08/02/2026

Set-2_ONLINE

Aakash

Medical | IIT-JEE | Foundations

Corporate Office : AESL, 3rd Floor, Incuspaze Campus-2, Plot No. 13, Sector-18, Udyog Vihar, Gurugram, Haryana - 122015, ***Ph.***+91-1244168300

MM : 180

AIATS For One Year JEE(Advanced)-2026 (XII Studying)_Test-4A_Paper-2_ONLINE

Time : 180 Min.

PHYSICS

Section-I

1. (B,D)

2. (A,D)

3. (A,C,D)

4. (B,D)

5. (A,D)

6. (A,C,D)

Section-II

7. (36.00)

8. (48.00)

9. (16.00)

10. (56.56)

11. (01.00)

12. (01.00)

Section-III

13. (D)

14. (C)

15. (B)

16. (D)

Section-IV

17. (8)

18. (9)

19. (10)

CHEMISTRY

Section-I

20. (A,B,C)

21. (B,C,D)

22. (A,B,C)

23. (B,C,D)

24. (A,D)

25. (A,C)

Section-II

26. (03.00)

27. (04.00)

28. (04.90)

29. (02.00)

30. (10.00)

31. (04.56)

Section-III

32. (B)

33. (D)

34. (B)

35. (B)

Section-IV

36. (8)

37. (6)

38. (7)

MATHEMATICS

Section-I

39. (A,B,C)

40. (A,C)

41. (A,B,C,D)

42. (A,C)

43. (A,B,C)

44. (A,B)

Section-II

45. (04.00)

46. (12.00)

47. (04.00)

48. (04.00)

49. (01.00)

50. (00.00)

Section-III

51. (B)

52. (B)

53. (C)

54. (A)

Section-IV

55. (15)

56. (22)

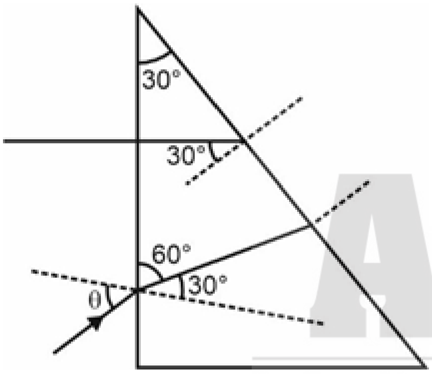
57. (12)

Hints and Solutions

PHYSICS

Section-I

(1) **Answer :** (B,D)
Hint:
Velocity inside the shell is constant.
Solution:
Velocity inside the shell is constant.
At surface,
$-\frac{GMm}{2R} = -\frac{GMm}{R} + \frac{1}{2}mv^2$
$\frac{1}{2}mv^2 = \frac{GMm}{2R}$
$v = \sqrt{\frac{GM}{R}}$
$\therefore\ t = \frac{2R}{\sqrt{\frac{GM}{R}}} = \frac{2R^{3/2}}{\sqrt{GM}}$

(2) **Answer :** (A,D)
Hint:
μsin30° = 1 sin90°
Solution:


μsin30° = 1 sin90°
μ = 2
1 sinθ = μsin30°
sinθ = 1
θ = 90°

(3) **Answer :** (A,C,D)
Hint:
$\frac{\Sigma x_i}{n} = < x >$
Solution:
Mean value
$= \frac{12.5+12.3+11.8+12.4+12.2+12.6}{6}$
= 12.3
Mean absolute error = $\frac{|\Delta x_1| + ... + |\Delta x_6|}{6}$
Relative error = $\frac{\text{Mean absolute error}}{\text{Mean value}}$

(4) **Answer :** (B,D)
Hint:
$\Delta v = Bvl$
Solution:
Due to rotation, no emf is induced between the ends.
$\therefore v_{AB} = Bvl$

(5) **Answer :** (A,D)
Hint:
$v = \sqrt{2gh}$
Solution:

$v = \sqrt{2g_{eff}h} = \sqrt{4gh} = 2\sqrt{gh}$

$t_f = \sqrt{\frac{2H}{2g}} = \sqrt{\frac{H}{g}}$

$R = vt_f = 2\sqrt{gh}\sqrt{\frac{H}{g}} = 2\sqrt{Hh}$

When it is coming down with acceleration g

$g_{\text{eff}} = 0$

$\therefore v = 0$

(6) **Answer :** (A,C,D)

Hint:

Use KVL

Solution:

Time taken for capacitor to charge,

$\frac{V_0}{2} = V_0(1-e)^{-t/RC}$

$t = RC\ln 2$

Time taken for discharging,

$\frac{V_0}{2} = Kt$

$t = \frac{V_0}{2K}$

$\therefore$ Time period = $RC\ln 2 + \frac{V_0}{2K}$

Section-II

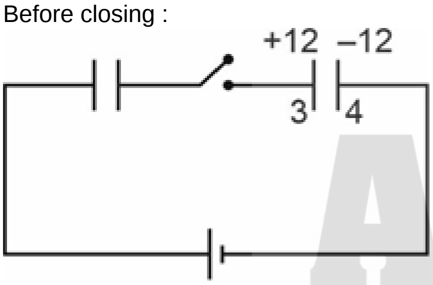
(7) **Answer :** 36.00

Hint:

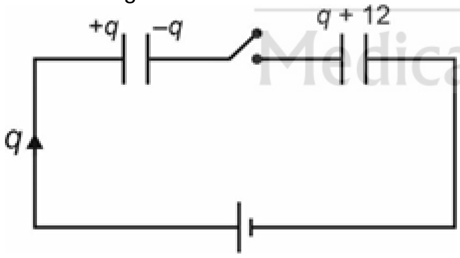
$q = CV$

Solution:

Before closing :



After closing :



$20 - \frac{q}{3} - \frac{q+12}{6} = 0$

$q = 36\ \mu C$

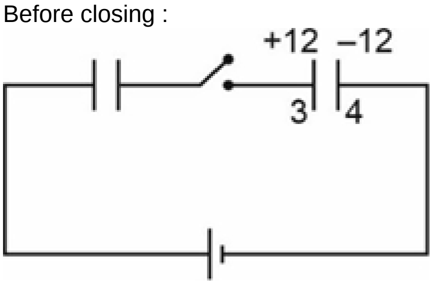
(8) **Answer :** 48.00

Hint:

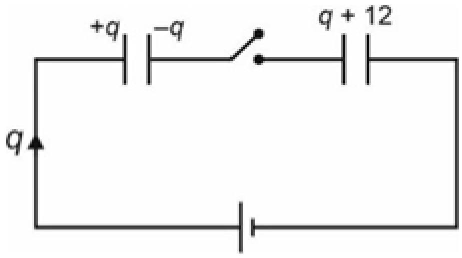
$q = CV$

Solution:

Before closing :



After closing :



$20-\frac{q}{3}-\frac{q+12}{6}=0$

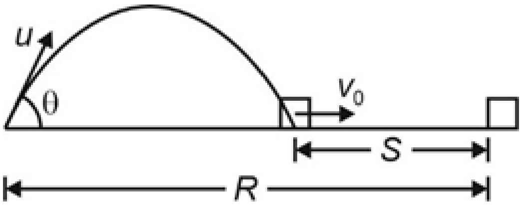
q = 36 μC

(9) **Answer :** 16.00

Hint:

$S_X = \mu_X t$

Solution:



$\int N dt = mu\sin\theta$

$v_0 = \mu\cos\theta - \sqrt{3}\mu\sin\theta$

⇒ θ ≤ 30°

$S_1 = \frac{u^2}{2a} = \frac{40\times40}{2\times\sqrt{3}\times10}$

$S_2 = \frac{u^2}{g}$

$S_2 > S_1$

(10) **Answer :** 56.56

Hint:

$\int F dt = \Delta P$

Solution:

$\int f dt = \sqrt{3}mu\sin\theta$

$f = mu\cos\theta = 56.56$

(11) **Answer :** 01.00

Hint:

$qvB = \frac{mv^2}{r}$

Solution:

$evB_0 = \frac{mv^2}{r}$

$\frac{v}{r} = \frac{eB_0}{m}$

(12) **Answer :** 01.00

Hint:

$mvr = \frac{nh}{2\pi},\ r = \frac{vm}{Bq}$

Solution:

$mvr = \frac{nh}{2\pi},\ r = \frac{vm}{Bq}$

$(mv)\left(\frac{mv}{Bq}\right) = \frac{nh}{2\pi}$

$\frac{1}{2}mv^2 = \frac{nhBq}{4\pi m}$

Potential energy :

$U = -\overrightarrow{M}\cdot\overrightarrow{B}$

$M = IA = \frac{q\omega}{2\pi}\times\pi r^2 = \frac{q\omega r^2}{2}$

$U = -MB\cos 18°$

= $MB = \frac{qvrB}{2}$

= $\frac{B}{2}\times\frac{nh}{2\pi m} = \frac{nhB}{4\pi m}$

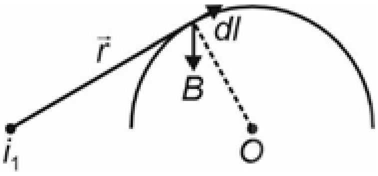
Total energy = $K + U = \frac{nhB}{2\pi m}$

$T = \frac{2\pi m}{Bq}$

Section-III

(13) **Answer :** (D)
Hint:
$\vec{B} = \vec{B}_1 + \vec{B}_2$
Solution:
$\vec{B} = \vec{B}_1 + \vec{B}_2$
$\vec{B} = \frac{\mu_0 i_1}{2\pi\left(R+\frac{2R}{3}\right)}(-\hat{j}) + \frac{\mu_0 i_2}{4R}(-\hat{k})$
$B = \frac{\mu_0}{2R}\sqrt{\frac{9i_1^2}{25\pi^2} + \frac{i_2^2}{4}}$

(14) **Answer :** (C)
Hint:
$B = \frac{\mu_0 i}{2r}$
Solution:
Force :



$B = \frac{\mu_0 i_1}{2\pi r}$
$dF = i_2 \vec{dl} \times \vec{B} = i_2(dl_1)B$
$dl_1 = dr$
$-\int_0^F dF = \int_{2R/3}^{8R/3} i_2 B dr$
$F = \frac{\mu_0 i_1 i_2}{2\pi}\ln 4 = \frac{\mu_0 i_1 i_2}{\pi}\ln 2$

(15) **Answer :** (B)
Hint:
PV^γ = constant
Solution:
PV^γ = constant
$TV^{\gamma-1}$ = constant
$T^{\gamma/\gamma-1}V^\gamma$ = constant
$\frac{P}{T^{\gamma/\gamma-1}}$ = constant
$\frac{P}{T^{5/2}}$ = constant
$\frac{P_0}{T_0^{5/2}} = \frac{32P_0}{T^{5/2}}$
$T^{5/2} = T_0^{5/2}2^5$
$T^{1/2} = T_0^{1/2}2$
$T = 4T_0$
(i) $W_{gas} = \frac{\mu R\Delta T}{1-\alpha} = \frac{\mu R3T_0}{1-\frac{5}{3}} = -\frac{9}{2}\mu RT_0$
$W = \frac{9}{2}\mu RT_0$
(ii) $32\mu C_V T_0 + \mu C_V 4T_0 = 33\mu C_V T$
$\frac{36}{33}T_0 = T \Rightarrow \frac{12}{11}T_0 = T$

(16) **Answer :** (D)
Hint:
PV^γ = constant
Solution:
PV^γ = constant

$TV^{\gamma-1}$ = constant

$T^{\gamma/\gamma-1}V^{\gamma}$ = constant

$\frac{P}{T^{\gamma/\gamma-1}}$ = constant

$\frac{P}{T^{5/2}}$ = constant

$\frac{P_0}{T_0^{5/2}} = \frac{32P_0}{T^{5/2}}$

$T^{5/2} = T_0^{5/2}2^5$

$T^{1/2} = T_0^{1/2}2$

$T = 4T_0$

(i) $W_{gas} = \frac{\mu R\Delta T}{1-\alpha} = \frac{\mu R3T_0}{1-\frac{5}{3}} = -\frac{9}{2}\mu RT_0$

$W = \frac{9}{2}\mu RT_0$

(ii) $32\mu C_V T_0 + \mu C_V 4T_0 = 33\mu C_V T$

$\frac{36}{33}T_0 = T \Rightarrow \frac{12}{11}T_0 = T$

Section-IV

(17) Answer : 8

Hint:

Force similar to gravitation ∝ $\frac{1}{x^2}$

Solution:

Force similar to gravitation ∝ $\frac{1}{x^2}$

$\therefore V_{escape} = \sqrt{2}V_{\text{orbit}}$

(18) Answer : 9

Hint:

Path difference in water = Path difference in air

Solution:

Frequency of oscillation of source is equal to the frequency of oscillation of the central bright.

Path difference in water = Path difference in air

$\frac{(A_w)d}{D} \times \mu_w = \frac{Ad}{\left(\frac{D}{3}\right)}$

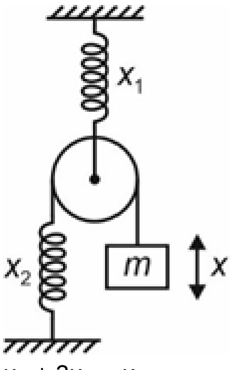
$A_w = \frac{3A}{\mu_w} = \frac{3A}{4} \times 3 = \frac{9A}{4}$

(19) Answer : 10

Hint:

$x_2 + 2x_1 = x$

Solution:



$x_2 + 2x_1 = x$

$kx_1 = 2kx_2$

$x_1 = 2x_2$

$x_2 + 4x_2 = x$

$x_2 = \frac{x}{5},\ x_1 = \frac{2x}{5}$

$F = kx_2 = \frac{kx}{5}$

$\therefore\ T = 2\pi\sqrt{\frac{5m}{k}}$

Finding amplitude,

$mg(A) = \frac{1}{2}k\left(\frac{A}{5}\right)^2 + \frac{1}{2}k\left(\frac{2A}{5}\right)^2$

$$mg = k\left(\frac{A}{50} + \frac{4A}{50}\right)$$

$$mg = \frac{kA}{10} \quad \Rightarrow \quad A = \left(\frac{10mg}{k}\right)$$

$$v_{\max} = Aw = \sqrt{\frac{k}{5m}} \times \frac{10mg}{k} = g\sqrt{\frac{20m}{k}}$$

CHEMISTRY

Section-I

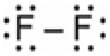
(20) **Answer :** (A,B,C)

Hint:

F_2 due to weak F–F bond and more hydration enthalpy of F^-, it has more oxidising power.

Solution:

Cl > F > Br > I (Magnitude of electron gain enthalpy)



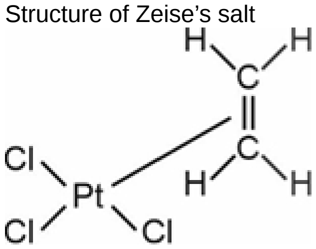
is weak due to lp–lp repulsion.

(21) **Answer :** (B,C,D)

Hint:

Charge on Pt = +2

Structure of Zeise's salt



Solution:

As π-bond of ethene molecule is in plane of platinum and 3Cl atoms, hence ethene molecule's plane will be perpendicular to plane containing Pt and 3Cl atoms.

(22) **Answer :** (A,B,C)

Hint:

With $SOCl_2$, PCl_3 is obtained.

Solution:

(A) $2\,NaN_3 \xrightarrow{\Delta} 2\,Na + \underset{(\text{Very pure})}{3N_2}$

(C) $Cu + \underset{(\text{Conc.})}{4\,HNO_3} \rightarrow Cu\,(NO_3)_2 + \underset{(\text{Brown})}{2\,NO_2} + 2H_2O$

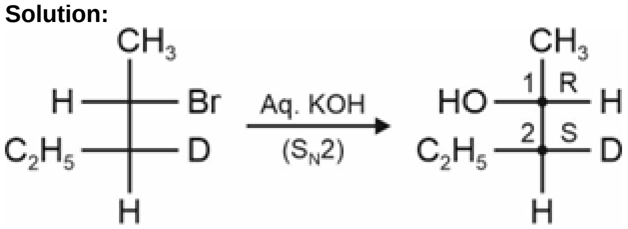
(D) $P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 4SO_2 + 2S_2Cl_2$

(23) **Answer :** (B,C,D)

Hint:

During S_N2 reaction, inversion in configuration takes place.

Solution:



Options (C) and (D) have same configuration at carbon-1 and carbon-2 as that of option (B).

(24) **Answer :** (A,D)

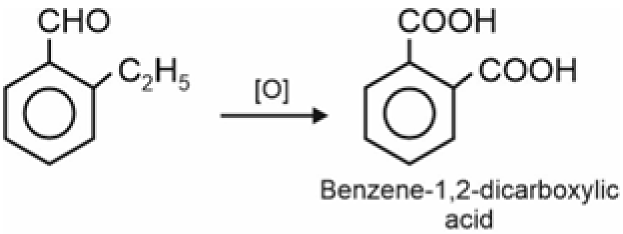
Hint:

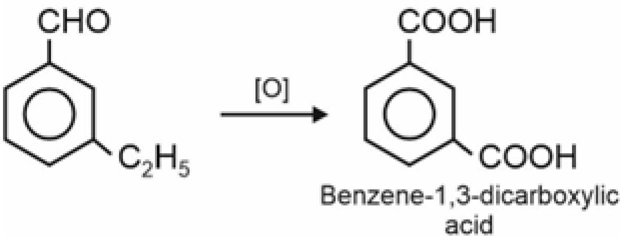
Aldehyde and ketones can give 2,4 DNP test.

Solution:

Compound "X" must contain aldehyde group without any α-hydrogen.

Possible structures :





(25) **Answer :** (A,C)

Hint:

$\Delta S = nC_p \ln \frac{T_2}{T_1} - nR\ln \frac{P_2}{P_1}$

Solution:

(A) A → B is isobaric process.

$\Delta S_{A\to B} = nC_p \ln \frac{T_2}{T_1} - nR\ln \frac{P_2}{P_1}$

$= (1)\left(\frac{5R}{2}\right)\ln 2$

$\Delta S_{A\to B} = \frac{5R}{2}\ln 2$

(B) B → C is isochoric process.

$\Delta S_{B\to C} = nC_v \ln \frac{T_2}{T_1} + nR\ln \frac{V_2}{V_1}$

$= \left(1\right)\left(\frac{3R}{2}\right)\ln \frac{1}{2}$

$\Delta S_{B\to C} = \frac{-3R}{2}\ln 2$

(C) $W_{C\to A} = \frac{1}{2}\left(0.5 \times 20\right) + \left(20\right) \times \left(0.5\right)$

⇒ 5 + 10 = 15 atm. L

(D) For process A → B → C → A

$\Delta S_{system} = 0$

As entropy of system is state function.

Section-II

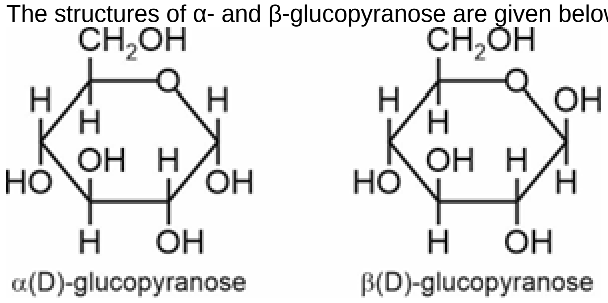
(26) **Answer :** 03.00

Hint:

Anomers have different orientation of –OH group in C_1 carbon.

Solution:

The structures of α- and β-glucopyranose are given below.



Number of trans OH groups in α-anomer = 3

Number of trans OH groups in β-anomer = 4

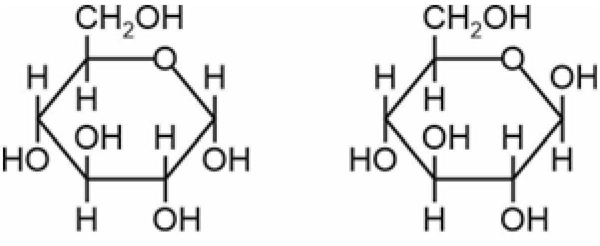
(27) **Answer :** 04.00

Hint:

Anomers have different orientation of –OH group in C_1 carbon.

Solution:

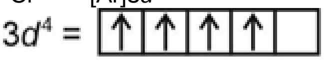
The structures of α- and β-glucopyranose are given below.



α(D)-glucopyranose β(D)-glucopyranose

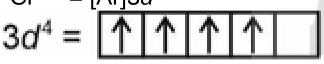
Number of trans OH groups in α-anomer = 3
Number of trans OH groups in β-anomer = 4

(28) **Answer :** 04.90
Hint:
$\mu_{spin\ \ only} = \sqrt{n\left(n+2\right)}\ \text{BM}$, n is number of unpaired electron.
Solution:
(1) $\mu_{spin\ \ only} = \sqrt{n\left(n+2\right)}\ \text{BM}$
Cr^{2+} = [Ar]$3d^4$



n = 4
$\mu = \sqrt{4(4+2)}\ \text{BM}$
= $\sqrt{24}\ \text{BM}$
= 4.9 BM
(2) μ = 5.92 BM
n = 5
Mn^{2+} = [Ar]$3d^5$
x = 2

(29) **Answer :** 02.00
Hint:
$\mu_{spin\ \ only} = \sqrt{n\left(n+2\right)}\ \text{BM}$, n is number of unpaired electrons.
Solution:
(1) $\mu_{spin\ \ only} = \sqrt{n\left(n+2\right)}\ \text{BM}$
Cr^{2+} = [Ar]$3d^4$



n = 4
$\mu = \sqrt{4(4+2)}\ \text{BM}$
= $\sqrt{24}\ BM$
= 4.9 BM
(2) μ = 5.92 BM
n = 5
Mn^{2+} = [Ar]$3d^5$
x = 2

(30) **Answer :** 10.00
Hint:
If solution contain weak part and its conjugate part, then solution behaves as buffer solution.
Solution:
At end point, meq of CH_3COOH = meq of NaOH
x × 0.1 = 25 × 0.04
x = 10 mL
When 10 mL of NaOH is added
mmoles of CH_3COOH left = 1 – 0.4 = 0.6 mmol
mmoles of CH_3COONa formed = 0.4 mol
$\text{pH} = \text{pK}_a + \log \frac{[CH_3\ COONa]}{[CH_3\ COOH]}$
= $4.74 + \log \frac{4}{6}$
= 4.56

(31) **Answer :** 04.56
Hint:
If solution contain weak part and its conjugate part, then solution behaves as buffer solution.
Solution:
At end point, meq of CH_3COOH = meq of NaOH
x × 0.1 = 25 × 0.04

x = 10 mL
When 10 mL of NaOH is added
mmoles of CH_3COOH left = 1 – 0.4 = 0.6 mmol
mmoles of CH_3COONa formed = 0.4 mol

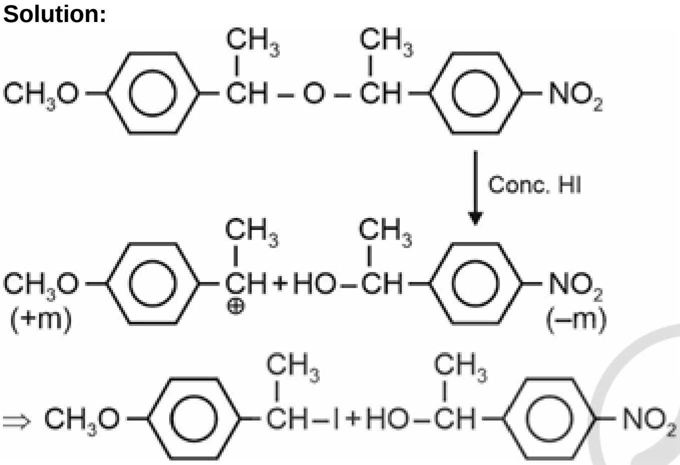
$$\mathrm{pH} = \mathrm{pK_a} + \log\frac{[CH_3\ COONa]}{[CH_3\ COOH]}$$

$= 4.74 + \log\frac{4}{6}$

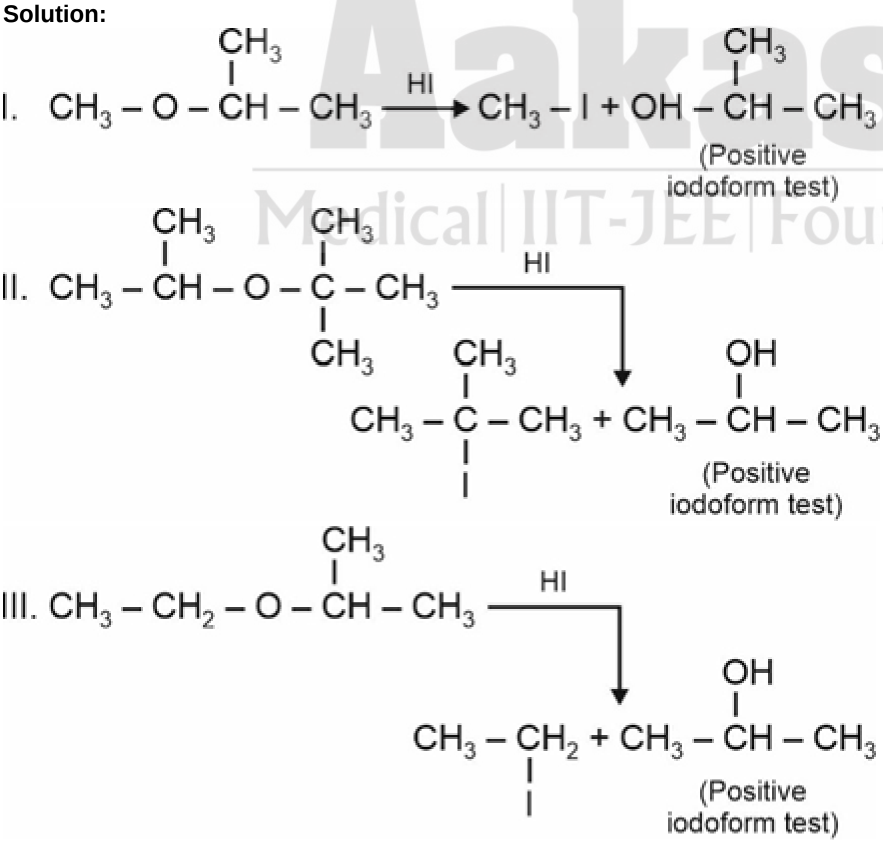
= 4.56

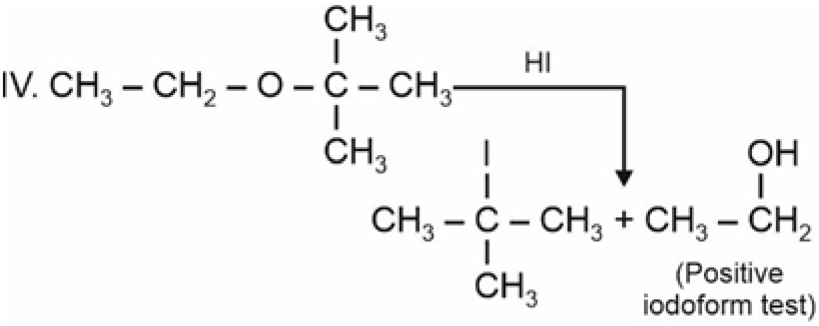
Section-III

(32) **Answer :** (B)
Hint:
If carbocation is stable, then reaction goes through S_N1 mechanism.
Solution:



(33) **Answer :** (D)
Hint:
Ethanol can give iodoform test.
Solution:



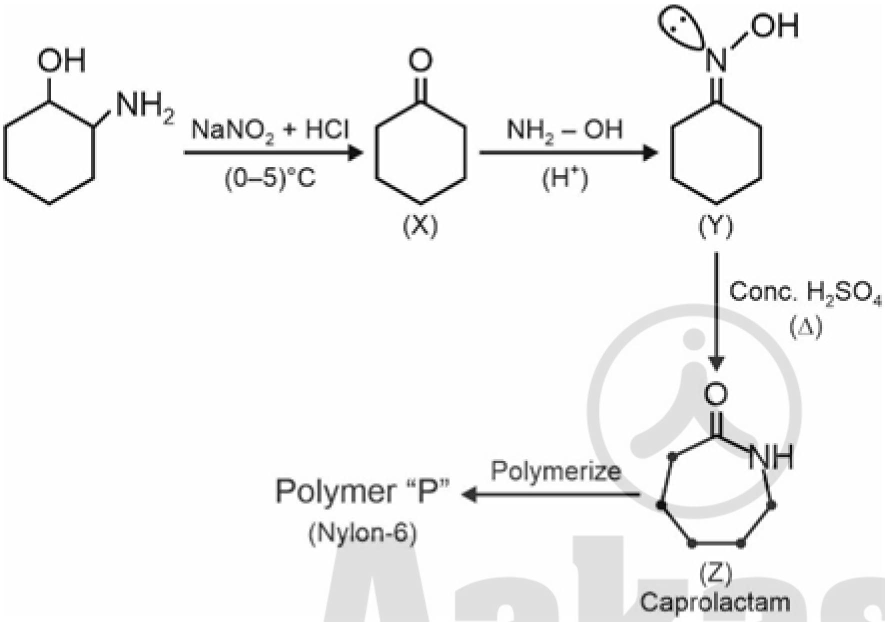


(34) **Answer :** (B)

Hint:

Monomer of P is caprolactam.

Solution:

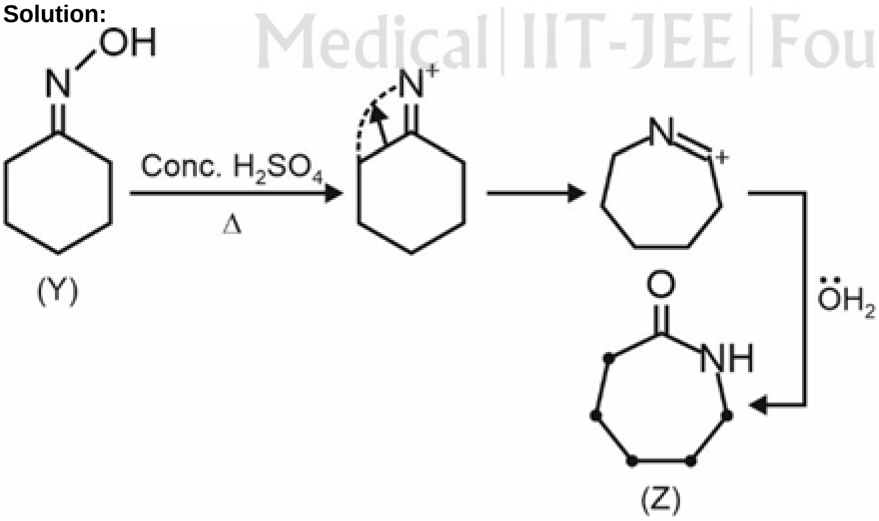


(35) **Answer :** (B)

Hint:

Wolff rearrangement takes place.

Solution:



Section-IV

(36) **Answer :** 8

Hint:

Compounds having all paired electrons are diamagnetic.

Solution:

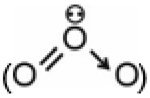
$KMnO_4$: diamagnetic (Mn^{7+} : d^0)

K_2MnO_4 : paramagnetic (Mn^{6+} : d^1)

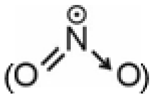
K_2CrO_4 : diamagnetic (Cr^{6+} : d^0)

C_2 : diamagnetic (all electrons are paired)

B_2 : paramagnetic (2 unpaired electrons in ($\pi 2p_x = \pi 2p_y$))
O_2 : paramagnetic (2 unpaired electrons in ($\pi^* 2p_x = \pi^* 2p_y$))

O_3 : diamagnetic 

$KO_2 = O_2^-$: paramagnetic (1 unpaired electron in ($\pi^* 2p_x = \pi^* 2p_y$))

NO_2 (monomer) : paramagnetic 

$K_2[PtCl_4]$: diamagnetic (Pt^{2+} : $6s^0 5d^8$) pairing will take place hybridisation : dsp^2)
$K_2[Ni(CN)_4]$: diamagnetic due to pairing
$[Ni(CO)_4]$: diamagnetic (Hybridisation : sp^3)
$K_2[NiCl_4]$: paramagnetic (2 unpaired electrons)
$[Zn(NH_3)_4]^{2+}$: diamagnetic (Hybridisation : sp^3)

(37) **Answer :** 6
Hint:
At anode oxidation takes place.
Solution:
Anode : $2Hg(l) + 2Cl^-(aq) \rightarrow Hg_2Cl_2(s) + 2e^-$
Cathode : $Q + 2H^+ + 2e^- \rightarrow QH_2$
Net cell reaction :
$Hg(l) + 2Cl^-(aq) + Q + 2H^+ \rightarrow Hg_2Cl_2(s) + QH_2$

$$E^o_{cell} = \left(E^o_{Hg\,|\,Hg_2\,Cl_2\,|\,Cl^-}\right) + \left(E^o_{H^+|Q|\,QH_2}\right)$$

= –0.24 + 0.70
= 0.46 V

$$E_{cell} = E^o_{cell} - \frac{0.06}{2}\log\frac{1}{[H^+]^2}$$

$$E_{cell} = E^o_{cell} - 0.06\,(pH)$$

⇒ 0.10 = 0.46 – 0.06 (pH)
⇒ $\frac{0.10-0.46}{-0.06} = pH$
⇒ pH = 6

(38) **Answer :** 7
Hint:
Partial pressure at vapour phase
$P_A = y_A P_T$
Solution:
$x_A = 0.3$; $x_B = 0.7$
$y_A = 0.6$; $y_B = 0.4$

$$\frac{y_A}{y_B} = \frac{P^o_A x_A}{P^o_B x_B}$$

⇒ $\frac{6}{4} = \left(\frac{P^o_A}{P^o_B}\right)\left(\frac{3}{7}\right)$

⇒ $\frac{P^o_A}{P^o_B} = \frac{7}{2} = 3.5$

⇒ $\frac{2P^o_A}{P^o_B} = 7$

MATHEMATICS

Section-I

(39) **Answer :** (A,B,C)
Hint:

$$\left|\vec{a}\right|^2 = \left|\vec{b}\right|^2 = \left|\vec{c}\right|^2 = 3$$

Solution:

$$\vec{a}\cdot\vec{a} = \vec{b}\cdot\vec{b} = \vec{c}\cdot\vec{c} = 3$$

$\left|\vec{a}\right|^2 = \left|\vec{b}\right|^2 = \left|\vec{c}\right|^2 = 3$

$\left|\vec{a}-\vec{b}\right|^2 + \left|\vec{b}-\vec{c}\right|^2 + \left|\vec{c}-\vec{a}\right|^2 = 27$

⇒ $\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\vec{a}\cdot\vec{b} + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2$

$-2\vec{b}\cdot\vec{c} + \left|\vec{c}\right|^2 + \left|\vec{a}\right| - 2\vec{a}\cdot\vec{c} = 27$

⇒ $2(3+3+3) - 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 27$

$\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{9}{2}$

Now, $3(\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2) - \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 - \left|\vec{c}\right|^2$

$-2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 27$

⇒ $3(3+3+3) - (\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2$

$+2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 27$

⇒ $27 - (\vec{a}+\vec{b}+\vec{c})^2 = 27$

⇒ $\vec{a}+\vec{b}+\vec{c} = 0$

$\vec{a}$, $\vec{b}$, $\vec{c}$ are co-planar and form a triangle.

$\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} \geq -\frac{9}{2}$

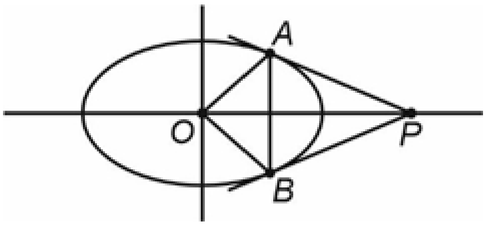
(40) **Answer :** (A,C)
Hint:
$\cos^2 x = \sin 2x$
Solution:
$2\sin^2\left(\frac{\pi}{2}\cos^2 x\right) = 2\sin^2\left(\frac{\pi}{2}\sin 2\pi\right)$
⇒ $\cos^2 x = \sin 2x$ ⇒ $\cos x(\cos x - 2\sin x) = 0$
⇒ $1 - 2\tan x = 0$ as $\cos x \neq 0$
⇒ $\tan x = \frac{1}{2}$ and $\cos 2x = \frac{3}{5}$

(41) **Answer :** (A,B,C,D)
Hint:
$|z| = |\bar{z}|$
Solution:
(a) $z^3 = \bar{z}$ …(i)
$|z^3| = |\bar{z}| = |z|$
$|z|(|z|^2 - 1) = 0$, $|z| = 0$, $|z|^2 = 1$
$z\bar{z} = 1 \Rightarrow \bar{z} = \frac{1}{z}$
From (i),
$z^3 = \frac{1}{z} \Rightarrow z^4 = 1$
(b) Let $z = \alpha$ be a real root of equation
$z^3 + (3 + i)z^2 - 3z - (m + i) = 0$
$(\alpha^3 + 3\alpha^2 - 3\alpha - m) + i(\alpha^2 - 1) = 0$
$\alpha = \pm 1$
If $\alpha = 1$, $m = 1$ and if $\alpha = -1$, $m = 5$
(c) $(z - \bar{z})(z + \bar{z} - 4) = 0$
$z = \bar{z},\ x^2 = 4x + x^2 + \frac{16}{|x|^3}$

⇒ $x = \frac{-4}{|x|^3}$

$x = -\sqrt{2},\ z = -\sqrt{2},\ |z|^4 = 4$

(42) **Answer :** (A,C)
Hint:
Equation of AB is $\frac{xh}{4} + \frac{yk}{1} = 1$
Solution:



Let the point of intersection of tangents A and B be $P(h, k)$, then equation AB is

$\frac{xh}{4}+\frac{yk}{1}=1$

Now, $\frac{x^2}{4}+\frac{y^2}{1}=\left(\frac{xh}{4}+\frac{yk}{1}\right)^2$

⇒ $x^2\left(\frac{h^2-4}{16}\right)+y^2(k-1)+\frac{2hk}{4}xy=0$ …(i)

Given equation of OA and OB is

$x^2 + y^2 + \alpha xy = 0$ …(ii)

Equation (i) and (ii) represent same line.

$\because\ \frac{h^2-4}{16}=\frac{k^2-1}{4}=\frac{hk}{2\alpha}$

$h^2 - 4 = 4(k^2 - 1)$

⇒ $h^2 - 4k^2 = 0$

$(h - 2k)(h + 2k) = 0$

∴ Locus is $(x - 2y)(x + 2y) = 0$

(43) **Answer :** (A,B,C)

Hint:

$AB = BA = 0$

Solution:

$AB = BA = 0$

∴ $(A + B)^n = A^n + B^n$

$(A - B)^n = A^n + (-B)^n$

$(3A + 7B)^n = 3^nA^n + 7^nB^n$

$= 3^n\, 3^{n-1}A + 7^n\, 3^{n-1}B$

$\because A^n = 3^{n-1}A,\ B^n = 3^{n-1}B$

(44) **Answer :** (A,B)

Hint:

$P(A)=\frac{4}{8},\ P(B)=\frac{4}{8},\ P(C)=\frac{4}{8}$

Solution:

$P(A)=\frac{4}{8},\ P(B)=\frac{4}{8},\ P(C)=\frac{4}{8}$

$P(A\cap B)=\frac{2}{8},\ P(A/B)=\frac{1}{2},\ P(A/C)=\frac{1}{2}$

$P(A \cap B \cap C) = 0$

Section-II

(45) **Answer :** 04.00

Hint:

$g(x)$ is inverse of $f(x)$.

Solution:

$g(f(x)) = x$

⇒ $g(x)$ is inverse of $f(x)$

⇒ $g'(f(x))\, f'(x) = 1$

⇒ $g'(f(x))=\frac{1}{f'(x)}$ …(i)

$g'(f(0))=\frac{1}{f'(0)}=\frac{1}{3}$

⇒ $g'(1)=\frac{1}{3}$

Now, $h(g(g(x))) = x$

⇒ $h(g(g(f(x)))) = f(x)$

⇒ $h(g(x)) = f(x)$ …(ii)

⇒ $h(g(1)) = f(1) = 5$

From (ii),

$h(g(x)) = f(x)$

⇒ $h(g(f(x))) = f(f(x))$

⇒ $h(x) = f(f(x))$

⇒ $h'(x) = f'(f(x))\times f'(x)$

$h'(0) = f'(f(0))\times f'(0) = f'(1) \times 3 = 18$

and $g(h(g(x))) = g(f(x)) = x$

⇒ $g(h(g(7))) = 7$

$\therefore\ 3g'(1)+\frac{1}{6}h'(0)=3\times\frac{1}{3}+\frac{1}{6}\times 18=1+3=4$

$h(g(1)) + g(h(g(7))) = 5 + 7 = 12$

(46) **Answer :** 12.00

Hint:

$g(x)$ is inverse of $f(x)$.

Solution:

$g(f(x)) = x$

⇒ $g(x)$ is inverse of $f(x)$

⇒ $g'(f(x))\, f'(x) = 1$

⇒ $g'(f(x))=\frac{1}{f'(x)}$ …(i)

$g'(f(0))=\frac{1}{f'(0)}=\frac{1}{3}$

⇒ $g'(1)=\frac{1}{3}$

Now, $h(g(g(x))) = x$

⇒ $h(g(g(f(x)))) = f(x)$

⇒ $h(g(x)) = f(x)$ …(ii)

⇒ $h(g(1)) = f(1) = 5$

From (ii),

$h(g(x)) = f(x)$

⇒ $h(g(f(x))) = f(f(x))$

⇒ $h(x) = f(f(x))$

⇒ $h'(x) = f'(f(x))\times f'(x)$

$h'(0) = f'(f(0))\times f'(0) = f'(1) \times 3 = 18$

and $g(h(g(x))) = g(f(x)) = x$

⇒ $g(h(g(7))) = 7$

$\therefore\ 3g'(1)+\frac{1}{6}h'(0)=3\times\frac{1}{3}+\frac{1}{6}\times 18=1+3=4$

$h(g(1)) + g(h(g(7))) = 5 + 7 = 12$

(47) **Answer :** 04.00

Hint:

$f(x)=\left(\frac{\lambda}{2-\lambda}+1\right)\sin x=2$

Solution:

$$f(x)-\lambda\sin x\int_0^{\pi/2}\cos t\, f(t)\,dt=\sin x$$

⇒ $f(x) - P\sin x = \sin x$

⇒ $f(x) = (P + 1)\sin x$

where $P=\lambda\int_0^{\pi/2}\cos t\, f(t)\,dt$

$$=\lambda\int_0^{\pi/2}\cos t(P+1)\sin t\,dt$$

$$=\frac{\lambda(P+1)}{2}\int_0^{\pi/2}\sin 2t\,dt$$

$P=\frac{\lambda(P+1)}{2}\ \Rightarrow\ P=\frac{\lambda}{2-\lambda}$

$f(x)=\left(\frac{\lambda}{2-\lambda}+1\right)\sin x=2$

⇒ $\sin x = 2 - \lambda$

⇒ $\lambda \in [1, 3]$

$\therefore \alpha + \beta = 1 + 3 = 4$

(48) **Answer :** 04.00

Hint:

$f(x)=\left(\frac{\lambda}{2-\lambda}+1\right)\sin x$

Solution:

$$\int_0^{\pi/2}\frac{2}{2-\lambda}\sin x\,dx=3$$

⇒ $\lambda=\frac{4}{3}$

⇒ $3\lambda = 4$

(49) **Answer :** 01.00

Hint:

y-Intercept of parabola is at $x = 0$

Solution:

y-Intercept of parabola is at $x = 0$

$y = 0^2 + a(0) + 1 = 1$

∴ Point of intersection is (0, 1).

Slope of tangent is $y'(0) = a$

Equation of tangent line is

$y - 1 = a(x - 0)$

⇒ $y = ax + 1$

which is tangent to $x^2 + y^2 = r^2$, if

$$r = \frac{|a(0)-1(0)+1|}{\sqrt{a^2+(-1)^2}} = \frac{1}{\sqrt{a^2+1}}$$

$r_{max} = 1$ for $a = 0$

∴ Slope of tangent = 0

(50) **Answer :** 00.00

Hint:

y-Intercept of parabola is at $x = 0$

Solution:

y-Intercept of parabola is at $x = 0$

$y = 0^2 + a(0) + 1 = 1$

∴ Point of intersection is (0, 1).

Slope of tangent is $y'(0) = a$

Equation of tangent line is

$y - 1 = a(x - 0)$

⇒ $y = ax + 1$

which is tangent to $x^2 + y^2 = r^2$, if

$$r = \frac{|a(0)-1(0)+1|}{\sqrt{a^2+(-1)^2}} = \frac{1}{\sqrt{a^2+1}}$$

$r_{max} = 1$ for $a = 0$

∴ Slope of tangent = 0

Section-III

(51) **Answer :** (B)

Hint:

$$f(x) = \begin{cases} 2a^{-x}, & x < 0 \\ a^{-x} + a^{x}, & x \geq 0 \end{cases}$$

Solution:

$$f(x) = \begin{cases} 2a^{-x}, & x < 0 \\ a^{-x} + a^{x}, & x \geq 0 \end{cases}$$

Clearly, $f(x)$ is continuous in R.

For $x > 0$,

$f'(x) = -a^{-x} \log_e a + a^x \log_e a = (a^x - a^{-x})\log_e a$

$= \left(\frac{a^{2x}-1}{a^x}\right)\log_e a > 0$ for $x > 0$

⇒ $f(x)$ is monotonically increasing for $x > 0$.

For $x < 0$,

$f(x) = 2a^{-x}$

⇒ $f'(x) = -2a^{-x} \log_e a < 0$

⇒ $f(x)$ is monotonically decreasing for $x < 0$.

(52) **Answer :** (B)

Hint:

Concept of area between two curves.

Solution:

Required area

$$= \int_{-1}^{0} 2a^{-x}dx + \int_{0}^{1} (a^{-x} + a^{x})dx$$

$$= \left[\frac{-2a^{-x}}{\log_e a}\right]_{-1}^{0} + \left[\frac{-a^{-x}+a^{x}}{\log_e a}\right]_{0}^{1}$$

$$= \frac{1}{\log_e a}[-2-(-2a)] + \frac{1}{\log_e a}\left[\left(\frac{-1}{a}+a\right)-0\right]$$

$= \frac{3a^2-2a-1}{a\log_e a}$

(53) **Answer :** (C)
Hint:
Equation of tangent is $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

Normal is $ax\cos\theta + by\cot\theta = a^2 + b^2$
Solution:
Equation of tangent is
$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

Normal is $ax\cos\theta + by\cot\theta = a^2 + b^2$
The normal at P meets the coordinate axes at

$G\left(\frac{a^2+b^2}{a}\sec\theta,\ 0\right)$ and $g\left(0,\ \frac{a^2+b^2}{a}\tan\theta\right)$

$\therefore\ PG^2 = \left(\frac{a^2+b^2}{a}\sec\theta - a\sec\theta\right)^2 + (b\tan\theta - 0)^2$

$PG^2 = \frac{b^2}{a^2}(b^2\sec^2\theta + a^2\tan^2\theta)$

(54) **Answer :** (A)
Hint:
PG is minimum, when $\tan\theta = 0$
Solution:
When $\tan\theta = 0$

$PG = \frac{b^2}{a}$

Section-IV

(55) **Answer :** 15
Hint:
If a relation is reflexive, symmetric and transitive, then the relation is equivalence.
Solution:
R' = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (1, 2), (2, 1), (4, 6), (6, 4), (5, 6), (6, 5), (4, 5), (5, 4)}

(56) **Answer :** 22
Hint:
$2(x + y + z) = 20 - t$ = even ⇒ t is even
Solution:
t should be even, say $2k$.
So, the equation reduces to $x + y + z + k = 10$, whose non-negative integral solution is $^{13}C_3 = 286$

(57) **Answer :** 12
Hint:
Use $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$

and $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Solution:
The given differential equation can be written as

$\frac{d}{dx}\left(\frac{1}{y}\frac{dy}{dx}\right) = \left(\frac{1}{y}\frac{dy}{dx}\right)^2$

⇒ $-\frac{1}{y\left(\frac{dy}{dx}\right)} = x + c$, whose solution is $(x + c)y = k$

The curve passes through the shortest distance between the curves $xy = 8$ and director circle of $x^2 + y^2 = 4$ is
$(4 - 2\sqrt{2})$ units = $(4 - \sqrt{8})$
$\therefore p = 4, q = 8 \Rightarrow p + q = 12$