All India Aakash Test Series for JEE (Main)-2025

TEST - 2

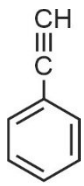
Test Date : 03/03/2024

ANSWERS

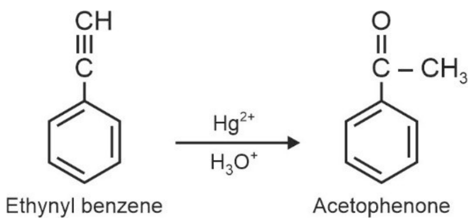
CHEMISTRY	MATHEMATICS	PHYSICS
1. (2)	31. (4)	61. (3)
2. (3)	32. (3)	62. (1)
3. (1)	33. (3)	63. (4)
4. (2)	34. (1)	64. (3)
5. (3)	35. (1)	65. (1)
6. (2)	36. (3)	66. (1)
7. (3)	37. (1)	67. (4)
8. (3)	38. (3)	68. (3)
9. (3)	39. (1)	69. (1)
10. (2)	40. (2)	70. (3)
11. (4)	41. (3)	71. (3)
12. (4)	42. (1)	72. (2)
13. (3)	43. (2)	73. (3)
14. (4)	44. (4)	74. (2)
15. (2)	45. (3)	75. (1)
16. (3)	46. (2)	76. (3)
17. (2)	47. (2)	77. (4)
18. (3)	48. (1)	78. (2)
19. (1)	49. (2)	79. (3)
20. (2)	50. (1)	80. (3)
21. (75.00)	51. (15.00)	81. (03.00)
22. (00.00)	52. (05.00)	82. (15.00)
23. (22.00)	53. (03.00)	83. (02.00)
24. (05.00)	54. (09.00)	84. (06.00)
25. (03.00)	55. (00.00)	85. (02.00)
26. (06.00)	56. (25.00)	86. (09.00)
27. (05.00)	57. (02.00)	87. (06.64)
28. (65.00)	58. (02.00)	88. (03.00)
29. (12.00)	59. (09.00)	89. (68.00)
30. (05.00)	60. (19.00)	90. (03.00)

PART - A (CHEMISTRY)

1. Answer (2)

Hint : Ethynylbenzene is 

Sol. :



2. Answer (3)

Hint : Halogen group is ortho-para directing.

Sol. : Halogen group acts as weakly deactivating.

3. Answer (1)

Hint : Melting point of lead (Pb) is more than that of tin (Sn).

Sol. : The correct order of first ionization energy is C > Si > Ge > Pb > Sn.

4. Answer (2)

Hint : Alkene is given higher priority over substituents.

Sol. : Correct IUPAC name is :

5-bromo-3-methylhept-3-ene

5. Answer (3)

Hint : Hydrocarbons on complete combustion give CO_2 and H_2O gases.

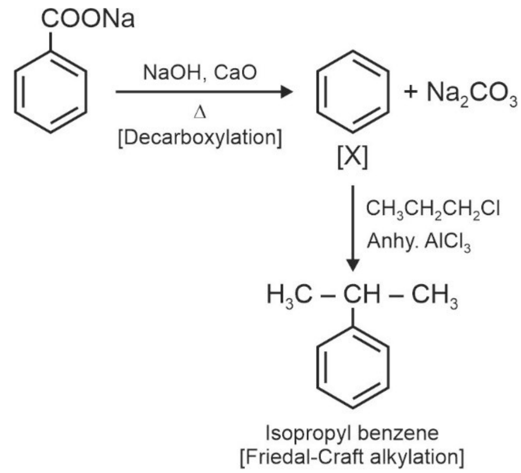
Sol. :

- $C_2H_6(g)+\frac{7}{2}O_2(g)\rightarrow 2CO_2(g)+3H_2O(g)$
- $C_2H_2(g)+\frac{5}{2}O_2(g)\rightarrow 2CO_2(g)+H_2O(g)$
- $C_2H_4(g)+3O_2(g)\rightarrow 2CO_2(g)+2H_2O(g)$
- $CH_4(g)+2O_2(g)\rightarrow CO_2(g)+2H_2O(g)$

6. Answer (2)

Hint : Rearrangement is possible in Friedel-Craft alkylation.

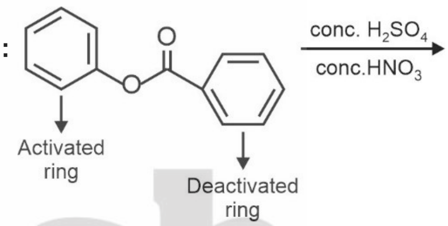
Sol. :

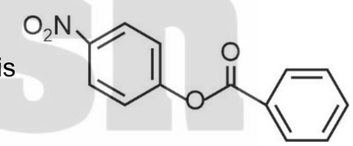


7. Answer (3)

Hint : Major product will be formed as per activating group.

Sol. :



Product is 

8. Answer (3)

Hint : 20 six-membered rings, 12 five-membered rings are present in a molecule of fullerene (C_{60}).

Sol. : In fullerene (C_{60})

x = 20

y = 12

$\therefore\ \frac{6x}{y}=\frac{6\times 20}{12}=10$

9. Answer (3)

Hint : Compound X is $(BN)_x$ and Y is $B_3N_3H_6$.

Sol. : In compound X, which is boron nitride, all the B and N atoms are sp^2 hybridised. It has graphite like structure.

10. Answer (2)

Hint : The first four members of alkanes are gaseous at 25°C.

Sol. : All four hydrocarbons propane, ethene, propyne and butane are gaseous at 25°C.

11. Answer (4)

Hint : A reaction in which the atoms of same element are simultaneously oxidised as well as reduced is called disproportionation reaction.

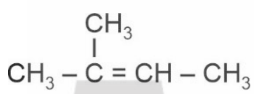
Sol. : $2\overset{+1}{H_2}\overset{-1}{O_2} \longrightarrow 2\overset{+1}{H_2}\overset{-2}{O}(l) + \overset{0}{O_2}(g)$

is an example of disproportionation reaction.

12. Answer (4)

Hint : To show geometrical isomerism, the two atoms or groups attached to each doubly bonded carbon should be different.

Sol. : 2-methylbut-2-ene does not show geometrical isomerism as it has two methyl groups on same doubly bonded carbon.



13. Answer (3)

Hint : Meq of $KMnO_4$ = Meq of Fe^{2+}

Sol. : $5e^- + Mn^{7+} \longrightarrow Mn^{2+}$

$Fe^{2+} \longrightarrow Fe^{3+} + e^-$

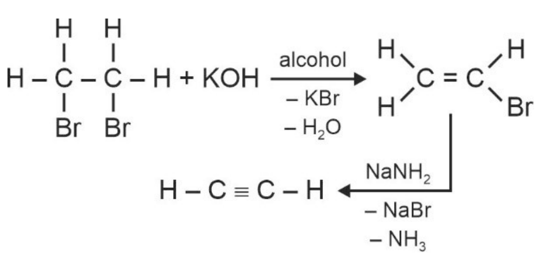
V × 0.6 × 5 = 0.40 × 20 × 1

$\therefore \quad V = \dfrac{0.40 \times 20}{0.6 \times 5} = 2.67 \text{ mL}$

14. Answer (4)

Hint : Alc. KOH is used for dehydrohalogenation.

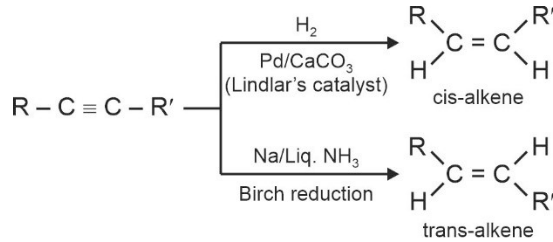
Sol. :



15. Answer (2)

Hint : Alkyne on reaction with Lindlar's catalyst produces cis-alkene.

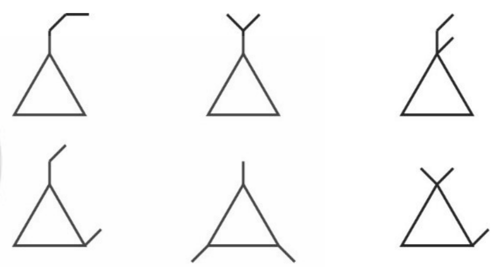
Sol. :



16. Answer (3)

Hint : Degree of unsaturation in C_6H_{12} is one.

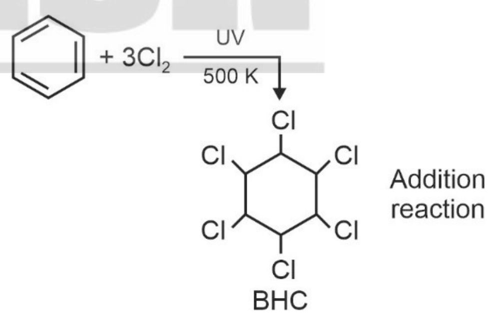
Sol. : Possible isomers are



17. Answer (2)

Hint : SO_3 acts as a neutral electrophile during sulphonation of benzene.

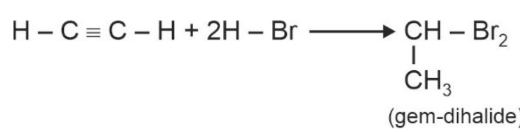
Sol. :



18. Answer (3)

Hint : Terminal alkynes give Tollen's test and ammonical cuprous chloride test.

Sol. :



19. Answer (1)

Hint : $CH_4 + O_2 \xrightarrow[\Delta]{Mo_2O_3} HCHO + H_2O$

Sol. : $CH_4 + O_2 \xrightarrow[523\,K/100\,atm]{Cu} CH_3OH$

$2CH_3 - CH_3 + 3O_2 \xrightarrow[\Delta]{(CH_3COO)_2Mn}$

$2CH_3COOH + 2H_2O$

$(CH_3)_3CH \xrightarrow[Oxidation]{KMnO_4} (CH_3)_3C - OH$

20. Answer (2)

Hint : Chain isomers have different chain of carbon atoms; position isomers have different position of C = C, C ≡ C or functional group; functional isomers have different functional groups, metamers have different alkyl groups on either side of functional group.

Sol. :

(I) $\underset{\text{But-1-ene}}{CH_3 - CH_2 - CH = CH_2}$ and

$\underset{\text{But-2-ene}}{CH_3 - CH = CH - CH_3}$ are position isomers.

(II) $\underset{\text{(Propanal)}}{CH_3 - CH_2 - CHO}$ and $\underset{\text{(Propanone)}}{CH_3 - \overset{\overset{O}{\|}}{C} - CH_3}$ are functional isomers.

(III) $\underset{\text{Ethoxyethane}}{CH_3 - CH_2 - O - CH_2 - CH_3}$ and

$\underset{\text{Methoxypropane}}{CH_3 - CH_2 - CH_2 - O - CH_3}$ are metamers.

(IV) $CH_3 - \overset{\overset{CH_3}{|}}{CH} - CH_2 - CH_3$ and $CH_3 - \overset{\overset{CH_3}{|}}{\underset{\underset{CH_3}{|}}{C}} - CH_3$

Isopentane Neopentane

are chain isomers.

21. Answer (75.00)

Hint : Milliequivalent of MnO_4^-

= Milliequivalent of Mn^{2+}

Sol. : $Mn^{2+} \longrightarrow Mn^{4+} + 2e^-$

$Mn^{7+} + 3e^- \longrightarrow Mn^{4+}$

∴ 0.1 × 3 × 25 = M × 2 × 50

$$M = \frac{0.1 \times 3 \times 25}{2 \times 50} = 0.075\ M$$

22. Answer (00.00)

Hint : Producer gas is formed.

Sol. : $2C(s) + O_2(g) + 4N_2(g) \xrightarrow{1273\,K}$

$\underset{(X)}{2CO(g)} + \underset{(Y)}{4N_2(g)}$

23. Answer (22.00)

Hint : Double bond contains 1σ and 1π bond while triple bond contains 1σ and 2π bonds.

Sol. : $CH_3 - CH = \underset{\underset{CH_3}{|}}{C} - \overset{\overset{CH_2 - CH_3}{|}}{\underset{\underset{H}{|}}{C}} - C \equiv C - CH_3$

4-Ethyl-3-methylhept-2-en-5-yne

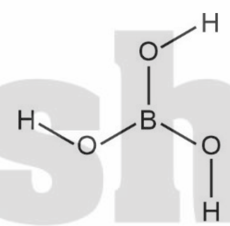
σ = 25

π = 3

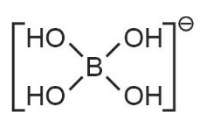
24. Answer (05.00)

Hint : Product B is $[B(OH)_4]^-$

Sol. :

H_3BO_3  sp^2 hybridisation B-atom

$H_3BO_3 + 2H_2O \rightleftharpoons H_3O^+ + [B(OH)_4]^-$

 sp^3 hybridised B-atom

n = 2, m = 3

n + m = 2 + 3 = 5

25. Answer (03.00)

Hint : $R_f = \frac{\text{Distance travelled by component}}{\text{Distance travelled by solvent}}$

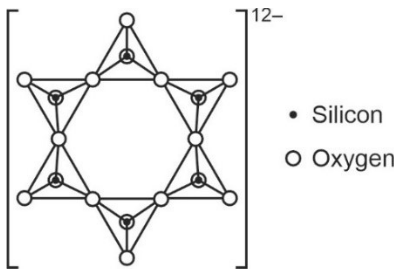
Sol. : $0.6 = \frac{\text{Distance travelled by component}}{5}$

Distance travelled by component = 3 cm

26. Answer (06.00)

Hint : Beryl is an example of cyclic silicate.

Sol. : Anionic part of the beryl.



27. Answer (05.00)

Hint : If a conjugated system has an electron withdrawing group, it shows –R or –M effect.

Sol. : –CHO, –COOH, –COOR, –CN, $-NO_2$ groups shows –R effect when attached to benzene ring.

28. Answer (65.00)

Hint : % of I = $\frac{127 \times \text{Weight of AgI}}{235 \times \text{W}} \times 100$

Sol. : W = 0.320

Weight of AgI = 0.384

% of I = $\frac{127 \times 0.384}{235 \times 0.320} \times 100 = 64.8\% \simeq 65\%$

29. Answer (12.00)

Hint : X is $[BF_4]^-$

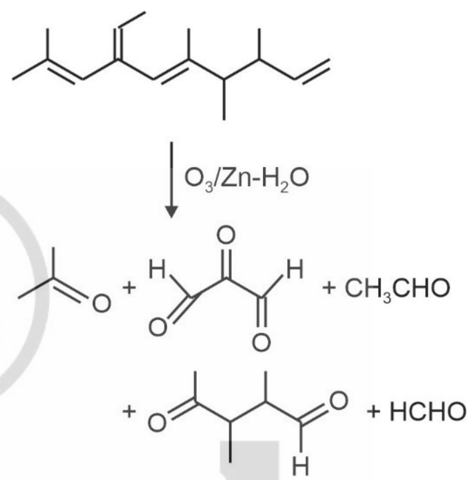
Sol. : $BF_3 + HF \longrightarrow H^+[BF_4]^-$

B atom is sp^3 hybridised.

30. Answer (05.00)

Hint : The reagent for reductive ozonolysis is $O_3/Zn\text{-}H_2O$.

Sol. :



PART - B (MATHEMATICS)

31. Answer (4)

Hint : Write $7\cos^2 x + \sin x \cdot \cos x - 3(\cos^2 x + \sin^2 x) = 0$

Sol. : Since $7\cos^2 x + \sin x \cos x - 3 = 0$

Dividing the equation by $\cos^2 x$, we get

$7 + \tan x - 3\sec^2 x = 0$

$\Rightarrow 3\tan^2 x - \tan x - 4 = 0$

$\Rightarrow \tan x = -1$ or $\tan x = \frac{4}{3}$

$\Rightarrow x = n\pi + \frac{3\pi}{4}$ or $k\pi + \tan^{-1}\left(\frac{4}{3}\right)$

32. Answer (3)

Hint : $\tan^2\alpha + 1 + \cot^2\alpha = 3$

Sol. : $\tan^2\alpha + 1 + \cot^2\alpha = 3$

$\Rightarrow \tan^2\alpha + \cot^2\alpha = 2$

$\Rightarrow \tan^2\alpha = \cot^2\alpha = 1$

$\Rightarrow \alpha = n\pi \pm \frac{\pi}{4}$

$\Rightarrow \alpha = \frac{\pi}{4}, \pi \pm \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\Rightarrow$ Sum = 4π

33. Answer (3)

Hint : Points (3, 2) and (5, –4) are end-points of diameter of circle on which *P* lies.

Sol. : Multiplying given lines we get locus

$(y-2)(y+4) = m_1 m_2 (x-3)(x-5)$

$y^2 + 2y - 8 = -(x^2 - 8x + 15)$ [$\because m_1m_2 = -1$]

$x^2 + y^2 - 8x + 2y + 7 = 0$

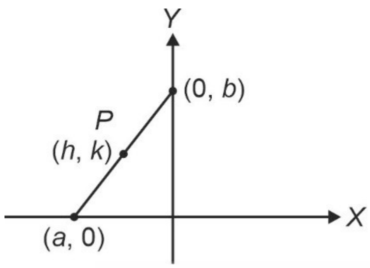
which is a circle.

$\therefore$ $g = -8, f = 2, c = 7$

34. Answer (1)

Hint : Intercepts : a, b, but length of intercept $-a$, b.

Sol. :



Length of x intercept = $-a$

Length of y intercept = b

$\therefore$ Given $b - a = l$ …(i)

Let circumcentre is $P(h, k)$.

$\therefore$ $h = \frac{a}{2}, k = \frac{b}{2}$

$2(k - h) = b - a$ …(ii)

From (i) and (ii),

Locus is $2(y - x) = l$

35. Answer (1)

Hint : Algebraic distance are signed distances.

Sol. : Let the line is $ax + by + c = 0$

Sum of algebraic distance = 0

$$\frac{a(0)+b(-2)+c}{\sqrt{a^2+b^2}} + \frac{2a+0b+c}{\sqrt{a^2+b^2}} + \frac{a+b+c}{\sqrt{a^2+b^2}} = 0$$

$(0 + 2 + 1)a + (-2 + 0 + 1)b + 3c = 0$

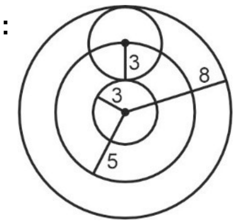
$$\left(\frac{0+2+1}{3}\right)a + \left(\frac{1+0-2}{3}\right)b + c = 0$$

The line passes through the centroid of triangle ABC.

36. Answer (3)

Hint : Any point in set lies on circle, so maximum minimum distance $C + 3$, $C - 3$.

Sol. :



From figure, it is clear that point will lie between two concentric circles

$x^2 + y^2 = 4$ and $x^2 + y^2 = 64$

$\therefore$ Locus is $4 \le x^2 + y^2 \le 64$

37. Answer (1)

Hint : Use orthogonality condition.

Sol. : Let circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

It passes through origin and intersects $x^2 + y^2 - 5x + 3y - 1 = 0$ orthogonally.

$\therefore$ $c = 0$

$$2\left(-\frac{5}{2}\right)g + 2\left(\frac{3}{2}\right)f = c - 1$$

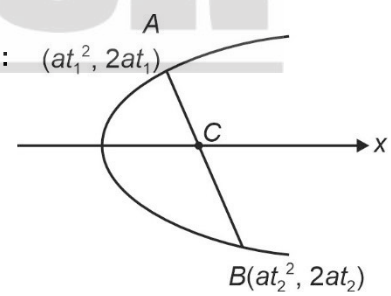
$\Rightarrow$ $-5g + 3f = -1$

Hence, locus is $5x - 3y = -1$

38. Answer (3)

Hint : Use section formula.

Sol. :



$AC : AB = 1 : 3$

$AC : CB = 1 : 2$

$$C = \left(\frac{2at_1^2 + at_2^2}{3}, \frac{4at_1 + 2at_2}{3}\right)$$

$\because$ C lies on x-axis

$$\therefore \frac{4at_1 + 2at_2}{3} = 0 \Rightarrow t_2 + 2t_1 = 0$$

39. Answer (1)

Hint : Use parametric form for the coordinates of point.

Sol. : Any point on the ellipse is

$(2\cos\theta, \sqrt{3}\sin\theta)$

The focus on the positive x-axis is (1, 0).

Given that

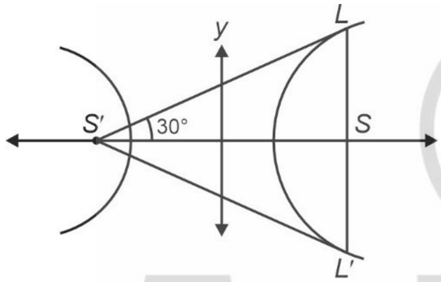
$$(2\cos\theta - 1)^2 + 3\sin^2\theta = \frac{25}{16}$$

$$\Rightarrow \cos\theta = \frac{3}{4}$$

40. Answer (2)

Hint : Find the SL by tan30° and the triangle will be symmetrical about x-axis.

Sol. :



In $\Delta SS'L$,

$$\tan 30^\circ = \frac{LS}{SS'}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\frac{b^2}{a}}{2ae}$$

$$\Rightarrow 2a^2e = \sqrt{3}b^2$$

$$\Rightarrow 2a^2e = \sqrt{3}a^2(e^2 - 1)$$

$$\Rightarrow \sqrt{3}e^2 - 2e - \sqrt{3} = 0$$

$$\Rightarrow e = \sqrt{3}$$

41. Answer (3)

Hint : Use formula for normal of a parabola.

Sol. : Let parabola $y^2 = 4ax$

Equation of normal $y = mx - 2am - am^3$

It passes through (h, k).

$am^3 + m(2a - h) + k = 0$ …(i)

Let slope of normal are

$m_1 = \tan\theta_1$

$m_2 = \tan\theta_2$

$m_3 = \tan\theta_3$

Let m_1 and m_2 make complementary angle it means

$\theta_2 = 90^\circ - \theta_1$

$$\tan\theta_2 = \cot\theta_1 = \frac{1}{\tan\theta_1}$$

$$m_2 = \frac{1}{m_1}$$

From (i),

$$m_1 \times m_2 \times m_3 = -\frac{k}{a}$$

$$m_3 = -\frac{k}{a}$$

m_3 is root of equation (i) put value of m_3 in equation (i).

$k^2 = a(h - a)$

The locus of point P is $y^2 = a(x - a)$

42. Answer (1)

Hint : AM ≥ GM

Sol. : $$\frac{2\sin x + 2\cos x + \sec x + \operatorname{cosec} x + \sqrt{2}\tan x + \sqrt{2}\cot x}{6} \ge \sqrt{2}$$

(∵ AM ≥ GM)

$\Rightarrow$ 2sin$x$ = 2cos$x$ = sec$x$ = cosec$x$

$$= \sqrt{2}\tan x = \sqrt{2}\cot x = \sqrt{2}$$

$\Rightarrow$ Variance = 0

43. Answer (2)

Hint : Use binomial theorem.

Sol. : $17^2 = 289 = 290 - 1$

$17^{256} = (17^2)^{128} = (290 - 1)^{128}$

$= {}^{128}C_0(290)^{128} - \ldots + {}^{128}C_{126}(290)^2 - {}^{128}C_{127}(290) + {}^{128}C_{128}$

$= 1000\lambda + 128.127.290.145 - 128 \times 290 + 1$

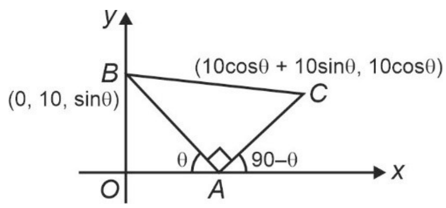
$= 1000(\lambda + 1) + 681$

Last two digits are 81.

44. Answer (4)

Hint : Mid-point of BC is circumcentre of ΔABC.

Sol. :



Circumcentre is mid-point of BC is

$(5\cos\theta + 5\sin\theta, 5\cos\theta + 5\sin\theta)$

Locus is $y = x$

45. Answer (3)

Hint : $|\cos x| = -\cos x$; $|\sin x| = \sin x$ at $x = \frac{2\pi}{3}$

Sol. : Around $x = \frac{2\pi}{3}$,

$|\cos x| = -\cos x$ and $|\sin x| = \sin x$

$\therefore \quad y = -\cos x + \sin x$

$$\therefore \quad \frac{dy}{dx} = \sin x + \cos x$$

At $x = \frac{2\pi}{3}$,

$$\frac{dy}{dx} = \sin\frac{2\pi}{3} + \cos\frac{2\pi}{3}$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{1}{2}(\sqrt{3}-1)$$

46. Answer (2)

Hint : Limit form 1^∞

Sol. : $f(n) = (1)^\infty$

Apply the formula,

$$f(n) = \lim_{x\to\infty} e^{\frac{1}{x}\left(\left(1+\sin\frac{x}{2}\right)\left(1+\sin\frac{x}{2^2}\right)\left(1+\sin\frac{x}{2^n}\right)-1\right)}$$

$$= \lim_{x\to\infty} e^{\frac{\left(1+\left(\sin\frac{x}{2}+\sin\frac{x}{2^2}+...+\sin\frac{x}{2^n}\right)...\left(\sin\frac{x}{2}\sin\frac{x}{2^2}+...\right)-1\right)}{x}}$$

$$= \lim_{x\to 0} e^{\left(\frac{\sin\frac{x}{2}}{x}+\frac{\sin\left(\frac{x}{2^2}\right)}{x}+...+\frac{\sin\frac{x}{2^n}}{x}\right)}$$

$$= e^{\left(\frac{1}{2}+\frac{1}{2^2}+...+\frac{1}{2^n}\right)}$$

Sum of infinite G.P. $= e^{\frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)}} = e$

47. Answer (2)

Hint : Put $z = x + iy$

Sol. : $\text{Re}\left(\frac{z-2i+1-1}{z+1}\right) = 1$

$$\Rightarrow \quad \text{Re}\left(\frac{-1-2i}{z+1}+1\right) = 1$$

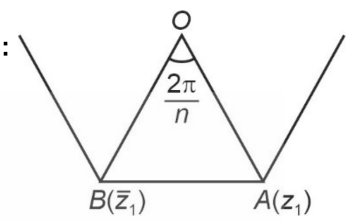
$$\Rightarrow \quad \text{Re}\left(\frac{-1-2i}{z+1}\right) = 0$$

$\Rightarrow \quad x + 1 + 2y = 0$

48. Answer (1)

Hint : $z_1 = re^{i\theta}$; $\bar{z}_1 = re^{-i\theta}$ etc.

Sol. :



$$\frac{z_1-0}{\bar{z}_1-0} = \frac{|z_1|}{|\bar{z}_1|}\cdot e^{\frac{2\pi i}{n}} \Rightarrow \frac{z_1}{\bar{z}_1} = e^{\frac{2\pi i}{n}}$$

Let $z_1 = re^{i\theta}$

$$\therefore \quad \frac{\text{Im}(z_1)}{\text{Re}(z_1)} = \sqrt{2}-1$$

$$\Rightarrow \quad \frac{r\sin\frac{\pi}{n}}{r\cos\frac{\pi}{n}} = \sqrt{2}-1$$

$$\tan\frac{\pi}{n} = \tan\frac{\pi}{8}$$

$\therefore \quad n = 8$

49. Answer (2)

Hint : Required probability $= \frac{\text{Favourable}}{\text{Total}}$

Sol. : Let one of quantities is x.

And other quantities is $(2n - x)$.

Their product will be greatest when they are equal.

So, product is n^2

According to the problem,

$$x(2n-x) \geq \frac{3}{4}n^2$$

$$(2x - 3n) \cdot (2x - n) \leq 0$$

$$\frac{n}{2} \leq x \leq \frac{3}{2}n$$

Favourable cases = $\frac{3}{2}n - \frac{n}{2} = n$

$\therefore$ Required problem = $\frac{n}{2n} = \frac{1}{2}$

50. Answer (1)

Hint : Put $z = x + iy$

Sol. : $z = re^{i\theta} = x + iy$, $\arg z = \theta$

$$\sqrt{(x-3)^2 + (y-2)^2} = \sqrt{x^2 + y^2}\sin\left(\frac{\pi}{4} - \theta\right)$$

$$\Rightarrow \sqrt{(x-3)^2 + (y-2)^2} = \frac{x-y}{\sqrt{2}}$$

Distance between (x, y) and $(3, 2)$.

$\therefore$ Locus is parabola with focus $(3, 2)$ and directrix $x - y = 0$

51. Answer (15.00)

Hint : Use formula of mean and variance.

Sol. : Mean $\bar{x} = \frac{72}{8} = 9$

$\Rightarrow$ Variance = σ^2

$$= \frac{1}{8}[0 + 36 + 1 + 1 + 0 + 1 + 0 + 81]$$

$= 15$

52. Answer (05.00)

Hint : Common chord of the two circle is

$6x + 14y + c + d = 0$

Sol. : Common chord of the two circle is

$6x + 14y + c + d = 0$

This should pass through the centre of second circle

$\Rightarrow$ $6(1) + 14(-4) + c + d = 0$

$$\Rightarrow c + d = 50 \Rightarrow \frac{c+d}{10} = 5$$

53. Answer (03.00)

Hint : $e^2 = 1 + \frac{b^2}{a^2}$

Sol. : Foci of the ellipse are $(3, 0)$ and $(-3, 0)$ for hyperbola

$$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{7e^2}{9} \text{ (as } ae = 3)$$

$$\Rightarrow e^2 - \frac{7e^2}{9} = 1 \Rightarrow e = \frac{3}{\sqrt{2}}$$

$\therefore$ $k = 3$

54. Answer (09.00)

Hint : The semi-latus rectum is the H.M. of SP and SQ.

Sol. : The length of the latus rectum of $y = ax^2 + bx + c$ is $\frac{1}{a}$. Now, the semi-latus rectum is the H.M. of SP and SQ.

$\therefore$ We have,

$$\frac{1}{SP} + \frac{1}{SQ} = \frac{2}{\frac{1}{2a}}$$

$$\Rightarrow 4a = \frac{1}{4} + \frac{1}{6} = \frac{5}{12} \Rightarrow a = \frac{5}{48}$$

Hence, latus rectum $L = \frac{1}{a} = \frac{48}{5}$

$\therefore$ $[L] = 9$

55. Answer (00.00)

Hint : Use binomial theorem.

Sol. : $(37)^{n+2} + 16^{n+1} + 30^n$

$= (35 + 2)^{n+2} + (14 + 2)^{n+1} + (28 + 2)^n$

$= 7k + 2^{n+2} + 2^{n+1} + 2^n$

$= 7k + 2^n \cdot 7$

$\therefore$ Remainder is 0.

56. Answer (25.00)

Hint : $N_1 = {}^{n+3}C_3$, $N_2 = {}^{n+2}C_2$

Sol. : $N_1 = {}^{n+3}C_3$, $N_2 = {}^{n+2}C_2$

$$\frac{N_1}{N_2} = \frac{(n+3)}{3} \quad [\because n \text{ will be a multiple of 3}]$$

as $\frac{N_1}{N_2}$ is a natural number.

$$\therefore \quad \frac{n+3}{3} > 9$$

$n > 24$

57. Answer (02.00)

Hint : $\sin^2\alpha + \cos^2\alpha + \cos^2 2\alpha = 1$

Sol. : $\sin^2\alpha + \cos^2\alpha + \cos^2 2\alpha = 1$

$\Rightarrow \cos 2\alpha = 0$

$$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{2}},\ \sin\alpha = \pm\frac{1}{\sqrt{2}}$$

$\Rightarrow$ Direction cosines are

$$\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle, \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right\rangle,$$

$$\left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle, \left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right\rangle$$

58. Answer (02.00)

Hint : Use half angle formula.

Sol. : $$16 \lim_{x\to 0} \frac{1-\cos(1-\cos x)}{x^4}$$

$$= 16 \lim_{x\to 0} \frac{1-\cos\left(2\sin^2\frac{x}{2}\right)}{x^4}$$

$$= 16 \lim_{x\to 0} \frac{2\sin^2\left(\sin^2\frac{x}{2}\right)}{x^4}$$

$$= 16\cdot 2 \lim_{x\to 0} \left[\frac{\sin^2\left(\sin^2\frac{x}{2}\right)}{x^4}\right]^2$$

$$= 32 \lim_{x\to 0} \left[\frac{\sin\left(\sin^2\frac{x}{2}\right)}{\sin^2\frac{x}{2}} \cdot \frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2} \cdot \frac{1}{4}\right]^2$$

$$= 32\left(\frac{1}{4}\right)^2 = \frac{32}{16} = 2$$

59. Answer (09.00)

Hint : Make cases

Sol. : $n(S)$ = Number of element in sample Space = 6^4

$n(A) = 6 \times 5 \times (1 \times 5 + 4 \times 4)$

So, probability = $\frac{6\times 5\times 21}{6^4} = \frac{630}{1296}$

60. Answer (19.00)

Hint : Use discriminant ≥ 0.

Sol. : $S = \{x^2 + bx + c = 0 \mid b, c \in \{1, 2, 3, 4, 5, 6\}\}$

E = Equation has real roots

We know that E is possible when for

$Ax^2 + Bx + C = 0$, $B^2 - 4AC \geq 0$

Here, $A = 1$, $B = b$, $C = c$

$\therefore$ E occurs when $b^2 - 4c \geq 0 \Rightarrow b^2 \geq 4c$

This is possible for following values of (b, c).

(2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (3, 2), (4, 2), (5, 2), (6, 2), (4, 3), (5, 3), (6, 3), (4, 4), (5, 4), (6, 4), (5, 5), (6, 5), (5, 6), (6, 6)

i.e., for 19 different values of (b, c), E can occur.

But, $b, c \in \{1, 2, 3, 4, 5, 6\}$

$\therefore$ There are a total of 6 × 6 = 36 possible values of (b, c).

$$\therefore \quad P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{\text{Number of possible values of } (b, c) \text{ for } E \text{ to occur}}{\text{Total number of possible values of } (b, c)}$$

$$= \frac{19}{36}$$

$\therefore$ The probability that an equation chosen at random will have real roots is $\frac{19}{36}$.

PART - C (PHYSICS)

61. Answer (3)

Hint : $g_h = g\left(1-\frac{2h}{R_e}\right)$ and $g_d = g\left(1-\frac{d}{R_e}\right)$

Sol. : $g_h = g\left(1-\frac{2h}{R_e}\right)$ and $g_d = g\left(1-\frac{d}{R_e}\right)$

where R_e is the radius of earth and h = 2 km.

Here, $gh = gd$

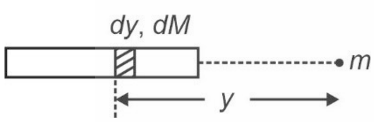
$$g\left(1-\frac{2\times 2\text{ km}}{R_e}\right) = g\left(1-\frac{d}{R_e}\right)$$

$\therefore$ d = 4 km

62. Answer (1)

Hint : Gravitational field = $\frac{GM}{r^2}$

Sol. :



Gravitational force due to small mass (dm),

$$d_F = \frac{G(dM)\cdot m}{y^2}$$

Here, $dM = \frac{M}{\ell}\cdot dy$

So, gravitational force due to rod on mass,

$$F = \int dF$$

$$F = \int_x^{\ell+x} \frac{GM\cdot m\cdot dy}{y^2}$$

$$\therefore \quad F = \frac{GMm}{x(\ell+x)}$$

63. Answer (4)

Hint : $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \text{constant}$

Sol. : $v = \omega\sqrt{A^2 - x^2}$

At $x = 0$, $v \Rightarrow$ maximum

At $x = \pm A$, $v \Rightarrow$ minimum

64. Answer (3)

Hint : $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

Sol. : $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$$P_1 - P_2 = \frac{1}{2}\rho\,(v_2^2 - v_1^2)$$

$$\rho_0 gh - \rho gh = \frac{1}{2}\rho\,(v_2^2 - v_1^2) \quad ...(i)$$

$$\pi R^2 v_1 = \pi r^2 v_2 \quad ...(ii)$$

Solving, we get

$$v_2^2 = \frac{2(\rho_0 - \rho)}{\rho}\frac{R^4}{(R^4 - r^4)}gh$$

65. Answer (1)

Hint : $PV = nRT$

For $P_{\max}$ = $\frac{dP}{dT} = 0$

Sol. : $T = T_0 + \gamma V^2$ $\quad$ ($V = \frac{RT}{P}$ for 1 mole)

$$\Rightarrow \quad T = T_0 + \gamma\frac{R^2T^2}{P^2}$$

$$\Rightarrow \quad P = \sqrt{\gamma}RT(T - T_0)^{-1/2}$$

For minimum P,

$$\frac{dP}{dT} = 0 \;\Rightarrow\; T = 2T_0$$

So, $T = T_0 + \gamma V^2$

$$\Rightarrow \quad T_0 = \gamma V^2 \;\Rightarrow\; V = \sqrt{\frac{T_0}{\gamma}}$$

66. Answer (1)

Hint : Beat = $|n_1 - n_2|$

Sol. : For 1st wave,

$$\text{Frequency} = \frac{\text{Speed}}{\text{Wavelength}}$$

$$n_1 = \frac{V}{0.6}$$

For 2nd wave,

$n_2 = \frac{V}{0.62}$

Beat = $n_1 - n_2$

$18 = \frac{V}{0.6} - \frac{V}{0.62}$

V = 335 m/s

67. Answer (4)

Hint : Wave travelling along same axis will produce stationary wave.

Sol. : $y_1 \rightarrow$ travelling along +ve x-axis

$y_2 \rightarrow$ travelling along –ve x-axis

$y_3 \rightarrow$ travelling along –ve z-axis

So, $y_1 + y_2$ will produce stationary wave.

68. Answer (3)

Hint : $\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$

Sol. : $\frac{I_{\max}}{I_{\min}} = \frac{(4+2)^2}{(4-2)^2} = \frac{36}{4} = 9:1$

69. Answer (1)

Hint : $U = \frac{f}{2}PV$

Sol. : Volume of gas = $\frac{2}{8} = \frac{1}{4}\text{ m}^3$

Degree of freedom = 5

Then, internal energy (U) = $\frac{f}{2}PV$

$= \frac{5}{2} \times 10 \times 10^4 \times \frac{1}{4}$

$= 10^5$ J

70. Answer (3)

Hint : For isothermal process, PV = constant

Sol. : $P \cdot V$ = constant

So, option (3) is correct.

71. Answer (3)

Hint : $\Delta Q = nC_p\Delta T \rightarrow$ isobaric

$\Delta Q = \Delta W = nRT\ \ln\left(\frac{V_2}{V_1}\right) \rightarrow$ isothermal

Sol. : $A \rightarrow B$ isobaric process

$\Delta Q = 1 \times \frac{5R}{2}(2T_0 - T_0) = \frac{5}{2}RT_0$

$B \rightarrow C$ isothermal process

$\Delta Q = W = 1 \times R \times 2T_0\ \ln\left(\frac{3P_0}{P_0}\right) = 2RT_0\ \ln 3$

72. Answer (2)

Hint : Radiation power, $E = \sigma AT^4$

Sol. : According to Stefan-Boltzman law, rate of energy radiation by a black body is given as

$E = \sigma AT^4$

$= \sigma 4\pi R^2 T^4$

So, $\frac{E_1}{E_2} = \left(\frac{R_1}{R_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4$

$\frac{300}{E_2} = \left(\frac{R_1}{2R_1}\right)^2 \left(\frac{T_1}{T_{1/2}}\right)^4$

E_2 = 75 watt

73. Answer (3)

Hint : Force by surface tension = $2LS$

Sol. : At equilibrium,

$2LS = W$

$S = \frac{5 \times 10^{-2}}{2 \times 0.8}$

$S = 3.1 \times 10^{-2}$ N/m

74. Answer (2)

Hint : Theoretical.

Sol. : Archimede's upward thrust ($B = \rho_\ell vg$) depends on gravity.

75. Answer (1)

Hint : Use Bernoulli's theorem,

$P + \rho vg + \frac{1}{2}\rho v^2 = \text{constant}$

Sol. : From Bernoulli's theorem,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2,$$

P_1 = Pressure (inside)

So, $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$

$$\Delta P = \frac{1}{2}\times 1.2(400-0)$$

$$\Delta P = \frac{1}{2}\times 1.2\times 400 = 240,\ P_1 > P_2$$

Force acting on roof,

$F = \Delta P \times$ Area

$= 240 \times 200$

$= 48 \times 10^3$

76. Answer (3)

Hint : $Y = \frac{\text{Stress}}{\text{Strain}}$

Sol. : Young modulus (Y) = $\frac{\text{Stress}}{\text{Strain}}$ = Slope

$Y_A = \tan 45°$

$Y_B = \tan 30°$

$$\frac{Y_A}{Y_B} = \frac{\tan 45°}{\tan 30°}$$

$$Y_A = \sqrt{3}Y_B$$

77. Answer (4)

Hint : Energy stored

$= \frac{1}{2}$ stress × strain × volume

Sol. : Energy stored in stretched wire,

$E = \frac{1}{2}$ × stress × strain × volume

Energy per unit volume,

$\left(\frac{E}{V}\right) = \frac{1}{2}$ × stress × strain × volume

$$\frac{E}{V} = \frac{1}{2}\times Y \times (\text{strain})^2$$

$$= \frac{1}{2}\times 2.0\times 10^{11}\left(\frac{2.0\times 10^{-3}}{10}\right)^2$$

$= 10^{11} \times 4 \times 10^{-8}$

$= 4 \times 10^3$ J/m^3

78. Answer (2)

Hint : Net potential = Potential due to original mass – Potential due to removed mass

Sol. : Potential at point *P* due to complete solid sphere

$$V_s = \frac{GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right] = -\frac{11GM}{8R}$$

Mass of removed part = $\frac{M}{8}$

Potential at point *P* due to removed part

$$V_e = -\frac{3}{2}\frac{GM\times 2}{8R} = -\frac{3GM}{8R}$$

So, potential due to remaining part, $V = V_s - V_c$

$$V = -\frac{GM}{R}$$

79. Answer (3)

Hint : $mgh = nC_v\Delta T$

Sol. : At maximum temperature, speed of ball is zero. Whole P.E. is converted to internal energy $mgh = nC_v\Delta T$

$$mgh = \frac{(P_0 + \Delta P)V}{RT}C_v(T_{\max} - T)$$

80. Answer (3)

Hint : $v_e = \sqrt{2gR}$

Sol. : Escape velocity $(v_e = \sqrt{2gR})$ independent an angle of projection.

81. Answer (03.00)

Hint : $T^2 \propto r^3$

Sol. : From Kepler's third law,

$$\left(\frac{r_1}{r_2}\right)^3 = \left(\frac{T_1}{T_2}\right)^2$$

$$\left(\frac{10^{14}}{10^{12}}\right)^3 = \left(\frac{T_1}{T_2}\right)^2$$

$$10^6 = \left(\frac{T_1}{T_2}\right)^2$$

$$\therefore\ \frac{T_1}{T_2} = 10^3$$

82. Answer (15.00)

Hint : $T = 2\pi\sqrt{\frac{M}{k}}$

Sol. : $\frac{ML^2}{3}\alpha = -k\frac{L}{2}\theta \cdot \frac{L}{2} - kL\theta L$

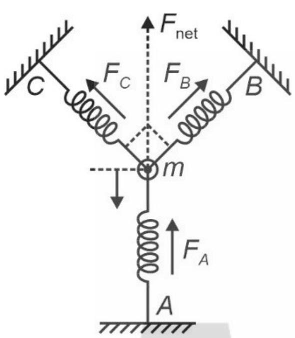
$$\alpha = -\left(3 \times \frac{5k}{4M}\right)\theta$$

$$T = 2\pi\sqrt{\frac{4M}{15k}} = 4\pi\sqrt{\frac{M}{15k}}$$

83. Answer (02.00)

Hint : $T = 2\pi\sqrt{\frac{m}{k'}}$

Sol. :



Let particle displaced x against spring A.

Then, $F_{net} = F_A + F_B\cos45° + F_C\cos45°$

$F_{net} = F_A + 2ky'\cos45°$

$F_{net} = ky + 2k(y\cos45°)\cos45°$

$F_{net} = 2ky$

$k'y = 2ky$, $k' = 2k$

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

$n = 2$

84. Answer (06.00)

Hint : $v = \sqrt{\frac{T}{\mu}}$

Sol. : Tension at a distance x from its lower end is

$$\int_0^x \lambda_0 x^2 dx\, g = \frac{\lambda_0 x^3 g}{3}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\lambda_0 x^3 g}{3\lambda_0 x^2}} = \frac{\sqrt{xg}}{\sqrt{3}}$$

$$a = \frac{v\,dv}{dx} = \frac{1}{\sqrt{3}}\sqrt{g} \cdot \frac{1}{2\sqrt{x}}\frac{\sqrt{xg}}{\sqrt{3}} = \frac{g}{6}$$

85. Answer (02.00)

Hint : Fundamental frequency in open tube,

$$n_0 = \frac{\ell}{2\ell}$$

Fundamental frequency in closed tube,

$$n_0 = \frac{\ell}{4\ell}$$

Sol. : Open tube → $n_A = \frac{\ell}{2\ell}$

Closed tube → $n_B = \frac{\ell}{4\ell}$

$$\frac{n_A}{n_B} = 2:1$$

86. Answer (09.00)

Hint : $PV = nRT$

Sol. : Equation of straight line from given coordinates,

$$P = 3P_0 - \frac{P_0}{V_0}V \quad \text{...(i)}$$

$$nRT = 3P_0V - \frac{P_0}{V_0}V^2$$

Differentiate w.r.t. volume,

$$3P_0 - 2\frac{P_0}{V_0}V = 0$$

$$V = \frac{3}{2}V_0$$

From equation (i),

$$P = \frac{3P_0}{2}$$

Then, $PV = nRT$

$$T = \frac{9P_0V_0}{4nR}$$

87. Answer (06.64)

Hint : Total work done by gas.

Sol. : $W_{BC} = 2R(T_C - T_B)$

$W_{AB} = W_{CD} = 0$ (Isochoric process)

$W_{DA} = 2R(T_A - T_D)$

Total work done,

$W = 2R(T_C - T_B + T_A - T_D)$

$W = 2R(1200 - 400 + 200 - 600)$

$= 2R(400)$

$= 6640$ J

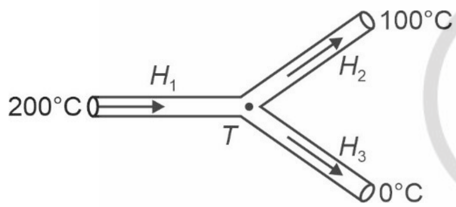
$= 06.64$ kJ

88. Answer (03.00)

Hint : Total incoming heat current

= Total outgoing heat current

Sol. :



Let temperature at junction is T.

Then, $H_1 = H_2 + H_3$

$$\frac{200 - T}{\left(\frac{\ell}{2KA}\right)} = \frac{T - 0}{\left(\frac{\ell}{6KA}\right)} + \frac{T - 100}{\left(\frac{\ell}{4KA}\right)}$$

$2(200 - T) = 6T + 4(T - 100)$

$200 - T = 3T + 2T - 200$

$$T = \frac{400}{6} = 66.67°\text{C}$$

89. Answer (68.00)

Hint : Heat lost = Heat gain

Sol. : $\Delta Q = \frac{mgh}{2}$

$$mL = \frac{mgh}{2}$$

$$h = \frac{2L}{g}$$

$$h = \frac{2 \times 3.4 \times 10^5}{10} = 68 \text{ km}$$

90. Answer (03.00)

Hint : Work done

= Surface tension × charge in area

Sol. : Work done = Surface tension of film

× Change in area of film

$W = T[2(A_2 - A_1)]$

$$T = \frac{4 \times 10^{-4}}{2(6 \times 10^{-4})}$$

$$T = \frac{1}{3} \text{ N/m}$$

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