09/11/2025

Online

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MM : 300 AIATS For One Year JEE(Main)-2026 (XII Studying)_Test-02_ONLINE **Time : 180 Min.**

CHEMISTRY

Section-I

1. (2)

2. (1)

3. (3)

4. (2)

5. (1)

6. (3)

7. (2)

8. (2)

9. (1)

10. (1)

11. (1)

12. (4)

13. (3)

14. (2)

15. (4)

16. (3)

17. (1)

18. (1)

19. (2)

20. (2)

Section-II

21. (90)

22. (29)

23. (5)

24. (10)

25. (4)

MATHEMATICS

Section-I

26. (3)

27. (4)

28. (1)

29. (2)

30. (1)

31. (4)

32. (2)

33. (2)

34. (4)

35. (1)

36. (1)

37. (4)

38. (2)

39. (2)

40. (3)

41. (1)

42. (3)

43. (1)

44. (3)

45. (2)

Section-II

46. (6)

47. (4)

48. (4)

49. (32)

50. (10)

PHYSICS

Section-I

51. (2)

52. (3)

53. (2)

54. (2)

55. (3)

56. (1)

57. (3)

58. (1)

59. (3)

60. (3)

61. (3)

62. (1)

63. (3)

64. (3)

65. (3)

66. (3)

67. (2)

68. (3)

69. (4)

70. (2)

Section-II

71. (9)

72. (58)

73. (14)

74. (13)

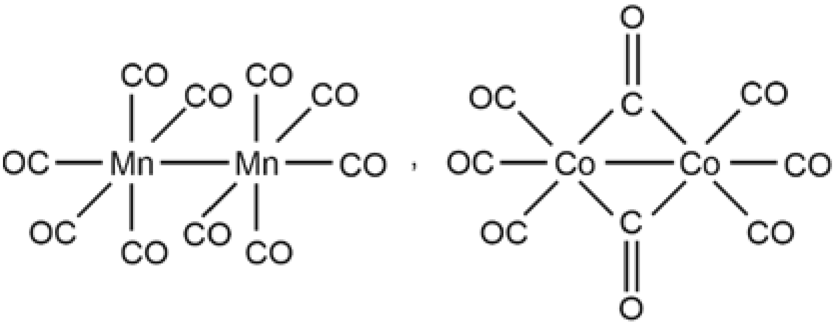
75. (14)

Hints and Solutions

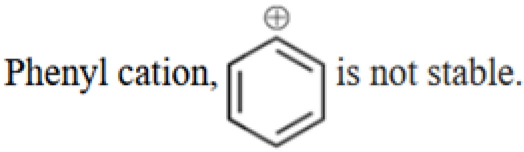
CHEMISTRY

Section-I

(1) **Answer :** (2)
Hint:
Mn has octahedral geometry in $[Mn_2(CO)_{10}]$
Solution:



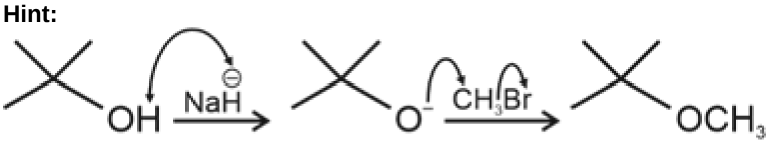
(2) **Answer :** (1)
Hint:



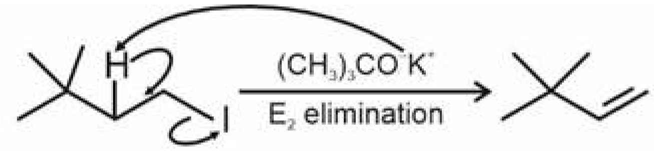
Solution:
Both statements are correct and reason explains assertion.

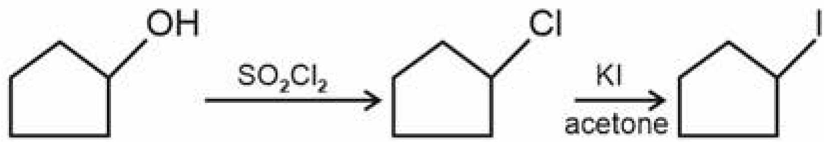
(3) **Answer :** (3)
Hint:
Chelation causes more stability of complexes.
Solution:
$[Co(en)_2(NH_3)_2]^{3+}$ is more stable than $[Co(NH_3)_6]^{3+}$
$[Fe(CN)_6]^{3-}$ is more stable due to higher charge on Fe

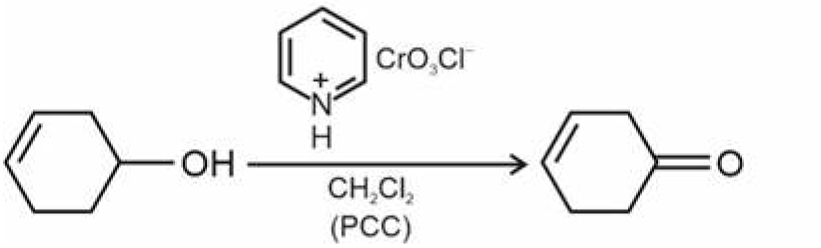
(4) **Answer :** (2)
Hint:



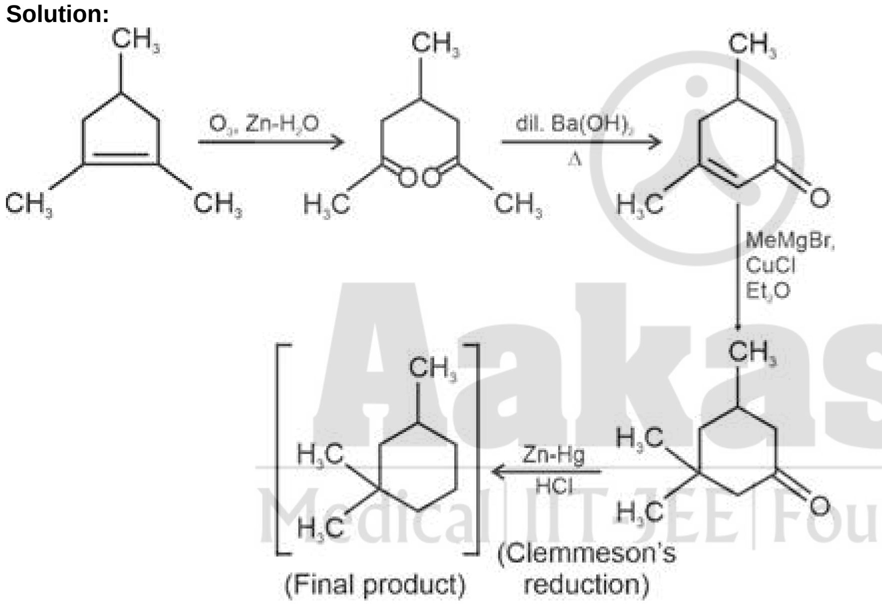
Solution:







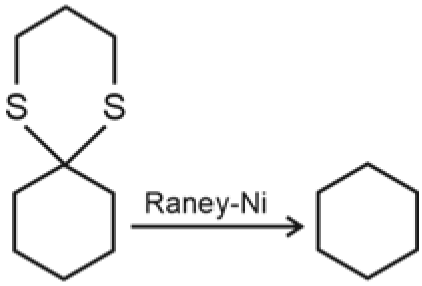
(5) **Answer :** (1)
Hint:
dil. $Ba(OH)_2$ causes aldol reaction in the ketone produced after ozonolysis.
Solution:



(6) **Answer :** (3)
Hint:
Cl^- act as WFL in presence of Ni^{2+} ion.
F^- act as WFL in presence of Co^{3+} ion.
Solution:
$[NiCl_4]^{2-}$ = 2 unpaired e^-
$[Ni(CN)_4]^{2-}$ = 0 unpaired e^-
$[CoF_6]^{3-}$ = 4 unpaired e^-
$[Fe(CN)_6]^{3-}$ = 1 unpaired e^-

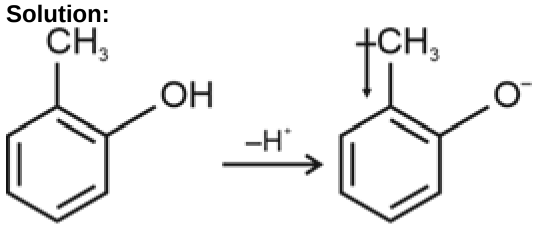
(7) **Answer :** (2)
Hint:
A will produce optically inactive compound.
Solution:
B and C are already optically inactive hence they will also produce optically inactive compounds.

(8) **Answer :** (2)
Hint:



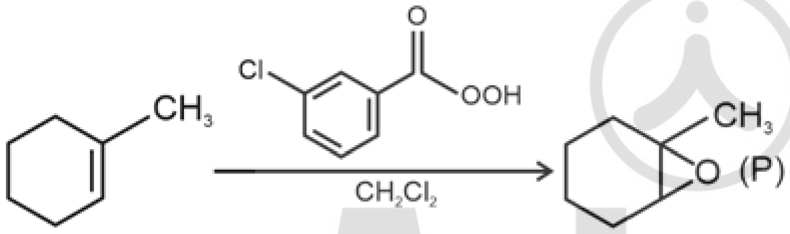
Solution:
A, B and D reagents used to convert cyclohexanone to cyclohexane.

(9) **Answer :** (1)
Hint:
Negative charge of phenoxide ion is stabilised by electron withdrawing groups.
Solution:



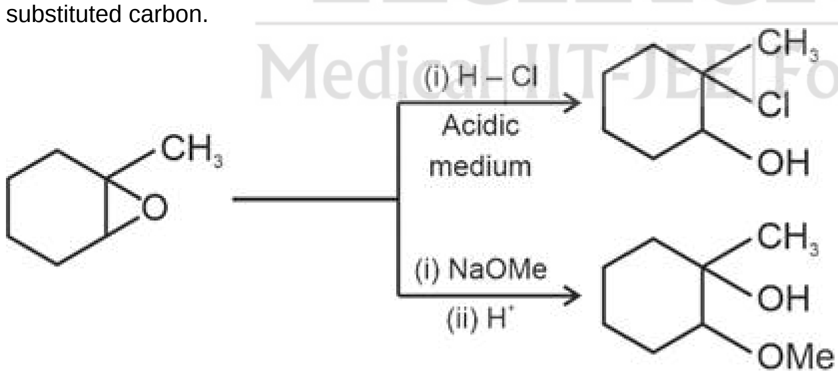
$-CH_3$ being electron releasing group, destablises phenoxide ion an hence, resulting in decrease in acidic strength.

(10) **Answer :** (1)
Hint:



(Epoxide is formed)

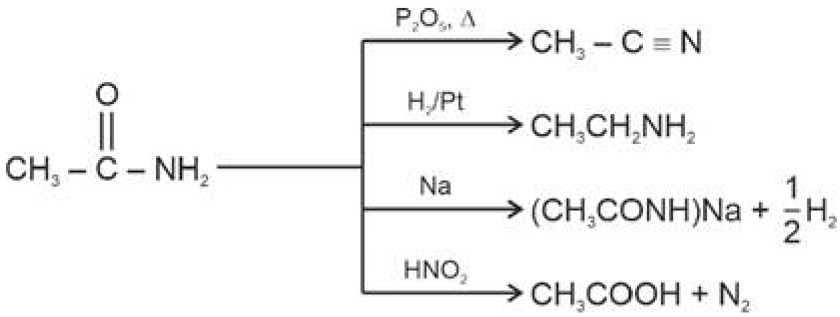
Solution:
Attack of $Nu^{\ominus}$ in acidic medium occur at more substituted carbon while in basic medium, attack of $Nu^{\ominus}$ is on less substituted carbon.



(11) **Answer :** (1)
Hint:

$$R-\overset{\overset{\displaystyle O}{\|}}{C}-NH_2 \xrightarrow{NaOBr} R-NH_2 \text{ (Hoffman Bromamide)}$$

Solution:

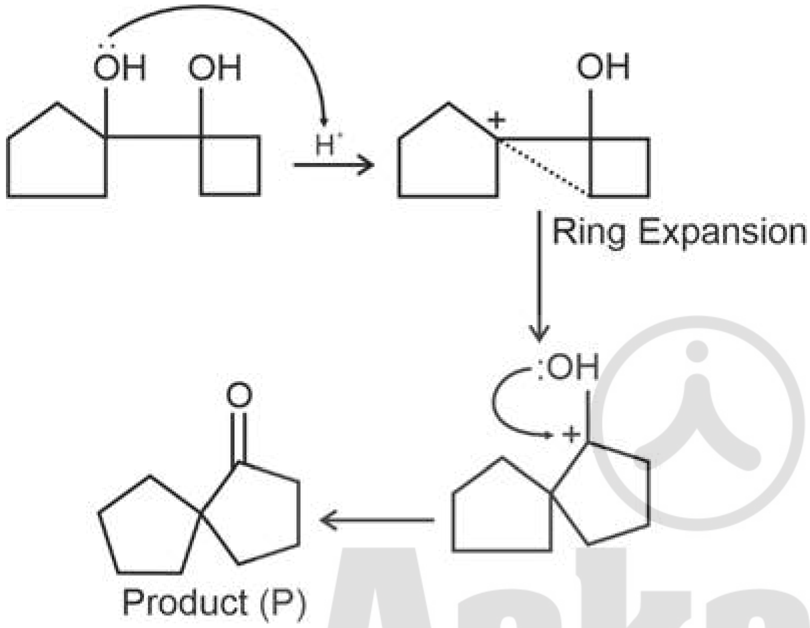


(12) **Answer :** (4)

Hint:

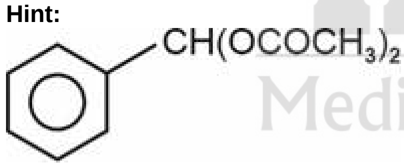
Ring expansion will take place.

Solution:



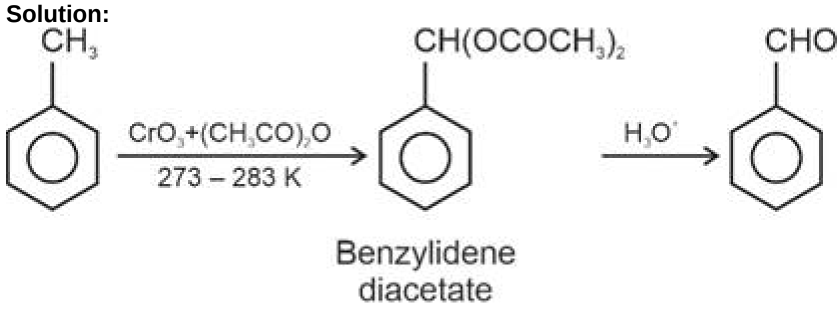
(13) **Answer :** (3)

Hint:



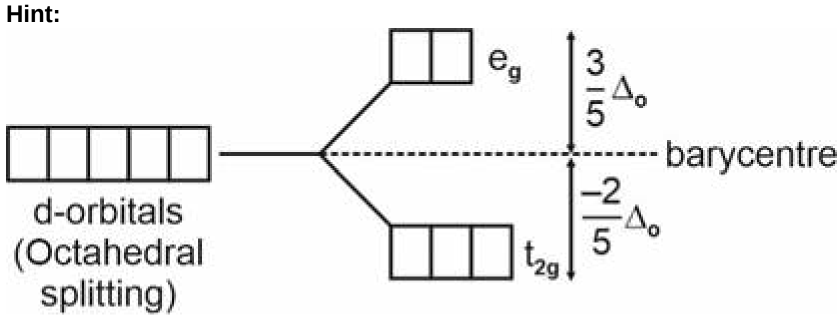
is the intermediate

Solution:



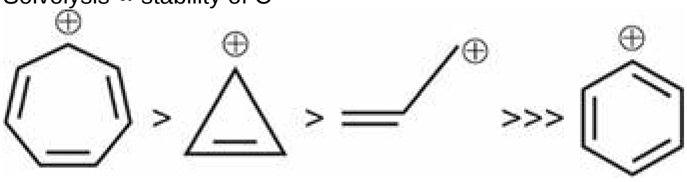
(14) **Answer :** (2)

Hint:

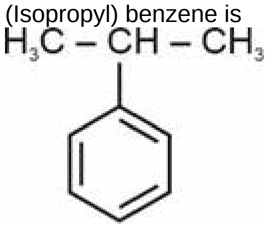


Solution:
Both statements are correct, but (R) does not explain (A).

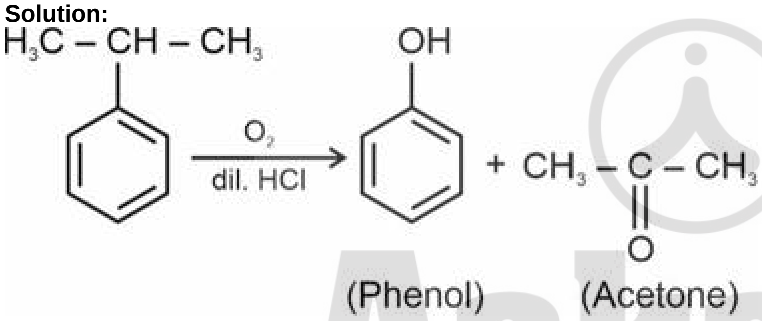
(15) **Answer :** (4)
Hint:
Tropylium carbocation is most stable due to Aromaticity
Solution:
Solvolysis ∝ stability of $C^{\oplus}$



(16) **Answer :** (3)
Hint:
(Isopropyl) benzene is

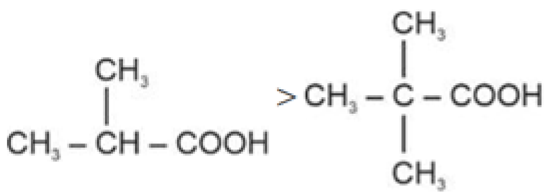


also known as cumene.
Solution:



It is a method of preparation of phenol.

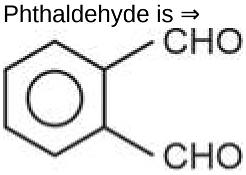
(17) **Answer :** (1)
Hint:
HCOOH is most reactive among given acids.
Solution:
The correct order of reactivity of carboxylic acids towards esterification reaction is
$HCOOH > CH_3COOH > CH_3CH_2COOH >$



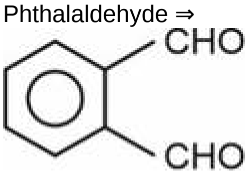
(18) **Answer :** (1)
Hint:
More stable carbocation formed in 3° alkyl halide.
Solution:
+I effect of alkyl group make 2° alkyl halide more reactive than 1° towards hydrolysis.

(19) **Answer :** (2)
Hint:
Cl^-, WFL in presence of Fe^{3+}.
Solution:
$[FeCl_6]^{3-} \Rightarrow Fe^{3+} \rightarrow d^5 \rightarrow$ Outer orbital complex (sp^3d^2 and octahedral)
$[Fe(CN)_6]^{4-} \Rightarrow Fe^{2+} \rightarrow d^6 \rightarrow$ Inner orbital complex (d^2sp^3 and octahedral)
$[Ni(CO)_4] \Rightarrow Ni^0 \rightarrow 4s^2 3d^8 \rightarrow 4s^0 3d^{10} \rightarrow sp^3 \rightarrow$ tetrahedral.
$[Ni(CN)_4]^{2-} \Rightarrow Ni^{2+} \Rightarrow 3d^8 \Rightarrow dsp^2$ with square planar shape.

(20) **Answer :** (2)
Hint:
Phthaldehyde is ⇒



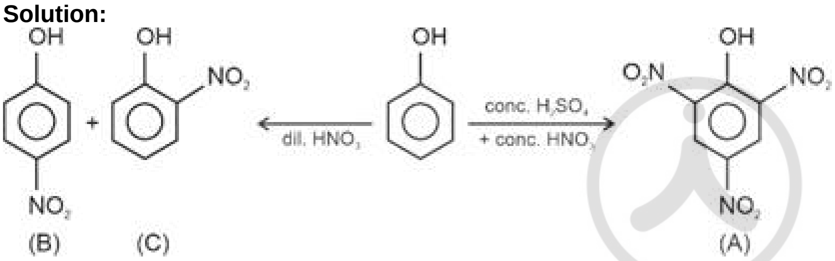
Solution:
Acrolein ⇒ CH_2 = CH – CHO ⇒ Prop-2-enal
Phthalaldehyde ⇒



⇒ Benzene-1, 2-dicarbaldehyde

Section-II

(21) **Answer :** 90
Hint:
Trisubstitution takes place in case of conc. HNO_3
Solution:

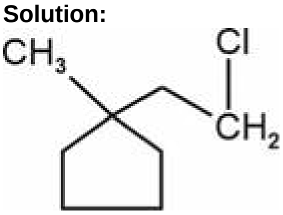


Molar mass of A = 229 g/mol
Molar mass of B = 139 g/mol
Difference = 229 – 139 = 90

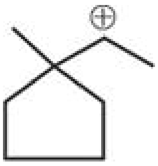
(22) **Answer :** 29
Hint:
$V_{N_2} = \frac{T_2 \times (P_1 - Aq.\ tension) \times V_1}{T_1 \times P_2}$
Solution:
V_{N_2} at STP = $\frac{273 \times (725 - 25) \times 50}{300 \times 760}$
= 41.9 mL
$M_{N_2} = \frac{41.9}{22400} \times 28 = 0.052$
$\% N = \frac{0.052}{0.18} \times 100 = 28.8$

(23) **Answer :** 5
Hint:
2N atoms and 3 – O atoms are available for coordination.
Solution:
Total 5 coordination sites are present in $EDTA^{3-}$

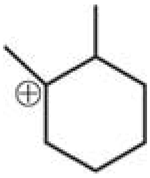
(24) **Answer :** 10
Hint:
Elimination reaction will take place.
Solution:

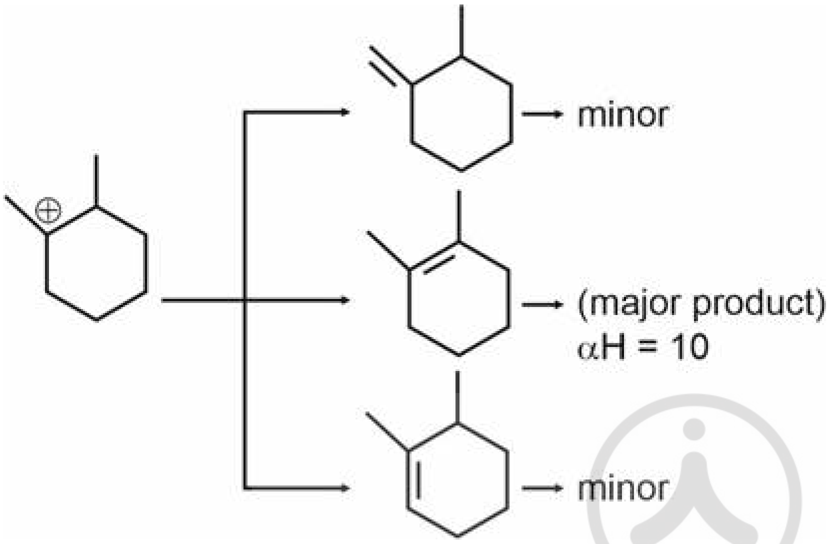


1-(2-chloroethyl)-1-methylcyclopentane



is formed in this reaction, which rearranges to give





(25) **Answer :** 4

Hint:

Zero unpaired e^-, zero magnetic moment.

Solution:

$[Co(CN)_6]^{3-}$, $[Fe(NO_2)_6]^{4-}$, $[Ni(dmg)_2]^{2+}$ and $[Co(NH_3)_6]^{3+}$ have zero unpaired e^-.

MATHEMATICS

Section-I

(26) **Answer :** (3)

Hint:

Convert in definite integration

$\lim\limits_{n\to\infty}\sum\limits_{K=1}^{2n}\frac{k}{n^2}\sin\left(\frac{K^2+n^2}{n^2}\right)$ $\left\{\text{Put } l=\frac{K}{n}\right\}$

Solution:

$\lim\limits_{n\to\infty}\left[\frac{1}{n^2}\sin\left(\frac{1+n^2}{n^2}\right)+\frac{2}{n^2}\sin\left(\frac{4+n^2}{n^2}\right) + \frac{3}{n^2}\sin\left(\frac{9+n^2}{n^2}\right)+...+\frac{2}{n}\sin 5\right]$

$=\lim\limits_{n\to\infty}\sum\limits_{K=1}^{2n}\frac{K}{n^2}\sin\left(\frac{K^2+n^2}{n^2}\right)$

$=\lim\limits_{n\to\infty}\sum\limits_{K=1}^{2n}\frac{K}{n^2}\sin\left(\frac{K^2}{n^2}+1\right)$

Put $l=\frac{K}{n}$

$=\int\limits_0^2 l\sin(l^2+1)dt$

Now, put $u = \lambda^2 + 1$

$du = 2\lambda dt$

$= \int_1^5 \frac{1}{2}\sin u du = -\frac{1}{2}[\cos u]_1^5$

$= \frac{1}{2}[\cos 5 - \cos 1]$

$= -\frac{1}{2}\left(-2\sin\frac{6}{2}\sin\frac{4}{2}\right)$

= sin3 · sin2

(27) Answer : (4)

Hint:

$f'(x) > 0$

Solution:

$f(x) = 6ax - a\sin 4x - 6x - \sin 6x$

$f'(x) = 6a - 4a\cos 4x - 6 - 6\cos 6x$

$f'(x) > 0\ \ \forall x \in R$

$\because -1 \le \cos 6x \le 1$

$\because -1 \le \cos 5x \le 1$

If $a > 0$, $(f'(x))_{min} = 6a - 6 - 4a - 6$

$\Rightarrow 2a - 12 > 0$

$\Rightarrow a > 6$

$\Rightarrow a \in (6, \infty)$

If $a < 0$

$f'(x) = 4a(1 - \cos 4x) + 2a - 6(1 + \cos 6x)$

$f'(x) < 0$

(28) Answer : (1)

Hint:

$f(x) = \begin{cases} 0 & , \quad x < 0 \\ 2x^2 & , \quad x \ge 0 \end{cases}$

Solution:

When $t \ge 0$, then

$x = 2t - t = t$ and $y = t^2 + t^2 = 2t^2$

Thus $y = 2x^2$

When $t < 0$, then

$x = 2t + t = 3t$ and $y = t^2 - t^2 = 0$

Thus, $y = 0$, $x < 0$

$\therefore \quad f(x) = \begin{cases} 0 & , \quad x < 0 \\ 2x^2 & , \quad x \ge 0 \end{cases}$

Now, $f'(0^-) = 0$

$f'(0^+) = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0}$

$\lim_{x \to 0^+} \frac{2x^2 - 0}{x - 0}$

$= \lim_{x \to 0^+} \frac{2x^2}{x} = 0$

$\therefore f(x)$ is differentiable at $x = 0$

Hence it is also continuous at $x = 0$

(29) Answer : (2)

Hint:

Take x^{2025} common from D^r

Solution:

$\frac{dx}{x^{2025}\left(1 + \frac{1}{x^{2024}}\right)}$

Let $1 + \frac{1}{x^{2024}} = t$

$-2024\, x^{-2025}\, dx = dt$

$I = \frac{-1}{2024}\int \frac{dt}{t}$

$= \frac{-1}{2024}\ln\left|1 + \frac{1}{x^{2024}}\right| + c$

$= \frac{-1}{2024}\ln\left|\frac{x^{2024} + 1}{x^{2024}}\right| + c$

$= \frac{1}{2024}\ln\left(\frac{x^{2024}}{1 + x^{2024}}\right) + c$

$\Rightarrow a = b = c = 2024$

(30) **Answer :** (1)
Hint:
Put $e^{1/k} = \lambda$
Solution:

$$P = \lim_{k\to\infty}\left[\frac{e^{\frac{1}{k}}}{k^2} + \frac{2\left(e^{\frac{1}{k}}\right)^2}{k^2} + \frac{3\left(e^{\frac{1}{k}}\right)^3}{k^2} + \ldots + \frac{\left(e^{\frac{1}{k}}\right)^k}{k}\right]$$

Let $e^{\frac{1}{k}} = \lambda$

$\Rightarrow P = \lim_{k\to\infty}\frac{1}{k^2}\left[\lambda + 2\lambda^2 + 3\lambda^3 + \ldots + k\cdot\lambda^k\right]$

Let $P_1 = \lambda + 2\lambda^2 + 3\lambda^3 + \ldots + k\lambda^k$

$\Rightarrow \lambda P_1 = \lambda^2 + 2\lambda^3 + \ldots + k\lambda^{k+1}$

$\Rightarrow P_1(1-\lambda) = \left(\lambda + \lambda^2 + \lambda^3 + \ldots + \lambda^k\right) - k\lambda^{k+1} \quad = \frac{\lambda\left(\lambda^k - 1\right)}{(\lambda-1)} - k\lambda^{k+1}$

$\Rightarrow P_1 = \frac{-\lambda\left(\lambda^k-1\right)}{(\lambda-1)^2} + \frac{k\lambda^{k+1}}{(\lambda-1)}$

$$\therefore P = \lim_{k\to\infty}\left[\frac{-e^{\frac{1}{k}}(e-1)}{k^2\left(e^{\frac{1}{k}}-1\right)^2} + \frac{e^{1+\frac{1}{k}}}{k\left(e^{\frac{1}{k}}-1\right)}\right]$$

$$= -(e-1)\lim_{k\to\infty}\frac{e^{\frac{1}{k}}}{k^2\left(e^{\frac{1}{k}}-1\right)^2} + e\lim_{k\to\infty}\frac{e^{\frac{1}{k}}}{k\left(e^{\frac{1}{k}}-1\right)}$$

$$= -(e-1)\lim_{k\to\infty}\frac{e^{\frac{1}{k}}}{\left(\frac{e^{\frac{1}{k}}-1}{\frac{1}{k}}\right)^2} + e\lim_{k\to\infty}\frac{e^{\frac{1}{k}}}{\left(\frac{e^{\frac{1}{k}}-1}{\frac{1}{k}}\right)} \quad = -e + 1 + e = 1$$

$\therefore \quad P^{P^{P\ldots\ldots(2016\text{ times})}} = 1$

(31) **Answer :** (4)
Hint:
$\Rightarrow 2a^2 + a + 1 > 3a^2 - 4a + 1$
Solution:
$f(2a^2 + a + 1) < f(3a^2 - 4a + 1)$
$\Rightarrow 2a^2 + a + 1 > 3a^2 - 4a + 1$
$\Rightarrow a \in (0, 5)$
Also, $2a^2 + a + 1 > 0$ and $3a^2 - 4a + 1 > 0$
$\Rightarrow\ a < \frac{1}{3}$ or $a > 1$
So, $a \in \left(0,\ \frac{1}{3}\right) \cup (1,\ 5)$

(32) **Answer :** (2)
Hint:
$1 + e^x = t^2$
Solution:
$1 + e^x = t^2$

$$= 2\int \ln(t^2-1)dt = 2t\ln\left(t^2-1\right) - 4\int\frac{t^2}{t^2-1}dt$$

$= 2t\ln\left(t^2-1\right) - 4\left(1 + \frac{1}{t^2-1}\right)dt$

$= 2t\left[\ln(t^2-1) - 2\right] - 2\ln\left(\frac{t-1}{t+1}\right) + c$

$= \sqrt{1+e^x}(2x-4) + 2\ln\left(\frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}\right) + c$

$A = 2, C = 2$
$\therefore A + 4 + C = 2 + 4 + 2 = 8$

(33) **Answer :** (2)

Hint:

$y_{\min} = f(b) = b^3 - 3b^2 + 3$

Solution:

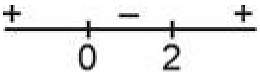
$y = (x - 1)^2 + b^3 - 3b^2 + 3$

$\therefore y_{\min} = f(b) = b^3 - 3b^2 + 3$

Now, $f'(b) = 3b^2 - 6b$

$= 3b(b - 2)$

For maximum or minimum, $f'(b) = 0$



Also, $f(0) = 3$

$f(2) = -1$

$f(4) = 19$

$f(b)_{\max} = 19$

(34) **Answer :** (4)

Hint:

$f(x) = \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$

Solution:

$f(x) = \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\lim\limits_{h\to 0} \frac{f(x)+f(h)+2xh(x+h)-\frac{1}{3}-\left(f(x)+f(0)-\frac{1}{3}\right)}{h}$

$= \lim\limits_{h\to 0} \frac{f(h)-f(0)}{h} + 2x^2 = f'(0) + 2x^2$

$\lim\limits_{h\to 0} \frac{3f(h)-1}{6h} \lim\limits_{h\to 0} \frac{f(h)-\frac{1}{3}}{2h} = \lim\limits_{h\to 0} \frac{f(h)-f(0)}{2h}$

$\Rightarrow \frac{f'(0)}{2} = \frac{2}{3}$

$f'(0) = \frac{4}{3}$

$\therefore \; f'(x) = \frac{4}{3} + 2x^2$

$f(x) = \lambda + \frac{4}{3}x + \frac{2x^3}{3}$ or $f(0) = \lambda = \frac{1}{3}$

$\therefore \; f(x) = \frac{2x^3}{3} + \frac{4x}{3} + \frac{1}{3}$ or $f(2) = \frac{25}{3}$

$\therefore \; [f(2)] = \left[\frac{25}{3}\right] = 8$

(35) **Answer :** (1)

Hint:

Use substitute

Solution:

Rewrite $I = \int\limits_0^{\pi/4} x\sec^2 x dx$

Integration by parts

Let $u = x \Rightarrow du = dx$, $dv = \sec^2 x dx \Rightarrow v = \tan x$

$\int x\sec^2 x dx = x\tan x - \int \tan x dx = x\tan x - \ln|\sec x|$

Apply limit 0 to $\frac{\pi}{4}$

· At $x = \frac{\pi}{4}$: $x \tan x - \ln(\sec x) = \frac{\pi}{4} - \frac{1}{2}\ln 2$

· At $x = 0$: 0

$I = \frac{\pi}{4} - \frac{1}{2}\ln 2$

(36) **Answer :** (1)

Hint:

If $f'(x)$ changes from positive to negative about $x = a$ then, $x = a$ is a point of maxima

Solution:

$h(x) = f^2(x) + (f'(x))^2$

$h'(x) = 2f(x)f'(x) + 2f'(x) \cdot f''(x)$

$= 2f'(x)[f(x) + f''(x)]$

$= -2xg(x)(f'(x))^2$
$h'(x)$ changes from positive to negative about $x = 0$ so $x = 0$ is a point of maxima.

(37) Answer : (4)
Hint:
Use AM ≥ GM
Solution:
Continuous at $x = 1$
$\therefore \cos^2\theta(\alpha^2 + 4) = 1 - 4\beta^2$
$\Rightarrow \alpha^2\cos^2\theta + 4\beta^2 + 4\cos^2\theta = 1$ …(i)
continuous at $x = 3$
$\therefore 3 - 4\beta^2 = \alpha\beta - \beta^2 - 3\beta^2 + 2$
$\Rightarrow \alpha\beta = 1$
Use AM ≥ GM
$\frac{\alpha^2\cos^2\theta+4\beta^2}{2} \geq 2\alpha\beta\,|\cos\theta|$
$1 - 4\cos^2\theta \geq 4|\cos\theta|$
⇒ Maximum value of $\cos\theta = \frac{\sqrt{2}-1}{2}$
$c = 2, d = 4$
$c + d = 6$

(38) Answer : (2)
Hint:
Put $\frac{\ln x}{x} = t \Rightarrow \left(\frac{1-\ln x}{x^2}\right)dx = dt$
Solution:
$I = \int \frac{x^2(1-\ln x)}{x^4\left(\left(\frac{\ln x}{x}\right)^4-1\right)}dx = \int \frac{1-\ln x}{x^2\left(\left(\frac{\ln x}{x}\right)^4-1\right)}dx$
Put $\frac{\ln x}{x} = t \Rightarrow \left(\frac{1-\ln x}{x^2}\right)dx = dt$
$I = \int \frac{dt}{(t^4-1)} = \int \frac{dt}{(t^2+1)(t^2-1)}$
$= \frac{1}{2}\int \frac{(t^2+1)-(t^2-1)}{(t^2+1)(t^2-1)}\,dt$
$I = \frac{1}{2}\left(\int \frac{dt}{t^2-1} - \int \frac{dt}{t^2+1}\right) = \frac{1}{2}\left(\frac{1}{2}\ln\frac{t-1}{t+1} - \tan^{-1}t\right)$
$= \frac{1}{4}\ln\left(\frac{\ln x - x}{\ln x + x}\right) - \frac{1}{2}\tan^{-1}\left(\frac{\ln x}{x}\right) + c$

(39) Answer : (2)
Hint:
Put $f'(x) = 0$
Solution:
$f(x) = x^3 - 3x - 1$
$f'(x) = 3x^2 - 3$
$= 3(x + 1)(x - 1)$
Put $f'(x) = 0$
$\therefore x = -1, 1$
$f''(x) = 6x$
$f''(-1) = -6$ (local maximum)
$f''(1) = 6$ (local minimum)
$f(-1) = (-1)^3 - 3(-1) - 1 = 1$
$f(1) = (1)^3 - 3(1) - 1 = -3$
Positive difference $= |f(-1) - f(1)|$
$= |1 - (-3)|$
$= 4$

(40) Answer : (3)
Hint:
Let $x = \frac{1}{K},\ y = \frac{1}{K+1}$
Solution:
$T_K = \cos^{-1}\left(\frac{1}{K(K+1)} + \frac{\sqrt{(K-1)K(K+1)(K+2)}}{K(K+1)}\right)$

Let $x=\frac{1}{K},\ y=\frac{1}{K+1}$

$\sqrt{1-y^2}=\sqrt{1-\frac{1}{(K+1)^2}}=\sqrt{\frac{(K+1)^2-1}{(K+1)^2}}=\frac{\sqrt{K(K+2)}}{K+1}$

$T_K=\cos^{-1}\left(\frac{1}{K+1}\right)-\cos^{-1}\frac{1}{K}$, substituting n = 2, 3, 4

$S=\lim_{n\to\infty}\cos^{-1}\left(\frac{1}{n+1}\right)-\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$

$S_n=\frac{\pi}{6}=\frac{120\pi}{K}$

∴ K = 720

(41) **Answer :** (1)

Hint:

$$I=\int_{-\infty}^{0}e^x dx+\int_{0}^{\ln 2}(e^x-1)dx+\int_{\ln 2}^{\ln 3}(e^x-2)dx$$

Solution:

$$I=\int_{-\infty}^{0}e^x dx+\int_{0}^{\ln 2}(e^x-1)dx+\int_{\ln 2}^{\ln 3}(e^x-2)dx$$

On solving, I = 3 + ln2 – 2ln3

(42) **Answer :** (3)

Hint:

$f'(0) < 0$

Solution:

For local maxima at $x \in R^+$ & minima at certain $x \in R^-$ the roots of $f'(x) = 0$ are of opposite sign

⇒ $f'(0) < 0$

⇒ $b < 0$

(43) **Answer :** (1)

Hint:

Use substitution

Solution:

Substitution u = arctan$x$ ⇒ $du = \frac{dx}{1+x^2}$

Transform limits :

When $x = 0$, $u = 0$

When $x = 1$, $u=\frac{\pi}{4}$

Integral becomes :

$$I=\int_0^{\frac{\pi}{4}}u\,du=\left.\frac{u^2}{2}\right|_0^{\frac{\pi}{4}}=\frac{(\pi/4)^2}{2}=\frac{\pi^2}{32}$$

(44) **Answer :** (3)

Hint:

Consider $f(x)=f\left(\frac{x}{2^n}\right)$

Solution:

$f(2x) = f(x) = f\left(\frac{x}{2}\right)=f\left(\frac{x}{4}\right)=\ ...=f\left(\frac{x}{2^{n-1}}\right)=f\left(\frac{x}{2^n}\right)$ consider $f(x)=f\left(\frac{x}{2^n}\right)$

$\lim_{n\to\infty}f(x)=\lim_{n\to\infty}f\left(\frac{x}{2^n}\right)$

$\Rightarrow\ f(x)=f(0)\ \forall x\in R$

(Since $f(x)$ is given to be a continuous function)

$\Rightarrow\ f(x)=f(2010)=\frac{\pi}{2}$

$\Rightarrow\ \lim_{x\to 0}\frac{\cos\frac{\pi}{2}-\sqrt[3]{\cos\frac{\pi}{2}}}{\sin^2 x}=0$

(45) **Answer :** (2)

Hint:

Let $g(x)=\sin^{-1}(x)+\frac{x}{1-x^2}$

Solution:

$f'(x)=\frac{2}{\sqrt{1-x^2}}\left(\sin^{-1}x+\frac{x}{1-x^2}\right)$

Now, let $g(x)=\sin^{-1}(x)+\frac{x}{1-x^2}$

$g'(x)=\frac{1}{\sqrt{1-x^2}}+\frac{(1-x^2)-(x)(-2x)}{(1-x^2)^2}$

$g'(x)=\frac{1}{\sqrt{1-x^2}}+\frac{1+x^2}{(1-x^2)^2}>0$

for all $x \in (-1, 1)$

$\Rightarrow g(x)$ is increasing for all $x \in (-1, 1)$

Put $g(0) = 0$, $g(x) > 0$ for $x \in (0, 1)$

So, $f'(x) > 0$ for $x \in (0, 1)$

Section-II

(46) **Answer :** 6

Hint:

Differentiate w.r.t. x

Solution:

$f^3(x)=\int_0^x t\cdot f^2(t)dt$ …(i)

Differentiate w.r.t. x

$3f^2(x)\cdot f'(x)=x\cdot f^2(x)\Rightarrow f'(x)=\frac{x}{3}$

$\Rightarrow\ f(x)=\frac{x^2}{6}+c$ …(ii)

From (i) $f^3(0) = 0 \Rightarrow f(0) = 0$

$\therefore$ from (ii) $c = 0$

$\therefore\ f(x)=\frac{x^2}{6}$

$f(6)=\frac{(6)^2}{6}=6$

(47) **Answer :** 4

Hint:

$g(f(x)) = y$

Solution:

$f(x) = x + 1$ …(i)

$g^{-1}(x) = x^3 + x + 1$ …(ii)

$h(x) = g(f(x))$ …(iii)

$g(f(x)) = y$

$f(x) = g^{-1}(y)$

$x \to y$ (from (ii))

$g^{-1}(y) = y^3 + y + 1$

$f(x) = y^3 + y + 1 = x + 1$

$x = y^3 + y$

$h(x) = y$

$x = h^{-1}(y) = y^3 + y$

$y \to x$

$h^{-1}(x) = x^3 + x$

$\lim_{x\to1}\frac{h^{-1}(x)-h^{-1}(1)}{x-1}$

$\lim_{x\to1}\frac{x^3+x-2}{x-1}$ $\left(\frac{0}{0}\text{form}\right)$

Apply L'Hospital Rule

$\lim_{x\to1}\frac{3x^2+1}{1}$

$= 3(1)^2 + 1 = 4$

(48) **Answer :** 4

Hint:

$I=4\int_0^{\pi}(\cos 2\alpha\cdot\cos\alpha)dx$

Solution:

Consider the numerator $\cos 4x - \cos 4\alpha$

= $(2\cos^2 2x - 1) - (2\cos^2 2\alpha - 1)$
= $2(\cos^2 2x - \cos^2 2\alpha)$
= $2(\cos 2x + \cos 2\alpha)(\cos 2x - \cos 2\alpha)$
= $4(\cos 2x + \cos 2\alpha)(\cos x - \cos\alpha)(\cos x + \cos\alpha)$

$$\therefore\ I = 4\int_0^{\pi}(\cos 2x + \cos 2\alpha)\,(\cos x + \cos\alpha)dx$$

$$= 4\int_0^{\pi}(\cos 2\alpha \cdot \cos\alpha)dx$$

= $4\pi\cos\alpha\cos 2\alpha$
$\therefore K = 4$

(49) **Answer :** 32
Hint:
Let $f''(x) = 6a(x - 1)$, $a > 0$
Solution:
Let $f''(x) = 6a(x - 1)$, $(a > 0)$
then $f'(x) = 6\left(\frac{x^2}{2} - x\right) + b = 3a(x^2 - 2x) + b$
Now, $f'(-1) = 0 \Rightarrow 9a + b = 0 \Rightarrow b = -9a$
$\therefore f'(x) = 3a(x^2 - 2x - 3) = 0$
$\Rightarrow x = -1$ & 3
So, $y = f(-1)$ & $y = f(3)$ are two horizontal tangents
$\therefore$ Distance = $|f(3) - f(-1)| = |-22 - 10| = 32$

(50) **Answer :** 10
Hint:
$f(x) = x^3 - 5x^2 + 2x + 6$
Solution:
$\because f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$
$\therefore f'(x) = 3x^2 + 2xf'(1) + f''(2)$
$f'(1) = 3 + 2f'(1) + f''(2)$
$f'(1) + f''(2) = -3$ …(i)
$f''(x) = 6x + 2f'(1)$
$f''(2) = 12 + 2f'(1)$ …(ii)
From (i) & (ii)
$f'(1) = -5$, $f''(2) = 2$
$f(x) = x^3 - 5x^2 + 2x + 6$
$f(1) = 1 - 5 + 2 + 6 = 4$
$f(0) = 6$
$\therefore f(1) + f(0) = 4 + 6 = 10$

PHYSICS

Section-I

(51) **Answer :** (2)
Hint:
$\delta = i + e - A$
Solution:
$\alpha = i + e - A$
= $90 + \sin^{-1}\left(\frac{1}{\sqrt{2}}\sin 15^\circ\right) - 60 = 51.5^\circ$
$\beta = 45 + 45 - 60 = 30^\circ$

(52) **Answer :** (3)
Hint:
$\Delta q = \frac{\Delta\phi}{R}$
Solution:
$\Delta\phi = \frac{\mu_0 M}{2}\frac{d}{dx}\frac{x}{\sqrt{a^2+x^2}}$

$\Rightarrow\ \Delta\phi = \frac{\mu_0 M}{2} \frac{a^2}{[a^2+x^2]^{3/2}}$

$\Rightarrow\ \Delta\phi = \frac{32\mu_0 M}{125a}$

$\Rightarrow\ q = \frac{32\mu_0 M}{125aR}$

(53) Answer : (2)

Hint:

Snell's law

Solution:

φ = 2θ

$\sin\theta = \mu \sin\frac{\theta}{2}$

$\mu = 2\cos\frac{\theta}{2}$

(54) Answer : (2)

Hint:

Δ*x* = *n*λ for maxima

Solution:

$2\mu t = N\lambda$

$\lambda = \frac{2\mu t}{N}$

$\lambda = 9000\text{Å},\ \ 4500\text{Å},\ 3000\text{Å}$

(55) Answer : (3)

Hint:

Current maximize of minimize in series & parallel LCR circuit respectively.

Solution:

Theoretical

(56) Answer : (1)

Hint:

Refraction at curved surface

Solution:

$\frac{\mu}{v_1} - \frac{3\mu}{\propto} = \frac{\mu-3\mu}{R}$

$\frac{2\mu}{f} - \frac{\mu}{v_1} = \frac{2\mu-\mu}{-2R}$

$\Rightarrow\ \frac{2\mu}{f} = -\frac{2\mu}{R} - \frac{\mu}{2R}$

$\Rightarrow\ \frac{2\mu}{f} = -\frac{5\mu}{2R}$

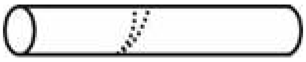
$\Rightarrow\ f = -\frac{4R}{5}$

(57) Answer : (3)

Hint:

$\sigma\omega R = \frac{di}{dx} = \frac{idN}{dx} = ni$

Solution:



$\frac{\sigma 2\pi R dx}{\frac{2\pi}{\omega}}$

$\sigma\omega R dx = d_i$

$\sigma\omega R = \frac{di}{dx} = \frac{idN}{dx} = ni$

$B = \mu_0 \sigma\omega R$

$B = \mu_0(\sigma)(\omega)(2R)$ + $\mu_0(2\sigma)(2\omega)R$

= $6\mu_0\sigma\omega R$

(58) Answer : (1)

Hint:

$\oint \vec{B}\cdot\vec{dl} = \mu_0 i$

Solution:

$\int_0^r B_0 r^2 dr = \mu_0 \int_0^r J(r) 2\pi r dr$

$\frac{B_0 r^3}{3} = J2\pi\mu_0 = \frac{r^2}{2}$

$B_0 r^2 2\pi r = \mu_0 \int J 2\pi r dr$

diff.

$3 \times 2\pi B_0 r^2 = \mu_0 2\pi J(x) r$

$J = \frac{3B_0 r}{\mu_0}$

(59) Answer : (3)

Hint:

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Solution:

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-30} = \frac{1}{10}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{30} = \frac{2}{30}$

v = 15 cm

Ray must be parallel to principal axis between middle and right lens

⇒ d = 15 – 10 = 5 cm

(60) Answer : (3)

Hint:

Lenz's law

Solution:

$\varepsilon = \frac{-d\left(\int \vec{B} \cdot \vec{dA}\right)}{dt} = 0$

(61) Answer : (3)

Hint:

$\varepsilon = \frac{-d\phi}{dt}$

Solution:

$B = \mu_0 n i$

φ = $BA\cos\theta$

$\phi = \mu_0 n i A \sin \omega t \cos \omega t$

$\phi = \frac{1}{2} \mu_0 i_0 n A \sin(2\omega t)$

(62) Answer : (1)

Hint:

Motional EMF is produced because of flow of charge

Solution:

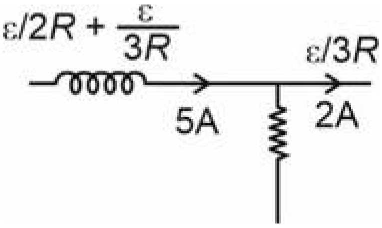
Theoretical

(63) Answer : (3)

Hint:

Flux through inductor doesn't change instantaneously

Solution:



$U = \frac{1}{2} \times 10^{-3} \times 5^2 + \frac{1}{2} \times 2 \times 10^{-3} \times 2^2$

= 10^{-3}{12.5 + 4}

= 16.5 mJ

(64) Answer : (3)

Hint:

δ = 180 – 2i

δmi = μsin$r$

δ = i–r

Solution:

Theoretical

(65) Answer : (3)

Hint:

Loss = W_B – K

Solution:
F = *ilB*
I = *mu* = *qlB*
$\frac{mu}{lB} = q$
Loss = $W_B - K$
$= \frac{muV_0}{lB} - \frac{1}{2}mu^2$

(66) **Answer :** (3)
Hint:
f ∝ λ
Solution:
f ∝ λ

(67) **Answer :** (2)
Hint:
$i = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$
Solution:
$\Delta q = \frac{V}{R}\left[t + \frac{L}{R}e^{-t/\tau}\right]_0^\tau$
$= \frac{V}{R}\left[\tau + \frac{L}{eR} - \frac{L}{R}\right]$
$= \frac{VL}{eR^2}$

(68) **Answer :** (3)
Hint:
$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$
Solution:
From spherical side
$\frac{1}{v} + \frac{3}{2\times 8} = \frac{1}{2\times 16}$
$v = \frac{-32}{5}$ cm
From flat side
$v = \frac{-16}{3}$cm
$\Delta x = 16 - \frac{16}{3} - \frac{32}{5} = \frac{64}{15}$ cm

(69) **Answer :** (4)
Hint:
Resolving power does **not** depend on the eyepiece focal length.
Solution:
$\theta = \frac{1.22\lambda}{d}$

(70) **Answer :** (2)
Hint:
cos*idi* = cos*r*μ*dr*
Solution:
cos*idi* = cos*r*μ*dr*

Section-II

(71) **Answer :** 9
Hint:
$I = \frac{V}{Z}$
Solution:
$I = \frac{\sum}{\left[R^2+x^2\right]^{1/2}}$
$|dI| = \left|\sum \frac{1}{2}\frac{2R}{\left(R^2+x^2\right)^{3/2}}\right|$
$\frac{dI}{I} = \frac{R\Delta R}{\left(R^2+x^2\right)}$
$\because \frac{\Delta R}{R} \times 100 = 1$
$\Delta R = \frac{R}{100}$
$\frac{dI}{I} \times 100 = \frac{R^2}{R^2+x^2}$

$\Rightarrow \frac{9}{25}$

(72) Answer : 58

Hint:

$\varepsilon = \left(B\frac{dA}{dt}\right)$

Solution:

$\phi = Blb$

$= B6v\{a + 4vt)$ + $B4v(a + 6vt)$

At $t = \frac{a}{v}$

$B6v \times 5a + B4v7a$

$\varepsilon = (30 + 28)Bav$

= 58 aBv

(73) Answer : 14

Hint:

$F = ilB$

Solution:

$= \frac{\mu_0 i^2}{2\pi x}dx$

$F = \frac{\mu_0 i^2}{2\pi}\ln 2$

$= 2 \times 10^{-7} \times 10^{2}(0.7)$

$= 1.4 \times 10^{-5}$

$= 14 \times 10^{-6}$

(74) Answer : 13

Hint:

$\frac{\Delta P}{\Delta V} = \frac{I}{C^2}$

Solution:

$\frac{\Delta p}{\Delta V} = \frac{F\Delta t}{AC\Delta T} = \frac{P}{C} = \frac{I}{C^2}$

$= \frac{10^{-3}}{10^{-6} \times 9 \times 10^{16}}$

$= \frac{1}{9} \times 10^{-13}$

(75) Answer : 14

Hint:

$\sqrt{3}\, V_0 = \sqrt{(V_L - V_C)^2 + V_R^2}$

Solution:

$(V_L - V)^2 + V^2 = 3V^2$

$(V_L - V)^2 = 2V^2$

$V_L - V = \pm\sqrt{2}\, V$

$V_L = \pm\sqrt{2}\, V + V$

V_L = 2.4 Volt

V_{LC} = 1.4 Volt