow \quad \omega = \frac{V_0L}{4}\times\frac{48}{10L^2} = \frac{6}{5}\cdot\frac{V_0}{L}$$

$$V_A = \frac{V_0}{2}+\frac{6V_0}{5L}\times\frac{L}{4} = V_0\left[\frac{1}{2}+\frac{3}{10}\right] = \frac{8V_0}{10}$$

Loss in KE

$$= \frac{1}{2}MV_0^2 - \left[\frac{1}{2}2M\cdot\left(\frac{V_0}{2}\right)^2 + \frac{1}{2}\frac{10}{48}ML^2\cdot\frac{36}{25}\frac{V_0^2}{L^2}\right]$$

$$= \frac{1}{2}MV_0^2\left[1-\frac{1}{2}-\frac{3}{4}\times\frac{2}{5}\right]$$

$$= \frac{1}{2}MV_0^2\left[\frac{10-6}{20}\right] = \frac{MV_0^2}{10}$$

6. Answer (A, C, D)

Hint :

For null deflection $E = V_{AJ}$

Solution :

When S_1 and S_2 are open, $I = \frac{12}{30} = 0.4$ A

$V_{AB} = 4\,V \quad \Rightarrow \quad E = 2\,V$

When S_1 and S_2 are closed, $I = \frac{12}{10} = 1.2$ A

V_{AB} = 12 V, $\Rightarrow \quad 3\times\frac{2}{3+r} = \frac{25}{200}\times 12 = 1.5$

$\Rightarrow \quad 4.5+1.5r = 6$

$\Rightarrow \quad r = 1\,\Omega$

$P_{R_2} = (0.5)^2\times 3 = 0.75$ W

7. Answer (A, C)

Hint :

$T(x) = \frac{Mgx}{L} \quad \Rightarrow \quad \sigma(x) = \frac{Mgx}{AL}$

Solution :

$$U_{\text{Total}} = \int_0^L \frac{1}{2}\cdot\left(\frac{Mgx}{AL}\right)^2\frac{1}{Y}Adx$$

$$= \frac{M^2g^2}{2AL^2Y}\frac{L^3}{3} = \frac{M^2g^2L}{6AY}$$

$$U_{UH} = \int_{\frac{L}{2}}^{L}\frac{1}{2}\frac{(Mgx)^2}{AL^2Y}dx = \frac{M^2g^2}{2AL^2Y}\cdot\frac{1}{3}\left[L^3-\frac{L^3}{8}\right]$$

$$= \frac{7}{48}\frac{M^2g^2L}{AY}$$

$$\Delta L = \frac{MgL}{2AY}$$

8. Answer (B, C)

Hint :

$$\vec{F} = q\left[E_0\hat{k} + \left(V_x\hat{i}+V_z\hat{k}\right)\times B_0\hat{j}\right]$$

$$= q\left[E_0\hat{k} + B_0V_x\hat{k} - B_0V_z\hat{i}\right]$$

Solution :

$$\vec{F} = q\left[E_0\hat{k} + \left(V_x\hat{i}+V_z\hat{k}\right)\times B_0\hat{j}\right]$$

$$= q\left[E_0\hat{k} + B_0V_x\hat{k} - B_0V_z\hat{i}\right]$$

$$\Rightarrow \quad a_x = \frac{-qB_0V_z}{m} = \frac{-qB_0}{m}\cdot\frac{dz}{dt}$$

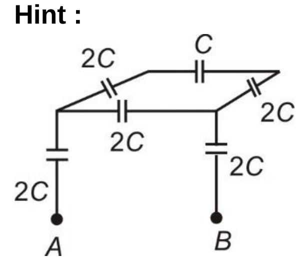
$$\Rightarrow \quad V_x = \frac{-qB_0z}{m} \qquad \ldots \text{(i)}$$

$$qE_0z = \frac{1}{2}mV^2$$

$$\Rightarrow \quad V = \sqrt{\frac{2qE_0z}{m}} \qquad \ldots \text{(ii)}$$

9. Answer (70)

Hint :



Solution :

$$C_{eq} = \frac{C\,\frac{5C}{2}}{C+\frac{5C}{2}} = \frac{5C}{7}$$

$$= \frac{5}{7}\times 49 = 35\ \mu\text{F}$$

$Q = C_{eq.}$ V = 35 × 2 = 70 μC

10. Answer (48)

Hint :

$$\phi = \int_0^a ady5t^3y$$

$$= \frac{5a^3t^3}{2}$$

Solution :

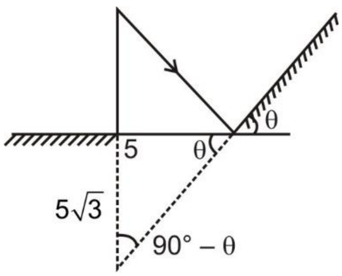
$$|\varepsilon|_{t=1\text{ s}} = \left.\frac{d\phi}{dt}\right|_{t=1\text{ s}}$$

$$= \frac{15}{2} \times 64 \times 10^{-6} \times 1$$

$$= 48 \times 10^{-5}\text{ V}$$

11. Answer (60)

Hint :



Solution :

$$\tan(90° - \theta) = \frac{1}{\sqrt{3}} = \tan 30°$$

$$\Rightarrow \theta = 60°$$

12. Answer (80)

Hint :

$$\beta = \frac{D}{d}\lambda = \frac{D}{d}\cdot\frac{h}{mV}$$

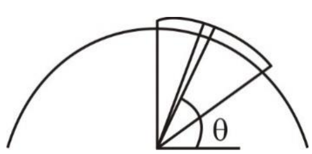
Solution :

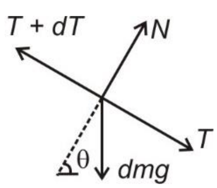
$$\beta = \frac{D}{d}\lambda = \frac{D}{d}\cdot\frac{h}{mV}$$

$$= \frac{8}{0.02} \times \frac{6.6\times10^{-3}}{330\times10^{-3}\times10} = \frac{800}{20} \times \frac{1}{5\times10} = 0.8\text{ m}$$

13. Answer (22)

Hint :





$dT = dmg\cos\theta$

Solution :

$$\int dT = \int_{30°}^{60°} \lambda R d\theta\, g\cos\theta$$

$$T = \lambda Rg \cdot \left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$$

$$= \frac{3mg}{\pi}\left[\frac{\sqrt{3}-1}{2}\right] = 21.96\text{ N}$$

14. Answer (26)

Hint :

On α-emission $_{Z}X^{A} \longrightarrow _{Z-2}X^{A-4} + _{2}\text{He}^{4}$

β-emission $_{Z}X^{A} \longrightarrow _{Z+1}X^{A} + _{-1}e^{0}$

Solution :

$$N_\alpha = \frac{24}{4} = 6$$

$$N_\alpha = \frac{24}{4} = 6,\quad N_\beta = 8 + 6\times2 = 20$$

15. Answer (D)

Hint :

$$\frac{3}{4} = e^{-\lambda t} \quad \Rightarrow t = \frac{1}{\lambda_0}\ln\left(\frac{4}{3}\right)$$

Solution :

$$\frac{3}{4} = e^{-\lambda t} \quad \Rightarrow t = \frac{1}{\lambda_0}\ln\left(\frac{4}{3}\right)$$

$$t = \frac{\ln 2}{\lambda_0}$$

16. Answer (A)

Hint :

$\lambda = 4\lambda_0$

Solution :

$\lambda = 4\lambda_0$

$$\frac{1}{4} = 1 - e^{-\lambda_0 t}$$

$$\frac{4}{3} = e^{\lambda_0 t}$$

$$\frac{1}{2} = 1 - e^{\lambda_0 t}$$

$$\Rightarrow \ln 2 = 3\lambda_0 t$$

$$t = \frac{\ln 2}{3\lambda_0}$$

17. Answer (B)

Hint :

$$\theta = 0,\quad mg\frac{\ell}{2} = \frac{m\ell^2}{3}\cdot\alpha \Rightarrow \alpha = \frac{3g}{2\ell}$$

Solution :

$$a_B = \alpha\ell = \frac{3g}{2} = 15\text{ m/s}^2$$

$$mg\frac{\ell}{2} = \frac{1}{2}\frac{m\ell^2}{3}\cdot\omega^2 \Rightarrow \omega^2 = \frac{3g}{\ell}$$

$$\omega^2\ell = \frac{3g}{2} = 30\text{ m/s}^2$$

F_H at $\theta = 0$,

$$F_H = \frac{mg}{4} = \frac{1}{4}\times 10 = 2.5\text{ N}$$

At $\theta = \frac{\pi}{2}$, $F_H = mg + m\omega^2\frac{\ell}{2} = 10 + 15 = 25\text{ N}$

18. Answer (C)

Hint :

$$\theta = 0,\quad a_B = \frac{3g}{2} = 15\text{ m/s}^2$$

Solution :

$$F_H = \frac{mg}{4} = \frac{1\times 10}{2\times 4} = 1.25\text{ N}$$

$$\theta = \frac{\pi}{2},\ a_B = \omega^2\ell = \frac{3g}{\ell}\cdot\ell = 30\text{ m/s}^2$$

$$F_H = mg + m\omega^2\frac{\ell}{2} = 5 + \frac{1}{2}\times\frac{3g}{\ell}\times\frac{\ell}{2}$$

$$= 5 + \frac{15}{2} = 12.5\text{ N}$$

PART - II (CHEMISTRY)

19. Answer (A, C)

Hint: Electrons with greater value of principal quantum number generally have lower ionization energy.

Solution:

As effective nuclear charge increases, ionization energy increases.

20. Answer (B, D)

Hint: $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

$$\Rightarrow\ P = \frac{RT}{V-b} - \frac{a}{V^2}$$

$$\Rightarrow\ \frac{PV}{RT} = \frac{V}{(V-b)} - \frac{a}{VRT}$$

Solution:

At P_1, $Z < 1$

So, $\frac{V}{V-b} < 1 + \frac{a}{VRT}$

At P_2, $Z = 1$

So, $\frac{V}{V-b} = 1 + \frac{a}{VRT}$

At P_3, $Z > 1$

So, $\frac{V}{V-b} > 1 + \frac{a}{VRT}$

21. Answer (A, C)

Hint: $H_Q < H_P$

$H_R > H_Q$

Solution:

For $P \rightarrow Q$, $\Delta S < 0$.

So, $S_Q < S_P$

For $Q \rightarrow R$, $\Delta S > 0$.

So, $S_R > S_Q$

22. Answer (B, C)

Hint: Down the group, basic character of oxides increases. Basic character increases with decrease in oxidation state.

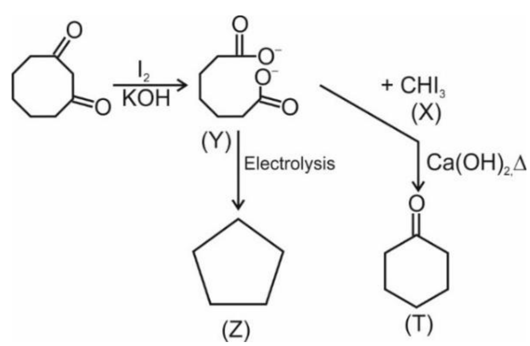
Solution:

In perhalic acids, acidic strength decreases from $HClO_4$ to HIO_4.

23. Answer (A, B, C)

Hint: Active methylenes respond to iodoform test.

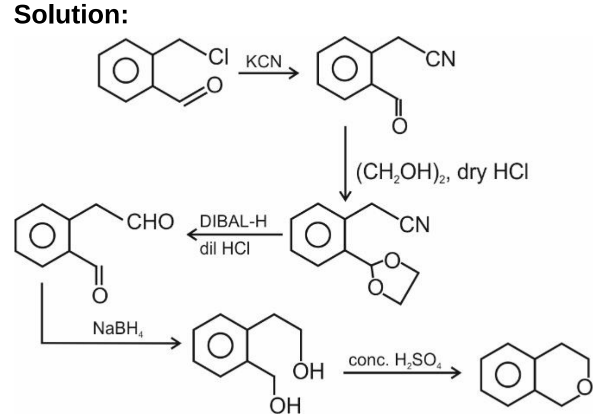
Solution:



24. Answer (A, C)

Hint: Selective reagents and protection of carbonyl group.

Solution:



25. Answer (B)

Hint: Glycine is the only α-amino acid without a chiral centre.

Solution:

Since, X has —COOH on glycine, so the other 3 amino acids can be arranged in $^3P_1$ ways.

26. Answer (B, C, D)

Hint: $\alpha = \frac{10^{-4}}{10^{-2}} = 10^{-2}$

$$\Rightarrow \frac{\Lambda_m}{\Lambda_m^\circ} = 10^{-2} \Rightarrow \Lambda_m^\circ = 100\Lambda_m$$

Solution:

K = G × G*

$= 5 \times 10^{-7} \times 10$

$= 5 \times 10^{-6}$ S cm^{-1}

$$\therefore \quad \Lambda_m = \frac{K \times 1000}{C} = \frac{5 \times 10^{-6} \times 10^{3}}{10^{-2}}$$

$= 5 \times 10^{-1}$ S cm^{-2} mol^{-1}

$$\therefore \quad \Lambda_m^\circ = 50 \text{ S cm}^2 \text{ mol}^{-1}$$

27. Answer (35)

Hint: x = 7

Solution:

$H_2S_4O_6$, $H_2S_2O_8$, $H_4P_2O_6$, H_2O_2, H_2SO_5, $O_2[PtF_6]$, $H_2S_2O_3$

28. Answer (05)

Hint: $MnO_4^- \xrightarrow[(x=5)]{H^+} Mn^{2+}$

Solution:

$$\left[Fe(C_2O_4)_2(H_2O)_2\right]^{2-} \xrightarrow[x=5]{} Fe^{3+} + 2CO_2$$

n-factor of the complex and MnO_4^- is same. Hence x = 1.

29. Answer (06)

Hint: $AB \rightleftharpoons A^+ + B^-,\ K_{sp}$

$$B^- + H^+ \rightleftharpoons HB,\ \frac{1}{K_a}$$

Solution:

$$\underset{\substack{- \\ -}}{AB} + \underset{\substack{2\times10^{-3} \\ 2\times10^{-3}}}{H^+} \rightleftharpoons \underset{\substack{- \\ s}}{HB} + \underset{\substack{- \\ s}}{A^-},\quad \frac{K_{sp}}{K_a} = \frac{2\times10^{-12}}{10^{-9}} = 2 \times 10^{-3}$$

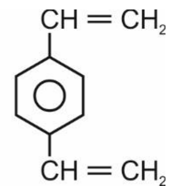
$$\frac{S^2}{2\times10^{-3}} = 10^{-3} \times 2$$

$S^2 = 4 \times 10^{-6}$

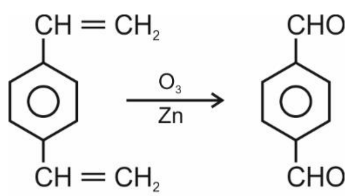
$S = 2 \times 10^{-3}$

30. Answer (11)

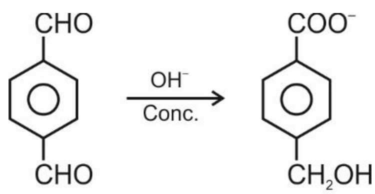
Hint: DU of X = 6 and it forms terephthalic acid upon oxidation with $KMnO_4$, H^+, Δ

X is 

Solution:



(a = 6)



(b = 5)

31. Answer (12)

Hint: x = 2, y = 4

Solution:

x + y = 6

So, 2(x + y) = 12

32. Answer (26)

Hint: Mass balance and charge balance.

Solution:

$$^{63}_{29}Cu + ^{1}_{1}H \longrightarrow 6^{1}_{0}n + ^{4}_{2}\alpha + 2^{1}_{1}H + ^{A}_{Z}Y$$

29 + 1 = 2+ 2 + Z

$\Rightarrow$ Z = 26

33. Answer (B)

34. Answer (A)

Hints and Solutions for Q. 33 & Q. 34 :

Hints :

(I) $W_{X\to Y\to Z} = \frac{RT_0}{2}\ln 2$ (R)

(III) $\Delta S_{X\to Y\to Z} = \frac{3R}{2}\ln 2 + R\ln 2$

$= \frac{5R}{2}\ln 2$ (S)

Solution :

(II) $q_{X\to Y\to Z} = \frac{RT_0}{2}\ln 2 + \frac{3RT}{4}$ (Q)

(IV) $\Delta U_{X\to Y\to Z} = \frac{3R}{2}\left(\frac{T_0}{2}\right)$

$= \frac{3RT_0}{4}$ (P)

35. Answer (C)

36. Answer (D)

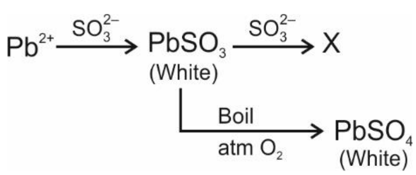
Hints and Solution for Q. 35 & Q. 36:

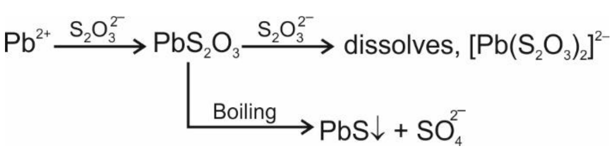
Hint:

$SO_3^{2-} \xrightarrow{H^+} SO_2$

$S_2O_3^{2-} \xrightarrow{H^+} S\downarrow + SO_2$

Solution:





PART - III (MATHEMATICS)

37. Answer (A, C)

Hint : $f(x) = 0$

Solution :

$f'(x) - \alpha \cdot f(x) \le 0$

$e^{-\alpha x}\cdot f'(x) - \alpha\cdot e^{-\alpha x}\cdot f(x) \le 0$

$d\left|e^{-ax}\cdot f(x)\right| \le 0$

$n(x) = e^{-\alpha x}\cdot f(x)$

$n'(x) \le 0$

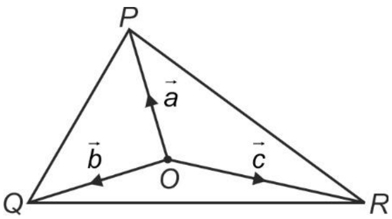
Hence $f(x)$ is always zero

Similarly $g(x)$ is always zero.

38. Answer (B)

Hint : $\Delta = \frac{1}{2}|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}|$

Solution :



$\overrightarrow{OP} + k_1\overrightarrow{OQ} + k_2\overrightarrow{OR} = 0$

$\vec{a} + k_1\vec{b} + k_2\vec{c} = 0$

($\because$ O is inside so k_1, k_2 are positive)

Area of $\Delta PQR = \frac{-1}{2}|(\bar{a} - \bar{b})\times(\bar{c} - \bar{b})|$

$= \frac{1}{2}|(-k_1\bar{b} - k_2\bar{c} - b)\times(c - \bar{b})|$

Area of $\Delta OQR = \frac{1}{2}|\vec{b}\times\vec{c}|$

$\Rightarrow \frac{\Delta PQR}{\Delta OQR}$

$= \frac{|k_1 + k_2 + 1||\bar{b}\times\bar{c}|}{|\bar{b}\times\bar{c}|} = 4$

$k_1 + k_2 + 1 = 4$

$k_1 + k_2 = 3$

39. Answer (C, D)

Hint : Apply LMVT.

Solution :

(A) Option is correct for some α

(B) Option is correct for some α

(C) $g(x) = f^2(x)$

Applying L.M.V.T,

$$\frac{f^2\left(\frac{\pi}{2}\right) - f(0)}{\frac{\pi}{2}} = 2f(x)\cdot f'(\alpha)$$

We get $f(x)\cdot f'(x) = \frac{1}{x}$

(D) Take a function $g(x) = f(x) - \frac{f}{\pi^2} \cdot x^2$

By applying L.M.V.T. we get *f*.

40. Answer (A, B)

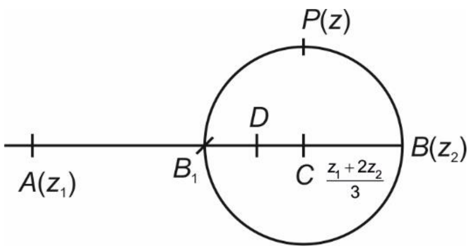
Hint : Section formula.

Solution :

$$\left|z - \frac{z_1 + 2z_2}{3}\right| = \frac{1}{3}|z_1 - z_2|$$

And $\arg\left(\frac{z_1 - z_2}{z - (kz_1 + (1-k)z_2)}\right) = \pm\frac{\pi}{2}$

Angle between the line segment joining z_1 and z_2; z and $kz_1 + (1 - k)z_2$ is $\frac{\pi}{2}$ $\frac{z_1 + 2z_2}{3}$ is a point on segment AB such that $AC : CS = 2 : 1$ and D $(kz_1 + (1 - k)z_2)$ is a point on segment AB such that $AD : DB = 1 - k \Rightarrow AD = k|(z_1 - z_2|$



(A) For no sol.

$$BD > BB_1 \Rightarrow k|z_1 - z_2| > \frac{2}{3}|z_1 - z_2| \Rightarrow k > \frac{2}{3}$$

(B) For more than one sol. $0 < BD < BB_1$

$$\Rightarrow \quad 0 < k|z_1 - z_2| < \frac{2}{3}|z_1 - z_2|$$

$$0 < k < \frac{2}{3}$$

41. Answer (A, C)

Hint : LMVT

Solution : By the mean value theorem,

$$\frac{f(0) + f(2)}{2} = f(c) \qquad 0 < c < 2$$

$f(0) + f(2) = 2f(c)$

and by LMVT,

$$\frac{f(1) - f(0)}{1 - 0} = f'(c_1) \qquad 0 < c_1 < 1$$

$f(1) - f(0) = f'(c_1) \qquad 0 < c_1 < 1$

and by LMVT,

$$\frac{f(2) - f(1)}{2 - 1} = f'(c_2) \qquad 1 < c_2 < 2$$

$f(1) - f(0) = f'(c_1) \qquad \ldots(i)$

$f(2) - f(1) = f'(c_2) \qquad \ldots(ii)$

Subtracting (ii) – (i),

$f(2) + f(0) - 2f(1) = f'(c_2) - f'(c_1)$

Since $f''(x) < 0$ in $[0, 2]$

$f'(x)$ is decreasing function

$c_2 > c_1$

$f'(c_2) < f'(c_1)$

So, $f'(c_2) - f'(c_1) < 0$

So, $f(2) + f(0) - 2f(1) < 0$

$f(2) + f(0) < 2f(1)$

So, options (A) and (C) are correct.

42. Answer (A, B, C, D)

Hint :

Application of inequality.

Solution :

(A) By applying A.M, G.M. we get $A = B = C$.

(B) Equality is true for $\sin C = 1$ and $A = B$

(C) A.M, G.M., H.M inequality

(D) Jensen's inequality

43. Answer (A, C, D)

Hint :

Separate into two integrals and then substitute $\tan x$ and $\cot x$.

Solution :

$$f(\theta) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left[\frac{\sec^2 x}{\sqrt{\sin\theta + \cos\theta\tan x}} + \frac{\text{cosec}^2 x}{\sqrt{\cos\theta + \sin\theta\cot x}}\right] dx$$

$$\Rightarrow f(\theta) = 2\sqrt{\sqrt{3}\cos\theta + \sin\theta}\left(\frac{1}{\cos\theta} - \frac{1}{3^{\frac{1}{4}}\sin\theta}\right) +$$

$$2\sqrt{\sqrt{3}\sin\theta + \cos\theta}\left(\frac{1}{\sin\theta} - \frac{1}{3^{\frac{1}{4}}\cos\theta}\right)$$

$$f(\theta) = f\left(\frac{\pi}{2} - \theta\right)$$

44. Answer (B, D)

Hint : $a_6 = 1$

Solution : $N = {}^{11}C_5 = 462$ as $a_6 = 1$

45. Answer (04)

Hint :

$[\log_5 x] = 4$, M is possible integral values of x.

Solution : $L = \text{antilog}_{32}(0.6)$

$\log_{32} L = 0.6$

So, $L = (32)^{0.6} = (2^5)^{6/10} = 2^3 = 8$

M = number of positive integers having characteristic 4 so has base is 5 will be 2500

and $N = 49^{(1-\log_7 2)} + 5^{-\log_5 4}$

$$= 7^{2(\log_7 7 - \log_7 2)} + \frac{1}{4}$$

$$= 7^{\log_7\left(\frac{7}{2}\right)^2} + \frac{1}{4}$$

$$= \frac{49}{4} + \frac{1}{4} = \frac{50}{4} = \frac{25}{2}$$

So, $\dfrac{LM}{400N} = \dfrac{2500 \times 8}{400 \times 25} \times 2 = 4$

46. Answer (07)

Hint : Multinomial theorem.

Solution :

Finding out coefficient of

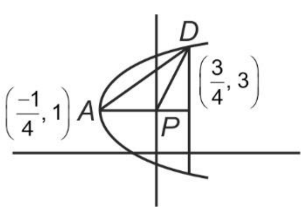
x^{20} in $(1 + x + n^2 + ...x^9)^{21}$

is required answer we get $k = 21$.

47. Answer (10)

Hint : $\tan\theta = \dfrac{|m_1 - m_2|}{1 + m_1 m_2}$

Solution :



$(y - k)^2 = 4(x - h)$

It passes through origin and (0, 2).

So, $h = \dfrac{-1}{4}$, $k = 1$

The equation of latus rectum

$x = \dfrac{3}{4}$

So, coordinate of D is $\left(\dfrac{3}{4}, 3\right)$ and coordinate of P is (0, 1).

So, slope of $AD = \left(\dfrac{3-1}{\dfrac{3}{4} + \dfrac{1}{4}}\right) = 2$

Slope of $DP = \dfrac{3-1}{\left(\dfrac{3}{4} - 0\right)} = \dfrac{8}{3}$

So, $\tan\theta = \dfrac{\dfrac{8}{3} - 2}{1 + \dfrac{8}{3} \times 2} = \dfrac{2}{3\left(1 + \dfrac{16}{3}\right)} = \dfrac{2}{19}$

$\tan\theta = \dfrac{2}{19} \Rightarrow 95\tan\theta = 95 \times \dfrac{2}{19} = 10$

48. Answer (20)

Hint : $r = 4R \sin\dfrac{A}{2} \sin\dfrac{B}{2} \sin\dfrac{C}{2}$

Solution : $64\Delta^2 = 6abcs$

$64\Delta^2 = 6 \times 4\Delta Rs$

$64\Delta = 6 \times 4 \times R \times s$

$16\Delta = 6 \times R \times s$

$16 \times r \times s = 6R \times s$

$8r = 3R$

$r = 4R \sin\dfrac{A}{2} \sin\dfrac{B}{2} \sin\dfrac{C}{2}$

$\dfrac{3R}{8} = 4R \sin\dfrac{A}{2} \sin\dfrac{B}{2} \sin\dfrac{C}{2}$

$3 = 32 \sin\dfrac{A}{2} \sin\dfrac{B}{2} \sin\dfrac{C}{2}$

$\dfrac{3}{16} = \left[\dfrac{\cos(A-B)}{2} - \dfrac{\cos(A+B)}{2}\right] \sin\dfrac{C}{2}$

$\dfrac{3}{16} = \dfrac{\sqrt{3}}{2} \sin\dfrac{C}{2} - \sin^2\dfrac{C}{2}$

$\Rightarrow \sin^2\dfrac{C}{2} - \dfrac{\sqrt{3}}{2} \sin\dfrac{C}{2} + \dfrac{3}{16} = 0$

$\Rightarrow \left(\sin\dfrac{C}{2} - \dfrac{\sqrt{3}}{4}\right)^2 = 0$

$\Rightarrow \quad \sin\frac{C}{2} = \frac{\sqrt{3}}{4} \quad \cos C = \frac{5}{8}$

$\Rightarrow \quad 32 \cos C = 20$

49. Answer (09)

Hint : Pair of tangent from origin to ellipse.

Solution :

Here the locus of centre of the given circle will be an ellipse whose equation is $\frac{x^2}{100} + \frac{(y-12)^2}{75} = 1$. If we drop pair of tangents from origin

$$m^2 = \frac{69}{100} = \frac{p}{q}$$

Hence $p + q - 160 = 9$.

50. Answer (04)

Hint : Series expansion.

Solution :

By applying series on position of cos we get

$n = 4$.

51. Answer (D)

Hint : Parametric point on line.

Solution :

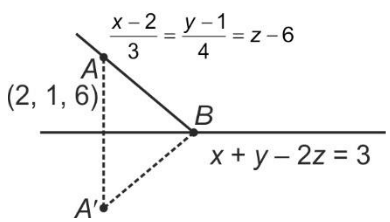


Image point of (2, 1, 6) will be (6, 5, –2)

Point B will be (–10, –15, –14) and equation of reflected ray will join the point A' and B.

So, equation of reflected ray

$$\frac{x+10}{16} = \frac{y+15}{20} = \frac{z+14}{12}$$

$$\frac{x+10}{4} = \frac{y+15}{5} = \frac{z+14}{3}$$

52. Answer (D)

Hint : Counting

Solution :

(A) All possible position of D

$\_\,\_\,\_\,\underline{D}\,\_\,\_ = \underline{|3} \times \underline{|2} = 12$

$\_\,\_\,\_\,\_\,\underline{D}\,\_ = {}^4C_3 \times \underline{|3} \times \underline{|2} = 48$

$\_\,\_\,\_\,\_\,\_\,\underline{D} = \underline{|5} = 120$

12 + 48 + 120 = 180

So, A, B, C all perform before D = 180 number of ways

(B) Number of ways when C perform in 3rd place = $\underline{|5} = 120$

(C) Number of ways when F performs in first and A perform in last = $\underline{|4} = 24$

(D) Number of ways when A always perform before B = ${}^6C_2 \times \underline{|4} = 360$

53. Answer (B)

54. Answer (B)

❑ ❑ ❑