03/05/2026

Code-A_(ONLINE)

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MM : 180

AIATS For One Year JEE(Advanced)-2026 (XII Studying/XII Passed)-Test-7A_Paper-1_Online

Time : 180 Min.

PHYSICS

Section-I

1. (A,B,C,D)

2. (B,D)

3. (A,B,C)

Section-II

4. (D)

5. (D)

6. (B)

7. (B)

Section-III

8. (3)

9. (11)

10. (5)

11. (4)

12. (3)

13. (7)

Section-IV

14. (B)

15. (B)

16. (B)

17. (D)

CHEMISTRY

Section-I

18. (A,B,C)

19. (B,C)

20. (A,C)

Section-II

21. (C)

22. (A)

23. (C)

24. (B)

Section-III

25. (6)

26. (1)

27. (14)

28. (6)

29. (6)

30. (12)

Section-IV

31. (B)
32. (B)
33. (C)
34. (A)

MATHEMATICS

Section-I

35. (A,B,C,D)
36. (A,B)
37. (B,C)

Section-II

38. (C)
39. (B)
40. (D)
41. (A)

Section-III

42. (3)
43. (15)
44. (55)
45. (3)
46. (5)
47. (0)

Section-IV

48. (D)
49. (A)
50. (D)
51. (C)

Hints and Solutions

PHYSICS

Section-I

(1) **Answer :** (A,B,C,D)
Hint:
F sinα = mg
Solution:
At breaking off

$F\sin\alpha=mg,\ t=\sqrt{\frac{mg}{a\sin\alpha}}=\sqrt{\frac{2mg}{a}}$

Speed $v=\int_0^{t_0}\frac{at^2\cos\alpha}{m}\approx dt$

$v=\sqrt{\frac{mg^3}{9a\tan^2\alpha\sin\alpha}}=\sqrt{\frac{2mg^3}{3a}}$

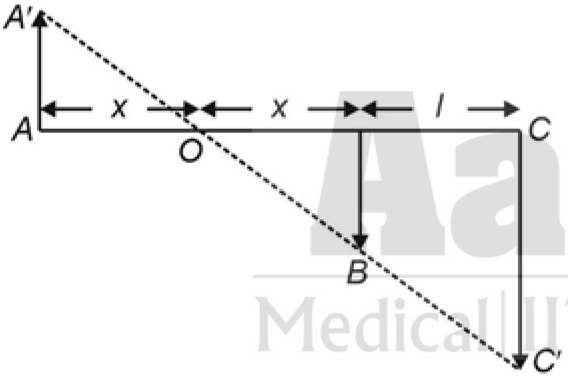
accleration (a) = $\frac{F\cos\alpha}{m}=g\cot\alpha=\sqrt{3}g$

$s=\int v\cdot dt=\int_0^{t_0}\frac{at^3\cos\alpha}{3m}dt=\frac{mg^2}{12a\tan\alpha\cdot\sin\alpha}=\frac{mg^2}{2\sqrt{3}a}$

(2) **Answer :** (B,D)
Hint:

$\frac{ut}{l-x}=\frac{u_Bt+\frac{1}{2}a_Bt^2}{x}=\frac{\frac{1}{2}at^2}{l+x}$

Solution:
Let at time t.



$AA' = ut$

$BB' = u_Bt+\frac{1}{2}a_Bt^2$

$CC' = \frac{1}{2}at^2$

$\Delta AOA'$, BOB' and COC' are similar.

$\frac{ut}{l-x}=\frac{u_Bt+\frac{1}{2}a_Bt^2}{x}=\frac{\frac{1}{2}at^2}{l+x}$

$\frac{2u}{l-x}=\frac{at}{l+x}$

$x=\left(\frac{at-2u}{at+2u}\right)t$

$u_Bt+\frac{1}{2}a_Bt^2=\left(-\frac{u}{2}\right)t+\frac{1}{2}\left(\frac{a}{2}\right)t^2$

$u_B=-\frac{u}{2}$→upward

$a_B=\frac{a}{2}$→downward

(3) **Answer :** (A,B,C)
Hint:

$\frac{1}{2}mv^2=-mgx_0+\frac{1}{2}kx_0^2$

Solution:
At equilibrium, $x=\frac{mg}{k}$

From energy conservation, $v=\sqrt{2gh}$

$\frac{1}{2}mv^2=-mgx_0+\frac{1}{2}kx_0^2$

$kx_0^2 - 2mgx_0 - mv^2 = 0$

$x_0 = \frac{2mg \pm \sqrt{4m^2g^2 + 4kmv^2}}{2k}$

$= \frac{mg}{k} \pm \left(\sqrt{1+\frac{2kh}{mg}}\right) \cdot \frac{mg}{k} = \frac{mg}{k}\left[1+\sqrt{1+\frac{2kh}{mg}}\right]$

Amplitude = $x_0 - x = \frac{mg}{k}\sqrt{1+\frac{2kh}{mg}}$

$\omega = \sqrt{\frac{k}{m}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

$E = \frac{1}{2}m\omega^2A^2 = \frac{m^2g^2}{2k}\left(1+\frac{2kh}{mg}\right)$

Section-II

(4) **Answer :** (D)

Hint:

$\frac{d^2x}{dt^2} = \alpha a$

$\frac{d^2y}{dt^2} = -2\alpha y$

Solution:

$\frac{d^2x}{dt^2} = \alpha a$

$x = \frac{1}{2}\alpha a t^2$ …(i)

$\frac{d^2y}{dt^2} = -2\alpha y$

Multiplying this equation by $2\frac{dy}{dt}$

$\frac{2dy}{dt} \times \frac{d^2y}{dt^2} = -4\alpha y \cdot \frac{dy}{dt}$

$\left(\frac{dy}{dt}\right)^2 = -2\alpha y^2 + C$

$\frac{dy}{dt} = 0$ at $y = a,\ C = 2\alpha a^2$

$\frac{dy}{dt} = \sqrt{2\alpha(a^2 - y^2)}$

$\int_a^y \frac{dy}{\sqrt{a^2-y^2}} = \sqrt{2\alpha}\int_0^t dt$

$y = a\ \cos\left(\sqrt{2\alpha}\ t\right)$ …(ii)

From (i) and (ii),

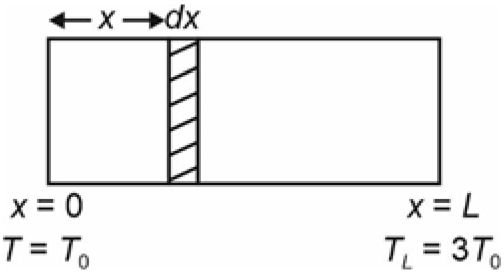
$y = a\ \cos\left(2\sqrt{\frac{x}{a}}\right)$

(5) **Answer :** (D)

Hint:

$P\left(A \cdot dx\right) = dnR\left[T_0 + \frac{T_L - T_0}{L}x\right]$

Solution:



$T = T_0 + \left(\frac{T_L - T_0}{L}\right)x$

$P(A \cdot dx) = dnRT$

$P\left(A \cdot dx\right) = dnR\left[T_0 + \frac{T_L - T_0}{L}x\right]$

$\int_0^L \frac{dx}{T_0 + \frac{T_L - T_0}{L}x} = \int_0^n \frac{R}{PA} \cdot dn$

$n = \frac{PV}{(T_L - T_0)R}\ln\left(\frac{T_L}{T_0}\right)$

$= \frac{PV}{2T_0R}\ln(3)$

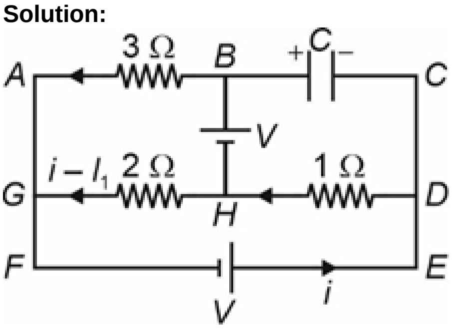
(6) **Answer :** (B)
Hint:
$dQ = mcdT = \frac{T_0 - T}{R}$
Solution:
$R = \frac{R_2 - R_1}{4\pi k R_1 R_2} = 2 \times 10^{-2}\ \mathrm{K/W}$
$dQ = mcdT = \frac{T_0 - T}{R}$
$t = mcR\ \ln\left|\frac{T_0}{T_0 - T}\right|,\ m = \rho v = \frac{125\pi}{6}$
$= \frac{125\pi}{6} \times 4200 \times 2 \times 10^{-2}\ln(2)$
$= 1750\pi\ln(2)\ \mathrm{s}$

(7) **Answer :** (B)
Hint:
Average force $= \frac{\text{change in momentum at support}}{\text{time between two collision at support}}$
Solution:
Average force $= \frac{\text{change in momentum at support}}{\text{time between two collision at support}}$
$<F> = \frac{2mv}{\frac{L-10r}{v} \times 2} = \frac{mv^2}{r}$

Section-III

(8) **Answer :** 3
Hint:
$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$
Solution:
For m_1 :
$\frac{1}{v} - \frac{1}{10} = \frac{1}{40}$
$v_1 = 8\ \mathrm{cm}$
For m_2 :
$\frac{1}{v} - \frac{1}{10} = -\frac{1}{40}$
$v_2 = \frac{40}{3}\ \mathrm{cm}$
Distance between image $= \frac{40}{3} - 8 = \frac{16}{3}\ \mathrm{cm}$

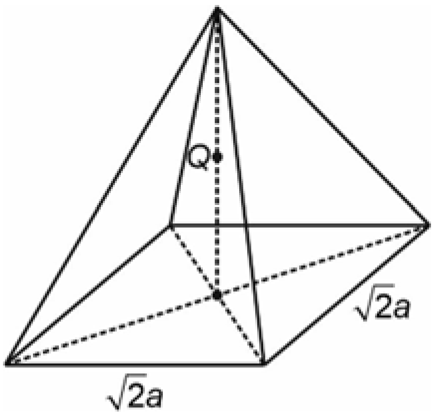
(9) **Answer :** 11
Hint:
Apply KVL
Solution:



$(V_C)_{\max} = 2 \times 10^6 d$
In loop *BCDHB*,
$V - (V_C)_{\max} - i = 0$
$V = i + 2d \times 10^6$
In loop *ABHGA*,
$V - 3i_1 + 2(i - i_1) = 0\ ,\ V = -2i + 5i_1 — (i)$
In loop *DGFED*,
$-i - 2(i - i_1) + V = 0\ ,\ V = 3i - 2i_1 — (ii)$
From (i) and (ii),
$i = \frac{7}{11}V$
So, $V = \frac{7}{11}V + 2d \times 10^6$

$\frac{4V}{11} = 2 \times 2 \times 10^{-3} \times 10^{6}$
$V = 11\ \text{kV}$

(10) **Answer :** 5
Hint:
$4\phi_{\text{triangle}} + \phi_{\text{square}} = \frac{Q}{60}$
Solution:

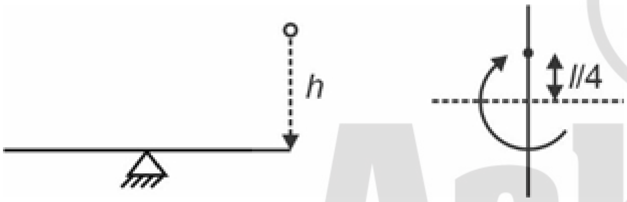


Flux through square $= \frac{Q}{6\epsilon_0}$ because point charge Q will be on centre of cube of side $\sqrt{2}a$.

Now, $\frac{Q}{6\epsilon_0} + 4\phi = \frac{Q}{\epsilon_0}$

$\phi = \frac{5Q}{24\epsilon_0}$

(11) **Answer :** 4
Solution:



$v = \sqrt{2gh}$

$m\sqrt{2gh} \cdot \frac{l}{2} = \left(\frac{ml^2}{12} + \frac{ml^2}{4}\right)\omega$

$\omega = \frac{3}{2l}\sqrt{2gh}$

$\left(2mg \cdot \frac{l}{4}\right) = \frac{1}{2}\left[\frac{ml^2}{12} + \frac{ml^2}{4}\right]\left(\frac{9}{4l^2} \times 2gh\right)$

$h = \frac{2l}{3} = 4\ \text{m}$

(12) **Answer :** 3
Hint:
$E_1 = (12 - 2n)E + 2E$
$E_2 = (12 - 2n)E - 2E$
Solution:
Let *n* cells connected wrongly.
Then $(12 - n)E - \text{nE} = (12 - 2n)E$
When cells aid the battery
$E_1 = (12 - 2n)E + 2E$, $I_1 = 2 = \frac{(12-2n)E+2E}{R}$
When cells oppose the battery
$E_2 = \frac{(12-2n)E-2E}{R}$, $I_2 = 1 = \frac{(12-2n)E-2E}{R}$
$\frac{2}{1} = \frac{(12-2n)+2}{(12-2n)-2}$, $n = 3$

(13) **Answer :** 7
Hint:
$(\Delta Q)_{AB} = nC_p\,\Delta T$
$(\Delta Q)_{AC} = \Delta U + \Delta W$
Solution:
$(\Delta Q)_{AB} = nC_p\,\Delta T = \frac{\gamma}{\gamma-1}\left(nR\,\Delta T\right) = \frac{\gamma}{\gamma-1}\left(3P_0V_0 - P_0V_0\right) = \frac{2P_0V_0\gamma}{\gamma-1}$

$56 = \frac{2P_0V_0\gamma}{\gamma-1}$

$(\Delta Q)_{AC} = \Delta U + \Delta W = \frac{nR\Delta T}{\gamma-1} + \frac{1}{2}(3V_0)(P_0 + 4P_0)$

$360 = \frac{15P_0V_0(\gamma+1)}{2(\gamma-1)}$

$\gamma = \frac{7}{5}$

Section-IV

(14) **Answer :** (B)
Solution:
For p: rod is non conducting, so no current in the circuit.
For Q: No energy dissipation as no resistance
$F = ilB$ to left, $f_{ext} \rightarrow$ to right.
$F = ilB,\ Bvl = L\frac{di}{dt} \rightarrow$ not possible to move with constant velocity v.

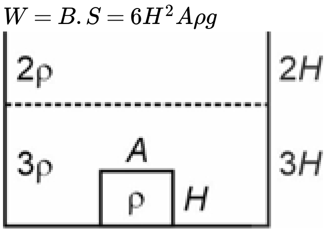
For R: $i = \frac{V - \frac{B}{v}}{R}$

$F = \frac{V - \frac{B}{v}}{R}\, Bl$

For s: energy is dissipated during the motion rod will stop work done by F is equal to energy dissipated during the motion.

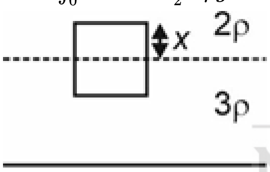
(15) **Answer :** (B)
Solution:
(P) $B = m_f g = (AH)(3\rho)g,$

$W = B.S = 6H^2A\rho g$



(Q) $B = (m_{f_1} + m_{f_2})g = Ax(2\rho)g + A(H-x)3\rho g$

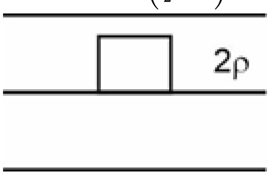
$W = \int_0^H B dx = \frac{5}{2}A\rho g H^2$



(R) $B = AH(2\rho)g$

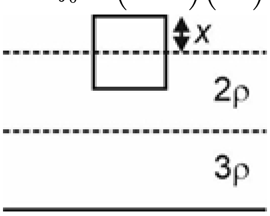
$W = AH(2\rho)gH = 2H^2A\rho g$

Total work = $\left(\frac{5}{2} + 2\right)H^2A\rho g = \frac{9}{2}H^2A\rho g$



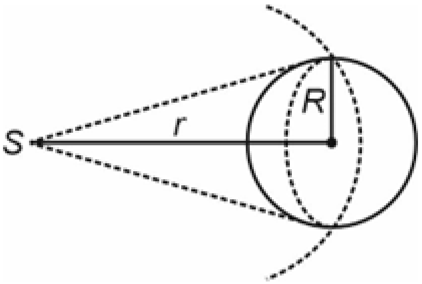
(S) $B = A(H-x)(2\rho g)$

$W = \int_0^H A(H-x)(2\rho g)dx = A\rho g H^2$



(16) **Answer :** (B)
Hint:
$I = \frac{P}{4\pi r^2},\ E = \frac{P}{4\pi r} \text{ x } \pi R^2$
Solution:
$I = \frac{P}{4\pi r^2},\ E = \frac{P}{4\pi r} \text{ x } \pi R^2$

If e is the energy of single electron and η is the efficiency of photon to liberate an electron, the number of ejected electron is $\eta\frac{PR^2}{4r^2e}=10^5$ electron/s



When electron emission stop

$\mathrm{eV}=(\mathrm{KE})_{\max},\ V=\frac{(KE)_{\max}}{e}$

$(\mathrm{KE})_{\max}=h\nu-\phi=2\ \mathrm{eV}$

$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{ne}{R}\right)=2$

$n=\frac{4\pi\varepsilon_0(2R)}{e}=1.11\,\mathrm{x}\,10^7$

$t=\frac{1.11\,\mathrm{x}\,10^7}{10^5}=111\ \mathrm{s}$

(17) **Answer :** (D)
Hint:

$Q=\frac{L\omega_0}{R}$

Solution:

(P) $\omega=\frac{1}{\sqrt{LC}}\approx 4167\ \mathrm{rad/s}$

$f=663.5\ \mathrm{Hz},\ I_0=14.14\ \mathrm{A}$

(Q) $P=E_{\mathrm{rms}}\cdot I_{\mathrm{rms}}\cos\phi=2300\ \mathrm{W}$

(R) $\omega_1=\omega_0+\frac{R}{2L}\Rightarrow 4263\ \mathrm{rad/s}\ \text{or}\ 4071\ \mathrm{rad/s}$

(S) $Q=\frac{L\omega_0}{R}=21.74$

CHEMISTRY

Section-I

(18) **Answer :** (A,B,C)
Hint:

Kaolinite is $Al_2Si_2O_5(OH)_4$.

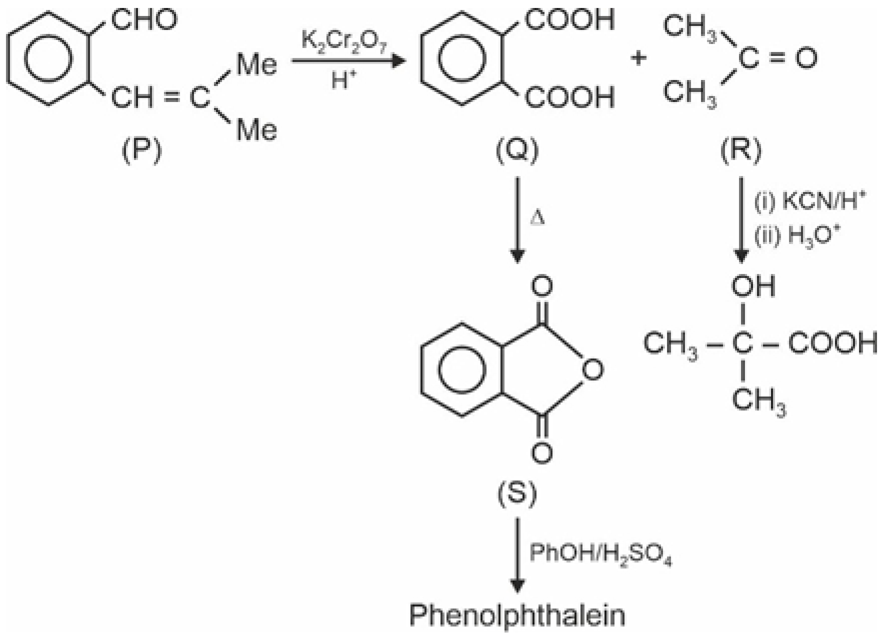
Solution:

Slag formation during extraction of iron is $CaSiO_3$ and for copper is $FeSiO_3$.

(19) **Answer :** (B,C)
Hint:

Aldehydes and ketones both gives 2,4-DNP test.

Solution:



(20) **Answer :** (A,C)

Hint:

$W = -nRT\ln\frac{V_2}{V_1}$

Solution:

$P_A = \frac{1\times R\times 300}{V_0}$

$P_B = \frac{1\times R\times 300}{2V_0} = \frac{150\,R}{V_0}$

$W_{AB} = -nRT\ \ln\frac{V_2}{V_1}$

$= -1\times R\times 300\ \ln\frac{2V_0}{V_0}$

= –300R ln2

= –208R

From B to C,

$\left(\frac{150\,R}{V_0}\right)^{1-1.5} T_B^{3/2} = \left(\frac{150\,R}{4V_0}\right)^{1-1.5} T_C^{3/2}$

$300 = \left(\frac{1}{4}\right)^{-1/3} T_C$

189 K = T_C

$W_{BC} = \frac{nR[T_2 - T_1]}{\gamma - 1}$

$= \frac{1\times R(189-300)}{\frac{3}{2}-1}$ = –222R

W_{AC} = –208R – 222R = –430R

Section-II

(21) **Answer :** (C)

Hint:

$[PbCl_4]^{2-}$ is water soluble.

Solution:

$Pb^{2+}(aq) \xrightarrow{Cl^-} PbCl_2(s) \xrightarrow{HCl(ex)} [PbCl_4]^{2-}(aq)$

White ppt.

(22) **Answer :** (A)

Hint:

Talc is an example of sheet silicate.

Solution:

It has empirical formula $Mg_2(Si_2O_5)_2Mg(OH)_2$ which is $(Si_2\,O_5)_n^{2n-}$ units of two dimensional sheet silicates.

(23) **Answer :** (C)

Hint:

On differentiation,

2AdA = αdt

Solution:

$A^2 = \alpha t + \beta$

On differentiation,

2AdA = αdt

$\frac{dA}{dt} = \frac{\alpha}{2A} = \frac{\alpha}{2}[A]^{-1}$

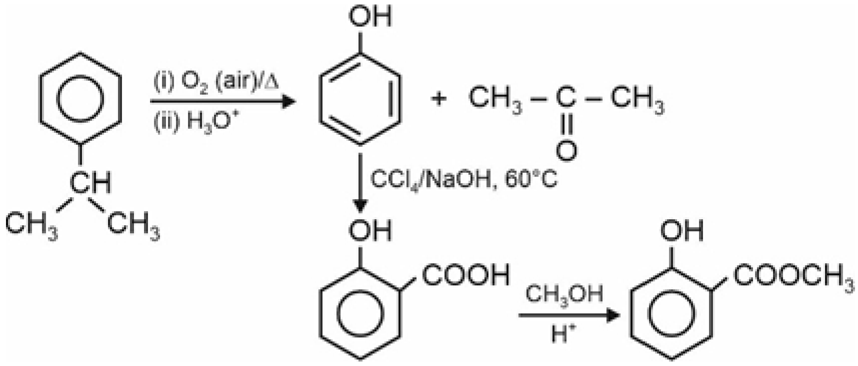
Hence order is –1.

(24) Answer : (B)

Hint:

Cumene forms phenol on aerial oxidation followed by acidic hydrolysis.

Solution:



Section-III

(25) Answer : 6

Hint:

Pyrogallol absorbs O_2.

Solution:

$$C_xH_y(g) + \left(x + \frac{y}{4}\right)O_2(g) \rightarrow xCO_2(g) + \frac{y}{2}H_2O(l)$$

5	30	0	
0	$30 - 5\left(x + \frac{y}{4}\right)$	5x	

$5x = 10 \qquad x = 2 \qquad 5\left(x + \frac{y}{4}\right) = 15 \qquad y = 4$

The formula for hydrocarbon = C_2H_4

(26) Answer : 1

Hint:

$K_{a_1}K_{a_2} = \frac{[H^+]^2[S^{2-}]}{[H_2S]}$

Solution:

$ZnS \rightleftharpoons Zn^{2+} + S^{2-}$

$K_{sp} = [Zn^{2+}][S^{2-}]$

$[S^{2-}] = \frac{10^{-21}}{10^{-2}} = 10^{-19}$

$H_2S \rightleftharpoons 2H^+ + S^{2-}$

$K_{a_1}K_{a_2} = \frac{[H^+][S^{2-}]}{[H_2S]}$

$10^{-20} = \frac{[H^+]^2 \times 10^{-19}}{0.1}$

$[H^+] = 0.1$

pH = 1

(27) Answer : 14

Hint:

Equation of line

$\frac{1}{\Lambda_m} = \frac{1}{K_a(\Lambda_m^o)^2}\left(c\Lambda_m\right) + \frac{1}{\Lambda_m^o}$

Solution:

Equation of line

$\Rightarrow \frac{1}{\Lambda_m} = \frac{1}{K_a(\Lambda_m^o)^2}\left(c\Lambda_m\right) + \frac{1}{\Lambda_m^o}$

Slope = $\frac{5}{16} = \frac{1}{K_a(\Lambda_m^o)^2}$

$K_a = 2 \times 10^{-5}$

$K_a = \frac{c\alpha^2}{1-\alpha}$

$2 \times 10^{-5} = 0.1 \times \alpha^2$

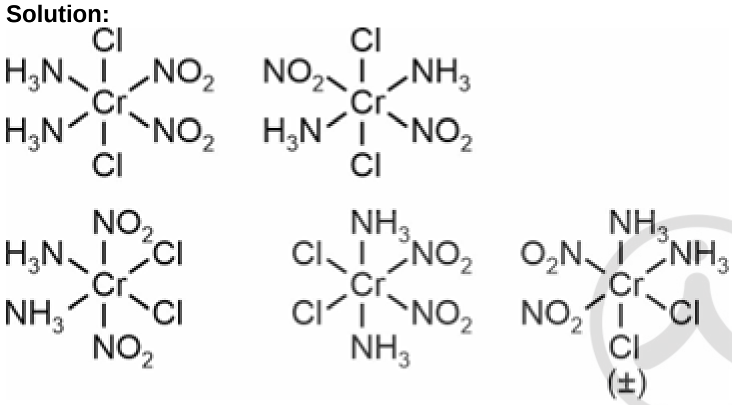
$1.4 \times 10^{-2} = \alpha$

(28) **Answer :** 6

Hint:

It is complex of type $[Ma_2b_2c_2]$.

Solution:



Last will show optical isomerism.

(29) **Answer :** 6

Hint:

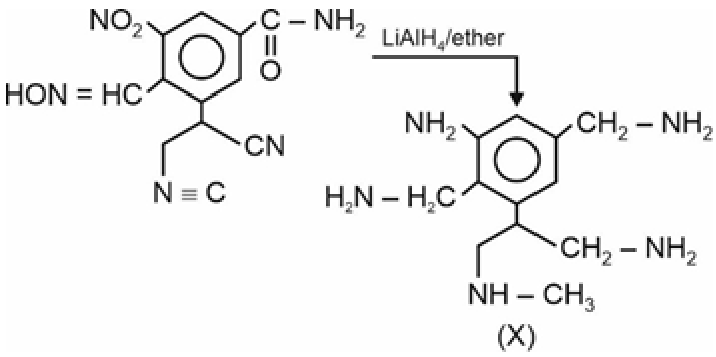
$LiAlH_4$ can reduce

$-NO_2,\ -\underset{\underset{O}{\|}}{C}-NH_2,\ -C\equiv N$, oximes and –N ≡ C

Solution:

$LiAlH_4$ can reduce

$-NO_2,\ -\underset{\underset{O}{\|}}{C}-NH_2,\ -C\equiv N$, oximes and –N ≡ C



(30) **Answer :** 12

Hint:

$\Delta G_r^o = \Delta H_r^o - T\Delta S_r^o$

Solution:

$\Delta G_r^o = \Delta H_r^o - T\Delta S_r^o$

$\Delta G_r^o = \left(80 - 120\right) \times 10^3 - 300\left(170 - 210\right)$

$\Delta G_r^o = -28 \times 10^3\ J$

$-RT\ lnK = -28 \times 10^3$

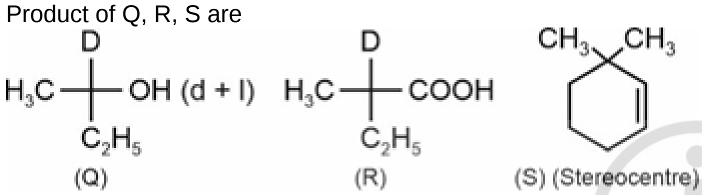
$8 \times 300\ lnK = 28 \times 10^3$

lnK = 11.66

Section-IV

(31) **Answer :** (B)
Hint:
$NaNO_2 + FeSO_4 + 3H_2SO_4 \rightarrow Fe(SO_4)_3 + NaHSO_4 + H_2O + NO$
Solution:
$NaNO_2 + FeSO_4 + 3H_2SO_4 \rightarrow Fe(SO_4)_3 + NaHSO_4 + H_2O + NO$
$HNO_3 + P_4O_{10} \rightarrow HPO_3 + N_2O_5$
$Cu + HNO_3$ (conc.) $\rightarrow Cu(NO_3)_2 + NO_2 + H_2O$
$NH_4NO_3 \xrightarrow{\Delta} N_2O + 2H_2O$

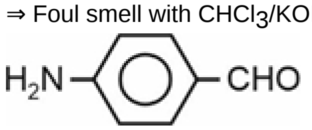
(32) **Answer :** (B)
Hint:
Deuterium exchange is possible in (P).
Solution:
In (P), given compound exchanges –H with –D and become racemic.
In (Q), compound reacts through carbocation and gives racemic mixture.
In (R), no bond attached to stereocentre breaks, so retention.
In (S), compound give Hoffman elimination and does not have stereogenic centre.
Product of Q, R, S are



(33) **Answer :** (C)
Hint:
ZnS ⇒ shows Frenkel defect and has CN = 4
Solution:
ZnS ⇒ shows Frenkel defect and has CN = 4
NaCl ⇒ has FCC so limiting $\frac{r_+}{r^-} = 0.414$
CsCl ⇒ 8 nearest neighbour
CaF_2 ⇒ CN = 4 & 8, and insoluble in water

(34) **Answer :** (A)
Hint:
1° amine gives isocyanide test.
Solution:
$CH_3 – C \equiv CH$
⇒ White ppt with Tollens reagent
⇒ Decolourises Br_2/water
HCOOH
⇒ Positive test with Tollens, Fehling's, $NaHCO_3$ solution

⇒ Foul smell with $CHCl_3$/KOH



⇒ Positive test with Tollens reagent and isocyanide test

MATHEMATICS

Section-I

(35) **Answer :** (A,B,C,D)
Hint:
Use extended sine rule $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}$
Solution:

Let $R=\frac{\sqrt{3}}{6}$ be the circumradius. From the extended sine rule,

$\sin A=2a\sin B\Rightarrow\frac{a}{2R}=\frac{2ab}{2R}\Rightarrow b=\frac{1}{2}$

Therefore, $\sin B=\frac{b}{2R}=\frac{\frac{1}{2}}{2\cdot\frac{\sqrt{3}}{6}}=\frac{\sqrt{3}}{2}$

Since, $\angle A,\ \angle C,\ \angle B<\frac{\pi}{2},\ \angle B=\frac{\pi}{3}$

and $\angle A=\angle(A+C)-\angle C=\frac{2\pi}{3}-\angle C>\frac{2\pi}{3}-\frac{\pi}{2}=\frac{\pi}{6}$

⇒ $\angle A>\frac{\pi}{6}$

So, $\angle A\in\left(\frac{\pi}{6},\frac{\pi}{2}\right)$. Thus, the perimeter l of the Δ*ABC* is given by

$I\ =a+b+c=2R\sin A+2R\sin C+b$

$=\frac{1}{2}+\frac{\sqrt{3}}{3}(\sin A+\sin C)$

$=\frac{1}{2}+\frac{\sqrt{3}}{3}\left(\sin A+\sin\left(\frac{2\pi}{3}-A\right)\right)$

$=\frac{1}{2}+\frac{\sqrt{3}}{3}\left(\sin A+\frac{\sqrt{3}}{2}\cos A+\frac{1}{2}\sin A\right)$

$=\frac{1}{2}+\left(\frac{\sqrt{3}}{2}\sin A+\frac{1}{2}\cos A\right)$

$=\frac{1}{2}+\sin\left(A+\frac{\pi}{6}\right)$

Since the range of $A+\frac{\pi}{6}$ is $\left(\frac{\pi}{3},\ \frac{2\pi}{3}\right)$, so that $\frac{\sqrt{3}}{2}<\sin\left(A+\frac{\pi}{6}\right)\leq 1$

The range of l is $\left(\frac{1+\sqrt{3}}{2},\frac{3}{2}\right]$.

(36) Answer : (A,B)

Hint:

Use $(2A)^{-1}=\frac{adj(2A)}{|2A}$

Solution:

$\because\ A=\begin{bmatrix}2&3\\0&-1\end{bmatrix},\ A^2=\begin{bmatrix}4&3\\0&1\end{bmatrix},\ A^3=\begin{bmatrix}8&9\\0&-1\end{bmatrix},\ \ldots$

$A^n=\begin{bmatrix}2^n&2^n-(-1)^n\\0&(-1)^n\end{bmatrix}$

Also, $2A\cdot(\text{adj}(2A))=|2A|I$

$\Rightarrow A\cdot\text{adj}(2A)=-4I$

Now, $\left|A^{10}-(\text{adj}2A)^{10}\right|=\frac{\left|A^{20}-A^{10}(\text{adj}(2A))^{10}\right|}{|A|^{10}}$

$=\frac{|A^{20}-2^{20}I|}{|A^{10}|}$ ...(i)

$A^{20}-2^{20}\cdot I=\begin{bmatrix}2^{20}&2^{20}-1\\0&1\end{bmatrix}-2^{20}\begin{bmatrix}1&0\\0&1\end{bmatrix}$

$=\begin{bmatrix}0&2^{20}-1\\0&1-2^{20}\end{bmatrix}$

$\Rightarrow\left|A^{20}-2^{20}I\right|=0$

From (i), $\left|A^{10}-(\text{adj}(2A))^{10}\right|=0$

Hence, $\det\left(A^4\right)+\det\left(A^{10}-(\text{adj}(2A)^{10})\right)$

$=\ |A|^4+\ 0$

$=(-2)^4=16$

Now for *B*,

$\det(2\text{Adj}(2\text{Adj}(\text{Adj}\cdot 2B)))=2^{41}$

$\Rightarrow\det\left(2\text{Adj}\left(2\text{Adj}\left(2^2\cdot\text{Adj}B\right)\right)\right)=2^{41}$

$\Rightarrow\det\left(2\text{Adj}\left(2^5\text{Adj}(\text{Adj}B)\right)\right)=2^{41}$

$\Rightarrow\det\cdot\left(2^{11}\text{Adj}(\text{Adj}(\text{Adj}B))\right)=2^{41}$

$\Rightarrow 2^{33}\cdot\det(\text{Adj}(\text{Adj}(\text{Adj}B)))=2^{41}$

$\Rightarrow\ |B|^8=2^8\ \Rightarrow\ |B|\ =2$

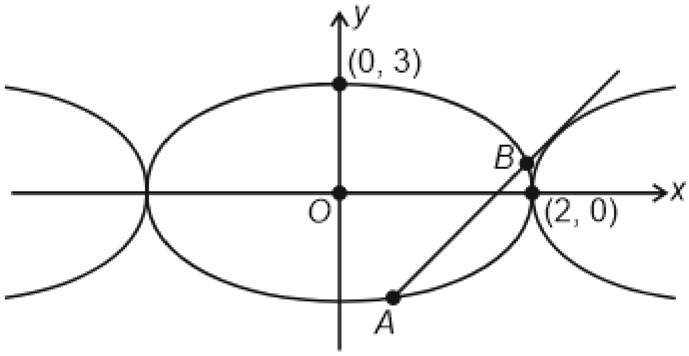
(37) Answer : (B,C)

Hint:

Write the equation of chord and apply the condition for tangent.

Solution:

Let the mid-point of the chord *AB* be (*h*, *k*).



$\left(\frac{xh}{4}+\frac{yk}{9}-1\right)=\frac{h^2}{4}+\frac{k^2}{9}-1$ ...(i)

As (i) touches the hyperbola hence $c^2 = a^2m^2 - b^2$

$\Rightarrow \frac{81}{k^2}\left(\frac{h^2}{4}+\frac{k^2}{9}\right)^2=\frac{81}{k^2}\left(\frac{h^2}{4}-\frac{k^2}{9}\right)$

Hence, locus is $\left(\frac{x^2}{4}+\frac{y^2}{9}\right)^2=\left(\frac{x^2}{4}-\frac{y^2}{9}\right)$.

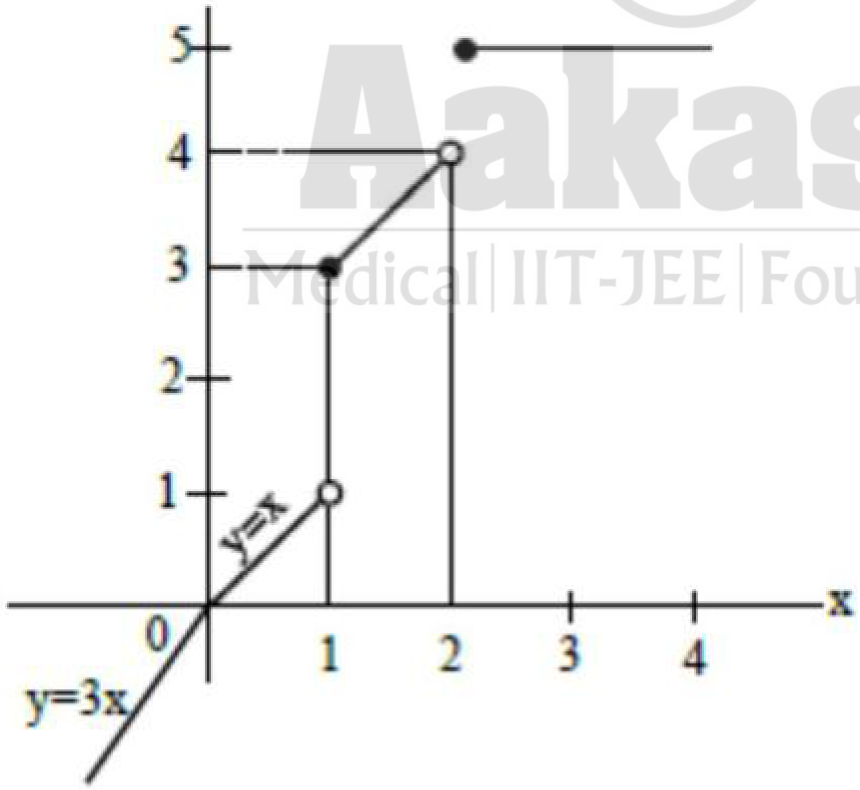
Section-II

(38) Answer : (C)

Hint:

Function will be discontinuous if LHL $f(a) \neq$ RHL $f(a) \neq f(a)$

Solution:



$f(x)=[x]+|x-2|,\ -2<x<3$

$$f(x)=\begin{cases}-x, & -2<x<-1\\ -x+1, & -1\le x<0\\ -x+2, & 0\le x<1\\ -x+3, & 1\le x<2\\ x, & 2\le x<3\end{cases}$$

So, $f(x)$ is not continuous for $x \in \{-1, 0, 1, 2\}$.

$$g(x) = \begin{cases} 3x & ; & x < 0 \\ \min\{1+x, x\} & ; & 0 \le x < 1 \\ \min\{2+x, x+2\} & ; & 1 \le x < 2 \\ 5 & ; & x > 2 \end{cases}$$

$$g(x) = \begin{cases} 3x & ; & x < 0 \\ x & ; & 0 \le x < 1 \\ x+2 & ; & 1 \le x < 2 \\ 5 & ; & x > 2 \end{cases}$$

Not continuous at $x \in \{1, 2\}$

(39) **Answer :** (B)
Hint:
$(2+\sqrt{3})^n + (2-\sqrt{3})^n = 2m,\ m \in I^+$
Solution:
$(2+\sqrt{3})^n + (2-\sqrt{3})^n = 2m,\ m \in I^+$

$\therefore \lim\limits_{n\to\infty} \sin\left(\left(2+\sqrt{3}\right)^n \pi\right)$

$= \lim\limits_{n\to\infty} \sin\left(\left(2m - \left(2-\sqrt{3}\right)^n\right)\pi\right)$

$= \lim\limits_{n\to\infty} \sin\left(2m\pi - \left(2-\sqrt{3}\right)^n \pi\right)$

$= -\lim\limits_{n\to\infty} \sin\left(\left(2-\sqrt{3}\right)^n \pi\right)$

$= -\lim\limits_{n\to\infty} \sin\left(\frac{\pi}{(2+\sqrt{3})^n}\right) = 0$

If n is odd $\left\{(\sqrt{2}+1)^n\right\} = (\sqrt{2}-1)^n$ and $\left[(\sqrt{2}+1)^n\right] =$ an even integer

If n is even $\left\{(\sqrt{2}+1)^n\right\} = 1-(\sqrt{2}-1)^n$ and $\left[(\sqrt{2}+1)^n\right] =$ an odd integer

⇒ limit is '0' if n is odd number.
⇒ limit is –1 if n is even number.

(40) **Answer :** (D)
Hint:
For continuity $f(a^-) = f(a) = f(a^+)$ and for differentiability LHD $f(a)$ = RHD $f(a)$.
Solution:

$$f\left(x\right) = \begin{cases} -\int_0^1 (x-1)t\,dt, & x \le 0 \\ \int_0^x (x-t)t\,dt - \int_x^1 (x-t)t\,dt, & 0 < x < 1 \\ -\int_0^1 (x-1)t\,dt, & x \le 0 \end{cases}$$

$$\Rightarrow\ f(x) = \begin{cases} \frac{-3x+2}{6}, & x \le 0 \\ \frac{2x^3-3x+2}{6}, & 0 < x < 1 \\ \frac{3x-2}{6}, & x \ge 1 \end{cases}$$

For continuity at $x = 0$

$f(0) = \frac{1}{3}$

$\lim\limits_{x\to 0^-} f(x) = \frac{1}{3}$

$\lim\limits_{x\to 0^+} f(x) = \frac{1}{3}$

Hence, f is continuous at $x = 0$.
For continuity at $x = 1$,

$f(1) = \frac{1}{6}$

$\lim\limits_{x\to 1^-} f(x) = \frac{1}{6},\ \lim\limits_{x\to 1^+} f(x) = \frac{1}{6}$

Hence, f is continuous at $x = 1$
⇒ f is continuous ' $x \in R$

$$f'(x) = \begin{cases} -\frac{1}{2}, & x < 0 \\ \frac{2x^2-1}{2}, & 0 < x < 1 \\ \frac{1}{2}, & x > 1 \end{cases}$$

$$\lim_{x \to 0^-} f'(x) = -\frac{1}{2} = \lim_{x \to 0^+} f'(x)$$

Hence, f is differentiable at $x = 0$.
Also, $\lim_{x \to 1^-} f'(x) = \frac{1}{2} = \lim_{x \to 1^+} f'(x)$
Hence, f is differentiable at $x = 1$.
⇒ f is differentiable ' $x \in R$.

(41) **Answer :** (A)
Hint:
Evaluate determinant
Solution:

$$\Delta = \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

$x = -4$ makes all three row identical hence $(x + 4)^2$ will be factor.
Also, $C_1 \to C_1 + C_2 + C_2$

$$\Delta = \begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix}$$

⇒ $5x - 4$ is a factor.

$$\Delta = \lambda(5x-4)(x+4)^2$$

Section-III

(42) **Answer :** 3
Hint:
For isosceles Δ with sides a, a, b such that $2a > b$.
Solution:
The possible cases for isosceles triangle are given by
(1, 1, 1), (1, 1, 2), …, (1, 1, 6)
(2, 2, 1), (2, 2, 2), …, (2, 2, 6)
…………………………………………
(6, 6, 1), (6, 6, 2), …, (6, 6, 6)
But when the two equal sides are 1 only possible value of the third side of the triangle is 1.
Similarly, when two equal sides are 2. Only possible values of third side of the triangle are (1, 2, 3).
When the two equal sides are 3, the third side can takes values (1, 2, 3, 4 and 5).
When the two equal sides are 4, 5 or 6, the third side can takes values (1, 2, 3, 4, 5, 6).
So, total isosceles triangles possible = 1 + 3 + 5 + 6 + 6 + 6 = 27.
For the triangle to be obtuse cosine of the angle at vertex is negative.
This is true only for triangle with sides (2, 2, 3), (3, 3, 5) and (4, 4, 6) only.
So, the total number of obtuse isosceles triangles is 3.

(43) **Answer :** 15
Hint:
Use Baye's theorem.
Solution:
Let E_1, E_2 and E_3 be the events that there are exactly two, three and four wrongly addressed envelopes. And let A be the event that two opened envelopes are wrongly addressed, then the number of ways of being exactly two wrongly addressed envelopes

$$= {}^4C_2 \times 2!\left(1 - \frac{1}{1!} + \frac{1}{2!}\right) = 6 \Rightarrow P(E_1) = \frac{6}{4!}$$

Number of ways of being exactly three wrongly addressed envelopes

$$= {}^4C_3 \times 3!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right) = 8 \Rightarrow P(E_2) = \frac{8}{4!}$$

Number of ways of being exactly four wrongly addressed envelopes

$$= {}^4C_4 \times 4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = 9 \Rightarrow P(E_3) = \frac{9}{4!}$$

Now, $P\left(\frac{A}{E_1}\right) = \frac{{}^2C_2}{{}^4C_2},\ P\left(\frac{A}{E_2}\right) = \frac{{}^3C_2}{{}^4C_2},\ P\left(\frac{A}{E_3}\right) = \frac{{}^4C_2}{{}^4C_2}$

Then, $P\left(\frac{E_1}{A}\right)$

$$= \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{6}{4!}\times\frac{^2C_2}{^4C_2}}{\frac{6}{4!}\times\frac{^2C_2}{^4C_2}+\frac{8}{4!}\times\frac{^3C_2}{^4C_2}+\frac{9}{4!}\times\frac{^4C_2}{^4C_2}} = \frac{1}{14}$$

(44) Answer : 55

Hint:

$a_n = S_n - S_{n-1}$

Solution:

$S_n = \frac{1}{2}\left(a_n + \frac{1}{a_n}\right) = \frac{1}{2}\left(S_n - S_{n-1} + \frac{1}{S_n - S_{n-1}}\right)$

$\Rightarrow S_n^2 = S_{n-1}^2 + 1$

Since, $S_1 = a_1 = 1$, so $S_n^2 = n,\ S_n = \sqrt{n},\ n \in \mathbb{N}$.

$$\sum_{k=1}^{5}\left(\sqrt{k}\right)^4 = \sum_{k=1}^{5} k^2 = 55$$

(45) Answer : 3

Hint:

Analyse the values of function at various points around critical points.

Solution:

Here, $f(x) = x^3 + x^2 - 5x - 1$

$f'(x) = 3x^2 + 2x - 5 = 0$

$\Rightarrow\ x = -\frac{5}{3}$

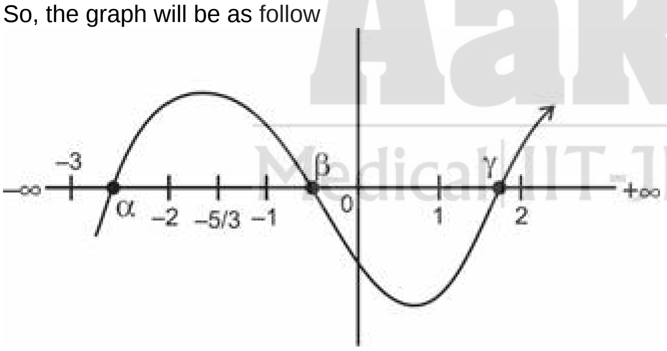
$\Rightarrow f(x)$ has 3 real roots.

Now we see that $f(-\infty) = -\infty < 0$

$f(\infty) = \infty > 0$

$f(1) = -4 < 0,\ f\left(-\frac{5}{3}\right) = \frac{148}{27} > 0$

So, the graph will be as follow



Now also $f(-3) = -27 + 9 + 15 - 1 = -4 < 0$

$f(-2) = -8 + 4 + 10 - 1 > 0$

$f(-1) = 4 > 0,\ f(0) = -1 < 0$

$f(2) = 1 > 0$

So, we observe that $\alpha \in (-3, -2)$, $\beta \in (-1, 0)$, $y \in (1, 2)$

$\Rightarrow [\alpha] = -3,\ [\beta] = -1,\ [y] = 1$

$\Rightarrow |-3 - 1 + 1| = 3$

(46) Answer : 5

Hint:

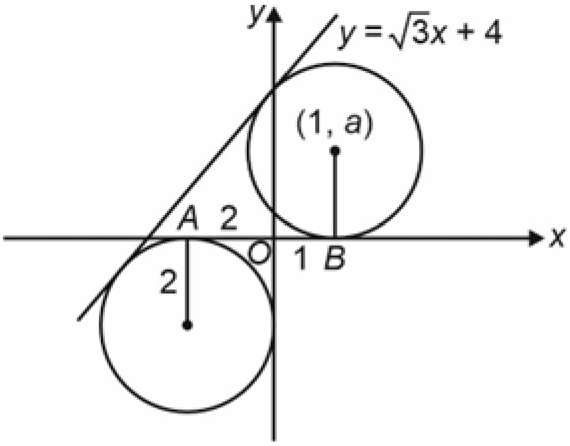
For equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$, centre is $\sqrt{f^2 + g^2 - c}$.

Solution:

Given circle $x^2 + y^2 + 4x + 4y + 4 = 0$

Its centre $C = (-2, -2)$

$r = \sqrt{9^2 + f^2 - c} = \sqrt{4 + 4 - 4} = 2$



∴ The circle lies in the 3rd quadrant touching both the axes given common tangent is

$y = \sqrt{3}x + 4$

Given distance between two circles along *x*-axis is = 3 units

i.e., *AB* = 3

2 + *OB* = 3 ⇒ *OB* = 1

∴ The required circle touches the *x*-axis at (1, 0).

Let centre = (1, *a*) of the required circle.

Since $\sqrt{3}x - y + 4 = 0$ is a tangent to the required circle.

$\frac{|\sqrt{3}\cdot 1 - a + 4|}{\sqrt{3+1}}$ = radius = *a*

$\sqrt{3} - a + 4 = 2a$

$a = \frac{\sqrt{3}+4}{3}$

Since *a* > 0

∴ Centre $C = (1\ ,\ a) = \left(1\ ,\ \frac{\sqrt{3}+4}{3}\right),\ r = \frac{\sqrt{3}+4}{3}$

Hence, the equation of the required circle is $(x-1)^2 + \left(y - \frac{\sqrt{3}+4}{3}\right)^2 = \left(\frac{\sqrt{3}+4}{3}\right)^2$.

(47) **Answer :** 0

Hint:

Use $(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} \cdot b^r$

Solution:

$(101)^{202} + (202)^{101} - (107)^{99}$

$(101)^{202} = (100 + 1)^{202}$

$= {}^{202}C_0\, 100^{202} + \ldots + {}^{202}C_{201} 100 + 1$

⇒ Unit digit is 1

⇒ $(202)^{101} = (200 + 2)^{101}$

$= {}^{101}C_0 200^{101} + \ldots + {}^{101}C_{100}(200)2^{100} + 2^{101}$

$= M(100) + 2^{101}$

$2^{101} = 2(2^4)^{25} = 2[(2^4)(2^4)\ldots\ldots(2^4)25$ times$]\cdot 2[6\cdot 6\ldots\ldots 6)25$ times

⇒ Units digit is 2.

∴ Unit digit in $2(2^4)^{25}$ is 2.

Also, $(107)^{99} = (100 + 7)^{99}$

$= {}^{99}C_0 100^{99} + \ldots + {}^{99}C_{98}(100)7^{98} + 7^{99}$

Now $7^{99} = 7^3(7^4)^{24} = 7^3(7^4\cdot 7^4\ldots\ldots 24$ times)

⇒ Unit digit is 3.

⇒ Unit digit in $(101)^{202} + (202)^{101} - (107)^{99} = 1 + 2 - 3 = 0$

Section-IV

(48) **Answer :** (D)

Hint:

Replace *x* with 1 – *x*

Solution:

Given

$f(x^2 + x) + 2f(x^2 - 3x + 2) = 9x^2 - 15x \quad \ldots$(i)

By replacing x with $1 - x$ in (i), it follows that

$f(x^2-3x+2)+2f(x^2+x)=9x^2-3x-6$ ...(ii)

By eliminating $f(x^2-3x+2)$ from the two simultaneous equations (i) and (ii), we obtain

$3f(x^2+x)=9x^2+9x-12=9(x^2+x)-12$

i.e., $f(x^2+x)=3(x^2+x)-4,\ x\in\mathbb{R}$. Since the equation $x^2+x=2011$ has real roots, so there is x_0 such that $x_0^2+x_0=2011$.

Hence,

$f(2011)=f(x_0^2+x_0)=3(2011)-4=6029$

$f(2025)=f(x_0^2+x_0)=3(2025)-4=6071$

(49) Answer : (A)

Hint:

Make some base on both side of inequality, use concept of completing the square.

Solution:

(P) Change both sides to let them have the same power, then compare their bases.

$12^{200}<n^{300}\Leftrightarrow(144)^{100}<(n^3)^{100}\Leftrightarrow 144<n^3$

Then $5^3<144<6^3$ implies that $n = 6$.

(Q) Setting $2^x=a$ and $3^x=b$, the given equation becomes

$1+a^2+b^2-a-b-ab=0$

Multiplying both sides of the last equation by 2 and completing squares, then

$(1-a)^2+(a-b)^2+(b-1)^2=0$

Therefore, $a = b = 1$, namely $2^x=3^x=1$. So, $x = 0$ is the unique solution.

(R) The substitution $y=\sqrt{5^{2x}-26\cdot5^x+26}$ yields $y^2+y-2=0$, therefore $y = 1$. Then

$\sqrt{5^{2x}-26\cdot5^x+26}=1$

$\Rightarrow(5^x-1)(5^x-25)=0$

$\Rightarrow x_1=0,\ x_2=2$

(S) Since $7^{\log_{11}13}=13^{\log_{11}7}$,

$13^{\log_{11}(x^2-10x+23)}=7^{\log_{11}13}$

$\Leftrightarrow 13^{\log_{11}(x^2-10x+23)}=13^{\log_{11}7}$

$\Leftrightarrow \log_{11}(x^2-10x+23)=\log_{11}7$

$\Leftrightarrow x^2-10x+23=7$

$\Leftrightarrow x^2-10x+16=0$

$\Leftrightarrow (x-2)(x-8)=0$

$\Leftrightarrow x=2$ or 8

(50) Answer : (D)

Hint:

Consider a as a variable and x as a parameter.

Solution:

For the sake of factorizing the left hand side, if considering a as the variable and x as a parameter, it follows that

$x^4-9x^3+2(10-a)x^2+9ax+a^2$

$=a^2-(2x^2-9x)a+(x^4-9x^3+20x^2)$

$=a^2-x(2x-9)a+x^2(x^2-9x+20)$

$=a^2-x(2x-9)a+x^2(x-4)(x-5)$

$=[a-x(x-5)][a-x(x-4)]=(a-x^2+5x)(a-x^2+4x)$

$=(x^2-5x-a)(x^2-4x-a)$

Therefore, the given equation can be written in the form

$(x^2-5x-a)(x^2-4x-a)=0$

Then,

$x^2-5x-a=0\Rightarrow x_1=\frac{5-\sqrt{25+4a}}{2}$,

$x_2=\frac{5+\sqrt{25+4a}}{2}$

$x^2-4x-a=0\Rightarrow x_3=2-\sqrt{4+a}$,

$x_4=2+\sqrt{4+a}$

(51) Answer : (C)

Hint:

Property of definite integration.

Solution:

(P) $I=\int_0^{\pi}\frac{x\sin x}{3+\sin^2 x}dx=\int_0^{\pi}\frac{(\pi-x)\sin x}{3+\sin^2 x}dx$

$2I=\int_0^{\pi}\frac{\pi\sin x}{3+\sin^2 x}=2\pi\int_0^{\frac{\pi}{2}}\frac{\sin x}{4-\cos^2 x}dx=2\pi\int_1^{0}\frac{-dt}{4-t^2}$

⇒ $I=\pi\int_0^{1}\frac{dt}{4-t^2}=-\pi\int_0^{1}\frac{1}{t^2-4}dt=-\pi\cdot\frac{1}{2\times 2}\left[\ln\left|\frac{t-2}{t+2}\right|\right]_0^1$

⇒ $I=-\frac{\pi}{4}\ln\left(\frac{1}{3}\right)=\frac{\pi}{4}\ln 3$

(Q) $I=\int_0^{\pi}\frac{x|\sin 2x|}{3+\sin^2 x}dx=\int_0^{\pi}\frac{(\pi-x)|\sin 2x|}{3+\sin^2 x}dx$

$2I=\int_0^{\pi}\frac{2\pi|\sin 2x|}{3+\sin^2 x}dx\Rightarrow I=\pi\int_0^{\frac{\pi}{2}}\frac{|\sin 2x|}{3+\sin^2 x}dx$

⇒ $I=\pi\int_0^{\frac{\pi}{2}}\frac{2\sin x\cos x}{4-\cos^2 x}dx=\pi\int_0^{1}\frac{2t}{4-t^2}dt$

$=-\pi\left[\ln\left|t^2-4\right|\right]_0^1=\pi\ln\left(\frac{4}{3}\right)$

(R) $I=\int_0^{\frac{\pi}{2}}\frac{\sin x+\cos x}{3+\sin 2x}dx=\int_0^{\frac{\pi}{2}}\frac{d(\sin x-\cos x)}{4-(\sin x-\cos x)^2}$

⇒ $I=\frac{1}{4}\left[\ln\left|\frac{2+(\sin x-\cos x)}{2-(\sin x-\cos x)}\right|\right]_0^{\frac{\pi}{2}}=\frac{1}{4}\ln 3$

(S) $I=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\ln(1+\cot x)dx$

$I=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\ln\left(1+\cot\left(\frac{3\pi}{4}-x\right)\right)dx$

$2I=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\ln\left((1+\cot x)\cdot\left(1+\cot\left(\frac{3\pi}{4}-x\right)\right)\right)dx$

$2I=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\ln 2dx=\frac{\pi}{4}\ln 2$

⇒ $I=\frac{\pi}{8}\ln 2$