01/02/2024
Morning

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2024 (Online) Phase-1

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.

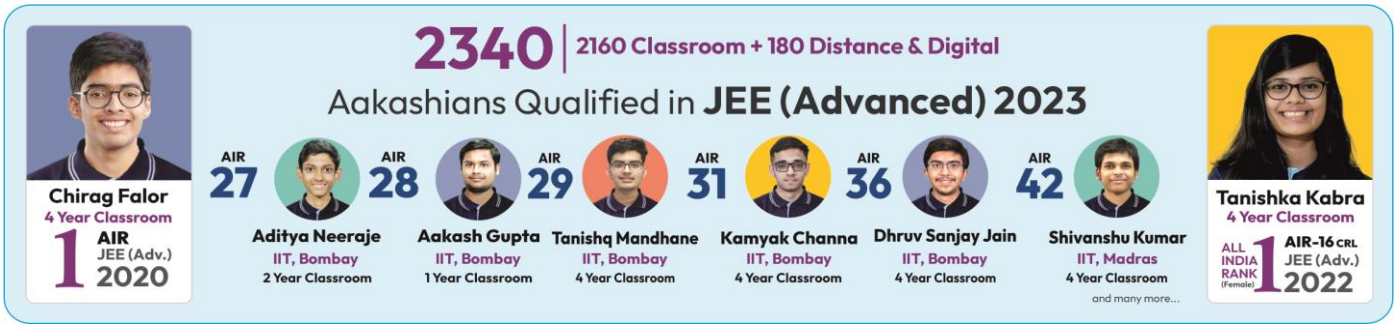
(3) This question paper contains **Three Parts**. **Part-A** is Mathematics, **Part-B** is Physics and **Part-C** is Chemistry. Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt any 05 questions out of 10 Questions.

(6) **Section - A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section - B (21 – 30) contains** 10 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.



MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let $S = \{z \in C : |z - 1| = 1$ and $(\sqrt{2}-1)(z+\bar{z}) - i(z-\bar{z}) = 2\sqrt{2}\}$. Let $z_1, z_2 \in \mathbf{S}$ be such that $|z_1| = \max_{z \in S} |z|$ and $|z_2| = \min_{z \in S} |z|$. Then $|\sqrt{2}z_1 - z_2|^2$ equals :

(1) 2 (2) 4

(3) 3 (4) 1

Answer (1)

Sol. Let $z = x + iy$

$|z - 1| = 1 \Rightarrow |x + iy - 1| = 1$

$(x-1)^2 + y^2 = 1$ …(1)

$(\sqrt{2}-1)(z+\bar{z}) - i(z-\bar{z}) = 2\sqrt{2}$ (Given)

$(\sqrt{2}-1)(2x) - i(2iy) = 2\sqrt{2}$

$(\sqrt{2}-1)x + y = \sqrt{2}$ …(2)

Solving (1) and (2), we get

$(x-1)^2 + \left(\sqrt{2} - (\sqrt{2}-1)x\right)^2 = 1$

$(x^2 - 2x + 1) + 2 + (\sqrt{2}-1)^2 x^2 - 2\sqrt{2}(\sqrt{2}-1)x = 0$

$x^2\left(1 + (\sqrt{2}-1)^2\right) + x\left(-2 - 2\sqrt{2}(\sqrt{2}-1)\right) + 2 = 0$

$x^2(4 - 2\sqrt{2}) + x(2\sqrt{2} - 6) + 2 = 0$

$x^2(2 - \sqrt{2}) + x(\sqrt{2} - 3) + 1 = 0$

$\Rightarrow x = 1$ and $x = \frac{1}{2-\sqrt{2}}$ …(3)

When $x = 1$, $y = 1 \Rightarrow z_2 = 1 + i$

When $x = \frac{1}{2-\sqrt{2}}$, $y = \sqrt{2} - \frac{1}{\sqrt{2}}$

$\Rightarrow z_1 = \left(1 + \frac{1}{\sqrt{2}}\right) + \frac{i}{\sqrt{2}}$

Now,

$|\sqrt{2}\, z_1 - z_2|^2$

$= \left|\left(\frac{1}{\sqrt{2}} + 1\right)\sqrt{2} + i - (1+i)\right|^2$

$= \left(\sqrt{2}\right)^2$

$= 2$

2. Let $S = \{x \in \mathbf{R} : (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10\}$. Then the number of elements in S is :

(1) 2 (2) 1

(3) 4 (4) 0

Answer (1)

Sol. $\left(\sqrt{3}+\sqrt{2}\right)^x = t$

$t + \frac{1}{t} = 10$

$\Rightarrow t^2 - 10t + 1 = 0$

$\Rightarrow t = \frac{10 \pm \sqrt{96}}{2}$

$\Rightarrow t = 5 \pm 2\sqrt{6}$

$\Rightarrow \left(\sqrt{3}+\sqrt{2}\right)^x = 5 \pm 2\sqrt{6}$

$\Rightarrow x = \pm 2$

3. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a > b$ be an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latusrectum is $\sqrt{14}$. Then the square of the eccentricity of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is :

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$

(3) 3 (4) $\frac{7}{2}$

Answer (1)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $a > b$

$e=\frac{1}{\sqrt{2}}$

$\Rightarrow 1-\frac{b^2}{a^2}=\frac{1}{2}$

$\Rightarrow \frac{b^2}{a^2}=\frac{1}{2}$

Also, $\frac{2b^2}{a}=\sqrt{14}$

$\Rightarrow \frac{2b^2}{a^2}\times a=\sqrt{14}$

$\Rightarrow 2\times\frac{1}{2}\times a=\sqrt{14}$

$\Rightarrow a=\sqrt{14}$

$\Rightarrow b^2 = 7$

Now, $\frac{x^2}{14}-\frac{y^2}{7}=1$

$e_1^2=1+\frac{b^2}{a^2}$

$e_1^2=1+\frac{7}{14}$

$\boxed{e_1^2=\frac{3}{2}}$

4. For $0 < \theta < \pi/2$, if the eccentricity of the hyperbola $x^2 - y^2\text{cosec}^2\theta = 5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^2\text{cosec}^2\theta + y^2 = 5$, then the value of θ is :

(1) $\frac{\pi}{4}$ (2) $\frac{5\pi}{12}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

Answer (3)

Sol. $x^2-y^2\text{cosec}^2\theta=5$

$\Rightarrow \frac{x^2}{5}-\frac{y^2}{5\sin^2\theta}=1$

$e_H=\sqrt{1+\sin^2\theta}$

and $x^2\text{cosec}^2\theta + y^2 = 5$

$\Rightarrow \frac{x^2}{5\sin^2\theta}+\frac{y^2}{5}=1$

$e_E=\sqrt{1-\sin^2\theta}=\cos\theta$

$e_H=\sqrt{7}e_E$

$\sqrt{1+\sin^2\theta}=\sqrt{7}\cos\theta$

$\Rightarrow 1 + \sin^2\theta = 7\cos^2\theta$

$\Rightarrow 1 + \sin^2\theta = 7 - 7\sin^2\theta$

$\Rightarrow 8\sin^2\theta = 6$

$\Rightarrow \sin\theta=\frac{\sqrt{3}}{2}$

$\Rightarrow \theta=\frac{\pi}{3}$ $\left[\because 0<\theta<\frac{\pi}{2}\right]$

So, option (3) is correct

5. Let $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ be defined as $f(x) = \begin{cases} \log_e x, & x > 0 \\ e^{-x}, & x \le 0 \end{cases}$ and $g(x) = \begin{cases} x, & x \ge 0 \\ e^x, & x < 0 \end{cases}$. Then, $gof : \mathbf{R} \to \mathbf{R}$ is :

(1) Neither one-one nor onto

(2) Onto but not one-one

(3) Both one-one and onto

(4) One-one but not onto

Answer (1)

Sol. Given $f(x) = \begin{cases} e^{-x} & ; \quad x \le 0 \\ \ln x & ; \quad x > 0 \end{cases}$

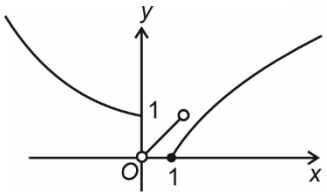
and $g(x) = \begin{cases} x & ; \quad x \ge 0 \\ e^x & ; \quad x < 0 \end{cases}$

then $gof(x)$

$= g(f(x))$

$= \begin{cases} f(x) & ; \quad f(x) \ge 0 \\ e^{f(x)} & ; \quad f(x) < 0 \end{cases}$

$= \begin{cases} e^{-x} & ; \quad x \le 0 \\ x & ; \quad 0 < x < 1 \\ \ln x & ; \quad x \ge 1 \end{cases}$



$gof(x)$

$\therefore$ $f(x)$ is neither one-one nor onto

6. Let the median and the mean deviation about the median of 7 observation 170, 125, 230, 190, 210, a, b be 170 and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is :

(1) 31 (2) 32

(3) 28 (4) 30

Answer (4)

Sol. Mean deviation about median

$$= \frac{0+45+60+20+40+170-a+170-b}{7} = \frac{205}{7}$$

$\Rightarrow$ $a + b = 300$

$$\text{Mean} = \frac{170+125+230+190+210+a+b}{7} = 175$$

Mean deviation about mean

$$= \frac{50+175-a+175-b+5+15+35+55}{7}$$

$= 30$

7. If $\tan A = \frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C = (x^{-3} + x^{-2} + x^{-1})^{1/2}$, $0 < A, B, C < \frac{\pi}{2}$, then $A + B$ is equal to

(1) $2\pi - C$

(2) C

(3) $\frac{\pi}{2} - C$

(4) $\pi - C$

Answer (2)

Sol. $\tan A = \frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$

and $\tan C = \left(x^{-3}+x^{-2}+x^{-1}\right)^{\frac{1}{2}}$

$A = \tan^{-1}\left(\frac{1}{\sqrt{x(x^2+x+1)}}\right)$ and

$B = \tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2+x+1}}\right)$

$$A+B=\tan^{-1}\left(\frac{\frac{1+x}{\sqrt{x(x^2+x+1)}}}{\left(1-\frac{1}{x^2+x+1}\right)}\right)$$

$$=\tan^{-1}\left(\frac{\frac{1+x}{\sqrt{x(x^2+x+1)}}}{\frac{x^2+x}{\left(x^2+x+1\right)}}\right)$$

$$=\tan^{-1}\left(\frac{1}{\sqrt{x(x^2+x+1)}}\times\frac{x^2+x+1}{x}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{x^2+x+1}}{x^{3/2}}\right)$$

$$=\tan^{-1}C$$

$\therefore$ $A + B = C$

8. If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:

(1) 43 (2) 53

(3) 47 (4) 51

Answer (4)

Sol. Since rooms are identical so we can distribute in following way

	(1)	(2)	(3)	(4)
1 way = 1	0	0	0	5
$\frac{5!}{4!1!}$ ways = 5	0	0	1	4
$\frac{5!}{2!3!}$ ways = 10	0	0	2	3
$\frac{5!}{3!1!1!}\times\frac{1}{2!}=10$	0	0	1	3
$\frac{5!}{1!2!2!2!}=15$	0	1	2	2
$\frac{5!}{1!1!!2!}\times\frac{1}{3!}=10$	1	1	1	2

$\therefore$ Total ways = 1 + 5 + 10 + 10 + 15 + 10 = 51

9. If the system of equations

$2x + 3y - z = 5$

$x + \alpha y + 3z = -4$

$3x - y + \beta z = 7$

has infinitely many solutions, then $13\alpha\beta$ is equal to _____ .

(1) 1220 (2) 1110

(3) 1120 (4) 1210

Answer (3)

Sol. Given $2x + 3y - z = 5$

$x + \alpha y + 3z = -4$

$3x - y + \beta z = 7$

$$\Delta_2=\begin{vmatrix}2 & -1 & 5\\ 1 & 3 & -4\\ 3 & \beta & 7\end{vmatrix}$$

$\Delta_2 = 2(21 + 4\beta) + 1(7 + 12) + 5(\beta - 9)$

$\Delta_2 = 42 + 8\beta + 19 + 5\beta - 45$

$\Delta_2 = 13\beta + 16$

$\Delta_2 = 0$

$\therefore$ $\boxed{\beta=-\frac{16}{13}}$

$$\Delta_3=\begin{vmatrix}2 & 3 & 5\\ 1 & \alpha & -4\\ 3 & -1 & 7\end{vmatrix}$$

$\Delta_3 = 2(7\alpha - 4) - 3(7 + 12) + 5(-1 - 3\alpha)$

$\Delta_3 = 14\alpha - 8 - 57 - 5 - 15\alpha$

$\Delta_3 = -\alpha - 70$

$\Delta_3 = 0$

$\boxed{\alpha = -70}$

$13\alpha\beta = (13)(-70)\left(-\frac{16}{13}\right)$

$= +1120$

10. A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is :

(1) $\frac{2}{5}$ (2) $\frac{1}{7}$

(3) $\frac{1}{5}$ (4) $\frac{2}{7}$

Answer (4)

Sol. $P(2W \text{ and } 2B) = P(2B, 6W) \times P(2W \text{ and } 2B) + P(3B, 5W) \times P(2W \text{ and } 2B) + P(4B, 4W) \times P(2W \text{ and } 2B) + P(5B, 3W) \times P(2W \text{ and } 2B) + P(6B, 2W) \times P(2W \text{ and } 2B)$

$$\frac{1}{9}\left(0+0+\frac{{}^2C_2 \times {}^6C_2}{{}^8C_4}+\frac{{}^3C_2 \times {}^5C_2}{{}^8C_4}+\frac{{}^4C_2 \times {}^4C_2}{{}^8C_2}+\frac{{}^5C_2 \times {}^3C_2}{{}^8C_4}+\frac{{}^8C_2 \times {}^2C_2}{{}^8C_4}+0+0\right)$$

$$=\frac{1}{9}\times\frac{1}{{}^8C_4}(15+30+36+30+15)$$

$$=\frac{1}{9}\times\frac{1}{{}^8C_4}\times 126$$

$$P\left(\frac{4B \text{ and } 4W}{3W \text{ and } 2B}\right)=\frac{\frac{1}{9}\times\frac{{}^4C_2 \times {}^4C_2}{{}^8C_4}}{\frac{1}{9}\times\frac{1}{{}^8C_4}\times 126}$$

$$=\frac{36}{126}$$

$$=\frac{6}{21}$$

$$\frac{2}{7}$$

11. The value of the intergral $\int_0^{\pi/4}\frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$ equals :

(1) $\frac{\sqrt{2}\pi^2}{64}$ (2) $\frac{\sqrt{2}\pi^2}{8}$

(3) $\frac{\sqrt{2}\pi^2}{32}$ (4) $\frac{\sqrt{2}\pi^2}{16}$

Answer (3)

Sol. $\int_0^{\pi/4}\frac{xdx}{\sin^4(2x)+\cos^4(2x)}$

Take $I = \int_0^{\pi/4}\frac{xdx}{\sin^4(2x)+\cos^4(2x)}$

Let $2x = t$

$2dx = dt$

$dx = \frac{dt}{2}$

$$I = \int_0^{\pi/2}\frac{t/2 \cdot 1/2\,dt}{\sin^4 t+\cos^4 t}$$

$$I = \frac{1}{4}\int_0^{\pi/2}\frac{tdt}{\sin^4 t+\cos^4 t}$$

$$=\frac{1}{4}\int_0^{\pi/2}\frac{\left(\frac{\pi}{2}-t\right)dt}{\sin^4(\pi/2-t)+\cos^4(\pi/2-t)}$$

$$=\frac{1}{4}\int_0^{\pi/2}\frac{\left(\frac{\pi}{2}-t\right)}{\sin^4 t+\cos^4 t}$$

$$2I = \frac{1}{4}\int_0^{\pi/2} \frac{\frac{\pi}{2}}{\sin^4 t + \cos^4 t} dt$$

$$2I = \frac{\pi}{8}\int_0^{\pi/2} \frac{dt}{\sin^4 t + \cos^4 t}$$

$$2I = \frac{\pi}{8}\int_0^{\pi/2} \frac{\sec^4 t}{1+\tan^4 t} dt$$

Put $\tan t = y$

$\sec^2 t dt = dy$

$$2I = \frac{\pi}{8}\int_0^{\infty} \frac{(1+y^2)dy}{1+y^4}$$

$$= \frac{\pi}{8}\int_0^{\infty} \frac{1+\frac{1}{y^2}}{y^2+\frac{1}{y^2}-2+2} dy$$

$$\frac{\pi}{8}\int_0^{\infty} \frac{\left(1+\frac{1}{y^2}\right)dy}{2+\left(y-\frac{1}{y}\right)^2}$$

Put, $y - \frac{1}{y} = 4$

$$2I = \frac{\pi}{8}\int_{-\infty}^{\infty} \frac{du}{2+u^2}$$

$$= \frac{\pi}{8\sqrt{2}}\left[\tan^{-1}\frac{y}{\sqrt{2}}\right]_{-\infty}^{\infty}$$

$$= \frac{\sqrt{2}\pi^2}{32}$$

12. Let 3, a, b, c be in A.P. and 3, $a-1$, $b+1$, $c+9$ be in G.P. Then, the arithmetic mean of a, b and c is :

(1) –4 (2) –1

(3) 11 (4) 13

Answer (3)

Sol. 3, a, b, c be in A.P.

$a - 3 = b - a = c - b$

3, $a-1$, $b+1$, $c+9$ in G.P.

$$\frac{a-1}{3} = \frac{b+1}{a-1} = \frac{c+9}{b+1} \quad (a \neq 1, b = -1)$$

$(a-1)^2 = 3(b+1)$

$a^2 + 1 - 2a = 3b + 3$

$a^2 - 8a + 7 = 0$

$(a-1)(a-7) = 0$

$a = 7$ as $a \neq 1$

$b = 2a - 3$

$= 14 - 3$

$= 11$

$c = 2b - a = 22 - 7 = 15$

Mean of a, b, c

$$\frac{a+b+c}{3} = \frac{7+11+15}{3} = \frac{33}{3} = 11$$

13. If $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2, \forall x \neq 0$ and $y = 9x^2 f(x)$, then y is strictly increasing in :

(1) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

(2) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

(3) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(4) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

Answer (4)

Sol. $5f(x) + 4f(1/x) = x^2 - 2$ …(1)

Replace x by $1/x$

$$5f(1/x) + 4f(x) = \frac{1}{x^2} - 2 \quad \ldots(2)$$

Multiply equation (1) by 5 and multiply equation (2) by 4 and then subtract equation (2) from (1)

$$25f(x) - 16f(x) = 5x^2 - 10 - \frac{4}{x^2} + 8$$

$$9f(x) = 5x^2 - \frac{4}{x^2} - 2$$

$$9f(x) = \frac{5x^4 - 4 - 2x^2}{x^2}$$

$y = 9x^2 f(x)$

$y = 5x^4 - 2x^2 - 4$

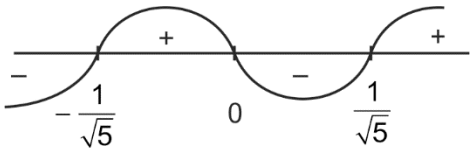
$y' = 20x^3 - 4x$

Put $y' > 0$

$20x^3 - 4x > 0$

$5x^3 - x > 0$

$x(5x^2 - 1) > 0$



$$x \in \left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$$

14. Let $\vec{a} = -5\hat{i} + \hat{j} - 3k, \vec{b} = \hat{i} + 2\hat{j} - 4k$ and $\vec{c} = (((\vec{a} \times \vec{b}) \times \hat{i}) \times \hat{i}) \times \hat{i}$. Then $\vec{c} \cdot (-\hat{i} + \hat{j} + k)$ is equal to :

(1) –10 (2) –15

(3) –12 (4) –13

Answer (3)

Sol. $\vec{a} = -5 \cdot \hat{i} + \hat{j} - 3\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{c} = (((\vec{a} \times \vec{b}) \times \hat{i}) \times \hat{i}) \times \hat{i}$

$= (((\vec{a} \cdot \hat{i})\vec{b} - (\vec{b} \cdot \hat{i})\vec{a}) \times \hat{i}) \times \hat{i}$

$= ((-5\vec{b} - \vec{a}) \times \hat{i}) \times \hat{i}$

$= (\vec{u} \times \hat{i}) \times \hat{i}$

$\{\vec{u} = -(5\vec{b} + \vec{a}) = 0 \cdot \hat{i} - 11\hat{j} + 23\hat{k}\}$

$= (\vec{u} \cdot \hat{i})\hat{i} - (\hat{i} \cdot \hat{i}) \cdot \vec{u}$

$= 0 - \vec{u} = 0 \cdot \hat{i} + 11\hat{j} - 23\hat{k}$

$\vec{c} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 0 + 11 - 23$

$= -12$

15. If $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $C = ABA^T$ and $X = A^T C^2 A$, then det X is equal to

(1) 243 (2) 891

(3) 729 (4) 27

Answer (3)

Sol. $C = A \cdot B \cdot A^T \Rightarrow |C| = |A| \cdot |B| \cdot |A^T|$

$= |A|^2 \cdot |B|$ …(i)

$X = A^T \cdot C^2 \cdot A \Rightarrow |X| = |A^T| \cdot |C|^2 \cdot |A|$

$= (|A| \cdot |C|)^2$ …(ii)

From (i) and (ii)

$|X| = (|A| \cdot |A|^2 \cdot |B|)^2$

$= (|A|^3 \cdot |B|)^2$ …(iii)

$$|A| = \begin{vmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{vmatrix} = 2 + 1 = 3 \quad \text{…(iv)}$$

$$|B| = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 \quad \text{…(v)}$$

From (iii), (iv) and (v)

$|X| = 729$

16. The area enclosed by the curves $xy + 4y = 16$ and $x + y = 6$ is equal to :

(1) $32 - 30 \log_e 2$ (2) $30 - 32 \log_e 2$

(3) $30 - 28 \log_e 2$ (4) $28 - 30 \log_e 2$

Answer (2)

Sol. $xy + 4y = 16$; $x + y = 6$

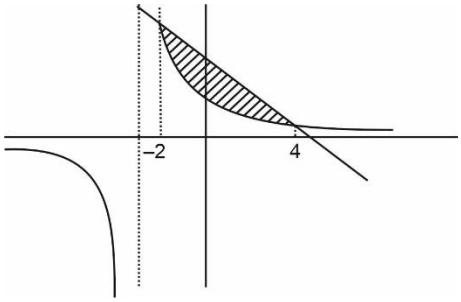
$y(x + 4) = 16$; $x + y = 6$

$\Rightarrow (6 - x)(x + 4) = 16$ $y = 6 - x$

$\Rightarrow -x^2 + 2x + 24 = 16$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = 4, x = -2$



$$\text{Area} = \int_{-2}^{4}\left[(6-x)-\left(\frac{16}{x+4}\right)\right]dx$$

$= 30 - 32 \ln 2$

17. If the shortest distance between the lines $\frac{x-\lambda}{-2}=\frac{y-2}{1}=\frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1}=\frac{y-1}{-2}=\frac{z-2}{1}$ is 1, then the sum of all possible values of λ is :

(1) 0 (2) $2\sqrt{3}$

(3) $3\sqrt{3}$ (4) $-2\sqrt{3}$

Answer (2)

Sol. Shortest distance

$$= \left|\frac{\vec{V}_1\times\vec{V}_2}{|\vec{V}_1\times\vec{V}_2|}\cdot\overrightarrow{AB}\right| = 1$$

where $V_1 = -2\hat{i}+\hat{j}+\hat{k}$, $\overrightarrow{AB} = (\lambda-\sqrt{3}, 2-1, 1-2)$

$V_2 = \hat{i}-2\hat{j}+\hat{k}$

$$\vec{V}_1\times\vec{V}_2 = \begin{vmatrix} i & j & k \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 3\hat{i}+3\hat{j}+3\hat{k}$$

$\Rightarrow \vec{V}_1\times\vec{V}_2 = 3\left(\hat{i}+\hat{j}+\hat{k}\right)$

$\Rightarrow |\vec{V}_1\times\vec{V}_2| = 3\sqrt{3}$

$$\Rightarrow \left|\frac{(\hat{i}+\hat{j}+\hat{k}).((\lambda-\sqrt{3})\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}\right| = 1$$

$\left|(\lambda-\sqrt{3})+1-1\right| = \sqrt{3}$

$\Rightarrow |\lambda-\sqrt{3}| = \sqrt{3}$

$\Rightarrow \lambda-\sqrt{3} = \sqrt{3}$ or $\lambda-\sqrt{3} = -\sqrt{3}$

$\Rightarrow \lambda = 2\sqrt{3}$ or 0

$\Rightarrow$ Sum of $\lambda = 2\sqrt{3}$

18. Let $f : \mathbf{R} \to \mathbf{R}$ be define as :

$$f(x) = \begin{cases} \frac{a-b\cos 2x}{x^2}; & x<0 \\ x^2+cx+2; & 0\le x\le 1 \\ 2x+1 & ; x>1 \end{cases}$$

If f is continuous every where in **R** and m is the number of points where f is **NOT** differential then $m + a + b + c$ equal :

(1) 4 (2) 2

(3) 1 (4) 3

Answer (2)

Sol. $f(x)$ is continuous $\forall\ x \in R$

$\Rightarrow f(0) = f(0^-)$

$$\Rightarrow 2 = \lim_{x\to 0^-}\frac{a-b\cos 2x}{x^2}$$

$\Rightarrow a - b = 0 \Rightarrow a = b$

$$\lim_{x\to 0^-}\frac{2b\sin 2x}{2x} = 2b = 2 \Rightarrow b = 1 = a$$

Also $f(1) = f(1^+)$

$\Rightarrow 1 + c + 2 = 3$

$\Rightarrow c = 0$

$$\Rightarrow f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, & x<0 \\ x^2+2, & x\in[0,1] \\ 2x+1, & x>1 \end{cases}$$

$$f'(x) = \begin{cases} \frac{d}{dx}\left[2\left(\frac{\sin x}{x}\right)^2\right], & x<0 \\ 2x, & x\in[0,1] \\ 2, & x>1 \end{cases}$$

Differentiability at $x = 1$ holds

$\Rightarrow m = 0$

$m + a + b + c = 0 + 1 + 1 + 0 = 2$

19. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$, $y(0) = 1$. Then $\left(\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals:

(1) $\frac{2}{1+\sqrt{e}}$ (2) $\frac{3}{3-\sqrt{e}}$

(3) $\frac{1}{2-\sqrt{e}}$ (4) $\frac{4}{4+\sqrt{e}}$

Answer (3)

Sol. $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$

Put $x + y = t$

$\Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

$\frac{dt}{dx} - 1 = 2x(t)^3 - xt$

$\Rightarrow \frac{dt}{2t^3 - t} = xdx$

$\int \frac{1}{2t^3 - t} dt = \int xdx$

$\Rightarrow \int \frac{t}{2t^4 - t^2} dt = \int xdx$

$t^2 = z$

$2tdt = dz$

$\frac{1}{2}\int \frac{dz}{2z^2 - z} = \int xdx$

$\ln\left|\frac{z - \frac{1}{2}}{z}\right| = x^2 + c$

$\ln\left|\frac{(x+y)^2 - \frac{1}{2}}{(x+y)^2}\right| = x^2 + c$

$y(0) = 1 \Rightarrow c = \ln\left(\frac{1}{2}\right)$

$\Rightarrow \frac{(x+y)^2 - \frac{1}{2}}{(x+y)^2} = e^{x^2} \times \frac{1}{2}$

$\frac{(x+y)^2 - \frac{1}{2}}{(x+y)^2} = \sqrt{e} \times \frac{1}{2}$

$\Rightarrow (x+y)^2 = \frac{1}{2-\sqrt{e}}$

20. Let C : $x^2 + y^2 = 4$ and C' : $x^2 + y^2 - 4\lambda x + 9 = 0$ be two circles. If the set of all values of λ so that the circles C and C' intersect at two distinct points, is **R** – $[a, b]$, then the point $(8a + 12, 16b - 20)$ lies on the curve :

(1) $5x^2 - y = -11$ (2) $x^2 + 2y^2 - 5x + 6y = 3$

(3) $x^2 - 4y^2 = 7$ (4) $6x^2 + y^2 = 42$

Answer (4)

Sol. $C : x^2 + y^2 = 4 \Rightarrow C(0, 0), r_1 = 2$

$C' : x^2 + y^2 - 4\lambda x + 9 = 0 \Rightarrow C'(2\lambda, 0), r_2 = \sqrt{4\lambda^2 - 9}$

$|r_1 - r_2| < CC' < |r_1 + r_2|$

$|2 - \sqrt{4\lambda^2 - 9}| < |2\lambda| < 2 + \sqrt{4\lambda^2 - 9}$

$|2\lambda| > |2 - \sqrt{4\lambda^2 - 9}|$

$\Rightarrow 4\lambda^2 > 4 + 4\lambda^2 - 9 - 4\sqrt{4\lambda^2 - 9}$

$4\sqrt{4\lambda^2 - 9} + 5 > 0 \Rightarrow \lambda \in R$

$|2\lambda| < 2 + \sqrt{4\lambda^2 - 9}$

$\Rightarrow 4\lambda^2 < 4 + 4\lambda^2 - 9 + 4\sqrt{(4\lambda^2) - 9}$

$5 < 4\sqrt{4\lambda^2 - 9} \Rightarrow \lambda^2 \geq \frac{9}{4} \rightarrow$ Domain

$$\frac{25}{16} < 4\lambda^2 - 9$$

$$\Rightarrow \lambda^2 > \frac{169}{64}$$

$$\lambda \in \left(-\infty, \frac{-13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$$

$$\lambda \in R - \left[\frac{-13}{8}, \frac{13}{8}\right]$$

$$a = \frac{-13}{8}, b = \frac{13}{8}$$

$\Rightarrow (8a + 12, 16b - 20) = (-1, 6)$

$\Rightarrow 6(-1)^2 + (6)^2 = 42$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

21. If $x = x(t)$ is the solution of the differential equation $(t + 1)dx = (2x + (t + 1)^4)dt$, $x(0) = 2$, then, $x(1)$ equals_____.

Answer (14)

Sol. $(t + 1)dx = (2x + (t + 1)^4)dt$.

$$\frac{dx}{dt} - \frac{2x}{(t+1)} = (t+1)^3$$

$$\text{IF} = e^{-\int \frac{2}{t+1}dt} = \frac{1}{(t+1)^2}$$

$$\frac{x}{(t+1)^2} = \int (t+1)dt$$

$$\frac{x}{(t+1)^2} = \frac{t^2}{2} + t + C$$

Now $x(0) = 2$

$\Rightarrow C = 2$

$$\therefore x = \left(\frac{t^2}{2} + t + 2\right)(t+1)^2$$

$$x(1) = \left(\frac{1}{2} + 1 + 2\right)(1+1)^2$$

$$= \frac{7}{2} \times 4 = 14$$

22. Let $P = \{z \in \mathbb{C}: |z + 2 - 3i| \le 1\}$ and $Q = \{z \in \mathbb{C}: z(1 + i) + \bar{z}(1 - i) \le -8\}$. Let in $P \cap Q$, $|z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z_2|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then $\alpha + \beta$ equals _____.

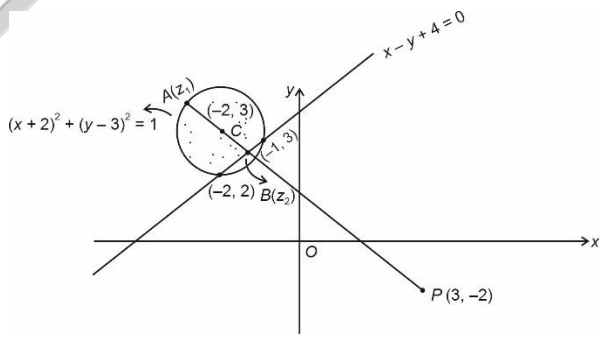
Answer (36)

Sol. $P : (x + 2)^2 + (y - 3)^2 \le 1$

$\because (x + iy)(1 + i) + (x - iy)(1 - i) \le -8$

$2(x - y) \le -8$

$\Rightarrow Q : (x - y) \le -4$

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$m_{CP} = -1$, eq^n of line AP is $y = -x + 1$

$\Rightarrow |z - 3 + 2i|$ be maximum, when z is at point A.

$|z - 3 + 2i|$ be minimum when z is at point B.

A is intersection point of $(x + 2)^2 + (y - 3)^2 = 1$ and $y = -x + 1$

$$\Rightarrow A\left(-2-\frac{1}{\sqrt{2}}, 3+\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow z_1 = \left(-2-\frac{1}{\sqrt{2}}\right)+i\left(3+\frac{1}{\sqrt{2}}\right)$$

B is intersection point of $x - y + 4 = 0$ and $y = -x + 1$

$$\Rightarrow B\left(-\frac{3}{2}, \frac{5}{2}\right) \Rightarrow z_2 = -\frac{3}{2}+\frac{5}{2}i$$

$$\Rightarrow |z_1|^2 + 2|z_2|^2 = 31 + 5\sqrt{2} \Rightarrow \alpha + \beta = 36$$

23. The number of elements in the set $S = \{(x, y, z) : x, y, z \in \mathbb{Z}, x + 2y + 3z = 42, x, y, z \geq 0\}$ equal _____.

Answer (169)

Sol. $x + 2y + 3z = 42$

$x, y, z \geq 0$

as

$z = 0$ $x + 2y = 42 \Rightarrow$ 22 cases

$z = 1$ $x + 2y = 39 \Rightarrow$ 20 cases

$z = 2$ $x + 2y = 36 \Rightarrow$ 19 cases

$z = 3$ $x + 2y = 33 \Rightarrow$ 17 cases

$z = 4$ $x + 2y = 30 \Rightarrow$ 16 cases

$z = 5$ $x + 2y = 27 \Rightarrow$ 14 cases

$z = 6$ $x + 2y = 24 \Rightarrow$ 13 cases

$z = 7$ $x + 2y = 21 \Rightarrow$ 11 cases

$z = 8$ $x + 2y = 18 \Rightarrow$ 10 cases

$z = 9$ $x + 2y = 15 \Rightarrow$ 8 cases

$z = 10$ $x + 2y = 12 \Rightarrow$ 7 cases

$z = 11$ $x + 2y = 9 \Rightarrow$ 5 cases

$z = 12$ $x + 2y = 6 \Rightarrow$ 4 cases

$z = 13$ $x + 2y = 3 \Rightarrow$ 2 cases

$z = 14$ $x + 2y = 0 \Rightarrow$ 1 case

169 cases.

24. Let 3, 7, 11, 15, …, 403 and 2, 5, 8, 11, …, 404 be two arithmetic progressions. Then the sum of the common terms in them, is equal to _____.

Answer (6699)

Sol. 3, 7, 11, 15….403

2, 5, 8, 11….404.

So common term AP

11, 23, 35…., 395

$\Rightarrow 395 = 11 + (n - 1)12$

$\Rightarrow n = 33$

$$\text{Sum} = \frac{33}{2}\left[2\times 11 + (32)12\right]$$

$$= \frac{33}{2}[22 + 384]$$

= 6699

25. Let $A = \{1, 2, 3, \ldots, 20\}$. Let R_1 and R_2 two relation on A such that

$R_1 = \{(a, b) : b \text{ is divisible by } a\}$

$R_2 = \{(a, b) : a \text{ is an integral multiple of } b\}$.

Then, number of elements in $R_1 - R_2$ is equal to ____.

Answer (46)

Sol. $S = \{1, 2, 3\ldots., 20\}$

$$R_1 : \begin{Bmatrix} (1,1),(1,2)....,(1,20), \\ (2,2),(2,4)....,(2,20), \\ (3,3),(3,6)....,(3,18), \\ (4,4),(4,8)....,(4,20), \\ (5,5)(5,10)....,(5,20), \\ (6,6),(6,12),(6,18),(7,7),(7,14), \\ (8,8),(8,16),(9,9),(9,18)(10,10), \\ (10,20),(11,11),(12,12)....,(20,20) \end{Bmatrix}$$

$n(R_1) = 66$

$R_2 : \{a \text{ is integral multiple of } b.\}$

$R_2 : \{(1, 1), (2, 2)\ldots,(20, 20)\}$

$n(R_2) = 20$

$n(R_1 - R_2) = 66 - 20 = 46$

26. If $\int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x\, dx}{(1+e^{\sin x})(1+\sin^4 x)} = \alpha\pi + \beta\log_e(3+2\sqrt{2})$,

where α, β are integers, then $\alpha^2 + \beta^2$ equals_____.

Answer (8)

Sol.

$$= \int_0^{\frac{\pi}{2}} \left\{ \frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)} + \frac{8\sqrt{2}\cos x}{\left(1+e^{-\sin x}\right)\left(1+\sin^4 x\right)} \right\} dx$$

$$= 8\sqrt{2}\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^4 x}\, dx$$

Let $\sin x = t$

$$I = 8\sqrt{2}\int_0^1 \frac{dt}{1+t^4}$$

$$= 4\sqrt{2}\int_0^1 \frac{\left(1+\frac{1}{t^2}\right)-\left(1-\frac{1}{t^2}\right)}{t^2+\frac{1}{t^2}}\, dt$$

$$= 4\sqrt{2}\int_0^1 \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t^2}\right)^2+2} - 4\sqrt{2}\int_0^1 \frac{\left(1-\frac{1}{t^2}\right)dt}{\left(t+\frac{1}{t^2}\right)^2-2}$$

$$= 4\sqrt{2}\cdot\frac{1}{\sqrt{2}}\left(\tan^{-1}\frac{t-\frac{1}{t}}{\sqrt{2}}\right)_0^1 - 4\sqrt{2}\cdot\frac{1}{2\sqrt{2}}\left[\log\left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right|\right]_0^1$$

$$= 2\pi - 2\log\left|\frac{2-\sqrt{2}}{2+\sqrt{2}}\right|$$

$$= 2\pi + 2\log(3+2\sqrt{2}) = \alpha\pi + \beta\log_e(3+2\sqrt{2})$$

$\Rightarrow \alpha = 2, \beta = 2$

$\Rightarrow \alpha^2 + \beta^2 = 8$

27. Let $\{x\}$ denoted the fractional part of x and $f(x) = \frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x^3\}}$, $x \neq 0$. If L and R respectively denotes the left hand limit and the right hand limit of $f(x)$ at $x = 0$, then $\frac{32}{\pi^2}(L^2 + R^2)$ is equal to _____.

Answer (18)

Sol. RHL $\Rightarrow \lim\limits_{x\to 0^+} \dfrac{\cos^{-1}(1-x^2)\sin^{-1}(1-x)}{x-x^3}$

$\Rightarrow \lim\limits_{x\to 0^+} \dfrac{\pi}{2}\cdot\dfrac{\cos^{-1}(1-x^2)}{x}$

$\Rightarrow \dfrac{\pi}{2}\lim\limits_{x\to o^+} \dfrac{-1}{\sqrt{(1-(1-x^2)^2}}(-2x)$

$= \dfrac{\pi}{2}\lim\limits_{x\to 0^+} \dfrac{2x}{\sqrt{2x^2-x^4}} = \pi \lim\limits_{x\to 0^+} \dfrac{x}{x\sqrt{2-x^2}}$

$= \dfrac{\pi}{\sqrt{2}} = R$

LHL $\Rightarrow \lim\limits_{x\to 0^-} \dfrac{\cos^{-1}(1-(1+x)^2)\sin^{-1}(1-(1+x))}{1\cdot(1-(1+x)^2)}$

$= \lim\limits_{x\to 0^-} \dfrac{\cos^{-1}(-x^2-2x)\cdot\sin^{-1}(-x)}{-x^2-2x}$

$= \dfrac{\pi}{2}\lim\limits_{x\to 0^-} \dfrac{-\sin^{-1}x}{-x(x+2)} = \dfrac{\pi}{2}\times\dfrac{1}{2} = \dfrac{\pi}{4} = L$

Required value = $\dfrac{32}{\pi^2}(L^2+R^2)$

$= \dfrac{32}{\pi^2}\left(\dfrac{\pi^2}{2}+\dfrac{\pi^2}{16}\right) = 18$

28. If the coefficient of x^{30} in the expansion of $\left(1+\dfrac{1}{x}\right)^6$ $(1 + x^2)^7$ $(1 - x^3)^8$; $x \neq 0$ is α, then $|\alpha|$ equals_____.

Answer (678)

Sol. Coefficient of x^{30} in $\dfrac{(1+x)^6\,(1+x^2)^7\,(1-x^3)^8}{x^6}$

$\Rightarrow$ Coefficient of x^{36} in $(1 + x)^6 (1 + x^2)^7 (1 - x^3)^8$

$\Rightarrow$ General term = ${}^6C_{r_1}\ {}^7C_{r_2}\ {}^8C_{r_3}\ (-1)^{r_3}\, x^{r_1+2r_2+3r_3}$

$\Rightarrow r_1 + 2r_2 + 3r_3 = 36$

Then possible value of r_1, r_2 and r_3 are

$r_1\ r_2\ r_3$

$\begin{matrix}0&6&8\\2&5&8\\4&4&8\\6&3&8\end{matrix}$ ${}^8C_8 \times \left[\begin{matrix}{}^6C_0\times{}^7C_6 + {}^6C_2\times{}^7C_5\\ + {}^6C_4\times{}^7C_4 + {}^6C_6\times{}^7C_3\end{matrix}\right]$

$\begin{matrix}1&7&7\\3&6&7\\5&5&7\end{matrix}$ $- {}^8C_7 \times \left[\begin{matrix}{}^6C_1\times{}^7C_7 + {}^6C_3\times{}^7C_6\\ + {}^6C_5\times{}^7C_5\end{matrix}\right]$

$\begin{matrix}4&7&6\\6&6&6\end{matrix}$ ${}^8C_6 \times \left[{}^6C_4\times{}^7C_7 + {}^6C_6\times{}^7C_6\right]$

$\therefore\ \alpha = 882 - 8\times 272 + 28\times 22$

$\alpha = -678$

$|\alpha| = 678$

29. Let the line L : $\sqrt{2x}+y=\alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the y-axis, then the square of the area of the triangle PQ_1Q_2 is equal to_____.

Answer (72)

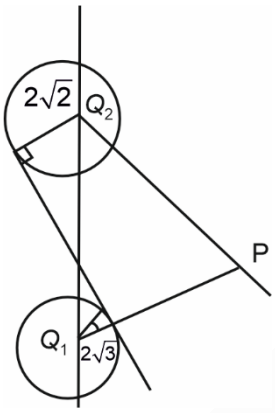
Sol Solving $x^2+y^2=3$ and $x^2=2y$

$\Rightarrow$ $y=1$ or $y=-3$ (Rejected)

Then $P:\left(\sqrt{2},1\right) \Rightarrow \alpha=3$

Let centre of C_1 or C_2 be $(0, \beta)$

If touches $\sqrt{2}x+y=3$



$\Rightarrow \left|\frac{\beta-3}{\sqrt{2+1}}\right|=2\sqrt{3}$

$\beta=9$ or $\beta=-3$

Area of $\Delta PQ_1Q_2=\frac{1}{2}\sqrt{2}\,|\beta_1-\beta_2|$

$=6\sqrt{2}$

Required $=\left(6\sqrt{2}\right)^2=72$

30. Let the line of the shortest distance between the lines

$L_1:\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and

$L_2:\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(\hat{i}+\hat{j}-\hat{k})$

Intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the mid point of the line segment PQ, then $2(\alpha+\beta+\gamma)$ is equal to_____.

Answer (21)

Sol. $L_1\equiv\vec{r}=(1,2,3)+\lambda(1,-1,1) \quad \left(\vec{r}=\vec{a}_1+\lambda\vec{b}_1\right)$

$L_2\equiv\vec{r}=(4,5,6)+\mu(1,-1,1) \quad \left(\vec{r}=\vec{a}_2+\lambda\vec{b}_2\right)$

$P\equiv(\lambda+1,-\lambda+2,\lambda+3)$

$Q\equiv(\mu+4,\mu+5,-\mu+6)$

$\overrightarrow{PQ}=(\mu-\lambda+3,\mu+\lambda+3,-\mu,-\lambda+3)$

$\overrightarrow{PQ}\cdot\vec{b}_1=0\Rightarrow 3\lambda+\mu=3$ ...(i)

$\overrightarrow{PQ}\cdot\vec{b}_2=0\Rightarrow 3\mu+\lambda=3$ ...(ii)

From (i) and (ii),1

$P\equiv\left(\frac{5}{2},\frac{1}{2},\frac{9}{2}\right)$ & $Q\equiv\left(\frac{5}{2},\frac{7}{2},\frac{15}{2}\right)$

$\alpha=\frac{5}{2},\beta=\frac{4}{2},\gamma=\frac{12}{2}$

$2(\alpha+\beta+\gamma)=21$

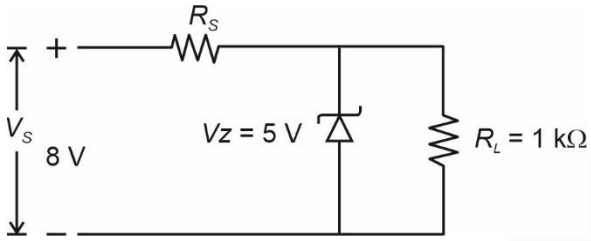
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. In the given circuit if the power rating of Zener diode is 10 mW, the value of series resistance R_S to regulate the input unregulated supply is :



(1) 5 kΩ

(2) 10 Ω

(3) 10 kΩ

(4) 1 kΩ

Answer (4)

Sol. $\frac{3}{R_S}+\frac{10\text{ mW}}{5\text{ V}}=\frac{5}{1\text{k}\Omega}$

$\frac{3}{R_S}=3\text{ mA}$

$\Rightarrow R_S = 1\text{ k}\Omega$

32. A particle moving in a circle of radius R with uniform speed takes time T to complete one revolution. If this particle is projected with the same speed at an angle θ to the horizontal, the maximum height attained by it is equal to $4R$. The angle of projection θ is then given by:

(1) $\cos^{-1}\left[\frac{2gT^2}{\pi^2R}\right]^{\frac{1}{2}}$

(2) $\sin^{-1}\left[\frac{2gT^2}{\pi^2R}\right]^{\frac{1}{2}}$

(3) $\sin^{-1}\left[\frac{\pi^2R}{2gT^2}\right]^{\frac{1}{2}}$

(4) $\cos^{-1}\left[\frac{\pi R}{2gT^2}\right]^{\frac{1}{2}}$

Answer (2)

Sol. $v=\frac{2\pi R}{V}$

$H_{\max}=\frac{v^2\sin^2\theta}{2g}\Rightarrow 4R=\frac{v^2\sin^2\theta}{2g}$

$\sin^2\theta=\frac{8gR}{v^2}=\frac{2gT^2}{\pi^2R}$

$\theta=\sin^{-1}\left(\frac{2gT^2}{\pi^2R}\right)^{1/2}$

33. Two identical capacitors have same capacitance C. One of them is charged to the potential V and other to the potential $2V$. The negative ends of both are connected together. When the positive ends are also joined together, the decrease in energy of the combined system is :

(1) $\frac{1}{4}CV^2$

(2) $\frac{3}{4}CV^2$

(3) $2CV^2$

(4) $\frac{1}{2}CV^2$

Answer (1)

Sol. $E_i = \frac{1}{2}CV^2 + \frac{1}{2}C(2V)^2 = \frac{5}{2}CV^2$

$E_f = 2\times\left(\frac{Q^2}{2C}\right) = \frac{9C^2V^2}{4C} = \frac{9}{4}CV^2$

Decrease in energy

$= E_i - E_f = \left(\frac{5}{2} - \frac{9}{4}\right)CV^2 = \frac{1}{4}CV^2$

34. A galvanometer has a resistance of 50 Ω and it allows maximum current of 5 mA. It can be converted into voltmeter to measure upto 100 V by connecting in series a resistor of resistance:

(1) 20050 Ω (2) 5975 Ω

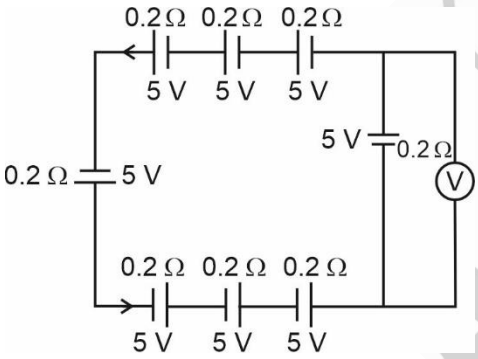
(3) 19950 Ω (4) 19500 Ω

Answer (3)

Sol. $V = i_g(r_g + s) \Rightarrow 100 = 5\times10^{-3}\ (50 + s)$

$2 \times 10^4 = 50 + s \Rightarrow s = 20000 - 50 = 19950\ \Omega$

35. The reading in the ideal voltmeter (V) shown in the given circuit diagram is :



(1) 0 V (2) 5 V

(3) 10 V (4) 3 V

Answer (1)

Sol. Total 8 cells : $40 - i\times(8\times0.2) = 0 \Rightarrow i = 25$ A

$V_B - (25\times0.2) + 5 = V_A$

$V_B + 0 = V_A \Rightarrow V_A - V_B = 0$ (Reading)

36. A ball of mass 0.5 kg is a string of length 50 cm. The ball is rotated on a horizontal circular path about its vertical axis. The maximum tension that the string can bear is 400 N. The maximum possible value of angular velocity of the ball in rad/s is, :

(1) 1000 (2) 40

(3) 20 (4) 1600

Answer (2)

Sol. $T\cos\theta = mg$, $T\sin\theta = m\omega^2\ell\sin\theta$

$\Rightarrow T = m\omega^2\ell$

$\Rightarrow 400 = 0.5 \times \omega^2 \times 0.5$

$\Rightarrow \omega^2 = 1600$

$\Rightarrow \omega = 40$ rad/s

37. The minimum energy required by a hydrogen atom in ground state to emit radiation in Balmer series is nearly :

(1) 13.6 eV (2) 1.5 eV

(3) 12.1 eV (4) 1.9 eV

Answer (3)

Sol. Balmer emission: transition to $n = 2$

To transition to $n = 2$, minimum state to transition from is $n = 3$

$\Delta E = E_{\text{ground}} - E_{n=3} = 12.1$ eV

38. The dimensional formula of angular impulse is :

(1) $[ML^2T^{-2}]$ (2) $[ML^{-2}T^{-1}]$

(3) $[ML^2T^{-1}]$ (4) $[MLT^{-1}]$

Answer (3)

Sol. Angular impulse = Impulse × distance from axis

[Angular impulse] = [Force] [Time] [Length]

$= [ML^2T^{-1}]$

39. The pressure and volume of an ideal gas are related as $PV^{\frac{3}{2}} = K$ (Constant). The work done when the gas is taken from state $A(P_1, V_1, T_1)$ to state $B(P_2, V_2, T_2)$ is :

(1) $2\left(P_2\sqrt{V_2} - P_1\sqrt{V_1}\right)$ (2) $2(P_1V_1 - P_2V_2)$

(3) $2(P_2V_2 - P_1V_1)$ (4) $2\left(\sqrt{P_1}V_1 - \sqrt{P_2}V_2\right)$

Answer (2)

Sol. $W = \frac{P_FV_f - P_iV_i}{1-Y} = \frac{P_2V_2 - P_1V_1}{1-\frac{3}{2}} = -2(P_2V_2 - P_1V_1)$

$W = 2(P_1V_1 - P_2V_2)$

40. With rise in temperature, the Young's modulus of elasticity:

(1) Decreases (2) Remains unchanged

(3) Increases (4) Changes erratically

Answer (1)

Sol. With rise in temperature, Young's modulus of elasticity decreases due to the increase in atomic vibrations which leads to decrease in atomic forces.

41. Two moles a monoatomic gas is mixed with six moles of a diatomic gas. The molar specific heat of the mixture at constant volume is:

(1) $\frac{3}{2}R$ (2) $\frac{9}{4}R$

(3) $\frac{7}{4}R$ (4) $\frac{5}{2}R$

Answer (2)

Sol. $C_{V_{mix}} = \frac{n_1C_{v_1} + n_2C_{v2}}{n_1 + n_2} = \frac{\frac{2\times 3R}{2} + \frac{6\times 5R}{2}}{2+6}$

$CV_{mix} = \frac{9R}{4}$

42. A simple pendulum of length 1 m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of 2×10^2 ms^{-1}. The bullet gets embedded into the bob. The height to which the bob rises before swinging back is (use g = 10 m/s^2)

(1) 0.20 m

(2) 0.35 m

(3) 0.30 m

(4) 0.40 m

Answer (1)

Sol. Linear momentum conservation:

$10^{-2} \times 2 \times 10^2 = (1 + 0.01)v$

v = 2 m/s

Energy conservation:

$\frac{1}{2}\times 1\times 2^2 = 1\times 10\times h$

h = 0.20 m

43. A parallel plate capacitor has a capacitance C = 200 pF. It is connected to 230 V ac supply with an angular frequency 300 rad/s. The rms value of conduction current in the circuit and displacement current in the capacitor respectively are:

(1) 14.3 μA and 143 μA (2) 13.8 μA and 138 μA

(3) 13.8 μA and 13.8 μA (4) 1.38 μA and 1.38 μA

Answer (3)

Sol. $i_{displacement} = \frac{C\cdot dv}{dt} : (i_{displacement})_{rms} = C\cdot\omega E_{rms}$

$= 230 \times 200 \times 10^{-12} \times 300$

$= 13.8 \times 10^{-6}A = 13.8\ \mu A$

$(i_{conduction})_{rms} = E_{rms}\cdot\omega C = 230 \times 200 \times 10^{-12} \times 300$

$= 13.8 \times 10^{-6}$A $= 13.8\ \mu A$

44. The de Broglie wavelengths of a proton and an α particle are λ and 2λ respectively. The ratio of the velocities of proton and α particle will be:

(1) 4 : 1

(2) 1 : 8

(3) 8 : 1

(4) 1 : 2

Answer (3)

Sol. $\therefore \quad \lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda} : v \propto \frac{1}{m\lambda}$

$$\frac{V_P}{V_\alpha} = \frac{m_\alpha \lambda_\alpha}{m_P \lambda_P} = \frac{4m_P \cdot 2\lambda}{m_P \cdot \lambda} = \frac{8}{1}$$

45. In series *LCR* circuit, the capacitance is changed from *C* to 4*C*. To keep the resonance frequency unchanged, the new inductance should be:

(1) Increased to 4*L*

(2) Increased by 2*L*

(3) Reduced by $\frac{1}{4}L$

(4) Reduced by $\frac{3}{4}L$

Answer (4)

Sol. $w_i = \frac{1}{\sqrt{LC}}; w_f = \frac{1}{\sqrt{4C \cdot L'}}$

For $w_i = w_f$, $L' = \frac{L}{4}\left(\text{reduced by } \frac{3L}{4}\right)$

46. If *R* is the radius of the earth and the acceleration due to gravity on the surface of earth is $g = \pi^2$ m/s^2, then the length of the second's pendulum at a height $h = 2R$ from the surface of earth will be:

(1) $\frac{4}{9}$ m

(2) $\frac{8}{9}$ m

(3) $\frac{1}{9}$ m

(4) $\frac{2}{9}$ m

Answer (3)

Sol. $g_H = \frac{GM}{(R+2R)^2} = \frac{GM}{9R^2} = \frac{g}{9}$

$$2 = 2\pi\sqrt{\frac{l}{g_H}} \Rightarrow i = \frac{\pi}{\pi}\sqrt{9l} \Rightarrow l = \frac{1}{9}\text{m}$$

47. A monochromatic light of wavelength 6000Å is incident on the single slit of width 0.01 mm. If the diffraction pattern is formed at the focus of the convex lens of focal length 20 cm, the linear width of the central maximum is:

(1) 12 mm (2) 120 mm

(3) 60 mm (4) 24 mm

Answer (4)

Sol. Linear width = $\frac{2\lambda 0}{d}$

$$= \frac{2(6000\times10^{-10})(20\times10^{-2})}{0.01\times10^{-3}}$$

$= 24 \times 10^{-3}$ m

= 24 mm

48. 10 divisions on the main scale of a Vernier calliper coincide with 11 divisions on the Vernier scale. If each division on the main scale is of 5 units, the least count of the instrument is:

(1) $\frac{10}{11}$ (2) $\frac{1}{2}$

(3) $\frac{5}{11}$ (4) $\frac{50}{11}$

Answer (3)

Sol. $10\,M = 11\,v \Rightarrow 1v = \frac{10}{11};\ v' = \frac{10}{11} + 5 = \frac{50}{11}$

$$\text{Least count} = M - v' = 5 - \frac{50}{11} = \frac{5}{11}$$

49. The radius (r), length (l) and resistance (R) of a metal wire was measured in the laboratory as

$r = (0.35 \pm 0.05)$ cm

$R = (100 \pm 10)$ ohm

$l = (15 \pm 0.2)$ cm

The percentage error in resistivity of the material of the wire is:

(1) 35.6%

(2) 39.9%

(3) 25.6%

(4) 37.3%

Answer (2)

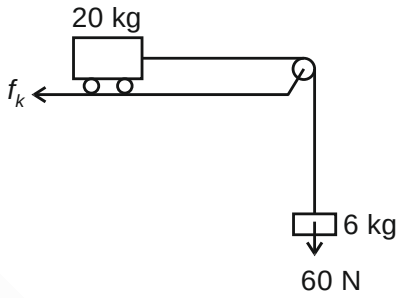
Sol. $l = \frac{RA}{l} = \frac{R.\pi r^2}{l}$

$$\therefore \frac{dl}{l} \times 100 = \frac{2dr}{r} \times 100 + \frac{dR}{R} \times 100 + \frac{dl}{l} \times 100$$

$$= 2 \times \frac{0.05}{0.35} \times 100 + \frac{10}{100} \times 100 + \frac{0.2}{15} \times 100 = 39.9\%$$

50. Consider a block and trolley system as shown in figure. If the coefficient of kinetic friction between the trolley and the surface is 0.04, the acceleration of the system in ms^{-2} is:

(Consider that the string is massless and unstretchable and the pulley is also massless and frictionless):



(1) 4 (2) 3

(3) 2 (4) 1.2

Answer (3)

Sol. $60 - f_k = (20 + 6)a$

$60 - 200 \times 0.04 = 26a \Rightarrow a = 2\ m/s^2$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

51. A rectangular loop of sides 12 cm and 5 cm, with its sides parallel to the x-axis and y-axis respectively, moves with a velocity of 5 cm/s in the positive x-axis direction, in a space containing a variable magnetic field in the positive z direction. The field has a gradient of 10^{-3} T/cm along the negative x direction and it is decreasing with time at the rate of 10^{-3} T/s. If the resistance of the loop is 6 mΩ, the power dissipated by the loop as heat is _____ $\times 10^{-9}$ W.

Answer (216)

Sol. $I_1 = \frac{-dB}{dx} \times \frac{dx}{dt} \times A$

$= 30 \times 10^{-6}$ V

$I_2 = \frac{-dB}{dt} A$

$= 6 \times 10^{-6}$ V

$P = \frac{(I_1 + I_2)^2}{R} = 216 \times 10^{-9}$ W

52. A plane is in level fight at constant speed and each of its two wings has an area of 40 m^2. If the speed of the air is 180 km/h over the lower wing surface and 252 km/h over the upper wing surface, the mass of the plane is _____ kg. (Take air density to be 1 kg m^{-3} and g = 10 ms^{-2})

Answer (9600)

Sol. $P_1 - P_2 = \frac{1}{2} e(V_2^2 - V_1^2)$

$\Rightarrow P_1 - P_2 = \frac{1}{2} \times 1 \times (70^2 - 50^2)$

$P_1 - P_2 = 1200$ N/m^2

$\therefore F = mg = (P_1 - P_2)A.$

$\Rightarrow m = \frac{(P_1 - P_2)A}{g} = \frac{1200 \times (2 \times 40)}{10} = 9600$ kg

53. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle θ with each other. When suspended in water the angle remains the same. If density of the material of the sphere is 1.5 g/cc, the dielectric constant of water will be _____ (Take density of water = 1 g/cc)

Answer (3)

Sol. $\therefore \tan\theta = \frac{qE}{mg} = \frac{qE'}{mg - F_B} \Rightarrow \frac{E}{g} = \frac{E/K}{mg - 1.vg}$

$\frac{E}{mg} = \frac{E/K}{mg/3} \Rightarrow 1 = \frac{3}{K} \Rightarrow K = 3$

54. A particle is moving in one dimension (along x-axis) under the action of a variable force. It's initial position was 16 m right of origin. The variation of its position (x) with time (t) is given as $x = -3t^3 + 18t^2 + 16t$, where x is in m and t is in s. The velocity of the particle when its acceleration becomes zero is _____ m/s.

Answer (52)

Sol. $x = -3t^3 + 18t^2 + 16t \Rightarrow v = -9t^2 + 36t + 16$

$a = -18t + 36 = 0 \Rightarrow t = 2$ sec

$v_{(t=2)} = -9(2)^2 + 36(2) + 16 = 52$ m/s

55. The current in a conductor is expressed as $I = 3t^2 + 4t^3$, where I is in Ampere and t is in second. The amount of electric charge that flows through a section of the conductor during t = 1 s to t = 2 s is _____ C.

Answer (22)

Sol. $\int dq = \int i.dt \Rightarrow q = \int_1^2 (3t^2 + 4t^3) \cdot dt$

$q = \left[t^3 + t^4\right]_1^2 = [8 + 16 - 2] = 22$ C.

56. The distance between object and its 3 times magnified virtual image as produced by a convex lens is 20 cm. The focal length of the lens used is _____ cm.

Answer (15)

Sol. $|m| = \frac{v}{u} \Rightarrow v = 3u; |v| - |u| = 20$ cm

$u = -10$ cm, $v = 30$ cm

$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{-1}{30} + \frac{1}{10} \Rightarrow f = 15$ cm

57. The identical spheres each of mass 2M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 4 m each. Taking point of intersection of these two sides as origin, the magnitude of position vector of the centre of mass of the system is $\frac{4\sqrt{2}}{x}$, where the value of x is _____

Answer (2)

Sol. $x_{com} = \frac{-2M \times 4 + 2M \times 0}{4M} = -2\hat{i}$

$y_{com} = \frac{2M \times 4 + 2M \times 0}{4M} = 2\hat{j}$

$r_{com} = \sqrt{4+4} = 2\sqrt{2} = \frac{4\sqrt{2}}{x} \Rightarrow x = 2$

58. A tuning fork resonates with a sonometer wire of length 1 m stretched with a tension of 6 N. When the tension in the wire is changed to 54 N, the same tuning fork produces 12 beats per second with it. The frequency of the tuning fork is _____ Hz.

Answer (6)

Sol. Let f_0 = tuning fork frequency.

$\therefore \quad f_0 = \frac{1}{2l}\sqrt{\frac{6}{\mu}}$

And $\frac{l}{2l}\sqrt{\frac{54}{\mu}} - \frac{1}{2l}\sqrt{\frac{6}{\mu}} = 12$

$\therefore \quad \frac{l}{2l}\sqrt{\frac{6}{\mu}} = 6$

$\therefore \quad \boxed{f_0 = 6}$

59. The radius of a nucleus of mass number 64 is 4.8 fermi. Then the mass number of another nucleus having radius of 4 fermi is $\frac{1000}{x}$, where x is _____.

Answer (27)

Sol. $R = R_0 (A)^{\frac{1}{3}} \Rightarrow \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}} \Rightarrow \frac{4.8}{4} = \left(\frac{64}{A_2}\right)^{\frac{1}{3}}$

$1.2 = \frac{4}{(A_2)^{\frac{1}{3}}} \Rightarrow A_2^{\frac{1}{3}} = \frac{10}{3} \Rightarrow A_2 = \frac{1000}{27}$

$\Rightarrow \quad \boxed{x = 27}$

60. A regular polygon of 6 sides is formed by bending a wire of length 4 π meter. If an electric current of $4\pi\sqrt{3}$ A is flowing through the sides of the polygon, the magnetic field at the centre of the polygon would be $x \times 10^{-7}$ T. The value of x is _____.

Answer (72)

Sol. $B = 6 \times \left(\frac{\mu_0 \left(4\pi\sqrt{3}\right)}{4\pi \times \left(\frac{\pi}{\sqrt{3}}\right)} (\sin 30 + \sin 30) \right)$

$B = 6 \times \frac{\mu_0 \times 3}{\pi} = \frac{6 \times 4\pi \times 10^{-7} \times 3}{\pi} = 72 \times 10^{-7}$ T

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : PH_3 has lower boiling point than NH_3.

Reason (R) : In liquid state NH_3 molecules are associated through van der Waal's forces, but PH_3 molecules are associated through hydrogen bonding.

In the light of above statements, choose the most appropriate answer from the options given below.

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(3) (A) is correct but (R) is not correct

(4) (A) is not correct but (R) is correct

Answer (3)

Sol. PH_3 has lower boiling point than NH_3 due to lower electronegativity of larger PH_3 molecules. They are unable to form hydrogen bonds among themselves.

62. Which of the following complex is homoleptic?

(1) $[Co(NH_3)_4Cl_2]^+$ (2) $[Ni(NH_3)_2Cl_2]$

(3) $[Fe(NH_3)_4Cl_2]^+$ (4) $[Ni(CN)_4]^{2-}$

Answer (4)

Sol. Homoleptic complexes are compounds in which all the ligands attached to metal centre are the same.

e.g., $[Ni(CN)_4]^{2-}$

63. Match List - I with List - II.

	List - I (Reactions)		**List - II (Reagents)**
(A)	$CH_3(CH_2)_5-\underset{\underset{O}{\|}}{C}-OC_2H_5 \longrightarrow CH_3(CH_2)_5CHO$	(I)	CH_3MgBr, H_2O
(B)	$C_6H_5COC_6H_5 \rightarrow C_6H_5CH_2C_6H_5$	(II)	Zn(Hg) and conc. HCl
(C)	$C_6H_5CHO \rightarrow C_6H_5CH(OH)CH_3$	(III)	$NaBH_4$, H^+
(D)	$CH_3COCH_2COOC_2H_5 \rightarrow CH_3\underset{\underset{H}{\|}}{C}(OH)CH_2COOC_2H_5$	(IV)	DIBAL-H, H_2O

Choose the correct answer from the options given below.

(1) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Answer (3)

Sol. $CH_3(CH_2)_5-\underset{\underset{O}{\|}}{C}-OC_2H_5 \xrightarrow{\text{DIBAL-H}} CH_3(CH_2)_5CHO + C_2H_5OH$

$C_6H_5COC_6H_5 \xrightarrow[\text{Conc. HCl}]{\text{Zn(Hg) and}} C_6H_5CH_2C_6H_5$

$C_6H_5CHO \xrightarrow{CH_3MgBr,\ H_2O} C_6H_5CH(OH)CH_3$

$CH_3COCH_2COOC_2H_5 \xrightarrow{NaBH_4/H^+} CH_3-\underset{\underset{H}{\|}}{C}(OH)CH_2COOC_2H_5$

64. We have three aqueous solutions of NaCl labelled as 'A', 'B' and 'C' with concentration 0.1 M, 0.01 M and 0.001 M, respectively. The value of van't Hoff factor (i) for these solutions will be in the order :

(1) $i_A > i_B > i_C$ (2) $i_A < i_B < i_C$

(3) $i_A = i_B = i_C$ (4) $i_A < i_C < i_B$

Answer (2)

Sol. As concentration of solution decreases degree of dissociation (α) increases and van't Hoff factor (i) depends on degree of dissociation.

[A] = 0.1 M

[B] = 0.01 M

[C] = 0.001 M

$i_A < i_B < i_C$

65. Which of the following reactions are disproportionation reactions?

(A) $Cu^+ \rightarrow Cu^{2+} + Cu$

(B) $3MnO_4^{2-} + 4H^+ \longrightarrow 2MnO_4^- + MnO_2 + 2H_2O$

(C) $3KMnO_4 \longrightarrow K_2MnO_4 + MnO_2 + O_2$

(D) $2MnO_4^- + 3Mn^{2+} + 2H_2O \longrightarrow 5MnO_2 + 4H^+$

Choose the correct answer from the options given below.

(1) (B), (C), (D) (2) (A), (D)

(3) (A), (B) (4) (A), (B), (C)

Answer (3)

Sol. Disproportionation reaction is a reaction in which, the same element is simultaneously oxidised and reduced.

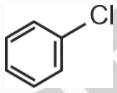
(A) $\overset{\text{Oxidation}}{\overset{\frown}{Cu^+ \longrightarrow Cu^{2+}}} + \underset{\text{Reduction}}{\underset{\smile}{Cu^0}}$

(B) $\overset{+6}{3MnO_4^{2-}} + 4H^+ \longrightarrow \overset{+7}{K_2MnO_4} + \overset{+4}{MnO_2} + O_2$ (Oxidation: $+6 \rightarrow +7$; Reduction: $+6 \rightarrow +4$)

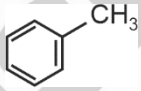
(C) $\overset{+7}{2KMnO_4} \longrightarrow \overset{+6}{K_2MnO_4} + \overset{+4}{MnO_2} + O_2$ (Reduced only)

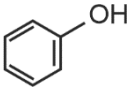
(D) $\overset{+7}{2MnO_4^-} + 3Mn^{2+} + 2H_2O \longrightarrow \overset{+4}{5MnO_2} + 4H^+$

66. Which of the following compound will most easily be attacked by an electrophile?

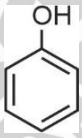
(1) 

(2) 

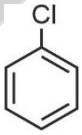
(3) 

(4) 

Answer (4)

Sol. 

Phenol is most easily attacked by an electrophile because of presence of –OH group which increases electron density at ortho and para position mainly.



Here, chlorine atom shows +R effect o-, p-directive. But deactivate benzene ring.

–OH and $–CH_3$ are activating and ortho para directing groups.

–OH activates more than $–CH_3$.

67. Given below are two statements :

Statement (I) : The NH_2 group in Aniline is ortho and para directing and a powerful activating group.

Statement (II) : Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation).

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Answer (1)

Sol.
- NH_2 group in aniline is ortho and para directing and a powerful activating group due to its strong electron donating nature.
- Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation) due to salt formation with aluminium chloride which is used as a Lewis acid catalyst.

68. Given below are two statements :

Statement (I) : A solution of $[Ni(H_2O)_6]^{2+}$ is green in colour.

Statement (II) : A solution of $[Ni(CN)_4]^{2-}$ is colourless.

In the light of the above statements, choose the most appropriate answer from options given below.

(1) Both Statement I and Statement II are incorrect

(2) Statement I is correct but Statement II is incorrect

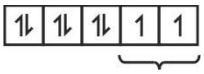
(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are correct

Answer (4)

Sol. $[Ni(H_2O)_6]^{2+}$ is green in colour.

Ni^{2+} has $3d^8$ configuration.



H_2O is weak ligand; no-pairing of unpaired electrons, d-d transition absorbs light and emits green light.

$[Ni(CN)_4]^{2-}$ is colourless.

As CN^- is strong ligand, causes pairing of electrons. Therefore, d-d transition is not possible.

69. In case of isoelectronic species the size of F^-, Ne and Na^+ is affected by

(1) Electron-electron interaction in the outer orbitals

(2) Principal quantum number (n)

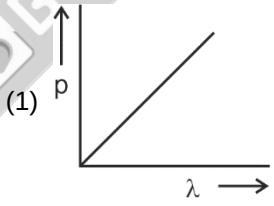
(3) Nuclear charge (z)

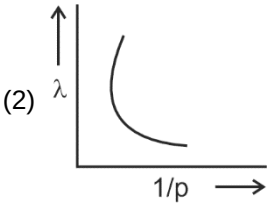
(4) None of the factors because their size is the same

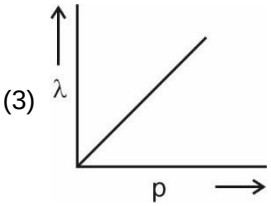
Answer (3)

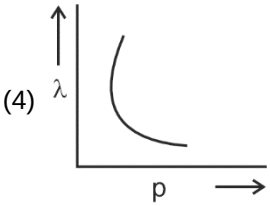
Sol. For isoelectronic species, size depends on nuclear charge. More nuclear charge, lesser will be the size of species, this is because the valence electron will experience greater attractive force due to increase in nuclear charge.

70. According to the wave-particle duality of matter by de-Broglie, which of the following graph plot presents most appropriate relationship between wavelength of electron (λ) and momentum of electron (p)?

(1) 

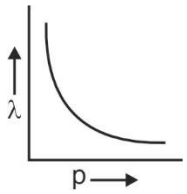
(2) 

(3) 

(4) 

Answer (4)

Sol.



$$\lambda = \frac{h}{p}$$

$$\lambda \propto \frac{1}{p}$$

So, graph between λ and p is rectangular hyperbola.

71. Arrange the bonds in order of increasing ionic character in the molecules. LiF, K_2O, N_2, SO_2 and ClF_3.

(1) $LiF < K_2O < ClF_3 < SO_2 < N_2$

(2) $N_2 < SO_2 < ClF_3 < K_2O < LiF$

(3) $ClF_3 < N_2 < SO_2 < K_2O < LiF$

(4) $N_2 < ClF_3 < SO_2 < K_2O < LiF$

Answer (2)

Sol. The ionic character of molecule depends on electronegativity difference between atoms of molecule. Greater the difference, greater will be ionic character.

On this basis, the order of increasing ionic character in the given molecule is –

$N_2 < SO_2 < ClF_3 < K_2O < LiF$

72. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Haloalkanes react with KCN to form alkyl cyanides as a main product while with AgCN form isocyanide as the main product.

Reason (R) : KCN and AgCN both are highly ionic compounds.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) **(A)** is not correct but **(R)** is correct

(2) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(3) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

(4) **(A)** is correct but **(R)** is not correct

Answer (4)

Sol. $R - X + KCN \longrightarrow R - CN + KX$

$R - X + AgCN \longrightarrow R - NC + AgX$

KCN is ionic so it provides cyanide ions in solution and attacks from carbon side on alkyl halide.

But AgCN is covalent it cannot form cyanide ion so it attacks from nitrogen side and isocyanide is formed predominantly.

A is correct R is not correct.

73. In acidic medium, $K_2Cr_2O_7$ shows oxidising action as represented in the half reaction :

$$Cr_2O_7^{2-} + XH^+ + Ye^{\ominus} \longrightarrow 2A + ZH_2O$$

X, Y, Z and A are respectively:

(1) 8, 4, 6 and Cr_2O_3

(2) 14, 6, 7 and Cr^{3+}

(3) 14, 7, 6 and Cr^{3+}

(4) 8, 6, 4 and Cr_2O_3

Answer (2)

Sol. In acidic medium

$$K_2Cr_2O_7 + 14H^+ + 6e^- \longrightarrow 2Cr^{+3} + 7H_2O$$

X = 14

Y = 6

Z = 7

A = Cr^{+3}

74. Given below are two statements :

Statement (I): Aminobenzene and aniline are same organic compounds.

Statement (II): Aminobenzene and aniline are different organic compounds.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) **Statement I** is correct but **Statement II** is incorrect

(2) Both **Statement I** and **Statement II** are incorrect

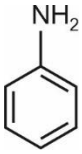
(3) **Statement I** is incorrect but **Statement II** is correct

(4) Both **Statement I** and **Statement II** are correct

Answer (1)

Sol. Aminobenzene and aniline are same organic compound.

Aniline is also known as aminobenzene or phenylamine.



because aniline has amino group in its structure.

75. In Kjeldahl's method for estimation of nitrogen, $CuSO_4$ acts as :

(1) Oxidising agent (2) Catalytic agent

(3) Reducing agent (4) Hydrolysis agent

Answer (2)

Sol. In Kjeldahl's Method $CuSO_4$ acts as a catalytic agent.

Organic compound + $H_2SO_4 \xrightarrow{CuSO_4} (NH_4)_2SO_4$

76. Given below are two statements :

Statement (I) : Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution.

Statement (II) : In this titration phenolphthalein can be used as indicator.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are incorrect

(2) Both Statement I and Statement II are correct

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Answer (2)

Sol. Potassium hydrogen phthalate is a primary standard, generally used to standardize a solution of NaOH.

Indicator used for titration of weak acids is phenolphthalein as it goes from colourless at acidic pH to pink at basic pH.

77. Choose the correct option for free expansion of an ideal gas under adiabatic condition from the following.

(1) $q = 0, \Delta T < 0, w \neq 0$

(2) $q \neq 0, \Delta T = 0, w = 0$

(3) $q = 0, \Delta T \neq 0, w = 0$

(4) $q = 0, \Delta T = 0, w = 0$

Answer (4)

Sol. In adiabatic free expansion, there is no external pressure for gas to expand.

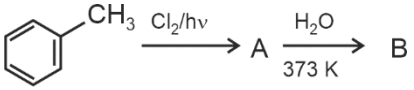
So, work done is zero.

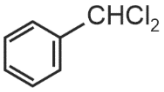
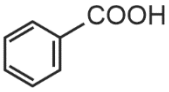
$w = 0, \Delta T = 0.$

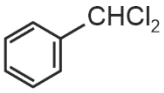
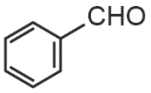
For adiabatic process, no heat is exchanged.

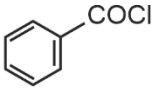
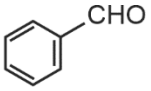
$q = 0$

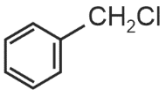
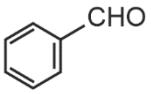
78. Identify A and B in the following sequence of reaction.



(1) A =  B = 

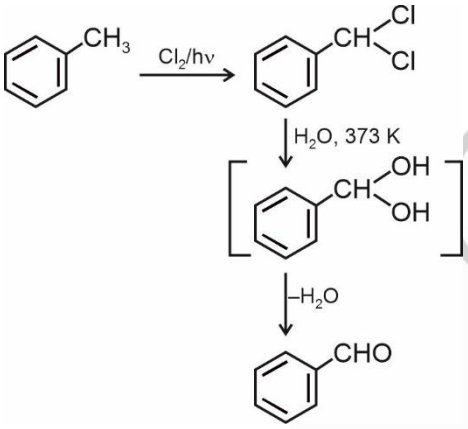
(2) A =  B = 

(3) A =  B = 

(4) A =  B = 

Answer (2)

Sol.



79. If one strand of a DNA has the sequence ATGCTTCA, sequence of the bases in complementary strand is

(1) TACGAAGT (2) CATTAGCT

(3) GTACTTAC (4) ATGCGACT

Answer (1)

Sol. Adenine forms hydrogen bonds with thymine whereas cytosine forms hydrogen bonds with guanine.

Given sequence → [A T G C T T C A]

Complementary strand → [T A C G A A G T]

80. Ionic reactions with organic compounds proceed through

(A) Homolytic bond cleavage

(B) Heterolytic bond cleavage

(C) Free radical formation

(D) Primary free radical

(E) Secondary free radical

Choose the correct answer from the options given below.

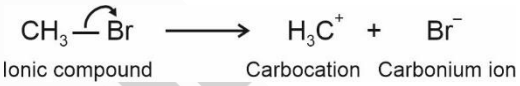
(1) (C) only (2) (D) and (E) only

(3) (A) only (4) (B) only

Answer (4)

Sol. Ionic reactions occur when covalent bond between two atoms undergoes heterolytic cleavage by transferring electrons and in process forms positively and negatively charged ions.

Heterolytic cleavage



Ionic compound Carbocation Carbonium ion

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

81. K_a for CH_3COOH is 1.8×10^{-5} and K_b for NH_4OH is 1.8×10^{-5}. The pH of ammonium acetate solution will be _____.

Answer (7)

Sol. K_a for $CH_3COOH = 1.8 \times 10^{-5}$

K_b for $NH_4OH = 1.8 \times 10^{-5}$

$pK_a = 4.74$; $pK_b = 4.74$

$$\text{pH of } CH_3COONH_4 = \frac{1}{2}(pK_w + pK_a - pK_b)$$

$$pH = \frac{1}{2}(14 + 4.74 - 4.74)$$

$$= 7$$

82. Among the following oxides of p-block elements, number of oxides having amphoteric nature is _____.

Cl_2O_7, CO, PbO_2, N_2O, NO, Al_2O_3, SiO_2, N_2O_5, SnO_2

Answer (3)

Sol. Amphoteric oxides are those which can react with both acids and bases.

Amphoteric oxides are: PbO_2, Al_2O_3, SnO_2

CO, NO and N_2O are neutral.

Cl_2O_7, SiO_2 and N_2O_5 are acidic

83. The lowest oxidation number of an atom in a compound A_2B is –2. The number of electrons in its valence shell is _____.

Answer (6)

Sol. In compound A_2B

Lowest oxidation state of B is –2

It means it has $6e^-$ in its valence shell.

84. The ratio of $\frac{^{14}C}{^{12}C}$ in a piece of wood is $\frac{1}{8}$ part that of atmosphere. If half life of ^{14}C is 5730 years, the age of wood sample is _____ years.

Answer (17190)

Sol. $N = N_0e^{-\lambda t}$

$$\frac{1}{8} = 1e^{-\lambda t}$$

$$e^{\lambda t} = 8$$

$$\lambda t = \ln 8$$

$$\frac{0.693}{t_{1/2}} t = \ln 8$$

$$t = \frac{\ln 8}{\ln 2} \times t_{1/2}$$

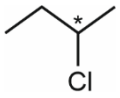
$t = 3 \times 5730$ years

= 17190 years

85. Number of optical isomers possible for 2-chlorobutane _____.

Answer (2)

Sol. 2-chlorobutane



2-chlorobutane contains only one chiral centre. So, it can show two optical isomers

86. Consider the following reaction:

$3PbCl_2 + 2(NH_4)_3PO_4 \rightarrow Pb_3(PO_4)_2 + 6NH_4Cl$

If 72 mmol of $PbCl_2$ is mixed with 50 mmol of $(NH_4)_3PO_4$, then the amount of $Pb_3(PO_4)_2$ formed is _____ mmol (nearest integer).

Answer (24)

Sol. $\underset{72\text{ mmol}}{3PbCl_2} + \underset{50\text{ mmol}}{2(NH_4)_3PO_4} \longrightarrow Pb_3(PO_4)_2 + 6NH_4Cl$

$PbCl_2$ is limiting reagent

3 mol $PbCl_2$ produces 1 mol of $Pb_3(PO_4)_2$

72 mmol of $PbCl_2$ will produce

$\frac{1}{3} \times 72$ mmol of $Pb_3(PO_4)_2$

= 24 mmol

87. The number of white coloured salts, among the following is _____.

(a) $SrSO_4$ (b) $Mg(NH_4)PO_4$

(c) $BaCrO_4$ (d) $Mn(OH)_2$

(e) $PbSO_4$ (f) $PbCrO_4$

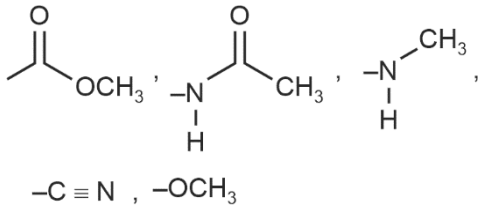
(g) AgBr (h) PbI_2

(i) CaC_2O_4 (j) $[Fe(OH)_2(CH_3COO)]$

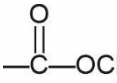
Answer (5)

Sol. $SrSO_4$, $PbSO_4$, $Mg(NH_4)PO_4$, $Mn(OH)_2$ and CaC_2O_4 are white coloured salts.

88. Total number of deactivating groups in aromatic electrophilic substitution reaction among the following is _____.



Answer (2)

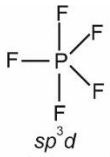
Sol.  are deactivating groups in aromatic electrophilic substitution reaction, because they are –R group which pulls electron density towards themselves.

89. The number of molecules/ion/s having trigonal bipyramidal shape is _____.

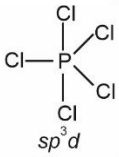
PF_5, BrF_5, PCl_5, $[Pt\ Cl_4]^{2-}$, BF_3, $Fe(CO)_5$

Answer (3)

Sol.



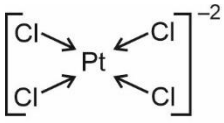
sp^3d
(trigonal bipyramidal)



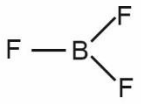
sp^3d
(trigonal bipyramidal)

sp^3d^2
(square pyamidal)

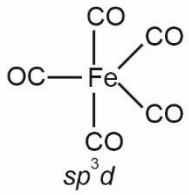
$[PtCl_4]^{-2}$



dsp^2
square planar



sp^2
triangular planar



sp^3d
(trigonal bipyramidal)

90. The potential for the given half cell at 298 K is (–) _____ × 10^{-2} V.

$2H^+_{(aq)} + 2e^- \rightarrow H_2(g)$

$[H^+] = 1M,\ P_{H_2} = 2$ atm

(Given : 2.303RT/F = 0.06 V, log2 = 0.3)

Answer (1)

Sol. $E^\circ_{H^+/H_2} = 0\,V$

$$2H^+_{(aq)} + 2e^- \longrightarrow H_2(g)$$

$$E_{Half\ Cell} = E^\circ_{Half\ Cell} - \frac{0.06}{n}\log\frac{p_{H_2}}{[H^+]^2}$$

$$E_{Half\ Cell} = 0 - \frac{0.06}{2}\log\frac{2}{[1]^2}$$

$$E_{Half\ Cell} = -0.03 \times 0.30$$

$$= -0.009$$

$$= 0.9 \times 10{-}2$$

$$0.9 \approx 1$$

❑ ❑ ❑