02/11/2025

PS_Set-2
ONLINE

Aakash
Medical|IIT-JEE|Foundations

Corporate Office : AESL, 3rd Floor, Incuspaze Campus-2, Plot No. 13, Sector-18,
Udyog Vihar, Gurugram, Haryana - 122015, ***Ph.***+91-1244168300

MM : 180 AIATS For One Year JEE(Advanced)-2026 (XII Passed)_Test-1A_Paper-2_Online **Time : 180 Min.**

PHYSICS

Section-I

1. (D)

2. (B)

3. (B)

4. (B)

Section-II

5. (A,B,C)

6. (A,C)

7. (B,C)

Section-III

8. (12)

9. (6)

10. (3)

11. (7)

12. (50)

13. (2)

Section-IV

14. (20.00)

15. (26.50)

16. (12.00)

17. (41.00)

CHEMISTRY

Section-I

18. (A)

19. (A)

20. (D)

21. (A)

Section-II

22. (A,B,C,D)

23. (A,B,C)

24. (B,D)

Section-III

25. (98)

26. (27)

27. (9)

28. (7)

29. (5)

30. (4)

Section-IV

31. (02.00)

32. (06.00)

33. (88.78)

34. (00.34)

MATHEMATICS

Section-I

35. (B)

36. (B)

37. (A)

38. (D)

Section-II

39. (C,D)

40. (B,C,D)

41. (A,C)

Section-III

42. (12)

43. (3)

44. (0)

45. (14)

46. (3)

47. (31)

Section-IV

48. (04.00)

49. (09.00)

50. (08.00)

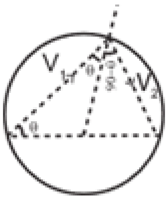
51. (13.00)

Hints and Solutions

PHYSICS

Section-I

(1) **Answer :** (D)
Hint:
$V_1 \sin\theta = V_2 \cos\theta$
Solution:

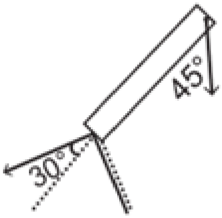


$V_1 \sin\theta = V_2 \cos\theta$

$e = \frac{V_2 \sin\theta}{V_1 \cos\theta}$

$e = \tan^2\theta = \frac{9}{16}$

(2) **Answer :** (B)
Hint:
Use of rod constraint
Solution:
Constraint on rod *A*.



$20\sqrt{3}\frac{1}{\sqrt{2}} = \omega\left(\frac{1}{10}\right)\frac{\sqrt{3}}{2}$

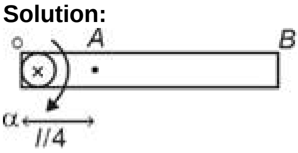
$w = 200\sqrt{2}$ rad/s

(3) **Answer :** (B)
Hint:
Work energy theorem
Solution:

$$mg\sin\theta l - \int_0^l mg\cos\theta.\mu_0 x dx = 0 - 0$$

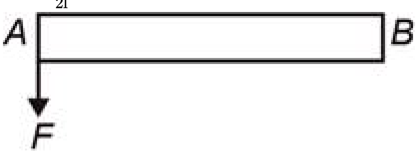
$l = \frac{2}{\mu_0}\tan\theta = \frac{2}{\sqrt{3}\mu_0}$

(4) **Answer :** (B)
Hint:
Shear stress $= \frac{F_t}{A}$
Solution:



$\frac{ml^2}{3}\alpha = mg\frac{l}{2}$

$\alpha = \frac{3g}{2l}$



$F + \frac{3mg}{4} = \left(\frac{3m}{4}\right)\frac{51}{8}\left(\frac{3g}{21}\right)$

$F = \frac{-3}{64}mg$

Shear stress = $\frac{3mg}{64A}$

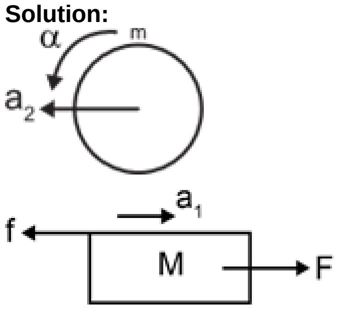
Section-II

(5) **Answer :** (A,B,C)

Hint:

$A = \alpha R$

Solution:



a_2 is acceleration of cylinder with respect to plank

For plank:

$F - f = ma_1$

For cylinder:

$ma_1 - f = ma_2$

$fR = I\alpha$

$fR = \left(\frac{1}{2}mR^2\right)\left(\frac{a_2}{R}\right)$

$f = \frac{1}{2}ma_2$

$a_1 = \frac{3F}{3M+m}$ = 6 m/s^2

$a_2 = \frac{2F}{3M+m}$ = 4 m/s^2

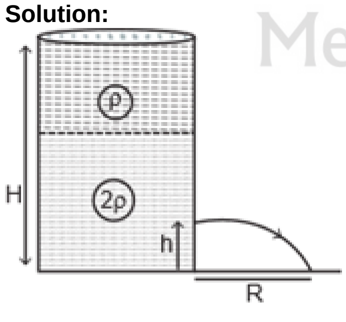
$\alpha = \frac{4}{0.1} = 40$

(6) **Answer :** (A,C)

Hint:

$P_1 + \frac{1}{2}\rho_1 V_1^2 = P_2 + \frac{1}{2}\rho_2 V^2$

Solution:



$P_1 + \frac{1}{2}\rho_1 V_1^2 = P_2 + \frac{1}{2}\rho_2 V^2$

$P_0 + \rho\frac{9H}{2} + 2\rho g\left(\frac{H}{2} - h\right) = P_0 + \frac{1}{2}(2\rho)V^2$

$V = \sqrt{\frac{g}{2}(3H - 4h)}$

$R = \sqrt{h(3H - 4h)}$

$\frac{dR}{dh} = 0$

$h = \frac{3}{8}H$ = 1.5 m

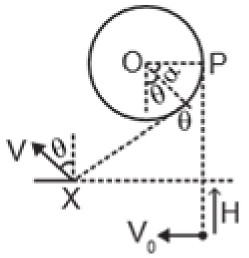
But if the liquid ejects from interface then R = 4 cm and h_0 = 2 m.

(7) **Answer :** (B,C)

Hint:

Work energy theorem

Solution:



$V_0 = \sqrt{\frac{7}{3}\pi gR}$

$\theta = \frac{\pi}{6}$

$\alpha = \frac{\pi}{3}$

$\theta x = 2\pi R - \frac{\pi}{3}R = \frac{5\pi}{3}R$

$h = 2\pi R - \left[R\sin\alpha + \frac{5\pi}{3}RG\right]$

$h = \frac{7}{6}\pi R - \frac{\sqrt{3}}{2}R$

$V^2 = V_0^2 - 2gh = \frac{7}{3}\pi gR - 2g\left(\frac{7}{6}\pi R - \frac{\sqrt{3}}{2}R\right)$

$V = \sqrt{\sqrt{3}gR}$

$\tau - mg\cos\alpha = \frac{mv^2}{r}$

$\tau = \frac{mg}{2} + \frac{m}{\pi 5R}.\sqrt{3}gR \times 3$

$= mg\left(\frac{1}{2} + \frac{3\sqrt{3}}{5\pi}\right)$

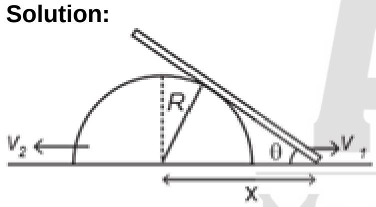
Section-III

(8) **Answer :** 12

Hint:

$\frac{d\theta}{dt} = \frac{V_1}{l}$

Solution:



$x = R$ cosecθ

$\frac{dx}{dt} = -R\cos ec\theta\ \cot\theta\frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \frac{(V_1+V_2)\tan\theta.\sin\theta}{R}$

$\omega = \frac{4.\sqrt{3}.\sqrt{3}}{0.5\times 2}$

= 12 rad/s

(9) **Answer :** 6

Hint:

$W = \Delta U$

Solution:

$E_i = \frac{-3}{5}\frac{GM^2}{R} - \frac{GM^2}{5R}$

$E_f = \frac{-3}{5}\frac{GM^2}{2R} - \frac{GM^2}{3R}$

$W = E_f - E_i$

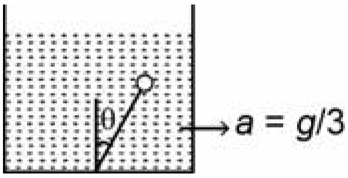
$W = \frac{GM^2}{6R}$

(10) **Answer :** 3

Hint:

$\tan\theta = \frac{a}{g}$

Solution:



$\tan\theta = a/g = \frac{1}{3}$

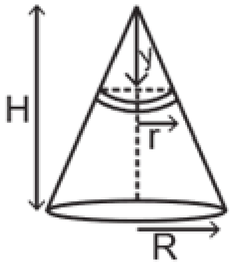
$\cot\theta = 3$

(11) **Answer :** 7

Hint:

$I = \int dmr^2$

Solution:



$\frac{y}{r} = \frac{H}{R}$

$dy = \frac{H}{R}dr$

$dm = \pi r^2 dy.\, s$

$I = \int \frac{1}{2}(dm)r^2$

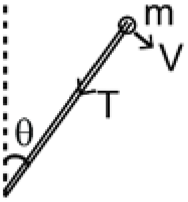
$I = \frac{3}{10}MR^2$

(12) **Answer :** 50

Hint:

$T = mg(2 - 3\cos\theta)$

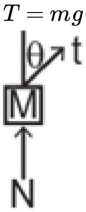
Solution:



$\frac{1}{2}mv^2 = mg(1 - 1\cos\theta)$

$T + mg\cos\theta = \frac{mv^2}{1}$

$T = mg(2 - 3\cos\theta)$



$N = Mg - mg\cos\ (2 - 3\ \cos\theta)$

$N_{\min} = mg - \frac{mg}{3}$

$\Rightarrow 60 - 10$

= 50

(13) **Answer :** 2

Hint:

$\frac{d}{dx}(Inx) = \frac{1}{x}$

Solution:

$Z = e^{\sin x}$

lnZ = sin*x*

$\frac{dZ}{Z} = (\cos x)dx$

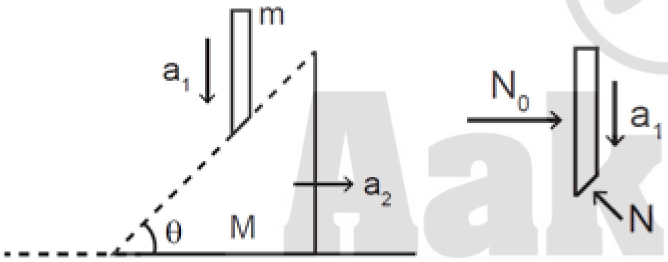
$\frac{dZ}{Z} = \frac{1}{2} \times 0.04$
$\frac{dZ}{Z} \times 100 = 2\%$

Section-IV

(14) Answer : 20.00
Hint:
$-h = R\tan\theta - \frac{gR^2}{2u^2\cos^2\theta}$
Solution:
For maximum range on ground
$\frac{dR}{d\theta} = 0 \Rightarrow \tan\theta = \frac{u}{\sqrt{u^2+2gh}}$
$R_{\max} = \frac{u}{g}\sqrt{u^2+2gh}$

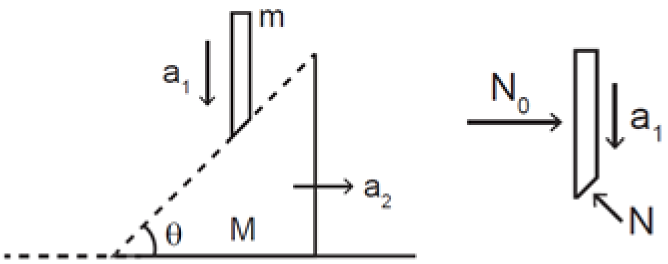
(15) Answer : 26.50
Hint:
$-h = R\tan\theta - \frac{gR^2}{2u^2\cos^2\theta}$
Solution:
For maximum range on ground
$\frac{dR}{d\theta} = 0 \Rightarrow \tan\theta = \frac{u}{\sqrt{u^2+2gh}}$
$R_{\max} = \frac{u}{g}\sqrt{u^2+2gh}$

(16) Answer : 12.00
Hint:
Wedge constraint
Solution:



$a_1 = a_2 \tan\theta$
$N \sin\theta = Ma_2$
let N_0 is normal by support on rod.
$N \sin\theta = N_0$
$Mg - \cos\theta = ma_1$
$a_1 = \frac{mg}{m+M\cot^2\theta} = \frac{50}{5+10\times\frac{16}{9}} = \frac{50\times9}{205} = \frac{90}{41}$
$N_0 = \frac{Mmg}{m\tan\theta+M\cot\theta} = \frac{5\times10\times10}{5\times\frac{3}{4}+10.\frac{4}{3}}$
$= \frac{1200}{41}$

(17) Answer : 41.00
Hint:
Wedge constraint
Solution:



$a_1 = a_2 \tan\theta$

$N\sin\theta = Ma_2$
let N_0 is normal by support on rod.
$N\sin\theta = N_0$
$Mg - \cos\theta = ma_1$

$a_1 = \frac{mg}{m+M\cot^2\theta} = \frac{50}{5+10\times\frac{16}{9}} = \frac{50\times9}{205} = \frac{90}{41}$

$N_0 = \frac{Mmg}{m\tan\theta+M\cot\theta} = \frac{5\times10\times10}{5\times\frac{3}{4}+10.\frac{4}{3}}$

$= \frac{1200}{41}$

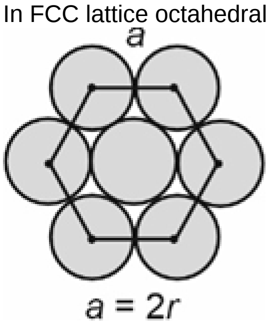
CHEMISTRY

Section-I

(18) **Answer :** (A)
Hint:
$6Fe_2O_3 \rightarrow 4Fe_3O_4 + O_2$
Solution:
Mass of O_2 produced
= 3 g – 2.92 g
= 0.08 g
Moles of O_2 formed $= \frac{0.08}{32}\ mol$
1 mol of O_2 formed by 6 mol of Fe_2O_3
$= \frac{0.08}{32}$ mol of O_2 will formed by $\frac{6\times0.08}{32}\ mol$ of Fe_2O_3
Mole of Fe_2O_3 present = 0.015 mol
Mass of Fe_2O_3 formed = 0.015 × 160
= 2.4 g
% Mass of SiO_2 in mixture
$= \frac{0.6}{3}\times100$
= 20%

(19) **Answer :** (A)
Hint:
Element having atomic number 81 is thalium, which have $1e^-$ in 6*p*-subshell
Solution:
n = 1
For 1^{st} order reaction
$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{k}$

(20) **Answer :** (D)
Hint:
Due to Schottky defect density of unit cell decreases.
Solution:
In FCC lattice octahedral voids are present at edge centre and body centre.



$P.E = \frac{3\times\frac{4}{3}\pi r^3}{6\times\frac{\sqrt{3}}{4}(2r)^2\times2r}\times100$
= 60.8%
When Ge is doped with P or As, n-type semiconductor is formed.

(21) **Answer :** (A)
Hint:

Compound having μ ≠ 0 are polar.
Solution:
NH_3, H_2O, ClF_3, NF_3, PCl_3, SF_4, BrF_5, BrF_3, I_3^+ are polar
H_2O, ClF_3, XeF_4, BrF_3, I_3^+ , CO_2, BF_3 are planar

Section-II

(22) **Answer :** (A,B,C,D)
Hint:
For phase transition, $\Delta S = \frac{\Delta H}{T}$
Solution:
For phase transition step, the change in Gibbs free energy is zero.

(23) **Answer :** (A,B,C)
Hint:
Complexing agent decreases the concentration of Fe^{3+} ion
Solution:
According to Nernst equation
$E_{cell} = E^o_{cell} - \frac{RT}{nF} \ln \frac{[Fe^{2+}]}{[Fe^{3+}]}$
Hence potential will depends on $\left(\frac{[Fe^{3+}]}{[Fe^{2+}]}\right)$

(24) **Answer :** (B,D)
Hint:
The strength of bonds being broken in P and Q affects the activation energy of reaction not frequency factor (A).
Solution:
The overall enthalpy change reflects the energy difference between reactants and products and does not directly influence the frequency of effective collision.
More complex molecule often have more stringent steric requirements for effective collisions. They might need to collide in a very specific way for the reactive sites to align properly. The increased complexity leads to a lower probability of effective collision and thus a lower frequency factor.

Section-III

(25) **Answer :** 98
Hint:
$MnO_4^- \,(aq.) + 8H^+(aq.) + 5e^- \rightarrow Mn^{2+}(aq.) + 4H_2O(l)$
Solution:
Using Nernst equation
$E_{MnO_4^-, H^+/Mn^{2+}} = E^o_{MnO_4^-, H^+/Mn^{2+}} - \frac{0.06}{5} \log \frac{[Mn^{+2}]}{[MnO_4^-][H^+]^8}$
= 1.51 – 0.012 log ($10^{-4} \cdot 10^{48}$)
= 1.51 – 0.012 log (10^{44})
= 1.51 – 0.012 × 44
= 0.982 V
= 98.2 × 10^{-2} V
x = 98.2
≈ 98

(26) **Answer :** 27
Hint:
$\Delta T_f = i \times k_f \times m$
Solution:
For A_2B
$2.5 = 25 \times \frac{3.5}{2a+b} \times \frac{1000}{50}$
⇒ 2a + b = 700 …(i)
And,
$1 = 25 \times \frac{1.5}{a+2b} \times \frac{1000}{50}$
a + 2b = 750 …(ii)
eq. (ii) × 2 – eq.(i)
2a + 4b = 1500
2a + b = 700
– – –
____

3b = 800
b = 266.67 g/mol

(27) Answer : 9

Hint:

$pOH = pK_b = \log\frac{[Salt]}{[Base]}$

Solution:

$$\begin{array}{cccc} 2NH_4OH & + \ H_2SO_4 & \rightarrow & (NH_4)_2SO_4 \\ 100\text{ m mol} & 25\text{ m mol} & & — \\ 50\text{ m mol} & — & & 25\text{ m mol} \end{array}$$

$$\begin{array}{cccc} NH_4OH & + \ HCl & \rightarrow & NH_4Cl \\ 50\text{ m mol} & 12.5\text{ m mol} & & 50\text{ m mol} \\ 37.5\text{ m mol} & — & & 62.5\text{ m mol} \end{array}$$

$pOH = pK_b + \log\frac{[NH_4^+]}{[NH_4OH]}$

$= 4.74 + \log\left(\frac{5}{3}\right)$

= 4.74 + 0.22
= 4.96
$\simeq$ 5
pH = 14 – 5 = 9

(28) Answer : 7

Hint:

Compound having μ ≠ 0 are polar.

Solution:

O_3, PCl_3, NO_2^-, BrF_5, SF_4, ClF_3 and $HClO_3$ contain one or more lone pair on central atom, and also have permanent dipole moment and hence polar.

(29) Answer : 5

Hint:

ΔH_{neut} for SA and SB = –57 kJ/mol

Solution:

$$\begin{array}{ccccc} 2HCN & + \ Ba(OH)_2 & \rightarrow & Ba(CN)_2 & + \ 2H_2O \\ 500\text{ m mol} & 200\text{ m mol} & & — & — \\ 100\text{ m mol} & — & & 200\text{ m mol} & \end{array}$$

ΔH_{ion} for 400 m mol of HCN = 45 × 0.4
= 18 kJ

ΔH_{neut} = (0.4 × 57) – 18
= 4.8 kJ
$\approx$ 5 kJ

(30) Answer : 4

Hint:

$\frac{r_1}{r_2} = \frac{n_1}{n_2}\sqrt{\frac{M_2}{M_1}}$

Solution:

$\frac{r_{SO_2}}{r_{CH_4}} = \sqrt{\frac{M_{CH_4}}{M_{SO_2}}} = \sqrt{\frac{16}{64}} = \frac{1}{2}$

$\frac{(n_f)_{SO_2}}{(n_f)_{CH_4}} = \left(\frac{(n_i)_{SO_2}}{(n_i)_{CH_4}}\right) \times \left(\frac{r_{SO_2}}{r_{CH_4}}\right)^n$

$\frac{1}{16} = \left(\frac{1}{2}\right)^n$

n = 4

Section-IV

(31) Answer : 02.00

Hint:

KHC_2O_4 behaves as acid in KOH solution.

Solution:

m_{eq} of $KMnO_4$ = m_{eq} of KHC_2O_4 + m_{eq} of $K_2C_2O_4$

10 × 0.06 × 5 = x × 2 + y × 2
2x + 2y = 3

m_{eq} of KOH = m_{eq} of KHC_2O_4
10 × 0.05 = x × 1
x = 0.5
2 × 0.5 + 2y = 3
2y = 2
y = 1
Molar ratio $\frac{y}{x} = \frac{1}{0.5}$
$= \frac{2}{1}$
10 ml of mixture require 3 milliequivalent of $Cr_2O_7^{2-}$
So 20 ml of mixture require
6 m_{eq} of $Cr_2O_7^{2-}$
a = 6

(32) **Answer :** 06.00
Hint:
KHC_2O_4 behaves as acid in KOH solution.
Solution:
m_{eq} of $KMnO_4$ = m_{eq} of KHC_2O_4 +
m_{eq} of $K_2C_2O_4$
10 × 0.06 × 5 = x × 2 + y × 2
2x + 2y = 3
m_{eq} of KOH = m_{eq} of KHC_2O_4
10 × 0.05 = x × 1
x = 0.5
2 × 0.5 + 2y = 3
2y = 2
y = 1
Molar ratio $\frac{y}{x} = \frac{1}{0.5}$
$= \frac{2}{1}$
10 ml of mixture require 3 milliequivalent of $\mathrm{Cr_2\,O_7^{2-}}$
So 20 ml of mixture require
6 m_{eq} of $\mathrm{Cr_2\,O_7^{2-}}$
a = 6

(33) **Answer :** 88.78
Hint:
Apply Nernst equation
$\mathrm{E_{cell} = E^o_{cell} - \frac{0.06}{n} \log \frac{[P]}{[R]}}$
Solution:
According to figure given
Ag is cathode and Cu is anode
Anode : Cu $\rightarrow$ Cu^{2+} + $2e^-$
Cathode : $2Ag^+$ + $2e^-$ $\rightarrow$ 2Ag
$\mathrm{E^o_{cell}}$ = (0.80 – 0.34) V
= 0.46
ΔG° = –nF E^o_{cell}
ΔG° = – 2 × 96500 × 0.46
= –88.78 kJ
$\mathrm{E_{cell} = 0.46 - \frac{0.06}{2} \log \frac{[Cu^{2+}]}{[Ag^+]^2}}$
$= 0.46 - 0.03 \ \log \left(\frac{1}{10^{-4}}\right)$
= 0.46 – 0.03 $\log 10^4$
= 0.46 – 0.12
= 0.34

(34) **Answer :** 00.34
Hint:
Apply Nernst equation
$\mathrm{E_{cell} = E^o_{cell} - \frac{0.06}{n} \log \frac{[P]}{[R]}}$
Solution:
According to figure given
Ag is cathode and Cu is anode

Anode : Cu $\rightarrow Cu^{2+} + 2e^-$

Cathode : $2Ag^+ + 2e^- \rightarrow 2Ag$

E^o_{cell} = (0.80 – 0.34) V

= 0.46

$\Delta G^\circ = -nF\, E^o_{cell}$

$\Delta G^\circ = -2 \times 96500 \times 0.46$

= –88.78 kJ

$$E_{cell} = 0.46 - \frac{0.06}{2}\log\frac{[Cu^{2+}]}{[Ag^+]^2}$$

$$= 0.46 - 0.03\ \log\left(\frac{1}{10^{-4}}\right)$$

= 0.46 – 0.03 $\log 10^4$

= 0.46 – 0.12

= 0.34

MATHEMATICS

Section-I

(35) **Answer :** (B)

Hint:

$f(x) > 0$ or $f(x) < 0, \forall x$

Solution:

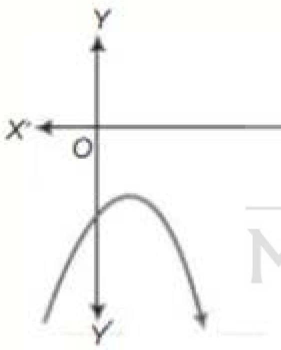
Since, $f(x) = ax^2 + bx + c = 0$ has non-real roots.

Thus, either

$f(x) > 0$ or $f(x) < 0, \forall x$

$a + c < b \Rightarrow f(-1) < 0$

As



i.e. $a - b + c < 0$ or $a + c < b$

$\therefore f(x) < 0, \forall x$

Thus, for all $x \in R$; $ax^2 + bx + c < 0$.

Now, for $x = -2$; $4a - 2b + c < 0$

$\Rightarrow 4a + c < 2b$

(36) **Answer :** (B)

Hint:

Principal argument

Solution:

Principal argument $z = \arg(i)^{1/2} + \arg(-1+i) - \arg(2+2\sqrt{3}i) - \arg(-1)$

$= \frac{1}{2} \times \frac{\pi}{2} + \pi - \frac{\pi}{4} - \frac{\pi}{3} - \pi$

$= -\frac{\pi}{3}$

(37) **Answer :** (A)

Hint:

$= \frac{1}{x}\left[\sqrt{a+rx} - \sqrt{a+(r-1)x}\,\right]$

Solution:

Let

$t_r = \frac{1}{\sqrt{a+rx}+\sqrt{a+(r-1)x}}$

$= \frac{\sqrt{a+rx}-\sqrt{a+(r-1)x}}{a+rx-a-(r-1)x}$

$= \frac{1}{x}\left[\sqrt{a+rx}-\sqrt{a+(r-1)x}\right]$

$\therefore\ t_1+t_2+\ldots+t_n = \frac{1}{x}[\{\sqrt{a+x}-\sqrt{a}\}+\sqrt{a+2x}$

$-\sqrt{a+x}\}+\ldots+\{\sqrt{a+nx}-\sqrt{a+(n-1)x}\}]$

$= \frac{1}{x}\left[\sqrt{a+nx}-\sqrt{a}\right]$

$= \frac{a+nx}{x(\sqrt{a+nx}+\sqrt{a})}$

$= \frac{n}{\sqrt{a}+\sqrt{a+nx}}$

Hence, (A) is the correct answer.

(38) Answer : (D)

Hint:

$3^{2008} = 3(3^{2007}) = 3(3^{3\times 669})$

Solution:

$3^{2008} = 3(3^{2007}) = 3(3^{3\times 669})$

$= 3 \times 27^{669} = 3 \times (28-1)^{669} = 3(28\lambda - 1)$

$= 7\mu - 3 = 7(\mu - 1) + 4$

$\therefore$ Remainder = 4

Section-II

(39) Answer : (C,D)

Hint:

Number of students who appeared in the examination = ${}^{20}C_r$

Solution:

Number of students who appeared in the examination = ${}^{20}C_r$

${}^{20}C_r$ will be maximum when

$r = \frac{20}{2} = 10$.

$\therefore$ Maximum number of students $= {}^{20}C_{10} = \frac{20!}{(10!)^2}$

(40) Answer : (B,C,D)

Hint:

$AB^2 = BA \Rightarrow B^2 = A^{-1}BA$ squaring both the sides

Solution:

$AB^2 = BA \Rightarrow B^2 = A^{-1}BA$ squaring both the sides

$B^4 = \left(A^{-1}BA\right)\left(A^{-1}BA\right) = A^{-1}B^2A = A^{-1}\left(A^{-1}BA\right)A\ \ B^4 = \left(A^{-1}\right)^2BA^2$

Again squaring

$B^8 = \left(\left(A^{-1}\right)^2BA^2\right)\left(\left(A^{-1}\right)^2BA^2\right) \Rightarrow B^8 = \left(A^{-1}\right)^2 B^2 A^2 = \left(A^{-1}\right)^2\left(A^{-1}BA\right)A^2 = \left(A^{-1}\right)^3BA^3$ Similarly

$B^{64} = \left(A^{-1}\right)^6BA^6 \Rightarrow B^{63} = I$

so $m = 63 = k, \frac{k}{9} = 7$

(41) Answer : (A,C)

Hint:

$\left(\sin^2 x + \frac{1}{\sin^2 x}\right) + \left(\cos^2 y + \frac{1}{\cos^2 y}\right) = 4$

Solution:

$\left(\sin^2 x + \frac{1}{\sin^2 x}\right) + \left(\cos^2 y + \frac{1}{\cos^2 y}\right) = 4 \quad \Rightarrow$

Only possible if $\sin x = 1$ and $\cos y = 1$

$\Rightarrow x = \frac{\pi}{2}$ and $y = 0$

$\therefore (x-y) = \frac{\pi}{2}$

Section-III

(42) Answer : 12

Hint:

$R_2 \to R_2 - (R_1 + R_3)$

Solution:

We have, $\Delta_r = \begin{vmatrix} r^2+1r & r+1 & r-2 \\ 2r^2+3r-1 & 3r & 3r-3 \\ r^2+2r+3 & 2r-1 & 2r-1 \end{vmatrix}$

Applying $R_2 \to R_2 - (R_1 + R_3)$, then

$$\Delta_r = \begin{vmatrix} r^2+r & r+1 & & & r-2 \\ \vdots & & & & \\ -4 & \cdots & 0 & \cdots & 0 \\ \vdots & & & & \\ r^2+2r+3 & 2r-1 & & & 2r-1 \end{vmatrix}$$

Expanding along R_2 , we get

$= 4\begin{vmatrix} r+1 & r-2 \\ 2r-1 & 2r-1 \end{vmatrix}$

$= 4\left[(r+1)(2r-1) - (r-2)(2r-1)\right]$

$= 24r - 12$

Now, $\sum_{r=1}^{n} \Delta_r = 24\sum_{r=1}^{n} r - 12\sum_{r=1}^{n} 1$

$= 24\frac{n(n+1)}{2} - 12n = 12n(n+1-1)$

$= 12n^2 = an^2 + bn + c$

[given]

For $n = 1$, we have

$a + b + c = 12$

(43) Answer : 3

Hint:

$ax^2 + bx + c = 0$

Solution:

Given that, $ax^2 + bx + c = 0$

and $bx^2 + cx + a = 0$ have a common root. Hence

$\left(bc - a^2\right)^2 = \left(ab - c^2\right)\left(ac - b^2\right)$

$\Rightarrow b^2c^2 + a^4 - 2a^2bc = a^2bc - ab^3 - ac^3 + b^2c^2$

$\Rightarrow a^4 + ab^3 + ac^3 = 3a^2bc$

$\Rightarrow \frac{a^3+b^3+c^3}{abc} = 3$

(44) Answer : 0

Hint:

$z_1 = \frac{\lambda z_2 + z_3}{\lambda + 1}$

Solution:

As $z_1 = \frac{\lambda z_2 + z_3}{\lambda + 1}$

which shows z_1 divides z_2, z_3 in the ratio of 1 : λ. Thus, the points are collinear.

∴ Distance of A from line BC is zero.

(45) Answer : 14

Hint:

$a_1 = 2, \frac{a_n}{a_{n-1}} = \frac{a_{n-1}}{a_{n-2}} \Rightarrow a_1, a_2, a_3, \ldots\ldots, a_n$ are in G.P.

Solution:

$a_1 = 2, \frac{a_n}{a_{n-1}} = \frac{a_{n-1}}{a_{n-2}} \Rightarrow a_1, a_2, a_3, \ldots\ldots, a_n$ are in G.P.

Let $a_2 = x$, then for $x = 3$,

$\frac{a_3}{a_2} = \frac{a_2}{a_1} = \frac{x^2}{2}$

$\Rightarrow {a_2}^2 = a_1a_3 \Rightarrow a_3 = \frac{x^2}{2}$

$\Rightarrow 2, x, \frac{x^2}{2}, \frac{x^3}{4}, \frac{x^4}{8}, \ldots.$ with common ratio,

$r = \frac{x}{2}$ given $\frac{x^4}{8} \le 162 \Rightarrow x \le 6$

Also $x \text{ and } \frac{x^4}{8}$ are integer's, so if x is even then only $\frac{x^4}{8}$ will be an integer. So only possible value of *x* is 4 and 6.

(46) Answer : 3

Hint:

Given, $\sum_{r=0}^{3n} a_r x^r = \left(1+x+x^2+x^3\right)^n$

Solution:

Given, $\sum_{r=0}^{3n} a_r x^r = \left(1+x+x^2+x^3\right)^n$

It is clear that a_r is the coefficient of x^r in the expansion of $\left(1+x+x^2+x^3\right)^n$.

On replacing x by $\frac{1}{x}$ in the given equation, we get

$\sum_{r=0}^{3n} a_r\left(\frac{1}{x}\right)^r = \frac{\left(1+x+x^2+x^3\right)^n}{x^{3n}}$

Here, a , represents the coefficient of 3^{3n-r} in $\left(1+x+x^2+x^3\right)^n$.

Thus,

$a_r = a_{3n-r}$ ....(i)

Let $I = \sum_{r=0}^{3n} r \times a_r = \sum_{r=0}^{3n}(3n-r)a_{3n-r}$

[replacing r by $3n-r$]

$= \sum_{r=0}^{3n}(3n-r)a_r$ [from Eq. (i)]

$= 3n\sum_{r=0}^{3n} a^r - \sum_{r=0}^{3n} ra^r$

$\Rightarrow\ 2I = 3nk \Rightarrow I = \frac{3nk}{2} \therefore \lambda = 3$

(47) **Answer :** 31

Hint:

$7 \times 31 = 217$

Solution:

Here, we have two sections A and B (say), the section A has 3 questions and section B has 5 questions and one question from each section is compulsory, according to the given direction.

$\therefore$ Number of ways selecting one or more than one question from section A is $2^3-1=7$ and number of ways selecting one or more than one question from section B is $2^5-1=31$

Hence, by the principle of multiplication, the required number of ways in which a candidate can select the questions.

$= 7 \times 31 = 217$

Section-IV

(48) **Answer :** 04.00

Hint:

$\Delta > 0 \Rightarrow\ m \in R,\ f(2)\ f(3) < 0$

Solution:

$\Delta > 0 \Rightarrow\ m \in R,\ f(2)\ f(3) < 0$

$\Rightarrow\ m \in \left(\frac{7-\sqrt{33}}{2}, \frac{11-\sqrt{73}}{2}\right) \cup \left(\frac{7+\sqrt{33}}{2}, \frac{11+\sqrt{73}}{2}\right)$

$\Rightarrow m = 1,\ 7,\ 8,\ 9$

(49) **Answer :** 09.00

Hint:

$\Rightarrow f(2) < 0$

Solution:

(i) Exactly one root is less than 2

$\Rightarrow f(2) < 0 \Rightarrow\ m \in \left(\frac{7-\sqrt{33}}{2}, \frac{7+\sqrt{33}}{2}\right) \to 0$

No negative integral m

(ii) Both roots less than 2

$\Rightarrow f(2) > 0$ and $2 > \frac{4m+2}{4}$

$\Rightarrow m \in \left(-\infty, \frac{7-\sqrt{33}}{2}\right)$

$\therefore$ Negative integral m = –9 to –1

(50) **Answer :** 08.00

Hint:

$$D_n = \begin{vmatrix} a & 1 & 0 & 0 & 0 & \ldots & 0 \\ 1 & a & 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & a & 1 & 0 & \ldots. & 0 \\ 0 & 0 & 1 & a & \ldots & \ldots & 0 \\ 0 & 0 & 0 & 1 & a & \ldots & 0 \\ 0 & 0 & 0 & 0 & 1 & \ldots & a \end{vmatrix}$$

Solution:

$$D_n = \begin{vmatrix} a & 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & a & 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & a & 1 & 0 & \dots. & 0 \\ 0 & 0 & 1 & a & \dots & \dots & 0 \\ 0 & 0 & 0 & 1 & a & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 & \dots & a \end{vmatrix}$$

expanding it from 1^{st} Row

$$D_n = aD_{n-1} - \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & \dots.. \\ 0 & a & 1 & 0 & 0 & \dots.. \\ 0 & 1 & a & 1 & 0 & \dots.. \\ \vdots & 0 & 1 & a & \dots & \dots.. \\ 0 & & & & & \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$

$$D_n = aD_{n-1} - \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & \dots.. & 0 \\ 0 & a & 1 & 0 & 0 & \dots.. & 0 \\ 0 & 1 & a & 1 & 0 & \dots.. & 0 \\ 0 & 0 & 1 & a & 1 & \dots.. & 0 \\ \vdots & \vdots & \vdots & \vdots & & & \\ 0 & 0 & 0 & \dots. & & & 001a \end{vmatrix} D_n = aD_{n-1} - D_{n-2}$$

$D_n = aD_{n-1} - D_{n-2}$

$a = 2 \quad \Rightarrow \quad D_n = 2D_{n-1} - D_{n-2}$

$D_n + D_{n-2} = 2D_{n-1}$

i.e., D_{n-2}, D_{n-1}, D_n are in A.P.

$D_1 = 2\ D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$

$D_3 = 4 \dots..D_{2017} = 2018$

(51) **Answer :** 13.00

Hint:

$D_n = D_{n-1} - D_{n-2}$

Solution:

$a = 1$

$D_n = D_{n-1} - D_{n-2}$

$D_1 = 1\ D_2 = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$D_3 = D_2 - D_1 = -1$

$D_4 = -1$

$D_5 = 0$

$D_6 = 1$

$D_7 = 1$

$D_8 = 0$

$D_9 = -1$

$D_{10} = -1$

$D_{11} = 0$

$D_{12} = 1$

$D_{13} = 1$

i.e., $D_1 = D_6 = D_7 = D_{12} = D_{13} = 1$

$D_2 = D_5 = D_8 = D_{11} = 0$

$D_3 = D_4 = D_9 = D_{10} = -1$

6λ, $6\lambda + 1$ type number gives 1

$6\lambda + 2$, $6\lambda + 5$ gives 0

6λ + 3, 6λ + 4 gives –1

Hence $\sum_{k=1}^{2017} |D_k| = \frac{4}{6}\left(2016\right) + 1$

= 2(672) + 1 = 1344 + 1 = 1345