All India Aakash Test Series for JEE (Main)-2024

TEST - 1

Test Date : 08/10/2023

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(4)	31.	(4)	61.	(1)
2.	(4)	32.	(1)	62.	(2)
3.	(4)	33.	(2)	63.	(3)
4.	(3)	34.	(1)	64.	(3)
5.	(3)	35.	(3)	65.	(1)
6.	(1)	36.	(4)	66.	(2)
7.	(1)	37.	(1)	67.	(4)
8.	(2)	38.	(2)	68.	(1)
9.	(1)	39.	(1)	69.	(3)
10.	(1)	40.	(1)	70.	(1)
11.	(1)	41.	(2)	71.	(3)
12.	(4)	42.	(4)	72.	(1)
13.	(1)	43.	(3)	73.	(1)
14.	(3)	44.	(4)	74.	(2)
15.	(3)	45.	(3)	75.	(2)
16.	(4)	46.	(4)	76.	(3)
17.	(2)	47.	(4)	77.	(1)
18.	(1)	48.	(3)	78.	(3)
19.	(3)	49.	(4)	79.	(2)
20.	(3)	50.	(4)	80.	(3)
21.	(22.00)	51.	(03.00)	81.	(05.00)
22.	(01.50)	52.	(05.00)	82.	(01.00)
23.	(04.00)	53.	(04.00)	83.	(01.00)
24.	(02.00)	54.	(07.00)	84.	(05.00)
25.	(24.00)	55.	(01.00)	85.	(96.00)
26.	(12.00)	56.	(10.31)	86.	(04.00)
27.	(13.50)	57.	(01.00)	87.	(01.00)
28.	(06.00)	58.	(03.00)	88.	(08.00)
29.	(04.00)	59.	(30.60)	89.	(13.00)
30.	(00.25)	60.	(01.00)	90.	(00.00)

PART - A (PHYSICS)

1. Answer (4)

Hint : Bernoulli's theorem

Sol. : As cross-sectional areas of both the tubes *A* and *C* are same and tube is horizontal, hence according to equation of continuity, $v_A = v_C$ and therefore, according to Bernoulli's equation: $P + \frac{1}{2}\rho v^2 = \text{constant}$

$P_A = P_C$

i.e., height of liquid is same in both the tubes *A′* and *C′*.

2. Answer (4)

Hint : Theoretical

Sol. : Theoretical

3. Answer (4)

Hint : Bernoulli theorem

Sol. : $P_0 = P + \frac{1}{2}\rho v^2$

$4.5\times10^5 = 4\times10^5 + \frac{1}{2}(10^3)v^2 \Rightarrow v = 10\text{ m/s}$

4. Answer (3)

Hint : $u = \frac{1}{2}(\text{stress})\times(\text{strain})$

Sol. : $u = \frac{1}{2}(\text{stress})\times(\text{strain})$

5. Answer (3)

Hint : $\text{Shear strain} = \frac{\text{Shear Stress}}{\text{Shear Modulus}}$

Sol. : $\text{Shear strain} = \frac{\text{Shear Stress}}{\text{Shear Modulus}}$

$= \frac{10\times10^3\times10^2}{2\times10^{11}}$

$= 5\times10^{-6}$

6. Answer (1)

Hint : Use formula of capillary rise

Sol. : $T(2\pi r)\cos\theta = \rho gh(\pi r^2)$

$\Delta P = \rho gh = \left(\frac{2T}{r}\right)\cos\theta$

7. Answer (1)

Hint : $\omega = \sqrt{\frac{3g}{l}}$

Sol. : $\omega = \sqrt{\frac{3g}{l}}$

$(a_m)_x = \omega^2 \times \frac{l}{2}$

$= \frac{3g}{2}$

8. Answer (2)

Hint : $I_{\text{sphere}} = \frac{2}{5}\rho(\text{Vol.})R^2$

Sol. : Radius of sphere $= \frac{L}{2}$

$\frac{M'}{M} = \frac{4\pi}{3}\left(\frac{L}{2}\right)^3 \times \frac{1}{L^3} \Rightarrow M' = \frac{\pi M}{6}$

$I = \frac{2}{5}\left(\frac{\pi M}{6}\right)\left(\frac{L}{2}\right)^2 = \frac{ML^2\pi}{60}$

9. Answer (1)

Hint : *J* = Area under curve

Sol. : *J* = Area under curve

$J = \frac{1}{2}(2+4)(50) = 150$

10. Answer (1)

Hint : Apply uniformly accelerated motion equation

Sol. : $v_0 = at$, $t = \frac{v_0}{a}$

$S_Q = \frac{v_0^2}{2a}$, $S_P = v_0\left(\frac{v_0}{a}\right)$

$\frac{S_P}{S_Q} = 2$

11. Answer (1)

Hint : $F = [\text{MLT}^{-2}]$

Sol. : We know

$\therefore \text{M} = [\text{FL}^{-1}\text{T}^2]$

12. Answer (4)

Hint : $a = \sqrt{a_t^2 + a_r^2}$

Sol.: $v^2 = 2a(2\pi R) \Rightarrow \dfrac{v^2}{R} = a_r = 4\pi a$

$a = \sqrt{a_t^2 + a_r^2} = a\sqrt{1+16\pi^2}$

13. Answer (1)

Hint : $V_{avg} = \dfrac{\text{Total distance}}{\text{Total time}}$

Sol. : $\dfrac{S}{2} = \dfrac{1}{2}at^2 \Rightarrow t = \sqrt{\dfrac{S}{a}} = 10$

$v_{\text{avg}} = \dfrac{100}{20} = 5 \text{ m/s}$

14. Answer (3)

Hint : $F_{\text{net}} = mg\sin\theta - f$

Sol.: $a = \dfrac{mg\sin\theta - \mu mg\cos\theta}{m}$

$a = g(\sin\theta - \mu\cos\theta)$

15. Answer (3)

Hint : $T_1\cos 53^\circ = T_2\cos 37^\circ$

Sol. : $T_1\cos 53^\circ = T_2\cos 37^\circ$

$\dfrac{T_1}{T_2} = \dfrac{4}{3}$

16. Answer (4)

Hint : Total energy = 0

Sol. : For any particle moving around the Earth, if it is just capable to go out of the gravitational pull,

T.E. = K.E. + P.E = 0

17. Answer (2)

Hint : W-E theorem

Sol. : $W_F + W_{mg} = 0$

$\Rightarrow \quad W_F = -W_{mg} = mgR$

18. Answer (1)

Hint : K.E. $\dfrac{P^2}{2m}$

Sol. : $\vec{P}_1 + \vec{P}_2 = 0$ or $P_1 = P_2$

$\dfrac{K_P}{K_Q} = \dfrac{P^2/2m_P}{P^2/2m_Q} = \dfrac{m_Q}{m_P}$

19. Answer (3)

Hint : P.E. = $-\int \vec{F}.d\vec{r}$

Sol. : $U_r = -\int_{\infty}^{r} \vec{F}_c \cdot d\vec{r} = \int_{\infty}^{r} \dfrac{K}{r^2}dr = \dfrac{-K}{r}$

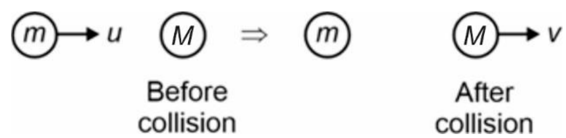
$\because \quad \dfrac{K}{r^2} = \dfrac{mv^2}{r} \Rightarrow K = \dfrac{1}{2}mv^2 = \dfrac{K}{2r}$

Total energy = $U + K = \dfrac{-K}{2r}$

20. Answer (3)

Hint : $e = \dfrac{V_{\text{sep.}}}{V_{\text{ap.}}}$

Sol. :



$\dfrac{1}{2}\left(\dfrac{1}{2}mu^2\right) = \dfrac{1}{2}Mv^2 \Rightarrow v = \dfrac{u}{2}$

$e = \dfrac{v}{u} = \dfrac{1}{2}$

21. Answer (22.00)

Hint : Error in case of Power

Sol.: $\dfrac{\Delta y}{y} \times 100\% = 4\left(\dfrac{\Delta a}{a} \times 100\right) + 2\left(\dfrac{\Delta b}{b} \times 100\right) + \dfrac{1}{3}\left(\dfrac{\Delta c}{c} \times 100\right) + \dfrac{4}{3}\left(\dfrac{\Delta d}{d} \times 100\right)$

$= 4(2) + 2(3) + \dfrac{1}{3}(4) + \dfrac{4}{3}(5) = 22\%$

22. Answer (01.50)

Hint : $P = \int F.dV$

Sol. : $P = ma \cdot v \Rightarrow P = mv\dfrac{dv}{dt}$

$\Rightarrow \quad \dfrac{v^2}{2} = \dfrac{P}{m}t \Rightarrow v = ct^{1/2} \Rightarrow ds = ct^{1/2}dt$

$\Rightarrow \quad s = c' t^{3/2}$

23. Answer (04.00)

Hint : $R = \dfrac{u^2 \sin 2\theta}{g}$

Sol. : $T = \dfrac{2u\sin\theta}{g} \Rightarrow 2T = \dfrac{2(2u)\sin\theta}{g}$

$\Rightarrow R' = (2u)^2 \dfrac{\sin 2\theta}{g} = 4R$

24. Answer (02.00)

Hint : $T = \left(\sum m.a\right)$

Sol. : $T_{PQ} = \dfrac{F}{3m} \times 2m = \dfrac{2F}{3}$

$T_{QR} = \dfrac{F}{3m} \times m = \dfrac{F}{3}$

25. Answer (24.00)

Hint : $W = \Delta$ K.E

Sol. : $E = \dfrac{1}{2}m(5)^2 - 0$

$\therefore \dfrac{1}{2}m(25)^2 - \dfrac{1}{2}m(5)^2 = \dfrac{1}{2}m(5)^2[25-1] = 24E$

26. Answer (12.00)

Hint : M.I. = $\dfrac{Mh^2}{6}$

Sol. : M.I. $= \dfrac{Mh^2}{6} = \dfrac{M(\ell\cos 45^\circ)^2}{6} = \dfrac{M\ell^2}{12}$

27. Answer (13.50)

Hint : $W = \int f dx$ = Area enclosed by force vs displacement graph

Sol. : $W = \int f dx$ = Area enclosed by force vs displacement graph

$W = 3 \times 3 + \dfrac{1}{2} \times 3 \times 3$

$= 9 + \dfrac{9}{2} = 13.5$ J

28. Answer (06.00)

Hint : $I = \dfrac{GM}{r^2}$

Sol. : $\dfrac{GM_{moon}}{(nR)^2} = \dfrac{GM_{Earth}}{(60R - nR)^2}$

$\Rightarrow \dfrac{1}{(nR)^2} = \dfrac{81}{(60R - nR)^2}$

$\Rightarrow 9n = 60 - n \Rightarrow n = 6$

29. Answer (04.00)

Hint : Breaking strength = tension in the wire = $m\omega^2 r$

Sol. : Breaking strength = tension in the wire = $m\omega^2 r$

$4.8\times10^7\times10^{-6} = 10\times0.3\times\omega^2 \Rightarrow \omega = 4$ rad/s

30. Answer (00.25)

Hint : $v = \sqrt{2gd}$ where d is the depth of water in barrel.

Sol. : $v = \sqrt{2gd}$ where d is the depth of water in barrel.

$\because \quad t = \sqrt{\dfrac{2h}{g}}$

$\therefore \quad R = vt \Rightarrow d = \dfrac{R^2}{4h}$

PART - B (CHEMISTRY)

31. Answer (4)

Hint : SO_4^{2-} is common ion

Sol. : The solution is saturated for $SrSO_4$ and $BaSO_4$

$$\frac{[Sr^{2+}][SO_4^{2-}]}{[Ba^{2+}][SO_4^{2-}]} = \frac{7.5\times10^{-7}}{1.5\times10^{-10}} = 5\times10^3 = \frac{y}{x}$$

$$BaSO_4\,(S) \rightleftharpoons \underset{x}{Ba^{2+}} + \underset{(x+y)}{SO_4^{2-}}$$

where y is the $[SO_4^{2-}]$ from $SrSO_4$

$[Ba^{2+}]$ = x can be calculated as follows.

$x(x + y) = 1.5 \times 10^{-10}$

$x^2 + xy = 1.5 \times 10^{-10}$

$x^2 + 5 \times 10^3 x^2 = 1.5 \times 10^{-10} \quad \left(\dfrac{y}{x} = 5\times10^3\right)$

$x = \left(3\times10^{-14}\right)^{\frac{1}{2}}$

$= 1.7 \times 10^{-7}$

32. Answer (1)

Hint : $\wedge_m$ of a given electrolyte depends on concentration and temperature.

Sol. : $\Lambda°_m$ of strong electrolyte can be calculated by graphical method.

33. Answer (2)

Hint : CrO_4^{2-} is common ion

Sol. : In the 0.1 M Na_2CrO_4 solution, the solubility of $PbCrO_4$ will be equal to $[Pb^{2+}]$

$$\left[Pb^{2+}\right] = \frac{K_{sp}}{\left[CrO_4^{2-}\right]} = \frac{10^{-16}\ M^2}{0.1M} = 10^{-15}\ M$$

34. Answer (1)

Hint : molecular velocity depends on temperature

Sol.: All molecular speed is directly proportional to square root of temperature.

35. Answer (3)

Hint : Graphical method for finding order

Sol.:

Graph-(i) : In[Reactant] vs time is linear
Hence, 1st order

Graph-(ii) : [Reactant] vs time is linear
Hence, zero order

36. Answer (4)

Hint : Surfactants have both lyophobic and lyophilic parts.

Sol. : The formation of micelle takes place above Kraft temperature and ΔS system is positive.

37. Answer (1)

Hint : Gram atoms of C in 11800 g of hydrocarbon will be equal to the gram atoms of C in 49980 g of urea.

Sol. : On applying P.O.A.C. for carbon.

$$\frac{11800 \times n}{12n + 2n + 2} = \frac{49980 \times 1}{60}$$

On calculation $n \simeq 12$

Hence alkane is $C_{12}H_{26}$.

38. Answer (2)

Hint : $\Delta H° - T\Delta S° \le 0$ for the reaction to be spontaneous.

Sol. : $Fe_3O_4 + 2C \rightarrow 3Fe + 2CO_2$

$\Delta H° = 320$ kJ

$\Delta S° = 360\ JK^{-1}$

$\Delta H - T\Delta S = 0$

$$T = \frac{320000}{360} = 889\ K$$

39. Answer (1)

Hint : For ideal gas PV = nRT

Sol. : At final state $nT = \frac{PV}{R} \Rightarrow$ equal for both flasks

Let n_1 moles in 300 K and n_2 moles in 400 K flask

$$\frac{n_1}{n_2} = \frac{4}{3} \ \&\ n_1 + n_2 = 0.7$$

$n_2 = 0.3$

40. Answer (1)

Hint : Higher order reactions (>3) are rare.

Sol. : Higher order greater than 3 for reaction is rare because there is low probability of simultaneous collision of all the reacting species.

41. Answer (2)

Hint : Ore Ag_2S consists of Ag_2S and impurities.

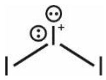
Sol. : Let us consider 100 g sample has m gram Ag_2S.

On applying POAC for Ag

$$\frac{2m}{248} = \frac{1.08 \times 1}{108},\ m = 1.24$$

Hence % of $Ag_2S = \frac{1.24 \times 100}{100} = 1.24$

42. Answer (4)

Hint : 

Sol. : I_3^- is sp^3d hybridised with one lone pairs at each equatorial position and each surrounding iodine (I) atom has three lone pairs.

I_3^+ is sp^3 hybridised with two lone pairs at central atom while the each surrounding iodine (I) atom has three lone pairs.

43. Answer (3)

Hint : The finding of black body radiation could not be explained satisfactorily on the basis of wave theory.

Sol. : At a given temperature, intensity of radiation emitted increases with the increase of wavelength and then starts decreasing with further increase of wavelengths.

44. Answer (4)

Hint : $[H^+]$ of the given HCl solution = 0.1 M

Sol. : Let x litre of water be added to 1 litre of the given HCl solution to get pH = 2 or $[H^+]$ = 10^{-2} M

$$\frac{0.1}{1+x} = 0.01 \Rightarrow x = 9 \text{ litre}$$

45. Answer (3)

Hint : First ionisation enthalpy of N is higher than that of O.

Sol. : Due to half filled subshell the electron affinity of N is lower than that of C and ionisation energy is higher than that of O. The oxygen has electron affinity lower than that of Po.

46. Answer (4)

Hint : NaCl has 6 : 6, CsCl has 8 : 8, CaF_2 has 8 : 4, Li_2O has 4 : 8 coordination number ratio.

Sol. : NaCl has FCC of either ion and second ion is present in octahedral voids.

CsCl has simple cube of either ion and other ion is present at body center.

CaF_2 has FCC of Ca^{2+} & F^- in tetrahedral voids.

Li_2O has FCC of O^{2-} & Li^+ in tetrahedral voids.

47. Answer (4)

Hint : MW. Of fluorine is less than that of chlorine.

Sol. : Each waer molecule forms 4 hydrogen bonds, while each HF molecule forms 2 Hydrogen bonds.

HN_3 has intermolecular hydrogen bond that is absent in CH_3N_3.

The intermolecular hydrogen bond of NH_3 is weaker.

48. Answer (3)

Hint : $3Fe + 2O_2 \rightarrow Fe_3O_4$ is thermochemical equation of formation of Fe_3O_4

Sol. : (i) $3Fe + 2O_2 \rightarrow Fe_3O_4 \quad \Delta H_1 = -1120$ kJ

(ii) $C + O_2 \rightarrow CO_2 \quad \Delta H_2 = -400$ kJ

(iii) $Fe_3O_4 + 2C \rightarrow 3Fe + 2CO_2$

$\Delta H_3 = 2\Delta H_2 - \Delta H_1$

Since chemical equation (iii) is obtained by
2 × equation (ii) – equation (i)
$\Delta H_3 = 2\Delta H_2 - \Delta H_1$

$= -800 + 1120$ kJ

$= 320$ kJ

49. Answer (4)

Hint : Bond order of O_2 is 2.0

Sol. : Higher is the bond order, shorter is the bond length. Bond order of O_2^{2+} is 3.0

50. Answer (4)

Hint : $Na_2S_2O_3 + I_2 \rightarrow Na_2S_4O_6 + 2NaI$

Sol. : In iodimetry, $Na_2S_2O_3$ reduces I_2 to I^- and get oxidised to $Na_2S_4O_6$.

51. Answer (03.00)

Hint : The value of n for cell reaction is 4.

Sol. : $E = E^\circ - \dfrac{0.06}{n}\log\dfrac{\left[Fe^{2+}\right]^2}{p_{O_2}\left[H^+\right]^4}$

$$\frac{0.06}{4}\log\frac{10^{-4}}{0.01\left[H^+\right]^4}=0.15$$

$$\log\frac{10^{-2}}{\left[H^+\right]^4}=10$$

$$\frac{10^{-2}}{\left[H^+\right]^4}=10^{10}$$

$[H^+] = 10^{-3}$

pH = 3

52. Answer (05.00)

Hint : The pH of 0.1 M solution of weak acid HX is 3.

Sol. : $pH=-\log\sqrt{cK_a}$

$10^{-3}=\sqrt{cK_a}$

$$K_a=\frac{10^{-6}}{0.1}=10^{-5}$$

$pK_a = 5$

53. Answer (04.00)

Hint : $KE=\frac{1}{2}mv^2$

Sol. : KE of emitted photoelectron

KE = Energy of incident photon–work function

$$\frac{KE_2}{KE_1}=\frac{4}{1}\ \text{ as }\ KE=\frac{1}{2}mv^2$$

$$KE_2=\frac{hc}{\lambda_2}-w$$

$$4KE_1=\frac{hc}{\lambda_2}-w=5-w$$

$$KE_1=\frac{hc}{\lambda_1}-w=4-w$$

$$w=\frac{11}{3}=3.66$$

Nearest integer = 4

54. Answer (07.00)

Hint : The angular node of d_{z^2} is conical node.

Sol. : For unielectronic atomic species like Li^{2+} the third shell has nine orbitals of equal energy in which *s* has zero angular node, *p* has one angular node and d has two angular nodes. The angular node of *p* and d except d_{z^2} is present in the form of planar node.

55. Answer (01.00)

Hint : Complex is $Ba_3[Co(CN)_5]_2$

$c_1i_1 = c_2i_2$

Sol. : 0.05 × i = 0.15

i = 3

= 1 + α(n – 1)

n = 5

The complex is $Ba_3[Co(CN)_5]_2$

Osmotic pressure = i c RT

= 5 × 0.01 × 0.082 × 300 atm

= 1.23 atm.

56. Answer (10.31)

Hint : $K_P=\frac{p^4_{NO_2}\ p^2_{H_2O}\ p_{O_2}}{p^4_{HNO_3}}$

Sol. : $K_P=\frac{p^4_{NO_2}\ p^2_{H_2O}\ p_{O_2}}{p^4_{HNO_3}}$

$$=\frac{\left(4p_{O_2}\right)^4\left(2p_{O_2}\right)^2\ p_{o_2}}{\left(P-7p_{O_2}\right)^4}$$

$$=\frac{1024\ p^7_{O_2}}{\left(P-7p_{O_2}\right)^4}$$

x + n = 1031

$$\frac{x+n}{100}=10.31$$

57. Answer (01.00)

Hint : $\int dS = \int \frac{dQ}{T}$

Sol. :

$$\underset{(\text{at } 273\text{ K})}{100 \text{ g water}} \xrightarrow{\Delta S_1} \underset{(\text{at } 373\text{ K})}{100 \text{ g water}} \xrightarrow{\Delta S_2} \underset{(\text{at } 373\text{ K})}{\text{Vapour}}$$

$$\Delta S_1 = 2.303 \times 100 \times 4.2 \log \frac{373}{273} = 125.74 \text{ JK}^{-1}$$

$$\Delta S_2 = \frac{100 \times 2257}{373} = 605 \text{ J K}^{-1}$$

$\Delta S_1 + \Delta S_2 = 730.74$

$\simeq 731$ J K^{-1} = 0.731 kJ K^{-1} nearest integer is 1 kJ K^{-1}

58. Answer (03.00)

Hint : $KMnO_4$ has +7 oxidation state of Mn.

Sol. : In neutral medium, H_2O_2 reduces $KMnO_4$ to MnO_2.

59. Answer (30.60)

Hint : KE = –Total mechanical energy

Sol. : KE = –Total energy

$$= \frac{13.6 \text{ eV} \times 9}{4}$$

= 30.6 eV

60. Answer (01.00)

Hint : One equivalent of H_3PO_4 consists of $\frac{1}{3}$ mol of H_3PO_4.

Sol. : Moles of H_3PO_4 in its one equivalent will be $\frac{1}{3}$.

The basicity of H_3PO_4 is 3, hence moles of H^+ obtained will be equal to $\frac{1}{3} \times 3$ and so the number of H^+ obtained will be N_A.

PART - C (MATHEMATICS)

61. Answer (1)

Hint : Rationalise $\frac{z-1}{2z+i}$

Sol.

$$\frac{x+iy-1}{2(x+iy)+1} = \frac{(x-1)+iy}{2x+i(2y+1)} \times \frac{2x-i(2y+1)}{2x-i(2y+1)}$$

$$= \frac{[(x-1)+iy] \times [2x-i(2y+1)]}{4x^2+(2y+1)^2}$$

$$= \frac{2x(x-1)+y(2y+1)+i[2xy-(2y+1)(x-1)]}{4x^2+(2y+1)^2}$$

$$= \frac{2x^2+2y^2-2x+y}{4x^2+(2y+1)^2} + \frac{i(2y-x+1)}{4x^2+(2y-1)^2}$$

$$\text{Re}\left(\frac{z-1}{2z+i}\right) = \frac{2x^2+2y^2-2x+y}{4x^2+(2y+1)^2} = 1$$

$\Rightarrow 2x^2 + 2y^2 - 2x + y = 4x^2 + (2y+1)^2$

$\Rightarrow 2x^2 + 2y^2 + 3y + 2x + 1 = 0$

$$\Rightarrow \left(x+\frac{1}{2}\right)^2 + \left(y+\frac{3}{4}\right)^2 = \left(\frac{\sqrt{5}}{4}\right)^2$$

$$\text{Centre} = \left(-\frac{1}{2}, -\frac{3}{4}\right)$$

62. Answer (2)

Hint : Algebra of complex number

Sol. : $B = -A^{-1}BA$

$\Rightarrow AB = -BA$

$\Rightarrow AB + BA = 0$

Now, $(A+B)^2 = (A+B)(A+B)$

$= A^2 + BA + AB + B^2$

$= A^2 + B^2$

63. Answer (3)

Hint : Special series

Sol. : $S = 1 + \frac{1}{2!} + \frac{1\cdot3}{4!} + \frac{1\cdot3\cdot5}{6!} + ...\infty$

$$\therefore T_n = \frac{1\cdot3\cdot5\,...\,(2n-1)}{(2n)!} \times \frac{2\cdot4\,...\,2n}{2\cdot4\,...\,2n}$$

$$= \frac{(2n)}{(2n)!\,2^n(n)!} = \frac{1}{2^n n!}$$

$$\therefore S = 1 + \sum T_n = 1 + \frac{1}{2(1)!} + \frac{1}{2^2(2!)} + ...\infty$$

$$= e^{\frac{1}{2}} = \sqrt{e}$$

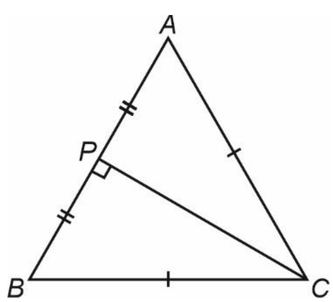
64. Answer (3)

Hint : Geometry of complex Numbers.

Sol. : Let $A = -z$, $B = iz$ and $C = z - iz$

Let $z = x + iy$

$|A - B|$, $|B - C|$ and $|C - A|$ forms an isosceles triangle with $AC = BC$



$$\text{Area} = \frac{1}{2} \times AB \times PC$$

P is the mid-point of $AB = \frac{A+B}{2} = \frac{-z+iz}{2}$

Now, $PC = \left|z - iz - \frac{(-z+iz)}{2}\right| = \left|\frac{3z-3iz}{2}\right|$

$AB = |iz - (-z)| = |z + iz|$

$$\text{Area of triangle} = \frac{1}{2} \times \frac{3|z-iz|}{2} \times |z+iz|$$

$$= \frac{3}{4} \times \left|z^2 + z^2\right| = \frac{3}{4} \times 2|z|^2 = \frac{3}{2}|z|^2$$

65. Answer (1)

Hint : Cramer's rule

Sol. : $x + ky - 2z = 0$

$2x + y - 3z = 0$

$4x + 2y - kz = 0$

For non-trivial solution

$$\begin{vmatrix} 1 & k & -2 \\ 2 & 1 & -3 \\ 4 & 2 & -k \end{vmatrix} = 0$$

$1(-k + 6) - k(-2k + 12) - 2(4 - 4) = 0$

$-k + 6 + 2k^2 - 12k = 0$

$2k^2 - 13k + 6 = 0$

$(k - 6)(2k - 1) = 0 \Rightarrow k = 6, \frac{1}{2}$

$$\Rightarrow k = 6 \quad \left\{\because \frac{1}{2} \notin z\right\}$$

66. Answer (2)

Hint : Principle of mathematical Induction.

Sol. : $P(n) = a^n + b^n$

$P(1) = a + b$, which is divisible by $a + b$

Now, Let $P(K) = a^K + b^K$ is divisible by $a + b$, where K is an odd integer

$\Rightarrow a^K + b^K = (a + b)\,f(a, b)$ …(1)

Now, $P(K + 2) = a^{K+2} + b^{K+2}$

$= a^2[(a + b)\,f(a, b) - b^K] + b^{K+2}$

$= a^2 f(a, b)(a + b) - a^2b^K + b^{K+2}$ (from (1))

$= a^2 f(a, b)(a + b) - b^K(a^2 - b^2)$

$= (a + b)[a^2 f(a, b) - b^K(a - b)]$, which is divisible by $(a + b)$

$\because$ $a^n + b^n$ is divisible by $(a + b)$ for all odd positive integral n.

67. Answer (4)

Hint : Polar form is $r(\cos\theta + i \sin\theta)$

Sol. : $(i^{25})^3 = i^{75} = i^{72 + 3} = i^{4 \times 18 + 3} = (i^4)^{18} \cdot i^3$

$= -i$

Now polar form of $(i^{25})^3 = r(\cos\theta + i \sin\theta)$

$$= 1\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$$

$$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$$

68. Answer (1)

Hint : Proportion of modulus of complex number.

Sol. : $\left|\dfrac{z_1 - 3z_2}{3 - z_1\bar{z}_2}\right| = 1$

$$|z_1 - 3z_2| = |3 - z_1\bar{z}_2|$$

Squaring,

$$\Rightarrow |z_1 - 3z_2|^2 = |3 - z_1\bar{z}_2|^2$$

$$\Rightarrow (z_1 - 3z_2)(\bar{z}_1 - 3\bar{z}_2) = (3 - z_1\bar{z}_2)(3 - \bar{z}_1 z_2)$$

$$\Rightarrow |z_1|^2 - 3z_1\bar{z}_2 - 3z_2\bar{z}_1 + 9|z_2|^2$$

$$= 9 - 3\bar{z}_1 z_2 - 3z_1\bar{z}_2 + |z_1|^2|z_2|^2$$

$$\Rightarrow |z_1|^2 - |z_1|^2|z_2|^2 - 9 + 9|z_2|^2 = 0$$

$$\Rightarrow |z_1|^2\left(1 - |z_2|^2\right) - 9\left(1 - |z_2|^2\right) = 0$$

$$\Rightarrow \left(1 - |z_2|^2\right)\left(|z_1|^2 - 9\right) = 0$$

$$\Rightarrow |z_2| = 1$$

69. Answer (3)

Hint : Product of Matrix

Sol. : $C^{-1}(AB^{-1})^{-1}(CA^{-1})^{-1}C^2$

$= C^{-1}(B^{-1})^{-1}A^{-1}(A^{-1})^{-1}C^{-1}C^2$

$= C^{-1}BA^{-1}AC^{-1}C^2$

$= C^{-1}BIC$

$= C^{-1}BC$

70. Answer (1)

Hint : Euler form is $z = re^{i\theta}$

Sol. : $\dfrac{2 + 6\sqrt{3}i}{5 + \sqrt{3}i}$

$$= \frac{\left(2 + 6\sqrt{3}i\right)\left(5 - \sqrt{3}i\right)}{\left(5 + \sqrt{3}i\right)\left(5 - \sqrt{3}i\right)}$$

$$= \frac{10 - 2\sqrt{3}i + 30\sqrt{3}i - 6 \times 3 \times i^2}{25 - 3i^2}$$

$$= \frac{10 + 28\sqrt{3}i + 18}{25 + 3} = \frac{28\left(1 + \sqrt{3}i\right)}{28} = 1 + \sqrt{3}i$$

$$r = |z| = \sqrt{(1)^2 + \left(\sqrt{3}\right)^2} = 2$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

Euler form $= 2e^{\frac{i\pi}{3}}$

71. Answer (3)

Hint : Combination

Sol. : Let x be the number of apples being selected

y be the number of mangoes being selected

z be the number of bananas being selected

Then, $x = 0, 1, 2, 3, 4, 5$

$y = 0, 1, 2, 3, 4$

$z = 0, 1, 2, 3$

Total number of triplets (x, y, z) is $6 \times 5 \times 4 = 120$

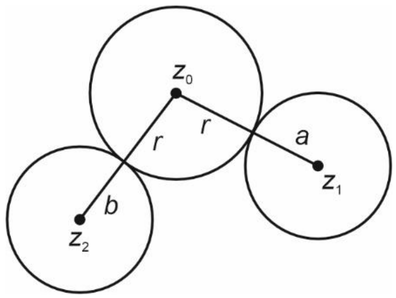
Exclude (0, 0, 0)

$\therefore$ Number of combinations = 120 – 1 = 119

72. Answer (1)

Hint : Variable circle is $|z - z_0| = r$

Sol. : Let the variable circle be $|z - z_0| = r$



Then, $|z_0 - z_1| = a + r$ and $|z_0 - z_2| = b + r$

Eliminating r, we get

$|z_0 - z_1| - |z_0 - z_2| = a - b$

$\because$ Locus is hyperbola.

73. Answer (1)

Hint : Calculate sum of square of terms in AP.

Sol. : $a_K + a_{K-2} = 2a_{K-1}$

Thus the terms are in A.P.

$\because$ Sum of square of the terms in A.P. is

$a^2 + (a + d)^2 + \ldots (a + 10d)^2$

$\Rightarrow 11a^2 + 110ad + 385d^2 = 900$

$\Rightarrow a^2 + 10ad + 35d^2 = 90$

$\Rightarrow 35d^2 + 150d + 225 - 90 = 0$

$7d^2 + 30d + 27 = 0$

$\Rightarrow (7d + 9)(d + 3) = 0$

$d = -3, \dfrac{-9}{7}$

$\because a_2 < 13.5, d = -3$

Thus, the average of 11 terms of an A.P.

$= a_6 = 15 + (6 - 1)(-3) = 0$

74. Answer (2)

Hint : Geometry of complex number.

Sol. $\because |CA| = |CB|$ and $\angle ACB = 90°$

$\therefore (z_2 - z_3) = \pm i(z_1 - z_3)$

$\Rightarrow (z_2 - z_3)^2 = -(z_1 - z_3)^2$

$\Rightarrow z_2^2 + z_3^2 - 2z_1z_2 = -z_1^2 - z_3^2 - 2z_1z_2$

$\Rightarrow z_1^2 + z_2^2 - 2z_1z_2 = 2(z_1z_2 - z_1z_3 - z_2z_3 + z_2^2)$

$\Rightarrow (z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$

$\therefore K = 2$

75. Answer (2)

Hint : Principle of Mathematical Induction.

Sol. : For $n = 1$, we have

$49n + 16n + \lambda = 49 + 16 + \lambda = 65 + \lambda = 64 + (\lambda + 1)$

which is divisible by 64 if $\lambda = -1$

For $n = 2$

$49n + 16n + \lambda = 49^2 + 2(16) + \lambda = 2433 + \lambda = (64 \times 38) + (\lambda + 1)$

which is divisible by 64 if $\lambda = -1$

$\because \lambda = -1$

76. Answer (3)

Hint : Use cosine rule $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Sol. : $\dfrac{1}{a+c} + \dfrac{1}{b+c} = \dfrac{3}{a+b+c}$

$\Rightarrow \dfrac{b+c+a+c}{(a+c)(b+c)} = \dfrac{3}{a+b+c}$

$\Rightarrow \dfrac{a+b+2c}{(a+c)(b+c)} = \dfrac{3}{a+b+c}$

$\Rightarrow a^2 + ab + ac + ba + b^2 + 1bc + 2ac + 2cb + 2c^2$

$= 3ab + 3ac + 3bc + 3c^2$

$\Rightarrow a^2 + b^2 - c^2 + 2ab + 3ca + 3cb$

$= 3ab + 3ac + 3bc$

$\Rightarrow a^2 + b^2 - c^2 = ab$

$\Rightarrow \dfrac{a^2 + b^2 - c^2}{ab} = 1$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab}=\frac{1}{2}$$

$$\Rightarrow \cos C=\frac{1}{2}$$

$\Rightarrow C = 60°$

77. Answer (1)

Hint : Number divisible by 3 when sum of digits is multiple of 3.

Sol. : $\because$ A five digit number is formed by using digits 0, 1, 2, 3, 4 & 5 divisible by 3 i.e., only possible when sum of digits is multiple of 3 which gives two cases.

Case I : Using digits 0, 1, 2, 4, 5 the number of ways = 4 × 4 × 3 × 2 × 1 = 96

Case II : Using digits 1, 2, 3, 4, 5 the number of ways = 5 × 4 × 3 × 2 × 1 = 120

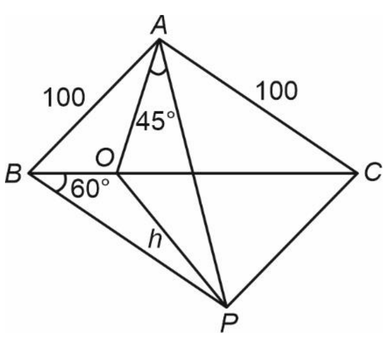
$\therefore$ Total number formed = 120 + 96

= 216

78. Answer (3)

Hint : Height and distance.

Sol. :



Tower $OP = h$

In ΔAOP

$$\tan 45° = \frac{OP}{OA}$$

$$OA = \frac{OP}{\tan 45°}$$

$OA = OP = h$

In ΔBOP

$$\tan 60° = \frac{OP}{OB}$$

$$OB = \frac{h}{\sqrt{3}}$$

In ΔABC, $AB = AC$

$AO \perp BC$

$(AO)^2 + (BO)^2 = (AB)^2$

$$h^2+\frac{h^2}{3}=10000$$

$$\frac{4h^2}{3}=10000 \Rightarrow h=50\sqrt{3}\text{ m}$$

79. Answer (2)

Hint : Expand the matrix

Sol. : $1(40-40)-3(20-24)$

$+(2\lambda+2)(10-12)=0$

$\Rightarrow 12+(\lambda+1)(-4)=0$

$\lambda = 3 - 1 = 2$

80. Answer (3)

Hint : Write general term

Sol. : $T_{r+1} = {}^{256}C_r\left(\sqrt{3}\right)^{256-r}\left(\sqrt[8]{5}\right)^r$

$\Rightarrow r$ is multiple of 8.

$\Rightarrow r = 0, 8, 16\ldots$

$\Rightarrow$ 33 terms

81. Answer (05.00)

Hint : If α & β are roots quadratic equation then

$$\alpha+\beta=\frac{-b}{a}$$

$$\alpha\beta=\frac{c}{a}$$

Sol. : Let the roots be α and 2α

$$\alpha+2\alpha=\frac{-(3a-1)}{a^2-5a+3}$$

$$3\alpha = \frac{-(3a-1)}{a^2-5a+3}$$

$$\alpha = \frac{-(3a-1)}{3(a^2-5a+3)} \qquad ...(1)$$

$$\alpha(2\alpha) = \frac{2}{a^2-5a+3}$$

$$\alpha^2 = \frac{1}{a^2-5a+3}$$

$$\Rightarrow \left[\frac{-(3a-1)}{3(a^2-5a+3)}\right]^2 = \frac{1}{a^2-5a+3}$$

$$\Rightarrow \frac{(3a-1)^2}{9(a^2-5a+3)^2} = \frac{1}{a^2-5a+3}$$

$$\Rightarrow 9a^2+1-6a = 9(a^2-5a+3)$$

$$\Rightarrow 9a^2+1-6a = 9a^2-45a+27$$

$$\Rightarrow 39a = 26$$

$$\Rightarrow a = \frac{2}{3}$$

$\lambda + \mu = 2 + 3 = 5$

82. Answer (01.00)

Hint : Concept of concurrent lines.

Sol. : $x + ay + a = 0$

$$a\left(\frac{x}{a}+y+1\right) = 0 \qquad ...(i)$$

$bx + y + b = 0$

$$x+\frac{y}{b}+1 = 0 \qquad ...(ii)$$

$cx + cy + 1 = 0$

$$x+y+\frac{1}{c} = 0 \qquad ...(iii)$$

Subtracting (i) from (iii) we get,

$$x-\frac{x}{a}+\frac{1}{c}-1 = 0$$

$$\Rightarrow x = \frac{c-1}{a-1}\cdot\frac{a}{c}$$

Subtracting (ii) from (iii), we get

$$\Rightarrow y = \frac{c-1}{b-1}\cdot\frac{b}{c}$$

Substituting values of x & y in (iii)

$$\frac{a}{a-1}+\frac{b}{b-1}+\frac{c}{c-1} = 1$$

83. Answer (01.00)

Hint : Expansion of Determinant.

Sol. : $x^4 + y^4 + z^4 = 0$

Since $x, y, z \in R$

$\therefore\ x = y = z = 0$

$$\begin{vmatrix} 1 & xy & yz \\ zx & 1 & xy \\ yx & zx & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

84. Answer (05.00)

Hint : $t_n = ar^{n-1}$

Sol. : Let the first three terms of G.P. be $\frac{a}{r}$,

a, ar

$$\left(\frac{a}{r}\right)(a)(ar) = 1000$$

$a^3 = 1000$

$a = 10$

Now, $T_4 + T_3 = 60$

$ar + ar^2 = 60$

$r^2 + r - 6 = 0$

$\Rightarrow (r + 3)(r - 2) = 0$

$\Rightarrow r = 2$

$T_7 = ar^5$

$= 10(2)^5 = 320$

85. Answer (96.00)

Hint : Permutation

Sol. : Total ways in which MEDICAL letters can be arranged if AE are taken as 1 unit is $6! \times 2! = 1440$

Now, out of these words in which *AEI* comes and *AE* together are 5! × 4 = 480

Total ways = 1440 – 480 = 960

$\Rightarrow$ 960 ways

86. Answer (04.00)

Hint : $\tan x = 1 \Rightarrow x = nx + \dfrac{\pi}{4}$

Sol. : $\tan x = 1$

$\Rightarrow x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \dfrac{13\pi}{4}$

87. Answer (01.00)

Hint : : Let α be the common root

$$\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

Sol. Here, $a_1 = 1$, $b_1 = b$, $c_1 = -a$

$a_2 = 1$, $b_2 = -a$, $c_2 = b$

Now, $\dfrac{\alpha^2}{b^2 - a^2} = \dfrac{\alpha}{-a-b} = \dfrac{1}{-a-b}$

$\alpha^2 = \dfrac{b^2 - a^2}{-a-b}$ ...(i)

& $\alpha = 1$ …(ii)

$1 = \dfrac{b^2 - a^2}{-a-b}$

$a^2 - b^2 = a + b$

$(a - b)(a + b) = (a + b)$

$a - b = 1$

88. Answer (08.00)

Hint : $y = a \sin x \pm b \cos x$

$y_{max} = \sqrt{a^2 + b^2}$

$y_{min} = -\sqrt{a^2 + b^2}$

Sol. : $|7\cos x + 5\sin x| \le \sqrt{7^2 + 5^2}$

$\Rightarrow -\sqrt{7^2 + 5^2} \le (7\cos x + 5\sin x) \le \sqrt{7^2 + 5^2}$

$\Rightarrow -8.6 \le 2K + 1 \le 8.6$

$\Rightarrow -4.8 \le K \le 3.8$

Integral values of K are –4, –3, –2, –1, 0, 1, 2, 3

Total 8 values

89. Answer (13.00)

Hint : Number of diagonal of a polygon of n sides = $\dfrac{n(n-3)}{2}$

Sol. : If a polygon has n vertices then it will have n sides and for every vertices we can draw n – 3 diagonals. So, total number of diagonals should be $n(n - 3)$ but his will mean that we have counted a diagonal twice.

So total number of diagonals should be

$\dfrac{n(n-3)}{2}$

$\because \dfrac{n(n-3)}{2} = 65$

$\therefore n = 13$

90. Answer (00.00)

Hint : Properties of modulus of complex number.

Sol. : $\dfrac{3}{|z_2 - z_3|} = \dfrac{4}{|z_3 - z_1|} = \dfrac{5}{|z_1 - z_2|} = K$

$$\frac{9}{|z_2 - z_3|^2} = \frac{16}{|z_3 - z_1|^2} = \frac{25}{|z_1 - z_2|^2} = K^2$$

$$\frac{9}{z_2 - z_3} = K^2(\bar{z}_2 - \bar{z}_3)$$

$\dfrac{16}{z_3 - z_1} = K^2(\bar{z}_3 - \bar{z}_1)$ & $\dfrac{25}{z_1 - z_2} = K^2(\bar{z}_1 - \bar{z}_2)$

So, $\dfrac{9}{z_2 - z_3} + \dfrac{16}{z_3 - z_1} + \dfrac{25}{z_1 - z_2}$

$= K^2(\bar{z}_2 - \bar{z}_3 - \bar{z}_3 - \bar{z}_1 + \bar{z}_1 - \bar{z}_2) = 0$

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