16/11/2025

Set-1

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MM : 180

AIATS For One Year JEE(Advanced)-2026 (XII Studying)_Test-2A_Paper-1
(ONLINE)

Time : 180 Min.

CHEMISTRY

Section-I

1. (A,C,D)

2. (A,C)

3. (A,B,D)

Section-II

4. (D)

5. (D)

6. (A)

7. (A)

Section-III

8. (3)

9. (58)

10. (6)

11. (4)

12. (16)

13. (5)

Section-IV

14. (A)

15. (A)

16. (B)

17. (A)

MATHEMATICS

Section-I

18. (A,C)

19. (B,C)

20. (A,B)

Section-II

21. (B)

22. (C)

23. (C)

24. (B)

Section-III

25. (3)

26. (1)

27. (12)

28. (2)

29. (1)

30. (3)

Section-IV

31. (D)

32. (C)

33. (A)

34. (B)

PHYSICS

Section-I

35. (A,D)

36. (B,C,D)

37. (B,C)

Section-II

38. (B)

39. (B)

40. (C)

41. (B)

Section-III

42. (112)

43. (16)

44. (3)

45. (6)

46. (5)

47. (5)

Section-IV

48. (B)

49. (B)

50. (B)

51. (C)

Hints and Solutions

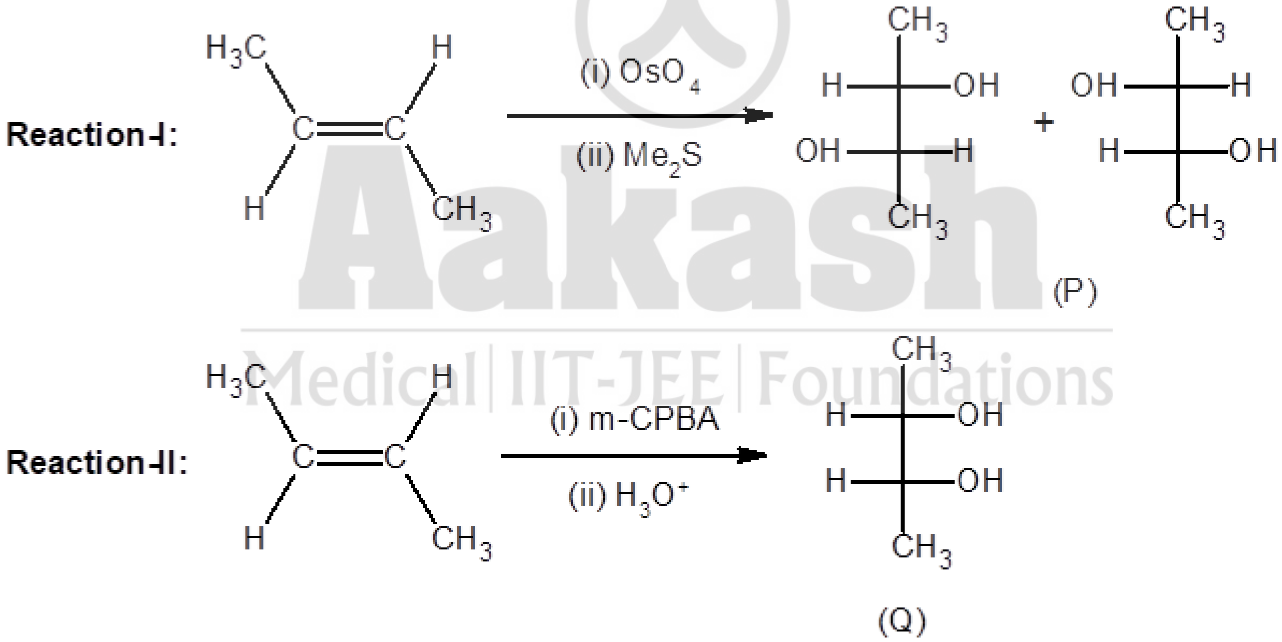
CHEMISTRY

Section-I

(1) **Answer :** (A,C,D)
Hint:
Beryl is $Be_3Al_2Si_6O_{18}$
Solution:
Cr^{3+} ions occupy octahedral void in beryl ($Be_3Al_2Si_6O_{18}$)

(2) **Answer :** (A,C)
Hint:
CH_3–CHO oxidises to CH_3–COO^-
Solution:
$CH_3–CHO + [Ag(NH_3)_2]^+ + OH^- \rightarrow CH_3COO^- + Ag + H_2O + NH_3$
$CH_3–CHO + Cu^{2+} + OH^- \rightarrow CH_3COO^- + Cu_2O + H_2O$

(3) **Answer :** (A,B,D)
Hint:
Trans alkene ⇒ Anti addition ⇒ Meso product
Trans alkene ⇒ syn addition ⇒ Racemic product
Solution:



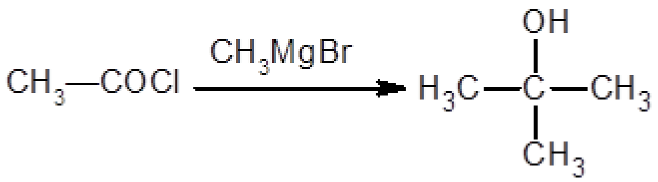
Section-II

(4) **Answer :** (D)
Hint:
Generally boiling point of alkane < ether < aldehyde
Solution:

$$CH_3-CH_2-Cl \xrightarrow[Dry\ ether]{Na} CH_3-CH_2-CH_2-CH_3 \quad (B.P.\ =\ 273\ K)$$

$$CH_3-CH_2-OH \xrightarrow[(ii)\ CH_3Cl]{(i)\ Na} CH_3-CH_2-O-CH_3 \quad (B.P.\ =\ 281\ K)$$

$$CH_3CH_2-CH_2-OH \xrightarrow{PCC} CH_3CH_2-CHO \quad (B.P.=322K)$$



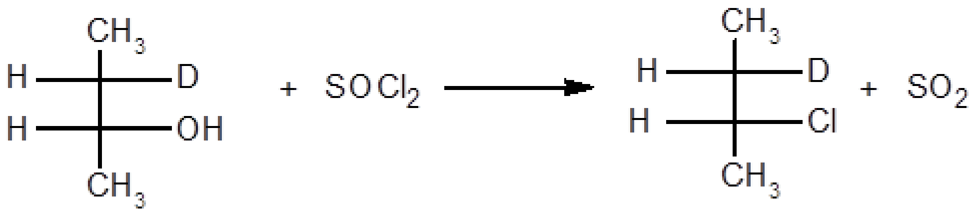
(BP = 355.2K)

(5) **Answer :** (D)

Hint:

In absence of base, retention in configuration is observed.

Solution:



(Retention)

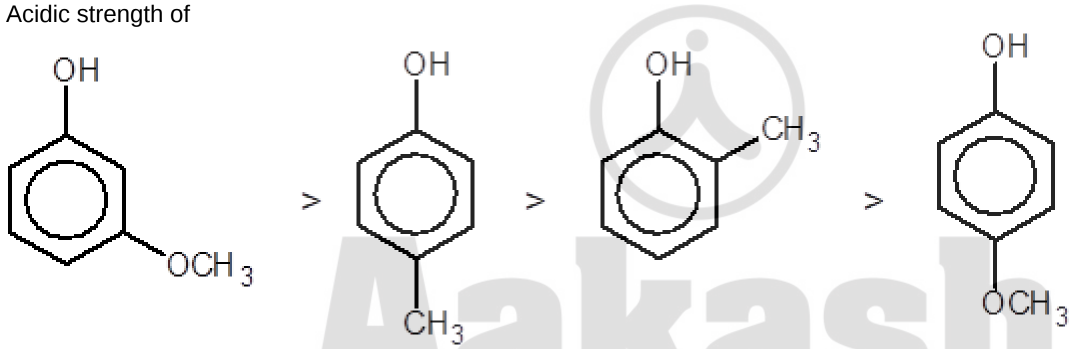
(6) **Answer :** (A)

Hint:

Acidic strength of phenol decreases due to electron donating groups.

Solution:

Acidic strength of

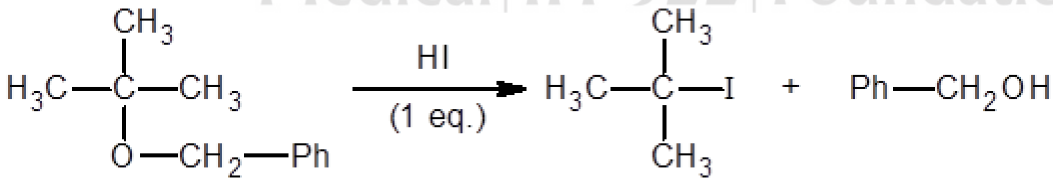


(7) **Answer :** (A)

Hint:

S_N1 mechanism is followed by reaction with HI if carbocation formed is stable.

Solution:



Section-III

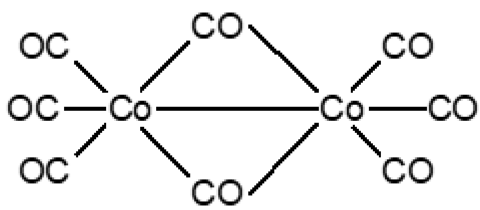
(8) **Answer :** 3

Hint:

Metal-metal bond exists in $[Co_2(CO)_8]$

Solution:

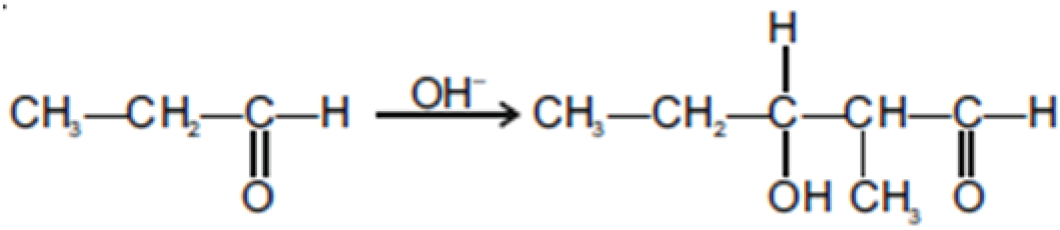
$[Co_2(CO)_8]$:



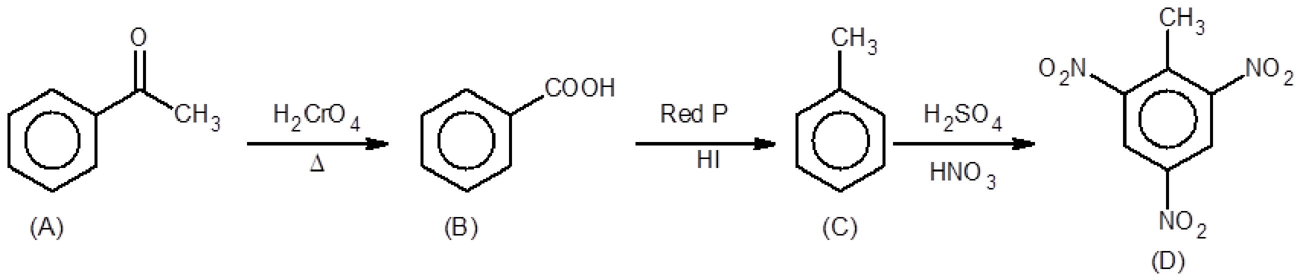
$p = 2$, $q = 1$, $r = 6$

$\frac{qr}{p} = \frac{1\times6}{2} = 3$

(9) **Answer :** 58
Hint:
W must be aldehyde.

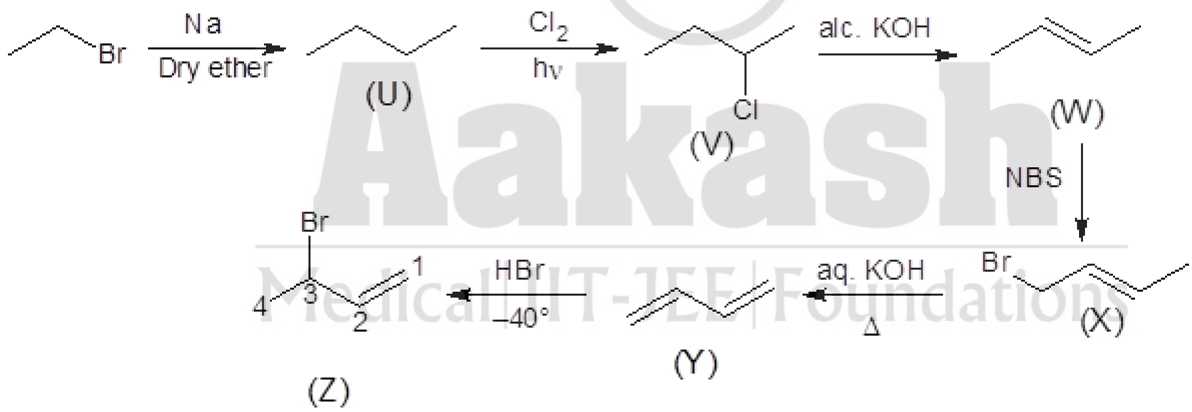


(10) **Answer :** 6
Hint:
Trinitrotoluene is formed
Solution:



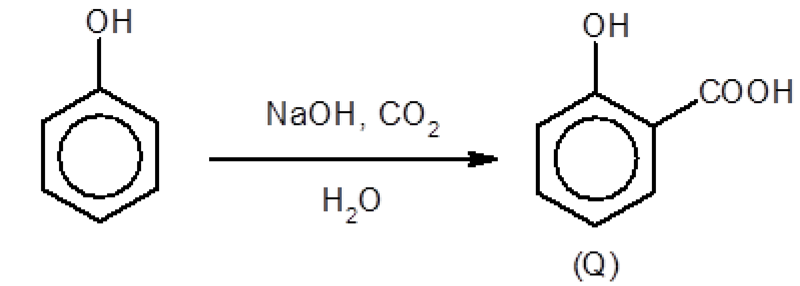
Number of π bonds in (D) = 6

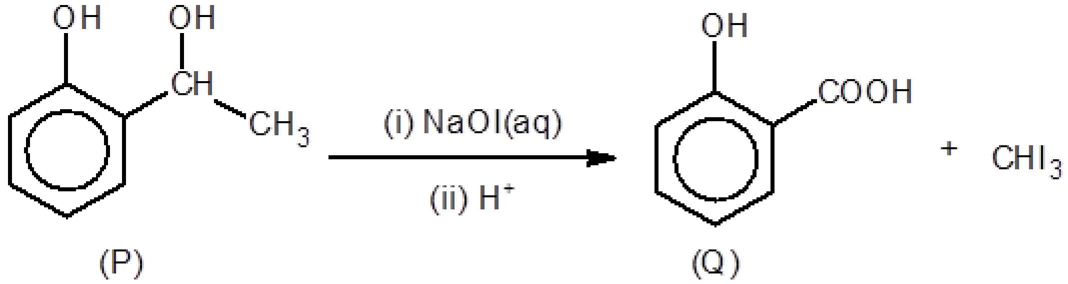
(11) **Answer :** 4
Hint:
Z formed is kinetic controlled product.
Solution:



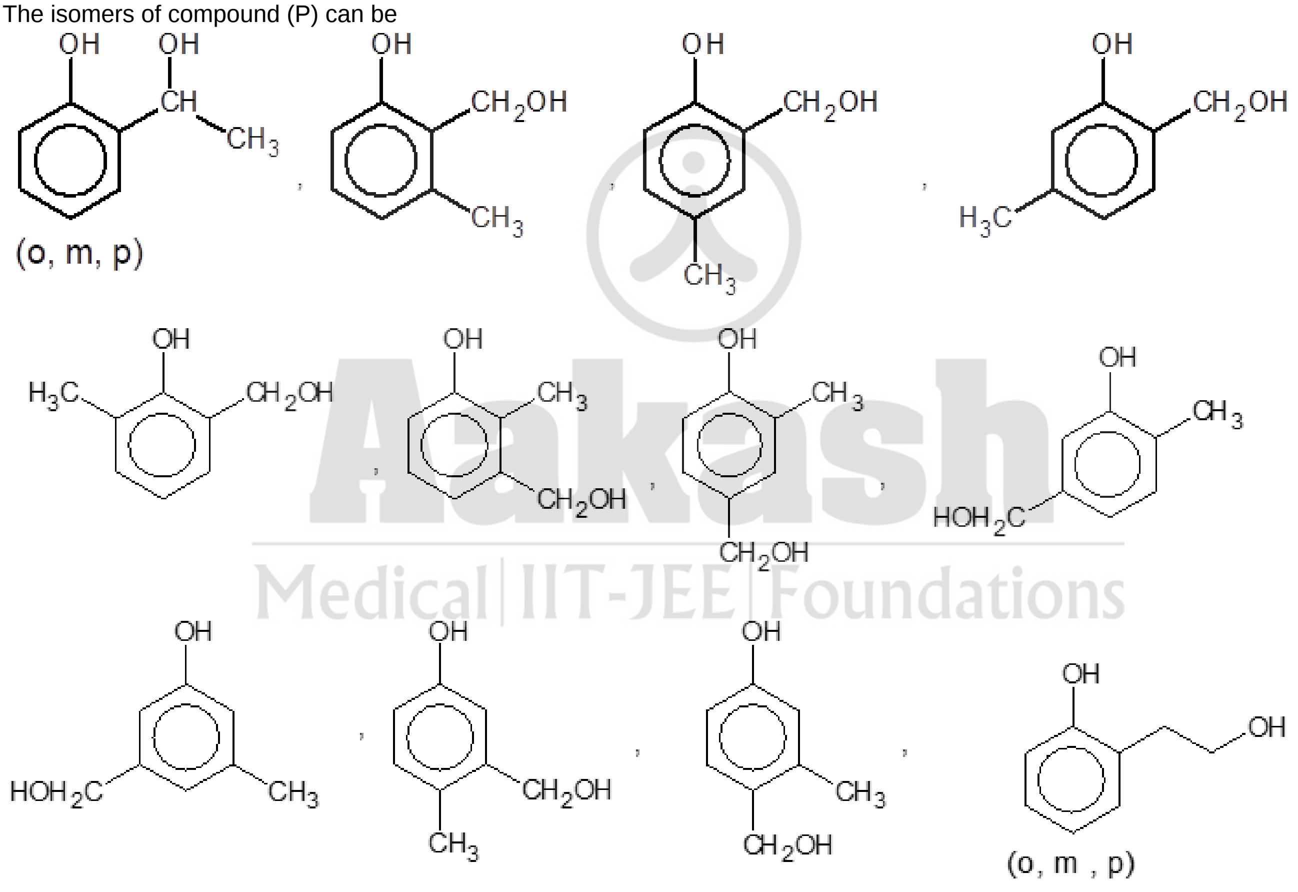
Sum = 1 + 3 = 4

(12) **Answer :** 16
Hint:
Kolbe-Schmitt reaction gives benzaldehyde
Solution:





The isomers of compound (P) can be

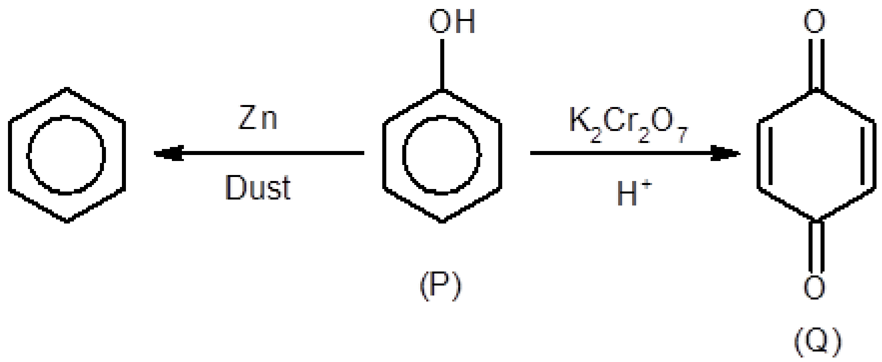


(13) **Answer :** 5

Hint:

Phenol on reduction with Zn gives benzene.

Solution:



Section-IV

(14) Answer : (A)

Hint:

When number of unpaired e^- = 0 : diamagnetic

When number of unpaired $e^- \neq 0$: paramagnetic

Solution:

$[Co(NH_3)_6]^{3+}$	: d^2sp^3	: Diamagnetic
$[NiCl_4]^{2-}$	: sp^3	: Paramagnetic
$[Ni(CN)_4]^{2-}$	: dsp^2	: Diamagnetic
$[CoF_6]^{3-}$	: sp^3d^2	: Paramagnetic

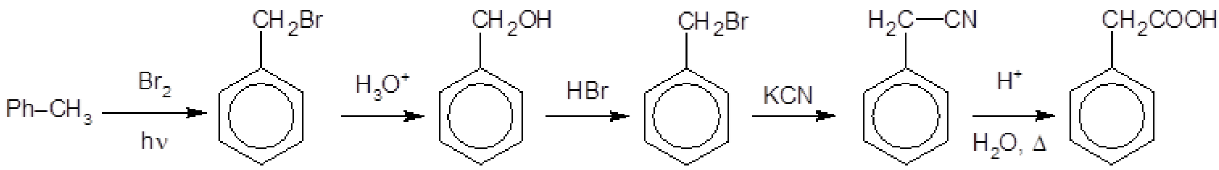
(15) Answer : (A)

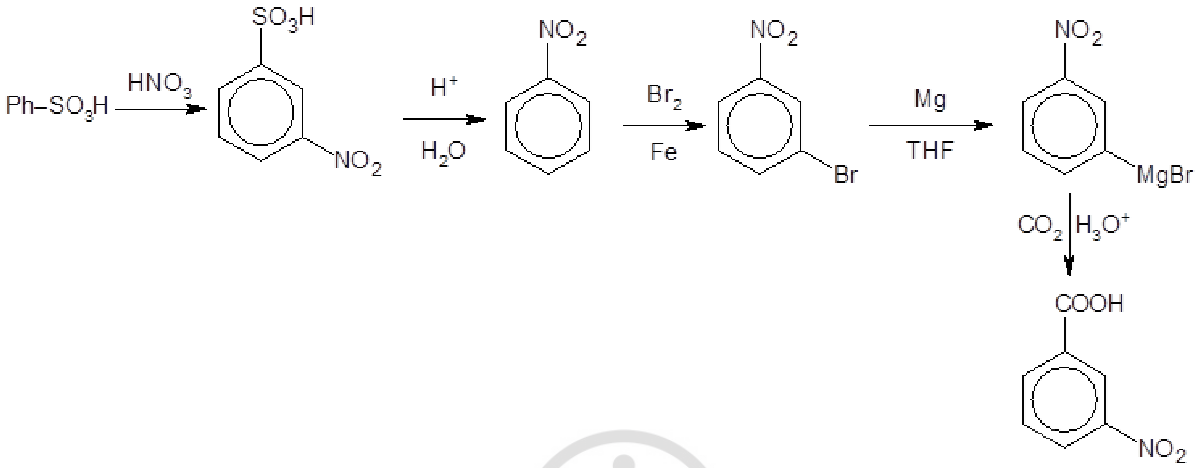
Hint:

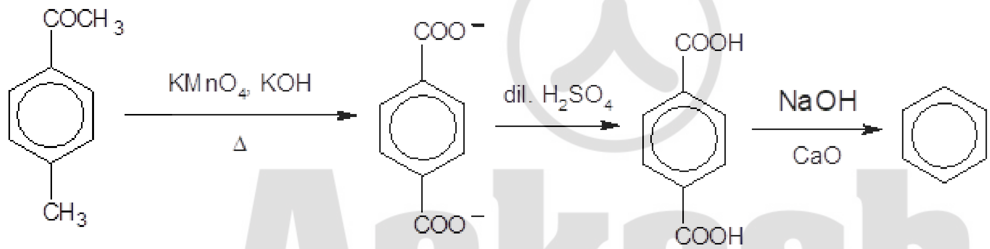
Reduction of aldehyde with LAH gives alcohol hydrolysis of cyanide gives carboxylic acid.

Solution:

(P) $CH_3CH_2CH_2-CHO \xrightarrow[\text{(ii) } H_2O]{\text{(i) } LiAlH_4} CH_3CH_2CH_2-CH_2OH \xrightarrow{\text{(iii) } CrO_3 - H_2SO_4(aq)} CH_3CH_2CH_2-COOH$

(Q) 

(R) 

(S) 

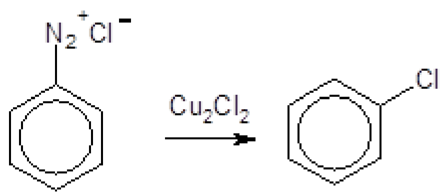
(16) Answer : (B)

Hint:

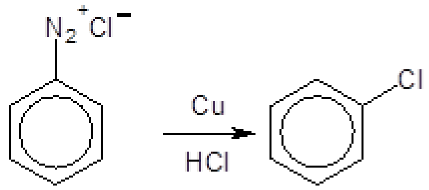
Alkyl iodide forms in Finkelstein reaction. Diazonium salt reacts with Cu_2Cl_2 in Sandmeyer reaction, while in gattermann, Cu/HCl is used.

Solution:

$CH_3-CH_2-Cl \xrightarrow[\text{Acetone}]{\text{NaI}} CH_3-CH_2-I$: Finkelstein reaction



: Sandmeyer reaction



: Gattermann reaction

$CH_3-COOAg \xrightarrow[CCl_4]{Br_2} CH_3Br + AgBr + CO_2$: Hunsdiecker reaction

(17) Answer : (A)

Hint:

Complementary color is shown by complex of the wavelength of light absorbed.
Solution:
535 nm → Violet
500 nm → Red
475 nm → Yellow orange
600 nm → Blue

MATHEMATICS

Section-I

(18) **Answer :** (A,C)
Hint:

$$f(x) = \begin{cases} \frac{x^2}{5} - \frac{4x}{5} + \frac{8}{5}; & x < 1 \\ 2 - x; & x \in [1, \ 2) \\ x - 2; & x \geq 2 \end{cases}$$

Solution:

$$f(x) = \begin{cases} \frac{x^2}{5} - \frac{4x}{5} + \frac{8}{5}; & x < 1 \\ 2 - x; & x \in [1, \ 2) \\ x - 2; & x \geq 2 \end{cases}$$

RHL at $x = 1 = \lim_{h \to 0} 2 - (1 + h) = 1$

LHL at $x = 1 = \lim_{h \to 0} \frac{(1-h)}{5}^2 - \frac{4(1-h)}{5} + \frac{8}{5} = 1$

$\therefore f(x)$ is continuous at $x = 1$

Now, $f'(x) = \begin{cases} \frac{2x}{5} - \frac{4}{5} \ ; & x < 1 \\ -1 \ ; & 1 \leq x < 2 \\ 1 \ ; & x \geq 0 \end{cases}$

RHD at $x = 1 \Rightarrow -1$

LHD at $x = 1 \Rightarrow \frac{-2}{5}$

$\therefore$ Non-differentiable at $x = 1$
$\Rightarrow$ Non-differentiable at $x = 2$

(19) **Answer :** (B,C)
Hint:
$f(x) > 0$ and $f'(x) > 0$ for $x < 0$
Solution:
$f(x) > 0$ and $f'(x) > 0$ for $x < 0$
$f'(x) < 0$ for $x > 0$
Now $g(x) = \frac{1}{f\left(\frac{1}{x}\right)}$

$g'(x) = \frac{-1}{f^2\left(\frac{1}{x}\right)} \times f\left(\frac{1}{x}\right) \times \left(\frac{-1}{x^2}\right)$

So, for $x < 0 \Rightarrow g'(x) > 0$
$x > 0 \Rightarrow g'(x) < 0$

(20) **Answer :** (A,B)
Hint:
Substitution method
Solution:

$I = \int_0^\alpha \frac{dx}{1 - \cos\alpha \ \cos x}$

$= \int_0^{\alpha} \frac{dx}{\left(\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)-\cos\alpha\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}$

$= \int_0^{\alpha} \frac{dx}{(1-\cos\alpha)\cos^2\frac{x}{2}+\left(1+\cos\alpha\sin^2\frac{x}{2}\right)}$

$= \int_0^{\alpha} \frac{dx}{2\sin^2\frac{\alpha}{2}\cos^2\frac{x}{2}+2\cos^2\frac{\alpha}{2}\sin^2\frac{x}{2}}$

$= \frac{1}{2}\int_0^{\alpha} \frac{\left(\sec^2\frac{\alpha}{2}\right)\sec^2\frac{x}{2}}{\tan^2\frac{\alpha}{2}+\tan^2\frac{x}{2}}\, dx$

Put $\tan\frac{x}{2} = t$

$I = \int_0^{\tan\frac{\alpha}{2}} \frac{\sec^2\frac{\alpha}{2}\, dt}{t^2+\tan^2\frac{\alpha}{2}}$

$= \sec^2\frac{\alpha}{2}\ \cot\frac{\alpha}{2}\left[\tan^{-1}\frac{t}{\tan\frac{\alpha}{2}}\right]_0^{\tan\frac{\alpha}{2}}$

$= \frac{2}{\sin\alpha}\cdot\frac{\pi}{4} = \frac{\pi}{2\sin\alpha}$

Thus, $\frac{A}{\sin\alpha}+B = \frac{\pi}{2\sin\alpha}$

$A = \frac{\pi}{2}$, $B = 0$ and $A = \frac{\pi}{4}$, $B = \frac{\pi}{4\sin\alpha}$

Section-II

(21) Answer : (B)

Hint:

$\frac{f(x)f'(x)}{1+f^4(x)} \ge 1$

Solution:

$\because f'(x) \ge f^3(x) + \frac{1}{f(x)}$

$f(x)f'(x) \ge f^4(x) + 1 \Rightarrow \frac{f(x)f'(x)}{1+f^4(x)} \ge 1$

On integrating w.r.t. x from $x = a$ to b

$\frac{1}{2}\left(\tan^{-1}f^2(x)\right)_a^b \ge b-a$

or $b - a \le \frac{1}{2}\left[\lim_{x\to b^-}\tan^{-1}\left(f^2(x)\right) - \lim_{x\to a^+}\tan^{-1}\left(f^2(x)\right)\right]$

or $b - a \le \frac{\pi}{24}$

(22) Answer : (C)

Hint:

Multiply and divide by $\sin x - x\cos x$

Solution:

$\lim_{x\to 0}\left(\frac{g(2+\sin x)-g(2+x\cos x)}{\sin x - x\cos x}\right)\cdot\left(\frac{\sin x - x\cos x}{x-\sin x}\right)$

$= g'(2)\lim_{x\to 0}\left(\frac{\sin x - x\cos x}{x-\sin x}\right)$ $\left(\frac{0}{0}\text{ form}\right)$

$= g'(2)\lim_{x\to 0}\left(\frac{x\sin x}{1-\cos x}\right)$

$= 2g'(2) = 2 \times 14 = 28$

(23) Answer : (C)

Hint:

$0 < \frac{3}{x^2+1} \le 3$

Solution:

$0 < \frac{3}{x^2+1} \le 3$, $\frac{3}{x^2+1} = 2 \Rightarrow x = \frac{1}{\sqrt{2}}$ and $\frac{3}{x^2+1} = 1 \Rightarrow x = \sqrt{2}$

$I = \int_0^{1/\sqrt{2}} 2dx + \int_{1/\sqrt{2}}^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\infty} 0\,dx$

$= \sqrt{2} + \sqrt{2} - \frac{1}{\sqrt{2}} = 2\sqrt{2} - \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

(24) **Answer :** (B)
Hint:
$f'(x) = 3x^2 + 2ax + b + 10\sin x \cos x \geq 0 \ \forall x \in R$
Solution:
$f(x) = x^3 + ax^2 + bx + 5\sin^2 x$
$f'(x) = 3x^2 + 2ax + b + 10\sin x \cos x \geq 0 \ \forall x \in R$
i.e., $3x^2 + 2ax + b + 5\sin 2x \geq 0 \ \forall x \in R$
if $3x^2 + 2ax + b - 5 \geq 0$
$4a^2 - 12(b - 5) \leq 0$
$\therefore a^2 - 3b + 15 \leq 0$

Section-III

(25) **Answer :** 3
Hint:

$$|f(g(x))| = \begin{cases} 1, & x < -1 \\ 0, & x = -1 \\ -1 & -1 < x < 0 \\ 0, & x = 0 \\ 1, & 0 < x < 1 \\ 0, & x = 1 \\ 1, & x > 1 \end{cases}$$

Solution:

$$|f(g(x))| = \begin{cases} 1, & x < -1 \\ 0, & x = -1 \\ -1 & -1 < x < 0 \\ 0, & x = 0 \\ 1, & 0 < x < 1 \\ 0, & x = 1 \\ 1, & x > 1 \end{cases}$$

Points of discontinuity are $x = -1, 0, 1$

(26) **Answer :** 1
Hint:
Multiply and divide by $e^{x-1} - 1$
Solution:

$$\lim_{x\to 1} \frac{\sin\left(e^{x-1}-1\right)}{\log x}$$

$$= \lim_{x\to 1} \frac{\sin\left(e^{x-1}-1\right)}{e^{x-1}-1} \times \frac{e^{x-1}-1}{\log x}$$

$$= \lim_{x\to 1} \frac{e^{x-1}-1}{\log x} \left\{ \begin{matrix} \text{as} & x & \longrightarrow & 1 \\ & e^{x-1}-1 & \longrightarrow & 0 \end{matrix} \right\}$$

$$= \lim_{h\to 0} \frac{e^h - 1}{h} \times \frac{h}{\log(1+h)}$$

$= 1$

(27) **Answer :** 12
Hint:
Substitution method
Solution:

$$I = \int \frac{\cos x \, dx}{\sin x(3 + \cos^2 x)}$$

$$I = \int \frac{\cos x \, dx}{\sin x(4 - \sin^2 x)}$$

$\sin x = t$

$I = \int \frac{dt}{t(4-t^2)}$

$= \int \frac{t^{-3}dt}{4t^{-2}-1}$

$4t^2 - 1 = z$

$-8t^2 dt = dz$

$\frac{-1}{8}\int \frac{dz}{z} = \frac{-1}{8}\ln z + C$

$= -\frac{1}{8}\ln(4t^{-2}-1) + C$

$= -\frac{1}{8}\ln\left|4\,\text{cosec}^2 x - 1\right| + C$

$A = 8$, $B = 4$

(28) **Answer :** 2

Hint:

$$f(x) = \begin{cases} x^3 - x^2 + x - 2, & x > 2 \\ x^3 - x^2 + 2 - x, & 1 < x \le 2 \\ x^2 - x^3 + 2 - x, & 0 < x \le 1 \\ x^3 - x^2 + 2 - x, & x \le 0 \end{cases}$$

Solution:

$$f(x) = \begin{cases} x^3 - x^2 + x - 2, & x > 2 \\ x^3 - x^2 + 2 - x, & 1 < x \le 2 \\ x^2 - x^3 + 2 - x, & 0 < x \le 1 \\ x^3 - x^2 + 2 - x, & x \le 0 \end{cases}$$

$$f'(x) = \begin{cases} 3x^2 - 2x + 1, & x > 2 \\ 3x^2 - 2x - 1, & 1 < x \le 2 \\ 2x - 3x^2 - 1, & 0 < x \le 1 \\ 3x^2 - 2x - 1, & x \le 0 \end{cases}$$

$f'(x) = 0$

$3x^2 - 2x + 1 = 0 \Rightarrow x = \varphi, x > 2$

$f'(x) = 0$

$3x^2 - 2x - 1 = 0 \Rightarrow x = \frac{2 \pm 4}{6} \Rightarrow x = 1, \frac{-1}{2}$; $1 < x < 2 \Rightarrow x = \varphi$

$f'(x) = 0$

$2x - 3x^2 - 1 = 0, 0 < x \le 1$

$\Rightarrow 3x^2 - 2x + 1 = 0, \Rightarrow x = \varphi$

$f'(x) = 0$

$3x^2 - 2x - 1, x \le 0$

$\Rightarrow x = 1, \frac{-1}{3} \Rightarrow x = \frac{-1}{3}$

Also, $x < 1 \Rightarrow f'(x) < 0$, $x > 1 \Rightarrow f'(x) > 0$

$\therefore$ minimum at $x = 1$

(29) **Answer :** 1

Hint:

Put, $\frac{1}{(\sqrt{x})^5} = y$

Solution:

$$\int \frac{dx}{(\sqrt{x})^2 + (\sqrt{x})^7} = \int \frac{dx}{(\sqrt{x})^7\left(1 + \frac{1}{(\sqrt{x})^5}\right)}$$

Put, $\frac{1}{(\sqrt{x})^5} = y$

$\frac{dy}{dx} = \frac{-5}{2(\sqrt{x})^7}$

$\therefore I = \int \frac{-2\,dy}{5\,(1+y)} = \frac{-2}{5}\ln|1+y| + c$

$= \frac{2}{5}\ln\left(\frac{1}{1+\frac{1}{(\sqrt{x})^5}}\right)$

∴ $a = \frac{2}{5}$, $k = \frac{5}{2}$
∴ $ak = 1$

(30) **Answer :** 3
Hint:

$$f(|x|) = \begin{cases} x-1, & x>1 \\ 1-x, & 0<x\leq 1 \\ 1+x, & -1\leq x\leq 0 \\ -x-1, & x<-1 \end{cases}$$

Solution:
$f(x) = |1 - x|$

$$f(|x|) = \begin{cases} x-1, & x>1 \\ 1-x, & 0<x\leq 1 \\ 1+x, & -1\leq x\leq 0 \\ -x-1, & x<-1 \end{cases}$$

Clearly, the domain of $\sin^{-1}(f\,|x|)$ is [–2, 2]
⇒ It is non-differentiate at points {–1, 0, 1}

Section-IV

(31) **Answer :** (D)
Hint:
Let $x\sqrt{t} = \mu$ then $dx = \frac{du}{\sqrt{t}}$
Solution:

$$I = \int_0^\infty e^{-tx^2} dt$$

Let $x\sqrt{t} = \mu$ then $dx = \frac{du}{\sqrt{t}}$

$$I = \int_0^\infty e^{-u^2}\frac{du}{\sqrt{t}} = \frac{1}{\sqrt{t}}\int_0^\infty e^{-u^2} du = \frac{\sqrt{\pi}}{2\sqrt{t}}$$

$$\int_0^\infty e^{-x^2} dx = 2\int_0^\infty e^{-x^2} dx = \sqrt{\pi}$$

(32) **Answer :** (C)
Hint:
$f'(x) = \frac{f(x)}{1+f(x)}$
Solution:
(P) $2(F(x) - f(x)) = f^2(x)$
⇒ $f(x) - f'(x) = f(x)\times f'(x)$
⇒ $f'(x) = \frac{f(x)}{1+f(x)} = 1 - \frac{1}{1+f(x)}$
⇒ $f(x)$ is an increasing function
(Q) $21 \leq f(5) - f(-2) \leq 28$
(R) $y = \frac{x+2}{2x^2+3x+6}$
⇒ $y \in \left[\frac{-1}{13}, \frac{1}{3}\right]$
(S) $y = e^{-x}\cos x$
$y_1 = -e^{x}\sin x - e^{-x}\cos x = -e^{-x}\sin x - y$
$y_2 = -e^{-x}\cos x + e^{-x}\sin x - y_1$
⇒ $y_4 + 4y = 0$
⇒ $k = 2$

(33) **Answer :** (A)
Hint:

$\lim\limits_{x\to 0}\frac{\sin x}{x}=1$

Solution:

(P) $I = \lim\limits_{x\to 0}\frac{e^{x^2}-1+x-e^x+1}{\frac{2\sin^2 x}{x^2}\cdot x^2}$

$= \frac{1}{2}\left[\lim\limits_{x\to 0}\frac{e^{x^2}-1}{x^2}+\lim\limits_{x\to 0}\frac{x-e^x+1}{x^2}\right]$

$= \frac{1}{2}\left[1-\lim\limits_{x\to 0}\frac{e^x-x-1}{x^2}\right]$

(Q) $I = \lim\limits_{x\to 0}\left(\frac{3+x}{3-x}\right)^{1/x} = e^{\lim\limits_{x\to 0}\frac{1}{x}\left(\frac{3+x}{3-x}-1\right)}$

$= e^{\lim\limits_{x\to 0}\frac{2x}{x(3-x)}} = e^{2/3}$

$\therefore p + q = 2 + 3 = 5$

(R) $\lim\limits_{x\to 0}\frac{(\tan^3 x - x^3)-(\tan x^3 - x^3)}{x^5}$

$= \lim\limits_{x\to 0}\frac{\tan^3 x - x^3}{x^5} - \lim\limits_{x\to 0}\frac{\tan x^3 - x^3}{x^5}$

$= \lim\limits_{x\to 0}\frac{\tan x - x}{x^3}\cdot\frac{\tan^2 x + x\tan x + x^2}{x^2} = \frac{1}{3}\times 3 = 1$

(S) By rationalising

$\lim\limits_{x\to 0}\frac{(x+2\sin x)\left[\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x - x+1}\right]}{(x^2+2\sin x+1)-(\sin^2 x - x+1)}$

$= 2\lim\limits_{x\to 0}\frac{x+\sin 2x}{-\sin^2 x+2\sin x+x} = 2\lim\limits_{x\to 0}\frac{1+\frac{\sin 2x}{x}}{x-\frac{\sin^2 x}{x}+2+1}$

$= 2\left(\frac{1+2}{3}\right) = 2$

(34) Answer : (B)

Hint:

$h'(x) = -2x(f'(x))^2 g(x)$

Solution:

(P) $(f(x))^2 + [f'(x)]^2 = h(x)$

$h'(x) = 2f(x)f'(x) + 2f'(x)f''(x)$

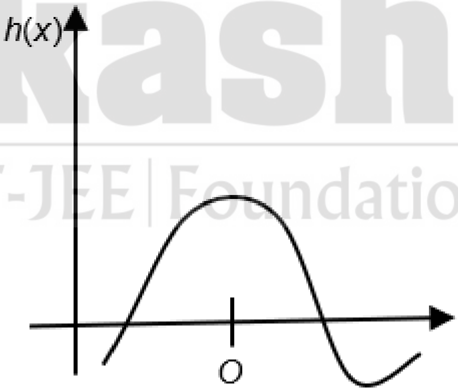
$= 2f'(x)(f(x) + f''(x))$

$= 2f'(x)\{-xg(x)f'(x)\}$

$= -2x(f'(x))^2 g(x)$

So, $h'(x) < 0$ for $x > 0$ and $h'(x) > 0$ for $x < 0$

Then $x = 0$ is a maxima



(Q) $f^2(x) + f(x) = g(x)$

$2f(x)f'(x) + f'(x) = g'(x)$ and $g'(x) = f'(x)\{2f(x) + 1\}$

So, $g'(x) \ge 0$ as $g(x)$ is increasing and for $f'(x) \ge 0$

$2f(x) + 1 \ge 0$

$\because f(x) \ge \frac{-1}{2}$

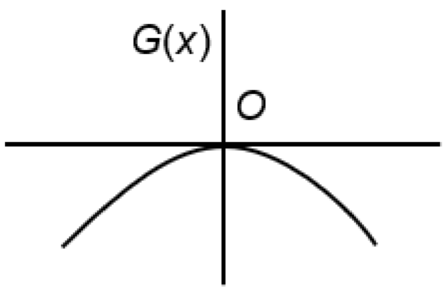
(R) Let $G(x) = \left[\int_0^x f(t)\,dt\right]^2$ ….(1)

$G'(x) = 2f(x)\int_0^x f(t)\,dt$ and $\frac{G'(x)}{2} = g(x)$

and $G'(0) = 0 = g(0)$

Also, $g'(x) \le 0$

$\because g(x) \ge 0,\ x \in (-\infty, 0),\ g(x) \le 0,\ x \in [0, \infty)$



i.e., $G'(x) \geq 0$ for $x \leq 0$, $G'(x) \leq 0$ for $x \geq 0$ and $G(0) = 0$
But $G(x) \geq 0$ {from (1)}
$\because G(x) = 0, x \in R^+$

$$\int_0^x f\ (t)\ dt = 0, \quad x\ \in\ R^+$$

i.e., $f(x) = 0, x \in R^+$
$\because f(1) = 0$

(S) $f(x) = x^2 + \int_0^x e^{-t} f(x-t)\ dt$ …(1)

$$f(x) = x^2 + e^{-x} \int_0^x e^t\ f\left(x - x + t\right)\ dt$$

$$f(x) = x^2 + e^{-x} \int_0^x e^t\ f\left(t\right)\ dt$$

$$f'(x) = 2x + e^{-x}\left[e^x f(x)\right] - e^{-x} \int_0^x e^t\ f\left(t\right)\ dt$$

$$f'(x) = 2x + f(x) - \int_0^x e^{-(x-t)}\ f\left(t\right)\ dt$$

$= 2x + f(x) + x^2 - f(x)$
$f'(x) = 2x + x^2$
$f(x) = x^2 + \frac{x^3}{3} + c$ and from (1), $f(0) = 0$
$\therefore c = 0$
$f(x) = x^2 + \frac{x^3}{3}$
$f(-1) = 1 - \frac{1}{3}$
$= \frac{2}{3}$
$\therefore \frac{k}{3} = \frac{2}{3} \Rightarrow k = 2$

PHYSICS

Section-I

(35) Answer : (A,D)
Hint:

$$i = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Solution:

$$i = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$V_L = \frac{V_0 \omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

V_L is maximum when

$\frac{d\left(\frac{R^2}{\omega^2}+\left(L-\frac{1}{\omega^2 C}\right)^2\right)}{d\omega}=0$

$\omega=\sqrt{\frac{2}{2LC-R^2C^2}}$

Similarly, $\frac{dV_C}{d\omega}=0$

$\omega=\sqrt{\frac{1}{LC}-\frac{1}{2}\frac{R^2}{L^2}}$

(36) **Answer :** (B,C,D)
Solution:
When magnet falls, due to variation of magnetic flux eddy currents develops in the cylinder. So, there will be loss of energy.

(37) **Answer :** (B,C)
Hint:
R = f θ
Solution:

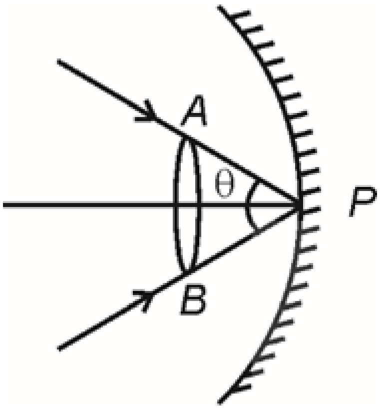


Image will be formed in focal plane.

$\theta=\frac{AB}{f} \quad \Rightarrow \quad AB=f\theta$

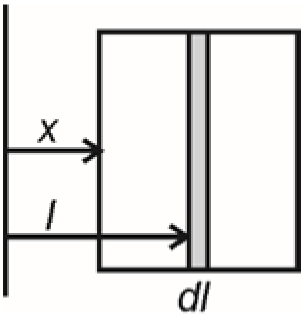
$2r=\frac{R}{2}\theta$

$r=\frac{R}{4}\theta$

$A=\pi\left(\frac{R}{4}\theta\right)^2$

Section-II

(38) **Answer :** (B)
Hint:
$\varepsilon=-\frac{d\phi}{dt}$
Solution:



$$\phi=\int_x^{L+x}\frac{B_0}{l}adl=B_0L\ln\left(\frac{L+x}{x}\right)$$

$$=B_0L\ln\left(1+\frac{L}{vt}\right)$$

$\varepsilon=-\frac{d\phi}{dt}=B_0L\frac{vt}{(L+vt)}\frac{L}{vt^2}$

$I=\frac{B_0L}{4(L+Vt)tR_0}=\frac{B_0L}{8R_0(L+2v)}$

(39) Answer : (B)

Hint:

$Pf = \frac{R}{z}$

Solution:

$$\left.\begin{aligned} i\omega_0 L = \frac{i}{\omega_0 C} = 40 \\ iR = 60 \end{aligned}\right\} \frac{\omega_0 L}{R} = \frac{2}{3}$$

at ω = 2ω_0

$X_L - X_C = 2\omega_0 L - \frac{1}{2\omega_0 C} = \frac{3}{2}\omega_0 L$

$\tan\phi = \frac{X_L - X_C}{R} = \frac{3}{2}\cdot\frac{\omega_0 L}{R} = 1$

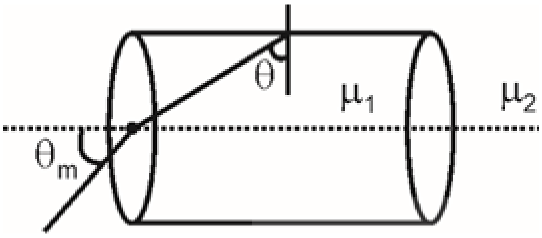
$\cos\phi = \frac{1}{\sqrt{2}}$

(40) Answer : (C)

Hint:

$\sin\theta_c = \frac{\mu_2}{\mu_1}$

Solution:



$\sin\theta = \frac{\mu_2}{\mu_1}$

$\mu_2 \sin\theta_m = \mu_1 \sin(90 - \theta)$

$\theta_m = \sin^{-1}\left(\frac{\mu_1}{\mu_2}\cos\left(\sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)\right)\right)$

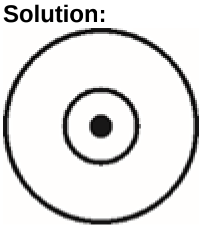
$\theta_m = \sin^{-1}\left(\frac{3}{4}\right)$

(41) Answer : (B)

Hint:

$B = \frac{B_0 I}{2\pi r} \quad R < r < 4R$

Solution:



$B = \frac{B_0 I}{2\pi r} \quad R < r < 4R$

Flux through rectangular strip of length *l* and width *dx*

$d\varphi = Bldx$

$\phi = \frac{\mu_0 Il}{2\pi}\int_R^{4R}\frac{dx}{x} \Rightarrow \frac{\mu_0 Il}{2\pi}\ln(4)$

$L = \frac{\phi}{Il} = \frac{l_0}{2\pi}\ln(4) = \frac{\mu_0 \ln(2)}{\pi}$

Section-III

(42) Answer : 112

Hint:

$P = I^2R$

Solution:

$I_1 = \frac{20}{10} = 2$ A

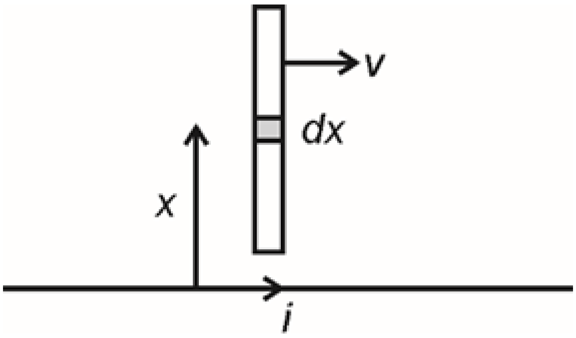
$I_2 = \frac{20}{5} = 4$ A

$I_3 = \frac{20}{10} = 2$ A

$I_4 = \frac{20}{5} = 4$ A

$P = 2^2 \times 10 + 2^2 \times 6 + 4^2 \times 3$
$P = 40 + 24 + 48$
$P = 112$

(43) **Answer :** 16
Hint:
V = BVl
Solution:

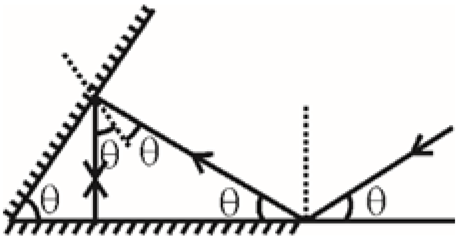


$$\varepsilon = \int_{\frac{l}{3}}^{\frac{4l}{3}} \frac{\mu_0 iv}{2\pi x} dx = \frac{\mu_0 iv}{2\pi} \ln\left(\frac{\frac{4l}{3}}{\frac{l}{3}}\right)$$

$= \frac{4\pi \times 10^{-7} \times 5 \times 2}{2\pi} \ln(4)$
$= 10^{-6} \times 2\ln(4)$
$= 10^{-6} \times \ln(16)$

(44) **Answer :** 3
Hint:
$\Delta = t \sec r \sin(i - r)$
Solution:
$\Delta = t \sec r \sin(i - r)$
3.14 = 90 × 1 × $i - \frac{i}{\mu}$
$3.14 = 90 \times 3 \times \frac{\pi}{180}\left(1 - \frac{1}{\mu}\right)$
$\frac{2}{3} = 1 - \frac{1}{\mu}$
$\mu = 3$

(45) **Answer :** 6
Hint:
Perpendicular incidence
Solution:



θ + θ + 90 + θ = 180°
θ = 30°

(46) **Answer :** 5
Hint:
$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$
Solution:
$v_e = \infty \Rightarrow u_e = f_e = 5$
$v_0 = 15 - 5 = 10$ cm
$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$
$u_0 = \frac{5}{2}$cm
$d = \frac{5}{2}$ cm
$2d = 5$

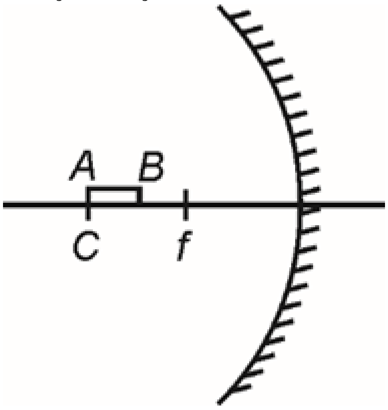
(47) Answer : 5

Hint:

$\frac{1}{v}+\frac{1}{\mu}=\frac{1}{f}$

Solution:

$f=\frac{10}{3} \quad l=\frac{f}{3}$



$v_A = u_A = -2f$

for end *B*

$u_B=2f-\frac{f}{3}=\frac{5f}{3}$

$\frac{1}{v_B}+\frac{1}{-\frac{5}{3}f}=\frac{-1}{f}$

$v_B=-\frac{5}{2}f$

Length of image = $|v_B - v_A|$

$=\frac{f}{2}=5$ cm

Section-IV

(48) Answer : (B)

Hint:

$i_L=i_0\left(1-e^{-t/\tau}\right)$

Solution:

(P) At $t \to 0^+$

$i=\frac{10}{5}=2$ A

$P=(2)^2\times 5=20$

(Q) At $t \to \infty$

$i=\frac{10}{4}=\frac{5}{2}$A

$P=\left(\frac{5}{2}\right)^2\times 4=25$

(R) $i_L=\frac{5}{6}\left(1-e^{-\frac{t\cdot 36}{25}}\right)$

$i_L\left(t=\frac{25}{36}\right)=\frac{5}{6}\left(1-\frac{1}{e}\right)=0.5$

(S) $i=\frac{6\left(\frac{5}{6}(1-e^{-t/\tau})\right)+\frac{5}{6}\cdot\frac{36}{25}e^{-t/\tau}}{3}$

$i = 1.2$

(49) Answer : (B)

Hint:

$\frac{1}{v}+\frac{1}{\mu}=\frac{1}{f}$

Solution:

For M_1

$u_1 = -20$ cm

$f_1 = -15$ cm

$\frac{1}{v_1}=\frac{1}{-15}-\frac{1}{-20}$

$v_1 = -60$ cm

$m_1 = -3$

For M_2

$u_2 = +10$ cm

f_2 = 20 cm

$\frac{1}{v_1} = \frac{1}{20} - \frac{1}{10}$

v_2 = –20 cm

m_2 = +2

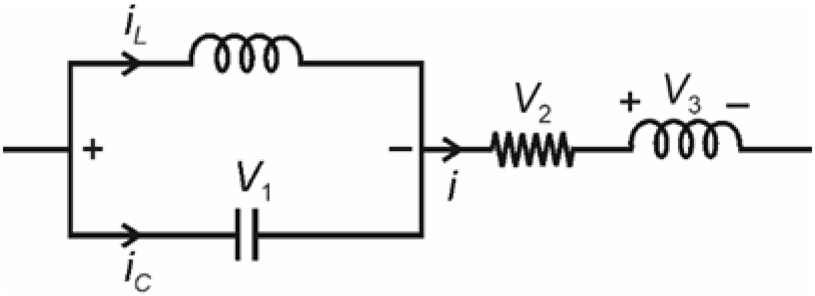
I_2(30, –14)

(50) Answer : (B)

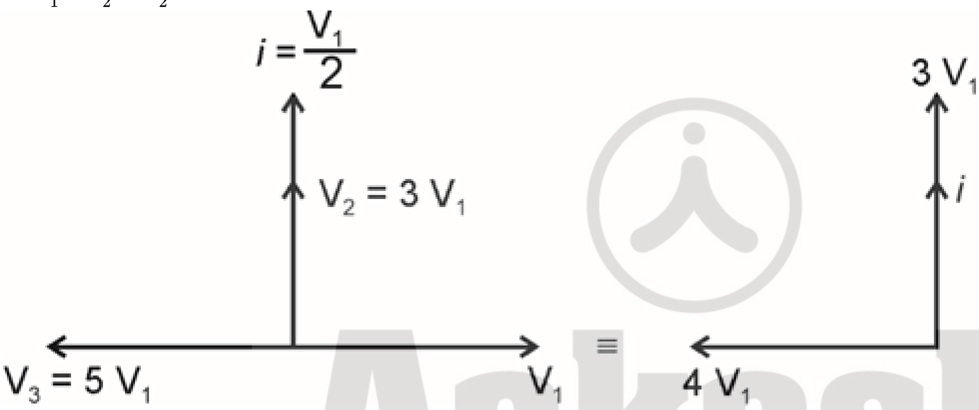
Hint:

$Pf = \frac{R}{z}$

Solution:



$i = \frac{V_1}{1} - \frac{V_1}{2} = \frac{V_1}{2}$



$\sqrt{(3\,V_1)^2 + (4\,V_1)^2} = 100$

V_1 = 20

i = 10 A

Power factor = $\frac{3}{5}$

V_1 = 20

$i_L = \frac{V_1}{2} = 10$

P = 100 × 10 × $\frac{3}{5}$ = 600

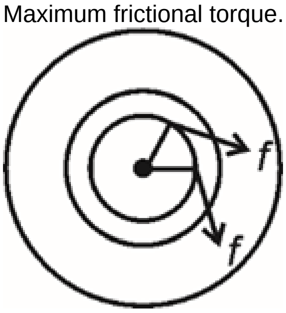
(51) Answer : (C)

Hint:

$d\tau = \mu \cdot \frac{M}{\pi R^2} 2\pi r dr \cdot g \cdot r$

Solution:

Maximum frictional torque.



$d\tau = \mu \cdot \frac{M}{\pi R^2} 2\pi r dr \cdot g \cdot r$

$\tau_f = \frac{2}{3}\mu M g R$

torque of indirect electric field

$\varepsilon = \frac{r}{2} \cdot \frac{dB}{dt} = \frac{r}{2} \cdot 6t = 3rt$

$d\tau = \frac{2}{\pi R^2} \cdot 2\pi r dr \cdot 3r \cdot t \cdot r$

$\tau_\varepsilon = \frac{3}{2} q R^2 t$

(P) $\frac{2}{3}\mu MgR = \frac{3}{2}qR^2 t \ \ \rightarrow \ \ t = 5$

(Q) $\tau_\varepsilon = \frac{3}{2} \times 2 \times 1^2 \times 2 = 6$ Nm

(S) $\frac{MR^2}{2}\alpha = \frac{3}{2} \times 2 \times (t-5)$

$\alpha = \frac{4}{3}(t-5)$

α = 4 rad/s^2