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KCET-2020 (Code-C3)
MATHEMATICS

Subject	No. Of Questions	Max. Marks	Maximum Time For Answering
Mathematics	60	60	70 Minutes

Choose the correct answer:

1. Then the smallest positive integer for which P(n) is true if

(A) 4 (B) 5
(C) 2 (D) 3

Sol. Answer (A)

$2^n < n!$

By hit & trial

$n = 4$ & 5 are to be true

So we require small $\Rightarrow n = 4$

2. If $z = x + iy$, then the equation $|z+1| = |z-1|$ represents

(A) x-axis (B) y-axis
(C) a circle (D) a parabola

Sol. Answer (B)

$|z+1| = |z-1|$

Locus of z is perpendicular bisector of line segment joining -1 and 1

3. The value of ${}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7$ is

(A) ${}^{17}C_{10}$ (B) ${}^{17}C_3$
(C) 0 (D) 1

Sol. Answer (C)

${}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7 = 0$

${}^nC_r = {}^nC_{n-r}$

4. The number of terms in the expansion of $(x+y+z)^{10}$ is

(A) 11 (B) 110
(C) 66 (D) 142

Sol. Answer (C)

$(x+y+z)^{10}$

no. of terms in the expansion $(x_1+x_2+....x_r)^n$ is

${}^{n+r-1}C_{r-1}$

$= {}^{10+3-1}C_{3-1}$

$= {}^{12}C_2 = \frac{12\times 11}{2} = 66$

5. If the sum of n terms of an A.P. is given by $S_n = n^2 + n,$ then the common difference of the A.P. is

(A) 2 (B) 6
(C) 4 (D) 1

Sol. Answer (A)

$S_n = n^2 + n$

$S_1 = 1 + 1 = T_1$

$S_2 = T_1 + T_2 = 2^2 + 2 = 6$

$T_1 = 2 = a$

$T_1 + T_2 = a + (a+d) = 6$

$\Rightarrow 2a + d = 6 \Rightarrow 4 + d = 6 \Rightarrow d = 2$

6. The two lines $lx+my=n$ and $l'x+m'y=n'$ are perpendicular if

(A) $lm+l'm'=0$ (B) $lm'+ml'=0$

(C) $ll'+mm'=0$ (D) $lm'=ml'$

Sol. Answer (C)

$lx+my=x$

$l'x+m'y=n'$

Product of their slopes is -1

$\Rightarrow\left(\frac{-l}{m}\right)\left(\frac{-l'}{m'}\right)=-1$

$\Rightarrow ll'=-mm'\Rightarrow ll'+mm'=0$

7. If the parabola $x^2=4ay$ passes through the point (2, 1), then the length of the latusrectum is

(A) 2 (B) 8

(C) 1 (D) 4

Sol. Answer (D)

$x^2=4ay$

It passes through (2, 1)

$\Rightarrow 2^2=4.a.1$

$\Rightarrow a=1$

So parabola $x^2=4y$

length of latusrectum is 4

8. $\lim\limits_{x\to0}\left(\frac{\tan x}{\sqrt{2x+4}-2}\right)$ is equal to

(A) 4 (B) 6

(C) 2 (D) 3

Sol. Answer: (C)

$\lim\limits_{x\to0}\left(\frac{\tan x}{\sqrt{2x+4}-2}\right)$

$\lim\limits_{x\to0}\frac{\tan x}{\left(\sqrt{2x+4}-2\right)}\times\frac{\left(\sqrt{2x+4}+2\right)}{\left(\sqrt{2x+4}+2\right)}$

$=\lim\limits_{x\to0}\frac{\tan x\left(\sqrt{2x+4}+2\right)}{2x+4-4}$

$=\lim\limits_{x\to0}\frac{\tan x}{2x}\times\left(\sqrt{2x+4}+2\right)$

$\left[\because\lim\limits_{\theta\to0}\frac{\tan\theta}{\theta}=1\right]$

$=\frac{1}{2}\times\left(\sqrt{4}+2\right)=\frac{1}{2}\times4=2$

9. The negation of the statement "For all real numbers x and y, $x+y=y+x$" is

(A) for some real numbers x and y, $x+y\ne y+x$

(B) for some real numbers x and y, $x-y=y-x$

(C) for all real numbers x and y, $x+y\ne y+x$

(D) for some real numbers x and y, $x+y=y+x$

Sol. Answer (A)

For some real numbers x and y, $x+y\ne y+x$

10. The standard deviation of the data 6,7,8,9,10 is

(A) 2 (B) 10

(C) $\sqrt{2}$ (D) $\sqrt{10}$

Sol. Answer (C)

6,7,8,9,10

$\bar{X}=\frac{6+7+8+9+10}{5}=8$

$\sigma=\sqrt{\frac{\sum\left(x_i-\bar{x}\right)^2}{N}}$

$=\sqrt{\frac{(-2)^2+(-1)^2+0+1^2+2^2}{5}}$

$=\sqrt{\frac{10}{5}}$

$=\sqrt{2}$

11. If A, B, C are three mutually exclusive and exhaustive events of an experiment such that P(A)=2 P(B)=3 P(C), then P(B) is equal to

(A) $\frac{3}{11}$ (B) $\frac{4}{11}$

(C) $\frac{1}{11}$ (D) $\frac{2}{11}$

Sol. Answer (A)

Let $P(A) = 2.P(B) = 3.P(C) = x$

Events are mutually exclusive and exhaustive

$P(A)+P(B)+P(C)=1$

$\Rightarrow x+\frac{x}{2}+\frac{x}{3}=1$

$\Rightarrow \frac{6x+3x+2x}{6}=1$

$\Rightarrow \frac{11x}{6}=1 \Rightarrow x=\frac{6}{11}$

$\Rightarrow P(B)=\frac{x}{2}=\frac{3}{11}$

12. If a relation R on the set $\{1,2,3\}$ be defined by $R=\{(1,1)\}$, then R is

(A) Symmetric and transitive

(B) Only symmetric

(C) Reflexive and symmetric

(D) Reflexive and transitive

Sol. Answer (A)

$R=\{(1,1)\}$

It is only symmetric & transitive

13. Let $f:[2,\infty)\to R$ be the function defined by $f(x)=x^2-4x+5,$ then the range of f is

(A) $(1,\infty)$ (B) $[5,\infty)$

(C) $(-\infty,\infty)$ (D) $[1,\infty)$

Sol. Answer (D)

$f(x)=x^2-4x+5$

$f(x)=(x-2)^2+1$

$f(x)\in[1,\infty)$

14. If $A=\{a,b,c\}$, then the number of binary operations on A is

(A) 3^3 (B) 3^9

(C) 3 (D) 3^6

Sol. Answer (B)

$A=\{a,b,c\}$

no. of binary operation n^{n^2} of set is having in elements = 3^9

15. The domain of the function defined by $f(x)=\cos^{-1}\sqrt{x-1}$ is

(A) $[-1,1]$ (B) $[0,1]$

(C) $[1,2]$ (D) $[0,2]$

Sol. Answer (C)

$f(x)=\cos^{-1}\sqrt{x-1}$

$x-1\geq 0 \Rightarrow x\geq 1 \to (1)$

$-1\leq\sqrt{x-1}\leq 1$

$\Rightarrow 0\leq\sqrt{x-1}\leq 1$

$\sqrt{x-1}\leq 1$

$x-1\leq 1$

$x\leq 2\to(2)$

From (1) & (2)

$x\in[1,2]$

16. The value of $\cos\left(\sin^{-1}\frac{\pi}{3}+\cos^{-1}\frac{\pi}{3}\right)$ is

(A) -1 (B) Does not exist

(C) 0 (D) 1

Sol. Answer (B)

For arc sin x, x should lie between -1 and 1

17. If $A=\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$, then A^4 is equal to

(A) I (B) 4A

(C) A (D) 2A

Sol. Answer (A)

$A=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$

$A^2=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$

$A^2=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I$

$A^2=I,\ A^4=A^2.A^2=I.I=I$

18. If A and B are square matrices of same order and B is a skew symmetric matrix, then $A'BA$ is

(A) Diagonal matrix

(B) Skew symmetric matrix

(C) Symmetric matrix

(D) Null matrix

Sol. Answer (B)

Let $P = A^TBA$

$P^T = (A^TBA)^T$

$= A^TB^T(A^T)^T$

$\left[(PQR)^T = R^TQ^TP^T\right]$

$= A^T(-B)\cdot A$

$= P^T = -A^T\cdot B\cdot A$

$P^T = -P$

Skew symmetric

19. If $\begin{pmatrix}2 & 1\\ 3 & 2\end{pmatrix}A = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, then the matrix A is

(A) $\begin{pmatrix}-2 & 1\\ 3 & -2\end{pmatrix}$ (B) $\begin{pmatrix}2 & -1\\ 3 & 2\end{pmatrix}$

(C) $\begin{pmatrix}2 & 1\\ 3 & 2\end{pmatrix}$ (D) $\begin{pmatrix}2 & -1\\ -3 & 2\end{pmatrix}$

Sol. Answer (D)

$$\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}^{-1}$$

$$= \frac{1}{4-3}\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$$

20. If $f(x) = \begin{vmatrix}x^3 - x & a+x & b+x\\ x-a & x^2-x & c+x\\ x-b & x-c & 0\end{vmatrix}$, then

(A) $f(0) = 0$ (B) $f(-1) = 0$

(C) $f(1) = 0$ (D) $f(2) = 0$

Sol. Answer (A)

$$f(x) = \begin{vmatrix}x^3 - x & a+x & b+x\\ x-a & x^2-x & c+x\\ x-b & x-c & 0\end{vmatrix}$$

$$f(0) = \begin{vmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{vmatrix} = 0$$

Determinant of skew symmetric matrix of odd order = 0

21. If $a_1, a_2, a_3, ..., a_9$ are in A.P, then the value of $\begin{vmatrix}a_1 & a_2 & a_3\\ a_4 & a_5 & a_6\\ a_7 & a_8 & a_9\end{vmatrix}$ is

(A) $\log_e(\log_e e)$ (B) 1

(C) $\frac{9}{2}(a_1 + a_9)$ (D) $a_1 + a_9$

Sol. Answer (A)

Apply $C_1 \to C_1 + C_3 - 2C_2$

22. If A is a square matrix of order 3 and |A|=5, then $|A\cdot \text{adj}A|$ is

(A) 25 (B) 625

(C) 5 (D) 125

Sol. Answer (D)

$A.\text{adj}A = |A|.I_n$

$|A.\text{adj}A| = \left||A|.I_3\right|$

$= |A|^3 = 5^3$

23. If $f(x) = \begin{cases}\dfrac{1-\cos Kx}{x\sin x}, & \text{if } x \neq 0\\ \dfrac{1}{2}, & \text{if } x = 0\end{cases}$

is continuous at $x = 0$, then the value of K is

(A) ± 2 (B) ± 1

(C) $\pm\frac{1}{2}$ (D) 0

Sol. Answer (B)

$$f(x) = \begin{cases}\dfrac{1-\cos kx}{x\sin x} & ; x \neq 0\\ \dfrac{1}{2} & ; x = 0\end{cases}$$

$\lim_{x\to 0}\frac{1-\cos kx}{x.\sin x}$

$\lim_{x\to 0}\frac{2\sin^2\frac{kx}{2}}{x.\sin x}$

$\lim_{x\to 0}2\left(\frac{\sin\frac{kx}{2}}{\frac{kx}{2}}.\frac{kx}{2}\right)^2.\frac{1}{x^2\left(\frac{\sin x}{x}\right)}$

$=\lim_{x\to 0}\frac{2.k^2x^2}{4.x^2}.\left(\frac{\sin\frac{kx}{2}}{\frac{kx}{2}}\right)^2.\frac{1}{\left(\frac{\sin x}{x}\right)}$

$=\frac{k^2}{2}$

Since $f(x)$ is continuous at x=0

So $f(0)=LHL=RHL$

$\frac{1}{2}=\frac{k^2}{2}\Rightarrow k^2=1\Rightarrow k=\pm 1$

24. The right hand and left hand limit of the function $f(x)=\begin{cases}\frac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$ are respectively

(A) -1 and -1 (B) -1 and 1

(C) 1 and 1 (D) 1 and -1

Sol. Answer (D)

RHL

Put $x=0+u$

$\lim_{u\to 0}\frac{e^{\frac{1}{u}}-1}{e^{\frac{1}{u}}+1}$

Take $e^{\frac{1}{u}}$ common

$\lim_{u\to 0}\frac{1-e^{-\frac{1}{u}}}{1+e^{-\frac{1}{u}}}=1$

LHL, Put $x=0-u$

$\lim_{u\to 0}\frac{e^{\frac{-1}{u}}-1}{e^{-\frac{1}{u}+1}}=-1$

25. If $2^x+2^y=2^{x+y}$, then $\frac{dy}{dx}$ is

(A) 2^{x-y} (B) $\frac{2^y-1}{2^x-1}$

(C) 2^{y-x} (D) -2^{y-x}

Sol. Answer (D)

$2^x+2^y=2^{x+y}$

$2^x+2^y=2^x.2^y$

$\Rightarrow 2^{-x}+2^{-y}=1$

Differentiating with respect to 'x'

$2^{-x}.\log 2.(-1)+2^{-y}.\log 2.(-1)\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-2^y}{2^x}$

26. If $f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f'(\sqrt{3})$ is

(A) $\frac{1}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$

(C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

Sol. Answer (C)

$\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\begin{cases}\pi-2\tan^{-1}x; & x\geq 1\\ 2\tan^{-1}x; & -1\leq x\leq 1\\ -\pi-2\tan^{-1}x; & x<-1\end{cases}$

$f'(x)=\begin{cases}\frac{-2}{1+x^2}; x>1\\ \frac{2}{1+x^2}; -1<x<1\\ \frac{-2}{1+x^2}; x<-1\end{cases}$

$f'(\sqrt{3})=\frac{-2}{1+3}=-\frac{1}{2}$

27. If $(xe)^y=e^x$, then $\frac{dy}{dx}$ is

(A) $\frac{\log x}{(1+\log x)}$ (B) $\frac{e^x}{x(y-1)}$

(C) $\frac{\log x}{(1+\log x)^2}$ (D) $\frac{1}{(1+\log x)^2}$

Sol. Answer (C)

$(xe)^y = e^x$

Take log on both sides

$y.\ln_e(xe) = x.\ln_e e$

$y.(\ln_e x + \ln e) = x$

$y = \frac{x}{\ln x + 1}$

Differentiating with respect to 'x'

$y' = \frac{1.(\ln x+1) - \frac{1}{x}.x}{(\ln x+1)^2} = \frac{\ln x}{(\ln x+1)^2}$

28. If $y = 2x^{n+1} + \frac{3}{x^n}$, then $x^2\frac{d^2y}{dx^2}$ is

(A) $x\frac{dy}{dx} + y$ (B) y

(C) $6n(n+1)y$ (D) $n(n+1)y$

Sol. Answer (D)

$y = 2.x^{n+1} + \frac{3}{x^n}$

$y' = 2.(n+1).x^n - 3nx^{-n-1}$

$y'' = 2.n(n+1).x^{n-1} + 3n(n+1).x^{-n-2}$

Now according to question, multiply x^2 on both sides

$x^2.y'' = 2.n(n+1).x^2.x^{n-1} + 3n(n+1).x^2.x^{-n-2}$

$= 2.n(n+1).x^{n+1} + 3n(n+1).\frac{1}{x^n}$

$y''.x^2 = n(n+1)(y)$

29. If the curves $2x = y^2$ and $2xy = K$ intersect perpendicularly, then the value of K^2 is

(A) 2 (B) 8

(C) 4 (D) $2\sqrt{2}$

Sol. Answer (B)

Curves are $\perp$, so product of the slopes = -1

$2x = y^2 \Rightarrow 2 = 2y.y' \Rightarrow y' = \frac{1}{y} = m_1$

$2xy = k \Rightarrow y = \frac{k}{2x}$

$y' = \frac{-k}{2x^2} = m_2$

$m_1m_2 = -1$

$\Rightarrow \frac{1}{y}.\left(\frac{-k}{2x^2}\right) = -1$

$\Rightarrow k = 2x^2y$

Put $2xy = k$ in $k = 2x^2y$

$2xy = 2x^2y$

$\Rightarrow xy(x-1) = 0$

$x = 0, y = 0$ or $x = 1$

Put $x = 1$ in equation (1), to get y

$2x = y^2 \Rightarrow y^2 = 2 \Rightarrow y = \pm\sqrt{2}$

Now $2xy = k$ (Squaring on both sides)

$k^2 = 4x^2y^2 = 4.1.2 = 8$

30. The maximum value of $\frac{\log_e x}{x}$, if $x > 0$ is

(A) $\frac{1}{e}$ (B) $-\frac{1}{e}$

(C) e (D) 1

Sol. Answer (A)

$y = \frac{\log_e x}{x}$ Differentiating with respect to 'x'

$y' = \frac{x.\frac{1}{x} - \log_e x}{x^2}$

$y' = \frac{1 - \log_e x}{x^2}$

Maxima occurs at x = e as $\frac{dy}{dx}$ in the NBD changes from +ve to –ve

$f(e) = \frac{\log_e e}{e} = \frac{1}{e}$

31. If the side of a cube is increased by 5%, then the surface area of a cube is increased by

(A) 6% (B) 20%

(C) 10% (D) 60%

Sol. Answer (C)

$\frac{dx}{x}\times 100 = 5$

$\frac{ds}{s}\times 100 = ?$

$s = 6x^2$

$\log s = \log 6 + 2.\log x$

$\frac{1}{s}.ds = 2\frac{dx}{x}$

$\frac{ds}{s} = 2.\frac{dx}{x}$

$\frac{ds}{s}\times 100 = 2.\frac{dx}{x}\times 100 = 2\times 5 = 10\%$

32. The value of $\int \frac{1+x^4}{1+x^6}dx$ is

(A) $\tan^{-1}x - \frac{1}{3}\tan^{-1}x^3 + C$

(B) $\tan^{-1}x + \frac{1}{3}\tan^{-1}x^2 + C$

(C) $\tan^{-1}x + \tan^{-1}x^3 + C$

(D) $\tan^{-1}x + \frac{1}{3}\tan^{-1}x^3 + C$

Sol. Answer (D)

$\int \frac{1+x^4}{(1+x^6)}dx$

$\int \frac{(1+x^4)(1+x^2)}{(1+x^6)(1+x^2)}dx$

$\int \frac{(1+x^6)+x^2(1+x^2)}{(1+x^6)(1+x^2)}dx$

$\int \frac{dx}{1+x^2} + \int \frac{x^2dx}{1+x^6}$

$\int \frac{dx}{1+x^2} + \frac{1}{3}\int \frac{3x^2}{1+(x^3)^2}dx$

$= \tan^{-1}x + \frac{1}{3}.\tan^{-1}x^3 + C$

33. If $\int \frac{3x+1}{(x-1)(x-2)(x-3)}dx$

$= A\log|x-1| + B\log|x-2| + C\log|x-3| + \text{Constant}$,

then the values of A, B and C are respectively

(A) 5,-7,5 (B) 2,-7,5

(C) 5,-7,-5 (D) 2,-7,-5

Sol. Answer (B)

$\int \frac{3x+1}{(x-1)(x-2)(x-3)}dx$

$\frac{3x+1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

Take LCM

$3x+1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

Put $x = 1$

$\Rightarrow 4 = A(-1)(-2) \Rightarrow A = 2$

Put x=2

$7 = B(1)(-1) \Rightarrow B = -7$

Put x=3

$9+1 = C.2.1 \Rightarrow C = 5$

$\int \frac{3x+1}{(x-1)(x-2)(x-3)}dx = 2\int \frac{dx}{x-1} - 7\int \frac{dx}{x-2} + 5\int \frac{dx}{x-3}$

$= 2\ln|x-1| - 7.\ln|x-2| + 5.\ln|x-3| + C$

A=2, B=-7, C=5

34. The value of $\int e^{\sin x}\sin 2x\,dx$ is

(A) $2e^{\sin x}(\cos x + 1) + C$

(B) $2e^{\sin x}(\cos x - 1) + C$

(C) $2e^{\sin x}(\sin x - 1) + C$

(D) $2e^{\sin x}(\sin x + 1) + C$

Sol. Answer (C)

$\int e^{\sin x}.2.\sin x.\cos x.dx$

Put sinx = t

$\cos x\,dx = dt$

$2\int e^{t}.t\ dt$

$2\left[t\int e^{t}dt - \int\left(\frac{dt}{dt}\int e^{t}\right)dt\right]$

$= 2\left[e^{t}.t - e^{t}\right] + C$

$= 2e^{t}(t-1) + C$

Replace ‘t’ by sinx

$= 2e^{\sin x}(\sin x - 1) + C$

35. The value of $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos^{-1} x\,dx$ is

(A) 1 (B) $\frac{\pi^2}{2}$

(C) π (D) $\frac{\pi}{2}$

Sol. Answer (D)

$I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos^{-1} x dx \rightarrow (1)$

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos^{-1}(-x)dx$

$I = \int_{-\frac{1}{2}}^{\frac{1}{2}} (\pi - \cos^{-1} x)dx \rightarrow (2)$

Add both (1) and (2)

$2I = \int_{-\frac{1}{2}}^{\frac{1}{2}} (\cos^{-1} x + \pi - \cos^{-1} x)dx$

$2I = \pi\int_{-\frac{1}{2}}^{\frac{1}{2}} dx$

$2I = \pi.\left(\frac{1}{2} + \frac{1}{2}\right)$

$I = \frac{\pi}{2}$

36. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^{x}} dx$ is

(A) 1 (B) -2

(C) 2 (D) 0

Sol. Answer (A)

$I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^{x}} dx \rightarrow (1)$

Use $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int \frac{\cos x}{1+e^{-x}} dx$

$I = \int \frac{e^{x}.\cos x}{1+e^{x}} dx \rightarrow (2)$

Add (1) and (2)

$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx$

$2I = (\sin x)_{-\pi/2}^{\pi/2}$

$2I = 1 + 1 \Rightarrow I = 1$

37. The value of $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ is

(A) $\frac{1}{2}$ (B) $\frac{\pi}{8}\log 2$

(C) $\frac{\pi}{2}\log 2$ (D) $\frac{\pi}{4}\log 2$

Sol. Answer (B)

$\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

Put $x = \tan\theta$

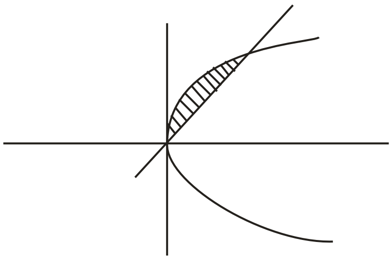
$\int_0^{\pi/4} \log(1+\tan\theta)d\theta = \frac{\pi}{8}\log 2$

38. The area of the region bounded by the curve $y^2 = 8x$ and the line $y = 2x$ is

(A) $\frac{3}{4}$ sq. units (B) $\frac{8}{3}$ sq. units

(C) $\frac{16}{3}$ sq. units (D) $\frac{4}{3}$ sq.units

Sol. Answer (D)



$y^2 = 8x$

y=2x

substitute $\Rightarrow (2x)^2 = 8x$

$4x^2 = 8x$

$x = 0, 2$

$\int_0^2 \left(\sqrt{8x} - 2x\right) dx$

$2\sqrt{2}\int_0^2 x^{\frac{1}{2}} dx - 2\int_0^2 x dx$

$$\left(\frac{2\sqrt{2}.x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2.x^2}{2}\right)_0^2$$

$$= \frac{4\sqrt{2}}{3}.2^{\frac{3}{2}} - 2^2$$

$$= \frac{4.2^2}{3} - 4 = \frac{4}{3}$$

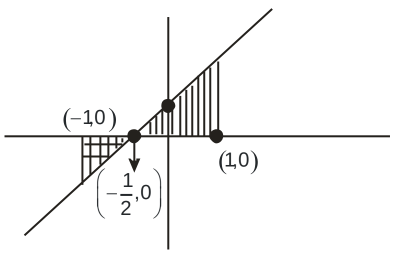
39. The area of the region bounded by the line $y = 2x + 1$, x-axis and the ordinates $x = -1$ and $x = 1$ is

(A) $\frac{5}{2}$ (B) 5

(C) $\frac{9}{4}$ (D) 2

Sol. Answer (A)

y=2x+1



$$= \int_{-1}^{-\frac{1}{2}} -(2x+1)dx + \int_{-\frac{1}{2}}^{1} (2x+1)dx$$

$$= -\left[x^2 + x\right]_{-1}^{-\frac{1}{2}} + \left[x^2 + x\right]_{-\frac{1}{2}}^{1}$$

$$= -\left[\frac{1}{4} - \frac{1}{2} - (1-1)\right] + \left[(1+1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$= \frac{1}{4} + 2 + \frac{1}{4} = \frac{5}{2}$$

40. The order of the differential equation obtained by eliminating arbitrary constants in the family of curves $c_1 y = (c_2 + c_3)e^{x+c_4}$ is

(A) 3 (B) 4

(C) 1 (D) 2

Sol. Answer (C)

$C_1 y = (C_2 + C_3).e^{x+C_4}$

$C_1 y = (C_2 + C_3).e^{C_4}.e^x$

$$y = \frac{(C_2 + C_3)}{C_1}.e^{C_4}.e^x$$

$y = C.e^x$

Only 1 arbitrary constant

$\therefore$ Order = 1

41. The general solution of the differential equation $x^2 dy - 2xy\,dx = x^4 \cos x\,dx$ is

(A) $y = \sin x + cx^2$

(B) $y = \cos x + cx^2$

(C) $y = x^2 \sin x + cx^2$

(D) $y = x^2 \sin x + c$

Sol. Answer (C)

$x^2 dy - 2xy.dx = x^4.\cos x dx$

$$\frac{dy}{dx} - \frac{2}{x}.y = x^2.\cos x$$

I.F $e^{\int -\frac{2}{x}dx}$

$$= e^{-2.\ln x} = \frac{1}{x^2}$$

Now solution

$$y.\frac{1}{x^2} = \int x^2.\cos x.\frac{1}{x^2}dx$$

$$\frac{y}{x^2} = \int \cos x \, dx$$

$$\frac{y}{x^2} = \sin x + C$$

42. The curve passing through the point (1, 2) given that the slope of the tangent at any point (x, y) is $\frac{2x}{y}$ represents

(A) Ellipse (B) Hyperbola

(C) Circle (D) Parabola

Sol. Answer (B)

$$\frac{dy}{dx} = \frac{2x}{y}$$

$$ydy = 2xdx$$

Variable separable

$$\frac{y^2}{2} = x^2 + C$$

Passes through (1, 2)

$$\frac{4}{2} = 1 + C \Rightarrow C = 1$$

$$\Rightarrow \frac{y^2}{2} = x^2 + 1$$

$\Rightarrow x^2 - \frac{y^2}{2} = -1$, Which is hyperbola

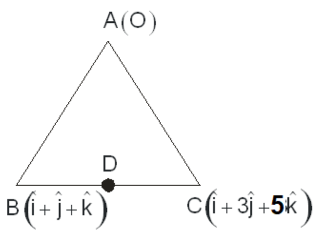
43.. The two vectors $\hat{i} + \hat{j} + k$ and $\hat{i} + 3\hat{j} + 5k$ represent the two sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of a ΔABC. The length of the median through A is

(A) 7 (B) $\sqrt{14}$

(C) $\frac{\sqrt{14}}{2}$ (D) 14

Sol. Answer (B)

Let A be origin



D is midpoint of B & C

$$\overrightarrow{AD} = \frac{\left(\hat{i} + \hat{j} + k\right) + \left(\hat{i} + 3\hat{j} + 5k\right)}{2}$$

$$\overrightarrow{AD} = \hat{i} + 2\hat{j} + 3k$$

$$\left|\overrightarrow{AD}\right| = \sqrt{1 + 4 + 9} = \sqrt{14}$$

44. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then $\sin\frac{\theta}{2}$ is

(A) $\frac{\left|\vec{a} - \vec{b}\right|}{2}$ (B) $\left|\vec{a} - \vec{b}\right|$

(C) $\left|\vec{a} + \vec{b}\right|$ (D) $\frac{\left|\vec{a} + \vec{b}\right|}{2}$

Sol. Answer (A)

$$\left|\vec{a} - \vec{b}\right| = \sqrt{\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\left|\vec{a}\right|\left|\vec{b}\right|.\cos\theta}$$

$$= \sqrt{1 + 1 - 2\cos\theta}$$

$$= \sqrt{2(1 - \cos\theta)}$$

$$\left|\vec{a} - \vec{b}\right| = 2.\sin\frac{\theta}{2}$$

45. If the vectors $2\hat{i} - 3\hat{j} + 4k, 2\hat{i} + \hat{j} - k$ and $\lambda\hat{i} - \hat{j} + 2k$ are coplanar, then the value of λ is

(A) -6 (B) 5

(C) 6 (D) -5

Sol. Answer (C)

$$\begin{vmatrix} 2 & -3 & 4 \\ 2 & 1 & -1 \\ \lambda & -1 & 2 \end{vmatrix} = 0$$

λ=6

46. If $\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a} \cdot \vec{b}\right|^2 = 144$ and $\left|\vec{a}\right| = 6$, then $\left|\vec{b}\right|$ is equal to

(A) 2 (B) 4

(C) 6 (D) 3

Sol. Answer (A)

$\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a} \cdot \vec{b}\right|^2 = \left|\vec{a}\right|^2 \left|\vec{b}\right|^2$, we get $\left|\vec{b}\right| = 2$

47. The point (1, -3, 4) lies in the octant

(A) Fourth (B) Eighth

(C) Second (D) Third

Sol. Answer (A)

Conceptual

48. If a line makes an angle of $\frac{\pi}{3}$ with each of x and y-axis then the acute angle made by z-axis is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Sol. Answer (C)

α, β, γ be angle with axes

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\left(\cos\frac{\pi}{3}\right)^2 + \left(\cos\frac{\pi}{3}\right)^2 + \cos^2\gamma = 1$$

$$\frac{1}{4} + \frac{1}{4} + \cos^2\gamma = 1$$

$$\Rightarrow \cos^2\gamma = 1 - \frac{1}{2}$$

$$\cos\gamma = \pm\frac{1}{\sqrt{2}}$$

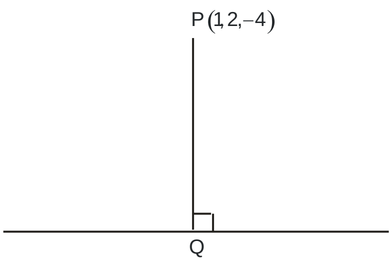
$$\gamma = \frac{\pi}{4}$$

49. The distance of the point (1, 2, -4) from the line $\frac{x-3}{2} = \frac{y-3}{3} = \frac{z+5}{6}$ is

(A) $\frac{293}{49}$ (B) $\frac{\sqrt{293}}{49}$

(C) $\frac{293}{7}$ (D) $\frac{\sqrt{293}}{7}$

Sol. Answer (D)



$$\frac{x-3}{2} = \frac{y-3}{3} = \frac{z+5}{6} = \lambda$$

General point on line is $(2\lambda+3, 3\lambda+3, 6\lambda-5)$

Direction ratio of PQ $(2\lambda+2, 3\lambda+1, 6\lambda-1)$

Since line & PQ are perpendicular

So, product of their direction ratios will be 0.

So $(2\lambda+2).2 + (3\lambda+1)3 + (6\lambda-1).6 = 0$

$$4\lambda + 4 + 9\lambda + 3 + 36\lambda - 6 = 0$$

$$49\lambda + 1 = 0 \Rightarrow \lambda = -\frac{1}{49}$$

Direction ratio PQ $(2\lambda+2, 3\lambda+1, 6\lambda-1)$

$$\left(2\left(\frac{-1}{49}\right)+2, 3\left(\frac{-1}{49}\right)+1, 6\left(\frac{-1}{49}\right)-1\right)$$

$$= \left(\frac{96}{49}, \frac{46}{49}, \frac{-55}{49}\right)$$

Distance

$$= \sqrt{\left(\frac{96}{49}\right)^2 + \left(\frac{46}{49}\right)^2 + \left(\frac{-55}{49}\right)^2} = \sqrt{\frac{14537}{492}}$$

$$\sqrt{\frac{293}{49}} = \frac{\sqrt{293}}{7}$$

50. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{3-y}{-4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

(A) $\frac{4}{5\sqrt{2}}$ (B) $\frac{\sqrt{2}}{10}$

(C) $\frac{3}{\sqrt{50}}$ (D) $\frac{3}{50}$

Sol. Answer (B)

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5} \;\&\; 2x - 2y + z = 5$$

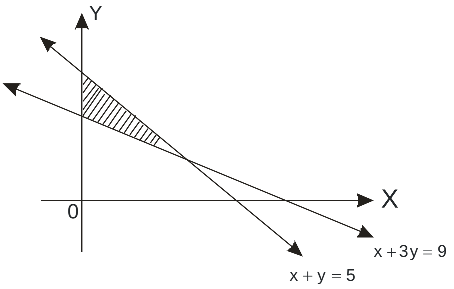
Angle between line & plane

$$\sin\theta = \frac{3.2 + 4(-2) + 5(1)}{\sqrt{3^2+4^2+5^2}\sqrt{2^2+(-2)^2+1^2}}$$

$$\sin\theta = \frac{6-8+5}{5\sqrt{2}.3}$$

$$= \frac{3}{3.5\sqrt{2}} = \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$$

51. The feasible region of an LPP is shown in the figure. If Z = 11x + 7y, then the maximum value of Z occurs at



(A) (5, 0) (B) (3, 2)

(C) (0, 5) (D) (3, 3)

Sol. Answer (B)

Solve

$x + y = 5$

$x + 3y = 9$

$-2y = -4$

$y = 2 \Rightarrow x = 3$

$x + 3y = 9$ meets y axis at (0, 3)

$x + y = 5$ meets at (0, 5)

Now check $z = 11x + 7y$

At (0,3) $\Rightarrow z = 0 + 7.3 = 21$

At (0,5) $\Rightarrow z = 0 + 7.5 = 35$

At (3,2) $\Rightarrow z = 11.3 + 7.2 = 47$

So, Z is maximum at (3,2)

52. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let z = px + qy, where p, q > 0. Condition on p and q so that the minimum of z occurs at (3, 0) and (1, 1) is

(A) $p = 3q$ (B) $p = q$

(C) $p = 2q$ (D) $p = \frac{q}{2}$

Sol. Answer (D)

$z = px + qy$

Minimum occurs at (3,0)

$z = 3p$

Also at $\Rightarrow z = p + q$

So 3p= p + q $\Rightarrow 2p = q$

53. If A and B are two events such that $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{6}$, then $P(A'/B)$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{12}$

(C) $\frac{2}{3}$ (D) $\frac{1}{3}$

Sol. Answer (C)

$P(A) = \frac{1}{3}$

$P(B) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{6}$

Since $P(A \cap B) = P(A).P(B)$

So events are independent

Now $P\left(\frac{A'}{B}\right) = P(A')$

$= 1 - \frac{1}{3} = \frac{2}{3}$

54. A die is thrown 10 times, the probability that an odd number will come up atleast one time is

(A) $\frac{11}{1024}$ (B) $\frac{1013}{1024}$

(C) $\frac{1}{1024}$ (D) $\frac{1023}{1024}$

Sol. Answer (D)

1 – P(no odd number comes)

$= 1 - \frac{3^{10}}{6^{10}} \Rightarrow 1 - \frac{1}{2^{10}} = \frac{1023}{1024}$

55. The probability of solving a problem by three persons A, B and C independently is $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{3}$ respectively. Then the probability of the problem is solved by any two of them is

(A) $\frac{1}{24}$ (B) $\frac{1}{8}$

(C) $\frac{1}{12}$ (D) $\frac{1}{4}$

Sol. Answer (D)

Solved by two implies one person should fail

$$= P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C)$$

Events are independent

$$= \frac{1}{2} \times \frac{1}{4} \times \frac{2}{3} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{3}$$

$$= \frac{6}{24} = \frac{1}{4}$$

56. Events E_1 and E_2 form a partition of the sample space S. A is any event such that $P(E_1) = P(E_2) = \frac{1}{2}$, $P(E_2/A) = \frac{1}{2}$ and $P(A/E_2) = \frac{2}{3}$, then $P(E_1/A)$ is

(A) 1 (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) $\frac{2}{3}$

Sol. Answer (C)

Using Baye's theorem

$$P(E_2/A) = \frac{P(E_2).P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2).P(A/E_2)}$$

$$\frac{1}{2} = \frac{\frac{1}{2}.\frac{2}{3}}{\frac{1}{2}.x + \frac{1}{2}.\frac{2}{3}}$$

$$\frac{1}{2} = \frac{2/3}{x + \frac{2}{3}} \Rightarrow x + \frac{2}{3} = \frac{4}{3}$$

$$x = \frac{2}{3} = P(A/E_1)$$

Now use again Baye's theorem

$$P(E_2/A) = \frac{P(E_2).P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2).P(A/E_2)}$$

$$= \frac{\frac{1}{2}.\frac{2}{3}}{\frac{1}{2}.\frac{2}{3} + \frac{1}{2}.\frac{2}{3}}$$

$$P(E_1/A) = \frac{1}{2}$$

57. If $A = \{1, 2, 3, 4, 5, 6\}$, then the number of subsets of A which contain atleast two elements is

(A) 57 (B) 58

(C) 64 (D) 63

Sol. Answer (A)

$$\Rightarrow {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$$

$$= 2^6 - {}^6C_0 - {}^6C_1$$

$$= 64 - 1 - 6 = 57$$

58. If n(A)=2 and total number of possible relations from set A to set B is 1024, then n(B) is

(A) 10 (B) 5

(C) 512 (D) 20

Sol. Answer (B)

No.of relations (maximum) = 2^{mn}

$$2^{mn} = 1024 = 2^{10}$$

$$n(A) = 2 \Rightarrow 2^{2.m} = 2^{10} \Rightarrow m = 5$$

59. The value of $\sin^2 51^0 + \sin^2 39^0$ is

(A) $\sin 12^0$ (B) $\cos 12^0$

(C) 1 (D) 0

Sol. Answer (C)

$$\sin^2\theta + \cos^2\theta = 1$$

60. If tan A + cot A = 2, then the value of $\tan^4 A + \cot^4 A =$

(A) 4 (B) 5

(C) 2 (D) 1

Sol. Answer (C)

$$\tan A + \cot A = 2$$

$$\tan A + \frac{1}{\tan A} = 2$$

$$\Rightarrow \tan^2 A + 2\tan A + 1 = 0$$

$$(\tan A - 1)^2 = 0 \Rightarrow \tan A = 1$$

$$\cot A = 1$$

So, $\tan^4 A + \cot^4 A = 1 + 1 = 2$

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