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Medical | IIT-JEE | Foundations

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MM : 180

AIATS For Two Year JEE(Advanced)-2027 (XI Studying-P1)_Test-4A_Paper-2_Online

Time : 180 Min.

PHYSICS

Section-I

- | | |
|--------|--------|
| 1. (C) | 3. (C) |
| 2. (B) | 4. (C) |

Section-II

- | | |
|----------|----------|
| 5. (A,C) | 7. (B,D) |
| 6. (B,D) | 8. (A,D) |

Section-III

- | | |
|-------------|-------------|
| 9. (12.80) | 13. (15.00) |
| 10. (05.00) | 14. (02.00) |
| 11. (00.80) | 15. (02.00) |
| 12. (03.00) | 16. (27.00) |

CHEMISTRY

Section-I

- | | |
|---------|---------|
| 17. (B) | 19. (A) |
| 18. (C) | 20. (D) |

Section-II

- | | |
|-----------|-------------|
| 21. (B,C) | 23. (A,B,C) |
| 22. (B,C) | 24. (A,D) |

Section-III

- | | |
|-------------|-------------|
| 25. (03.46) | 29. (08.24) |
| 26. (11.00) | 30. (08.00) |
| 27. (00.04) | 31. (03.00) |
| 28. (04.00) | 32. (12.50) |

Hints and Solutions

PHYSICS

Section-I

(1) Answer : (C)

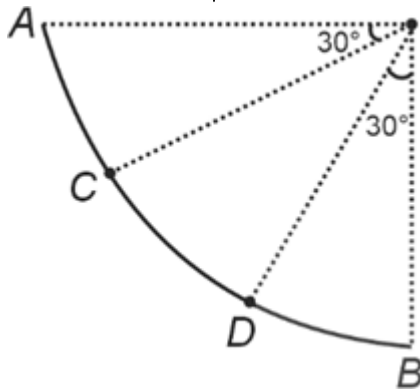
Hint:

$$V_C = \sqrt{gl}, \quad V_D = \sqrt{\sqrt{3} gl}$$

Solution:

$$t_{AP} > \sqrt{\frac{2\ell \sin 30}{g}} = \sqrt{\frac{\ell}{g}}$$

$$V_C = \sqrt{gl}, \quad V_D = \sqrt{\sqrt{3} gl}$$



The bob clearly covers the $\frac{\ell\pi}{6}$ long arcs CD and DB more rapidly than if it had moved along the first section with a constant speed V_C and along the later one with speed V_D .

$$t_{CB} < \frac{\ell\pi}{6V_C} + \frac{\ell\pi}{6V_D} \approx 0.92 \sqrt{\frac{\ell}{g}}$$

 So, $t_1 > t_2$

(2) Answer : (B)

Hint:

$$\vec{a} = \frac{d^2x}{dt^2} \hat{i} + \frac{d^2y}{dt^2} \hat{j} + \frac{d^2z}{dt^2} \hat{k}$$

Solution:

$$\frac{dz}{dt} = 3\alpha x^2 \frac{dx}{dt} + 2\beta y \frac{dy}{dt} = 3\alpha vx^2 + 2\beta vy$$

$$\frac{d^2z}{dt^2} = 6\alpha vx \left(\frac{dx}{dt}\right) + 2\beta v \left(\frac{dy}{dt}\right)$$

$$= 6\alpha v^2 + 2\beta v^2$$

Now, acceleration of particle is

$$\vec{a} = \frac{d^2x}{dt^2} \hat{i} + \frac{d^2y}{dt^2} \hat{j} + \frac{d^2z}{dt^2} \hat{k}$$

$$= (6\alpha v^2 x + 2\beta v^2) \hat{k}$$

(3) Answer : (C)

Hint:

Area = displacement

Solution:

Area = displacement

$$\Rightarrow v_0 = \frac{3}{2} \text{ m/s}$$

(4) Answer : (C)

Hint:

$$Y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

Solution:

Centre of gravity first goes down then rise up.

Section-II

(5) Answer : (A,C)

Hint:

$$a = \frac{dV}{dt}$$

Solution:

$$a = \frac{d}{dt} (\alpha \sqrt{x}) = \frac{\alpha^2}{2} = \text{constant}$$

$$a = \frac{dV}{dt}, v = \frac{\alpha^2}{2} t$$

$$\langle v \rangle = \frac{\int_0^t v \, dt}{\int_0^t dt} = \frac{5}{t}$$

$$a = \frac{\alpha^2}{2}, t = \frac{2\sqrt{3}}{\alpha}$$

$$\langle v \rangle = \frac{\alpha\sqrt{x}}{2}$$

(6) Answer : (B,D)

Hint:

$$[X] = \text{ML}^{-1}$$

Solution:

$$[X] = \frac{[F] [V^2]}{[Y^2]}$$

$$\text{and } \frac{[2\pi] [Y]}{[V^2]} = 1$$

$$[Y] = \text{L}^2 \text{T}^{-2}$$

$$[X] = \text{ML}^{-1}$$

(7) Answer : (B,D)

Hint:

$$Q = Q_{AB} + Q_{BC} + Q_{CA}$$

Solution:

$$Q = Q_{AB} + Q_{BC} + Q_{CA}$$

$$Q_{AB} = C_P \Delta T = C_P \frac{\Delta U}{C_V} = -5 V_0$$

$$Q_{BC} = \Delta U = 3U_0$$

$$Q_{CA} = nRT \ln \left(\frac{V_A}{V_C} \right) = \frac{10u_0}{3} \ln \left(\frac{\rho_C}{\rho_A} \right)$$

$$\rho_A = 2\rho_0, \rho_C = \rho_B = 5\rho_0$$

$$Q_{CA} = \frac{10u_0}{3} \ln \left(\frac{5}{2} \right)$$

$$Q = \left[\frac{10}{3} \ln \left(\frac{5}{2} \right) - 2 \right] u_0$$

$$W_{AB} = \Delta Q_{AB} - \Delta U_{AB} = -2u_0$$

(8) Answer : (A,D)

Hint:

$$mg(2x_0) = mgx_0 + \frac{1}{2} k x_0^2$$

Solution:

$$mg(2x_0) = mgx_0 + \frac{1}{2} k x_0^2$$

$$x_0 = \text{amplitude of block A} = \frac{2mg}{K}$$

$$mgx_0 = mg \left(\frac{x_0}{2} \right) + \frac{1}{2} K \left(\frac{x_0}{2} \right)^2 + \frac{1}{2} m V_m^2 + \frac{1}{2} m \left(\frac{V_m}{2} \right)^2$$

$$V_m = 2g \sqrt{\frac{m}{5K}}$$

$$\frac{V_m}{2} = \omega \left(\frac{x_0}{2} \right)$$

$$T = \frac{2\pi}{\omega} = \frac{1}{2\pi} \sqrt{\frac{K}{5m}}$$

Section-III

(9) Answer : 12.80

Hint:

$$\Delta T \sin \theta = \Delta mg$$

Solution:

Let change in tension is ΔT

$$2, \Delta T \sin \beta = \Delta mg, \Delta \ell = \frac{(\Delta T)L}{AY}$$

$$\text{But } \Delta \ell = L\alpha \Delta \theta$$

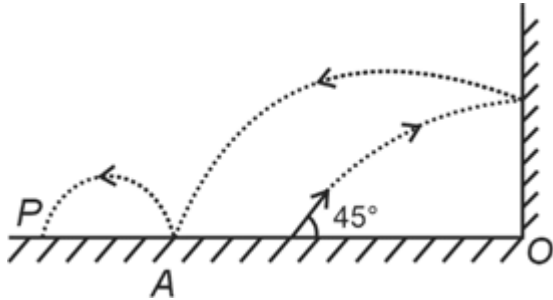
$$\Delta m = \frac{\alpha \Delta \theta \times A \times Y \times 2 \sin \beta}{g} = \frac{64}{5} \text{ kg}$$

(10) Answer : 05.00

Hint:

$$T = \frac{2u}{g}$$

Solution:



$$u_x = 10 \text{ m/s}, u_y = 10 \text{ m/s}$$

Due to the collision the direction of u_x changes, while u_y remains same.

Hence time of flight does not change.

$$T = \frac{2u_y}{g} = 2 \text{ sec}$$

Particle strikes at $t = 0.5$ sec, hence it remains in air for 1.5 sec more, and travels, $OA = u_x \times 1.5 = 15 \text{ m}$

Now when it strikes the ground and rebounds its horizontal velocity does not change while vertical becomes, $V_y' = e v_y =$

$$0.5 \times 10 = 5 \text{ m/s}$$

Now the range of this projectile,

$$AP = \frac{(2u_x V_y')}{g} = \frac{2 \times 10 \times 5}{10} = 10 \text{ m}$$

$$OP = 25 \text{ m}$$

(11) Answer : 00.80

Hint:

$$a = \alpha R$$

Solution:

About centre of disc

$$\frac{3mg}{2} + Mg = N \Rightarrow N = \frac{5Mg}{2}$$

$$f = \frac{5}{2} \mu mg$$

$$\alpha R = a_{com}, a_{com} = \frac{f}{m}$$

$$\frac{MR^2}{2} \alpha = \frac{3Mg}{2} \times 2R - \frac{2\mu MRg}{2}$$

$$\alpha = \frac{6g - 5\mu g}{R}$$

$$\alpha R = \frac{f}{m} = 6g - 5\mu g$$

$$\mu = \frac{4}{5} = 0.8$$

(12) Answer : 03.00

Hint:

$$\int F dt = \Delta p$$

Solution:

$$F = \frac{4F_0}{T_0^2} t (T_0 - t)$$

$$a = \frac{4F_0}{mT_0^2} t (T_0 - t)$$

$$V = \frac{4F_0}{mT_0^2} \left[T_0 \frac{t^2}{2} - \frac{t^3}{3} \right]$$

$$\int_0^s ds = \frac{4F_0}{mT_0^2} \left[\frac{T_0}{2} \int_0^{T_0} t^2 dt - \frac{1}{3} \int_0^{T_0} t^3 \cdot dt \right]$$

$$S = \frac{F_0 T_0^2}{3m}$$

(13) Answer : 15.00

Hint:

$$y = \frac{\omega^2}{2g} x^2$$

Solution:

$$\Delta h = \frac{\omega^2 (r_2^2 - r_1^2)}{2g}$$

$$= \frac{100 \times (0.2^2 - 0.1^2)}{2 \times 10}$$

$$= \frac{100 \times 3}{100 \times 2 \times 10} \times (100)$$

$$= 15 \text{ cm}$$

(14) Answer : 02.00

Hint:

$$\Delta P = \rho g \Delta h$$

Solution:

$$OM (\rho - d)g + OM (\rho + d)g = ON (\rho + d)g + ON (\rho)g$$

$$OM (\rho - d) + (R - OM) (\rho + d) = (R - ON) (\rho + d) + ON (\rho)$$

$$OM (\rho - d - \rho - d) = ON (\rho - \rho - d)$$

$$2OM = ON$$

$$\frac{ON}{OM} = 2$$

(15) Answer : 02.00

Hint:

$$\Delta P = -\beta \frac{\Delta V}{V}$$

Solution:

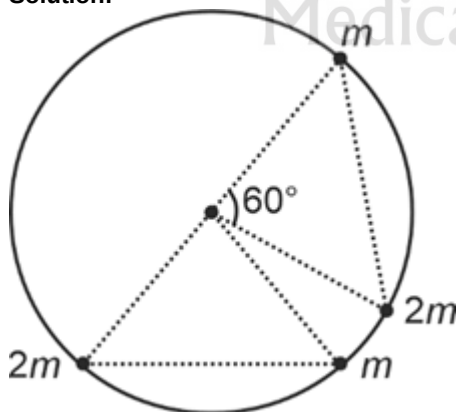
$$\frac{W}{V_0} = \frac{(P - P_0)^2}{2B}$$

(16) Answer : 27.00

Hint:

Work energy theorem

Solution:



$$\frac{1}{2} (2m)V^2 + \frac{1}{2} mV^2 = \Delta P \cdot E$$

$$\Delta P \cdot E = 2mg (\ell \cos 30 - \ell \sin 60^\circ)$$

$$+ mg \left(\ell \cos 30 + \frac{\ell}{2} \right)$$

$$= mg \ell \left[\frac{3\sqrt{3}-1}{2} \right]$$

$$V = \sqrt{\left(\frac{3\sqrt{3}-1}{3} \right) g \ell}$$

CHEMISTRY

Section-I

(17) Answer : (B)

Hint:

Acc. to Bohr model of atom, $r = \frac{n^2}{Z} a_o$

Solution:

Acc. to Bohr model of atom, $r = \frac{n^2}{Z} a_o$ A. For H-atom $r = \frac{2^2}{1} a_o = 4a_o$ B. For Li^{2+} ion $r = \frac{3^2}{3} a_o = 3a_o$ C. For He^+ ion $r = \frac{3^2}{2} a_o = 4.5a_o$ D. For H atom (3rd orbit) $r = \frac{3^2}{1} a_o = 9a_o$

(18) Answer : (C)

Hint:

Polluted water could have a BOD value of 17 ppm or more.

Solution:

Freons are non-reactive, non-flammable, non-toxic organic molecules and therefore used in refrigerators, air conditioners etc.

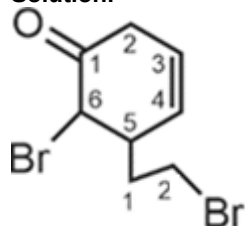
Polluted water could have a BOD value of 17 ppm or more.

(19) Answer : (A)

Hint:

Functional group is present in ring.

Solution:



6-Bromo-5-(2-bromo ethyl) cyclohex-3-enone

(20) Answer : (D)

Hint:

$$\Delta v \times \Delta x \geq \frac{h}{4\pi m}$$

Solution:

Given $\Delta x = 4 \times \Delta V$

$$\Delta v \times \Delta x \geq \frac{h}{4\pi m}$$

$$\Delta v \times 4\Delta v \geq \frac{h}{4\pi m}$$

$$\Delta v^2 \geq \frac{h}{16\pi m}$$

$$\Delta v \geq \frac{1}{4} \sqrt{\frac{h}{\pi m}}$$

Section-II

(21) Answer : (B,C)

Hint:

Since no heat exchange occurs

$$\Delta S_{\text{surrounding}} = 0$$

Solution:

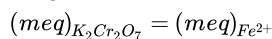
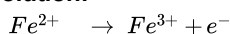
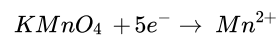
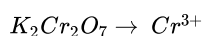
Since no heat exchange occurs

$$\Delta S_{\text{surrounding}} = 0$$

$$\Delta S_{\text{system}} = nC_v \ln \frac{T_2}{T_1} + nR \ln \frac{v_2}{v_1}$$

$$= 0 + 2 \times 8.3 \ln \frac{10}{4}$$

$$= 15.23 \text{ JK}^{-1}$$

(22) Answer : (B,C)**Hint:****Solution:** $n\text{-factor} = 1$  $n\text{-factor} = 5$  $n\text{-factor} = 6$

$$m.\text{eq of } Fe^{2+} = m.\text{eq of } K_2Cr_2O_7$$

$$m.\text{eq } Fe^{2+} = 0.1 \times 20 \times 6 = 12$$

$$m.\text{eq } Fe^{2+} = m.\text{eq } KMnO_4$$

$$12 = 0.1 \times V \times 5$$

$$V = 24 \text{ mL}$$

$$\% \text{ purity} = \frac{12 \times 10^{-3} \times 152 \times 100}{2}$$

$$= 91.2$$

(23) Answer : (A,B,C)**Hint:**Generally odd e^- species can form dimer**Solution:**NO₂ forms N₂O₄ICl₃ forms I₂Cl₃Al(CH₃)₃ forms Al₂(CH₃)₆ClO₂ does not form dimer.**(24) Answer : (A,D)****Hint:**

$$\% \text{ of free } SO_3 = \frac{40x}{9}$$

Solution:

$$\% \text{ of free } SO_3 = \frac{40x}{9} = \frac{40 \times 4.5}{9} = 20\%$$

Mass of H₂SO₄ in sample after addition of water = 261.25g eq of H₂SO₄ = g eq of NaOH

$$\frac{261.25}{98} \times 2 = 2 \times V$$

$$2.66 \text{ L} = V$$



Section-III

(25) Answer : 03.46**Hint:**[H⁺] coming from second dissociation of H₂CO₃ is negligible**Solution:**

$$[CH_3COOH]_{\text{after mixing}} = 0.001$$

$$[H_2CO_3]_{\text{after mixing}} = 0.2$$

$$[H^+] = \sqrt{K_{a_{CH_3COOH}} \times C + K_{a_{H_2CO_3}} \times C}$$

$$= \sqrt{2 \times 10^{-5} \times 0.001 + 0.2 \times 5 \times 10^{-7}}$$

$$= \sqrt{2 \times 10^{-8} + 1 \times 10^{-7}}$$

$$= \sqrt{12} \times 10^{-4}$$

$$pH = -\log(\sqrt{12} \times 10^{-4}) = [4 - \frac{1}{2}(2 \log 2 + \log 3)]$$

$$= 4 - \frac{1}{2}[2 \times 0.3 + 0.48]$$

$$= 3.46$$

(26) Answer : 11.00**Hint:**

$$\Delta_r G^\circ = \Delta_r H^\circ - T \Delta_r S^\circ$$

Solution:

$$\Delta_r G^\circ = \Delta_r H^\circ - T \Delta_r S^\circ$$

$$\Delta_r G^\circ = (80 - 120) \times 10^3 - 300 \times (70 - 110)$$

$$\Delta_r G^\circ = -40 \times 10^3 + 12 \times 10^3$$

$$\Delta_r G^\circ = -28 \times 10^3$$

$$-RT \ln K = -28 \times 10^3$$

$$-8.30 \times 300 \ln k = -28 \times 10^3$$

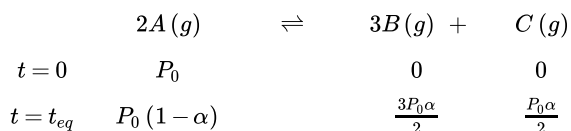
$$\ln k = 11.24$$

(27) Answer : 00.04

Hint:

$$K_p = \frac{(p_B)^3 (p_C)}{(p_A)^2}$$

Solution:



$$\alpha \ll 1$$

$$K_p = \frac{27\alpha^4}{16} P^2$$

$$\alpha = 2 \sqrt[4]{\frac{K_p}{27P^2}} = 2 \sqrt[4]{\frac{432 \times 10^{-12}}{27 \times (0.01)^2}}$$

$$= 2 \times 2 \sqrt{10^{-8}}$$

$$= 0.04$$

(28) Answer : 04.00

Hint:

PF₃ is pyramidal

Solution:

XeOF₄, BrF₅, SO₃ & PCl₃F₂

have at least 4 atom in same plane.

(29) Answer : 08.24

Hint:

$$K_{sp} = [Ag^+][Br^-]$$

Solution:

$$K_{sp} = [Ag^+][Br^-]$$

$$K_{sp} = [2 \times 10^{-5}][2 \times 10^{-5}]$$

$$K_{sp} = 4 \times 10^{-10} \text{ M}$$

For precipitation

$$Q_{ip} = K_{sp}$$

$$[Ag^+][Br^-] = 4 \times 10^{-10}$$

$$[Br^-] = \frac{4 \times 10^{-10}}{0.5} = 8 \times 10^{-10}$$

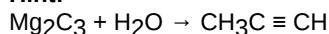
$$8 \times 10^{-10} = [NaBr] = \frac{w}{103 \times 10}$$

$$8 \times 103 \times 10 \times 10^{-10} = w$$

$$8.24 \times 10^{-7} \text{ g} = w$$

(30) Answer : 08.00

Hint:

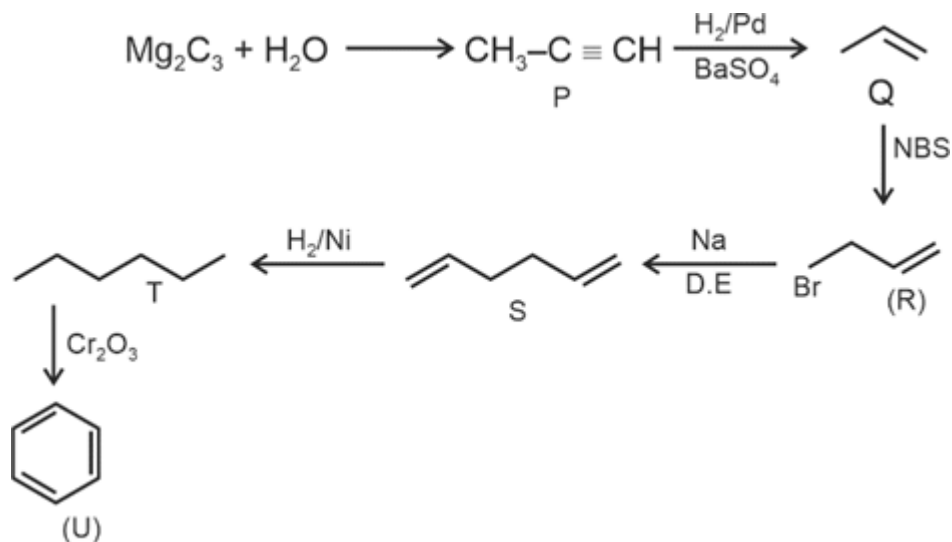


Solution:



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(31) Answer : 03.00

Hint:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Solution:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{\frac{710-26}{760} \times 45}{300} = \frac{1 \times V_{\text{mL}}}{273}$$

$$36.855 \text{ mL} = V_{\text{mL}}$$

$$\text{Moles of N}_2 = \frac{36.855}{22400}$$

$$\text{Mass of N atoms} = 0.0460 \text{ g}$$

$$\text{Molar mass of compound} = \frac{0.5 \times 28}{0.046}$$

$$= 304.347 \text{ g}$$

(32) Answer : 12.50

Hint:

$$b = 4 \times \frac{4}{3} \pi r^3 \times N_A$$

Solution:

$$b = 4\pi \times 10^{-4} \times 1000 = 4 \times \frac{4}{3} \pi r^3 \times 6 \times 10^{23}$$

$$r = 5 \times 10^{-9} \text{ cm}$$

$$\text{Distance of closest approach} = 2r$$

$$= 2 \times 5 \times 10^{-9} \text{ cm} = 10^{-8} \text{ cm} = 10^{-10} \text{ m}$$

$$y = 10$$

$$1.25y = 12.50$$



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MATHEMATICS

Section-I

(33) Answer : (A)

Hint:

$$e_H = 2$$

Solution:

$$\frac{x^2}{36} + \frac{y^2}{11} = 1$$

$$e^2 = 1 - \frac{11}{36} \Rightarrow e = \frac{5}{6}$$

$$\text{Foci of ellipse is } (\pm ae, 0) = (\pm 5, 0)$$

$$\text{hyperbola is } \frac{x^2}{\frac{100}{16}} - \frac{y^2}{\frac{P}{16}} = 1$$

$$a = \frac{10}{4} = \frac{5}{2}$$

$$b^2 = \frac{P}{16}$$

$$ae_H = 5$$

$$\frac{5}{2} \cdot e_H = 5$$

$$e_H = 2$$

$$4 = 1 + \frac{\frac{P}{16}}{\frac{100}{16}}$$

$$\Rightarrow 3 = \frac{P}{100}$$

$$\Rightarrow P = 300$$

Length of latus rectum

$$= \frac{2 \cdot 300}{16 \cdot \frac{5}{2}} = 15.$$

(34) Answer : (C)

Hint:

$$D = \left(\frac{1+5+x_1}{3}, \frac{3+7+y_1}{3} \right)$$

Solution:

Take point $A(x_1, y_1)$ & $D(\alpha, \beta)$

$$D = \left(\frac{1+5+x_1}{3}, \frac{3+7+y_1}{3} \right) = (\alpha, \beta)$$

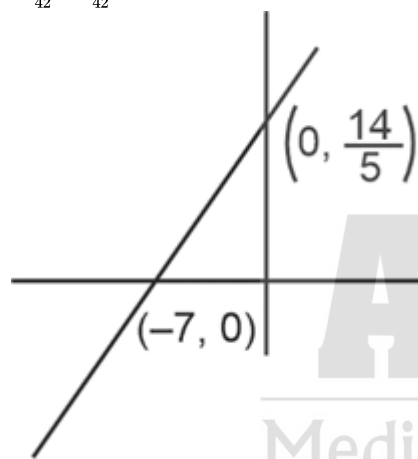
$$x_1 = 3\alpha - 6$$

$$y_1 = 3\beta - 10$$

$$\text{also } 2x_1 - 5y_1 + 4 = 0$$

$$2(3\alpha - 6) - 5(3\beta - 10) + 4 = 0$$

$$-\frac{6\alpha}{42} + \frac{15\beta}{42} = 0$$



$$\text{Area} = \frac{1}{2} \times 7 \times \frac{14}{5}$$

$$= \frac{49}{5}$$

(35) Answer : (C)

Hint:

$$\text{For } f(x) > 0 \quad \forall x \in R$$

$$D < 0$$

Solution:

$$\text{For } f(x) > 0 \quad \forall x \in R$$

$$\text{as } a > 0$$

$$\text{so } D < 0$$

$$[2(P+3)]^2 - 4(-6P+3) < 0$$

$$4(P^2 + 9 + 6P) + 24P - 12 < 0$$

$$4P^2 + 48P + 24 < 0$$

$$\text{so } P \in [-11, -1]$$

$$\text{probability} = \frac{11}{31}$$

(36) Answer : (C)

Hint:

$$\left(\frac{0}{0} \right) \text{ form, apply L'Hopital's}$$

Solution:

$$\left(\frac{0}{0} \right) \text{ form, so here we apply L'Hopital's}$$

$$\lim_{x \rightarrow 3} \frac{2x(1+f(3)) - 9f'(x) - 6}{1}$$

$$= 2 \times 3 (1 + 2) - 9f'(3) - 6$$

$$= 18 - 15$$

$$= 3$$

Section-II

(37) Answer : (A,C)

Hint:

$$n(A) = 2(50)^2 + 1$$

Solution:

$$A = S_1 \cup S_2 \cup \dots \cup S_{50} = S_{50}$$

$$B = S_1 \cap S_2 \cap \dots \cap S_{38} = S_1$$

$$S_1 < S_{50}$$

$$A < B$$

$$n(A) = 2(50)^2 + 1 = 5001$$

$$n(B) = 2(1)^2 + 1 = 3$$

$$A < B \Rightarrow n(A - B) = n(A) - n(B)$$

$$n(A \Delta B) = n(A - B)$$

(38) Answer : (A,D)

Hint:

$$\frac{a+b+c}{3} = 8$$

Solution:

$$a < b < c$$

$$\text{Median} = 10$$

$$\Rightarrow b = 10$$

$$\frac{a+b+c}{3} = 8$$

$$a + c = 14$$

As there is no mode.

$$0 < a < 10$$

$$c > 10$$

$$a + c = 14.$$

(a, c) can be (1, 13), (2, 12), (3, 11). So possible ordered triplet \rightarrow (1, 10, 13), (2, 10, 12), (3, 10, 11)

(39) Answer : (A,B)

Hint:

$$\text{Terms in } (8 + x^3)^{50} = 51$$

Solution:

$$(2 + x)^{52} (x^2 + 4 - 2x)^{50}$$

$$(2 + x)^2 [(2 + x) (x^2 + 4 - 2x)]^{50}$$

$$(2 + x)^2 (8 + x^3)^{50}$$

$$\text{Terms in } (8 + x^3)^{50} = 51$$

$$\text{Terms in } (2 + x)^2 = 3$$

$$\text{Total terms} = 153 = a$$

$$\sum_{r=1}^{12} r \cdot \sum_{r=1}^{15} C_r = \sum_{r=1}^{12} (16 - r) = 192 - 78$$

$$= 114 = b$$

(40) Answer : (A,C)

Hint:

$$2ae = 5$$

Solution:

$$x + y - 2 = 0$$

$$2x + y - 3 = 0$$

$$\text{P.O.I.} = (1, 1)$$

$$2x + 3y - 23 = 0$$

$$2x - y - 3 = 0$$

$$\text{P.O.I.} = (4, 5)$$

$$2ae = \sqrt{(4-1)^2 + (5-1)^2} = 5$$

$$2a = \sqrt{(-2-4)^2 + (5-5)^2} + \sqrt{(-2-1)^2 + (5-1)^2}$$

$$= 6 + 5 = 11$$

$$2a = 11$$

$$e = \frac{5}{11}$$

Area of triangle,

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 1 \\ -2 & 5 & 1 \end{vmatrix} = \frac{1}{2} [1(0) - 1(4+2) + 1(20+10)]$$

$$= 12 \text{ sq. units}$$

Section-III

(41) Answer : 04.00

Hint:

Put $x/y = t$

Solution:

$$\ln x + \ln y = \ln (x-2y)^2$$

$$xy = x^2 + 4y^2 - 4xy$$

$$x^2 + 4y^2 - 5xy = 0$$

$$\frac{x^2}{y^2} + 4 - 5\frac{x}{y} = 0$$

Put $\frac{x}{y} = t$

$$t^2 - 5t + 4 = 0$$

$$t = 1 \text{ \& } t = 4$$

$$\frac{x}{y} = 1 \text{ or } \frac{x}{y} = 4$$

$x \neq y$

$$\text{so } \frac{x}{y} = 4$$

(42) Answer : 02.00

Hint:

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1+\cos x}{2}}$$

Solution:

$$\frac{\sum_{K=1}^{44} \cos K^\circ}{\sum_{K=1}^{44} \sin K^\circ} = \frac{\cos 1^\circ + \cos 2^\circ + \dots + \cos 44^\circ}{\sin 1^\circ + \sin 2^\circ + \dots + \sin 44^\circ}$$

$$= \frac{\cos 1^\circ + \cos 2^\circ + \dots + \cos 44^\circ}{\cos 89^\circ + \cos 88^\circ + \dots + \cos 46^\circ}$$

$$\text{As } \cos x + \cos y = 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$$

$$= \frac{2 \cos \left(\frac{45^\circ}{2} \right) \left[\cos \left(\frac{43^\circ}{2} \right) + \cos \left(\frac{41^\circ}{2} \right) + \dots + \cos \left(\frac{1^\circ}{2} \right) \right]}{2 \cos \left(\frac{135^\circ}{2} \right) \left[\cos \left(\frac{43^\circ}{2} \right) + \cos \left(\frac{41^\circ}{2} \right) + \dots + \cos \left(\frac{1^\circ}{2} \right) \right]}$$

$$= \frac{\cos \left(\frac{45^\circ}{2} \right)}{\cos \left(\frac{135^\circ}{2} \right)}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1+\cos x}{2}}$$

$$= \frac{\sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}}}{\sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}} \Rightarrow t = 1 + \sqrt{2}$$

$$[t] = 2$$

(43) Answer : 00.00

Hint:

$$\alpha = \arg(0 + 1)$$

Solution:

$$\alpha = \arg(\omega^2 - \omega^3 + \omega^4 - \omega^5 + \omega^6 - \omega^7 + \dots + \omega^{676} - \omega^{677} + \omega^{678} - \omega^{679} + \omega^{681})$$

$$\alpha = \arg(0 + 1)$$

$$\alpha = 0$$

$$\Rightarrow 2\alpha = 0$$

(44) Answer : 35.00

Hint:

$$\text{Let } x_1 = a \Rightarrow a + x_2 + 7 = 42$$

Solution:

$$\text{Let } x_1 = a$$

$$\text{so } a + x_2 + 7 = 42$$

$$\Rightarrow x_2 = 35 - a$$

$$x_2 + x_3 + x_4 = 42$$

$$\Rightarrow 35 - a + 7 + x_4 = 42$$

$$\Rightarrow x_4 = a$$

$$x_3 + x_4 + x_5 = 42$$

$$\Rightarrow x_5 = 35 - a$$

$$x_6 = 7, x_7 = a, x_8 = 35 - a$$

$$\Rightarrow x_1 + x_8 = 35$$

(45) Answer : 05.00

Hint:

$$\lambda = -2, -3$$

Solution:

Lines are perpendicular, so

$$\lambda \cdot \lambda + (\lambda + 5) \cdot 1 + 2 \left(2\lambda + \frac{1}{2} \right) = 0$$

$$\lambda^2 + \lambda + 5 + 4\lambda + 1 = 0$$

$$\lambda^2 + 5\lambda + 6 = 0$$

$$\lambda = -2, -3$$

$$\text{so modulus of sum is } = |-2 - 3| = \boxed{5}$$

(46) Answer : 29.00

Hint:

$$(x - 2)(x^2 - x + 1) = 0$$

Solution:

Clearly, $x = 2$ is a root.

$$(x - 2)(x^2 - x + 1) = 0$$

Roots are $x = 2, -\omega, -\omega^2$

$$2^{30} + (-\omega)^{30} + (-\omega^2)^{30}$$

$$2^{30} + \omega^{30} + \omega^{60}$$

$$2^{30} + 1 + 1$$

$$2^{30} + 2$$

$$2(1 + 2^{29})$$

(47) Answer : 05.00

Hint:

$$e^{2x} = 16, e^x = \frac{3}{5}, \frac{4}{3}$$

Solution:

$$(e^{2x} - 16)(15e^{2x} - 29e^x + 12) = 0$$

$$(e^{2x} - 16)(5e^x - 3)(3e^x - 4) = 0$$

$$e^{2x} = 16, e^x = \frac{3}{5}, e^x = \frac{4}{3}$$

$$x = \frac{1}{2} \ln 16, \ln \frac{3}{5}, \ln \frac{4}{3}$$

Sum of all roots:

$$\ln \left[4 \times \frac{3}{5} \times \frac{4}{3} \right]$$

$$\Rightarrow K = 5$$

(48) Answer : 06.00

Hint:

Sum of x-coordinates of the 7th roots of unity is 0.

Solution:

$$1 + \cos \frac{3\pi}{7} + \cos \frac{\pi}{7} - \cos \frac{2\pi}{7}$$

As sum of x-coordinates of the 7th roots of unity is 0, we have

$$\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \cos \frac{8\pi}{7} + \cos \frac{10\pi}{7} + \cos \frac{12\pi}{7} + \cos \frac{14\pi}{7} = 0$$

$$\Rightarrow \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \cos \frac{8\pi}{7} + \cos \frac{10\pi}{7} + \cos \frac{12\pi}{7} = -1$$

$$\Rightarrow \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \cos \left(-\frac{6\pi}{7} \right) + \cos \left(-\frac{4\pi}{7} \right) + \cos \left(-\frac{2\pi}{7} \right) = -1$$



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$$\begin{aligned}\Rightarrow 2 \left(\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} \right) &= -1 \\ \Rightarrow \cos \frac{2\pi}{7} - \cos \frac{3\pi}{7} - \cos \frac{\pi}{7} &= -\frac{1}{2} \\ \Rightarrow \cos \frac{\pi}{7} - \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} &= \frac{1}{2} \\ \Rightarrow 1 + \cos \frac{3\pi}{7} + \cos \frac{\pi}{7} - \cos \frac{2\pi}{7} &= \frac{3}{2} = t \\ \Rightarrow 4t &= 4 \times \frac{3}{2} \\ &= 6\end{aligned}$$



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