

**Aakash**

Medical | IIT-JEE | Foundations

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MM : 180

AIATS For One Year JEE(Advanced)-2026 (XII Passed)_Test-1A_Paper-1_Online

Time : 180 Min.

PHYSICS

Section-I

1. (C)
2. (D)
3. (D)
4. (C)

Section-II



5. (B,C)
6. (B,C,D)
7. (B,C)

Section-III

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8. (2)
9. (1)
10. (6)
11. (21)
12. (12)
13. (10)

Section-IV

14. (B)
15. (C)
16. (D)
17. (D)

CHEMISTRY

Section-I

18. (A)
19. (B)
20. (B)

21. (D)

Section-II

22. (A,B)

23. (A,C)

24. (A,B,C,D)

Section-III

25. (4)

26. (8)

27. (75)

28. (3)

29. (5)

30. (3)

Section-IV

31. (A)

32. (B)

33. (C)

34. (A)



MATHEMATICS

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Section-I

35. (D)

36. (B)

37. (A)

38. (A)

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Section-II

39. (B,D)

40. (B,C,D)

41. (B,C)

Section-III

42. (2)

43. (1)

44. (5)

45. (3)

46. (3)

47. (39)

Section-IV

- 48. (A)
- 49. (B)
- 50. (B)
- 51. (B)



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Hints and Solutions

PHYSICS

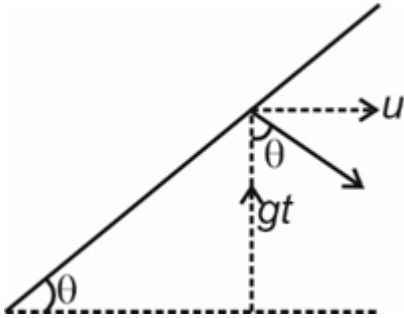
Section-I

(1) Answer : (C)

Hint:

Projectile motion

Solution:



$$\tan \theta = \frac{u}{gt} \text{ also } \tan \theta = \frac{H - \frac{1}{2}gt^2}{ut}$$

$$u = \sqrt{\frac{2gh}{2\cot^2 \theta}}$$

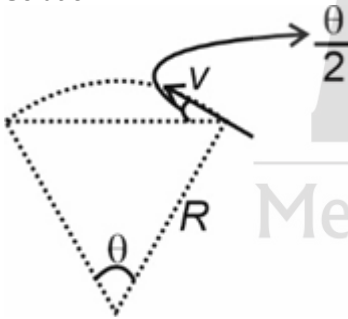
$$u = 4\sqrt{5} \text{ m/s}$$

(2) Answer : (D)

Hint:

Range equal to $2R \sin \frac{\theta}{2}$

Solution:



$$\frac{1}{2}mv^2 = mg \cdot \frac{5R}{2} - mgR \left(1 + \cos \frac{\theta}{2}\right)$$

$$\text{Range} = 2R \sin \frac{\theta}{2} = \frac{v^2 \sin \theta}{2g}$$

$$\frac{\theta}{2} = 60^\circ$$

$$\theta = 120^\circ$$

$$\frac{\theta}{3} = 40$$

(3) Answer : (D)

Hint:

$$Mg + T = F_B$$

Solution:

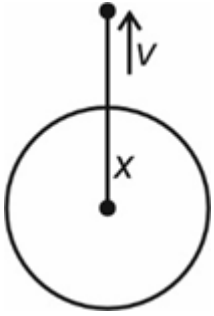
$$\frac{\pi R^3}{3} \rho g - \frac{\pi R^3 \rho g}{6} = T$$

(4) Answer : (C)

Hint:

$$\frac{dx}{dt} = \sqrt{\frac{2GM}{x}}$$

Solution:



$$\frac{1}{2}mv^2 - \frac{GMm}{x} = 0$$

$$v = \sqrt{\frac{2GM}{x}}$$

$$\frac{dx}{dt} = \sqrt{\frac{2GM}{x}}$$

$$\int_R^{4R} \sqrt{x} dx = \sqrt{2GM} \int_0^t dt$$

$$t = \frac{14}{3} \sqrt{\frac{R^3}{2GM}}$$

Section-II

(5) Answer : (B,C)

Hint:

$$a = 4(8 - v)$$

Solution:

$$a = 4(8 - v)$$

$$\int_4^v \frac{dv}{8-v} = \int_0^t 4 dt$$

$$v = 8 - 4e^{-4t}$$

$$v(\ln 2) = 8 - 4e^{-4 \ln 2}$$

$$= 8 - 4 \cdot \frac{1}{16}$$

$$= 8 - \frac{1}{4}$$

$$= \frac{31}{4} \text{ m/s}$$

(6) Answer : (B,C,D)

Hint:

Work energy theorem.

Solution:

Conceptual

(7) Answer : (B,C)

Hint:

$$\Delta z = \frac{2\Delta v}{v} + \frac{\Delta r}{r}$$

Solution:

$$\text{Let } t = \frac{v^2}{R} \quad z = \ln t$$

$$\Delta z = \frac{\Delta t}{t}$$

$$\Delta z = \frac{\Delta t}{t} = \frac{2\Delta v}{v} + \frac{\Delta z}{z}$$

$$\Delta z = 2 \times 0.02 + 0.04$$

$$= 0.08$$

Section-III

(8) Answer : 2

Hint:

$$\cos 2\theta + \mu \sin \theta = 0$$

Solution:

$$t = \sqrt{\frac{2l \sec \theta}{a}} = \sqrt{\frac{2l \sec \theta}{g \sin \theta - \mu g \cos \theta}}$$

$$\frac{dt}{d\theta} = 0$$

$$\cos 2\theta + \mu \sin 2\theta = 0$$

$$\tan 2\alpha = -\frac{1}{\mu} = -\frac{1}{0.4}$$

$$2\tan 2\alpha + 7 = 2$$

(9) Answer : 1

Hint:

$$\vec{F} = -\frac{\partial U}{\partial r}$$

Solution:

$$U = \sqrt{2} \sin(x^2 + y)$$

$$\vec{F} = -\frac{\partial U}{\partial x} \hat{i} - \frac{\partial U}{\partial y} \hat{j}$$

$$= -\sqrt{2} \cos(x^2 + y) \cdot 2x \hat{i} - \sqrt{2} \cos(x^2 + y) \hat{j}$$

$$\vec{F} = \sqrt{2} \cdot \frac{1}{\sqrt{2}} \hat{j}$$

$$\left| \vec{F} \right| = 1 \text{ N}$$

(10) Answer : 6

Hint:

$$rF \sin \theta = \tau$$

Solution:

$$\tau = \frac{1}{2} \times 20 \times \frac{3}{5}$$

$$= 6$$

(11) Answer : 21

Hint:

$$\text{stress} = \frac{F}{A}$$

Solution:

$$\frac{(\text{Stress})_A}{(\text{Stress})_B} = \left(\frac{M_A + M_B}{M_A} \right) \times \frac{A_B}{A_A}$$

$$= \left(\frac{\rho_A r_A^2 l_A}{\rho_B r_B^2 l_B} + 1 \right) = \frac{r_B^2}{r_A^2}$$

$$= \left(6 \times \frac{1}{9} \times 2 + 1 \right) \cdot 3^2$$

$$= \frac{7}{3} \times 3 \times 3$$

$$= 21$$

(12) Answer : 12

Hint:

$$\vec{E}_1 = \frac{\rho r}{3\epsilon_0}$$

Solution:

$$\vec{E}_1 = \frac{\rho r}{3\epsilon_0} = \frac{\rho R}{6\epsilon_0} \quad \vec{E}_2 = -\frac{\rho R}{24\epsilon_0}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\rho R}{6\epsilon_0} - \frac{\rho R}{24\epsilon_0} = \frac{\rho R}{8\epsilon_0}$$

$$F = \frac{3m}{8} \frac{GM}{R^2}$$

$$= \frac{3}{8} \times 16 \times 2$$

$$= 12$$

(13) Answer : 10

Hint:

Apply pseudo force on ball.

Solution:

$$mg l = ml^2 \alpha$$

$$\alpha = \frac{g}{l}$$

(14) Answer : (B)

Hint:

Use string constraint.

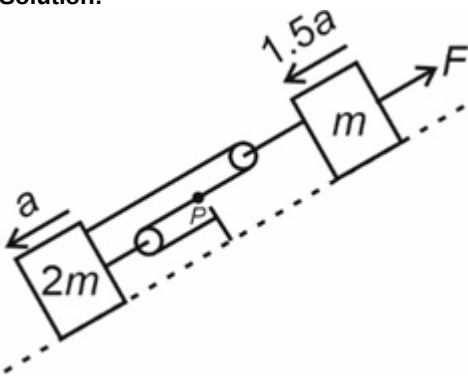


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Section-IV

Solution:



$$mg - 3T = 2ma$$

$$2T + \frac{mg}{2} - F = 1.5ma$$

$$\frac{7}{4}mg - \frac{3}{2}F = \frac{17}{4}ma$$

$$a = \frac{4g}{17}$$

$$T = \frac{mg - 2m \cdot \frac{4g}{17}}{3}$$

$$T = \frac{3mg}{17}$$

$$a_p = 2a = \frac{8g}{17}$$

(15) Answer : (C)

Hint:

Use wedge constraint.

Solution:

$$\sqrt{3}mg R \sin \theta = ml R(1 - \cos \theta) \Rightarrow \theta = 120^\circ$$

$$T = \sqrt{3}mg \cos 30^\circ - mg \cos 60^\circ$$

(1)		$mg \frac{1}{2} \cos 37$ $= N \sin 37$ $2 \times \frac{4}{5} = \frac{3}{5} N$
(2)		$N = \frac{mg}{2} = 1$
(3)		$f(\sin 37 + \cos 37)$ $= mg \frac{\cos 37}{2}$ $\frac{7}{5}f = \frac{4}{5}$
(4)		$5 \times \frac{3}{5} + 1 \times \frac{4}{5} = \frac{4}{5} N$

(16) Answer : (D)

Hint:

$$W_{mg} = \frac{\rho_0 l^4 g}{8}$$

Solution:

$$\frac{kl}{4} = \left(\frac{\rho_0}{2}\right) l^3 g \Rightarrow k = 2\rho_0 l^2 g$$

$$kx_2 = \omega_{app}$$

$$n_1 = \frac{l}{4}, x_2 = \frac{l}{4}$$

$$\omega_g = mg \frac{l}{2}$$

$$\omega_{sp} = 0$$

(17) Answer : (D)

Hint:

Central force is conservative force.

Solution:

Force is a central force centered at (1, 2).

CHEMISTRY

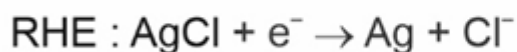
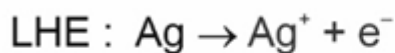
Section-I

(18) Answer : (A)

Hint:

$$\Delta G^\circ_{net} = \Delta G^\circ_{LHE} + \Delta G^\circ_{RHE}$$

Solution:



$$\text{Now, } -RT \ln K_{sp} = -nF \left[E^\circ_{\text{Ag}|\text{Ag}^+} + E^\circ_{\text{Cl}^-|\text{AgCl}|\text{Ag}} \right]$$

$$\Rightarrow -\frac{RT}{nF} \ln \frac{1}{K_{sp}} = E^\circ_{\text{Cl}^-|\text{AgCl}|\text{Ag}} - E^\circ_{\text{Ag}^+|\text{Ag}}$$

$$\Rightarrow E^\circ_{\text{Ag}^+|\text{Ag}} - \frac{RT}{F} \ln \frac{1}{K_{sp}} = E^\circ_{\text{Cl}^-|\text{AgCl}|\text{Ag}}$$

$$\text{As } K_{sp} \uparrow \frac{1}{K_{sp}} \downarrow \Rightarrow -\ln \frac{1}{K_{sp}} \uparrow \left[E^\circ_{\text{Cl}^-|\text{AgCl}|\text{Ag}} \right] \uparrow$$

(19) Answer : (B)

Hint:

Out of 12 tetrahedral voids 8 tetrahedral voids are inside the unit cell.

Solution:

Number of tetrahedral voids inside unit cell is 8.

$$Z = 8$$

(20) Answer : (B)

Hint:

$$[P]_t = [Q]_t$$

Solution:



$$t = 0 \quad P_0 \quad -$$

$$t = t \quad P_0 - x \quad nx$$

$$\Rightarrow P_0 - x = nx \text{ (at intersection)}$$

$$\text{So, } P_0 = x(n + 1)$$

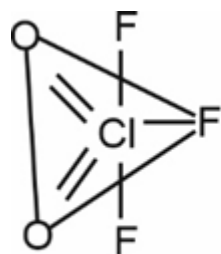
$$\text{So, } nx = \frac{nP_0}{(n+1)}$$

(21) Answer : (D)

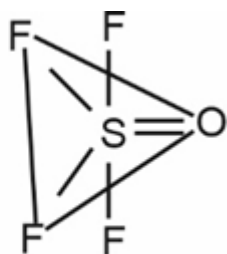
Hint:

Isostructural species have same shape.

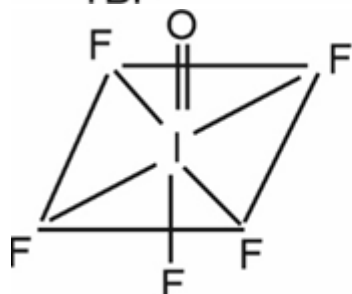
Solution:



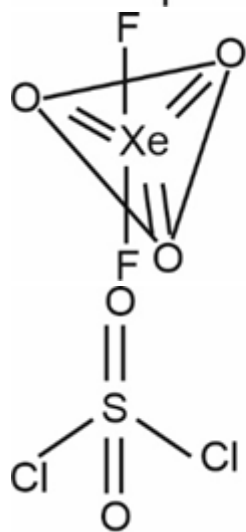
TBP



TBP



Polar, sp^3d^2



XeO₃F₂
(all O-atoms lie
in equatorial
position)

sp^3 hybridised
2($p\pi-d\pi$ bonds)

(22) Answer : (A,B)

Hint:

$$\Delta T_f = K_f \times m$$

Solution:

Ideal gases cannot be liquefied. ΔT_f depends on K_f – nature of solvent.

(23) Answer : (A,C)

Hint:

$$\text{Radial nodes} = n - l - 1$$

Solution:

Radial part of above function depends on n and l .

$$\text{Number of radial nodes} = n - l - 1$$

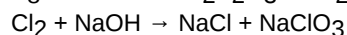
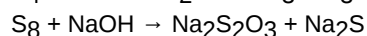
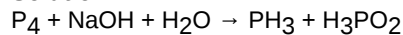
$$\text{Number of angular nodes} = l$$

(24) Answer : (A,B,C,D)

Hint:

Element having intermediate oxidation state can undergo disproportionation reaction.

Solution:



(25) Answer : 4

Hint:

Section-III

$$d = \frac{Z \times M}{N_A \times a^3}$$

Solution:

$$1.8125 = \frac{Z \times 58}{6 \times 10^{23} \times (6 \times 10^{-8})^3}$$

Solving, $Z = 4$

(26) **Answer : 8**

Hint:

$$K_f \propto \frac{T_f^2}{\Delta H_f}$$

Solution:

$$\frac{K_f}{K_b} = \frac{(T_f)^2}{(\Delta H_f)} \times \frac{(\Delta H_b)}{(T_b)^2} = \left(\frac{290}{580}\right)^2 \left(\frac{\Delta H_b}{\Delta H_f}\right)$$

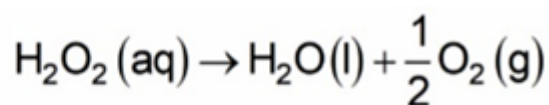
$$\frac{\Delta H_b}{\Delta H_f} = \left(\frac{580}{290}\right)^2 \times \frac{4.64}{2.32} = 8$$

(27) **Answer : 75**

Hint:

$$k = \frac{1}{t} \ln \frac{A_0}{A}$$

Solution:



After a long time A_0 $\frac{A_0}{2}$

So, initial concentration $\propto 200$ mL.

$$1 \text{ hr } 25 \text{ mins} = 85 \text{ mins} = 2t_{\frac{1}{2}}$$

$$\text{Concentration of H}_2\text{O}_2 \text{ remaining} \propto \frac{200}{(2)^2} = 50$$

Concentration of H_2O_2 consumed $\propto 150$

$$V_{\text{O}_2} \text{ produced} = \frac{150}{2} = 75 \text{ mL}$$

(28) **Answer : 3**

Hint:

$$w = -nR\Delta T$$

Solution:

$$W = -nR\Delta T = -1 \times 8 \times 75 = -600 = -0.6 \text{ kJ}$$

$$\Delta U = q + w = 1800 - 600 = 1200 \text{ J}$$

$$\gamma = \frac{\Delta H}{\Delta U} = \frac{1800}{1200} = 1.5$$

(29) **Answer : 5**

Hint:

Number of equivalents is same.

Solution:

Number of equivalents of $\text{CuSO}_4 \cdot x\text{H}_2\text{O}$ = number of equivalents of H_2SO_4 = Number of equivalents of NaOH

$$\frac{1.25}{\frac{159.5 + 18x}{2}} = 5 \times 2 \times 10^{-3}$$

$$\Rightarrow x = 5.03 \approx 5$$

(30) **Answer : 3**

Hint:

$$N_\alpha \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

Solution:

$$N_\alpha \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)} \Rightarrow \frac{N_{60^\circ}}{N_{90^\circ}} = \left[\frac{\sin\left(\frac{90^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}\right]^4$$

$$\frac{36}{N_{90^\circ}} = \frac{4}{1} \Rightarrow N_{90^\circ} = 9$$

So, $N = 15 \times 9 = 135$, scattered α - particles

Section-IV

(31) Answer : (A)

Hint:

Noble gases have high IE.

Solution:

Highest value of IE_1 refers to noble gas or group 18 element. Lowest the value of IE_1 , more reactive the metal.

(32) Answer : (B)

Hint:

Down the group IE decreases.

Solution:

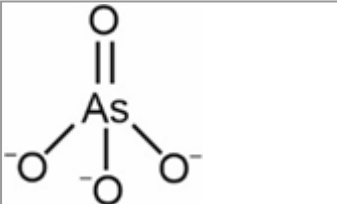
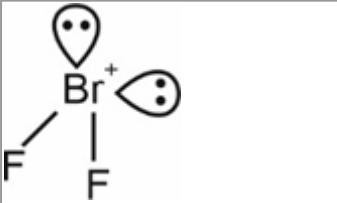
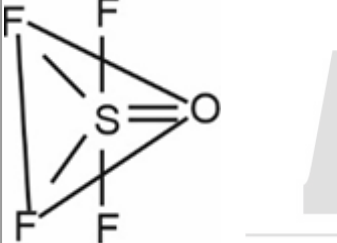
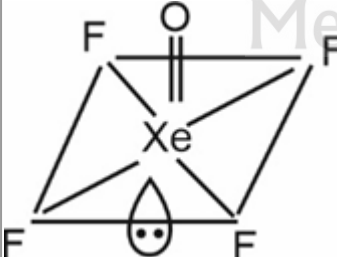
Down the group IE decreases. Electronegativity decreases generally. Cations have larger affinity for electrons $\Rightarrow Fe^{2+}$ has larger magnitude of electron gain enthalpy.

(33) Answer : (C)

Hint:

AsO_4^{3-} is having sp^3 hybridised As-atom.

Solution:

	Hybridisation $\rightarrow sp^3$ Tetrahedral Number of $p\pi-d\pi$ bonds = 1
	sp^3 Shape-V Br-F bonds have same bond lengths
	sp^3d_2 Trigonal bipyramid Number of $p\pi-d\pi$ bonds = 1.
	sp^3d^2 shape : square pyramidal. Number of $p\pi-d\pi$ bonds = 1

(34) Answer : (A)

Hint:

$$pH = pK_a + \log \frac{[\text{Salt}]}{[\text{Acid}]}$$

Solution:

$$(P) \quad pH = pK_{a_1} + \log \left[\frac{[H_2XO_4^-]}{[H_3XO_4]} \right] = 4$$

$$(Q) \quad pH = pK_{a_2} + \log \left[\frac{[HXO_4^{2-}]}{[H_2XO_4^-]} \right] = 8$$

$$(R) \quad pH = pK_{a_3} + \log \left[\frac{[XO_4^{3-}]}{[HXO_4^{2-}]} \right] = 12$$

$$(S) \quad pH = \frac{1}{2} [pK_{a_1} + pK_{a_2}] = \frac{1}{2} [4 + 8] = 6$$

MATHEMATICS

Section-I

(35) Answer : (D)

Hint:

 $P(x)$ divides both of them, hence $P(x)$ also divides

Solution:

 $\therefore P(x)$ divides both of them, hence $P(x)$ also divides

$$(3x^4 + 4x^2 + 28x + 5) - 3(x^4 + 6x^2 + 25)$$

$$= -14x^2 + 28x - 70$$

$$= -14(x^2 - 2x + 5); \therefore P(1) = 4$$

(36) Answer : (B)

Hint:

$$(z + w)^2 = zw \Rightarrow z^2 + zw + w^2 = 0$$

Solution:

$$(z + w)^2 = zw \Rightarrow z^2 + zw + w^2 = 0$$

$$\text{Let } \frac{z}{w} = t \Rightarrow \frac{z}{w} = \frac{-1 \pm \sqrt{3}i}{2}$$

$$\arg z - \arg w = \frac{2\pi}{3} \text{ OR } \arg z - \arg w = -\frac{2\pi}{3} .$$

(37) Answer : (A)

Hint:

3, 5, 7 \rightarrow A.P.

Solution:

3, 5, 7 \rightarrow A.P. but if they are in G.P. then, $y^2 = xz$ but y is prime. So not possible.

(38) Answer : (A)

Hint:

$$(-1)^{r-1} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{r}\right) \cdot {}^n C_r$$

Solution:

 T_r (r^{th} term of the series)

$$(-1)^{r-1} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{r}\right) \cdot {}^n C_r$$

$$\text{Now } (-1)^{r-1} (1 + x + x^2 + \dots + x^{r-1}) \cdot {}^n C_r = \left(\frac{1-x^r}{1-x}\right) (-1)^{r-1} \cdot {}^n C_r$$

Now

$$\frac{1}{(1-x)} \sum_{r=1}^n \left[(-1)^{r-1} \cdot {}^n C_r - (-1)^{r-1} \cdot x^r \cdot {}^n C_r \right]$$

$$= \frac{1}{(1-x)} \left[(-1) \sum_{r=1}^n (-1)^r \cdot {}^n C_r + \sum_{r=1}^n (-1)^r \cdot x^r \cdot {}^n C_r \right]$$

$$= \frac{1}{(1-x)} \left[(-1) \{ (1-1)^n - 1 \} + \{ (1-x)^n - 1 \} \right]$$

$$= \frac{(1-x)^n}{(1-x)} = (1-x)^{n-1}$$

$$\Rightarrow \sum_{r=1}^n (-1)^{r-1} (1 + x + x^2 + \dots + x^{r-1}) \cdot {}^n C_r = (1-x)^{n-1}$$

Integrating above equation, we get

$$\int_0^1 \left[\sum_{r=1}^n (-1)^{r-1} (1 + x + x^2 + \dots + x^{r-1}) \cdot {}^n C_r \right] dx = \int_0^1 (1-x)^{n-1} dx$$

$$\Rightarrow \sum_{r=1}^n (-1)^{r-1} \cdot {}^n C_r \int_0^1 (1 + x + x^2 + \dots + x^{r-1}) dx = \left[-\frac{(1-x)^n}{n} \right]_0^1$$

$$\Rightarrow \sum_{r=1}^n (-1)^{r-1} \cdot {}^n C_r \left[x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^r}{r} \right]_0^1 = \frac{1}{n}$$

$$\Rightarrow \sum_{r=1}^n (-1)^{r-1} \cdot {}^n C_r \left[1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{r} \right] = \frac{1}{n}$$

Section-II

(39) Answer : (B,D)

Hint:

$$\frac{6!}{2!2!1!1!2!2!}$$

Solution:

$$\text{Required ways} = \frac{6!}{2!2!1!1!2!2!} \times 4! = 1080$$

(40) **Answer :** (B,C,D)

Hint:

$$X^4 = I$$

Solution:

$$AX = B$$

$$\Rightarrow AX^2 = BX$$

$$\Rightarrow AX^2 = XB \quad (\because BX = XB)$$

$$\Rightarrow AX^2 = A$$

multiply both the side by A^{-1} ($\because A^{-1}$ exists)

$$\Rightarrow X^2 = I$$

$$\Rightarrow X^4 = I$$

$$= |X^4 + X^2 + I| = |3I| = 27$$

(41) **Answer :** (B,C)

Hint:

$$\left|z + \frac{1}{2}\right| \geq \frac{1}{2}$$

Solution:

$$|z^2 + z + 1| = 1$$

$$\Rightarrow \left|z + \frac{1}{2}\right|^2 + \frac{3}{4} = 1$$

$$\Rightarrow \left|z + \frac{1}{2}\right|^2 + \frac{3}{4} \leq \left|z + \frac{1}{2}\right|^2 + \frac{3}{4}$$

$$\Rightarrow 1 \leq \left|z + \frac{1}{2}\right|^2 + \frac{3}{4} \Rightarrow \left|z + \frac{1}{2}\right|^2 \geq \frac{1}{4}$$

$$\Rightarrow \left|z + \frac{1}{2}\right| \geq \frac{1}{2}$$

$$\text{also } \left|(z^2 + z) + 1\right| = 1 \geq \left||z^2 + z| - 1\right|$$

$$\Rightarrow |z^2 + z| - 1 \leq 1$$

$$\Rightarrow |z^2 + z| \leq 2$$

$$\Rightarrow \left||z^2| - |z|\right| \leq |z^2 + z| \leq 2$$

$$\Rightarrow |r^2 - r| \leq 2$$

$$\Rightarrow r = |z| \leq 2; \forall z \in S$$

Also we can always find root of the equation

$$z^2 + z + 1 = e^{i\theta}, \forall \theta \in \mathbb{R} \text{ Hence set 'S' is infinite}$$



Aakash

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Section-III

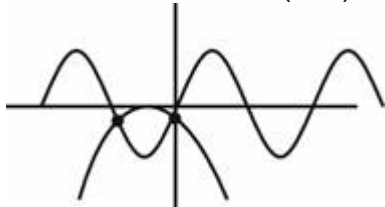
(42) **Answer :** 2

Hint:

$$-1 \leq \sin\theta \leq 1$$

Solution:

$$x^2 + 2x + 1 + \sin x = 0 \Rightarrow -(x + 1)^2 = \sin x$$



(43) **Answer :** 1

Hint:

$$20^2 - 4c \geq 0$$

Solution:

$$20^2 - 4c \geq 0$$

$$1 \leq c \leq 100$$

$$\text{Product} = 1.2.3 \dots 100 = 100!$$

$$n = 100$$

$$\text{Sum of digits} = 1$$

(44) Answer : 5

Hint:

Multiply equation (2) by i and add in equation (1)

Solution:

$$z_1^3 - 3z_1z_2^2 = 2 \quad \dots(1)$$

$$3z_1^2z_2 - z_2^3 = 11 \quad \dots(2)$$

Multiply equation (2) by i and add in equation (1)

$$z_1^3 - 3z_1z_2^2 + 3iz_2z_1^2 - iz_2^3 = 2 + 11i$$

$$z_1^3 + i^2 3z_1z_2^2 + 3iz_2z_1^2 + (iz_2)^3 = 2 + 11i$$

$$(z_1 + iz_2)^3 = 2 + 11i$$

$$\text{Similarly } (z_1 - iz_2)^3 = 2 - 11i$$

Multiply the two

$$((z_1 + iz_2)(z_1 - iz_2))^3 = 4 + 121i$$

$$(z_1^2 + z_2^2)^3 = 125$$

$$z_1^2 + z_2^2 = 5, 5w, 5w^2$$

$$|z_1^2 + z_2^2| = 5$$

(45) Answer : 3

Hint:

A.M. \geq G.M.

Solution:

$$0 < \sin \alpha \leq 1, 0 < \cos \beta \leq 1, \tan \gamma \in (0, \infty)$$

$$\frac{\sin \alpha + \cos \beta + \tan \gamma}{3} \geq (\sin \alpha \cdot \cos \beta \cdot \tan \gamma)^{\frac{1}{3}}$$

$$1 \geq 1$$

$$\Rightarrow \text{A.M.} = \text{G.M.}$$

$$\Rightarrow \sin \alpha = \cos \beta = \tan \gamma = 1$$

$$\Rightarrow \sin^2 \alpha + \cos^2 \beta + \tan^2 \gamma = 3$$

(46) Answer : 3

Hint:

$$\left(\sum_{k=1}^5 {}^{20}C_{2k-1} \right)^6 = \left(\frac{1}{2} ({}^{20}C_1 + {}^{20}C_3 + \dots + {}^{20}C_{19}) \right)^6$$

Solution:

$$\left(\sum_{k=1}^5 {}^{20}C_{2k-1} \right)^6 = \left(\frac{1}{2} ({}^{20}C_1 + {}^{20}C_3 + \dots + {}^{20}C_{19}) \right)^6$$

$$= 2^{108} = (2^5)^{21} \times 2^3 = (11 \times 3 - 1)^{21} \times 8$$

$$= 8(33^21 - 1) = 8 \times 33^21 - 8$$

$$= 11(8 \times 3^21 - 1) + 3$$

(47) Answer : 39

Hint:

$$1 \text{ ball for box 2 and 1 for box 4 in } {}^7C_1 \times {}^6C_1 = 42 \text{ ways}$$

Solution:

$$1 \text{ ball for box 2 and 1 for box 4 in } {}^7C_1 \times {}^6C_1 = 42 \text{ ways}$$

Remaining 5 balls are to be put in 3

$$\text{boxes} = 3 \times (2^5 - 2) + 3 \times 1 = 93$$

$$\text{Total} = 42 \times 93 = 3906$$

Section-IV

(48) Answer : (A)

Hint:

$$A^{50} = 25A^2 - 24I$$

Solution:

$$A^n - A^{n-2} = A^2 - I$$

$$\Rightarrow A^{50} = A^{48} + A^2 - I$$

$$A^{48} = A^{46} + A^2 - I$$

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$$A^4 = A^2 + A^2 - I$$

$$A^{50} = 25A^2 - 24I$$

$$\text{So, } A^{50} = \begin{pmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{pmatrix}$$

$$|A^{50}| = 1$$

$$\text{Trace } (A^{50}) = 3$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(49) Answer : (B)

Hint:

Use trigonometric identity

Solution:

$$(P) \cos^2 2x - \sin^2 x = 0$$

$$\Rightarrow \cos 3x \cos x = 0$$

$$\cos 3x = 0 \text{ or } \cos x = 0$$

$$3x = (2n+1)\frac{\pi}{2} \text{ or } x = (2n+1)\frac{\pi}{6}$$

Hence general solution of equation $(2n+1)\frac{\pi}{6}, n \in Z$

$$(Q) \cos x + \sqrt{3}\sin x = \sqrt{3}$$

$$\cos\left(x - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$x = \left(2n + \frac{1}{2}\right)\pi \text{ or } x = 2n\pi + \frac{\pi}{6}, n \in Z$$

$$(R) \sqrt{3}\tan^2 x - (\sqrt{3} + 1)\tan x + 1 = 0$$

$$\Rightarrow (\tan x - 1)(\sqrt{3}\tan x - 1) = 0$$

$$\tan x = \frac{1}{\sqrt{3}}, 1 \Rightarrow x = n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{6}, n \in Z$$

$$(S) \tan x \tan 2x \tan 3x = 0$$

$$x = n\pi \text{ or } \frac{n\pi}{2} \text{ (rejected) or}$$

$$\frac{n\pi}{3} \Rightarrow x = \frac{n\pi}{3}, n \in Z$$

(50) Answer : (B)

Hint:

Quadratic properties

Solution:

For the given conditions roots are 0, -1, -2

$$\therefore (a)_{\min} = 3$$

$$(b)_{\min} = 2$$

$$(c)_{\min} = 0$$

(51) Answer : (B)

Hint:

Permutation and combination

Solution:

$$(P) xyz = 3^5$$

$$\text{Number of solution } (x, y, z) \text{ is } {}^{3+5-1}C_3 = 21$$

$$(Q) \text{ Number of terms} = {}^{6+3-1}C_{3-1} = 28.$$

$$(R) x^2 + x - 400 \leq 0$$

$$x(x+1) \leq 400$$

solutions are 1, 2, 3, ..., 19

Number of solutions = 19.

$$(S) x + y + z = 10$$

$$\text{Number of solutions} = {}^{7+3-1}C_{3-1} = {}^9C_2 = 36$$