



Aakash

Medical | IIT-JEE | Foundations

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MM : 180

AIATS For Two Year JEE(Advanced)-2027 (XI Studying-P2)_Test-1A_Paper-2_Online

Time : 180 Min.

PHYSICS

Section-I

- | | |
|--------|--------|
| 1. (D) | 3. (D) |
| 2. (B) | 4. (B) |

Section-II

- | | |
|----------|----------|
| 5. (B,C) | 7. (A,C) |
| 6. (A,C) | |

Section-III

- | | |
|---------|----------|
| 8. (18) | 11. (10) |
| 9. (2) | 12. (5) |
| 10. (4) | 13. (62) |

Section-IV

- | | |
|-------------|-------------|
| 14. (16.00) | 16. (20.00) |
| 15. (30.00) | 17. (01.00) |

CHEMISTRY

Section-I

- | | |
|---------|---------|
| 18. (A) | 20. (B) |
| 19. (C) | 21. (D) |

Section-II

- | | |
|-------------|-----------|
| 22. (A,B,C) | 24. (A,C) |
| 23. (C,D) | |

Section-III

- | | |
|----------|----------|
| 25. (15) | 28. (13) |
| 26. (10) | 29. (4) |
| 27. (2) | 30. (6) |

Section-IV

31. (50.00)

33. (05.00)

32. (04.00)

34. (06.00)

MATHEMATICS

Section-I

35. (B)

37. (C)

36. (D)

38. (A)

Section-II

39. (B,D)

41. (A,C)

40. (A,B)

Section-III

42. (3)

45. (16)

43. (20)

46. (24)

44. (1)

47. (2)

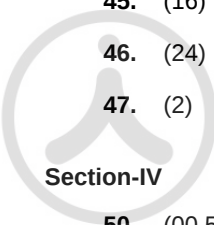
Section-IV

48. (02.00)

50. (00.50)

49. (02.00)

51. (01.00)


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Hints and Solutions

PHYSICS

Section-I

(1) Answer : (D)

Hint:

 Velocity after two collision will be $e(-\vec{v})$

Solution:

 Velocity after two collision will be $e(-\vec{v})$

 Impulse = ΔP

 Impulse = $20\sqrt{3}$

(2) Answer : (B)

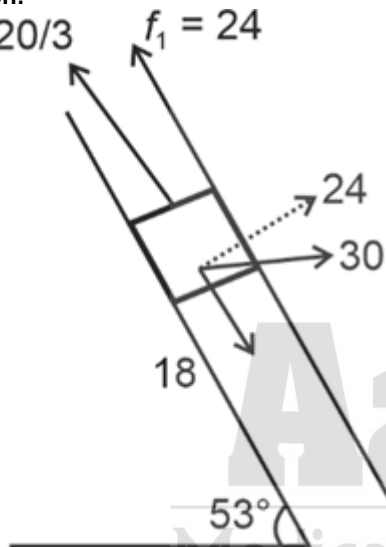
Hint:

Apply pseudo force on block

Solution:

$$f_2 = 20/3$$

$$f_1 = 24$$



$$a_r = 0$$

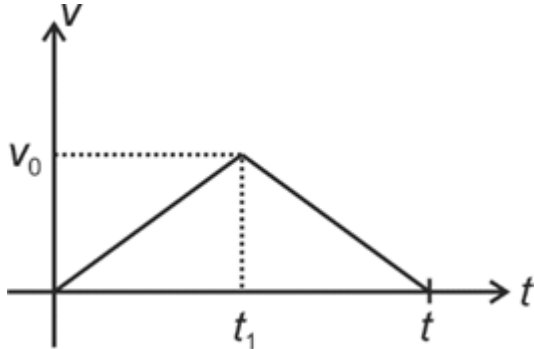
$$a = 15$$

(3) Answer : (D)

Hint:

 Area under $v-t$ curve is displacement.

Solution:



$$\frac{v_0}{t_1} = 5 \quad \frac{v_0}{t-t_1} = 4$$

$$d = \frac{1}{2} \cdot t \times \left(\frac{5 \times 4}{5+4} \right) t$$

$$d = \frac{1}{2} \cdot \frac{5 \cdot 4}{9} \cdot t^2 = 1000$$

$$t = 30 \text{ sec}$$

(4) Answer : (B)

Hint:

$$I = \int dm r^2$$

Solution:

$$I = \frac{m(a^2+b^2)}{2}$$

$$= \frac{m}{2} (R^2 + 4R^2)$$

$$I = \frac{5}{2} mR^2$$

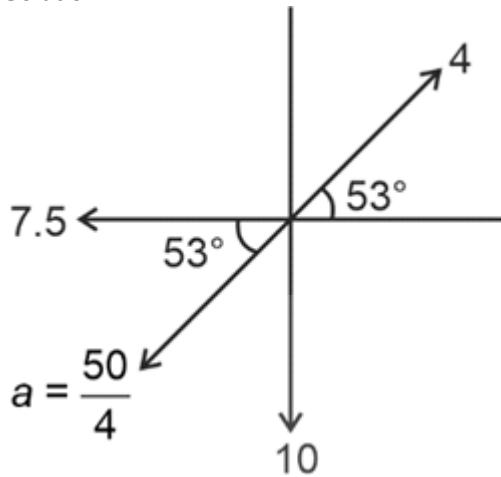
Section-II

(5) Answer : (B,C)

Hint:

Relative motion

Solution:



So path is straight line

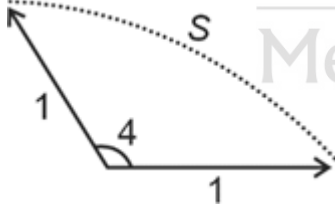
$$v = 0 \text{ at } t = \frac{100}{\frac{50}{4}} = 8 \text{ sec}$$

(6) Answer : (A,C)

Hint:

$\ell = RQ$

Solution:



$$\theta = \left(\frac{2}{1}\right) 2 = 4 \text{ rad}$$

$$S = \sqrt{1+1-2\cos 4} = 2\sin 2$$

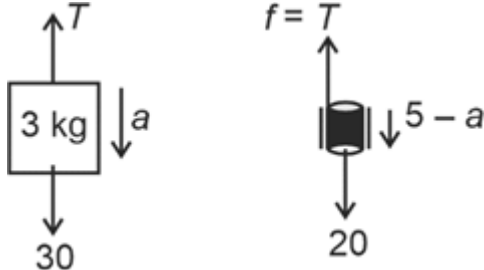
$$a_{\text{avg}} = \frac{|\Delta V|}{t} = \frac{\sqrt{2^2+2^2-2 \cdot 2 \cdot 2 \cos 4}}{2} = \frac{4\sin 2}{2} = 2\sin 2$$

(7) Answer : (A,C)

Hint:

Tension equal to friction

Solution:



$$30 - T = 3a$$

$$20 - T = 2(5 - a)$$

$$a = 4 \text{ m/s}^2$$

$$T = 18 \text{ N}$$

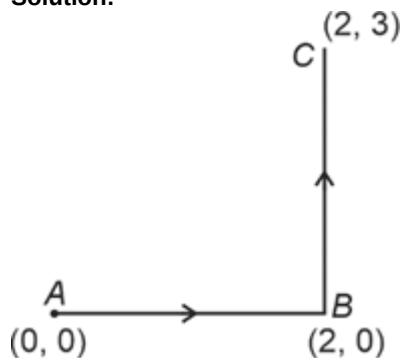
Section-III

(8) Answer : 18

Hint:

$$w = \int \vec{F} \cdot d\vec{s}$$

Solution:



$$W = W_{AB} + W_{BC}$$

$$W = 2xy \, dx + \int_0^3 2^2 \cdot y \, dy$$

$$W = 18$$

(9) Answer : 2

Hint:

$$\frac{\Delta V}{V} = \frac{3\Delta h}{h}$$

Solution:

$$V = \frac{1}{3} \pi r^2 h$$

$$\tan \theta = \frac{r}{h}$$

$$V \propto h^3$$

$$\frac{\Delta V}{V} = \frac{3\Delta h}{h}$$

$$= 3 \times \frac{0.04}{6} \times 100$$

$$= 2\%$$

(10) Answer : 4

Hint:

Work energy theorem

Solution:

$$x_i = 0 \cdot 12 = \frac{3\mu mg}{k}$$

at maximum speed $F_{\text{net}} = 0$

$$x_f = \frac{\mu mg}{k}$$

$$\frac{1}{2} m V^2 = \frac{1}{2} k \left(\frac{3\mu mg}{k} \right)^2 - \frac{1}{2} k \left(\frac{\mu mg}{k} \right)^2 - \mu mg \left(\frac{2\mu mg}{k} \right)$$

$$V = 2\mu g \sqrt{\frac{m}{k}}$$

$$5V_0 = 4 \text{ m/s}$$

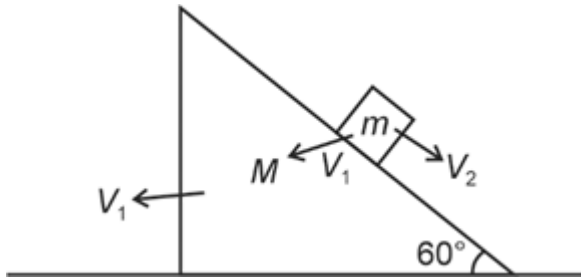
(11) Answer : 10

Hint:

Wedge constraint

Solution:





$V_2 = 10 \text{ m/s}$

$V_1 = 2 \text{ m/s}$

$$V = \sqrt{2^2 + 10^2 + 2 \cdot 2 \cdot 10 \cos(120^\circ)}$$

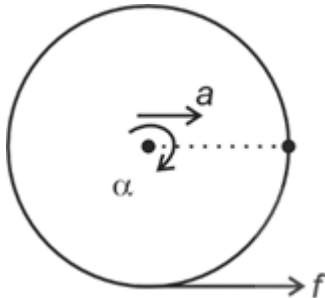
$$= \sqrt{84} = 2\sqrt{21}$$

(12) Answer : 5

Hint:

$a = \alpha R$

Solution:



$f = 4ma$

$(mg - f) R = (3mr^2 + mr^2) \alpha$

$mg - f = 4ma$

$\alpha R = \frac{5}{4} a$

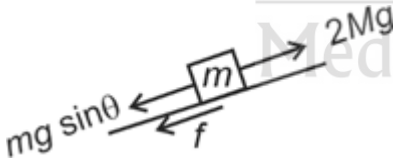
$4a_0 = 5$

(13) Answer : 62

Hint:

Balance forces

Solution:



At equilibrium of $M \Rightarrow kx = Mg$

$mgsin\theta + \mu mgcos\theta = 1Mg$

$M = \frac{62}{5}$

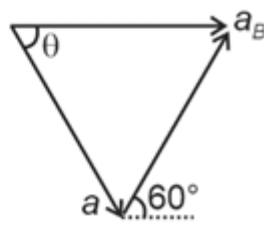
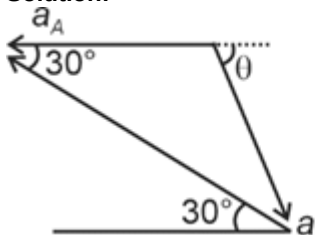
Section-IV

(14) Answer : 16.00

Hint:

Wedge constraint

Solution:



$a_A \sin 30^\circ = a \sin(\theta - 30^\circ)$

$a_B \sin 60^\circ = a \sin(\theta + 60^\circ)$

$a = a_A = 8 \text{ m/s}^2$

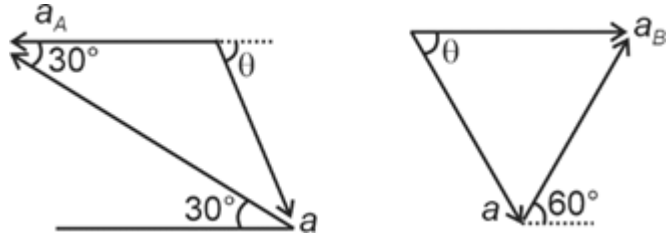
$\theta = 60^\circ$
 $\alpha = 30^\circ$

(15) Answer : 30.00

Hint:

Wedge constraint

Solution:



$a_A \sin 30^\circ = a \sin(\theta - 30^\circ)$

$a_B \sin 60^\circ = a \sin(\theta + 60^\circ)$

$a = a_A = 8 \text{ m/s}^2$

$\theta = 60^\circ$

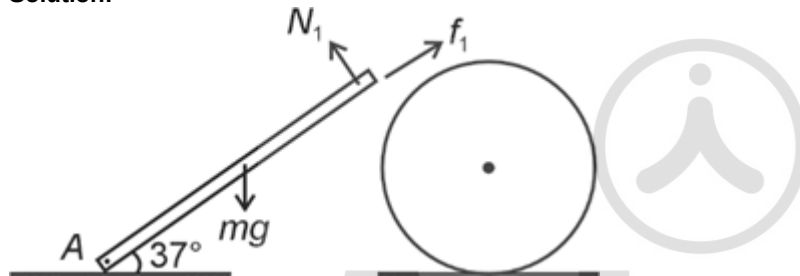
$\alpha = 30^\circ$

(16) Answer : 20.00

Hint:

Net torque on rod & sphere will zero.

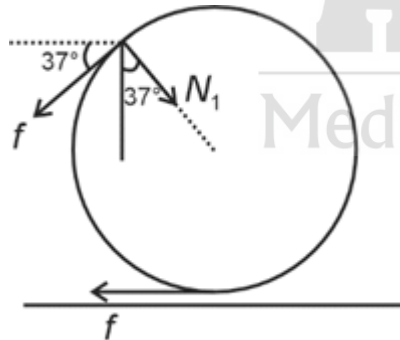
Solution:



Torque on rod about point A = 0

$mg \cdot \frac{1}{2} \cdot \frac{4}{5} = N_1 l$

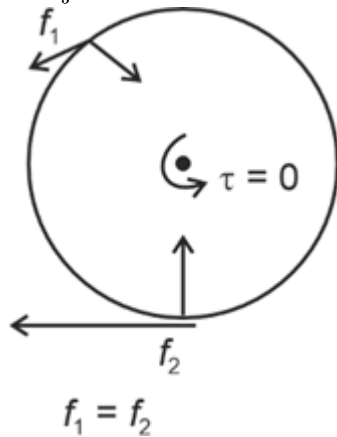
$N_1 = 20$



Net force on sphere in horizontal = 0

$f + f \cdot \frac{4}{5} = 20 \cdot \frac{3}{5}$

$f = \frac{20}{3}$

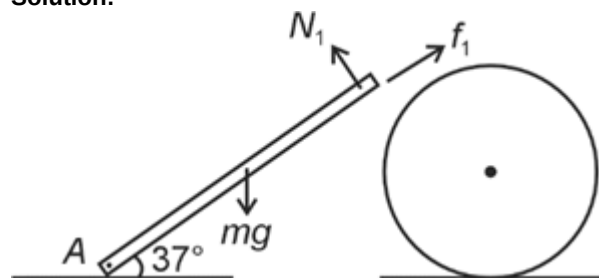


(17) Answer : 01.00

Hint:

Net torque on rod & sphere will zero.

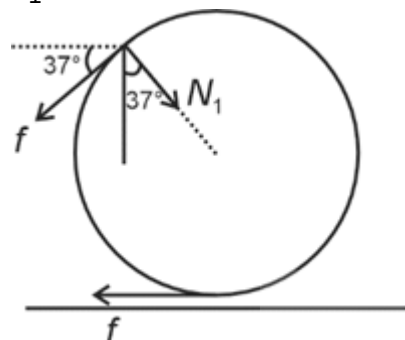
Solution:



Torque on rod about point A = 0

$$mg \cdot \frac{1}{2} \cdot \frac{4}{5} = N_1 l$$

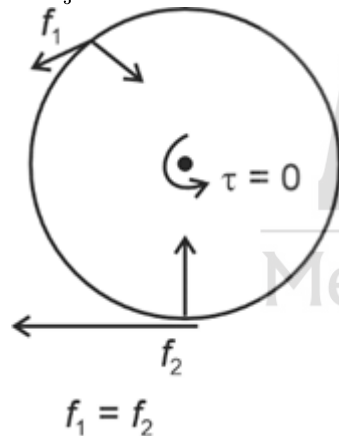
$$N_1 = 20$$



Net force on sphere in horizontal = 0

$$f + f \cdot \frac{4}{5} = 20 \cdot \frac{3}{5}$$

$$f = \frac{20}{3}$$



$$f_1 = f_2$$

CHEMISTRY

Section-I

(18) Answer : (A)

Hint:

$$r_n \text{ for H-atom} = a_0 \frac{n^2}{z}$$

Solution:

$$r_2 = a_0 \frac{(2)^2}{1}$$

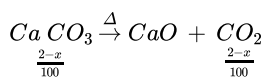
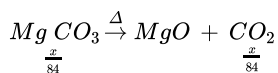
$$\Rightarrow x = 4a_0$$

$$\Rightarrow a_0 = \frac{x}{4}$$

$$r_3 \text{ of Be}^{3+} \text{ ion} = a_0 \frac{9}{4}$$

$$= \frac{9}{4} \times \frac{x}{4}$$

$$= \frac{9x}{16}$$

(19) Answer : (C)**Hint:**MgCO₃ and CaCO₃, both will produce CO₂ gas upon heating.**Solution:**

$$\text{Moles of } CO_2 = \frac{492.8}{22400} = 0.022 \text{ mol}$$

$$\frac{x}{84} + \frac{2-x}{100} = 0.022$$

$$100x + 168 - 84x = 184.8$$

$$16x = 16.8$$

$$x = 1.05 \text{ g}$$

$$\text{mass of } CaCO_3 = 2 - 1.05 = 0.95 \text{ g}$$

$$\% \text{ of } CaCO_3 = \frac{0.95}{2} \times 100$$

$$= 47.5\%$$

(20) Answer : (B)**Hint:**

Metallic character increases down the group.

Solution:Atomic size order $\Rightarrow Ca < Sr < Ba < K$

All other given orders are correct.

(21) Answer : (D)**Hint:**

$$\% s = \frac{\text{no. of } s \text{ orbitals}}{\text{Total orbitals}} \times 100$$

Solution:XeO₃, NH₃, NF₃ and PF₃ has 1 lone pair on central atom and sp³ hybridised having % s character is approximately 25.SO₃, BF₃, SnCl₂ $\Rightarrow sp^2 \Rightarrow \% s \approx 33\%$ XeO₄ $\Rightarrow sp^3$ but no lone pair on central atom.

Section-II

(22) Answer : (A,B,C)**Hint:**

$$\left(P + \frac{a}{V_m}\right)(V_m - b) = RT$$

Solution:

$$PV_m^3 - PbV_m^2 - RTV_m^2 + aV_m - ab = 0$$

$$V_m^3 - \left(b + \frac{RT}{P}\right)V_m^2 + \frac{aV_m}{P} - \frac{ab}{P} = 0$$

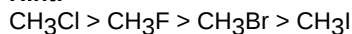
Coefficient of V_m³ is 1Coefficient of V_m² is $-\left(b + \frac{RT}{P}\right)$ Coefficient of V_m is $\frac{a}{P}$ **(23) Answer : (C,D)****Hint:**

$$B_2 = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^1 = \pi 2p_y^1, \underbrace{\sigma 2p_z}_{LUMO}$$

Solution:BO of N₂ = 3

$$N_2^+ = 2.5$$

BE \propto BOBE of N₂ > N₂⁺Li₂⁺ and Li₂⁻ both have BO = $\frac{1}{2}$ Li ion having less no. of e⁻ in ABMO is more stable Li₂⁺ > Li₂⁻ (stability)**(24) Answer : (A,C)**

Hint:

Order of dipole moment

Solution:

$$\text{BO} \propto \text{B.E}$$

(B.O)

$$\text{N}_2 = 3$$

$$\text{O}_2^+ = 2.5$$

$$\text{O}_2 = 2$$

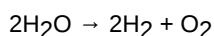
$$\text{O}_2^- = 1.5$$

**Section-III****(25) Answer : 15****Hint:**

$$w = -P_{\text{ext}}(V_2 - V_1)$$

Solution:

$$\text{Number of moles of } \text{H}_2\text{O} = \frac{72}{18} = 4 \text{ mol}$$



2 mol water form 3 mol gases

4 mol water will form 6 mol of gases

$$\text{Volume of gases formed} = \frac{6 \times R \times 300 \text{ m}^3}{P}$$

$$\text{Work done} = -P_{\text{ext}} dV$$

$$= -P_{\text{ext}} \left(\frac{6 \times 8.3 \times 300}{P} - 0 \right)$$

$$= -14.94 \text{ kJ}$$

$$\text{Expansion work} = 14.94 \text{ kJ}$$

(26) Answer : 10**Hint:**

$$[H^+] = \sqrt{K_a C + K_w}$$

Solution:

$$= \sqrt{2 \times 10^{-12} \times 4 \times 10^{-4} + 10^{-14}}$$

$$= \sqrt{8 \times 10^{-16} + 10^{-14}}$$

$$= \sqrt{1.08 \times 10^{-14}}$$

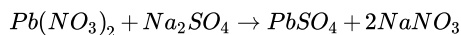
$$= 1.04 \times 10^{-7}$$

$$= 10.4 \times 10^{-8} = x \times 10^{-8}$$

$$x = 10.4$$

(27) Answer : 2**Hint:**

$$(K_{sp})_{\text{PbSO}_4} = [\text{Pb}^{2+}] [\text{SO}_4^{2-}]$$

Solution:

0.75 mmol

2.5 mmol

1.75 mmol

$$\text{Left } [\text{SO}_4^{2-}] = \frac{1.75 \text{ mmol}}{200 \text{ mL}} = 8.75 \times 10^{-3} \text{ M}$$

$$K_{sp} = [\text{Pb}^{2+}] [\text{SO}_4^{2-}]$$

$$1.8 \times 10^{-8} = [\text{Pb}^{2+}] \times 8.75 \times 10^{-3}$$

$$[\text{Pb}^{2+}] = \frac{1.8 \times 10^{-8}}{8.75 \times 10^{-3}}$$

$$= 2.06 \times 10^{-6}$$

$$x = 2.06$$

(28) Answer : 13**Hint:**

$$K_p = \frac{(X_B \cdot P_T)^2}{(X_A \cdot P_T)^3}$$

Solution:



$$n_T = 0.2 + \frac{1.6}{3}$$

$$\frac{0.6 + 1.6}{3} = \frac{2.2}{3}$$

$$K_p = \frac{\left(\frac{1.6 \times 3}{3 \times 2.2}\right)^2}{\left(\frac{0.2}{2.2} \times 3\right)^3 \times P}$$

$$K_p = \frac{0.53}{0.02 P} \Rightarrow P = \frac{0.53}{0.02 \times 2}$$

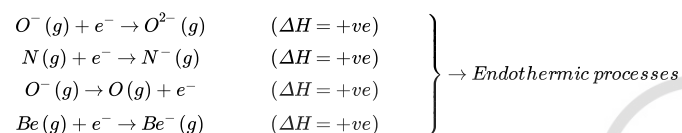
$$= 13.25 \text{ bar}$$

(29) Answer : 4

Hint:

- Electron gain enthalpy of N and Be are positive,
- 2^{nd} electron gain enthalpy of an element is positive
- Opposite of exothermic reaction is endothermic.

Solution:



(30) Answer : 6

Hint:

$$B.O. = \frac{N_b - N_a}{2} \text{ (For diatomic molecule/ions)}$$

For CO_3^{2-}

Solution:

$$B.O. = \frac{\text{Total no. of bonds in any resonating structure}}{\text{no. of atoms involved in resonance other than central atom}}$$

Molecule/Ions	Bond order
H ₂	1
He ₂ ⁺	0.5
N ₂	3
O ₂ ⁻	1.5
O ₂ ⁺	2.5
Br ₂	1
CO ₃ ²⁻	1.33
NO ⁺	3
NO ₂ ⁻	1.5

Section-IV

(31) Answer : 50.00

Hint:

If SA neutralised with SB than 57 kJ energy evolved per equivalent of SB and SA.

Solution:

Let the heat capacity of insulated beaker is C and mass of aqueous content in exp-1 = (50 + 50) × 1 = 100 g

Total heat capacity = (C + 100 × 4.2) J/K

Heat released in Exp-1 = 0.025 × 57

= 1.425 kJ

⇒ 1.425 × 1000 = (C + 100 × 4.2)ΔT

⇒ 1425 = (C × 3.2 + 100 × 4.2 × 3.2)

⇒ 1425 = C × 3.2 + 1344

⇒ C × 3.2 = 1425 - 1344

$$C = \frac{81}{3.2} = 25.3125 \text{ J/K}$$

In 2nd experiment

$$n_{\text{HCOOH}} = 0.1 \text{ mol}$$

$$n_{\text{NaOH}} = 0.05 \text{ mol}$$

Total mass of aqueous content = 200 g

$$\begin{aligned} \text{Total heat released} &= \\ &= 25.3125 \times 2.9 + 200 \times 4.2 \times 2.9 \\ &= 73.4 + 2436 \\ &= 2509.4 \text{ J} = 2.51 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \Delta H_{\text{new}} &= \frac{2.51}{0.05} \text{ kJ mol}^{-1} \\ &= 50.2 \text{ kJ} \\ &\approx 50 \text{ kJ} \end{aligned}$$

$$pH = pK_a + \log \frac{[\text{HCOO}^-]}{[\text{HCOOH}]}$$

$$pH = pK_a$$

$$pH = -\log(1.8 \times 10^{-4})$$

$$pH = 4 - \log 1.8$$

$$= 3.74$$

$$3.74 = y + \log 0.5$$

$$y = 3.74 - \log(0.5)$$

$$y = 3.74 + \log 2$$

$$= 3.74 + 0.3$$

$$= 4.04$$

(32) Answer : 04.00

Hint:

If SA neutralised with SB than 57 kJ energy evolved per equivalent of SB and SA.

Solution:

Let the heat capacity of insulated beaker is mass of aqueous content in exp-1 = (50 + 50) × 1 = 100 g

$$\text{Total heat capacity} = (C + 100 \times 4.2) \text{ J/k}$$

$$\text{Heat released in Exp-1} = 0.025 \times 57$$

$$= 1.425 \text{ kJ}$$

$$\Rightarrow 1.425 \times 1000 = (C + 100 \times 4.2)\Delta T$$

$$\Rightarrow 1425 = (C \times 3.2 + 100 \times 4.2 \times 3.2)$$

$$\Rightarrow 1425 = C \times 3.2 + 1344$$

$$\Rightarrow C \times 3.2 = 1425 - 1344$$

$$C = \frac{81}{3.2} = 25.3125 \text{ J/k}$$

In 2nd experiment

$$n_{\text{HCOOH}} = 0.1 \text{ mol} \quad n_{\text{NaOH}} = 0.05 \text{ mol}$$

Total mass of aqueous content = 200 g

$$\text{Total heat released} =$$

$$= 25.3125 \times 2.9 + 200 \times 4.2 \times 2.9$$

$$= 73.4 + 2436$$

$$= 2509.4 \text{ J} = 2.51 \text{ kJ}$$

$$\Delta H_{\text{new}} = \frac{2.51}{0.05} \text{ kJ mol}^{-1}$$

$$= 50.2 \text{ kJ}$$

$$\approx 50 \text{ kJ}$$

$$pH = pK_a + \log \frac{[\text{HCOO}^-]}{[\text{HCOOH}]}$$

$$pH = pK_a$$

$$pH = -\log(1.8 \times 10^{-4})$$

$$pH = 4 - \log 1.8$$

$$= 3.74$$

$$3.74 = y + \log 0.5$$

$$y = 3.74 - \log(0.5)$$

$$y = 3.74 + \log 2$$

$$= 3.74 + 0.3$$

$$= 4.04$$

(33) Answer : 05.00

Hint:

For 2nd period order of IE

$$\text{Li} < \text{B} < \text{Be} < \text{C} < \text{O} < \text{N} < \text{F}$$

Solution:

A and B are metals and H is noble gas other all elements are non-metals. C, D, E, F and G are non-metals. D is carbon, atomic no. = 6

(34) Answer : 06.00

Hint:

For 2nd period order of IE
 $Li < B < Be < C < O < N < F$

Solution:

A and B are metals and H is noble gas other all elements are non-metals. C, D, E, F and G are non-metals. D is carbon, atomic no. = 6

MATHEMATICS

Section-I

(35) Answer : (B)

Hint:

Vieta's formula

Solution:

$$x^3 - 2x^2 - 1 = 0$$

$$\alpha^2 + \beta^2 + \gamma^2$$

$$= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$$

$$= 4 - 0 = 4$$

(36) Answer : (D)

Hint:

Transformation formula

Solution:

$$\frac{\tan\left(\frac{\pi}{4} + \alpha\right)}{\tan\left(\frac{\pi}{4} + \beta\right)} = \frac{5}{3}$$

$$\therefore \frac{\cos(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{8}{2} = 4$$

$$\therefore \cos(\alpha + \beta) = 4\sin(\alpha - \beta)$$

(37) Answer : (C)

Hint:

$$\log_2 \left(\log_{\frac{1}{3}}(x) \right) < 1$$

Solution:

$$\log_{\frac{1}{2}} \left(\log_2 \left(\log_{\frac{1}{3}}(x) \right) \right) > 0$$

$$\log_2 \left(\log_{\frac{1}{3}}(x) \right) < 1$$

$$\log_{\frac{1}{3}}(x) < 2$$

$$x > \frac{1}{9} \dots (1)$$

$$\log_2 \left(\log_{\frac{1}{3}}(x) \right) > 0$$

$$\log_{\frac{1}{3}} x > 1$$

$$\Rightarrow x < \frac{1}{3} \dots (2)$$

From (1) and (2)

$$x \in \left(\frac{1}{9}, \frac{1}{3} \right)$$

(38) Answer : (A)

Hint:

$$\Rightarrow |z_1| |z_2| |z_3| \left| \frac{9\bar{z}_3}{|z_3|^2} + \frac{4\bar{z}_2}{|z_2|^2} + \frac{\bar{z}_1}{|z_1|^2} \right| = 12$$

Solution:

$$|9z_1z_2 + z_2z_3 + 4z_3z_1| = 12$$

$$\Rightarrow |z_1| |z_2| |z_3| \left| \frac{9\bar{z}_3}{|z_3|^2} + \frac{4\bar{z}_2}{|z_2|^2} + \frac{\bar{z}_1}{|z_1|^2} \right| = 12$$

$$\Rightarrow 1 \times 2 \times 3 |z_1 + z_2 + z_3| = 12$$

$$\Rightarrow |z_1 + z_2 + z_3| = 2$$

Section-II

(39) Answer : (B,D)

Hint:

Make different cases

Solution:

M, A, T \rightarrow 2

H, E, I, C, S \rightarrow 1

Case I: All different

Number of ways = ${}^8C_4 \cdot 4!$

$$= \frac{8!}{4!4!} \times 4! = 1680$$

Case II: 2 same, 2 different

$$\text{Number of ways} = {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$$

Case III: 2 same, 2 same

$$\text{Number of ways} = {}^3C_2 \times \frac{4!}{2!2!} = 18$$

$$\text{Total number of words} = 1680 + 756 + 18 = 2454$$

(40) Answer : (A,B)

Hint:

$$a_n = a_1 + (n-1)d$$

Solution:

$$a_1 = 3, d = 6, T_1 = 1$$

$$a_n = a_1 + (n-1)d$$

$$= 3 + (n-1)6$$

$$T_{n+1} - T_n = a_n$$

Put $n = 1$

$$T_2 - T_1 = a_1$$

$$\Rightarrow T_2 = T_1 + a_1$$

Put $n = 2$

$$T_3 - T_2 = a_2$$

$$\Rightarrow T_3 = T_2 + a_1 + a_2$$

\vdots

$$T_{n+1} = T_1 + a_1 + a_2 + a_3 \dots + a_n$$

$$= T_1 + \frac{n}{2} [2(3) + (n-1)6]$$

$$= T_1 + n(3 + 3n - 3)$$

$$= T_1 + 3n^2$$

Put $n = 9$

$$T_{10} = T_1 + 3(9)^2 = 1 + 243 = 244$$

Put $n = 19$

$$T_{20} = T_1 + 3(19)^2$$

$$= 1 + 1083$$

$$= 1084$$

$$\sum_{k=1}^{20} T_k = 1 + \sum_{k=1}^{19} T_k$$

$$= 1 + \sum_{k=1}^{19} (1 + 3k^2)$$

$$= 1 + (19) + \frac{3(19)(20)(39)}{6}$$

$$= 1 + 19 + 7410$$

$$= 7430$$

$$\sum_{k=1}^{15} T_k = 1 + \sum_{k=1}^{14} (1 + 3k^2)$$

$$= 1 + 1(14) + \frac{3(14)(15)(29)}{6}$$

$$= 1 + 14 + 3045$$

$$= 3060$$

(41) Answer : (A,C)

Hint:

Taking log both sides,

$$(x-1) \log_{10}(x^2 - x + 1) < 0$$

Solution:

$$x^2 - x + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} > 0 \forall x \in R$$

Given, $(x^2 - x + 1)^{x-1} < 1$

Taking log both sides,

$$(x-1) \log_{10}(x^2 - x + 1) < 0$$

Case I: $x-1 > 0$, $\log_{10}(x^2 - x + 1) < 0$

 i.e., $x-1 > 0$ and $0 < x < 1 \Rightarrow$ no solution

Case II: $x-1 < 0$ and $\log_{10}(x^2 - x + 1) > 0$

$$\Rightarrow x < 1 \text{ and } x > 1 \text{ or } x < 0$$

$$\Rightarrow x < 0$$

Section-III
(42) Answer : 3
Hint:

Operations on set

Solution:

$$B^C = \{1, 2, 8, 9, 10\}$$

$$A \cap B^C = \{1, 2\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

$$C - (A \cup B) = \{8\}$$

$$B \Delta C = \{2, 3, 5, 7, 8\}$$

$$(C - (A \cup B)) \cap (B \Delta C) = \{8\}$$

$$(A \cap B^C) \cup ((C - (A \cup B)) \cap (B \Delta C)) = \{1, 2, 8\}$$

 \therefore Number of elements = 3

(43) Answer : 20
Hint:

$$f(x) = x^3 - 3x$$

Solution:

$$\frac{f(x+y)}{x+y} - \frac{f(x-y)}{x-y} = 4xy = (x+y)^2 - (x-y)^2$$

$$\Rightarrow \frac{f(x+y)}{x+y} - (x+y)^2 = \frac{f(x-y)}{x-y} - (x-y)^2 = C$$

$$\Rightarrow \frac{f(x)}{x} - x^2 = C$$

$$\Rightarrow f(x) = x^3 + Cx$$

$$\therefore f(1) = -2$$

$$\Rightarrow C = -3$$

$$\therefore f(x) = x^3 - 3x$$

$$f(x) + f(-x) = 0$$

$$\therefore f(-5) + f(-4) + f(0) + f(2) + f(3) + f(4) + f(5)$$

$$= f(0) + f(2) + f(3)$$

$$= 0 + 8 - 6 + 27 - 9$$

$$= 20$$

(44) Answer : 1
Hint:

Transformation formula

Solution:

$$\sin^2 12^\circ + \sin^2 21^\circ + \sin^2 39^\circ + \sin^2 48^\circ - \sin^2 9^\circ - \sin^2 18^\circ$$

$$= \sin^2 12^\circ + \sin^2 21^\circ + (\sin^2 39^\circ - \sin^2 9^\circ) + (\sin^2 48^\circ - \sin^2 18^\circ)$$

$$= 1 - (\cos^2 12^\circ - \sin^2 21^\circ) + \sin 48^\circ \sin 30^\circ + \sin 66^\circ \sin 30^\circ$$

$$= 1 - \cos 33^\circ \cos 9^\circ + \frac{1}{2} \times 2 \times \sin 57^\circ \sin 9^\circ$$

$$= 1 - \cos 33^\circ \cos 9^\circ + \cos 33^\circ \cos 9^\circ$$

$$= 1$$

(45) Answer : 16
Hint:

 Replace x by $x-3$ in given equation

$$\Rightarrow x^3 - 10x^2 + 33x - 37 = 0$$

Solution:

Let $3 + y = x \Rightarrow y = x - 3$, then $f(y) = 0$

Replace x by $x - 3$ in given equation

$$\Rightarrow x^3 - 10x^2 + 33x - 37 = 0$$

$$\therefore 2a + b + c = 2(10) + 33 - 37$$

$$= 20 - 4$$

$$= 16$$

(46) Answer : 24

Hint:

Let x_1 be the number of times he took 1 step up the staircase

Let x_2 be the number of times he took 2 steps up the staircase.

$$x_1 + 2x_2 = 11$$

Solution:

Let x_1 be the number of times he took 1 step up the staircase

Let x_2 be the number of times he took 2 steps up the staircase.

$$x_1 + 2x_2 = 11$$

			No. of ways
$x_1 = 1,$	$x_2 = 5$	1, 2, 2, 2, 2, 2	$\frac{6!}{1!5!} = 6$
$x_1 = 3,$	$x_2 = 4$	1, 1, 1, 2, 2, 2, 2	$\frac{7!}{3!4!} = 35$
$x_1 = 5,$	$x_2 = 3$	1, 1, 1, 1, 1, 2, 2, 2	$\frac{8!}{3!5!} = 56$
$x_1 = 7,$	$x_2 = 2$	1, 1, 1, 1, 1, 1, 1, 2, 2	$\frac{9!}{2!7!} = 36$
$x_1 = 9,$	$x_2 = 1$		$\frac{10!}{9!1!} = 10$
$x_1 = 11,$	$x_2 = 0$		1

$$\therefore P = 144$$

$$\frac{P}{6} = 24$$

(47) Answer : 2

Hint:

$$\Rightarrow (a + 2d)^2 = a(a + 6d)$$

Solution:

Let r^{th} , $(r + 2)^{\text{th}}$ and $(r + 6)^{\text{th}}$ terms be a , $a + 2d$, $a + 6d$

$$\Rightarrow (a + 2d)^2 = a(a + 6d)$$

$$\Rightarrow a = 2d$$

Terms are $(2d, 4d, 8d)$ so common ratio of G.P is 2

(48) Answer : 02.00

Hint:

$$D \geq 0$$

Solution:

$$x + y = 4 - z$$

$$x^2 + y^2 = 6 - z^2$$

$$\therefore 2xy = (x + y)^2 - (x^2 + y^2) = (4 - z)^2 - (6 - z^2)$$

$$= 2z^2 - 8z + 10$$

Quadratic equation whose roots are x & y is

$$t^2 - (x + y)t + xy = 0$$

$$t^2 - (4 - z)t + z^2 - 4z + 5 = 0$$

$$D \geq 0$$

$$(4 - z)^2 - 4(z^2 - 4z + 5) \geq 0$$

$$\Rightarrow (3z - 2)(z - 2) \leq 0$$

$$\Rightarrow z \in \left[\frac{2}{3}, 2 \right]$$

$$p = \frac{2}{3}, q = 2$$

(49) Answer : 02.00

Hint:

$$f(2) \cdot f\left(\frac{2}{3}\right) < 0$$

Solution:

For exactly one root to lie in $\left(\frac{2}{3}, 2\right)$

$$f(2) \cdot f\left(\frac{2}{3}\right) < 0$$

$$\Rightarrow a \in \left(\frac{1}{3}, 3\right)$$

\therefore Number of integral values of a is 2

(50) Answer : 00.50

Hint:

Transformation formula

Solution:

$$B = A = 15^\circ$$

$$\sin(A + B) = \frac{1}{2}$$

$$\therefore \alpha^2 = 3$$

(51) Answer : 01.00

Hint:

Transformation formula

Solution:

$$\cos(A - B) = 1$$



Aakash

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