22/03/2026

SET-1_ONLINE

Aakash
Medical | IIT-JEE | Foundations

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MM : 300 AIATS For One Year JEE(Main)-2026 (SS & PS)_Test-7 & 5 (Online) **Time : 180 Min.**

Physics

Section-I

1. (3)
2. (4)
3. (4)
4. (1)
5. (3)
6. (2)
7. (4)
8. (4)
9. (1)
10. (2)
11. (4)
12. (2)
13. (2)
14. (2)
15. (4)
16. (2)
17. (3)
18. (1)
19. (4)
20. (3)

Section-II

21. (9)
22. (5)
23. (240)
24. (4)
25. (3)

Chemistry

Section-I

26. (4)
27. (1)
28. (3)
29. (2)
30. (4)
31. (4)
32. (4)
33. (1)
34. (2)
35. (4)
36. (4)
37. (1)
38. (2)
39. (3)
40. (4)
41. (3)
42. (2)
43. (3)
44. (1)
45. (4)

Section-II

46. (3)

47. (4)

48. (8)

49. (3)

50. (212)

Mathematics

Section-I

51. (4)

52. (4)

53. (2)

54. (3)

55. (4)

56. (1)

57. (1)

58. (3)

59. (1)

60. (3)

61. (1)

62. (1)

63. (2)

64. (4)

65. (1)

66. (4)

67. (2)

68. (4)

69. (4)

70. (3)

Section-II

71. (2)

72. (7)

73. (7)

74. (37)

75. (5)

Hints and Solutions

Physics

Section-I

(1) **Answer :** (3)
Hint:
$V_0 = \sqrt{gR}$
Solution:
For object on equator, $V = R\omega$ = 467 m/s
Orbital speed of satellite near earth surface
$V_0 = \sqrt{gR} = 8 \times 10^3$ m/s $= 8000$
So, Westerly launching speed = 8000 + 467
Easterly launching speed = 8000 – 467
$\frac{934}{7633} \times 100 = 12.23\%$

(2) **Answer :** (4)
Hint:
Balanced Wheatstone bridge
Solution:
$\frac{R}{l} = \frac{S}{100-l}$
$\frac{R_1}{\frac{1}{3}} = \frac{R_2}{\frac{2}{3}}$
$R_2 = 2R_1$
Again $\frac{R_1+5}{\left(\frac{2}{3}\right)} = \frac{R_2}{\frac{1}{3}}$
$R_1 + 5 = 2R_2$
$R_1 = \frac{5}{3}$
$R_2 = \frac{10}{3}$
$R_1 + R_2 = 5$

(3) **Answer :** (4)
Hint:
$\int \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$
Solution:
Total angle $\theta = \frac{\pi}{3}$
$\therefore \quad \int_A^B B \cdot dl = \frac{(\mu_0 I)}{2\pi} \times \frac{\pi}{3}$
$= \frac{\mu_0 I}{6}$

(4) **Answer :** (1)
Hint:
$\phi_{AHDE} = \phi_{EFGH} = \phi_{ABGH}$
Solution:
Initially $\phi_{ABCD} = 0$
$\phi_{AHDE} = \phi_{EFGH} = \phi_{ABGH} = \frac{1}{3}\left(\frac{q}{8\varepsilon_0}\right) = \frac{q}{24\varepsilon_0}$
When charge is inside the cube near the corner φ_{AHDE} , φ_{EFGH} and φ_{ABGH} remains same.
$3\phi_{ABCD} = \frac{q}{\varepsilon_0} - 3 \cdot \frac{q}{24\varepsilon_0}$
$\phi_{ABCD} = \frac{7q}{24\varepsilon_0}$

(5) **Answer :** (3)
Hint:
$\sum = \frac{-d\phi}{dt}$
Solution:
$\phi = B(t) \times A$

$= \frac{\mu_0 N I_0 e^{-\alpha t}}{L} \pi r^2$

$\varepsilon = -\frac{d\phi}{dt}$

$= \mu_0 n I_0 \alpha \pi r^2 e^{-\alpha t}$

$\varepsilon_{max} = \mu_0 n I_0 \alpha \pi r^2$

So, $\frac{y}{x} = 2$

(6) **Answer :** (2)

Hint:

$a_{com} = \frac{F}{2m}$

Solution:

(A) $F = (m + m)a_C$

$a_C = \frac{F}{2\,m}$

(B) $x = ut + \frac{1}{2}at^2$

$X_C = \frac{Ft^2}{4\,m}$

(C) $2mX_C = mX_A + mX_B$

$X_C = \frac{X_A + X_B}{2}$

$X_A + X_B = \frac{Ft^2}{2\,m}$

Extension in the spring $l_0 = X_B - X_A$

$X_A = \frac{1}{2}\left(\frac{Ft^2}{2\,m} - x_0\right)$

$X_B = \frac{1}{2}\left(\frac{Ft^2}{2\,m} + x_0\right)$

(7) **Answer :** (4)

Hint:

$\lambda_{\min} = \frac{hc}{eV}$

Solution:

$\frac{hc}{\lambda_{\min}} = K_{\max} = eV$

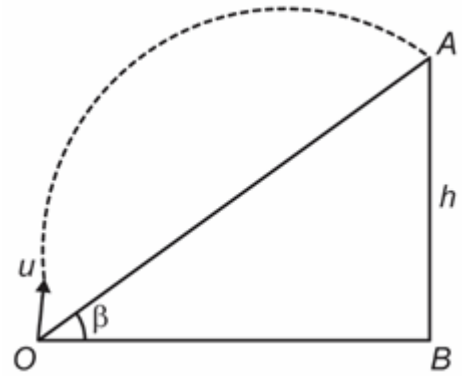
$\lambda_{\min} = \frac{hc}{eV}$

(8) **Answer :** (4)

Hint:

$R = \frac{\mu^2}{g(1+\sin\beta)}$

Solution:



$OA = h\,\text{cosec}\beta$

u is minimum when OA will be maximum range.

$h\,\text{cosec}\beta = \frac{u^2}{g(1+\sin\beta)}$

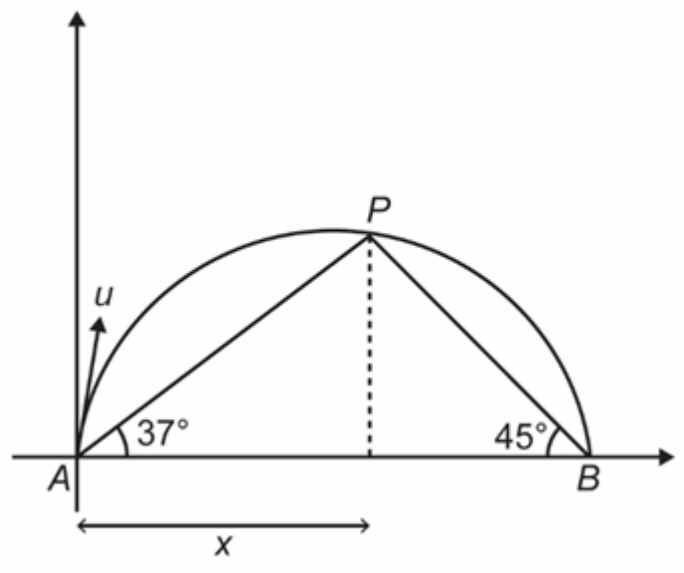
$u = \sqrt{gh(1+\text{cosec}\beta)} = 30$ m/s

(9) **Answer :** (1)

Hint:

$y = x\tan\theta\left(1 - \frac{x}{R}\right)$

Solution:



$\tan 37^\circ + \tan 45^\circ = \frac{y}{x} + \frac{y}{R-x}$

$= \frac{yR}{x(R-x)}$

$y = x\tan\theta\left(1 - \frac{x}{R}\right)$

$\tan\theta = \tan 37^\circ + \tan 45^\circ = \frac{7}{4}$

(10) **Answer :** (2)
Hint:

$L = 10\log\left(\frac{I}{I_0}\right)$

Solution:

$I = \frac{P}{2\pi rl}$

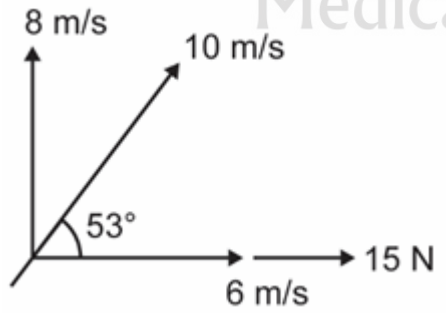
$\Delta I(dB) = 10\log\frac{P}{2\pi rl} - 10\log\frac{P}{2\pi(4r)l}$

= 10 log = 20 × 0.3 = 6 dB

(11) **Answer :** (4)
Hint:

$R = \frac{v^2}{a_N}$

Solution:



$\left|\vec{F}_n\right| = 15\sin(53^\circ) = 15 \times \frac{4}{5} = 12$ N

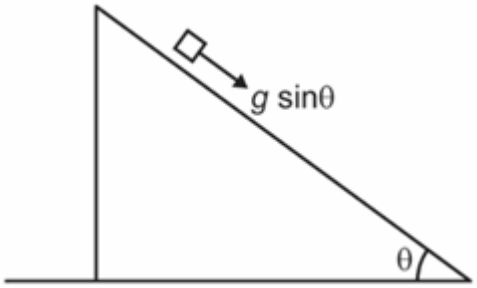
So, $\left|\vec{F}_n\right| = \frac{mv^2}{R}$

$R = \frac{2\times10\times10}{12} = \frac{50}{3}$ m

(12) **Answer :** (2)
Hint:

$a_X = g\sin\theta\cos\theta$

Solution:



$\Rightarrow a_X = g \sin\theta \cos\theta$

$\Rightarrow a_x = \frac{g}{2}\sin(2\theta)$

So, sin(2θ) will be maximum at 2θ = 90°

$\Rightarrow \theta = 45°$

(13) **Answer :** (2)

Hint:

$T = 2\pi\sqrt{\frac{2m+\frac{m}{3}}{K}}$

Solution:

$T = 2\pi\sqrt{\frac{2m+\frac{m}{3}}{K}}$

$= 2\pi\sqrt{\frac{7m}{3K}}$

(14) **Answer :** (2)

Hint:

$2h\nu = 2m_e c^2$

Solution:

$2h\nu = 2m_e c^2$

$2h\Delta\nu = mv^2$

(15) **Answer :** (4)

Hint:

$x = \frac{mg}{k}$

Solution:

$\frac{\sigma}{2\varepsilon_0} \times \sigma A = \frac{\sigma^2 A}{2\varepsilon_0}$

$x = \frac{mg}{k}$

(16) **Answer :** (2)

Hint:

$E = \frac{\sigma}{2\varepsilon_0}$

Solution:

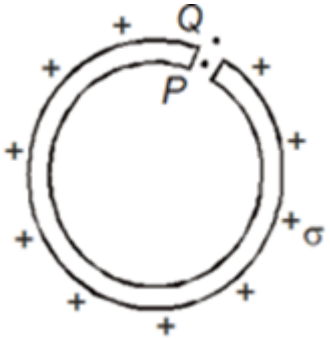
Let E_1 and E_2 represent field due to the shell (with hole) and field due to charge on material which has been punched out to form hole. When the shell has not been punched,

At *P* : Net field = 0 = $E_1 - E_2$

At *Q* : Net field = $\frac{\sigma}{\varepsilon_0}$

$= E_1 + E_2$

$\Rightarrow E_1 = \frac{\sigma}{2\varepsilon_0}$



(17) **Answer :** (3)

Hint:

CSR = *CSD* × *LC*

Solution:

CSR = *CSD* × *LC*

= 0.48 mm

And statement-II correctly explain how determine the contribution of the circular scale reading.

(18) **Answer :** (1)

Hint:

$Y = AB + \overline{A} + \overline{B} = 0$

Solution:

$Y = AB + \overline{A} + \overline{B} = 0$

(19) **Answer :** (4)

Hint:

Magnetic field due to arc

$= \frac{\mu_0 i}{2R} \times \frac{3}{4} = \frac{3}{8}\frac{\mu_0 i}{R}$

Solution:

Magnetic field due to arc

$= \frac{\mu_0 i}{2R} \times \frac{3}{4} = \frac{3}{8}\frac{\mu_0 i}{R}$

Magnetic field due to straight wire

$= \frac{\mu_0 i}{4\pi r}\sqrt{2} = \frac{\mu_0 i}{2\sqrt{2}\pi\left(\frac{R}{\sqrt{2}}\right)} = \frac{\mu_0 i}{2\pi R}$

$B_{\text{net}} = B_1 + B_2 = \frac{3}{8}\frac{\mu_0 i}{R} + \frac{\mu_0 i}{2\pi R}$

$= \frac{\mu_0 i}{2\pi R}\left(1 + \frac{3\pi}{4}\right)$

(20) **Answer :** (3)

Hint:

At time *t* distance between two cars are

$S = \sqrt{(100-3t)^2 + (100-4t)^2}$

Solution:

At time *t* distance between two cars are

$S = \sqrt{(100-3t)^2 + (100-4t)^2}$

$\frac{dS}{dt} = 0$

t = 28 sec

Section-II

(21) **Answer :** 9

Hint:

$\frac{\Delta E}{E_0} = \frac{4m_1 m_2}{(m_1+m_2)^2}$

Solution:

$\frac{\Delta E}{E_0} = \frac{4m_1 m_2}{(m_1+m_2)^2} = \frac{8}{9}$

After 1st collision:

$\Delta E_1 = \frac{8}{9}E_0$

$\Delta E_2 = \frac{8}{9}E_1$

$\Delta E_n = \frac{8}{9}E_{n-1}$

$\Delta E = \Delta E_1 + \Delta E_2 + \ldots\ldots \Delta E_n$

$\Delta E = E_0\left(1 - \frac{1}{9^n}\right)$

So, α = 9

(22) **Answer :** 5

Hint:

$I = \frac{v}{z}$

Solution:

$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}}$

and $\frac{I}{2}=\frac{V}{\sqrt{R^2+\left(\frac{3}{C\omega}\right)^2}}$

On re-arranging

$\frac{R}{X}=\sqrt{\frac{5}{3}}$

So, α = 5

(23) **Answer :** 240

Hint:

$\frac{1}{v}-\frac{1}{\mu}=\frac{1}{f}$

Solution:

In 1st case: $\frac{1}{40}+\frac{1}{60}=\frac{1}{f}$

In 2nd case: $\frac{1}{V}+\frac{1}{60}=\frac{1}{2f}$

On solving

V = 240 cm

(24) **Answer :** 4

Hint:

$r\propto\frac{1}{R_{TH}}$

Solution:

$r\propto\frac{1}{R_{TH}}$

In 1st case: $R_{TH}=\frac{R}{2}$

And in second case R_{TH} = 2R

$\frac{r_1}{r_2}=4$

(25) **Answer :** 3

Hint:

$Q=\Delta U+W$

Solution:

$T_A=T_C$

$\Delta U_{BC}=-Q$

$\Delta U_{AB}=Q$

$\frac{3}{2}nR\Delta T=Q$

and $W_{AB}=nR\Delta T=\frac{2Q}{3}$

So, n = 3

Chemistry

Section-I

(26) **Answer :** (4)

Hint:

Molarity = $\frac{n}{V(L)}$

Solution:

n_{NaCl} = 1.4 mol

$V_{solution}$ = 0.5 L

M_{NaCl} = 58.5 g/mol

M = $\frac{1.4\ mol}{0.5\ L}$

⇒ 2.8 M ⇒ N = M = 2.8 N(n_f = 1)

$\therefore\ n_{H_2O}=\frac{575\ g-81.9\ g}{18\ g}\approx 27.39\ mol$

$\therefore$ X_{NaCl} = $\frac{1.4}{1.4+27.39}$

= 0.0486

(27) **Answer :** (1)

Hint:
$\Delta T_b = iK_bm$
Solution:

Moles of solute = $\frac{1.85\ g}{74\ g}$

= 0.025 mol

Molality = $\frac{0.025\ mol}{0.250\ kg}$

= 0.10 molality

$i = 1 + \alpha\left(\frac{1}{n} - 1\right)$

$\Rightarrow\ 1 + 0.7\left(\frac{1}{2} - 1\right)$

= 0.65

$\Delta T_b = iK_bm = 0.65 \times 2.53 \times 0.1 = 0.16445°C$

$T_b = 80.1 + 0.16555°C = 80.26445°C$

= 80.26°C

(28) Answer : (3)
Hint:
$\mathrm{mvr} = \frac{\mathrm{nh}}{2\pi} = \mathrm{L}$
Solution:

- The probability of finding the electron is maximum along X-Y axes $\mathrm{d}_{x^2-y^2}$ orbitals.
- Rydberg's constant and wave number have same units
- $\mathrm{L} = \frac{\mathrm{nh}}{2\pi}\ \Rightarrow$ transition from $\frac{4h}{2\pi} \rightarrow \frac{2h}{2\pi}$ (n = 4 to n = 2).

(29) Answer : (2)
Hint:
MP : Mo > Ru > Tc
Solution:
(ii), (iii) and (v) are correct statements
$\therefore$ MP : Mo > Ru > Tc
$\therefore$ V_2O_5 reacts with alkalies and acids to give VO_4^{3-} and VO_2^{+} respectively.

(30) Answer : (4)
Hint:
Electrophilicity and steric factor affect hydrate formation.
Solution:
Extent of hydrate formation :

$$\underset{(\text{-}0\%)}{\mathrm{PhCHO}} < \underset{(\text{-}58\%)}{\mathrm{CH_3CHO}} < \underset{(99.99\%)}{\mathrm{HCHO}} < \underset{(\text{-}100\%)}{\mathrm{Cl_3CHO}}$$

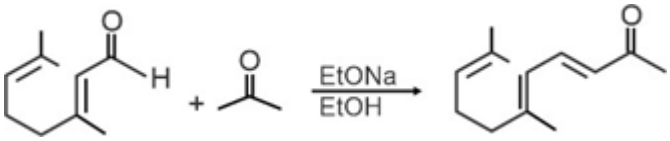
(31) Answer : (4)
Hint:
$W_{gas} = -P_{ext}\,\Delta V$
Solution:
$n_{O_2} = \frac{2000}{32}$
= 62.5 mol
$V_{final} = \frac{nRT}{P_f} = \frac{62.5 \times 0.082 \times 400}{1}$
= 2050 L
$\Delta V = 2050 - 10 = 2040$ L
$W_{gas} = -P_{ext}\,\Delta V = -1 \times 2040$ L atm = –2040 L atm
= –204 kJ

(32) Answer : (4)
Hint:
White P_4 + Alkali → NaH_2PO_2 (0 P – OH bond)
Solution:
White P_4 + Alkali → H_3PO_2 (1 P – OH bond)
$P_2O_3 + H_2O \rightarrow H_3PO_3$ (2 P – OH bonds)
$PCl_3 + H_3PO_3 \rightarrow H_4P_2O_5$ (2 P – OH bonds)
Red P_4 + Alkali → $H_4P_2O_6$ (4 P – OH bonds)

(33) Answer : (1)
Hint:
Semiconductors are used to make transistors.
Solution:

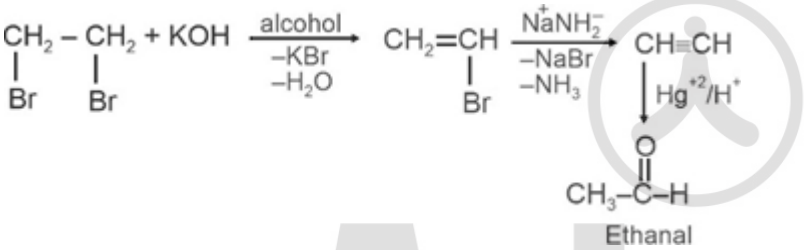
- Silicon is a very important component of ceramics, glass and cement.
- Tin occurs mainly as cassiterite.
- Ultrapure form of germanium and silicon are used to make transistors and semiconductor devices.

(34) Answer : (2)
Hint:
Acetone forms enolate ion
Solution:

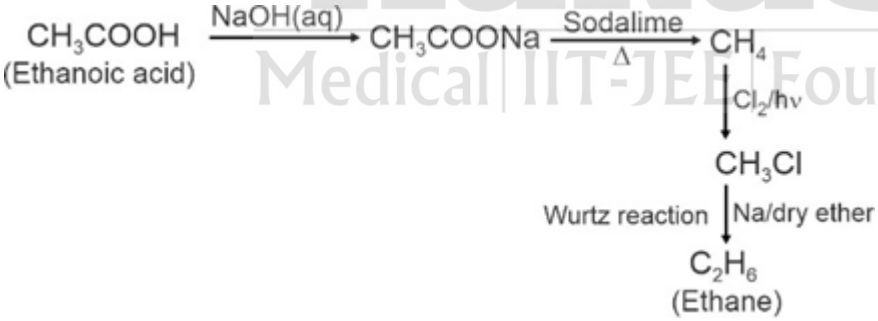


(35) Answer : (4)
Hint:
Orthoboric acid (H_3BO_3), on heating to 370 K forms metaboric acid (HBO_2).
Solution:
Borax → $Na_2B_4O_7.10H_2O$
Kernite → $Na_2B_4O_7.4H_2O$
Orthoboric acid → H_3BO_3
Metaboric acid → HBO_2

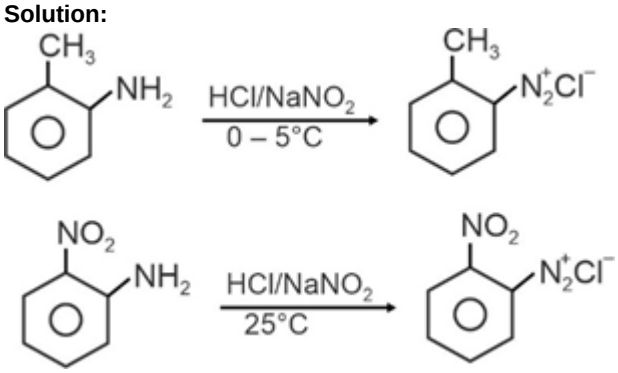
(36) Answer : (4)
Hint:
Alcoholic KOH cause elimination via E_2 in vicinal dihalides.
Solution:

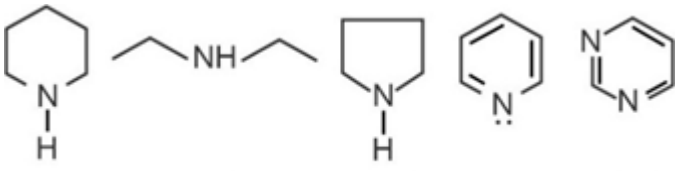


(37) Answer : (1)
Hint:
Alkyl halide with Na/DE gives Wurtz reaction.
Solution:



(38) Answer : (2)
Hint:
Ar – F > Ar – Cl (S_{NAR} – rate)
Solution:





(11.20) (10.98) (11.11) (5.23) (2.70)

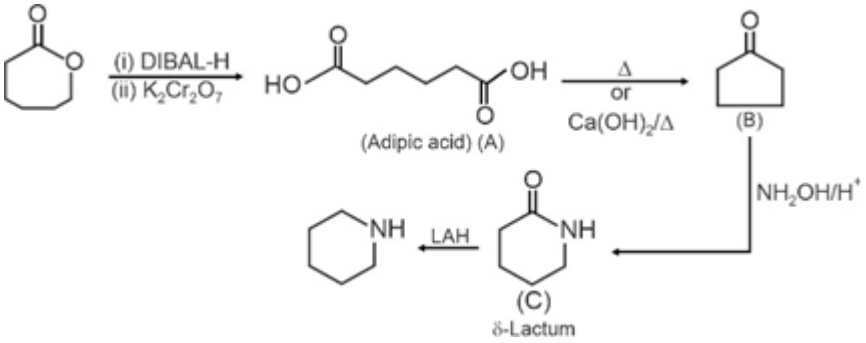
(pK_a of corresponding aminium ion)

(39) Answer : (3)

Hint:

DIBAL-H reduces esters

Solution:



(40) Answer : (4)

Hint:

(CO + N_2) is called producer gas

Solution:

$$C\left(s\right) + H_2O\left(g\right) \xrightarrow{473-1273\ K} \underset{\text{Water gas or Synthesis gas}}{CO(g) + H_2(g)}$$

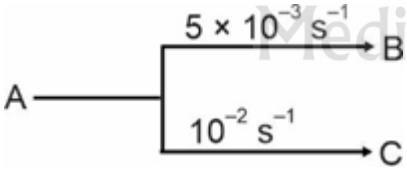
$$2C(s) + O_2(g) + 4N_2(g) \xrightarrow{1273\ K} \underset{\text{Producer gas}}{2\,CO(g) + 4N_2(g)}$$

(41) Answer : (3)

Hint:

$t_{1/2} = \frac{\ln 2}{K}$

Solution:



[B]% = $\frac{5\times10^{-3}}{15\times10^{-3}} \times 100\,\%$ ⇒ 33.34%

$(t_{1/2})_{net} = \frac{0.7}{K_{net}} = \frac{0.7}{15\times10^{-3}} \Rightarrow \frac{700}{15}$ = 46.67 s

$(E_a)_{net} = \frac{K_1\,E\,a_1 + K_2\,E\,a_2}{K_1 + K_2}$

$= \frac{5\times10^{-3}\times48 + 10^{-2}\times60}{15\times10^{-3}} = \frac{840}{15}$ = 56 kJ

$(t_{1/2})_{A\to C} = \frac{0.7}{10^{-2}}$⇒ 70 sec.

(42) Answer : (2)

Hint:

Zn is oxidised at anode

Solution:

In dry cell

Anode : $Zn(s) \rightarrow Zn^{2+} + 2e^-$

Cathode : $MnO_2 + NH_4^+ + e^- \rightarrow MnO(OH) + NH_3$

(43) Answer : (3)

Hint:

XeF_6 on hydrolysis gives $XeOF_4$, XeO_2F_2, XeO_3 and HF

Solution:

XeF_6 on hydrolysis gives $XeOF_4$, XeO_2F_2, XeO_3 and HF

(44) **Answer :** (1)

Hint:

n_f of $KMnO_4$ is 5 in acidic medium.

Solution:

eq. of $KMnO_4$ = eq. of $FeSO_4$

$\Rightarrow$ $0.1 \times V \times 5 = 0.01 \times 1$

$\Rightarrow$ V = 0.02 L = 20 mL

(45) **Answer :** (4)

Hint:

Molarity = $\frac{n}{V}$

Solution:

$\chi_P = \chi_Q = \frac{1}{2}$

$\therefore n_P = n_Q = a$ ($n_R = 0$)

Total initial moles = 2a

Initial molarity of P is K

$[P]_i = \frac{a}{V} = K \Rightarrow a = KV$

	P(aq)	$\rightleftharpoons$	Q(aq)	+	R(aq)
t = 0	KV		KV		0
$t = t_{eq}$	KV – x		KV + x		x

Total moles at eq^m = 2KV + x

At eq^{-m} $\chi_P = \frac{1}{3} = \frac{KV-x}{2KV+x} \Rightarrow x = \frac{KV}{4}$

$[P]_{eq} = \frac{KV-x}{V} = \frac{3K}{4}$, $[Q]_{eq} = \frac{5K}{4}$

$[R]_{eq} = \frac{x}{V} = \frac{5K}{4}$

$\therefore \quad K_C = \frac{\left(\frac{5K}{4}\right)\times\left(\frac{K}{4}\right)}{\left(\frac{3K}{4}\right)} = \frac{5K}{12}$

Section-II

(46) **Answer :** 3

Hint:

Check POS

Solution:

(A) No POS

(B) POS + diamagnetic; $Pt^{2+}\ [SFL] : 3d^8 4s^0$

(C) POS + diamagnetic; $Co^{3+} : t_2g^6 e_g^0$

(D) POS + diamagnetic; $Ni : t_2g^6 e_g^4$

(E) Paramagnetic

(F) Optical activity is present

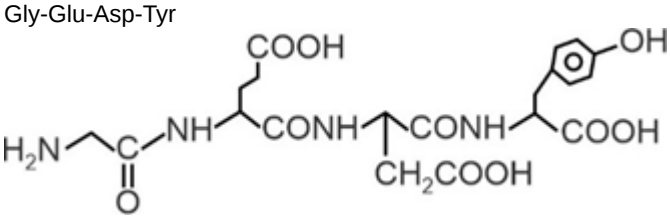
(47) **Answer :** 4

Hint:

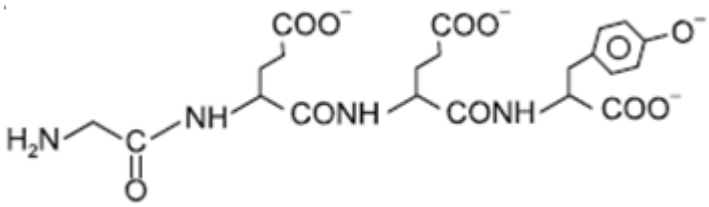
At high pH, molecule will loose acidic proton.

Solution:

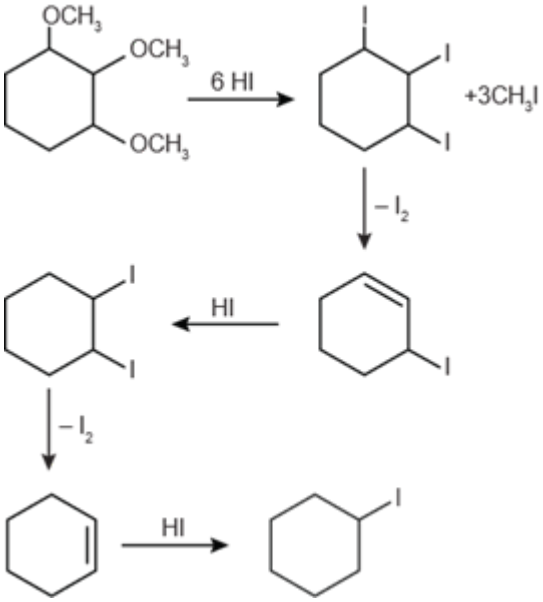
Gly-Glu-Asp-Tyr



–COOH, and phenol will lose its proton



(48) **Answer :** 8
Hint:
Ether are unstable in acidic medium.
Solution:



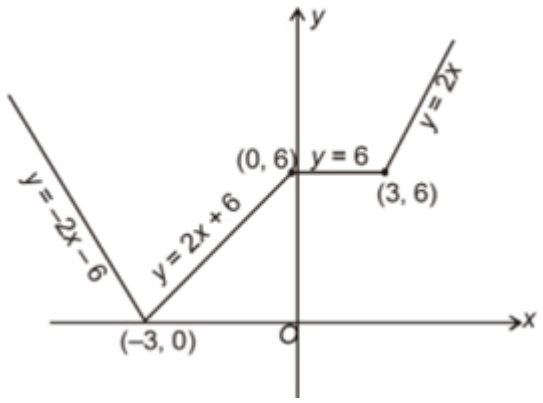
(49) **Answer :** 3
Hint:
PbI_2 is yellow
Solution:
(a) Chocolate brown precipitate
(b) Yellow
(c) White/bluish white
(d) Yellow
(e) White
(f) White

(50) **Answer :** 212
Hint:
Use thermochemistry
Solution:
$CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O(l)$
$\Delta^\circ H_C = -94 + 2 \times (-68) - (-18)$ kcal
$= -230 + 18 = -212$ kcal

Mathematics

Section-I

(51) **Answer :** (4)
Hint:
Draw graph
Solution:



(52) Answer : (4)

Hint:

$\sin 2x + 1 - 2\sin^2 x = 1 + \cos x - \sin x$

Solution:

$\sin 2x + 1 - 2\sin^2 x = 1 + \cos x - \sin x$

$2\sin x \cos x + 1 - 2\sin^2 x$

= 1 + cos*x* – sin*x*

$2\sin x\,(\cos x - \sin x) = (\cos x - \sin x)$

$(2\sin x - 1)\,(\cos x - \sin x) = 0$

$\Rightarrow \sin x = \frac{1}{2}\ or\ \tan x = 1$

$\sin x = \frac{1}{2}, x = n\pi + (-1)^n \frac{\pi}{6}, n \in I$

$\Rightarrow x = 2n\pi + \frac{\pi}{6}, n \in I$

And $x = 2n\pi + \frac{5\pi}{6}, n \in I$

Or $x = n\pi + \frac{\pi}{4},\ n \in I$

(53) Answer : (2)

Hint:

$\sum_{r=1}^{m} \left(\frac{m^r}{{}^mC_r} - \frac{m^{r-1}}{{}^mC_{r-1}}\right) = m^m - 1$

Solution:

$\sum_{r=1}^{m} \left(\frac{m^r}{{}^mC_r} - \frac{m^{r-1}}{{}^mC_{r-1}}\right) = m^m - 1$

(54) Answer : (3)

Hint:

$P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3), S(x_4, y_4)$

Solution:

$P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3), S(x_4, y_4)$

Let $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3), S(x_4, y_4)$

Now, $\frac{x_1 + x_2 + x_3 + x_4}{4} = 0$ and $x_2 + x_3 + x_4 = 6$

⇒ $x_1 = -6$

(55) Answer : (4)

Hint:

As $y^2 = P(x)$

Solution:

As $y^2 = P(x)$

⇒ $2y\frac{dy}{dx} = p'(x)$

$2y^3 \frac{d^2y}{dx^2} = p(x)p''(x) - p'(x)^2 \cdot \frac{1}{2}$

⇒ $2 \times \frac{d}{dx}\left\{y^3 \frac{d^2y}{dx^2}\right\}_{at\ x=5} = p(5) \cdot p'''(5) = 2$

(56) Answer : (1)

Hint:

Range of $\tan^{-1}x$ is $-\frac{\pi}{2} \le \tan^{-1}(x) \le \frac{\pi}{2}$

Solution:

Let $\theta = 2\tan^{-1}(-3) \Rightarrow -\pi < \theta < 0$

$\tan\frac{\theta}{2}=-3\Rightarrow\tan\theta=\frac{2\tan(\theta/2)}{1-\tan^2(\theta/2)}=\frac{3}{4}$

$\Rightarrow -\pi<\theta<-\frac{\pi}{2}$

Let $\alpha=\tan^{-1}\left(\frac{3}{4}\right)$ (principal value) then

$\theta=\alpha-\pi$

Now $\cos\alpha=\frac{4}{5}$

$\Rightarrow\theta=\cos^{-1}\left(\frac{4}{5}\right)-\pi=\frac{\pi}{2}-\sin^{-1}\left(\frac{4}{5}\right)-\pi$

= $-\sin^{-1}\left(\frac{4}{5}\right)-\frac{\pi}{2}$

$\cot\alpha=\left(\frac{4}{3}\right)$

$\Rightarrow\theta=\cot^{-1}\left(\frac{4}{3}\right)-\pi=\frac{\pi}{2}-\tan^{-1}\left(\frac{4}{3}\right)-\pi$

$=-\tan^{-1}\left(\frac{4}{3}\right)-\frac{\pi}{2}$

(57) Answer : (1)

Hint:

If a function is continuous then

LHL $f(a)$ = $f(a)$ = RHL $f(a)$

Solution:

$f(5^-)=\lim\limits_{x\to5^-}\sin\left(\frac{\pi}{2}(x-[x])\right)$

$=\lim\limits_{x\to5^-}\sin\left(\frac{\pi}{2}(x-4)\right)=1$

$f(5^+)=\lim\limits_{x\to5^+}\frac{ab^2|x^2-11x+24|}{x-3}$

$=\lim\limits_{x\to5^+}\frac{-ab^2(x-3)(x-8)}{x-3}=3ab^2$

$\Rightarrow 5(b-1)=1=3ab^2$

$\Rightarrow b=\frac{6}{5},\ a=\frac{25}{108}$

(58) Answer : (3)

Hint:

For increasing function $f'(x)\geq 0$

Solution:

We have $f(x)=ax+\cos2x+\sin x+\cos x$

$\Rightarrow$ $f'(x)=a-2\sin 2x+\cos x-\sin x$

As $f'(x)\geq 0$ for any real number $x\Rightarrow a\geq 2\sin2x+\sin x-\cos x$ [*]

Let $t=\sin x-\cos x=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$

$\Rightarrow\quad -\sqrt{2}\leq t\leq\sqrt{2}$, so the inequality can be written as $a\geq -2t^2+t+2$

Let $g(t)=-2t^2+t+2=-2(t-\frac{1}{4})^2+\frac{17}{8}$

then range of $g(t)$ for $-\sqrt{2}\leq t\leq\sqrt{2}$ is

$g\left(-\sqrt{2}\right)\leq g(t)\leq g\left(\frac{1}{4}\right)\Rightarrow -2-\sqrt{2}\leq g(t)\leq\frac{17}{8}$

So, the range of a can be found

$a\geq\max\limits_{|t|\leq\sqrt{2}}\Rightarrow a\geq\frac{17}{8}\Rightarrow a\in\left[\frac{17}{8},\infty\right)$

Hence, $(m+n)_{least}=17+8=25$

(59) Answer : (1)

Hint:

Use partial fraction & by parts

Solution:

$f(x)=-4\int\frac{dx}{\sqrt{x^4+x^3+1}}+\int\frac{(2x-1)(4x^3+3x^2)}{(x^4+x^3+1)^{3/2}}$

$=-4\int\frac{dx}{\sqrt{x^4+x^3+1}}-\frac{2(2x-1)}{(x^4+x^3+1)^{1/2}}+\int\frac{4dx}{(x^4+x^3+1)^{1/2}}=-\frac{2(2x-1)}{(x^4+x^3+1)^{1/2}}+c$

(60) Answer : (3)

Hint:

Substitute $t\to\frac{\pi}{4}-t$

Solution:

$a=\int\limits_0^{\pi/4}\ln\left(1+\frac{1-\tan t}{1+\tan t}\right)dt$

$2a=\frac{\pi}{4}\ln 2$

$a=\frac{\pi}{8}\ln 2$

$e^a=e^{\pi/8\ln 2}=2^{\pi/8}$

(61) Answer : (1)

Hint:

Transform it into a first order differential equation.

Solution:

$\frac{f(x)}{1+x^2}=1+\int\limits_0^x\frac{f^2(t)}{1+t^2}dt$

$\Rightarrow\frac{(1+x^2)f'(x)-2xf(x)}{(1+x^2)^2}=\frac{f^2(x)}{1+x^2}$

$\Rightarrow\frac{dy}{dx}-\left(\frac{2x}{1+x^2}\right)y=y^2$

Let $-\frac{1}{y}=t$

$\Rightarrow\frac{dt}{dx}+\left(\frac{2x}{1+x^2}\right)t=1$ solutions is

$-\frac{1}{y}\left(1+x^2\right)=\frac{x^3}{3}+x+c$

But $f(0)=1\Rightarrow c=-1\Rightarrow y=f(x)=\frac{-3\left(1+x^2\right)}{x^3+3x-3}$

$f(1)=-6$

(62) Answer : (1)

Hint:

Sum of *n* team of A.P. is

$S_n=\frac{n}{2}\left[2a+n-1\right)d]$

Solution:

$a^2+\frac{(n-1)}{2}\left[(a+4)+a+(n-1)4\right]\le 100$

$\Rightarrow a^2+(n-1)(a+2n)-100\le 0$

$\Rightarrow a^2+(n-1)a+2n(n-1)-100\le 0$

$(n-1)^2-4\left(2n^2-2n-100\right)\ge 0$

$7n^2-6n-401\le 0\Rightarrow n=8$

Now, $a^2+7a+12\le 0$

$\Rightarrow -4\le a\le -3$

(63) Answer : (2)

Hint:

Harmonic number $H_k=\int\limits_0^1\frac{1-x^k}{1-x}\,dx$

Solution:

r^{th} term of the series

$T_r=(-1)^{r-1}\cdot{}^nC_r\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{r}\right)$

clearly which can be obtained from $(-1)^{r-1}\cdot{}^nC_r\left(1+x+x^2+\cdots+x^{r-1}\right)$ by integrating within the limits 0 to 1

Let $a_r=(-1)^{r-1}{}^nC_r\left(1+x+x^2+\cdots+x^{r-1}\right)$

$a_r=(-1)^{r-1}{}^nC_r\left(\frac{1-x^r}{1-x}\right)$

$a_n=\frac{1}{x-1}\sum_{r=1}^n(-1)^{rn}C_r-\frac{1}{(1-x)}\sum_{r=1}^n(-1)^{rn}C_r\cdot x^r$

$a_n=(1-x)^{n-1}$

$S_n=\int\limits_0^1(1-x)^{n-1}dx=\frac{1}{n}$

(64) Answer : (4)

Hint:
If all roots *z* lie on this circle they are equidistant from the center of this circle. The complex number representing the centre is our answer.
Solution:
$(z + 2)^4 = -16z^4$
$\Rightarrow \left|z+2\right| = 2\left|z\right| \Rightarrow \left|\frac{z+2}{z}\right| = 2$
Which is circle of Apollonius whose centre is $\frac{2}{3}$ + 0i.

(65) Answer : (1)
Hint:
Let $A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}$, $adj(A) = \begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$
Solution:
Let $A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}$, $adj(A) = \begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$
Let $|A| = d$
$|A + d\ adjA| = \begin{vmatrix} m+qd & n(1-d) \\ p(1-d) & q+md \end{vmatrix} = 0$
$\Rightarrow d\left[(d-1)^2 + (m+q)^2\right] = 0 \Rightarrow d = 1,\ m+q = 0$

(66) Answer : (4)
Hint:
Favourable cases for bijection is $^{100}C_9 \times 9!$
Solution:
Favourable cases for bijection is $^{100}C_9 \times 9!$
For total cases
'*B*' is set of one element, number of function –1
'*B*' is set of two element, number of function -2^9
'*B*' is set of three element, number of function -3^9 …
'*B*' = Y Number of function -101^9
$\therefore$ Total cases $= 1 + {}^{100}C_1 \cdot 2^9 + {}^{100}C_2 \cdot 3^9 + \ \ldots \cdot {}^{100}C_{100} \cdot 101^9 = \sum_{r=1}^{101} r^9 \cdot {}^{100}C_{r-1}$
$\therefore$ Required probability = $\frac{{}^{100}C_9 \cdot 9!}{\sum_{r=1}^{101} r^9 \cdot {}^{100}C_{r-1}}$

(67) Answer : (2)
Hint:
Standard equation of focal chord is $y = \pm ax$
Solution:
$\frac{(x+2)^2}{16} + \frac{y^2}{12} = 1$ focus is (0, 0)
$\therefore$ equation of focal chord is $y - mx = 0$
$\frac{(x+2)^2}{16} + \frac{(mx)^2}{12} = 1$
$\Rightarrow \left(3 + 4m^2\right)x^2 + 12x - 36 = 0$
Length of chord is $\sqrt{1+m^2}\ |x_1 - x_2|$
$\therefore a = 24\sqrt{1+m^2}\ \frac{\sqrt{1+m^2}}{3+4m^2} = \frac{24(1+m^2)}{3+4m^2} \ldots (1)$
$b = \frac{24\left(1+\frac{1}{m^2}\right)}{3+\frac{4}{m^2}} = \frac{24(1+m^2)}{3m^2+4} \qquad \ldots (2)$
from (1) and (2)
$7\left(1+m^2\right) = 24\left(1+m^2\right)\left(\frac{1}{a} + \frac{1}{b}\right) \Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{7}{24}$
$\Rightarrow \quad \frac{2ab}{a+b} = \frac{48}{7}.$

(68) Answer : (4)
Hint:
Equivalence relations are reflexive, symmetric and transitive
Solution:
$R = \{(x, y) : y \in A_i, iff\ x \in A_1, 1 \le i \le k\}$

(1) Reflexive

$(a, a) \Rightarrow a \in A_i$ iff $a \in A_i$

(2) Symmetric

$(a, b) \Rightarrow a \in A_i$ iff $a \in A_i$

(b, a) $\in R$ as $b \in A$, iff $a \in A$,

(3) Transitive

$(a,\ b) \in R \& (b,\ c) \in R$

$\Rightarrow a \in A_i$ iff $b \in A_i \& b \in A_j$ iff $c \in A_j$

$\Rightarrow a \in A_i$ iff $c \in A_j$

$\Rightarrow (a,\ c) \in R$

⇒ Relation is equivalence

(69) Answer : (4)

Hint:

Use derived formulae from sum & difference formulae of trigonometric ratios is

Solution:

We have $a = \frac{4+\sec 20^\circ}{cosec 20^\circ} = \frac{\sin 20^\circ}{\cos 20^\circ}$

(4cos 20° + 1) = $\frac{2\sin 40^\circ + \sin 20^\circ}{\cos 20^\circ}$

= $\frac{\sin 40^\circ + (\sin 40^\circ + \sin 20^\circ)}{\cos 20^\circ}$

= $\frac{\sin 40^\circ + 2\sin 30^\circ \cos 10^\circ}{\cos 20^\circ}$

= $\frac{\sin 40^\circ + \sin 80^\circ}{\cos 20^\circ}$

= $\frac{2\sin 60^\circ \cos 20^\circ}{\cos 20^\circ} = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

(70) Answer : (3)

Hint:

Centroid $\alpha = \frac{\sum_{i=1}^{4} x_i}{4}$

Solution:

Let C = (λ_1 + 1, 2λ_1 + 2, 3λ_1 + 3) and

D = (λ_2 + 1, 2λ_2 + 2, 3λ_2 + 3)

$CD = \sqrt{14}$

⇒ $(\lambda_1 - \lambda_2)^2 + 4(\lambda_1 - \lambda_2)^2 + 9(\lambda_1 - \lambda_2)^2 = 14$

⇒ $\lambda_2 - \lambda_1 = 1 \Rightarrow \lambda_2 = 1 + \lambda_1$

Let G = (α, β, γ)

4α = 5 + 2λ_1

4β = 10 + 4λ_1

4γ = 9 + 6λ_1

Locus of G is $\frac{4x-5}{2} = \frac{4y-10}{4} = \frac{4z-9}{6}$

Section-II

(71) Answer : 2

Hint:

As product of roots = 72. Hence the divisor 2, –2, 3, –3, all as satisfied ⇒ confirms to be root.

Solution:

As product of roots = 72. Hence the divisor 2, –2, 3, –3, all as satisfied ⇒ confirms to be root.

$z^5 - 2z^4 - 13z^3 + 26z^2 + 36z - 72 = (z-2)$

$\left(z^4 - 13z^2 + 36\right)$

$\therefore$ Sum of real roots $= 2 + k + (-k) = 2$

(72) Answer : 7

Hint:

Number of required triangles = ${}^{n}C_3 - n(n-4) - n$

Solution:

Number of required triangles = ${}^{n}C_3 - n(n-4) - n$

$= \frac{n(n-4)(n-5)}{6}$

$= \frac{8\times 4\times 3}{6} = 16$

(73) Answer : 7

Hint:

$ax^2 + bx + c > 0$ if $a > 0$, $D < 0$

Solution:

$O(0, 0)$ lies above both the lines $x + y + 1 = 0$ and $-x + 2y + 3 = 0$

$\Rightarrow\ a + a^2 + 1 > 0$ and $-a + 2a^2 + 3 > 0$

$a \in R$

$2a^2 - a + 3 > 0$

$a^2 - \frac{a}{2} + \frac{3}{2} > 0$

$\left(a - \frac{1}{4}\right)^2 + \frac{3}{2} - \frac{1}{16} > 0,\ a \in R$

i.e. $a = \{-2, -1, 0, 1, 2, 3, 4\}$

(74) Answer : 37

Hint:

M_1 and M_2 are mirror image of each other

Solution:

M_1 and M_2 are mirror image of each other about PC, $PC = 13$

In triangle $PCM_1,\ 9 + PM_1{}^2 = 13^2$

In triangle $NCM_1,\ M_1N^2 + x^2 = 9$

In triangle $PNM_1, (13 - x)^2 + \left(9 - x^2\right) = 13^2 - 9$

$\Rightarrow 13^2 + x^2 - 26x + 9 - x^2 = 13^2 - 9$

$x = \frac{9}{13}$

$\Rightarrow M_1N = \frac{12}{13}\sqrt{10}$ and $M_1M_2 = \frac{24}{13}\sqrt{10}$ units

(75) Answer : 5

Hint:

Form : 1^∞

Solution:

$\lim\limits_{x \to 0} \left(1 + f(x) - 1\right)^{\frac{1}{f(x)-1} \times \frac{f(x)-1}{x}}$

$= \lim\limits_{x \to 0} e^{\frac{f(x)-1}{x}} \Rightarrow f'(0) = 4$

$f(0) = 1$