All India Aakash Test Series for JEE (Main)-2023

TEST - 8 - Code-A

Test Date : 26/03/2023

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(4)	31.	(3)	61.	(3)
2.	(1)	32.	(4)	62.	(2)
3.	(2)	33.	(3)	63.	(4)
4.	(3)	34.	(3)	64.	(2)
5.	(1)	35.	(3)	65.	(4)
6.	(4)	36.	(3)	66.	(3)
7.	(1)	37.	(2)	67.	(2)
8.	(4)	38.	(2)	68.	(1)
9.	(1)	39.	(2)	69.	(2)
10	(3)	40.	(1)	70.	(4)
11.	(4)	41.	(2)	71.	(1)
12.	(1)	42.	(3)	72.	(3)
13.	(3)	43.	(2)	73.	(2)
14.	(4)	44.	(4)	74.	(2)
15.	(2)	45.	(2)	75.	(4)
16.	(3)	46.	(2)	76.	(2)
17.	(1)	47.	(4)	77.	(1)
18.	(2)	48.	(4)	78.	(2)
19.	(4)	49.	(3)	79.	(1)
20.	(3)	50.	(1)	80.	(4)
21.	(04.80)	51.	(17.32)	81.	(04.00)
22.	(00.40)	52.	(09.00)	82.	(02.00)
23.	(04.00)	53.	(04.00)	83.	(71.04)
24.	(08.00)	54.	(09.00)	84.	(01.00)
25.	(00.70)	55.	(46.00)	85.	(04.00)
26.	(07.27)	56.	(10.94)	86.	(02.00)
27.	(00.40)	57.	(05.00)	87.	(23.00)
28.	(00.15)	58.	(05.00)	88.	(33.00)
29.	(00.51)	59.	(03.00)	89.	(01.00)
30.	(06.45)	60.	(02.00)	90.	(09.00)

PART - A (PHYSICS)

1. Answer (4)

Hint : Use Ohm's law

Sol. : At t = 1 s

$$r = R[2 - e^{-\ln(2)\times 1}]$$

$$= \frac{3R}{2}$$

So, $i = \frac{E}{R + \frac{3R}{2}} = \frac{2E}{5R}$

Thus, power $P = i^2R = \frac{4E^2}{25R}$

2. Answer (1)

Hint : Use Archimedes principle

Sol. : Let mass of copper = m

So, mass of gold = $x - m$

$$x - \frac{14x}{15} = 1\times\left\{\frac{m}{10} + \frac{x-m}{20}\right\}$$

$$\frac{x}{15} = \frac{x+m}{20}$$

$$\Rightarrow\ x + m = \frac{4x}{3}$$

Or $m = \frac{x}{3}$

3. Answer (2)

Hint : Number of moles remains constant.

Sol. : $\frac{P_0(2V_0)}{RT_0} + \frac{3P_0(V_0)}{R(2T_0)} = P_f \times\left[\frac{(2V_0)}{RT_0} + \frac{V_0}{2RT_0}\right]$

$$\Rightarrow\ \frac{7}{2}P_0 = P_f \times \frac{5}{2}$$

$$\Rightarrow\ P_f = \frac{7P_0}{5}$$

4. Answer (3)

Hint : $\omega = \frac{v_\perp}{r}$

Sol. : $\omega = \frac{v_B}{l\cos 37^\circ}$

$$\Rightarrow\ v_B = \frac{4}{5}\omega l = \frac{4}{5}\times\frac{1}{2}\times 5$$

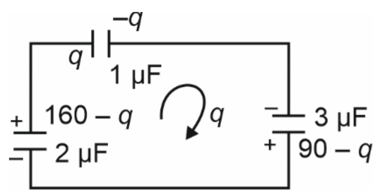
or $v_B = 2$ m/s

5. Answer (1)

Hint : Use KVL

Sol. : By KVL;

$$\frac{160-q}{2} - \frac{q}{1} + \frac{90-q}{3} = 0$$



$$\Rightarrow\ 110 = q\times\frac{11}{6}$$

$$\Rightarrow\ q = 60\ \mu C$$

So, final charge on 2 μF is

Q = 160 – 60

or Q = 100 μC

6. Answer (4)

Hint : $eV_s = \phi - \phi_0$

Sol. : $eV_S = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

$$\frac{eV_S}{k} = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$$

On dividing these equations

$$k = \frac{\frac{1}{\lambda} - \frac{1}{\lambda_0}}{\frac{1}{2\lambda} - \frac{1}{\lambda_0}}$$

$$\Rightarrow\ \frac{k}{2\lambda} - \frac{k}{\lambda_0} = \frac{1}{\lambda} - \frac{1}{\lambda_0}$$

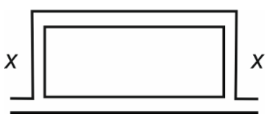
$$\Rightarrow \frac{1}{\lambda}\left(\frac{k}{2}-1\right)=\frac{1}{\lambda_0}(k-1)$$

$$\lambda_0=\frac{\lambda(k-1)}{\left(\frac{k}{2}-1\right)}$$

7. Answer (1)

Hint : Path difference = $n\lambda$ (for maxima)

Sol. : For maxima



Path difference = $n\lambda$

$$2x=\frac{nv}{f}$$

$$f=\frac{nv}{2x}$$

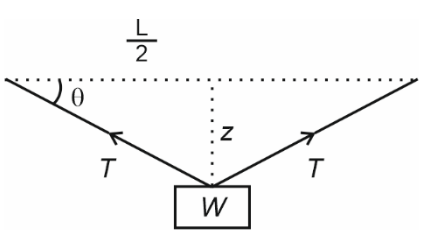
$$=\frac{n\times 300}{2\times 0.6}=n\times 250$$

Only n = 3, f = 750 Hz lies in given range

8. Answer (4)

Hint : $\left(y=\frac{FL}{A\Delta L}\right)$

Sol. : $\tan\theta=\frac{z}{\frac{L}{2}}=\frac{2z}{L}\simeq\sin\theta\simeq\theta$



Extension of wire, $\Delta l=2\sqrt{\frac{L^2}{4}+z^2}-L$

Or $\Delta l=L\left(1+\frac{4z^2}{L^2}\right)^{1/2}-L$

$$\simeq L\left(1+\frac{2z^2}{L^2}\right)-L=\frac{2z^2}{L}$$

For equilibrium

$2T\sin\theta = W$

$$\Rightarrow 2T\left(\frac{2z}{L}\right)=W$$

$$\Rightarrow T=\frac{WL}{4z}$$

Also, $y=\frac{\frac{T}{S}}{\frac{\Delta l}{L}}=\frac{TL}{S\Delta l}$

$$\Rightarrow 10^6\frac{W}{S}=\frac{WL}{4z}\times\frac{L}{S\times\frac{2z^2}{L}}$$

$$\Rightarrow z^3=10^{-6}\frac{L^3}{8}$$

$$\Rightarrow z=10^{-2}\frac{L}{2}=\frac{L}{200}$$

9. Answer (1)

Hint : Use $u=\sqrt{2gh}$, $F_{\text{centripetal}}=\frac{mv^2}{R}$

Sol. : $v_P^2=2g\times 2R$

$v_Q^2=2gR$

So, $N_P=\frac{mv_P^2}{R}=4\,mg$

and $N_Q=\frac{mv_Q^2}{R}-mg=mg$

10. Answer (3)

Hint : Apply kinetic friction on 3 kg block

Sol. : $F_{min}=M_{total}\times\mu g$

= 8 × 0.4 × 10 = 32 N

11. Answer (4)

Hint : $r_d=\frac{\Delta V}{\Delta i}$

Sol. : $r_d=\frac{\Delta V}{\Delta i}=\frac{0.04}{3\times 10^{-2}}$

$$\Rightarrow r_d=1.33\ \Omega$$

12. Answer (1)

Hint : Use breakdown of Zener Diode concept

Sol. : Current from battery

$$i_l = \frac{20}{1\,\text{k}\Omega} = 20\text{ mA}$$

Current through 2R

$$i_L = \frac{10}{2\,\text{k}\Omega} = 5\text{ mA}$$

So, current through diode is

$i_z = i_l - i_L = 15$ mA

13. Answer (3)

Hint : Average speed = $\frac{\text{Distance}}{\text{Time}}$

Sol. : Direction of motion will change at t = 2 s.

$$\frac{dx}{dt} = 2t - t^2$$

$$\Rightarrow \quad x = x_0 + t^2 - \frac{t^3}{3}$$

$t = 0 \qquad x = x_0$

$t = 2 \qquad x = x_0 + \frac{4}{3}$

$t = 4 \qquad x = x_0 - \frac{16}{3}$

So, distance travelled = $\frac{24}{3} = 8$ m

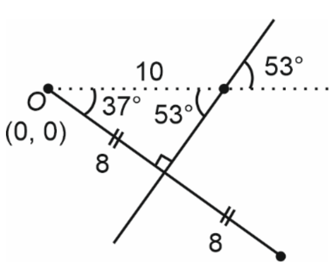
Average speed = $\frac{\text{Distance}}{\text{Time}} = 2$ m/s

14. Answer (4)

Hint : Distance of object along normal = distance of image along normal

Sol. : $x_I = 16\cos 37°$

$$= \frac{64}{5}$$



$y_I = -16\sin 37°$

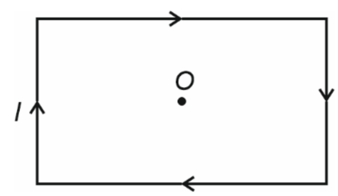
$$= \frac{-48}{5}$$

15. Answer (2)

Hint : $\beta = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$

Sol. : $B_0 = \frac{4\times\mu_0 I}{4\pi\left(\frac{a}{2}\right)}\left(2\times\frac{1}{\sqrt{2}}\right)$

$$= \frac{2\sqrt{2}\mu_0 I}{\pi a}$$



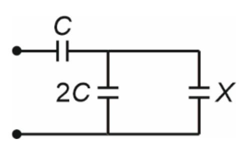
16. Answer (3)

Hint : Use infinite ladder concept.

Sol. : $X = \frac{C[2C+X]}{C+(2C+X)}$

$$\Rightarrow \quad X^2 + 3CX = 2C^2 + CX$$

$$\Rightarrow \quad X^2 + 2CX - 2C^2 = 0$$



So, $X = \left(\frac{-2+\sqrt{12}}{2}\right)C$

$$\Rightarrow \quad X = \left(\sqrt{3}-1\right)C$$

17. Answer (1)

Hint : $A = A_0 e^{-\lambda t}$

Sol. : $\begin{array}{c} A_0 = \lambda N_0 \\ A = \lambda N_0 e^{-\lambda t_0} \end{array} \Bigg| \ \frac{A_0 - A}{A_0}\times 100 = p$

$$\Rightarrow \quad (1 - e^{-\lambda t_0}) = \frac{p}{100}$$

$$\Rightarrow \quad e^{-\lambda t_0} = \left(1 - \frac{p}{100}\right)$$

$$\Rightarrow \quad \lambda t_0 = \ln\left[\frac{1}{1-\frac{p}{100}}\right]$$

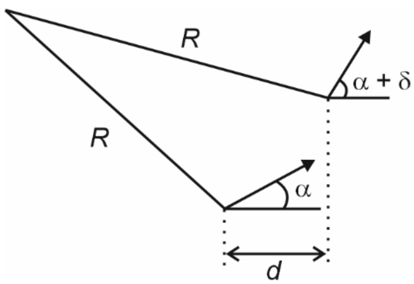
$$\Rightarrow \quad \lambda = \frac{1}{t_0}\ln\left(\frac{1}{1-\frac{p}{100}}\right)$$

18. Answer (2)

Hint : Find d using geometry

Sol. : $d = R[\sin(\alpha+\delta) - \sin(\alpha)]$

or $R = \dfrac{d}{2\sin\left(\frac{\delta}{2}\right)\cos\left(\alpha+\frac{\delta}{2}\right)}$



19. Answer (4)

Hint : $\theta = W + \Delta U$

Sol. : $\omega = \int_{V_0}^{2V_0} PdV = P_0 \int_{V_0}^{2V_0} \left(\frac{V}{V_0}\right)^2 dV$

$$\Rightarrow \quad W = \frac{P_0V_0}{3}\{8-1\} = \frac{7}{3}P_0V_0$$

$$\Delta U = \frac{3}{2}\{8P_0V_0 - P_0V_0\} = \frac{21}{2}P_0V_0$$

So, $Q = W + \Delta U$

$$= 7P_0V_0\left\{\frac{1}{3}+\frac{3}{2}\right\}$$

Or $Q = \dfrac{77}{6}P_0V_0$

20. Answer (3)

Hint : $E = \dfrac{kq}{r^2}$

Sol. : Electric field is zero somewhere between charges and changes direction on crossing any charge.

21. Answer (04.80)

Hint : Shift in path $\Delta x = (\mu - 1)t$

Sol. : $y = \dfrac{(\mu-1)tD}{d} = \dfrac{4\lambda D}{d}$

$$\Rightarrow \quad t = \frac{4\lambda}{(\mu-1)}$$

or $t = 8 \times 6 \times 10^{-7}$ m

= 4.80 μm

22. Answer (00.40)

Hint : $z = \sqrt{R^2 + X_C^2}$

Sol. : $z = \sqrt{R^2 + X_C^2} = 25\ \Omega$

$$i_0 = \frac{V_0}{z} = \frac{1}{5}\text{ A}$$

$$P_{\text{avg}} = \frac{1}{2}i_0^2R$$

$$= \frac{1}{2}\times(0.2)^2\times 20$$

= 0.4 W

23. Answer (04.00)

Hint : $\varepsilon = Blu$

Sol. : Effective length,

$l = 2x\tan 60° = 2\sqrt{3}ut$

so, emf $\varepsilon = Blu$

$$= 2\sqrt{3}Bu^2t$$

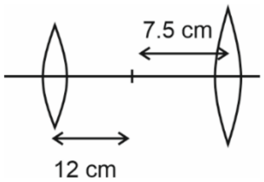
$$= 2\sqrt{3}\times\frac{1}{\sqrt{3}}\times 25\times 4$$

$$= 200\text{ V} = \frac{800}{4}$$

24. Answer (08.00)

Hint : Use lens formula

Sol. : $\frac{1}{(-30)} - \frac{1}{u_e} = \frac{1}{10}$



$\Rightarrow \frac{1}{u_e} = -\frac{4}{30}$

Or $u_e = -7.5$ cm

So, $\frac{1}{(12)} - \frac{1}{u_0} = \frac{1}{4}$

$\Rightarrow \frac{1}{u_0} = \frac{1}{4}\left(\frac{-2}{3}\right)$

$\Rightarrow u_0 = -6$ cm

So, $m = m_0 \times m_e$

$= \left(\frac{12}{6}\right) \times \left(\frac{30}{7.5}\right)$

Or $m = 8$

25. Answer (00.70)

Hint : $\tau = I\alpha$

Sol. : $F = ma$

$Fh = \frac{7}{5} mRa$

$\Rightarrow h = \frac{7R}{5} = 0.70$ m

26. Answer (07.27)

Hint : $\eta = 1 - \frac{T_2}{T_1}$

Sol. : $\frac{40}{100} = 1 - \frac{600}{T}$

$\Rightarrow T = 1000$ K

So, $\theta = T - 273 = 727°\text{C}$

Or $\frac{\theta}{100} = 7.27$

27. Answer (00.40)

Hint : Use $a = -\omega^2 x$

Sol. : $(\sigma A_H)\frac{d^2x}{dt^2} = -(\rho A_x)g$

$\Rightarrow \frac{d^2x}{dt^2} = -\left(\frac{\rho g}{\sigma H}\right)x$

So, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\sigma H}{\rho g}}$

$= 2\pi\sqrt{\frac{1}{3} \times \frac{12 \times 10^{-2}}{\pi^2}}$

Or $T = 0.40$ s

28. Answer (00.15)

Hint : Area = $6a^2$

Sol. : Area = $6a^2$

$= 6 \times 256 \times 10^{-4}$ m^2

$= 1536 \times 10^{-4}$ m^2

$\simeq 0.15$ m^2

We need to round off to two significant digits

29. Answer (00.51)

Hint : Use concept of inelastic collision

Sol. : Maximum loss of KE = $\frac{1}{2} \times \frac{4mv^2}{5}$

$= \frac{1}{5}K_i$

For inelastic collision

$\frac{1}{5}K_i \geq 10.2$ eV

$\Rightarrow K_i \geq 51$ eV

30. Answer (06.45)

Hint : Reading = $m + n$ (L.C.)

Sol. : $l = 6\text{ mm} + \frac{36}{80} \times 1\text{mm}$

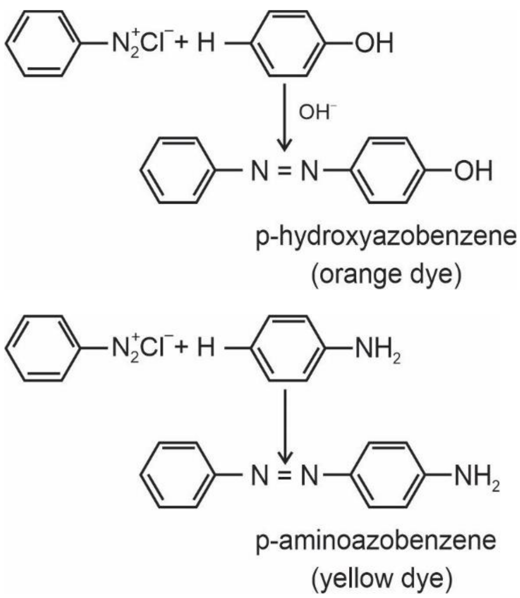
= 6.45 mm

PART - B (CHEMISTRY)

31. Answer (3)

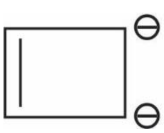
Hint : Diazonium salt can undergo coupling reaction with highly reactive aromatic compounds.

Sol. :

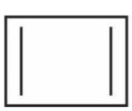


Chlorobenzene does not give this test.

32. Answer (4)

Hint :  $6\pi e^-$ in resonance
Aromatic in nature

Sol. :

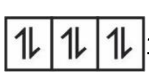
 $4\pi e^-$ in resonance

So, antiaromatic nature

33. Answer (3)

Hint : 13^{th} e^- of scandium is in 3*p* subshell.

Sol. : Sc(Z = 21); $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2\,3d^1$

 13^{th} e^- may have n, l, m and s values as 3, 1, –1, $+\frac{1}{2}$

34. Answer (3)

Hint : Inert pair effect

Sol. : Due to inert pair effect, $Pb^{+2} > Pb^{+4}$ and $Tl^{+1} > Tl^{+3}$

So, $PbO > PbO_2$ and $TlCl > TlCl_3$

35. Answer (3)

Hint : Colloidal particles have tendency to carry a charge.

Sol. :

Haemoglobin (blood)	–	Positively charged sols
Charcoal	–	Negatively charged sols
Collodion	–	Ultrafiltration
Protection of colloid	–	Gold number

36. Answer (3)

Hint : Critical parameter $(Z_c) = \frac{P_cV_c}{T_c} = \frac{3}{8}$

Sol. : $T_C = \frac{8a}{27Rb} < T_B = \frac{a}{Rb} < T_i = \frac{2a}{Rb}$

37. Answer (2)

Hint : A is sodium peroxoborate.

Sol. : $2NaBO_2 + 2H_2O_2 + 6H_2O \rightarrow$

$$Na_2\left[(OH)_2B(O-O)_2B(OH)_2\right]\cdot 6H_2O$$

Sodium peroxoborate

38. Answer (2)

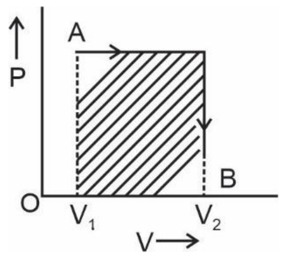
Hint : Both LiCl and $MgCl_2$ are deliquescent in nature.

Sol. : $LiHCO_3$ exists in aq. solution only, rest all are obtained in solid form.

39. Answer (2)

Hint : The area under the curve is the work done from V_1 to V_2.

Sol. : Thus, the graph with maximum area under the curve, has maximum value of work done



40. Answer (1)

Hint : Cheilosis can be caused by the deficiency of vitamin B_2.

Sol. : Hence, option (1) is correct.

41. Answer (2)

Hint : Maximum concentration of metal Al and Zn in drinking water are 0.2 ppm and 5.0 ppm respectively.

Sol. : Maximum concentration of metal Fe is 0.2 ppm

42. Answer (3)

Hint : Normality of H_2O_2 solution with volume strength V can be given by

Normality = $\frac{V}{5.6} \Rightarrow 4 = \frac{V}{5.6}$

V = 22.4

Sol. : So, volume strength of H_2O_2 solution is 22.4.

43. Answer (2)

Hint : Bond order of Be_2 is zero.

Sol. : Hence, compound with zero bond order doesn't exist.

44. Answer (4)

Hint : Sphalerite (also known as zinc blende) is an ore of zinc.

Sol. : Kaolinite is an ore of aluminium.

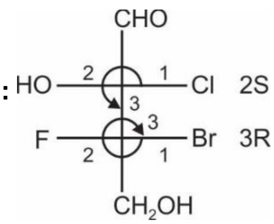
45. Answer (2)

Hint : $\underset{(A)}{MnO_2}(\text{fused}) + KOH \xrightarrow{O_2} \underset{\substack{\text{(Dark green)} \\ (B)}}{K_2MnO_4} + \underset{(C)}{H_2O}$

Sol. : Compound C is H_2O

46. Answer (2)

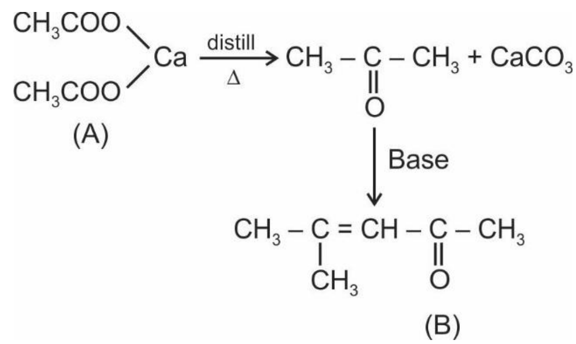
Hint :



Sol. : The correct option is (2).

47. Answer (4)

Hint :



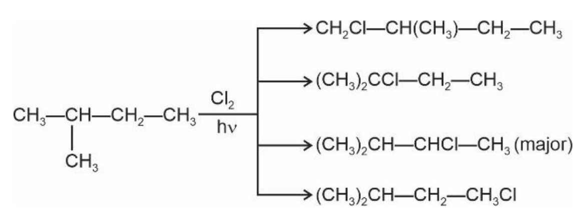
Sol. : Hence, option (4) is correct.

48. Answer (4)

Hint : Reactivity ratio of H-atoms towards chlorination

3°H : 2°H : 1°H = 5 : 3.8 : 1

Sol. :



Hence, option 4 is correct.

49. Answer (3)

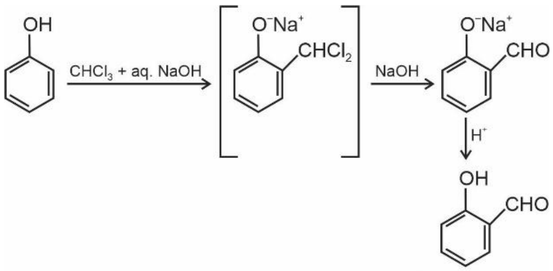
Hint : Alkyl fluorides $\longrightarrow$ Swarts reaction.

Sol. : Alkyl Iodides – Finkelstein reaction

50. Answer (1)

Hint : Reimer-Tiemann reaction, carbolic acid is phenol.

Sol. :



Salicylaldehyde

51. Answer (17.32)

Hint :

$$Pb(NO_3)_2 \rightarrow Pb^{+2}(aq.) + 2NO_3^-(aq.)$$

0.01	–	–
–	0.01	0.02

$$PbI_{2(S)} \rightleftharpoons Pb^{+2}(aq.) + 2I^-(aq.)$$

s 2s

≈ 0.01

Sol. :

$K_{sp} = 12 \times 10^{-10} = [Pb^{+2}]\,[I^-]^2$

$= 12 \times 10^{-10} = 0.01 \times (2\text{ s})^2$

$= \dfrac{12 \times 10^{-10}}{0.04} = s^2$

$s^2 = 3 \times 10^{-8}$

$x = s = 173.2 \times 10^{-6}$

$\Rightarrow \dfrac{x}{10} = 17.32$

52. Answer (09.00)

Hint : $3Br_2 + 6OH^- \rightarrow BrO_3^- + 5Br^- + 3H_2O$

Sol. : The value of (a + b + c) = (1 + 5 + 3) = 9

53. Answer (04.00)

Hint : CO and N_2O_5 are not greenhouse gases.

Sol. : The number of greenhouse gases is 4.

54. Answer (09.00)

Hint :

$(T_b - T_b^\circ) = i \times K_b \times m$ (Elevation in B.P)

$(T_f^\circ - T_f) = i \times K_f \times m$ (Depression in F.P)

$(T_b - T_f) - 100 = m(K_f + K_b)$

$\because \quad T_b^\circ = 100°C \qquad T_f^\circ = 0°C$

Sol. :

$$100.2372 - 100 = \frac{w_B \times 1000}{120 \times 750}(1.86 + 0.512)$$

$$\frac{0.2372}{2.372} = \frac{w_B \times 1000}{120 \times 750}$$

$w_B = 9$ g

$\therefore \quad x = 9$

55. Answer (46.00)

Hint : Percentage ionic character is given by $[16(X_A - X_B) + 3.5(X_A - X_B)^2]$

Sol. : $[16 \times (4 - 2) + 3.5(4 - 2)^2] = 46\%$

56. Answer (10.94)

Hint : 8.8 g of propane $\Rightarrow$ 0.2 mol

Sol. :

$CH_3 - CH_2 - \underset{\underset{O}{\|}}{C} - O^-Na^+$ obtained at 57% yield from propane = 0.2 × 0.57

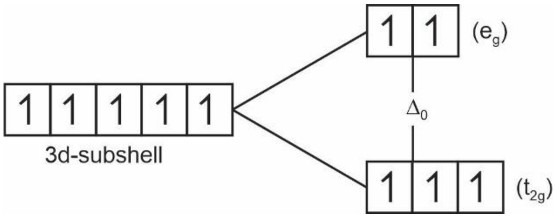
= 0.114 mol

Mass of sodium propionate = 0.114 × 96

= 10.94 g

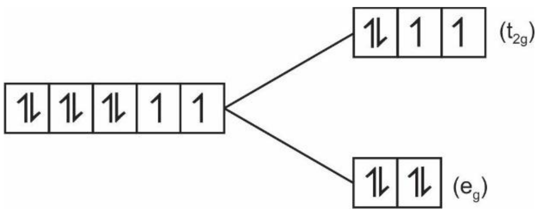
57. Answer (05.00)

Hint : For $[Mn(H_2O)_6]^{2+}$, Mn is in +2 oxidation state, So, according to CFT



The number of unpaired e^- in t_{2g} is 3

⇒ For $[NiCl_4]^{2-}$, Ni is in +2 oxidation state, So, according to CFT



The number of unpaired e^- in t_{2g} is 2

Sol. : So, the value of (x + y) is (3 + 2) = 5

58. Answer (05.00)

Hint : Compound having net resultant of dipoles equal to zero are non-polar in nature.

Sol. : SF_6, CO_2, I_3^-, XeF_4 and CH_4 are non-polar in nature.

59. Answer (03.00)

Hint : Unit of rate constant = $(\text{Conc.})^{1-n}\ s^{-1}$

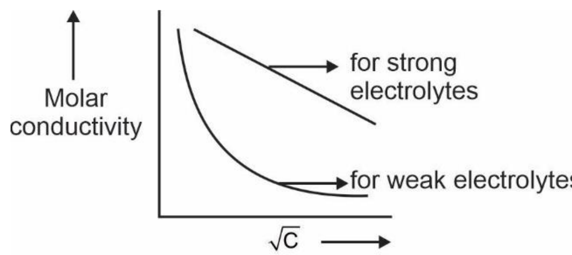
For 5th order kinetics

$(\text{mol L}^{-1})^{1-5}\ s^{-1} = \text{mol}^{-4}\ L^4\ s^{-1}$

Sol. : The value of [2 × (–4) + 3 × (4) + (–1)] is 3.

60. Answer (02.00)

Hint :



Sol. : NaCl and KCl being strong electrolytes possess the given linear plot

PART - C (MATHEMATICS)

61. Answer (3)

Hint: If a function is bijective then it is both one-one and onto

Sol. : Clearly $f(x)$ is onto as the range of $f(x)$ is $(-\infty, \infty)$.

$f(x)$ to be one-one,

$f'(x) = 6x^2 - 6x + 5a$

$D \le 0$

$36 - 120a \le 0$

$a \ge \frac{3}{10}$

62. Answer (2)

Hint: Use $\tan^{-1}a - \tan^{-1}b = \tan^{-1}\frac{(a-b)}{(1+ab)}$

Sol. : $\sum_{n=0}^{\infty}\tan^{-1}\left(\frac{6.3^n}{1+3^{2n+3}}\right)$

$= \sum_{n=0}^{\infty}\tan^{-1}\left(\frac{3^{n+2}-3^{n+1}}{1+3^{n+1}\,3^{n+2}}\right)$

$= \frac{\pi}{2} - \tan^{-1}3 = \cot^{-1}3 = \tan^{-1}\frac{1}{3}$

63. Answer (4)

Hint: Trace means sum of leading diagonal

Sol. : Let $X = 2A + 3B$

$Y = 3A - 2B$

$\text{Tr}(X) = 2\text{Tr}(A) + 3\text{Tr}(B) = 4$ …(i)

$\text{Tr}(Y) = 3\text{Tr}(A) - 2\text{Tr}(B) = -7$ …(ii)

By solving above two equations

$\text{Tr}(A) = -1$, $\text{Tr}(B) = 2$

$2\text{Tr}(A) + \text{Tr}(B) = 0$

64. Answer (2)

Hint: Put $x = 0$ and $x = -1$ in Δ

Sol. : Put $x = 0$,

$$D = \begin{vmatrix} -1 & 1 & -1 \\ -1 & 2 & 2 \\ 1 & -1 & -3 \end{vmatrix} = 4$$

Now put $x = -1$

$$-A + B - C + D = \begin{vmatrix} -3 & 0 & -4 \\ -2 & 1 & -1 \\ 0 & -2 & -7 \end{vmatrix} = 11$$

So, $A - B + C = 4 - 11 = -7$

65. Answer (4)

Hint: Simplify first then differentiate

Sol. :

$$y = \frac{x^4}{x^4 + x^3 + 1} + \frac{1}{1 + x^4 + x^3} + \frac{x^3}{x^3 + x^4 + 1}$$

$$y = \frac{x^4 + 1 + x^3}{x^4 + x^3 + 1} = 1$$

$$\frac{dy}{dx} = 0$$

66. Answer (3)

Hint: $g(x) = |(x - 1)(x - 2)^2|$

Sol. : $f(x) = (x - 1)(x - 2)^2$

$|f(x)|$ is differentiable at all those points where $f(x) \neq 0$.

$|f(x)|$ is also differentiable at $x = c$ if $f(c) = 0$ and $f'(c) = 0$.

$|f(x)|$ is non-differentiable at $x = c$ if $f(c) = 0$ and $f'(c) \neq 0$.

$f''(x) = 2(3x - 5) = 0 \Rightarrow x = \frac{5}{3}$

f has one point of inflection.

67. Answer (2)

Hint: If $f'(x) > 0 \ \forall \ x \in R$ then function is strictly increasing

Sol. : $f'(x) = \dfrac{4}{\left(4 + x^2\right)^{3/2}} + \dfrac{1}{\left(1 + (3 - x)^2\right)^{3/2}}$

$f'(x) > 0 \ \forall \ x \in R$

So, $f(x)$ is increasing $\forall \ x \in R$.

68. Answer (1)

Hint: Take $x^4 + 2x^3 - 8x = t$

Sol. : $\int x\left(x + 2 - 8x^{-2}\right)^{\frac{1}{3}}\left(2x^3 + 3x^2 - 4\right)dx$

$$= \int \left(x^4 + 2x^3 - 8x\right)^{\frac{1}{3}}\left(2x^3 + 3x^2 - 4\right)dx$$

Let $x^4 + 2x^3 - 8x = t$

$(4x^3 + 6x^2 - 8)dx = dt$

So, $\int t^{\frac{1}{3}} \cdot \frac{dt}{2} = \frac{3}{8}\left(x^4 + 2x^3 - 8x\right)^{\frac{4}{3}} + C$

$$= \frac{3}{8}x^4\left(x + 2 - 8x^{-2}\right)^{\frac{4}{3}} + C$$

69. Answer (2)

Hint: Use $\sec x + \tan x = e^t$

$\sec x - \tan x = e^{-t}$

Sol. :

Let $\left.\begin{matrix} \sec x + \tan x = e^t \\ \sec x - \tan x = e^{-t} \end{matrix}\right\} 2\sec x = e^t + e^{-t}$

$(\sec x \tan x + \sec^2 x)dx = e^t\, dt$

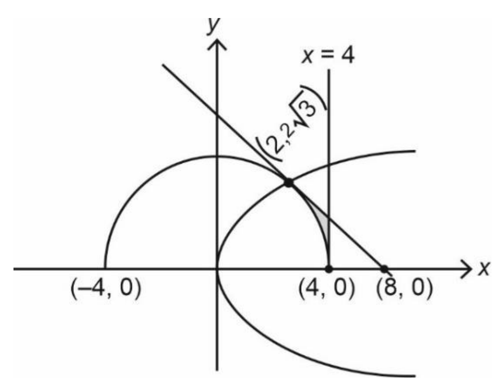
$$dx = \frac{2dt}{e^t + e^{-t}}$$

$$I = \int_0^{\ell n(1+\sqrt{2})} 2(e^t + e^{-t})^{12} dt$$

70. Answer (4)

Hint: Draw the graph and apply $\int_a^b (y_2 - y_1)dx$

Sol. :



Shaded region is

$$\int_2^4 \left(\frac{8-x}{\sqrt{3}} - \sqrt{16-x^2}\right)dx$$

or

$$\int_0^{2\sqrt{3}} \left(8 - y\sqrt{3} - \sqrt{16-y^2}\right)dy - \frac{1}{2}\times 4\times\frac{4}{\sqrt{3}}$$

71. Answer (1)

Hint: The lines are coplanar if $\vec{b}_1$ is parallel to $\vec{b}_2$

Sol. : $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ are coplanar lines if either $\vec{b_1}$ is parallel to $\vec{b_2}$

or $\left(\vec{a_1} - \vec{a_2}\right)\cdot\left(\vec{b_1}\times\vec{b_2}\right) = 0$

72. Answer (3)

Hint: $np = 3,\ npq = \frac{9}{4}$

Sol. : $np = 3,\ npq = \frac{9}{4}$

$p = \frac{1}{4},\ q = \frac{3}{4},\ n = 12$

$P(|x-6| \le 5) = P(1 \le x \le 11)$

$= 1 - P(x = 0) - P(x = 12)$

$$= 1 - \left(\frac{3}{4}\right)^{12} - \left(\frac{1}{4}\right)^{12} = \frac{2^{24} - 3^{12} - 1}{2^{24}}$$

73. Answer (2)

Hint: Use sum and product of roots.

Sol. : $\alpha + \beta = p$

$\frac{\alpha}{3} + 6\beta = q \Rightarrow \alpha + 18\beta = 3q$

$\beta = \frac{3q-p}{17},\ \alpha = \frac{3(6p-q)}{17}$

$\alpha\beta = r = \frac{3}{289}(6p-q)(3q-p)$

74. Answer (2)

Hint: Substitute $z = x + iy$

Sol. : $Im\left(\frac{a(x+iy)+1}{x+iy+b}\right) = y$

$$\frac{-y(ax+1)+ay(x+b)}{(x+b)^2+y^2} = y$$

$(x+b)^2 + y^2 = 1$

75. Answer (4)

Hint: Use exponential expansion

Sol. : $2n^2 - 6n + 3 = a(2n-1)(2n-2) + b(2n-1) + c$

$a = \frac{1}{2},\ b = \frac{-3}{2},\ c = \frac{1}{2}$

So,

$$\sum_{n=1}^{\infty}\frac{2n^2-6n+3}{(2n-1)!} = \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{(2n-3)!} - \frac{3}{2}\sum_{n=1}^{\infty}\frac{1}{(2n-2)!} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}$$

$$= \frac{1}{2}\left(\frac{e-e^{-1}}{2}\right) - \frac{3}{2}\left(\frac{e+e^{-1}}{2}\right) + \frac{1}{2}\left(\frac{e-e^{-1}}{2}\right)$$

$$= -\frac{e}{4} - \frac{5}{4}e^{-1}$$

76. Answer (2)

Hint: Sum of n term of $AP = \frac{n}{2}[2a + (n-1)d]$

$n \to$ number of terms

a = first term

d = common difference

Sol. : $\frac{\frac{p}{2}(2a+(p-1)d)}{\frac{q}{2}(2a+(q-1)d)} = \frac{p^2}{q^2}$

$2a(q-p) = d(q-p)$

$2a = d \quad (\because p \ne q)$

$$\frac{a_{17}}{a_{13}} = \frac{a+16d}{a+12d} = \frac{33}{25}$$

77. Answer (1)

Hint: Use binomial expansion

Sol. : $\left(1+\dfrac{x-2}{x-1}\right)^n = \left(\dfrac{2x-3}{x-1}\right)^n$

$$= \frac{1}{(x-1)^n}\sum_{r=0}^{n} C_r x^{n-r}(x-3)^r$$

$$= \sum_{r=0}^{n} C_r \left(\frac{x}{x-1}\right)^{n-r}\left(\frac{x-3}{x-1}\right)^r$$

So, $a_r = C_r$

78. Answer (2)

Hint: Find $a + b + c$, abc, $ab + bc + ca$ then form a cubic

Sol. : $a + b + c = 5$,

$a^2 + b^2 + c^2 = 59$,

$a^3 + b^3 + c^3 = 317$

$$ab+bc+ca = \frac{25-59}{2} = -17$$

$317 - 3abc = 5\{59 + 17\} \Rightarrow abc = -21$

So, a, b, c are the roots of equation

$x^3 - 5x^2 - 17x + 21 = 0$

$(x - 1)(x^2 - 4x - 21) = 0 \Rightarrow x = 1, -3, 7$

$a = -3, b = 1, c = 7$

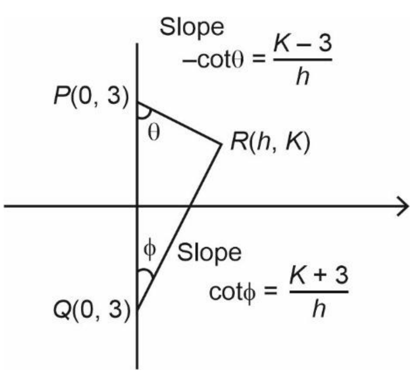
Required number of divisors = $(b + 1)(c + 1) - 1$

= 15

79. Answer (1)

Hint: Angle between lines.

Sol. : $\theta - \phi = \dfrac{\pi}{4}$



$$\frac{\tan\theta - \tan\phi}{1+\tan\theta\cdot\tan\phi} = 1$$

$$\frac{\dfrac{h}{K-3}+\dfrac{h}{K+3}}{1-\dfrac{h^2}{K^2-9}} = -1$$

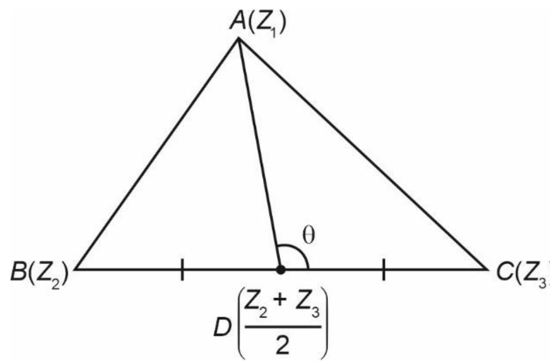
$\Rightarrow 2hK = -(K^2 - h^2 - 9)$

Locus of R is $x^2 - y^2 - 2xy + 9 = 0$

80. Answer (4)

Hint: Use rotation of complex numbers

Sol. :



The given equation is

$$\frac{Z_1 - \dfrac{Z_2+Z_3}{2}}{\dfrac{Z_3-Z_2}{2}} = i$$

From above equation, we can conclude that

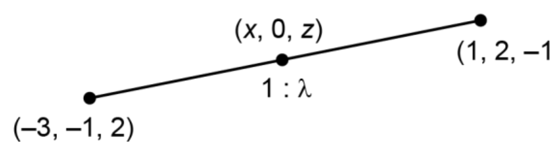
$$\angle ADC = \frac{\pi}{2} \text{ and } \frac{AD}{BC} = \frac{1}{2}$$

So, triangle ABC is right angled isosceles.

81. Answer (04.00)

Hint: Use section formula

Sol. :



$$\frac{2-\lambda}{1+\lambda} = 0 \Rightarrow \lambda = 2$$

$\frac{1-6}{1+2} = x = \frac{-5}{3}$

$\frac{-1+4}{1+2} = z = 1$

82. Answer (02.00)

Hint: 1^{∞} form

Sol. : $\lim\limits_{x\to 0}\left(1+\frac{(1-\cos x)}{x}\ell n|\sin\alpha+\cos\alpha|\right)^{\frac{1}{x}}$

$= e^{\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}\cdot \ell n|\sin\alpha+\cos\alpha|}$

$= \sqrt{|\sin\alpha+\cos\alpha|} = 2^{\frac{1}{4}}$

$|\sin\alpha+\cos\alpha| = \sqrt{2}$

$\sin 2\alpha = 1$

$\alpha = \frac{\pi}{4}, \frac{5\pi}{4}$

83. Answer (71.04)

Hint: Mean $= \frac{\Sigma x_i}{n}$, variance $= \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$

Sol. : $\frac{\sum x_i}{30} = 25$ & $\frac{\sum x_i^2}{30} - 25^2 = 75$

$\sum x_i = 750$ & $\sum x_i^2 = 21000$

$\sum\limits_{\text{corrected}} x_i = 750 - 13 + 31$

$= 768$

& $\sum\limits_{\text{corrected}} x_i^2 = 21000 - 13^2 + 31^2$

$= 21792$

Corrected variance $= \frac{21792}{30} - \left(\frac{768}{30}\right)^2$

$= 726.4 - 655.36$

$= 71.04$

84. Answer (01.00)

Hint:

$P(E) = \frac{\text{Number of element in favour}}{\text{Total number of possible outcomes}}$

Sol. : $P(1 \text{ or } 2) = \frac{1}{8}$

$P(1 \text{ or } 2 \text{ or } 3) = \frac{1}{4}$

So, $P(3) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$

$P(2 \text{ or } 3 \text{ or } 5) = \frac{7}{16}$

$P(2 \text{ or } 5) = \frac{7}{16} - \frac{1}{8} = \frac{5}{16}$

$P(2) = \frac{5}{16} - \frac{1}{4} = \frac{1}{16}$, $P(1) = \frac{1}{8} - \frac{1}{16} = \frac{1}{16}$

$P(4 \text{ or } 6) = 1 - \frac{1}{16} - \frac{1}{16} - \frac{1}{8} - \frac{1}{4} = \frac{1}{2}$

85. Answer (04.00)

Hint: $a^3 + b^3 + c^3 = 3abc$ if $(a + b + c = 0)$

Sol. : The equation can be rearranged as

$(-1)^3 + (\sin x)^3 + (-\cos x)^3 = 3(-1)\cdot\sin x\cdot(-\cos x)$

Which is possible if

either $\sin x - \cos x - 1 = 0$

or $\sin x = -\cos x = -1$

Now, $\sin x - \cos x = 1$ gives 4 solutions in $(-2\pi, 2\pi)$

& $\sin x = -\cos x = -1$ Not possible

86. Answer (02.00)

Hint: Use the range of LHS and RHS

Sol. : $\underbrace{\frac{1}{1-\frac{\sin 2x}{2}}}_{\left[\frac{2}{3}, 2\right]} = \underbrace{\left(\frac{4}{11}\right)^y + \left(\frac{4}{11}\right)^{-y}}_{[2,\infty)}$

Only possible when both sides are equal to 2.

So, $\sin 2x = 1$ & $y = 0$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

87. Answer (23.00)

Hint: Use partial fraction

Sol. :

$$\frac{2x^2-11x+13}{x^3-6x^2+11x-6}=\frac{2}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}$$

Integrating factor

$$=e^{\int\left(\frac{2}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}\right)dx}$$

$$=\frac{(x-1)^2|(x-2)|}{|(x-3)|}=\frac{(x-1)^2(2-x)}{3-x}$$

$$y\frac{(x-1)^2(2-x)}{3-x}=\int(x-1)(x-2)dx$$

$$=\frac{x^3}{3}-\frac{3x^2}{2}+2x+c$$

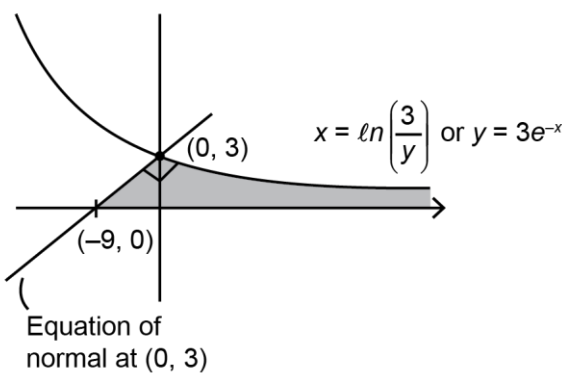
Passing through origin, $c = 0$

$$y(-1)=\frac{-\frac{1}{3}-\frac{3}{2}-2}{4\times\frac{3}{4}}=\frac{-23}{18}$$

88. Answer (33.00)

Hint: Area of triangle + $\int_0^\infty f(x)dx$

Sol. :



$x - 3y + 9 = 0$

Required area $=\frac{1}{2}\times3\times9+\int_0^\infty 3e^{-x}dx$

$$=\frac{27}{2}+3=\frac{33}{2}\text{ units}$$

89. Answer (01.00)

Hint: Plane passing through (x_1, y_1), (x_2, y_2), (x_3, y_3)

Sol. : $\begin{vmatrix}1 & 15 & 15\\ 3 & 4 & -2\\ 1 & -1 & K\end{vmatrix}=0$

$K = \frac{-137}{41}$

90. Answer (09.00)

Hint: $|\text{adj}A| = |A|^2$

Sol. : $|adj(adjA)|=49$

$|A|^4 = 49 \Rightarrow |A|^2 = 7$

$$\left|\sqrt{7}A\cdot adj(2A)\right|=\left(\sqrt{7}\right)^3|A|\cdot|adj(2A)|$$

$$=\left(\sqrt{7}\right)^3|A|\cdot|2A|^2$$

$$=\left(\sqrt{7}\right)^3\cdot|A|\cdot\left(2^3\cdot|A|\right)^2$$

$$=7\sqrt{7}\times\sqrt{7}\times2^6\cdot7$$

$$=7^3.2^6$$

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