All India Aakash Test Series for JEE (Advanced)-2024

TEST - 4A (Paper-2) - Code-B

Test Date : 02/04/2023

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(5)	19.	(7)	37.	(4)
2.	(4)	20.	(8)	38.	(2)
3.	(2)	21.	(7)	39.	(7)
4.	(2)	22.	(3)	40.	(8)
5.	(6)	23.	(1)	41.	(2)
6.	(5)	24.	(6)	42.	(7)
7.	(8)	25.	(4)	43.	(6)
8.	(6)	26.	(8)	44.	(2)
9.	(B, C)	27.	(A, B, D)	45.	(A, B)
10.	(B)	28.	(A, B, C, D)	46.	(A, D)
11.	(B, D)	29.	(A, B, C)	47.	(A, B, C)
12.	(A, C, D)	30.	(C)	48.	(A, B, D)
13.	(A, B)	31.	(A, C)	49.	(A, B, C)
14.	(B)	32.	(A, B, C, D)	50.	(A, C)
15.	(B)	33.	(D)	51.	(A)
16.	(C)	34.	(B)	52.	(A)
17.	(D)	35.	(C)	53.	(D)
18.	(C)	36.	(A)	54.	(A)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (5)

Hint : $T = 2\pi\sqrt{\dfrac{M + \dfrac{m}{3}}{K}}$

Sol. : $T = 2\pi\sqrt{\dfrac{M + \dfrac{m}{3}}{K}}$

$$T = 2\pi\sqrt{\frac{(20+5)\times 10^{-3}}{10}}$$

$$= 2\pi\sqrt{25}\times 10^{-2} \text{ seconds}$$

T = 10π × 10^{-2} seconds

$$= \frac{10\pi}{100} = \frac{\pi}{10} = \frac{11}{35} \text{ seconds}$$

2. Answer (4)

Hint : $f = \dfrac{v}{2(\text{length})}$

Sol. : Length = $\dfrac{\lambda}{2} = \dfrac{v}{2f}$

$$f = \frac{v}{2(\text{length})}$$

$$f_1 = \left(\frac{v_1}{2\times 2L}\right); f_2 = \frac{v_2}{2\times L}$$

$$\frac{f_1}{f_2} = \frac{v_1}{2v_2}$$

Also, $v = \sqrt{\dfrac{T}{\mu}}$ $\quad \mu_1 = \left(\rho\times\pi r^2\right) = \rho\pi r^2$

$$\mu_2 = \frac{\rho\times\pi r^2}{4}$$

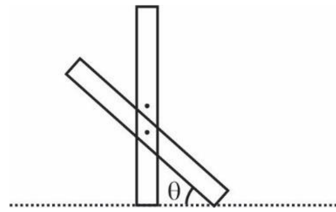
$$\frac{v_1}{v_2} = \sqrt{\frac{\mu_2}{\mu_1}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\frac{f_1}{f_2} = \left(\frac{1}{4}\right)$$

3. Answer (2)

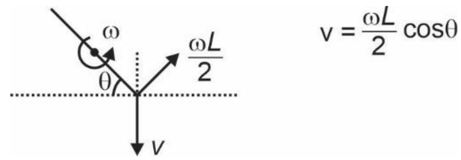
Hint : Apply energy conservation.

Sol. :



Energy Equation

$$mg\frac{l}{2}(1-\sin\theta) = \frac{1}{2}mv^2 + \frac{1}{2}i_{cm}\omega^2 \qquad \ldots(1)$$



Equation (1) can be written as

$$\Rightarrow mg\frac{L}{2}\left(1-\frac{1}{2}\right) = \frac{1}{2}m\frac{\omega^2L^2\times 3}{4\times 4} + \frac{1}{2}\times\frac{1}{12}mL^2\omega^2$$

$$= mL^2\omega^2\left[\frac{3}{32}+\frac{1}{24}\right]$$

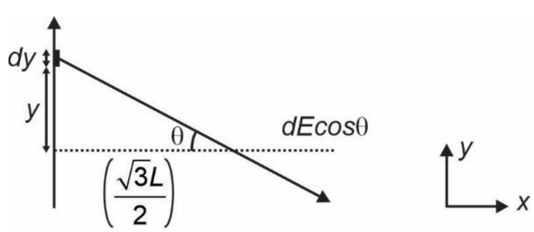
$$= mL^2\omega^2\left[\frac{9+4}{96}\right]$$

$$\Rightarrow \omega = \sqrt{\frac{24}{13}\frac{g}{L}}$$

4. Answer (2)

Hint : $E_x = \int dE\cos\theta$

Sol. :



$$E_x = \int dE\cos\theta$$

$$dE = \frac{Gdm}{\left(\dfrac{\sqrt{3}L}{2\cos\theta}\right)^2} = \frac{4Gdm\cos^2\theta}{3L^2}$$

$$y = \frac{\sqrt{3}L}{2}\tan\theta$$

$$dy = \frac{\sqrt{3}L}{2}\sec^2\theta d\theta$$

$$dm = \lambda dm = \frac{m}{L}\left(\frac{\sqrt{3}L}{2}\right)\sec^2\theta d\theta$$

$$= \frac{\sqrt{3}m}{2}\sec^2\theta d\theta$$

$$dE = \frac{4G\cos^2\theta}{3L^2}\times\frac{\sqrt{3}m}{2}\sec^2\theta d\theta$$

$$= \left(\frac{2\sqrt{3}Gm}{3L^2}\right)d\theta$$

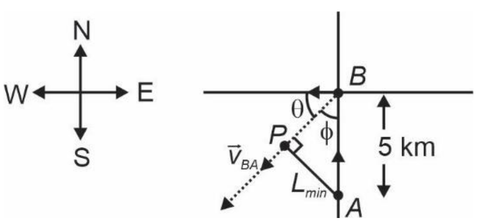
$$E_x = \int_{-\frac{\pi}{6}}^{+\frac{\pi}{6}}\frac{2\sqrt{3}Gm}{3L^2}\cos\theta d\theta = \frac{2\sqrt{3}Gm}{3L^2}\times 2\times\frac{1}{2}$$

$$= \left(\frac{2\sqrt{3}Gm}{3L^2}\right)$$

5. Answer (6)

Hint : $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

Sol. :



$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

$$|\vec{V}_{BA}| = \sqrt{20^2 + 10^2}$$

$$= 10\sqrt{5}\text{ km/hr}$$

$$\tan\theta = \left(\frac{V_A}{V_B}\right) = \frac{10}{20} = \frac{1}{2}$$

$$\tan\theta = 2 \Rightarrow \cos\phi = \left(\frac{1}{\sqrt{5}}\right)$$

$BP = 5\cos\phi$

$= 5 \times \frac{1}{\sqrt{5}}$

$= \sqrt{5}$ km

$$\text{time} = \frac{BP}{|\vec{V}_{BA}|} = \frac{\sqrt{5}}{10\sqrt{5}} = \frac{1}{10}\text{hr}$$

time = 6 min

6. Answer (5)

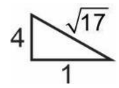
Hint : $y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$

Sol. : $y = 4x - x^2$

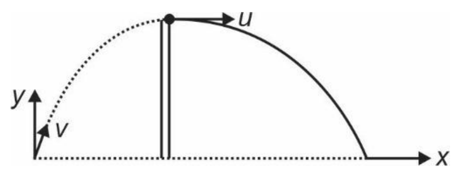
$$\Rightarrow y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

$x = 1, y = 3$

[$\tan\theta = 4$]



$$2\sin\theta\cdot\cos\theta = \left(\frac{8}{17}\right)$$



$$\frac{v^2\sin 2\theta}{g} = 4$$

$$= \frac{v^2}{g} = \left(\frac{17}{2}\right)$$

$u^2 = v^2 - 2g \times 3$

$$\frac{17g}{2} - 6g$$

$$u^2 = \frac{5g}{2}$$

$$u = \sqrt{25} = 5\text{ m/s}$$

7. Answer (8)

Hint : $X_{\text{mean}} = \frac{\overset{n}{\Sigma x}}{n}$

Sol. : $X_{\text{mean}} = \frac{10}{4}$ m

= 2.50 m

Mean absolute error = $\frac{0.08}{4}$

= 0.02 m

Percentage error in measurement

$$= \frac{0.02}{2.50}\times 100$$

$\approx 0.8\%$

8. Answer (6)

Hint : $\Sigma \vec{T} \cdot \vec{V} = 0$

Sol. : $V_A = V_B = 6$ m/s

as $3TV_A - 3TV_B = 0$

$V_A = V_B$

$V_A = 6$ m/s

9. Answer (B, C)

Hint : $\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$

Sol. : $\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} = (2xy)$

Force is conservative in nature.

10. Answer (B)

Hint : Normal reaction is independent of frame of reference

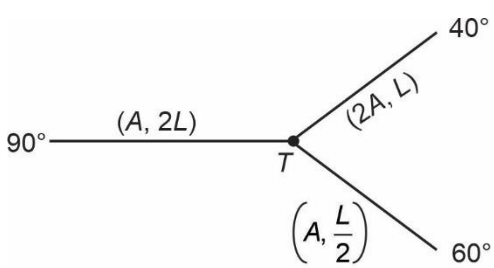
Sol. : Normal reaction is independent of frame of reference.

Acceleration of block in case-I is zero.

11. Answer (B, D)

Hint : $\frac{\Delta T}{R_{\text{th}}} = \text{constant}$

Sol. :



$$\frac{90-T}{\left(\frac{2L}{KA}\right)} = \frac{T-40}{\frac{L}{2KA}} + \frac{T-60}{\frac{L}{2KA}}$$

$$\frac{90-T}{2} = (T-40+T-60)\times 2$$

$\Rightarrow 90 - T = 8T - 100 \times 4$

$\Rightarrow 9T = 490$

$$T = \left(\frac{490}{9}\right)^\circ\text{C}$$

12. Answer (A, C, D)

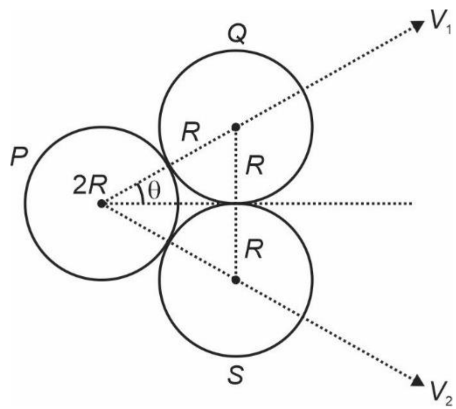
Hint : Temperature first increases then decreases.

Sol. : So, $W > 0$

13. Answer (A, B)

Hint : Apply linear momentum conservation.

Sol. :



$$\sin\theta = \frac{R}{3R} = \frac{1}{3}$$

$$\tan\theta = \frac{R}{2\sqrt{2}R} = \frac{1}{2\sqrt{2}}$$

$$\cos\theta = \frac{2\sqrt{2}}{3}$$

$M_Q = M$

$M_S = M$

$M_P = 8M$

Momentum conservation

$v_1 = v_2 = v_0$

Final velocity of P is v

$$\Rightarrow 8Mu = 2Mv_0 \times \frac{2\sqrt{2}}{3} + 8MV$$

$$8u = \frac{4\sqrt{2}}{3}v_0 + 8v$$

$$\Rightarrow 2u = \frac{\sqrt{2}}{3}v_0 + 2v \qquad \ldots(1)$$

$$e = \frac{v_0 v\cos\theta}{u\cos\theta} = 1$$

$$v_0 - \frac{v_2\sqrt{2}}{3} = \frac{u\times 2\sqrt{2}}{3}$$

$$\Rightarrow \frac{3 v_0}{2\sqrt{2}} - u = v \qquad \ldots(2)$$

On solving (1) and (2),

$$v_0 = \left(\frac{12\sqrt{2}}{11}\right)u$$

14. Answer (B)

Hint : $\left.\begin{array}{ll} F>0, & x<0 \\ F<0, & x>0 \end{array}\right\}$ stable equilibrium

Sol. : $\left.\begin{array}{ll} F<0, & x>0 \\ F>0, & x<0 \end{array}\right\}$ stable equilibrium

$\left.\begin{array}{ll} F>0, & x>0 \\ F<0, & x<0 \end{array}\right\}$ unstable equilibrium

15. Answer (B)

Hint : $l_1 \propto \dfrac{1}{nd^2}$

Sol. : $l_1 \propto \dfrac{1}{nd^2}$

n = number density per unit volume

d = diameter of molecule

$$\frac{l_1}{l_2} = \frac{n_2}{n_1}\left(\frac{d_2}{d_1}\right)^2$$

$$\frac{l_1}{l_2} = \left(\frac{3}{2}\right)\times\left(\frac{4}{3}\right)^2 = \frac{3}{2}\times\frac{16}{9} = \left(\frac{8}{3}\right)$$

16. Answer (C)

Hint : $\displaystyle\int_0^V \frac{dv}{2-v} = \int_0^V dt$

Sol. : $\dfrac{dv}{dt} = 2 - v$

$$\int_0^V \frac{dv}{2-v} = \int_0^V dt \quad \Rightarrow -\ln\left(\frac{2-v}{2}\right) = t$$

$$\Rightarrow 1 - \frac{v}{2} = e^{-t}$$

$$\Rightarrow v = 2(1 - e^{-t})$$

$$\frac{dx}{dt} = 2(1 - e^{-t})$$

$$\int dx = 2\int_0^{\ln 4} \left(1 - e^{-t}\right) dt$$

$$x = 2\left[t + e^{-t}\right]_0^{\ln 4} = 2\left[\ln 4 + \frac{1}{4} = 1\right]$$

$$= 2\left[\ln 4 - \frac{3}{4}\right]$$

17. Answer (D)

Hint : $I = \int x^2 dm$

Sol. : $I = \int x^2 dm = \int x^2 \lambda dx$

$$= \int x^2 \lambda_0 x^2 dx$$

$$= \int_0^2 \lambda_0 x^4 dx$$

$$= \frac{\lambda_0 x(2)^5}{5}$$

$$= \frac{32}{5}\lambda_0$$

18. Answer (C)

Hint : $\vec{a}_{\mathrm{P}} = \vec{a}_{\mathrm{P,\,com}} + \vec{a}_{\mathrm{com,\,A}}$

Sol. : $a_P = \omega^2 R + \omega^2 \times (2R)$

$= 3\omega^2 R$

PART - II (CHEMISTRY)

19. Answer (7)

Hint : NaCl + HCl $\rightarrow$ No reaction

$Na_2CO_3 + 2HCl \rightarrow 2NaCl + H_2O + CO_2$

Sol. : Since, 1 mole of $Na_2CO_3 \equiv$ 1 mole of CO_2

Let the mass of Na_2CO_3 be w gm

$$\therefore \quad \frac{w}{106} = \frac{4.8}{44}$$

$$w = \frac{4.8}{44}\times 106 = 11.56 \text{ g}$$

$$\%\,Na_2CO_3 = \frac{11.56}{20}\times 100 = 58\%$$

20. Answer (8)

Hint : $PV = \dfrac{1}{3}mnC^2$

Sol. : $P\times 10^{-3} = \dfrac{1}{3}\times 10^{-25}\times 10^{24}\times (10^2)^2$

$$= \frac{10^6}{3}\ \text{N/m}^2$$

$$x = P = \frac{10^6}{10^5\times 3}\ \text{bar} = 3.33 \text{ bar}$$

Kinetic energy of 1 molecule = $\frac{3}{2}k_BT$

$$\frac{3}{2}k_BT = \frac{1}{2}mC^2$$

$$y = T = \frac{mC^2}{3k_B} = \frac{10^{-25} \times (10^2)^2}{3 \times 1.38 \times 10^{-23}}$$

$$\frac{xy}{10} \simeq 8$$

21. Answer (7)

Hint : meq of O.A = meq of R.A

Sol. : $H_2O_2 + KI + H_2SO_4 \rightarrow I_2 + H_2O + K_2SO_4$

$I_2 + 2Na_2S_2O_3 \rightarrow 2NaI + Na_2S_4O_6$

Since, meq of I_2 = meq of $Na_2S_4O_6$

meq of I_2 = 25 × 0.1 = 2.5

meq of H_2O_2 = meq of I_2

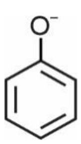
$$(N \times V)_{H_2O_2} = 2.5$$

$$N = \frac{2.5}{100} = \frac{25}{1000}$$

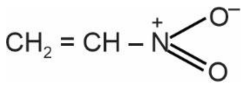
Volume strength of H_2O_2 = $\frac{25}{1000} \times 5.6$

x = 0.14 V

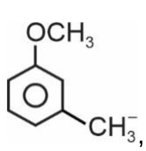
22. Answer (3)

Hint :  is more stable, since negative charge is more stable on more electronegative atom.

Sol. : $CH_3 - C \equiv \overset{+}{O}$ and $CH_3 - \overset{+}{O} = CH_2$ are more stable due to complete octet.



is more stable due to complete octet.

In 

–I effect is operating at Meta position, leads to stabilisation of carbanion.

23. Answer (1)

Hint : e^{-Zr/na_0}

$$\frac{-r}{3a_0} = \frac{-r}{na_0}$$

Sol. : n = 3

Maximum power of r = 1

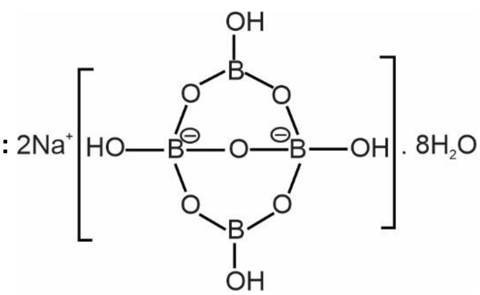
Hence, ℓ = 1

No. of radial nodes = n – ℓ – 1 = 3 – 1 – 1 = 1

24. Answer (6)

Hint : Sodium tetraborate decahydrate is Borax.

$Na_2[B_4O_5(OH)_4]\cdot 8H_2O$ has following structure.

Sol. : 

Hence x = 2, y = 2 and z = 2

25. Answer (4)

Hint : $\underset{C-Ch}{NH_4^+} + H_2O \rightleftharpoons \underset{Ch}{NH_4OH} + \underset{Ch}{H^+}$

$[H^+] = 10^{-pH}$

Sol. : $[H^+] = 10^{-5}$

$$K_h = Ch^2 = \frac{k_w}{k_b} = \frac{10^{-14}}{2 \times 10^{-5}} = 0.5 \times 10^{-9}$$

$$h = \frac{K_h}{Ch} = \frac{K_h}{[H^+]} = \frac{5 \times 10^{-10}}{10^{-5}} = 5 \times 10^{-5}$$

$$C = \frac{[H^+]}{h} = \frac{10^{-5}}{5 \times 10^{-5}} = 0.2$$

1 litre contains → 0.2 moles

800 ml contains → $\frac{0.2}{1000} \times 800$

⇒ 0.16 mole

Mass (in g) = 0.16 × 53.5 = 8.56 g

x = 8.56 g

26. Answer (8)

Hint : Work done by the system = –pΔV

$W = -\Delta nRT$

Sol. : $Ca + 2HCl \rightarrow CaCl_2 + H_2$

$$w = \frac{-m}{M}RT = \frac{-10}{40}\times 0.0821\times 400$$

$= -8.21$ L atm

27. Answer (A, B, D)

Hint : Li^+ Be^+ B^+ C^+

$1s^2$ $1s^22s^1$ $1s^22s^2$ $1s^22s^22p^1$

Sol. : I^- is more stable than other halide ions

$SiO_2 < P_2O_5 < SO_3 < Cl_2O_7$ (acidic strength)

As we move from left to right, acidic strength of oxides increases.

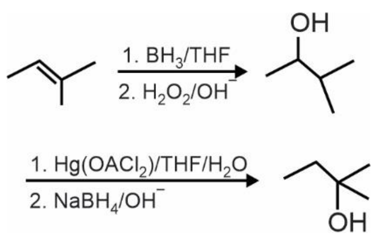
Although Ne^+ and F are iso-electronic, due to greater nuclear charge, the electron attracting tendency or electron affinity of Ne^+ ion is more than that of F.

28. Answer (A, B, C, D)

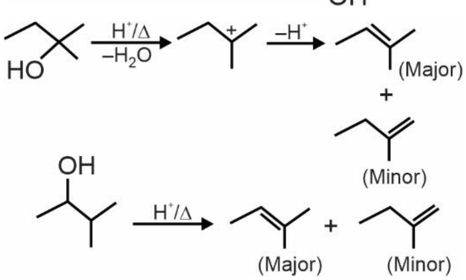
All statements are correct

29. Answer (A, B, C)

Hint :



Sol. :



30. Answer (C)

Hint : For exothermic reaction, increasing temperature moves the reaction in backward direction.

Sol. : Addition of inert gas at constant volume has no effect.

According to Le-Chatelier's principle,

On decreasing pressure, reaction moves in backward direction.

31. Answer (A, C)

Hint : $N_2 = \sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^2$

$O_2 = \sigma 1s^2, \sigma^* s^2, \sigma 2s^2, \sigma 2s^2, \sigma 2p_z^2, \pi 2p_x^2, = \pi 2p_y^2, \pi^*2p_x^1, \pi^*2p_y^1$

$$B.O = \frac{1}{2}[N_b - N_a]$$

Sol. : $N_2(B.O) = \frac{1}{2}[6] = 3$

$$N_2^+(B.O) = \frac{1}{2}[5] = 2.5$$

$$O_2^+, B.O. = \frac{1}{2}[6-1] = 2.5$$

$$C_2^- = \sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \pi 2p_x^2, \pi 2p_y^2, \sigma 2p_z^1$$

$$\text{Bond order} \propto \frac{1}{\text{Bond length}}$$

In F_2 molecule, HOMO is Anti bonding molecular orbital.

32. Answer (A, B, C, D)

Hint : Beryl contain $Si_6O_{18}^{12-}$ unit, hence a cyclic silicate.

Sol. : Pyroxenes are formed by sharing of O-atoms at two corners of each tetrahedron with other tetrahedra.

Two tetrahedral units are joined by sharing the O at one corner, thus giving the unit $(Si_2O_7)^{6-}$.

33. Answer (D)

Hint : Beryllium halides are essentially covalent and soluble in organic solvents.

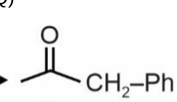
Sol. : Among alkaline earth metal hydroxides, the anion being common the cationic radius will influence lattice enthalpy. Since lattice enthalpy decreases much more than the hydration enthalpy with increasing ionic size, the solubility increases as we go down the group.

34. Answer (B)

Hint : $HC\equiv CH \xrightarrow[1\,eq]{NaNH_2,\ liq\ NH_3}$

$$\underset{(P)}{HC\equiv CNa} \xrightarrow[Ph-CH_2-Br]{} HC\equiv \underset{(Q)}{C}-CH_2-Ph$$

Sol.: $HC\equiv C-CH_2-Ph \xrightarrow{Hg^{2+}/H_2SO_4}$



(R)

35. Answer (C)

Hint : Percentage of nitrogen = $\frac{28 \times V \times 100}{22400 \times m}$

V is volume of N_2 in mL (at STP)

m is weight of organic compound.

Sol. : Actual pressure = 725 – 15 = 710 mm

Volume of nitrogen at STP

$= \frac{273 \times 710 \times 80}{300 \times 760}$

= 68 mL

Since, 22400 ml of N_2 at STP weighs 28 g

68 mL of nitrogen weighs = $\frac{28 \times 68}{22400}$ g

Percentage of nitrogen = $\frac{28 \times 68 \times 100}{22400 \times 0.8}$

= 10.6%

36. Answer (A)

Hint : $Mg_2C_3 \xrightarrow{H_2O} CH_3 - C \equiv CH$

Sol. : $\underset{(P)}{CH_3 - C \equiv CH} \xrightarrow{\text{2 mole HCl}} \underset{(Q)}{CH_3 - CCl_2 - CH_3}$

PART - III (MATHEMATICS)

37. Answer (4)

Hint : Remove modulus sign for different intervals and solve.

Sol. :

Case I : $\sin x \geq 0 \quad 0 < x \leq \pi$

$|\sin x| \sin y = \sin x \sin y$

$= \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

$\cos(x-y) - \cos(x+y) = -\frac{1}{2}$

$\cos(x+y) + \cos(x-y) = \frac{3}{2}$

$$\left.\begin{array}{l}\cos(x-y) = \frac{1}{2} \\ \cos(x+y) = 1\end{array}\right\} \Rightarrow \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right) \text{ and } \left(\frac{\pi}{6}, \frac{11\pi}{6}\right)$$

$\therefore$ Four ordered pairs.

38. Answer (2)

Hint : Use relation between roots and coefficients.

Sol. : $x^2 + px + q = 0$ with roots r, s

$x^2 + rx + s = 0$ with roots p, q

$r + s = -p$ …(i)

$p + q = -r$ …(iii)

$rs = q$ …(ii)

$pq = s$ …(iv)

Equation (i) × (ii),

$pqrs = qs \Rightarrow pr = 1$ …(v)

Equation (i) + (iii),

$p + q + r + s = -p - r$

$\Rightarrow 2(p + r) + q + s = 0$

$p^2 + rp + s = 0$ [$\because$ p is root of equation]

$r^2 + pr + q = 0$ [$\because$ r is root of equation]

$(p+r)^2 - 2(p+r) = 0 \Rightarrow p + r = 0, p + r = 2$

Also, $pr = 1 \Rightarrow p = 1, r = 1$

From equation (iv),

$pq = s \Rightarrow q = s$

$\therefore q + s = -2(p + r)$

$\Rightarrow q + s = -4 \Rightarrow q = s = -2$

$\therefore 4 \times 1 - 2 + 2 - 2 = 2$

39. Answer (7)

Hint : Use $\sum_{r=1}^{n}[(r+1)^{p+1} - r^{p+1}]$

Sol. : ${}^{p+1}C_1\sum_{r=1}^{n} r^p + {}^{p+1}C_2\sum_{r=1}^{n} r^{p-1} + \ldots + \sum_{r=1}^{n}(1)$

$\therefore a_0 = \frac{1}{p+1}$

$\frac{1}{8} = \frac{1}{p+1}$

$p + 1 = 8$

$\therefore p = 7$

40. Answer (8)

Hint : While applying summation on k, take i and j as constant.

Sol. : $S = 208\sum_{i=0}^{\infty}\sum_{\substack{j=0\\ i\neq j\neq k}}^{\infty}\sum_{k=0}^{\infty}\frac{1}{3^i 3^j 3^k}$

$$\frac{S}{208} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{3^i 3^j 3^k} - 3\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{9^i 3^k} + 2\sum_{i=0}^{\infty}\frac{1}{27^i}$$

$$= \left(\frac{3}{2}\right)^3 - 3\left(\frac{9}{8}\right)\left(\frac{3}{2}\right) + 2\left(\frac{27}{26}\right)$$

$$\frac{S}{208} = \frac{81}{208}$$

$S = 81 = 3^{a/2}$

$$\frac{a}{2} = 4$$

$\therefore\ a = 8$

41. Answer (2)

Hint : Make different cases and count.

Sol. : Case I : When we use 6, 7 or 8 at ten thousand place, then number of numbers

$= 3 \times {}^4P_4 = 72$

Case II : When we use 5 at ten thousand place and 6, 7 or 8 at thousand place, then number of numbers is

$= 1 \times 3 \times {}^3P_3 = 18$

Hence, required number of numbers is

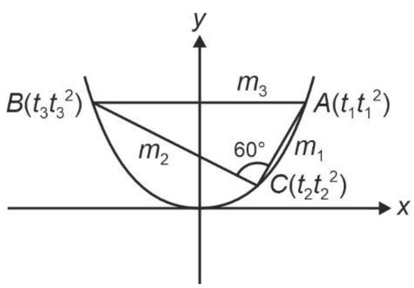
$72 + 18 = 90$

Now, $2 \times {}^{10}C_r = 90$

$\therefore\ r = 2$

42. Answer (7)

Hint :



$$m_1 = \frac{t_2^2 - t_1^2}{t_2 - t_1} = t_1 + t_2$$

Sol. : Similarly, $m_2 = t_2 + t_3$ and $m_3 = t_3 + t_1$

$$\sum t_1 = \frac{m_1 + m_2 + m_3}{2}$$

Now, $\tan 60 = \left|\frac{m-2}{1+2m}\right|$

$\Rightarrow\ \pm\sqrt{3}(1 + 2m) = m - 2$

$\sqrt{3}(1 + 2m) = m - 2$ $\quad$ $-\sqrt{3}(1 + 2m) = m - 2$

$m = \frac{(2+\sqrt{3})}{2\sqrt{3}-1}$ $\quad$ $m = \frac{2-\sqrt{3}}{2\sqrt{3}+1}$

$\therefore\ m_1 = \frac{-(2+\sqrt{3})}{2\sqrt{3}-1}$ $\quad m_2 = \frac{2-\sqrt{3}}{2\sqrt{3}+1}$ $\quad m_3 = 2$

$$\sum t_1 = \frac{3}{11}$$

43. Answer (6)

Hint : Use $\log_{a^k}(b)^m = \frac{m}{k}\log_a b$

Sol. : $5^{\log_{1/5}\frac{1}{2}} + \log_{\sqrt{2}}\frac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2}\frac{1}{10+2\sqrt{21}}$

$= 5^{\log_5 2} + 2\log_2\frac{4}{\sqrt{7}+\sqrt{3}} + \log_2(10+2\sqrt{21})$

$= 2 + 2\log_2(\sqrt{7}-\sqrt{3}) + \log_2(10+2\sqrt{21})$

$= 2 + \log_2(10-2\sqrt{21}) + \log_2(10+2\sqrt{21})$

$= 2 + \log_2(10^2 - (2\sqrt{21})^2)$

$= 2 + \log_2(100 - 84)$

$= 2 + \log_2 16$

$= 2 + 4 = 6$

44. Answer (2)

Hint : Use transformation formula to simplify and use standard limits.

Sol. :

$$\lim_{x\to 0^+}\frac{\left(\frac{1}{-\cos 5x + \cos x}\right)(5\sin 5x - 1\sin x)}{\left(\frac{1}{\sin x}\right)\cos x}$$

$$\lim_{x\to 0}\frac{\frac{1}{2\sin 3x\sin 2x}(5\sin 5x - \sin x)}{\frac{\cos x}{\sin x}}$$

$$\lim_{x\to 0}\frac{\sin x(5\sin 5x - \sin x)}{2\sin 3x\sin 2x\cos x}$$

45. Answer (A, B)

Hint : Put $z = x + iy$ and simplify.

Sol. : $|z - i\text{Re}(z)| = |z - \text{Im}(z)|$

Let $x = x + iy$

$|x + iy - ix| = |x + iy - y|$

i.e., $x^2 + (y - x)^2 = (x - y)^2 + y^2$

i.e., $x^2 = y^2$

$x = \pm y$

$\therefore \quad x = y$

$x = -y$

46. Answer (A, D)

Hint : Use parametric point as diametrically end points.

Sol. : The circles with $\left(2t_1, \frac{2}{t_1}\right)$ and $\left(2t_2, \frac{2}{t_2}\right)$ as diameter.

$$(x - 2t_1)(x - 2t_2) + \left(y - \frac{2}{t_1}\right)\left(y - \frac{2}{t_2}\right) = 0$$

Also, slope $-\frac{1}{t_1 t_2} = 1 \Rightarrow t_1 t_2 = -1$

So, equation of circle

$$\underbrace{(x^2 + y^2 - 8)}_{S} \underset{+}{-} \underset{\lambda \, L = 0}{(t_1 + t_2)(x - y)} = 0$$

Fixed point

$$\left.\begin{array}{l} x - y = 0 \\ x^2 + y^2 = 8 \end{array}\right\} \begin{array}{l} (2, 2) \\ (-2, -2) \end{array}$$

47. Answer (A, B, C)

Hint : Express $\cos 2x$ in terms of a.

Sol. : $4\sin\left(x + \frac{\pi}{3}\right)\cos\left(x - \frac{\pi}{6}\right)$

$$= a^2 + \sqrt{3}\sin 2x - \cos 2x$$

$$4\left(\frac{\sin x}{2} + \frac{\sqrt{3}}{2}\cos x\right)\left(\frac{\sqrt{3}}{2}\cos + \frac{\sin x}{2}\right)$$

$$= a^2 + \sqrt{3}\sin 2x - \cos 2x$$

$$\sqrt{3}\sin 2x + 3\cos^2 x + \sin^2 x$$

$$= a^2 + \sqrt{3}\sin 2x - \cos 2x$$

$\cos 2x + 2 = a^2 - \cos 2x$

$$\cos 2x = \frac{a^2 - 2}{2}$$

$$-1 \le \frac{a^2 - 2}{2} \le 1$$

$0 \le a^2 \le 4$

$a \in [-2, 2]$

48. Answer (A, B, D)

Hint : Try to represent as different of 2 terms.

Sol. : $T_n = \frac{n^2 + n - 1}{(n+2)!}$

$$= \frac{(n^2 + 2n) - (n+1)}{(n+2)!}$$

$$= \frac{n}{(n+1)!} - \frac{n+1}{(n+2)!}$$

$$T_{100} = \frac{100^2 + 100 - 1}{102!} = \frac{10099}{102!}$$

$$S_n = \left(\frac{1}{2!} - \frac{2}{3!}\right) + \left(\frac{2}{3!} - \frac{3}{4!}\right) + \ldots + \left(\frac{n}{(n+1)!} - \frac{n+1}{(n+2)!}\right)$$

$$\therefore \quad S_n = \frac{1}{2} - \frac{n+1}{(n+2)!}$$

$$S_{2009} = \frac{1}{2} - \frac{2010}{(2011)!}$$

$$= \frac{1}{2} - \frac{1}{2011} \cdot \frac{1}{2009!}$$

$$S_\infty = \frac{1}{2}$$

49. Answer (A, B, C)

Hint : Sum of roots = Product of roots and roots are real

Sol. :

$$\sec^2\theta + \text{cosec}^2\theta = -\frac{b}{a} = \frac{1}{\sin^2\theta\cos^2\theta} \quad \ldots\text{(i)}$$

$$\sec^2\theta + \text{cosec}^2\theta = \frac{c}{a} = \frac{1}{\cos^2\theta\sin\theta} \quad \ldots\text{(ii)}$$

$$\sin^2 2\theta = \frac{4a}{c} \le 1 \Rightarrow c \ge 4a$$

Combining equations (i) and (ii),

$b + c = 0$

50. Answer (A, C)

Hint : $3p = (4 - 1)^p = 4l + (-1)p$

$5q = (4 + 1)^q = 4m + 1$

$7r = (7 - 1)^r = 8m + (-1)r$

Sol. : Any positive power of 5 will be of form $4l + 1$.

Even power of 3 and 7 will form $4l + 1$ and odd power of 3 and 7 will be $4l - 1$.

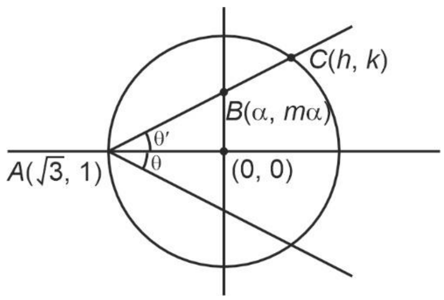
Hence, both p and r must be even.

Thus, $p + r$ is always even.

Also, $p + q + r$ can be odd or even.

51. Answer (A)

Hint :



The line can be written $y = mx$ and curve as $x^2 + y^2 = 4$.

Sol. : Let $C(h, k)$ be a point on circles and $A(\sqrt{3}, 1)$ be given point, then

$$\frac{h+2\sqrt{3}}{3} = \alpha$$

$$\Rightarrow \quad h = 3\alpha - 2\sqrt{3}$$

$$\frac{k+2}{3} = m\alpha$$

$$\Rightarrow \quad k = 3m\alpha - 2$$

Now, this point (h, k) lie on circle.

$$\Rightarrow \quad (3\alpha - 2\sqrt{3})^2 + (3m\alpha - 2)^2 = 4$$

$$= 3(1+m^2)\alpha^2 - 4\alpha(\sqrt{3}+m) + 4 = 0$$

$$D > 0$$

$$16(\sqrt{3}+m)^2 - 4\times 3(1+m^2)(4) > 0$$

$$(\sqrt{3}+m)^2 - 3(1+m^2) > 0$$

$$3 + m^2 + 2\sqrt{3}m - 3 - 3m^2 > 0$$

$$\therefore \quad 2m^2 - 2\sqrt{3}m < 0$$

$$m \in (0, \sqrt{3})$$

52. Answer (A)

Hint : Squaring and adding.

Sol. : $x^2 + y^2 = a^2 + b^2$

53. Answer (D)

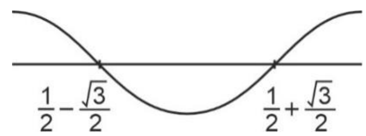
Hint : $\left(x^2 - x + \frac{1}{2}\right)^{x^2-3x+2} > 1$

Sol. :

(i) $x^2 - x + \frac{1}{2}$

$$\left(x - \frac{1}{2}\right)^2 + \frac{1}{4} = 1$$

$$\Rightarrow \quad x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$$



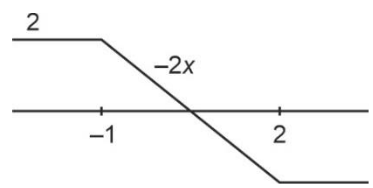
(ii) $x^2 - 3x + 2$

(iii) $|x-1| - |x+1| - \frac{1}{2}$

$\therefore$ Set X can be

$$\left(-\infty, -\frac{1}{4}\right) - \left\{-\frac{3}{4}\right\}$$

$-2x > \frac{1}{2}$	$-2x - \frac{1}{2} = 1$
$\Rightarrow \quad x < \frac{-1}{4}$	$\Rightarrow \quad x = \frac{-3}{4}$



54. Answer (A)

Hint : $\text{Im}(z) = 4$

Sol. : $z = |z| - 2 + 4i$

$$\Rightarrow \quad \text{Im}(z) = 4$$

$$z = a + 4i$$

$$\Rightarrow \quad a = \sqrt{a^2+16} - 2$$

$$\Rightarrow \quad (a+2)^2 = a^2 + 16 \Rightarrow a = 3$$

$$\therefore \quad z = 3 + 4i$$

$$\text{Amp}(z) = \tan^{-1}\left(\frac{4}{3}\right)$$

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