All India Aakash Test Series for JEE (Advanced)-2023

TEST - 4A (Paper-1) - Code-A

Test Date : 03/04/2022

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(C)	20.	(B)	39.	(D)
2.	(A)	21.	(B)	40.	(B)
3.	(C)	22.	(B)	41.	(A)
4.	(D)	23.	(D)	42.	(B)
5.	(21.00)	24.	(05.00)	43.	(07.00)
6.	(50.20)	25.	(00.00)	44.	(29.00)
7.	(00.33)	26.	(12.00)	45.	(33.00)
8.	(01.50)	27.	(03.00)	46.	(09.00)
9.	(08.00)	28.	(09.00)	47.	(07.00)
10.	(04.00)	29.	(02.00)	48.	(06.00)
11.	(A, B)	30.	(A, B)	49.	(A, B)
12.	(C, D)	31.	(A, B, D)	50.	(A, B, D)
13.	(B, D)	32.	(B, C)	51.	(A, C, D)
14.	(A, B, C)	33.	(A, C, D)	52.	(B, C, D)
15.	(A, C)	34.	(A, B)	53.	(A, C, D)
16.	(C, D)	35.	(A, C, D)	54.	(A, B, D)
17.	(50)	36.	(19)	55.	(08)
18.	(04)	37.	(07)	56.	(16)
19.	(03)	38.	(75)	57.	(04)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (C)

Hint : $\Delta Q = \Delta U + \Delta W$

Sol. : $\Delta Q_{B\to A} = (U_A - U_B) + \Delta W = 0$

$\Rightarrow (U_A - U_B) + (-30) = 0$

$\Rightarrow U_A - U_B = 30$ J …(i)

$\Delta Q_{A\to B} = (U_B - U_A) + \Delta W_{A\to B} = 20$

$-30 + \Delta W_{A\to B} = 20$

$\Rightarrow \Delta W_{A\to B} = 50$ J

2. Answer (A)

Hint : $\tan\theta = \frac{a}{g}$

Sol. : $\tan\theta = \frac{(12-8)}{(12+8)} = \frac{4}{20} = \frac{1}{5}$

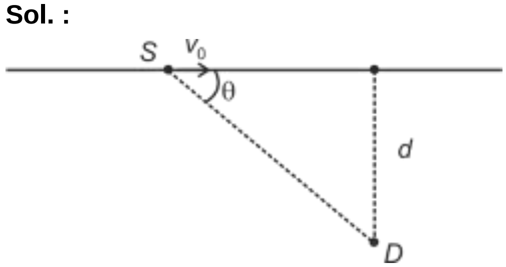
$\Rightarrow \frac{a}{g} = \frac{1}{5}$

$\Rightarrow a = \frac{g}{5} = 2$ m/s^2

3. Answer (C)

Hint : Use Doppler effect

Sol. :



$n' = \frac{v}{v - v_0\cos\theta} \times n_0$

and, $\frac{SD\cos\theta}{v_0} = \frac{SD}{v} \Rightarrow \cos\theta = \frac{v_0}{v}$

$n' = \frac{v^2}{v^2 - v_0^2} n_0$

$\therefore\ n' = \frac{340^2}{340^2 - 170^2} \times 1200$

$= 1600$ Hz

4. Answer (D)

Hint : Heat gained = Heat lost

Sol. : $Q_1 = S \times \frac{1}{2} \times 20 + 5 \times 80 = 450$ cal

$Q_2 = 10 \times 1 \times 30 = 300$ cal

$\Rightarrow$ Final temperature = 0°C

5. Answer (21.00)

6. Answer (50.20)

Solution for Q. Nos. 5 and 6

$$W_{\text{gas}} = P\Delta \cdot V + \frac{1}{2}Kx^2$$

$$= 10^5 \times \left(20 \times 10 \times 10^{-6}\right) + \frac{1}{2} \times 200 \times (0.1)^2$$

$= 20 + 1$

$= 21$ joules

$P_iV_i = nRT_i$

$V_i = \frac{2 \times R \times 300}{10^5}$ m^3

$V_i = \frac{2 \times \frac{25}{3} \times 300}{10^5} \times 10^6$ cm^3

$= 50000$ cm^3

$\Delta V = 20 \times 10$ cm^3

$= 200$ cm^3

$V_f = V_i + \Delta V = 50200$ cm^3

$= 50.2 \times 10^3$ cm^3

7. Answer (00.33)

8. Answer (01.50)

Solution for Q. Nos. 7 and 8

Hint : $F = \frac{mv^2}{r}$

Sol. : $dl = \frac{T(r)}{\left(\frac{YA}{dx}\right)}$

$$\Rightarrow \int dl = \int \frac{1}{2}\frac{\rho\omega^2\left(L^2 - r^2\right)dr}{Y}$$

$$\Rightarrow \Delta L = \frac{\rho\omega^2 L^3}{3Y}$$

$$= \frac{1}{3}\times\frac{10^4\times(400)^2\times(0.5)^3}{2\times10^{11}} = 0.33 \text{ mm}$$

Stress = $\frac{3\rho\omega^2 l^2}{8}$ = 1.5×10^8 Pa

9. Answer (08.00)

10. Answer (04.00)

Sol. of Q. 9 and Q. 10

Hint : $dK = \frac{1}{2}(dm)\cdot v_y^2$

Sol. : $y = (A \sin Kx)(\sin\omega t)$

$$\therefore\ dK = \frac{1}{2}\times(dm)\cdot\left(\frac{dy}{dt}\right)^2$$

$$\therefore\ K_v = \frac{\int K(x)\cdot dx}{l} = \frac{m\omega^2 A^2}{8}$$

$\therefore N = 8$

Total energy is always constant = $\frac{m\omega^2A^2}{4}$.

11. Answer (A, B)

Hint : Use principle of homogeneity

Sol. : $[a] = [PV^2] = ML^5T^{-2}$

$[b] = [V] = L^3$

$$[R] = \frac{[PV]}{[T]} = ML^2T^{-2}K^{-1}$$

12. Answer (C, D)

Hint : $v = u + at$, $s = ut + \frac{1}{2}at^2$

Sol. : $a_1 = \frac{8-0}{10} = 0.8 \text{ m/s}^2$

$$s_1 = \frac{1}{2}\times0.8\times(10)^2 = 40 \text{ m}$$

$$a_3 = \frac{8^2-0}{2\times64} = 0.5 \text{ m/s}^2$$

$s_3 = 64$ m $\Rightarrow s_2 = 584 - 40 - 64 = 480$ m

$\therefore t = 10 + 60 + 16 = 86$ s

13. Answer (B, D)

Hint : $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

Sol. : $U_{rel} = V + V\cos60° = \frac{3V}{2} = \frac{3}{2}\times10$

$= 15$ m/s

$\therefore\ \Delta t = \frac{30}{15} = 2$ s

$\therefore s = 10 \times 2 = 20$ m

14. Answer (A, B, C)

Hint : Use constraint relation

Sol. : $T_1 - mg = ma_A$

$$T_1 = 4\times\frac{m\times2m}{3m}\times(g - a_A)$$

$mg - T_2 = ma_B$ and $T_1 = 2T_2$

$$\Rightarrow a_A = \frac{5g}{11},\ T_1 = \frac{16mg}{11},\ T_2 = \frac{8mg}{11}$$

$$a_B = \frac{3g}{11}$$

15. Answer (A, C)

Hint : $a_{cm} = \frac{F_{ext}}{\text{mass}}$

Sol. : Block moves downward and rightward. Friction acts rightward on the wedge.

16. Answer (C, D)

Hint : At pure rolling slipping stops.

Sol. : $mv_0R + \frac{2}{5}mR^2\times\frac{v_0}{2R} = \frac{7}{5}mR^2\times\frac{v}{R}$

$$\Rightarrow \frac{6v_0}{5} = \frac{7v}{5} \Rightarrow v = \frac{6v_0}{7}$$

$$\therefore\ \frac{\left(v_0 - \frac{6v_0}{7}\right)}{\mu g} = t_0 \Rightarrow t_0 = \frac{v_0}{7\mu g}$$

17. Answer (50)

Hint : $\Delta KE = W_{ext}$

Sol. : $l_1 = \sqrt{3^2 + \left(3\sqrt{3}\right)^2} = 6$ m

$$l_2 = \sqrt{3^2+4^2} = 5 \text{ m}$$

$\therefore \Delta KE = F\cdot\Delta S = 50 \times 1 = 50$ J

18. Answer (04)

Hint : $F = \eta A\frac{dv}{dz}$

Sol. : $F = mg\sin\theta = \eta A\dfrac{dv}{dz}$

$\Rightarrow 20\times10\times\frac{1}{2} = 10^{-2}\times2^2\times\left(\frac{10}{h}\right)$

$\Rightarrow \frac{10^4}{10\times4} = \frac{1}{h}$

$\Rightarrow h = 4\times10^{-3}$ m = 4 mm

19. Answer (03)

Hint : $\frac{dT}{dx} = 0$

Sol. : $I = \frac{ml^2}{12} + my^2$, $y = \frac{\ell}{2} - x$

$\therefore T = 2\pi\sqrt{\frac{I}{mg\,y}}$

and $\frac{dT}{dy} = 0 \Rightarrow y = \frac{\ell}{\sqrt{12}} = \frac{\ell}{2\sqrt{3}}$

$\therefore x = \frac{\ell}{2} - y = \frac{\ell}{2} - \frac{\ell}{2\sqrt{3}}$

$= \frac{\ell}{2}\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$

PART - II (CHEMISTRY)

20. Answer (B)

Hint : An ideal gas undergoing reversible adiabatic expansion follows the following equation.

$TV^{\gamma-1}$ = constant

Sol : $TV^{\gamma-1}$ = constant

$500(10)^{\gamma-1} = T_2(40)^{\gamma-1}$

$T_2 = 500\left(\frac{1}{4}\right)^{\gamma-1} = \frac{500}{(4)^{2/3}} = 200\text{ K}$

q = 0 = ΔE – w

w = ΔE = $nC_{vm}(T_2 - T_1)$

$= \frac{3}{2}R\,(200-500)$

= –3 × 150R

= –3735 J

21. Answer (B)

Hint : $\ln K = \frac{\Delta S^\circ}{R} - \frac{\Delta H^\circ}{RT}$, where K is equilibrium constant.

Sol : $\ln K = \frac{\Delta S^\circ}{R} - \frac{\Delta H^\circ}{RT}$

$\frac{\Delta H^\circ}{R} = 2062.65 \quad\Rightarrow\quad \Delta H^\circ = 17.12\text{ kJ}$

22. Answer (B)

Hint : Acidic buffer

$$pH = pK_a + \log\frac{[Salt]}{[Acid]}$$

Sol : Initial mmol of C_6H_5COOH = 14

Initial mmol of C_6H_5COONa = 7

$pH_1 = pK_a + \log\frac{7}{14} = pK_a - \log 2$

After adding 2 mmol NaOH, the mmol of C_6H_5COOH = 12 mmol and C_6H_5COONa = 9

$pH_2 = pK_a + \log\frac{9}{12}$

$= pK_a + \log\frac{3}{4}$

23. Answer (D)

Hint : $Be^{2+} < Mg^{2+} < Ca^{2+} < Sr^{2+} < Ba^{2+}$

[Ionic radius M^{2+}]

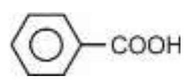
Be > Mg > Ca > Sr > Ba (IE_1)

Sol : Melting point: Be > Ca > Sr > Ba > Mg

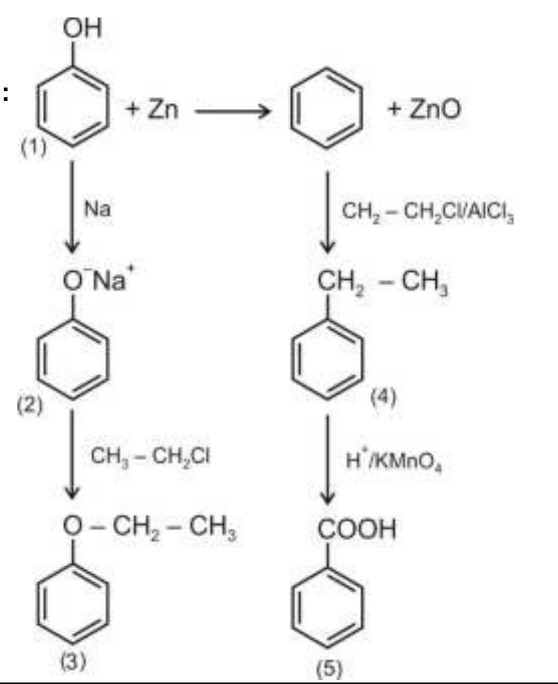
24. Answer (05.00)

25. Answer (00.00)

Hint and Sol. for Q.Nos. 24 and 25

Hint :  is most acidic among the products.

Sol. :

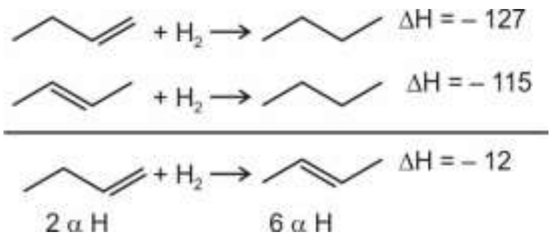


26. Answer (12.00)

27. Answer (03.00)

Hint and Sol. for Q.Nos. 24 and 25

Hint :



Sol. : $\Rightarrow$ Energy of stabilisation by

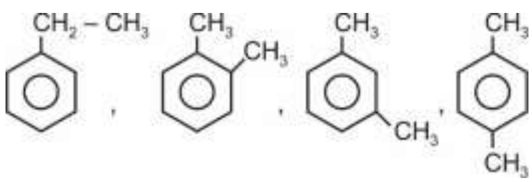
$1\ C - H = \frac{12}{4} = 3$ kJ/mole

28. Answer (09.00)

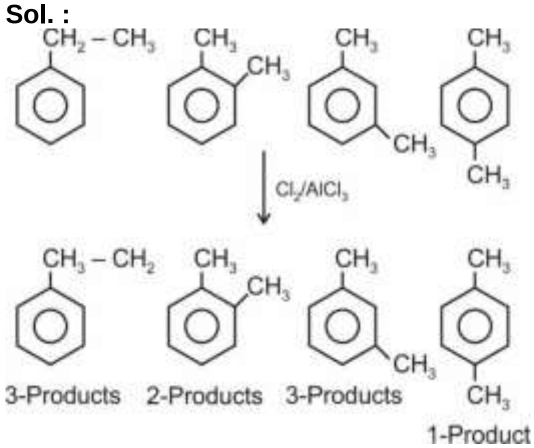
29. Answer (02.00)

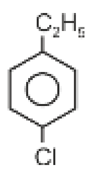
Hint and Sol. for Q.Nos. 28 and 29

Hint: The possible benzenoid compounds are :



Sol. :



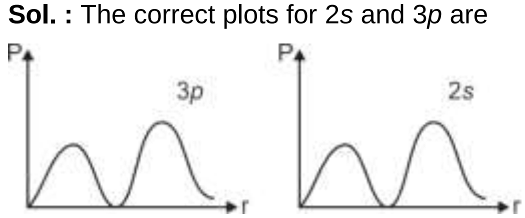


→ compound having highest dipole moment (it has 2 benzylic hydrogens)

30. Answer (A, B)

Hint : Total number of nodes = n – l – 1

Sol. : The correct plots for 2*s* and 3*p* are



31. Answer (A, B, D)

Hint : As, $\Delta n_g = 0 \Rightarrow K_P = K_C$

Sol. : $2XY(g) \rightleftharpoons X_2(g) + Y_2(g)$

	$2XY(g)$	$X_2(g)$	$Y_2(g)$
t = 0	2	–	–
t = eq.	2 – x	$\frac{x}{2}$	$\frac{x}{2}$

$$\frac{(x/2)^2}{(2-x)^2} = 0.16$$

$$\frac{x}{2(2-x)} = \frac{2}{5}$$

$$\Rightarrow \quad x = \frac{8}{9}$$

32. Answer (B, C)

Hint : $K_1K_2 = \frac{[H^+]^2[S^{2-}]}{[H_2S]} = 10^{-21}$

Sol. : Saturated solution of H_2S = 0.1 M H_2S

I. $\frac{(0.2)^2[S^{2-}]}{0.1} = 10^{-21}$

$\therefore$ $[S^{2-}] = 2.5 \times 10^{-21}$

$K_{sp} = [Fe^{2+}][S^{2-}] = 3.7 \times 10^{-19}$

Since $(0.1) \times (2.5 \times 10^{-21}) < 3.7 \times 10^{-19}$

FeS will not precipitate.

II. $\frac{(0.001)^2[S^{2-}]}{0.1} = 10^{-21}$ $\quad [S^{2-}] = 10^{-16}$

Since $(0.01)(10^{-16}) > 3.7 \times 10^{-19}$

FeS will precipitate.

33. Answer (A, C, D)

Hint : $\Delta U = q + w$

Sol. :

I. $H_2O(l) \rightleftharpoons H_2O(s)$

It is at equilibrium at 273 K and 1 atm.

So, ΔS_{system} is negative

$\Delta G = 0$

II. Mixing of two ideal gas at constant T

q = 0, ΔU = 0, w = 0

34. Answer (A, B)

Hint : Generally thermal stability increases down the group.

Sol. : $BeCO_3 > MgCO_3 > CaCO_3 > SrCO_3 > BaCO_3$ (Solubility order)

$Be(OH)_2 < Mg(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2$ (Thermal stability order)

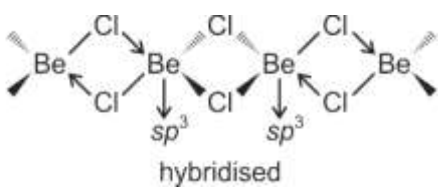
35. Answer (A, C, D)

Hint :

$$BeO + C + Cl_2 \xrightleftharpoons{600-800\text{ K}} \underset{(P)}{BeCl_2} + \underset{(Q)}{CO}$$

$$2BeCl_2 + LiAlH_4 \longrightarrow \underset{(R)}{2BeH_2} + \underset{(S)}{LiCl} + \underset{(T)}{AlCl_3}$$

Sol. : Polymeric structure of $(BeCl_2)_n$



Polymeric BeH_2 $\Rightarrow$ Be is sp^3 hybridised

36. Answer (19)

Hint : $HA + NaOH \longrightarrow H_2O + NaA$

Sol. : Millimoles of salt of NaA or A^- = 40 × 0.1 = 4

$$A^- + H^+ \longrightarrow HA$$

	A^-	H^+	HA
Initial millimoles	4	2	
Final millimoles	2	–	2

Acidic buffer solution is formed and $[A^-] = [HA]$

$$pH = pK_a + \log\frac{[A^-]}{[HA]} \Rightarrow pK_a = 6$$

Now, $HA + NaOH \longrightarrow NaA + H_2O$

Hydrolysis of A^- will take place.

$$[NaA] = \frac{\text{Millimoles of acid}}{\text{Total volume}} = \frac{20 \times 0.2}{40} = 0.1$$

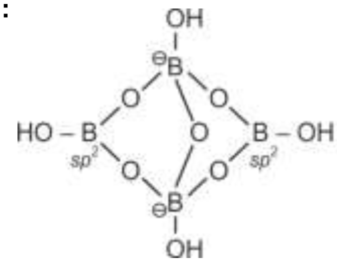
$$pH = \frac{1}{2}(pK_w + pK_a + \log C)$$

$$= \frac{1}{2}(14 + 6 - 1) = 9.5$$

37. Answer (07)

Hint : Tetranuclear unit $[B_4O_5(OH)_4]^{2-}$

Sol. :



P = 2
Q = 5

38. Answer (75)

Hint : $2NaN_3 \longrightarrow 2Na + 3N_2$

$10Na + 2KNO_3 \longrightarrow K_2O + 5Na_2O + N_2$

Sol. : Assume reaction initially starts with 'a' moles of NaN_3

$\therefore$ Moles of Na formed = a

Moles of N_2 formed = $\frac{3a}{2}$

Moles of KNO_3 required for 'a' moles of Na = $\frac{a}{5}$

Moles of N_2 formed in 2nd reaction = $\frac{a}{10}$

$$\therefore \frac{3a}{2} + \frac{a}{10} = \frac{8 \times 18.45}{0.082 \times 300} = 6$$

$$\Rightarrow a = \frac{10 \times 6}{16} = 3.75 \text{ moles}$$

$\therefore$ Mass of KNO_3 required =

$$\frac{a}{5} \times 101 = 75.75 \text{ g}$$

PART - III (MATHEMATICS)

39. Answer (D)

Hint : If $A + B + C = \pi$ then $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

Sol. : $\because$ 50° + 60° + 70° = 180°

tan 50° + tan 60° + tan 70° = tan 50° · tan 60° · tan 70°

$$= \sqrt{3} \cdot \tan(60° - 10°) \cdot \tan(60° + 10°)$$

$$= \sqrt{3} \cdot \frac{\sin(60° - 10°) \cdot \sin(60° + 10°)}{\cos(60° - 10°) \cdot \cos(60° + 10°)}$$

$$= \sqrt{3}\cdot\frac{\sin^2 60^\circ - \sin^2 10^\circ}{\cos^2 60^\circ - \sin^2 10^\circ}$$

$$= \sqrt{3}\cdot\frac{3-4\sin^2 10^\circ}{1-4\sin^2 10^\circ} = \sqrt{3}\cdot\frac{3-4\sin^2 10^\circ}{4\cos^2 10^\circ - 3}$$

$$= \sqrt{3}\cdot\frac{3\sin 10^\circ - 4\sin^3 10^\circ}{4\cos^3 10^\circ - 3\cos 10^\circ}\cdot\frac{\cos 10^\circ}{\sin 10^\circ}$$

$$= \sqrt{3}\cdot\tan 30^\circ\cdot\cot 10^\circ = \sqrt{3}\cdot\frac{1}{\sqrt{3}}.\tan 80^\circ$$

$$= \tan 80^\circ$$

40. Answer (B)

Hint : For maximum value of $\angle MPN$ a circle with centre at perpendicular bisector of MN and touches the x axis at P

Sol. : Equation of perpendicular bisector of MN is

$y - 3 = -1\ (x - 0)$

$\therefore\ y = 3 - x$

Let centre of circle be $(x_1, 3 - x_1)$

Then equation of circle passing though points M and N is

$$(x - x_1)^2 + (y - 3 + x_1)^2 = (x_1 - 1)^2 + (3 - x_1 - 4)^2$$

$$\therefore\ (x - x_1)^2 + (y - 3 + x_1)^2 = 2(x_1^2 + 1) \quad \ldots(1)$$

If this circle touches the x axis then

$$|3 - x_1|^2 = 2(x_1^2 + 1)$$

$$\therefore\quad x_1^2 + 6x_1 - 7 = 0$$

$$\therefore\quad x_1 = -7 \text{ or } 1.$$

For maximum angle radius must be minimum.

$\therefore\ x_1 = 1$ and point of contact $P = (1, 0)$.

41. Answer (A)

Hint : Interchange the x and y in given equation to get function.

Sol. : $\because\ f(xy + 1) = f(x)\cdot f(y) - f(y) - x + 2 \ldots(1)$

On interchanging x and y we get

$f(xy + 1) = f(y)\cdot f(x) - f(x) - y + 2 \quad \ldots(2)$

From. eq (1) and eq. (2) :

$f(x)\cdot f(y) - f(y) - x + 2 = f(y)\cdot f(x) - f(x) - y - 2$

$\therefore\ f(x) + y = f(y) + x$

On replacing $y = 0$ we get

$$\therefore\ \sum_{r=1}^{49} f(r) = f(1) + f(2) + \ldots + f(49)$$

$= 2 + 3 + 4 + \ldots + 50$

$$= \frac{50\times 51}{2} - 1 = 1274$$

42. Answer (B)

Hint : Property of ellipse

Sol. : $(S_1M_1)(S_2M_2) = b^2 = 4$

43. Answer (07.00)

Hint : Convert the given equation in quadratic in x and y respectively to get range of y and x.

Sol. : $9x^2 + 2xy + y^2 - 92x - 20y + 244 = 0$

$\therefore\ 9x^2 + (2y - 92)x + (y^2 - 20y + 244) = 0$

$$\therefore\ x = \frac{(92-2y) \pm \sqrt{(2y-92)^2 - 4.9(y^2 - 20y + 244)}}{18}$$

Here $(y - 46)^2 - 9\ (y^2 - 20y + 244) \geq 0$

$\Rightarrow\quad -8y^2 + 88y - 80 \geq 0$

$\Rightarrow\quad y^2 - 11y + 10 \leq 0$

$\therefore\ y \in [1, 10]$

$\therefore$ minimum value of $y = 1$

And again

$y^2 + (2x - 20)y + (9x^2 - 92x + 244) = 0$

$$\therefore\ y = \frac{(20-2x) \pm \sqrt{4(x-10)^2 - 4(9x^2 - 92x + 244)}}{18}$$

$\therefore\ (x - 10)^2 - (9x^2 - 92x + 244) \geq 0$

$\Rightarrow\quad x^2 - 9x + 18 \leq 0$

$\therefore\quad x \in [3, 6]$

$\therefore$ Maximum value of $x = 6$

$\therefore$ Required sum = 6 + 1 = 7

44. Answer (29.00)

Hint : For two linear factors the discriminant must be equal to zero.

Sol. : The expression

$-3x^2 - 8xy + 3y^2 - kx + 3y - 18$ is compared with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ we get

$$a = -3, h = -4, b = 3, g = -\frac{k}{2}, f = \frac{3}{2}, c = -18$$

For linear factors :

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\therefore\ (-3)\cdot 3\cdot(-18)+2\cdot\frac{3}{2}\cdot\left(-\frac{k}{2}\right)\cdot(-4)-(-3)\cdot\left(\frac{3}{2}\right)^2$$

$$-3\cdot\left(-\frac{k}{2}\right)^2-(-18)\cdot(-4)^2=0$$

$$162+6k+\frac{27}{4}-\frac{3k^2}{4}+288=0$$

$\Rightarrow k^2 - 8k - 609 = 0$

$\therefore\ k = 29$ as $k \in N$.

45. Answer (33.00)

Hint : Use of Multinomial theorem

Sol. : Number of permutations = Coefficients of x^3 in $3!\,(1+x)^3\left(1+x+\frac{x^2}{2!}\right)$

= coefficients of x^3 in

$$3!(1+3x+3x^2+x^3)\left(1+x+\frac{x^2}{2}\right)$$

$$=6\left(\frac{3}{2}+3+1\right)=33$$

46. Answer (09.00)

Hint : Use of coefficients in multinomial theorem

Sol. : $\therefore\ \lambda$ = Coefficient of x^4 in

$$4!\,(1+x)^4\left(1+x+\frac{x^2}{2}\right)^2\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}\right)$$

= Coefficient of x^4 in 4! $(1 + 4x + 6x^2 + 4x^3 + x^4)$

$$\left(1+2x^2+\frac{x^4}{4}+2x+x^3\right)\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)$$

= 1422

$\therefore$ Sum of digits of λ = 9.

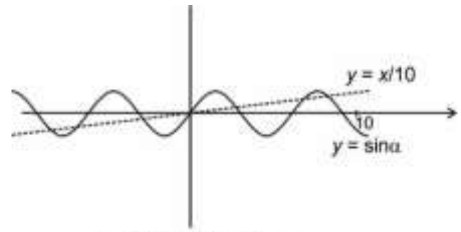
47. Answer (07.00)

48. Answer (06.00)

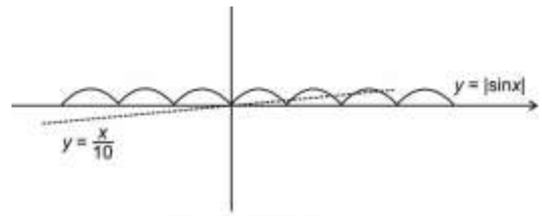
Hint and Sol. for Q.Nos. 47 and 48

Hint : Sketch graphs and count number of prints of intersection.

Sol. :



7 points of intersection



6 points of intersection

49. Answer (A, B)

Hint : Equation of normal to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at point (x_1, y_1) is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

Sol. : Equation of normal to ellipse at point with eccentric angle α is :

$$\frac{ax}{\cos\alpha}-\frac{by}{\sin\alpha}=a^2-b^2 \qquad \ldots(1)$$

And normal at point $(-a\sin\alpha, b\cos\alpha)$ is

$$\frac{ax}{\sin\alpha}+\frac{by}{\cos\alpha}=b^2-a^2 \qquad \ldots(2)$$

Slope of normal (1) $=m_1=\frac{a}{b}\tan\alpha$

Slope of normal (2) $=m_2=-\frac{a}{b}\cot\alpha$

$$\therefore\ \tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\Rightarrow \tan\theta=\left|\frac{\frac{a}{b}\tan\alpha+\frac{a}{b}\cot\alpha}{1-\frac{a^2}{b^2}}\right|$$

$$\Rightarrow \tan\theta=\left|\frac{\frac{1}{\sqrt{1-e^2}}\cdot\frac{1}{\sin\alpha\cdot\cos\alpha}}{1-\frac{1}{1-e^2}}\right|$$

$$\Rightarrow \frac{2\cot\theta}{\sin2\alpha}=\frac{e^2\sqrt{1-e^2}}{e^2-1}=\frac{-e^2}{\sqrt{1-e^2}}\text{ or }\frac{e^2}{\sqrt{1-e^2}}$$

50. Answer (A, B, D)

Hint : Let common ratio of G.P. be r then $a = a$, $b = ar$ and $c = ar^2$.

Sol. : $\because a + b + c = 70$

$\Rightarrow \quad a(1 + r + r^2) = 70 \quad \ldots(i)$

and $\quad 10b = 4a + 4c$

$5ar = 2a + 2ar^2$

$\therefore \quad 2r^2 - 5r + 2 = 0$

$\Rightarrow \quad r = 2 \text{ or } \frac{1}{2} \quad \ldots(ii)$

From (i) and (ii) : $r = 2$ then $a = 10$ and $r = \frac{1}{2}$ then $a = 40$.

$\therefore (a, b, c) = (10, 20, 40)$ or $(40, 20, 10)$

51. Answer (A, C, D)

Hint : L.H.L. $= \lim_{x\to a^-} f(x) = \lim_{x\to 0} f(a-h)$

and R.H.L. $= \lim_{x\to a^+} f(x) = \lim_{h\to 0} f(a+h)$

Sol. : $\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0}(1-h)^2 - 2 = -1$

and $\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0}(1+h)+1 = 2$

$\lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4-h) = \lim_{h\to 0} 4-h+1 = 5$

and $\lim_{x\to 4^+} f(x) = \lim_{h\to 0} f(4+h) = \lim_{h\to 0} 6-(4+h) = 2$

52. Answer (B, C, D)

Hint : z_1 lies on circle and z_2 lies on elliptical region.

Sol. : $|z-3|^2 + |z-7|^2 = 40$ then replace $z = x + iy$ then

$(x-3)^2 + y^2 + (x-7)^2 + y^2 = 40$

$\therefore x^2 + y^2 - 10x + 9 = 0$

$\therefore$ Centre of circle is (5, 0) and radius 4 units.

$|z-12| + |z-16| \le 6$ is elliptical region with centre at (14, 0)

The $\quad \max|z_1 - z_2| = 16$ units

$\min|z_1 - z_2| = 2$ units.

53. Answer (A, C, D)

Hint : Total number of words formed by 'TIKTANIC' is equal to $\frac{8!}{2!\cdot 2!} = 10080$

Sol. : The words start with T is $\frac{7!}{2!}$.

$\therefore$ Probability that words start with T

$$= \frac{\frac{7!}{2!}}{\frac{8!}{2!\cdot 2!}} = \frac{1}{4}$$

Probability that word start with vowel

$$= \frac{\frac{7!}{2!} + \frac{7!}{2!\cdot 2!}}{\frac{8!}{2!\cdot 2!}} = \frac{3}{8}$$

Probability that word start and end with I

$$= \frac{\frac{6!}{2!}}{\frac{8!}{2!\cdot 2!}} = \frac{1}{28}$$

Probability that word start with K and end with C is

$$= \frac{\frac{6!}{2!\cdot 2!}}{\frac{8!}{2!\cdot 2!}} = \frac{1}{8\times 7} = \frac{1}{56}$$

54. Answer (A, B, D)

Hint : $f(x) = \log_2\left(\frac{4}{\sqrt{x+2}+\sqrt{2-x}}\right)$

$$= 2 - \log_2(\sqrt{x+2}+\sqrt{2-x})$$

Sol. : For domain of $f(x)$,

$x + 2 \ge 0$ and $2 - x \ge 0$

$\therefore \; x \in [-2, 2]$

For range of $f(x)$ we have to get range of

$y = \sqrt{x+2}+\sqrt{2-x}$

$y^2 = 4 + 2\sqrt{4-x^2}$

$\therefore \; y^2 \in [4, 8] \Rightarrow y \in [2, 2\sqrt{2}]$

$\therefore$ Range of $f(x) = \left[\frac{1}{2}, 1\right]$

55. Answer (08)

Hint : For shortest chord it must be perpendicular to x axis.

Sol. : $\therefore$ Required equation of chord is $x = 2$

When $x = 2$ then $y = \pm 4$

56. Answer (16)

Hint : Differentiate $f(x)$ w.r.t. x and replace x by zero.

Sol. : $f(0) = 5 - 7 = -2$

and $f'(x) = 12x^3 + 5 + 2e^x + 2xe^x$

$$+ 7\sec^2 x - 5\sin x$$

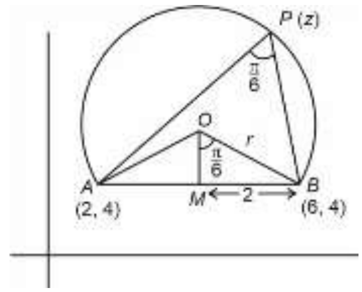
$\therefore \ f'(0) = 7 + 7 = 14$

$\therefore \ f'(0) - f(0) = 16$

57. Answer (04)

Hint : Length of chord is 4 units.

Sol. :



In ΔOMB

$$\sin\frac{\pi}{6} = \frac{2}{r}$$

$$\therefore \ r = \frac{2}{\sin\frac{\pi}{6}} = 4$$

Radius of circle be 4 units.

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