All India Aakash Test Series for JEE (Main)-2022

TEST - 11 - Code-A

Test Date : 03/04/2022

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(3)	31.	(2)	61.	(2)
2.	(2)	32.	(4)	62.	(3)
3.	(3)	33.	(4)	63.	(2)
4.	(4)	34.	(2)	64.	(1)
5.	(4)	35.	(1)	65.	(1)
6.	(4)	36.	(4)	66.	(2)
7.	(4)	37.	(3)	67.	(4)
8.	(1)	38.	(1)	68.	(4)
9.	(1)	39.	(2)	69.	(3)
10.	(4)	40.	(3)	70.	(2)
11.	(1)	41.	(2)	71.	(3)
12.	(4)	42.	(1)	72.	(2)
13.	(2)	43.	(4)	73.	(4)
14.	(4)	44.	(3)	74.	(1)
15.	(1)	45.	(3)	75.	(4)
16.	(1)	46.	(3)	76.	(4)
17.	(4)	47.	(4)	77.	(1)
18.	(1)	48.	(3)	78.	(2)
19.	(3)	49.	(3)	79.	(4)
20.	(1)	50.	(4)	80.	(3)
21.	(10.00)	51.	(90.00)	81.	(05.00)
22.	(00.50)	52.	(13.00)	82.	(03.00)
23.	(25.00)	53.	(32.00)	83.	(65.00)
24.	(02.00)	54.	(07.00)	84.	(11.00)
25.	(04.00)	55.	(06.00)	85.	(27.00)
26.	(20.00)	56.	(02.00)	86.	(91.00)
27.	(02.00)	57.	(08.00)	87.	(01.00)
28.	(03.00)	58.	(04.00)	88.	(12.00)
29.	(02.50)	59.	(02.00)	89.	(30.00)
30.	(02.00)	60.	(06.00)	90.	(32.00)

PART - A (PHYSICS)

1. Answer (3)

Hint : $a_x = 0,\ a_y = -2k\alpha$

Sol. : $\vec{V} = k\hat{i} + k(1-2\alpha t)\hat{j}$

$k(1-2\alpha t) = -k$

$t = \frac{1}{2\alpha}$

2. Answer (2)

Hint : $-2v_B - v_A + v_C = 0$

Sol. : $2v_B = v_C - v_A$

$2v_B = 4 - 2\times 4$

$v_B = 2$ m/s

3. Answer (3)

Hint : $\vec{v}_A = 24\hat{j}$

$\vec{v}_B = -32\hat{i}$

Sol. : $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$t = \frac{12}{24} = 0.5\ s$

$d = 24\times 0.5 - \frac{1}{2}\times 10\times \frac{1}{4}$

$d = 12 - \frac{5}{4} = 10.75$ m

4. Answer (4)

Hint : $\frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{3}$

Sol. : $R_{eq} = 2\ \Omega$

for max. power, $R = 2\ \Omega$

$\frac{E_{eq}}{2} = 3 \Rightarrow E_{eq} = 6$ V

$I = \frac{6}{4} = 1.5$ A

$P_0 = (1.5)^2 \times 2 = 4.5$ W

5. Answer (4)

Hint : $V_c = A + Be^{-\frac{t}{\tau}}$

Sol. : $\tau = \frac{9}{4}C$

At $t = 0,\ V_c = -4V,\ A + B = -4$

At $t = \infty,\ V_c = 9\ V,\ A = 9$

$V_c = 9 - 13e^{\frac{-t}{\tau}}$

$6 = 9 - 13e^{\frac{-4}{9c}}$

$t = \frac{9C}{4}\ln\left(\frac{13}{3}\right)$

6. Answer (4)

Hint : $C_{eq} = \frac{4}{7}\mu$ F

Sol. : $Q_1 = Q_2 = \frac{2}{7}\ V_{eq}$

$V_{CO} = \frac{Q_1}{C} - \frac{Q_2}{2C} = \frac{V_0}{7}$

7. Answer (4)

Hint : $\vec{B} = \vec{B}_1 + \vec{B}_2$

Sol. : $\vec{B} = 0 \Rightarrow \vec{B}_1 = -\vec{B}_2$

$\frac{\mu_0 I_0}{2\pi d} + \frac{\mu_0 I_1}{2R} = 0$

$I_1 = \frac{-I_0 R}{\pi d}$

8. Answer (1)

Hint : $T = \frac{mg}{2}$

Sol. : $a_2 = \frac{g}{4}$

$a_{m_1} = \frac{g}{8}$

9. Answer (1)

Hint : $mv_0 \frac{l}{4} = \frac{Ml^2}{12}\omega$

Sol. : $3mv_0 = M\omega l$ ...(i)

$\frac{mv_0}{M} + \frac{\omega l}{4} = v_0$

$\frac{mv_0}{M} + \frac{3mv_0}{4M} = v_0$

$7m = 4M$ ...(ii)

$v_A = \frac{mv_0}{M} - \frac{3mv_0}{2M}$

$= -\frac{mv_0}{2M}$

10. Answer (4)

Hint : $I(t=0) = \frac{V_0}{3R}$

Sol. : $I(t=\infty) = \frac{V_0}{R}$

11. Answer (1)

Hint : $V = 5 + 3\sin(\omega t)$

Sol. : $V = 5 + 3\sin(\omega t)$

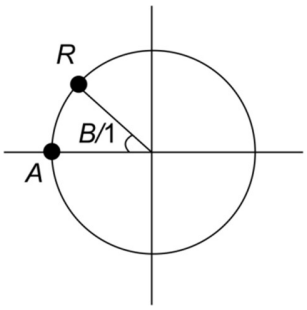
$V_{rms} = \sqrt{5^2 + \frac{3^2}{2}}$

$= \sqrt{\frac{59}{2}}$

12. Answer (4)

Hint : Draw phasor diagram

Sol. : $\omega t = \frac{\pi}{12}$



$\frac{2\pi}{T}t = \frac{\pi}{12}$

$t = \frac{T}{24}$

13. Answer (2)

Hint : Net field is along the angle bisector of *APB*.

Sol. : $\int dE = \int_0^{\frac{\pi}{2}} \frac{\lambda R d\theta \cos\theta}{4\pi\varepsilon_0 R^2}$

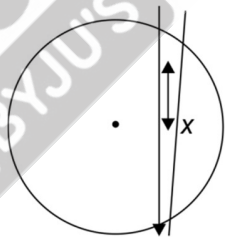
$E = \frac{\lambda}{4\pi\varepsilon_0 R} \sin\theta \Big|_0^{\frac{\pi}{2}}$

$E = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R} = \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 R}$

14. Answer (4)

Hint : $I(x) = \frac{GMd}{R^3}$

Sol. : $f(x) = \frac{GMmx}{R^3}$



$\omega^2 = \frac{GM}{R^3}$

$V = \frac{\omega\sqrt{3}R}{2} = \sqrt{\frac{GM}{R^3}} \times \frac{\sqrt{3}}{2} \times R$

$V = \sqrt{\frac{3GM}{4R}}$

15. Answer (1)

Hint : $C = C_v + \frac{R}{1-n}$

Sol. : $n = 3$

$C = \frac{5}{2}R + \frac{R}{1-3}$

$= 2R$

16. Answer (1)

Hint : Acceleration will change direction once it enters water.

Sol. : $a = +g$ [before entering water]

$a = -g$ [after entering water]

17. Answer (4)

Hint : $\frac{P}{P_0}+\frac{V}{V_0}=1$

Sol. : At that position where it turns from endothermic to exothermic is slope of adiabats.

$PV^r = \text{constant}$

$\frac{dP}{P}+r\frac{dV}{V}=0$

$+\frac{7}{5}\frac{P}{V}=+\frac{P_0}{V_0}$

$+\frac{7}{5}\left[\frac{P_0}{V}-\frac{P_0}{V_0}\right]=\frac{P_0}{V_0}$

$\frac{7}{5}\frac{P_0}{V}=\frac{12}{5}\frac{P_0}{V_0}$

$V=\frac{7V_0}{12}$

18. Answer (1)

Hint : $Q = Q_0\cos(\omega t)$

Sol. : $U_c=\frac{3}{4}U_T$

$Q_c=\frac{\sqrt{3}}{2}Q_0$

$\cos(\omega t)=\frac{\sqrt{3}}{2}$

$\omega t=\frac{\pi}{6}$

$t=\frac{\pi}{6}\sqrt{LC}$

19. Answer (3)

Hint : $\frac{1}{v}+\frac{1}{6}=\frac{1}{f}$

Sol. : $\frac{1}{v_1}+\frac{1}{12}=\frac{1}{f}$

$\frac{12f}{12-f}=\frac{6f}{f-6}$

$2f-12=12-f$

$f=8$ cm

20. Answer (1)

Hint : Theoretical

Sol. : $\vec{E}\cdot\vec{B}=0$

and $\vec{E}$ and B are in same phase

21. Answer (10.00)

Hint : $f = T$

Sol. : $2T = 0.50 \times 4 \times 10$

$T = 10$ N

22. Answer (00.50)

Hint : $f_0=\frac{v}{2l}$

Sol. : $f_1=\frac{v}{4l}$

$f_1=\frac{f_0}{2}$

23. Answer (25.00)

Hint : $\frac{1}{\lambda}=R\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

Sol. : $\frac{1}{\lambda}=R\left[\frac{1}{9}-\frac{1}{25}\right]$

$\lambda=\frac{225}{16R}$

24. Answer (02.00)

Hint : $3hv_0=3\phi+3eV_0$

Sol. : $2hv_0=\phi+3eV_0$

$hv_0=2\phi$

$\phi=\frac{hv_0}{2}$

25. Answer (04.00)

Hint : $R_{eq} = 6\ \Omega$

Sol. : $I = \frac{v_0}{R_{eq}} = 4\,A$

26. Answer (20.00)

Hint : $fx = \mu m_2 gx + \frac{1}{2}kx^2$

Sol. : $kx = \mu m_1 g$

$f = \mu m_2 g + \frac{\mu m_1 g}{2}$

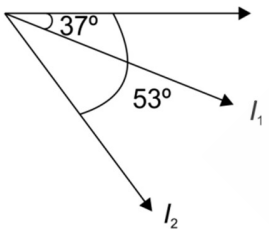
$f = 15 + 5$

$= 20$

27. Answer (02.00)

Hint : $I = I_1 + I_2$

Sol. : $\phi = 45^o$



Power factor = $\cos\phi = \frac{1}{\sqrt{2}}$.

28. Answer (03.00)

Hint : $\phi = \frac{\pi}{3} + \phi_1$

Sol. : $\phi_1 = \frac{2\pi}{3}$

$y = \frac{\lambda D}{3d}$

29. Answer (02.50)

Hint : $F = M^1L^1T^{-2}$

Sol. : $F^1A^{-\frac{1}{2}}T^2 = M^1$

$x = 1,\ y = -\frac{1}{2},\ z = 2$

30. Answer (02.00)

Hint : $\frac{\sin\theta}{\sin r} = n$

Sol. : $\sin\theta = \frac{n}{2}$

$\cos\theta\frac{d\theta}{dn} = \frac{1}{2}$

$\frac{d\theta}{dn} = \frac{1}{\sqrt{2}}$

PART - B (CHEMISTRY)

31. Answer (2)

Hint &Sol. : DMSO reacts with only 1º or 2º alkyl halides forming aldehydes or ketones after hydrolysis, 3º RX do not react with DMSO.

32. Answer (4)

Hint : Rate constant, k for a first order reaction is given by

$$k = \frac{2.303}{t}\log\frac{[A]_o}{[A]_t}$$

Sol. : Let the order of reaction be one

$\therefore$ k = $\frac{0.693}{15}$ min^{-1} = 0.0462 min^{-1}.

Also,

$R_{15} = k[A]_{15}$ at t = 15 min

$R_{45} = k[A]_{45}$ at t = 45 min

$0.92 = k[A]_{15}$

$0.23 = k[A]_{45}$

$\frac{[A]_{45}}{[A]_{15}} = \frac{0.23}{0.92} = \frac{1}{4}$

$[A]_{45} = [A]_o\, e^{-45\,k}$

$[A]_{15} = [A]_o\, e^{-15\,k}$

$\frac{[A]_{45}}{[A]_{15}} = e^{-30k} = \frac{1}{4}$

30k = ln4 = $\frac{2 \times 0.693}{2}$; k = 0.0462

33. Answer (4)

Hint : Reduced mass of positronium = $\frac{m}{2}$

Sol. : If a_0 is the radius of first orbit of H-atom, then radius of nth orbit of positronium is given by $r_n = 2a_0(n)^2$

n = 2 for first excited state;

$r_2 = 8\ a_0 = 8 \times 0.53$ Å

= 4.24 Å

34. Answer (2)

Hint : Ga is an element of Group-13. It shows two oxidation states, +1 and +3.

Sol. : Gallium dichloride exists as

$Ga^{I}\ [Ga^{III}\ Cl_4]$

35. Answer (1)

Hint : The oxidising power of perhalate ions is decided by their standard reduction potential values.

Sol. : The SRPs of perhlate ions

$$XO_4^- + 2H^+ + 2e \longrightarrow XO_3^- + H_2O$$

decrease as

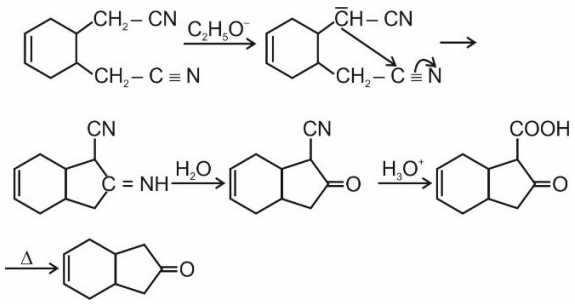
BrO_4^- (SRP = 1.76 V) > IO_4^- (SRP = 1.64 V) > ClO_4^- (SRP = 1.23 V)

This is also their decreasing order of their reducing power.

36. Answer (4)

Hint : The α hydrogen of cyanide is acidic.

Sol. :



37. Answer (3)

Hint : $\Delta T_f = iK_f\ m$

Sol. :

$K_f = 5.0$ K kg mol^{-1}, $K_b = 14.0$ K kg mol^{-1}

$\Delta T_f = -2 - (-2.75) = 0.75$ K

$\Delta T_f = i\ K_f\ m$

$0.75 = i \times 5 \times m$; $im = 0.15$

$\Delta T_b = i\ K_b\ m = i \times 14 \times 0.15 = 2.1$ºC

Boiling point of solution = 84 + 2.1ºC

= 86.1ºC

38. Answer (1)

Hint : $K_{sp}(CH_3COOAg) = [Ag^+]\ [CH_3COO^-]$ in a saturated solution.

Sol. :

Mass of solution = (100 + 1.04)g = 101.04 g

Volume of solution = $\frac{101.04}{1.01}$ mL

Molarity of saturated solution

$= \frac{1.04}{167} \times \frac{1000 \times 1.01}{101.04} = 0.0622$ M

$K_{SP} = (0.0622)^2 = 3.87 \times 10^{-3}$

39. Answer (2)

Hint :

Hybridisation of O-atom in each case is sp^3, size of Cl is large.

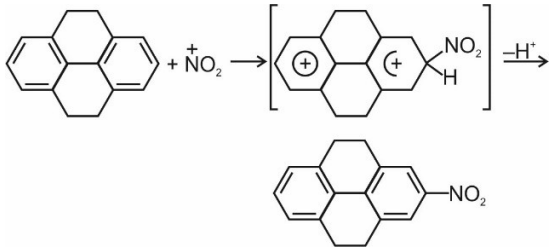
Sol. : Bond angle in Cl_2O is highest due to larger size of Cl^- atom and higher electronegativity of O-atom than that of Cl^-atom. Bond angle of F_2O is least due to higher electronegativity of F-atom than that of O–atom. Therefore, correct order of bond angle is $Cl_2O > H_2O > F_2O$

40. Answer (3)

Hint : Nitronium ion is an electrophile.

Sol. : Nitronium ion attacks at that C-atom of aromatic ring which produces more stable arenium ion.

$$HNO_3 + H^+ \longrightarrow \overset{+}{N}O_2 + H_2O$$



41. Answer (2)

Hint : π-Bond pair of electrons exert greater repulsion on axial bonds in SOF_4.

Sol. : Axial bond angle in SOF_4 is lower than that of SF_4 due to greater repulsion exerted by π bond pair of electrons than lone pair of electrons on S-atom of SF_4. However due to higher electron deficiency of S-atom in SOF_4, the equatorial bond angle in SOF_4 is higher than that of SF_4.

42. Answer (1)

Hint : $\Delta G^o = -nF\,E^o_{cell}$

Sol. : ΔG^o for the formation of 1 mol of Al

$= \frac{-594}{2}$ kJ

$\Delta G^o = -nF\,E^o_{cell}$

$E^o_{cell} = -\frac{\Delta G^o}{nF} = +\frac{594 \times 1000}{2 \times 3 \times 96500}$

= 1.02 V

43. Answer (4)

Hint : Pyrophosphoric acid on heating give metaphosphoric acid and orthophosphoric acid on hydrolysis

Sol. : $H_4P_2O_7 \xrightarrow{\text{Strong heating}} P_2O_5 + 2H_2O\uparrow$

$H_4P_2O_7 + H_2O \longrightarrow 2H_3PO_4$

44. Answer (3)

Hint : For a second order reaction

$$t = \frac{1}{K}\left[\frac{1}{a-x} - \frac{1}{a}\right]$$

Sol. : $t_{1/2} = K\left[\frac{2}{a} - \frac{1}{a}\right] = \frac{1}{Ka}$

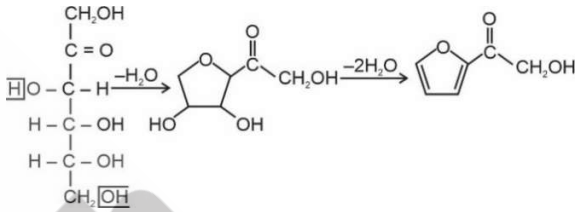
$t_{3/4} = \frac{1}{K}\left[\frac{4}{a} - \frac{1}{a}\right] = \frac{3}{Ka}$

$t_{3/4} : t_{1/2} = 3 : 1$

45. Answer (3)

Hint : D-fructose on treatment with conc. HCl undergoes dehydration to given Furan derivatives.

Sol. :



46. Answer (3)

Hint : Ce^{4+} ion oxidises I^- ion in presence of HCl to ICl and itself is reduced to Ce^{3+}.

Sol. :

Milli equivalents of Ce^{4+} = 20 × 0.05 = 1.00

Milli equivalents of I^- in original solution = 1.00

$I^- + Ce^{4+} \xrightarrow{HCl} ICl$

nf = 2

millimoles of I^- in 250 mL solution $= \frac{1}{2} = 0.5$

Conc of I^- in original solution = $\frac{0.5 \times 127}{250}$

= 0.254 g/L

47. Answer (4)

Hint : It is sequential reaction whose first reaction is zero order and the second reaction is first order.

Sol. : $\therefore \frac{d[B]}{dt} = k_1 - k_2[B]$

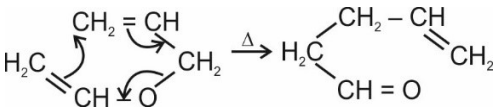
= 0 if [B] is maximum

$\therefore [B]_{max} = \frac{k_1}{k_2} = \frac{2 \times 10^{-3}}{3 \times 10^{-3}}$ = 0.67 mol/L

48. Answer (3)

Hint : This is an example of Claisen rearrangement which involves a six membered ring as the transition state.

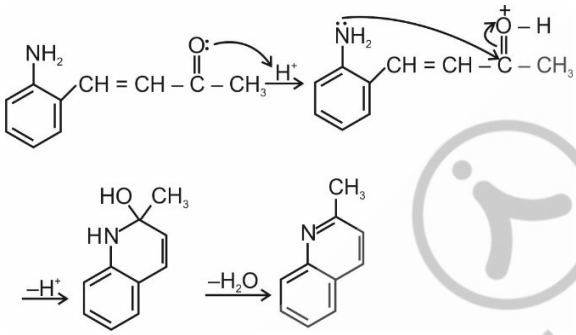
Sol. :



49. Answer (3)

Hint : This reaction involves intramolecular nucleophilic addition followed by elimination.

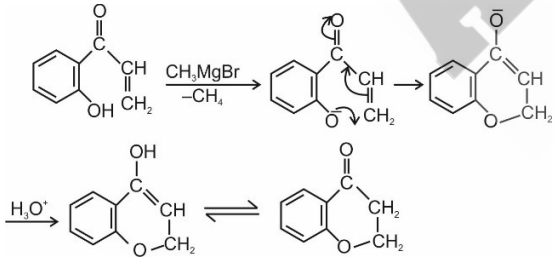
Sol. :



50. Answer (4)

Hint : This is based on intramolecular nucleophilic addition.

Sol. :



51. Answer (90.00)

Hint : Milliequivalents of oxalic acid is equal to Milliequivalent of $Ba(MnO_4)_2$ reacted.

Sol. :

Initial meq of $Ba(MnO_4)_2$ = 100 × 0.1 × 10 = 100

Excess meq of $Ba(MnO_4)_2$ = 60 × 1 = 60

Meq. Of $Ba(MnO_4)_2$ reacted = 100 – 60 = 40

Mass of oxalic acid = $\frac{40}{1000} \times \frac{96}{2} = 1.8$ g

Percentage purity = $\frac{1.8}{2} \times 100 = 90\%$

52. Answer (13.00)

Hint : Nitrogen of an organic compound is estimated as NH_3 by Kjeldahl method.

Sol. : Mass of organic compound = 0.303g

Meq of H_2SO_4 reacted with NH_3= 5 – 2.5 = 2.5

Mass of N = $\frac{2.5 \times 14}{1000}$

Percentage of N = $\frac{2.5 \times 14 \times 100}{1000 \times 0.303}$ = 11.55%

∴ Percentage of O in the organic compound

= 100 – (69.4 + 5.8 + 11.55) = 13.25%

≈ 13.00

53. Answer (32.00)

Hint : K_{sp} (CaF_2) = $[Ca^{2+}][F^-]^2$

Sol. :

Solubility of CaF_2 = $\frac{1.56 \times 10^{-2}}{78}$

= 2×10^{-4} molar

$$CaF_2(s) \rightleftharpoons \underset{x}{Ca^{2+}}(aq) + \underset{2x}{2F^-}(aq)$$

$K_{sp} = [Ca^{2+}][F^-] = 4x^3$

$= 32 \times 10^{-12}$

54. Answer (07.00)

Hint : $2CuSO_4 + KCN \longrightarrow 2K_3[Cu(CN)_4] + (CN)_2$

Sol. : $CuSO_4 + 2KCN \longrightarrow \underset{\text{Unstable}}{Cu(CN)_2} + K_2SO_4$

$2Cu(CN)_2 \longrightarrow 2CuCN + (CN)_2$

$CuCN + 3\,KCN \longrightarrow K_3[Cu(CN)_4]$

x = 3 and y = 4; x + y = 7

55. Answer (06.00)

Hint : 2,5-dibromohex-3-ene is a symmetrical molecule having 3 stereogenic units.

Sol. :

$$CH_3-\underset{\underset{Br}{|}}{\overset{*}{C}H}-CH=CH-\underset{\underset{Br}{|}}{\overset{*}{C}H}-CH_3\ (\text{Symmetrical})$$

n, number of stereocenters = 3 (odd number)

$\therefore$ Number of optically active isomers = 2^{n-1} = 4

Number of meso isomers = $2^{\frac{n-1}{2}}$ = 2 = m

Number of stereoisomers = a + m = 4 + 2 = 6

56. Answer (02.00)

Hint &Sol. :

$$A^{(x+n)+} + ne \longrightarrow A^{x+}$$

Oxidised form (OF) — Reduced form RF

Case-I $[A^{x+}] = 75$, $[A^{(x+n)+}] = 25$

$$E_{RP} = E^{o}_{RP} - \frac{0.059}{n}\log\frac{75}{25}$$

Case-II $E_{RP} = E^{o}_{RP} - \frac{0.059}{n}\log\frac{60}{40}$

$$0.115 - 0.1066 = \frac{0.059}{n}\left[\log 3 - \log\frac{3}{2}\right]$$

$$n = \frac{0.059\times\log 2}{0.0084} \simeq 2$$

57. Answer (08.00)

Hint : $NiCl_2 + 2NaOH \longrightarrow Ni(OH)_2\downarrow + 2NaCl$

Sol. :

	$NiCl_2$	$+2NaOH$	$\longrightarrow Ni(OH)_2$	$+ 2NaCl$
moles	0.025	0.044		
	0.003	0	0.022	

At equilibrium$[Ni^{2+}]$ = 0.012M

$K_{SP} = [Ni^{2+}][OH^-]^2$

$1.6 \times 10^{-14} = 0.012 \times [OH^-]^2$ M

$[OH^-] = 1.15 \times 10^{-6}$

pOH $\simeq$ 6; pH = 8

58. Answer (04.00)

Hint : Alcohols and phenol do not liberate CO_2 with $NaHCO_3$ but liberate H_2 with Na metal.

Sol. :

X = 7 (A, B, C, D, E, G, H)

Y = 3 (C, D, G)

X – Y = 4

59. Answer (02.00)

Hint : n-factor of N_2 in the given reaction = 6

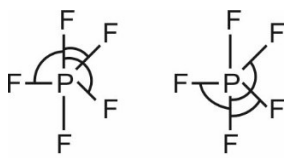
Sol. : Equivalent volume of N_2 at STP

$= x = \frac{22.4}{6}$ L

$\therefore \frac{3x}{5.6} = \frac{3\times 22.4}{5.6\times 6} = 2$

60. Answer (06.00)

Hint&Sol. :



(3) (3)

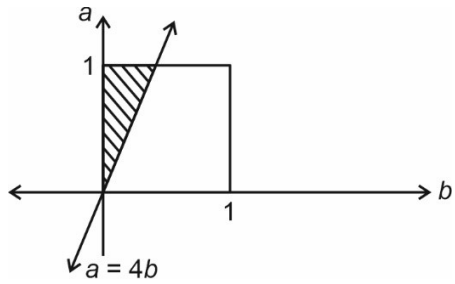
Angles less than 120º = 6

PART - C (MATHEMATICS)

61. Answer (2)

Hint : The discriminant of the equation is $a - 4b$ must be greater than or equal to zero.

Sol. :



The shaded part in the figure are all the points with coordinates (a, b) such that $0 < a, b < 1$ and $a \geq 4b$.

$\therefore$ Required Probability = area of shaded part $= \frac{1}{8}$

62. Answer (3)

Hint : Get the number of ordered triplet when $x = y$ and when $x \neq y$.

Sol. : Number of ordered triplet (x, y, z) with

$(x = y) = {}^{100}C_2$

and number of ordered triplet when

$x \neq y$ is $2 \times {}^{100}C_3$

$\therefore$ Total number of required ordered triplet

$= {}^{100}C_2 + 2 \times {}^{100}C_3$

$= {}^{101}C_3 + {}^{100}C_3.$

63. Answer (2)

Hint : The General term of the series is

$$t_n = \frac{1}{n\sqrt{n+2} + (n+2)\sqrt{n}}$$

Sol. : $\because \frac{1}{n\sqrt{n+2} + (n+2)\sqrt{n}}$

$$= \frac{1}{\sqrt{n}\sqrt{n+2}}\left(\frac{1}{\sqrt{n} + \sqrt{n+2}}\right)$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+2}}\right)$$

$$\therefore \frac{1}{9\sqrt{11} + 11\sqrt{9}} + \frac{1}{11\sqrt{13} + 13\sqrt{11}} + \ldots + \frac{1}{n\sqrt{n+2} + (n+2)\sqrt{n}}$$

$$= \frac{1}{2}\left(\frac{1}{3} - \frac{1}{\sqrt{n+2}}\right)$$

For sum of 36 terms $n = 79$ then required sum

$$= \frac{1}{2}\left(\frac{1}{3} - \frac{1}{\sqrt{81}}\right) = \frac{1}{9}$$

64. Answer (1)

Hint : $\frac{\sin 80°}{\sin 20°} - \frac{\sqrt{3}}{2\sin 80°} = \frac{\sin 80°}{\sin 20°} - \frac{\sin 60°}{\sin 80°}$

Sol. : $\frac{\sin^2 80° - \sin 20° \cdot \sin 60°}{\sin 20° \cdot \sin 80°}$

$$= \frac{1 - \cos 40° + \cos 80° - \cos 160°}{2\sin 20° \cdot \sin 80°}$$

$$= \frac{2\sin^2 20° + 2\sqrt{3}\sin 20° \cdot \cos 20°}{2\sin 20° \cdot \sin 80°}$$

$$= \frac{\sin 20° + \sqrt{3}\cos 20°}{\sin 80°} = \frac{2\sin 80°}{\sin 80°}$$

$= 2.$

65. Answer (1)

Hint : The only possible value of a, b, c are 1 each.

Sol. : $\because a^2 - b = (x - a)^2$ is a perfect square less than a^2.

Thus $a^2 - b \leq (a - 1)^2$. *i.e.* $b \geq 2a - 1$. Similarly $c \geq 2b - 1$ and $a \geq 2c - 1$. Combining these three inequalities, we have $a \geq 8a - 7$ or $a \leq 1$. Thus $a = 1$

Similarly $b = c = 1 \Rightarrow a \cdot b \cdot c = 1$.

66. Answer (2)

Hint : Replace x by t, $\frac{t-1}{t}$ and $\frac{1}{1-t}$, then get different equation.

Sol. : $\because x^2 f(x) + f\left(\frac{x-1}{x}\right) = 2x^2$ ...(1)

Put $x = t$ we get : $t^2 f(t) + f\left(\frac{t-1}{t}\right) = 2t^2$ ...(2)

Put $x = \frac{t-1}{t}$ we get :

$$\left(\frac{t-1}{t}\right)^2 f\left(\frac{t-1}{t}\right) + f\left(\frac{1}{1-t}\right) = 2\left(\frac{t-1}{t}\right)^2 \quad ...(3)$$

Put $x = \frac{1}{1-t}$ we get :

$$f(t) + \frac{1}{(1-t)^2} f\left(\frac{1}{1-t}\right) = \frac{2}{(1-t)^2} \quad ...(4)$$

On solving eq. (2), (3) and (4) we get :

$$f(t) = 1 + \frac{1}{(1-t)^2} - \frac{1}{t^2}$$

$$f(2) = 2 - \frac{1}{4} = \frac{7}{4}$$

67. Answer (4)

Hint : Let $y = m(x+1) + \frac{1}{m}$ is tangent to parabola $y^2 = 4(x + 1)$.

Sol. : $y = m'(x+2) + \frac{2}{m'}$ is tangent to parabola $y^2 = 8(x + 2)$

$\because m \cdot m' = -1 \Rightarrow m' = -\frac{1}{m}$

$\therefore y = -\frac{x}{m} + 2\left(-\frac{1}{m} - m\right)$ ...(i)

is tangent to second parabola and

$y = mx + \left(m + \frac{1}{m}\right)$ ...(ii)

is tangent of first parabola.

For locus eliminate m from eq.(i) and (ii) we get $x + 3 = 0$.

68. Answer (4)

Hint : From equation of medians get coordinate of centroid i.e. equal to $G(4, 1)$

Sol. : Now $C = (4, y_1)$ and B (x_2, y_2)

$\therefore \frac{1+4+x_2}{3} = 4 \Rightarrow x_2 = 7$

Also $x_2 + y_2 = 5 \Rightarrow y_2 = -2$

So, $B(7, -2)$ and $\frac{2+y_1-2}{3} = 1 \Rightarrow y_1 = 3$

$\therefore C = (4, 3)$ and B $(7, -2)$.

69. Answer (3)

Hint : Centre of circle is point of intersection of normals

Sol. : Centre and radius of circle $x(x-4) + y(y-3) = 0$ is $C_1 = \left(2, \frac{3}{2}\right)$ and $\frac{5}{2}$ respectively.

The centre of required circle $(C_2) = \left(-3, \frac{3}{2}\right)$.

$\because C_2$ just contain C_1, hence $C_1C_2 = r_2 - r_1$

$\Rightarrow 5 = r_2 - \frac{5}{2} \Rightarrow r_2 = \frac{15}{2}$

$\therefore$ Equation of circle is

$$x^2 + y^2 + 6x - 3y - 45 = 0$$

70. Answer (2)

Hint : Let $P(\theta)$ and $Q(\phi)$ be end points with eccentric angle of the chord of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Sol. : Point the intersection of tangents at P and

$$Q = R\left(\frac{a\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}, \frac{b\sin\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}\right)$$

$\because R$ lies on $x^2 + y^2 = r^2$

$$\therefore a^2\cos^2\left(\frac{\theta+\phi}{2}\right) + b^2\sin^2\left(\frac{\theta+\phi}{2}\right) = r^2\cos^2\left(\frac{\theta-\phi}{2}\right)$$

$\because$ M is the mid-point of P and Q let M be (x_1, y_1).

$$= \left(\frac{a}{2}(\cos\theta + \cos\phi), \frac{b}{2}(\sin\theta + \sin\phi)\right)$$

$$\therefore \cos^2\left(\frac{\theta+\phi}{2}\right) + \sin^2\left(\frac{\theta+\phi}{2}\right) = \frac{\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}}{\cos^2\left(\frac{\theta-\phi}{2}\right)}$$

$$\therefore \cos^2\left(\frac{\theta-\phi}{2}\right) = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$$

Also $a^2\cos^2\left(\frac{\theta+\phi}{2}\right) + b^2\sin^2\left(\frac{\theta+\phi}{2}\right)$

$$= r^2\cos^2\left(\frac{\theta-\phi}{2}\right) = \frac{x_1^2 + y_1^2}{\cos^2\left(\frac{\theta-\phi}{2}\right)}$$

$$\therefore (x_1^2 + y_1^2) = r^2\cos^4\left(\frac{\theta-\phi}{2}\right)$$

$\therefore$ Required locus is $r^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = x^2 + y^2$

71. Answer (3)

Hint : $\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$ is a discontinuous function at $x = 0$.

Sol. : L.H.L. and R.H.L. of $\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$ are different then to make it continuous $g(x)$ must be zero at $x = 0$ hence only possible option is *C*.

72. Answer (2)

Hint : $f''(x) > 0$ then f(x) is concave up.

Sol. : $\because$ given function is positive increasing and concave up then its inverse is also positive in given interval.

73. Answer (4)

Hint : $\lim_{x\to 0^+}\left(\frac{(1+\{x\})^{\frac{1}{\{x\}}}}{e}\right)^{\frac{1}{\{x\}}} = \lim_{h\to 0}\left(\frac{(1+h)^{\frac{1}{h}}}{e}\right)^{\frac{1}{h}}$

Sol. : $\lim_{h\to 0}\left(\frac{(1+h)^{\frac{1}{h}}}{e}\right)^{\frac{1}{h}} = e^{\frac{1}{e}\lim_{h\to 0}\left(\frac{e^{\frac{1}{h}\ln(1+h)} - e}{h}\right)}$

$$= e^{\lim_{h\to 0}\left(\frac{e^{\frac{\ln(1+h)}{h} - 1} - 1}{h}\right)}$$

$$= e^{\lim_{h\to 0}\left(\frac{e^{\frac{\ln(1+h)}{h} - 1} - 1}{\frac{\ln(1+h) - h}{h}}\right)\cdot\left(\frac{\ln(1+h) - h}{h^2}\right)}$$

$$= e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

74. Answer (1)

Hint : $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots$

Sol. : $g'(0) = b = \lim_{x\to 0}\frac{x^2 + x\tan x - x\tan 2x}{x(ax + \tan x - \tan 3x)}$

$$b = \lim_{x\to 0}\frac{x + \left(x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots\right) - \left(2x + \frac{8x^3}{3} + \frac{2}{15}\cdot 32x^5 + \ldots\right)}{ax + \left(x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots\right) - \left(3x + \frac{27x^3}{3} + \frac{2}{15}\cdot 243x^5 + \ldots\right)}$$

$$\therefore \quad a = 2 \text{ and } b = \frac{-\frac{7}{3}}{\frac{1}{3} - 9} = \frac{7}{26}$$

$$\therefore \quad \frac{b}{a} = \frac{7}{52}$$

75. Answer (4)

Hint : The tangent and line joining origin always remain constant.

Sol. : Let point $P(x, y)$ lies on the curve.

$\therefore$ on differentiating w.r.t. x the curve $\ln(x^2 + y^2) = 3\tan^{-1}\frac{y}{x}$ gives

$$\frac{dy}{dx} = \frac{2x + 3y}{3x - 2y} = m_1 \text{ (say)}$$

and slope of $OP = \frac{y}{x} = m_2$ (say)

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{2x+3y}{3x-2y} - \frac{y}{x}}{1 + \frac{2xy + 3y^2}{3x^2 - 2xy}}\right| = \frac{2}{3}$$

76. Answer (4)

Hint : $|z - a| + |z - b| = c$ represents an ellipse

Sol. : $|z - 1| + |z + 3| \le 8$

$\therefore$ z lies inside or on the ellipse with the foci (1, 0) and (–3, 0) and vertices at (–5, 0) and (3, 0).

Now minimum and maximum value of

$|z - 4|$ are 1 and 9.

$\therefore$ $|z - 4| \in [1, 9]$

77. Answer (1)

Hint : Let $e^x = t$ and rearrange the terms.

Sol. : $I = \int \frac{e^{2x} \cdot e^x dx}{(e^x \sin e^x + \cos e^x)^2}$ Let $e^x = t$

$\therefore$ $I = \int \frac{t^2 dt}{(t \sin t + \cos t)^2}$ Put $t = \tan\theta$

$$= \int \frac{\tan^2\theta \sec^2\theta \, d\theta}{(\tan\theta \sin(\tan\theta) + \cos(\tan\theta))^2}$$

$$= \int \frac{\tan^2\theta \cdot \cos^2\theta \cdot \sec^2\theta \, d\theta}{(\cos(\tan\theta) \cdot \cos\theta + \sin(\tan\theta) \cdot \sin\theta)^2}$$

$$= \int \frac{(\sec^2\theta - 1)\, d\theta}{\cos^2(\tan\theta - \theta)}$$

Put $\tan\theta - \theta = y \Rightarrow (\sec^2\theta - 1)\, d\theta = dy$

$= \int \sec^2 y \, dy = \tan y + c = \tan(\tan\theta - \theta) + c$

$= \tan(e^x - \tan^{-1} e^x) + c$

78. Answer (2)

Hint : Put $t = \frac{1}{u}$ in the first integral.

Sol. : $\int_{1/e}^{\tan x} \frac{t\,dt}{1+t^2} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)} = I$ Put $t = \frac{1}{u}$

$$\therefore I = \int_e^{\cot x} \frac{\frac{1}{u}\left(-\frac{1}{u^2}\right)du}{1+\frac{1}{u^2}} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$$

$$= \int_{\cot x}^{e} \frac{dt}{t(1+t^2)} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$$

$$= \int_{1/e}^{e} \frac{dt}{t(1+t^2)} = \int_{1/e}^{e} \left(\frac{1}{t} - \frac{t}{1+t^2}\right) dt$$

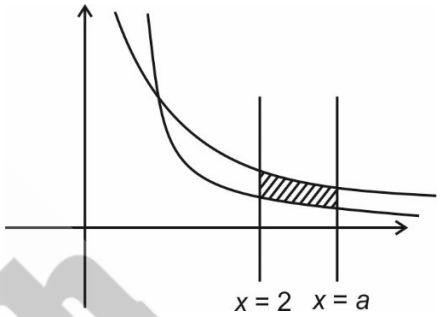
$$= \left[\ln t - \frac{1}{2}\ln(1+t^2)\right]_{1/e}^{e}$$

$= 1$

79. Answer (4)

Hint : Draw the graph and get area.

Sol. :



The area of shaded region

$$= \int_2^a \left(\frac{1}{x} - \frac{1}{2x-1}\right) dx = \ln\frac{4}{\sqrt{5}}$$

$$\left[\ln x - \frac{1}{2}\ln(2x-1)\right]_2^a = \ln\frac{4}{\sqrt{5}}$$

$$\Rightarrow \ln\left(\frac{a^2}{2a-1}\right) - \ln\frac{4}{3} = \ln\frac{16}{5}$$

$\Rightarrow 15a^2 - 128a + 64 = 0 \Rightarrow a = 8$ or $\frac{8}{15}$.

80. Answer (3)

Hint : Use differentiation then linear differential equations.

Sol. : $\int_0^x f(t)dt = x\sqrt{f(x)}$

$$\Rightarrow \quad f(x) = \frac{xf'(x)}{2\sqrt{f(x)}} + \sqrt{f(x)}$$

Let $f(x) = y^2$ then $y^2 = x\frac{dy}{dx} + y$

$$\Rightarrow \quad \int \frac{dy}{y(y-1)} = \int \frac{dx}{x}$$

$\Rightarrow \quad y = \dfrac{1}{1-cx} \Rightarrow \sqrt{f(x)} = \dfrac{1}{1-cx}$

$\therefore\ f(x) = \dfrac{1}{(1+(\sqrt{2}-1)x)^2} \Rightarrow f(2\sqrt{2}+2) = \dfrac{1}{9}$

81. Answer (05.00)

Hint : $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{a}\cdot\vec{b})\,\vec{c}$

Sol. : $\because\ (\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{a}\cdot\vec{b})\,\vec{c} + (\vec{a}\cdot\vec{b})\,\vec{b}$

$= (4-2x-\sin y)\,\vec{b} + (x^2-1)\,\vec{c}$

$\therefore\ \vec{a}\cdot\vec{c} + \vec{a}\cdot\vec{b} = 4-2x-\sin y$ ...(i)

and $\vec{a}\cdot\vec{b} = 1-x^2$ ...(ii)

$\because\ (\vec{c}\cdot\vec{c})(\vec{a}\cdot\vec{a}) = \vec{a}\cdot\vec{c} \Rightarrow \vec{a}\cdot\vec{c} = 1$

$\therefore\ (x-1)^2 + 1 - \sin y = 0 \Rightarrow x = 1$ and $\sin y = 1$

$\therefore\ x^{\sin y} + (4\sin y)^x = 1^1 + 4^1 = 5$

82. Answer (03.00)

Hint : A point on given line $A = (3 + \ ,\lambda 4 + 2\lambda,$ 5 $+ 2\lambda)$

Sol. : $\because$ Point A lies on the plane $x + y + z = 17$

$\therefore\ \lambda = 1$

And point $A = (4, 6, 7)$

$\therefore$ Required distance

$= \sqrt{(4-3)^2 + (6-4)^2 + (7-5)^2} = 3$

83. Answer (65.00)

Hint : The Given two lines are parallel.

Sol. : The lines are $L_1 : \dfrac{x-1}{2} = \dfrac{y+2}{3} = \dfrac{z}{6}$

and $L_2 : \dfrac{x+3}{1/3} = \dfrac{y-5}{1/2} = \dfrac{z+1}{1}$

$\Rightarrow \dfrac{x+3}{2} = \dfrac{y-5}{3} = \dfrac{z+1}{6}$

Or $L_1 : \bar{r} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$L_2 : \bar{r} = (-3\hat{i} + 5\hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\therefore$ Distance between L_1 and L_2

$= \left|\dfrac{(4\hat{i} - 7\hat{j} + \hat{k}) \times (2\hat{i} + 3\hat{j} + 6\hat{k})}{|2\hat{i} + 3\hat{j} + 6\hat{k}|}\right|$

$k = \sqrt{65} \Rightarrow k^2 = 65$.

84. Answer (11.00)

Hint : $|z|^2 = z\cdot\bar{z}$

Sol. : $(2z+5)(2\bar{z}+5) = (6z-9)(6\bar{z}-9)$

$\Rightarrow\ 32|z|^2 - 64(z+\bar{z}) + 56 = 0$

$\therefore\ |z|^2 = 4\,\mathrm{Re}(z) - \dfrac{7}{4}$

$\therefore\ a = 4, b = -\dfrac{7}{4} \Rightarrow a - 4b = 11$

85. Answer (27.00)

Hint : $(A + I)$ be a matrix with all its entries as 1.

Sol. : $A + I = \begin{bmatrix} 1 & 1 \ldots\ldots & 1 \\ 1 & 1 \ldots\ldots & 1 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ 1 & 1 \ldots\ldots & 1 \end{bmatrix} \Rightarrow (A+I)^2 = n(A+I)$

$\Rightarrow A^2 + (2-n)A + (1-n)\,I = 0$

$\Rightarrow A + (2-n)I = (n-1)A^{-1}$

$\therefore\ A^{-1} = \dfrac{A}{n-1} + \dfrac{2-n}{n-1}I \Rightarrow p = 2, q = 1$

86. Answer (91.00)

Hint : $(1-x)^{15}(1 + x + x^2 + \ldots + x^8)^{14}$

$= (1-x)(1-x^9)^{14}$

Sol. : Coefficients of x^{18} in

$(1-x)^{15}(1 + x + x^2 + \ldots + x^8)^{14}$

= Coefficients of x^{18} in $(1-x)(1-x^9)^{14}$

$= {}^{14}C_2 = \dfrac{14\times 13}{2} = 91$

87. Answer (01.00)

Hint : Solution exist if

$\cos(\pi\sqrt{x-4}) = \cos(\pi\sqrt{x}) = \pm 1$

Sol. : $\because$ x – 4 ≥ 0 and $x \geq 0 \Rightarrow x \geq 4$

Here $\pi\sqrt{x-4} = 2n\pi$ and $\pi\sqrt{x} = 2m\pi \Rightarrow x = 4$.

Only solution x = 4 will exist.

88. Answer (12.00)

Hint : $a \cos A + b \cos B + c \cos C$

$= R (\sin 2A + \sin 2B + \sin 2C)$

Sol. : $\dfrac{a\cos^2\frac{A}{2} + b\cos^2\frac{B}{2} + c\cos^2\frac{C}{2}}{a+b+c}$

$$= \frac{(a+b+c)+(a\cos A + b\cos B + c\cos C)}{2(a+b+c)}$$

$$= \frac{1}{2} + \frac{R}{4S}(\sin 2A + \sin 2B + \sin 2C)$$

$$= \frac{1}{2} + \frac{R}{4S}\left(4 \cdot \frac{a}{2R} \cdot \frac{b}{2R} \cdot \frac{c}{2R}\right) = \frac{1}{2} + \frac{abc}{8R^2S}$$

$$= \frac{1}{2}\left(1 + \frac{r}{R}\right) \leq \frac{1}{2}\left(1 + \frac{1}{2}\right)$$

$\therefore\ \lambda = \dfrac{3}{4} \Rightarrow 16\lambda = 12$

89. Answer (30.00)

Hint : Replace y by x to get

$f(2x) = xf'(2x) + xf'(0) + f(0)$

Sol. : Differentiate both sides w.r.t x we get,

$$f'(2x) = 2x \cdot f''(2x) + f'(0)$$

Put x by $\dfrac{x}{2}$ we get $f'(x) = xf''(x) + f'(0)$

Again diff. w.r.t x we get,

$$f'''(x) = 0 \Rightarrow f(x) = ax^2 + bx + c$$

But $f(0) = 0 \Rightarrow c = 0$, $f'(0) = 1$

$\Rightarrow\ b = 1$ and $a = 1$

$\therefore\quad f(x) = x^2 + x$

$\therefore\quad f(5) = 30$

90. Answer (32.00)

Hint : At point of maxima and minima horizontal tangents are formed.

Sol. : $\therefore$ Horizontal tangents are drawn at $x = -1$ and $x = 3$.

Distance between them = |–22 – 10| = 32

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