15/02/2026

Set-1_ONLINE

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**MM : 180**

AIATS For One Year JEE(Advanced)-2026 (XII Passed)_Test-4A_Paper-1_ONLINE

**Time : 180 Min.**

## PHYSICS

### Section-I

**1.** (D)

**2.** (A)

**3.** (A)

**4.** (A)

### Section-II

**5.** (03.20)

**6.** (50.00)

**7.** (06.40)

**8.** (00.48)

**9.** (00.03)

**10.** (00.60)

### Section-III

**11.** (B,C)

**12.** (B,C,D)

**13.** (A,C)

**14.** (A,C)

**15.** (A,B,C)

**16.** (A,C)

### Section-IV

**17.** (2)

**18.** (13)

**19.** (7)

## CHEMISTRY

### Section-I

**20.** (A)

**21.** (A)

**22.** (A)

**23.** (B)

### Section-II

**24.** (02.83)

**25.** (13.95)

**26.** (12.05)

**27.** (29.88)

**28.** (60.00)

**29.** (01.30)

### Section-III

**30.** (A,C)

**31.** (A,C)

**32.** (A,B,D)

**33.** (B,D)

**34.** (A,B,D)

**35.** (A,B)

### Section-IV

**36.** (8)

**37.** (4)

**38.** (14)



## MATHEMATICS

### Section-I

**39.** (C)

**40.** (B)

**41.** (A)

**42.** (A)

### Section-II

**43.** (60.00)

**44.** (01.00)

**45.** (03.00)

**46.** (08.00)

**47.** (01.00)

**48.** (03.00)

**Section-III**

**49.** (A,C,D)

**50.** (A,B,C)

**51.** (A,C)

**52.** (A,C,D)

**53.** (B,C)

**54.** (A,B)

**Section-IV**

**55.** (45)

**56.** (7)

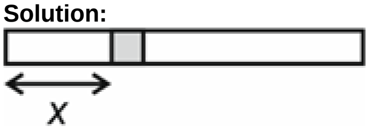
**57.** (3)

## Hints and Solutions

### PHYSICS

#### Section-I

**(1)** **Answer :** (D)
**Solution:**
$P = \frac{1}{F}$
$P_{eq} = \Sigma P_i$

**(2)** **Answer :** (A)
**Solution:**



$\int x^2 \lambda dx$

$\lambda = \frac{\lambda_0}{l} x + \lambda_0$

$I = \int_0^l x^2 \left(\frac{\lambda_0}{l} x + \lambda_0\right) dx$

$= \frac{\lambda_0}{l} \frac{l^4}{4} + \frac{\lambda_0 l^3}{3}$

$= \frac{7\lambda_0 l^3}{12}$

**(3)** **Answer :** (A)
**Solution:**
$\Sigma hi$ = constant

**(4)** **Answer :** (A)
**Solution:**
$U_i = 4\pi R^2 S$
$\frac{4}{3}\pi r^3 = 4\pi R^2 t$
$r = (3R^2 t)^{1/3}$
$U_f = 4\pi(3R^2 t)^{2/3} S$

#### Section-II

**(5)** **Answer :** 03.20
**Solution:**
$i_1 = 6$, $i_2 = 3$ A, $i_3 = 2$ A
$\Delta v = 6 + 44 = 50$ V
$i_5 = 22$ A
$i_x = 11$ A
$x = \frac{50}{11}\ \Omega$
$P = 22 \times 160\ \omega$
$\frac{P}{1100} = \frac{22 \times 160}{1100} = 3.2$

**(6)** **Answer :** 50.00
**Solution:**
$i_1 = 6$, $i_2 = 3$ A, $i_3 = 2$ A
$\Delta v = 6 + 44 = 50$ V
$i_5 = 22$ A

$i_X$ = 11 A

$x = \frac{50}{11}\ \Omega$

$P$ = 22 × 160 ω

$\frac{P}{1100} = \frac{22\times160}{1100} = 3.2$

**(7)** **Answer :** 06.40

**Solution:**

$\frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{v} + \frac{vd\theta}{dt} = |a\cos\theta|\hat{v} + |a\sin\theta|\hat{\theta}$

$a\cos\theta = 8\times\frac{4}{5} = 6.4$

$a\sin\theta = 8\times\frac{3}{5} = 4.8 = \frac{vd\theta}{dt}$

$\frac{d\theta}{dt} = \frac{4.8}{10} = 0.48$

**(8)** **Answer :** 00.48

**Solution:**

$\frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{v} + \frac{vd\theta}{dt} = |a\cos\theta|\hat{v} + |a\sin\theta|\hat{\theta}$

$a\cos\theta = 8\times\frac{4}{5} = 6.4$

$a\sin\theta = 8\times\frac{3}{5} = 4.8 = \frac{vd\theta}{dt}$

$\frac{d\theta}{dt} = \frac{4.8}{10} = 0.48$

**(9)** **Answer :** 00.03

**Solution:**

$\left|\vec{I}\right| = \left|\Delta\vec{p}\right| = mv = 0.1\times0.3 = 0.03$

$\vec{a} = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \frac{2v^2}{3a} = \frac{2\times0.3\times0.3}{3\times0.1}$

= 0.6

**(10)** **Answer :** 00.60

**Solution:**

$\left|\vec{I}\right| = \left|\Delta\vec{p}\right| = mv = 0.1\times0.3 = 0.03$

$\vec{a} = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \frac{2v^2}{3a} = \frac{2\times0.3\times0.3}{3\times0.1}$

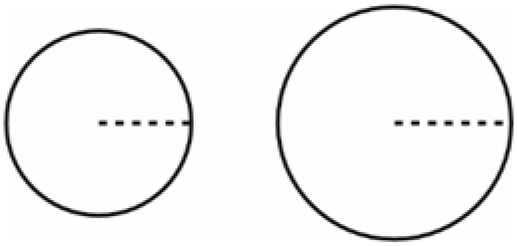
= 0.6



**Section-III**

**(11)** **Answer :** (B,C)

**Solution:**



$\Delta E = \frac{Q}{4\pi\varepsilon_0 R^2} - \frac{Q}{4\pi\varepsilon_0 4R^2}$

$= \frac{3Q}{16\pi\varepsilon_0 R^2}$

$U_i = \frac{3}{5}\frac{KQ^2}{R}$

$U_f = \frac{3}{5}\frac{KQ^2}{2R}$

$\Delta H = \frac{3}{5}\frac{KQ^2}{2R} = \frac{3}{40}\frac{Q^2}{\pi\varepsilon_0 R}$

**(12)** **Answer :** (B,C,D)

**Solution:**

$\Delta V = \frac{12\,CV}{3\,C} = 4$ V

$E = \frac{1}{2} 3\,C(2V)^2 = 6\,CV^2$

**(13)** **Answer :** (A,C)
**Solution:**

$\frac{M}{L} = \frac{\frac{2}{3}mR^2\omega}{\frac{2}{5}mR^2\omega}\frac{q}{2m}$

**(14)** **Answer :** (A,C)
**Solution:**

$U = \frac{1}{2}Li^2$

$= \frac{1}{2} \times 0.1 \times \left(\frac{0.2}{10}\right)^2$

$= \frac{0.1}{2} \times \frac{0.2\times 0.2}{10\times 10}$

$= \frac{4}{2} \times 10^{-5}$

$V$ = 0.2 volt

$i$ = 10 mA

$\Delta V_R$ = 100 mV

$\Delta V_L$ = 0.1

**(15)** **Answer :** (A,B,C)
**Solution:**
Theoretical

**(16)** **Answer :** (A,C)
**Solution:**

$\frac{Pr^3}{R^3} = -k4\pi r^2\frac{dT}{dr}$

$kr = -\frac{dT}{dr}$

$-k\int_0^r rdr = \int_{T_0}^{T} dT$

$-\frac{kr^2}{2} = T - T_0$



## Section-IV

**(17)** **Answer :** 2
**Solution:**

$\frac{V_1}{V_2} = \sqrt{\frac{Y_1\rho}{\rho_1 2Y_2}}$

**(18)** **Answer :** 13
**Solution:**
$|\Delta E|$ = |3 × 56 – 40 – 144|
= (168 – 184)
= –16 (negative means energy absorbed)
∴ 16 – 3 = 13 MeV

**(19)** **Answer :** 7
**Solution:**

$\Delta E = 18\left(\frac{1}{9} - \frac{1}{16}\right)$

$\Delta E = \frac{7\times 18}{144} = \frac{7}{8}$

## CHEMISTRY

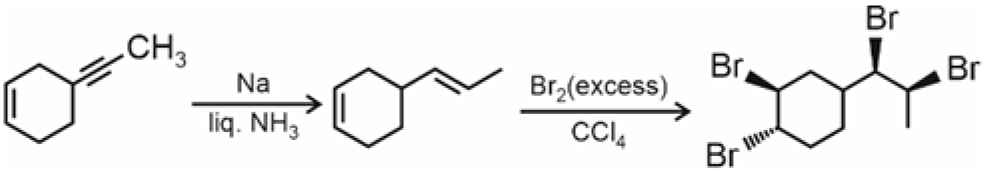
### Section-I

**(20) Answer :** (A)

**Hint:**

Birch reduction gives trans alkene.

**Solution:**

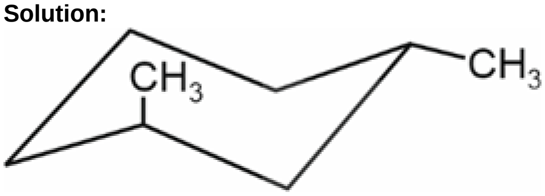



**(21) Answer :** (A)

**Hint:**

Heavier group on equatorial more stable.

**Solution:**




**(22) Answer :** (A)

**Hint:**

$r_Y + 2r_X + r_Y = a$

**Solution:**

As per the diagram

FCC – Y    O.V. – X(at edge centers)

Packing fraction $= \frac{4\times\frac{4}{3}\pi r_Y^3+3\times\frac{4}{3}\pi r_x^3}{(2r_x+2r_Y)^3}$

$= \frac{\frac{4}{3}\pi\left(4r_Y^3+3r_x^3\right)}{8(r_x+r_Y)^3}$

$= \frac{\pi}{6}\frac{\left(4+3\left(\frac{r_x}{r_Y}\right)^3\right)}{\left(1+\frac{r_x}{r_Y}\right)^3}$

$= \frac{\pi}{6}\left(\frac{4+3\times0.42}{5.36}\right)$

= 0.51

**(23) Answer :** (B)

**Hint:**

$\mu = \sqrt{n(n+2)}$ where, n = number of unpaired $e^{\ominus}$.

**Solution:**

$[CrCl_6]^{3-}$ : $Cr^{3+}$ : $[Ar]4s^0 3d^3$

$\mu = \sqrt{3(3+2)} = 3.87$

$[Co(CN)_6]^{4-}$ : $Co^{2+}$ : $[Ar]4s^0 3d^7$

n = 1

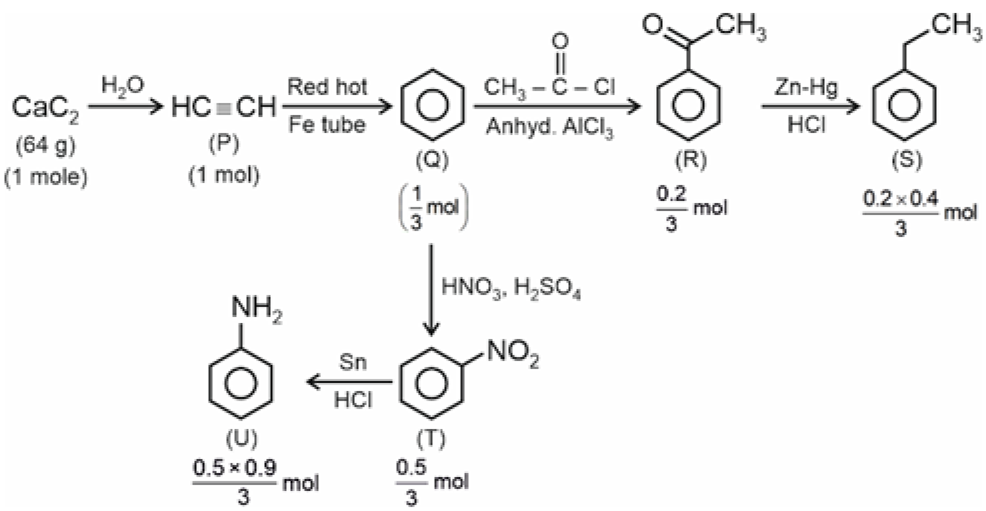
### Section-II

**(24) Answer :** 02.83

**Hint:**

Hydrolysis of $CaC_2$ gives acetylene.

**Solution:**




Mass of $U = \frac{0.5\times0.9}{3}\times 93$

= 13.95 g

Mass of $S = \frac{0.2\times0.4}{3}\times 106$

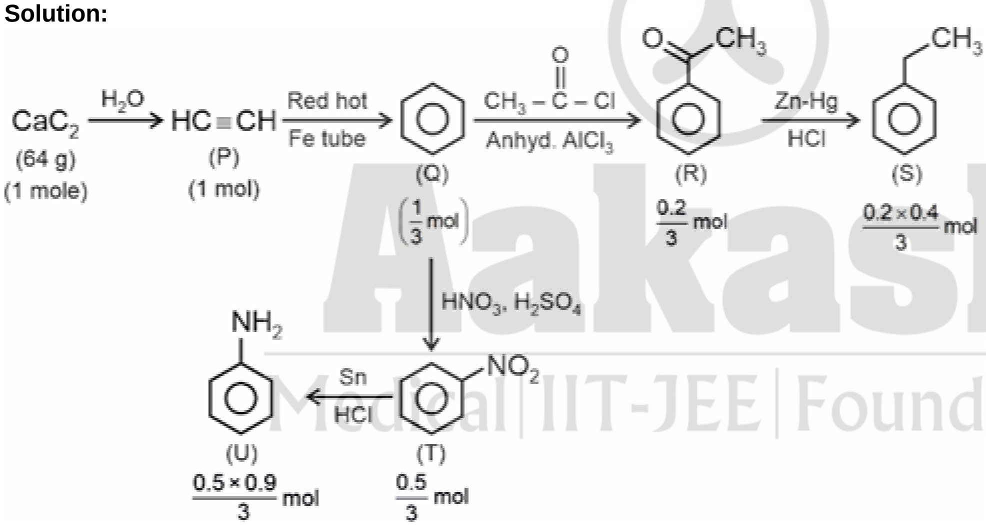
= 2.83 g

**(25)** **Answer :** 13.95

**Hint:**

Hydrolysis of $CaC_2$ gives acetylene.

**Solution:**






Mass of $U = \frac{0.5\times0.9}{3}\times 93$

= 13.95 g

Mass of $S = \frac{0.2\times0.4}{3}\times 106$

= 2.83 g

**(26)** **Answer :** 12.05

**Hint:**

$\Delta G = \Delta G^\circ + RT \ln Q$

**Solution:**

$\Delta G^\circ = -RT \ln P_{Total}$

$\tan\theta = -RT_1$

$1 = 8.3 \times T_1$

$T_1 = 0.1205$

At 24 K,

$K_{eq} = 6$ atm

So, $\Delta G^\circ = -RT_2 \ln 6$

$\Delta G^\circ = -8.3 \times 24 \times 1.8$

$\Delta G^\circ = -358.56$ J

$\frac{m}{12} = \frac{358.56}{12} = 29.88$

**(27)** **Answer :** 29.88
**Hint:**
$\Delta G = \Delta G° + RT \ln Q$
**Solution:**
$\Delta G° = -RT \ln P_{Total}$
$\tan\theta = -RT_1$
$1 = 8.3 \times T_1$
$T_1 = 0.1205$
At 24 K,
$K_{eq} = 6$ atm
So, $\Delta G° = -RT_2 \ln 6$
$\Delta G° = -8.3 \times 24 \times 1.8$
$\Delta G° = -358.56$ J
$\frac{m}{12} = \frac{358.56}{12} = 29.88$

**(28)** **Answer :** 60.00
**Hint:**
$\text{Molarity} = \frac{\text{Moles}}{\text{Volume } (\ln \text{ L})}$
**Solution:**
$\text{molality} = \frac{n_{CH_3COOH}}{1} \Rightarrow n_{CH_3COOH} = \frac{x}{60} = 1$
x = 60 g

| $CH_3COOH$ | $\rightleftharpoons$ | $CH_3COO^-$ | + | $H^+$ |
|---|---|---|---|---|
| $C(1-\alpha)$ | | $C\alpha$ | | $C\alpha$ |

$\Delta T_b = iK_b m$
$0.525 = (1 + \alpha)0.5 \times 1$
$\Rightarrow \frac{0.525}{0.5} = (1+\alpha)$
$\Rightarrow (1 + \alpha) = 1.05$
$\alpha = 0.05$
$[H^+] = C\alpha = 1 \times 0.05 = 5 \times 10^{-2}$
$pH = -\log(5 \times 10^{-2})$
$= 2 \times 0.7 = 1.3$

**(29)** **Answer :** 01.30
**Hint:**
$\text{Molarity} = \frac{\text{Moles}}{\text{Volume } (\ln \text{ L})}$
**Solution:**
$\text{molality} = \frac{n_{CH_3COOH}}{1} \Rightarrow n_{CH_3COOH} = \frac{x}{60} = 1$
x = 60 g

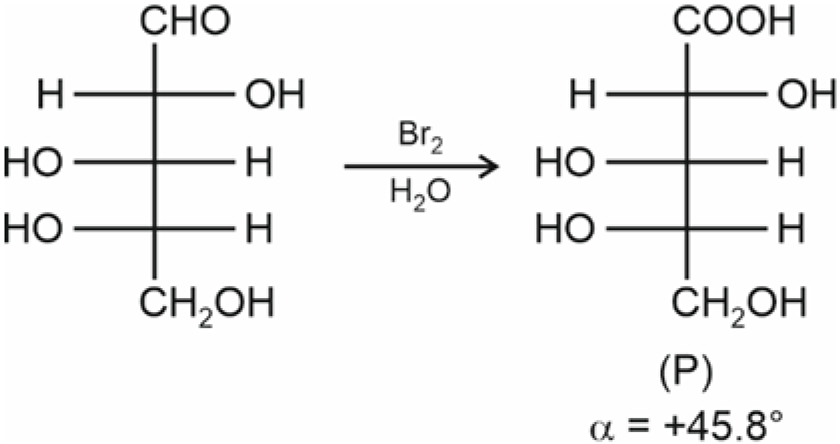
| $CH_3COOH$ | $\rightleftharpoons$ | $CH_3COO^-$ | + | $H^+$ |
|---|---|---|---|---|
| $C(1-\alpha)$ | | $C\alpha$ | | $C\alpha$ |

$\Delta T_b = iK_b m$
$0.525 = (1 + \alpha)0.5 \times 1$
$\Rightarrow \frac{0.525}{0.5} = (1+\alpha)$
$\Rightarrow (1 + \alpha) = 1.05$
$\alpha = 0.05$
$[H^+] = C\alpha = 1 \times 0.05 = 5 \times 10^{-2}$
$pH = -\log(5 \times 10^{-2})$
$= 2 \times 0.7 = 1.3$

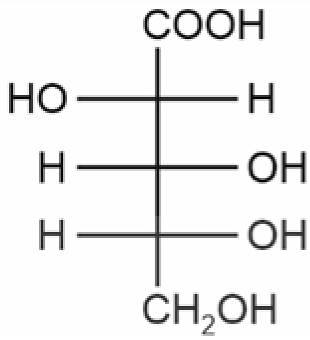
**Section-III**

**(30)** **Answer :** (A,C)
**Hint:**
$Br_2$, $H_2O$ oxidises glucose.

**Solution:**




$\alpha$ = –45.8° :

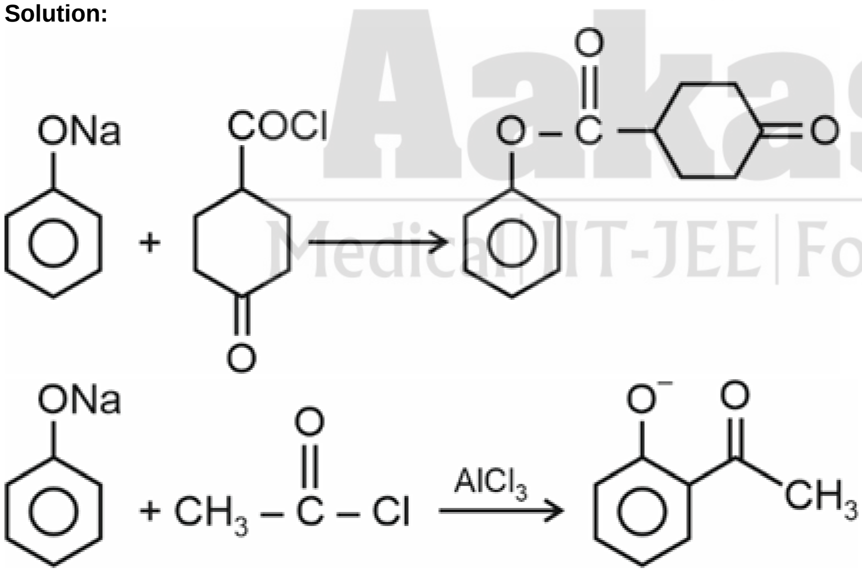



**(31)** **Answer :** (A,C)

**Hint:**

Ketone and aldehyde give 2,4-DNP Test.

**Solution:**




**(32)** **Answer :** (A,B,D)

**Hint:**

Diastase converts starch to maltose.

**Solution:**

Diastase converts starch to maltose.

**(33)** **Answer :** (B,D)

**Hint:**

For reversible adiabatic expansion:

q = 0, dV = +ve, dS = 0

**Solution:**

For reversible adiabatic expansion:

q = 0, dV = +ve, dS = 0

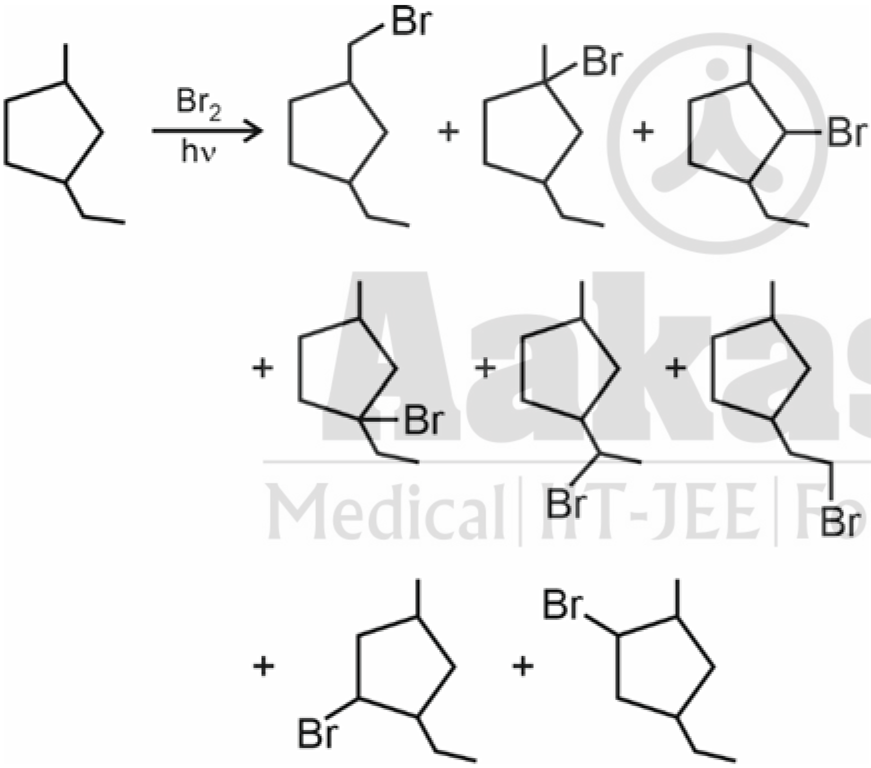
For reversible isothermal expansion
dT = 0, dV = +ve, dS = +ve, dH = 0

**(34)** **Answer :** (A,B,D)
**Hint:**
Wrought Iron is the purest form of commercial iron.
**Solution:**
Wrought Iron is the purest form of commercial iron.

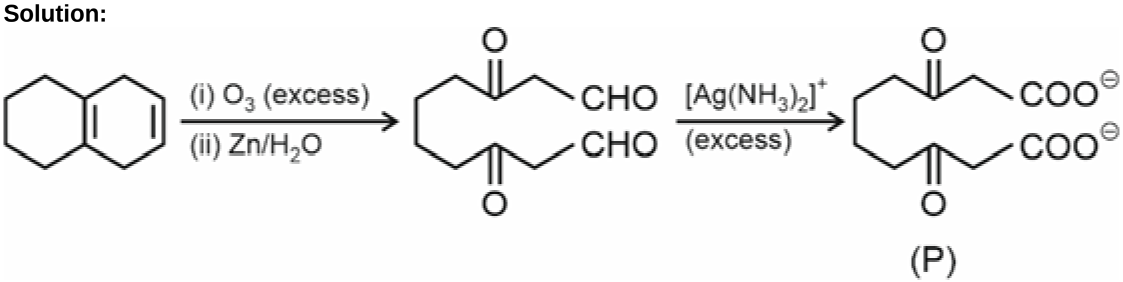
**(35)** **Answer :** (A,B)
**Hint:**
$CO_3^{2-}$ , $S^{2-}$, $SO_2$ and $CH_3COO^{\ominus}$ gives colourless gas with dil. $H_2SO_4$.
**Solution:**
$CO_3^{2-}$ , $S^{2-}$, $SO_2$ and $CH_3COO^{\ominus}$ gives colourless gas with dil. $H_2SO_4$.
$Pb^{2+}$, $Cu^{2+}$, $As^{3+}$ precipitates as sulphides on reaction with dil. HCl followed by $H_2S$.

**Section-IV**

**(36)** **Answer :** 8
**Hint:**
Bromination of alkane follows radical mechanism.
**Solution:**




**(37)** **Answer :** 4
**Hint:**
Tollen's reagent oxidises aldehyde and not ketones.
**Solution:**




Number of $sp^2$ hybridised C-atom = 4

**(38) Answer :** 14
**Hint:**
$[Ma_2b_2c_2]$ type complex has 6 stereoisomers.
**Solution:**
$[CoBr_2(py)_2(NH_3)_2]Cl$ : Number of stereoisomers = 6
$[CoBrCl(py)_2(NH_3)_2]Br$ : Number of stereoisomers = 8

**MATHEMATICS**

**Section-I**

**(39) Answer :** (C)
**Solution:**

$$S_n = \sum_{r=1}^{n} \tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right) = \sum_{r=1}^{n} \tan^{-1}\frac{(2-1)2^{r-1}}{1+2^{2r-1}}$$

$$= \sum_{r=1}^{n} \tan^{-1}\frac{\left(2^{r}-2^{r-1}\right)}{1+2^{r}\cdot 2^{r-1}}$$

$$= \sum_{r=1}^{n}\left(\tan^{-1}2^{r} - \tan^{-1}2^{r-1}\right)$$

$= (\tan^{-1}2 - \tan^{-1}1) + (\tan^{-1}2^2 - \tan^{-1}2) + \ldots + (\tan^{-1}2^n - \tan^{-1}2^{n-1})$

$= \tan^{-1}2^n - \tan^{-1}1 = \tan^{-1}2^n - \frac{\pi}{4}$

Now $S_\infty = \lim_{n\to\infty}\left(\tan^{-1}2^n - \frac{\pi}{4}\right) = \frac{\pi}{4}$

$\therefore S_n + S_\infty = \tan^{-1}2^n$

Hence $\tan(\tan^{-1}2^n) = 2^n$

**(40) Answer :** (B)
**Solution:**

$$f\left(x\right) = \begin{cases} 0 & \text{if} \quad 0 \le x < 1 \\ x & \text{if} \quad 1 \le x < 2 \\ 2(x-1) & \text{if} \quad 2 \le x < 3 \\ 6 & \text{if} \quad x = 3 \end{cases}$$

$f$ is discontinuous at $x = 1$ and $x = 3$ only
Also $f'(0) = 0$ and $f$ is not derivable at $x = 2$
So, $m = 2,\ n = 2$
$\therefore\ m + n = 4$

**(41) Answer :** (A)
**Solution:**
Equation of tangent $y = mx + \frac{2}{m}$ passes through $(\alpha,\ \beta)$

$\therefore\ \beta = m\alpha + \frac{2}{m}$

$\Rightarrow m^2\alpha - \beta \text{m} + 2 = 0$

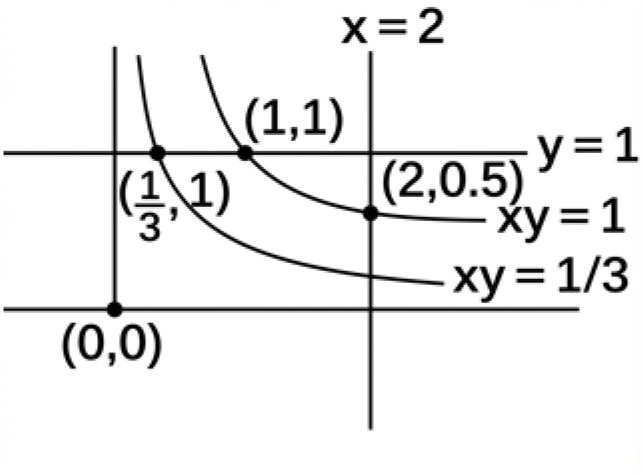
$\Rightarrow -m^2\alpha + \beta\text{m} - 2 = 0$

By comparing with roots of the equation
$\alpha = -3,\ \beta = 3$
Equation of *AB*
$3y = 4(x-3)$
$3y = 4x - 12$

Area of $\Delta PAB = \frac{(S)^{3/2}}{2a} = \frac{(9+24)^{3/2}}{4}$

$= \frac{(33)^{3/2}}{4}$

**(42) Answer :** (A)
**Solution:**




Required area $= \int_{1/3}^{1} \left(1 - \frac{1}{3x}\right) dx + \int_{1}^{2} \left(\frac{1}{x} - \frac{1}{3x}\right) dx$

$= \frac{2}{3} - \frac{1}{3}\ln 3 + \frac{2}{3}\ln 2$

**Section-II**

**(43) Answer :** 60.00

**Solution:**

$I = \int \cot^3 x \ \text{cosec}^3 x \, dx$

$= \int \frac{\cos^3 x}{\sin^6 x} dx$

Let $u = \sin x$

$du = \cos x \, dx$

$I = \int \frac{\cos^2 x}{u^6 x} du = \int \frac{1-u^2}{u^6} du$

$= \frac{-1}{5u^5} + \frac{1}{3u^3} + c$

$= \frac{-1}{5\sin^5 x} + \frac{1}{3\sin^3 x} + c$

$= \frac{-\cos^5 x}{5\sin^5 x \ \cos^5 x} + \frac{\cos^3 x}{3\sin^3 x \ \cos^3 x} + c$

$= \frac{-\sec^5 x}{5\tan^5 x} + \frac{\sec^3 x}{3\tan^3 x} + c$

$= \frac{\sec^3 x}{3\tan^3 x} - \frac{\sec^3 x \ (1+\tan^2 x)}{5\tan^5 x} + c$

$= \frac{5\sec^3 x \ \tan^2 x - 3\sec^3 x - 3\sec^3 x \ \tan^2 x}{15\tan^5 x} + c$

$= \frac{\sec^3 x \ (2\tan^2 x - 3)}{15\tan^5 x} + c$

Now, $\frac{4A\sec^3 x(4B\tan^2 x - 6)}{\tan^5 x} = \frac{\sec^3 x(2\tan^2 x - 3)}{15\tan^5 x}$

$\Rightarrow 4A\left(4B\tan^2 x - 6\right) = \frac{2\tan^2 x - 3}{15}$

$\Rightarrow 16AB\tan^2 x - 24A = \frac{12}{15}\tan^2 x - \frac{3}{15}$

$\Rightarrow 16AB = \frac{2}{15}$ and $-24A = -\frac{3}{15}$

$\Rightarrow A = \frac{1}{120}$ and $B = 1$

$\therefore \frac{B}{2A} = \frac{1}{2\times\frac{1}{120}} = 60$

$\left[A+B\right] = \left[\frac{1}{120} + 1\right] = 1$

**(44) Answer :** 01.00

**Solution:**

$I = \int \cot^3 x \ \text{cosec}^3 x \, dx$

$= \int \frac{\cos^3 x}{\sin^6 x} dx$

Let $u = \sin x$

$du = \cos x\, dx$

$I = \int \frac{\cos^2 x}{u^6 x} du = \int \frac{1-u^2}{u^6} du$

$= \frac{-1}{5u^5} + \frac{1}{3u^3} + c$

$= \frac{-1}{5\sin^5 x} + \frac{1}{3\sin^3 x} + c$

$= \frac{-\cos^5 x}{5\sin^5 x\ \cos^5 x} + \frac{\cos^3 x}{3\sin^3 x\ \cos^3 x} + c$

$= \frac{-\sec^5 x}{5\tan^5 x} + \frac{\sec^3 x}{3\tan^3 x} + c$

$= \frac{\sec^3 x}{3\tan^3 x} - \frac{\sec^3 x\ (1+\tan^2 x)}{5\tan^5 x} + c$

$= \frac{5\sec^3 x\ \tan^2 x - 3\sec^3 x - 3\sec^3 x\ \tan^2 x}{15\tan^5 x} + c$

$= \frac{\sec^3 x\ (2\tan^2 x - 3)}{15\tan^5 x} + c$

Now, $\frac{4A\sec^3 x(4B\tan^2 x - 6)}{\tan^5 x} = \frac{\sec^3 x(2\tan^2 x - 3)}{15\tan^5 x}$

$\Rightarrow 4A\left(4B\tan^2 x - 6\right) = \frac{2\tan^2 x - 3}{15}$

$\Rightarrow 16AB\tan^2 x - 24A = \frac{12}{15}\tan^2 x - \frac{3}{15}$

$\Rightarrow 16AB = \frac{2}{15}$ and $-24A = -\frac{3}{15}$

$\Rightarrow A = \frac{1}{120}$ and $B$ = 1

$\therefore \frac{B}{2A} = \frac{1}{2\times\frac{1}{120}} = 60$

$\left[A+B\right] = \left[\frac{1}{120} + 1\right] = 1$

**(45) Answer :** 03.00

**Solution:**

$x+y+z = 15$

$x+y+z = \frac{3}{2}\left(a+b\right)$

$\Rightarrow a+b = 10$

Now, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{3}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

$\Rightarrow ab = 9$

$\Rightarrow a(10-a) = 9 \Rightarrow (a-1)(a-9) = 0$

$\Rightarrow a = 1$ or $9$

If $a = 1,\ b = 9$ and $a = 9,\ b = 1$

Geometric mean of *a* and *b* $= \sqrt{ab} = \sqrt{9} = 3$

Greatest value of *a* – *b* = 9 – 1 = 8

**(46) Answer :** 08.00

**Solution:**

$x+y+z = 15$

$x+y+z = \frac{3}{2}\left(a+b\right)$

$\Rightarrow a+b = 10$

Now, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{3}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

$\Rightarrow ab = 9$

$\Rightarrow a(10-a) = 9 \Rightarrow (a-1)(a-9) = 0$

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If $a = 1,\ b = 9$ and $a = 9,\ b = 1$

Geometric mean of *a* and *b* $= \sqrt{ab} = \sqrt{9} = 3$

Greatest value of *a* – *b* = 9 – 1 = 8

**(47) Answer :** 01.00

**Solution:**

For $x < 0$

$$f(x)=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{e^x}{r(r+1)}$$

{∵ $r^2 + r - 1$ is an integer and $\{x + m\} = \{x\}$, $m \in I$}

$$=e^x\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

$$=e^x\lim_{n\to\infty}\left(\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right)$$

$= e^x$

For $x > 0$

$f\left(x\right)=\lim_{n\to\infty}\left\{\frac{px}{n}\sum_{r=1}^{n}\left(\frac{r(r+1)-1}{r(r+1)}\right)+\lambda\right\}$ {∵ $x > 0$, $e^{-x} \in (0, 1)$, $[e^{-x}] = 0$}

$=px\lim_{n\to\infty}\left(\frac{1}{n}\sum_{r=1}^{n}\left(1-\frac{1}{r(r+1)}\right)\right)+\lambda$

$=px\lim_{n\to\infty}\left(\frac{1}{n}\sum_{r=1}^{n}\left(1-\left(\frac{1}{r}-\frac{1}{r+1}\right)\right)\right)+\lambda$

$=px\lim_{n\to\infty}\left(\frac{1}{n}\left(n-\left(1-\frac{1}{n+1}\right)\right)\right)+\lambda$

$=px+\lambda$

$$\therefore\ f\left(x\right)=\begin{cases}e^x & , & x<0\\ q & , & x=0\\ px+\lambda & , & x>0\end{cases}$$

∴ $f(x)$ is differentiable in $R$.

∴ It is continuous at $x=0$

∴ $1=q=\lambda$

Derivable at $x=0$

∴ $p = 1$

∴ $p + q + \lambda = 3$

**(48) Answer :** 03.00

**Solution:**

For $x < 0$

$$f(x)=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{e^x}{r(r+1)}$$

{∵ $r^2 + r - 1$ is an integer and $\{x + m\} = \{x\}$, $m \in I$}

$$=e^x\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

$$=e^x\lim_{n\to\infty}\left(\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right)$$

$= e^x$

For $x > 0$

$f\left(x\right)=\lim_{n\to\infty}\left\{\frac{px}{n}\sum_{r=1}^{n}\left(\frac{r(r+1)-1}{r(r+1)}\right)+\lambda\right\}$ {∵ $x > 0$, $e^{-x} \in (0, 1)$, $[e^{-x}] = 0$}

$=px\lim_{n\to\infty}\left(\frac{1}{n}\sum_{r=1}^{n}\left(1-\frac{1}{r(r+1)}\right)\right)+\lambda$

$=px\lim_{n\to\infty}\left(\frac{1}{n}\sum_{r=1}^{n}\left(1-\left(\frac{1}{r}-\frac{1}{r+1}\right)\right)\right)+\lambda$

$=px\lim_{n\to\infty}\left(\frac{1}{n}\left(n-\left(1-\frac{1}{n+1}\right)\right)\right)+\lambda=px+\lambda$

$\therefore\ f\left(x\right)=\begin{cases} e^x & , & x<0 \\ q & , & x=0 \\ px+\lambda & , & x>0 \end{cases}$

$\therefore$ $f(x)$ is differentiable in $R$.
$\therefore$ It is continuous at $x=0$
$\therefore$ $1=q=\lambda$
Derivable at $x=0$
$\therefore p = 1$
$\therefore p + q + \lambda = 3$

**Section-III**

**(49) Answer :** (A,C,D)
**Solution:**

$f\left(x\right)=y=\frac{32}{4+x^2+x^4}$
$y\left(4+x^2+x^4\right)=32$
$x^4y+x^2y+4y-32=0$
$D\geq 0$
$y^2-4y\left(4y-32\right)\geq 0$
$0<y\leq\frac{128}{15}$
Number of integers = 8
$h(f(x)) > 0$
$-(f(x))^2 - 3f(x) + K > 0$
$\therefore 0 < f(x) \leq 8$, the minimum value of $-(f(x))^2 - 3f(x)$ occurs at $f(x) = 8$
Minimum value = $-(8)^2 - 3(8) = -88$
For $h(f(x)) > 0$
$K > 88$
Now, $h(g(x)) < 0$
$-(g(x))^2 - 3(g(x)) + K < 0$
$g(x) = 9 + x^2 \geq 9$
The minimum value of $g(x)$ is 9.
$h(g(x)) = -(9 + x^2)^2 - 3(9 + x^2) + K < 0$
Minimum value occurs at $x = 0$
$\therefore$ For $h(g(x)) < 0$
$K < 108$
$\therefore 88 < K < 108$
$\therefore$ Number integral value of $K$ is 19
Now, $g(f(x)) = 9 + (f(x))^2$
Maximum value of $f(x)$ is 8
$\therefore$ Maximum value of $g(f(x)) = 9 + 8^2 = 73$

**(50) Answer :** (A,B,C)
**Solution:**

$$\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$$

$$\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}=\mu$$

$\lambda+3=3\mu+1 \quad \ldots(\text{i})$
$2\lambda+1=\mu+2 \quad \ldots(\text{ii})$
$3\lambda+2=2\mu+3 \quad \ldots(\text{iii})$
Solving any two equations, we get
$\lambda=1$, $\mu=1$
$\therefore$ Point of intersection is (4, 3, 5)
Normal vector is parallel to the line connecting the origin (0, 0, 0) and (4, 3, 5)
$\vec{n}=\left(4,\ 3,\ 5\right)$
$\therefore$ Equation of plane is $4x+3y+5z=d$ is passing through (4, 3, 5)
$\Rightarrow 4(4) + 3(3) + 5(5) = d$
$\Rightarrow d = 50$

The equation of plane is $4x + 3y + 5z - 50 = 0$
(a) When $a = 4$, $d = 50$
The positive divisors of 50 are 1, 2, 5, 10, 25, 50.
Total 6 divisors

(b) $\dfrac{\left[5+\frac{1}{b}\right]+\left[5+\frac{2}{b}\right]}{\sum_{r=0}^{4}\left[b+\frac{r}{5}\right]}$

$= \dfrac{\left[5+\frac{1}{3}\right]+\left[5+\frac{2}{3}\right]}{\left[3+\frac{0}{5}\right]+\left[3+\frac{1}{5}\right]+\left[3+\frac{2}{5}\right]+\left[3+\frac{3}{5}\right]+\left[3+\frac{4}{5}\right]}$

$= \dfrac{5+5}{3+3+3+3+3}$

$= \dfrac{10}{15} = \dfrac{2}{3}$

(c) $(-2, 1, 3)$
$4(-2) + 3(1) + 5(3) - 50 = -40 < 0$
$(0, 1, 2)$
$4(0) + 3(1) + 5(2) - 50 = -37 < 0$
Both points are on the same side of the plane
(d) $a + b + c = 4 + 3 + 5 = 12$
$d = 50$
50 is not divisible by 12.

**(51)** **Answer :** (A,C)
**Solution:**

$8\sin x = \frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x}$

$\Rightarrow 8\sin^2 x\cos x = \sqrt{3}\sin x + \cos x$

$\Rightarrow 4\left(1-\cos 2x\right)\cos x = \sqrt{3}\sin x + \cos x$

$\Rightarrow \cos x - 2\cos 3x = \sqrt{3}\sin x$

$\Rightarrow \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \cos 3x$

$\cos\left(x+\frac{\pi}{3}\right) = \cos 3x$

$3x = 2nx \pm \left(x+\frac{\pi}{3}\right)$

$\therefore \quad 3x = 2nx + x + \frac{\pi}{3}$

$x = n\pi + \dfrac{\pi}{6}$

$[0, 2\pi]$

$\dfrac{\pi}{6}, \dfrac{5\pi}{6}$

or $3x = 2nx - x - \frac{\pi}{3}$

$x = \dfrac{n\pi}{2} - \dfrac{\pi}{12}$

$\dfrac{\pi}{2} - \dfrac{\pi}{12} = \dfrac{5\pi}{12}$

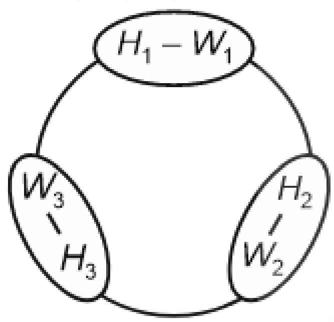
$\pi - \dfrac{\pi}{12} = \dfrac{11\pi}{12}$

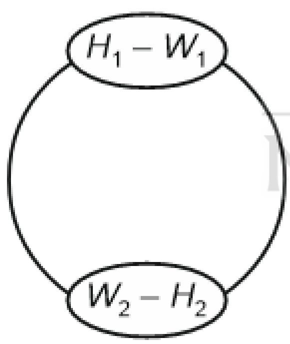
$\frac{3\pi}{2} - \frac{\pi}{12} = \frac{17\pi}{12}$

$2\pi - \frac{\pi}{12} = \frac{23\pi}{12}$

**(52)** **Answer :** (A,C,D)
**Solution:**
As the matrix is skew-symmetric, hence $|A| = 0$ if *n* is odd
⇒*A* is not invertible matrix when *n* is odd

**(53)** **Answer :** (B,C)
**Solution:**
Total number of people $N = 3 \times 2 = 6$
Total arrangement $= (6-1)! = 5! = 120$
Total arrangement $= (6-1)! = 5! = 120$
Number of ways where all couples sit together $= \left(3-1\right)! \times (2!)^3$
$= 2 \times 2^3 = 16$
*x* = Total arrangement – Number of ways all 3 couples sit together
$x = 120 - 16 = 104$
Let $H_1,\ W_1,\ H_2,\ W_2$ sit in $2 \times 2 = 4$ ways




Now, $H_3$ and $W_3$ can sit in 2 ways.
$\therefore\ y = 4 \times 2 = 8$

**(54)** **Answer :** (A,B)
**Solution:**
$S \geq 0$ and $S = 0$ for $Z_1 = Z_2 = Z_3$
Thus $\alpha = 0$
$\because |a-b|^2 = |a|^2 + |b|^2 - \bar{a}b - a\bar{b}$
$\because |Z_2 - Z_3|^2 + |Z_3 - Z_1|^2 + |Z_1 - Z_2|^2$
$= 6 - \overline{Z_1}\left(Z_2 + Z_3\right) - \overline{Z_2}\left(Z_3 + Z_1\right) - \overline{Z_3}\left(Z_1 + Z_2\right)$ …(i)
Also, $0 \leq |Z_1 + Z_2 + Z_3|^2$
$= |Z_1|^2 + |Z_2|^2 + |Z_3|^2 + \overline{Z_1}\left(Z_2 + Z_3\right) + \overline{Z_2}\left(Z_3 + Z_1\right) + \overline{Z_3}\left(Z_1 + Z_3\right)$
$\Rightarrow Z_1\left(Z_2 + Z_3\right) + \overline{Z_2}\left(Z_3 - Z_1\right) + \overline{Z_3}\left(Z_1 + Z_2\right) \geq -3$ …(ii)
From (i) and (ii)

$|Z_2 - Z_3|^2 + |Z_3 - Z_1|^2 + |Z_1 - Z_2|^2 \le 9$
$\beta = 9$

**Section-IV**

**(55)** **Answer :** 45
**Solution:**
Equation of chord of contact of a point $P(3\sec\theta,\ 2\tan\theta)$ on the hyperbola w.r.t the circle is $3\sec\theta x + 2\tan\theta y = 9$ …(i)
Let $M(h,\ k)$ be the mid-point of (i), then equation of (i) in terms of the mid-point is $hx + ky = h^2 + k^2$ … (ii)
Since (i) and (ii) represents same line
$\sec\theta = \frac{3h}{h^2+k^2},\ \tan\theta = \frac{9k}{2(h^2+k^2)}$
∴ Locus of (*h*, *k*) is
$\frac{9x^2}{(x^2+y^2)^2} - \frac{81y^2}{4(x^2+y^2)^2} = 1$
$4(x^2+y^2)^2 = 36x^2 - 81y^2$
*b* = 36, *c* = 81, *c* – *b* = 45

**(56)** **Answer :** 7
**Solution:**
**Case I:** 2 bowler + 2 batsman
Number of ways = ${}^4C_2 \times {}^4C_2 = 36$
**Case II:** 2 bowler + 1 batsman + 1 all-rounder
Number of ways = ${}^4C_2 \times {}^4C_1 \times {}^1C_1 = 24$
**Case III:** 1 bowler + 2 batsman + 1 all-rounder
Number of ways = ${}^4C_1 \times {}^4C_2 \times {}^1C_1 = 24$
Probability $= \frac{24+24}{24+24+36} = \frac{4}{7}$
$\therefore\ p = 7$

**(57)** **Answer :** 3
**Solution:**
$y\left(2x^4 + y\right)\frac{dy}{dx} = \left(1 - 4xy^2\right)x^2$
$\Rightarrow 2x^4y\ dy + y^2\ dy + 4x^3y^2\ dx - x^2\ dx = 0$
$\Rightarrow 2x^2y\left(x^2\ dy + 2xy\ dx\right) + y^2\ dy - x^2\ dx = 0$
$\Rightarrow 2x^2y\ d\left(x^2y\right) + y^2\ dy - x^2\ dx = 0$
Integrating, we get,
$\left(x^2y\right)^2 + \frac{y^3}{3} - \frac{x^3}{3} = C$
$3(x^2y)^2 + y^3 - x^3 = C$
$\therefore\ k = 3$