Aakash
Medical|IIT-JEE|Foundations

Corporate Office : AESL, 3rd Floor, Incuspaze Campus-2, Plot No. 13, Sector-18, Udyog Vihar, Gurugram, Haryana - 122015, ***Ph.***+91-1244168300

MM : 180

AIATS For One Year JEE(Advanced)-2026 (XII Studying & XII Passed)_Test-7A_Paper-2_Online

Time : 180 Min.

PHYSICS

Section-I

1. (B)

2. (B)

3. (D)

4. (B)

Section-II

5. (A,C)

6. (A,C)

7. (A,C)

Section-III

8. (16)

9. (25)

10. (11)

11. (3)

12. (13)

13. (12)

Section-IV

14. (15.00)

15. (27.00)

16. (02.25)

17. (50.00)

CHEMISTRY

Section-I

18. (C)

19. (B)

20. (B)

21. (C)

Section-II

22. (B,C,D)

23. (A,B)

24. (B,C)

Section-III

25. (2)

26. (19)

27. (4)

28. (10)

29. (2)

30. (10)

Section-IV

31. (70.00)

32. (14.00)

33. (04.00)

34. (05.00)

MATHEMATICS

Section-I

35. (A)

36. (D)

37. (B)

38. (D)

Section-II

39. (A,B,D)

40. (A,B,C)

41. (A,B,C)

Section-III

42. (3)

43. (5)

44. (18)

45. (9)

46. (0)

47. (36)

Section-IV

48. (01.00)

49. (02.00)

50. (10.00)

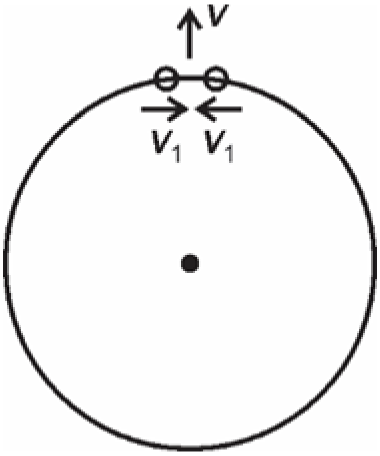
51. (95.00)

Hints and Solutions

PHYSICS

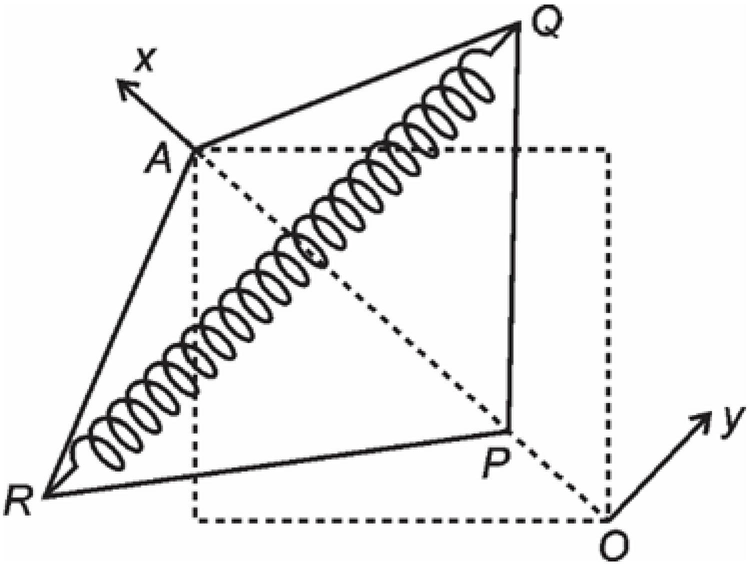
Section-I

(1) **Answer :** (B)
Hint:
Momentum conservation
Solution:



Just before collision
4*mv* = 2*mu*
$v = \frac{u}{2}$
$2\left(\frac{1}{2}\cdot m\cdot u^2\right) = \frac{1}{2}2m\cdot\frac{u^2}{4} + 2\cdot\left(\frac{1}{2}m\left(\frac{u^2}{4}+v_1^2\right)\right)$
$v_1 = \frac{u}{\sqrt{2}}$
Speed = $\sqrt{v^2+v_1^2}$ = $\frac{\sqrt{3}u}{2}$

(2) **Answer :** (B)
Hint:
$T = 2\pi\sqrt{\frac{m}{k}}$
Solution:



OP = *x*
$QR = \sqrt{2}l + 2y$
$\left(\frac{a}{\sqrt{2}}+y\right)^2 + \left(\frac{a}{\sqrt{2}}-\frac{x}{2}\right)^2 = a^2$
$y \simeq \frac{x}{2}$

$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E$

$T = 2\pi\sqrt{\frac{m}{k}}$

(3) **Answer :** (D)

Hint:

q = *CV*

Solution:

q = *CV*

$t = \frac{d}{v_0}$

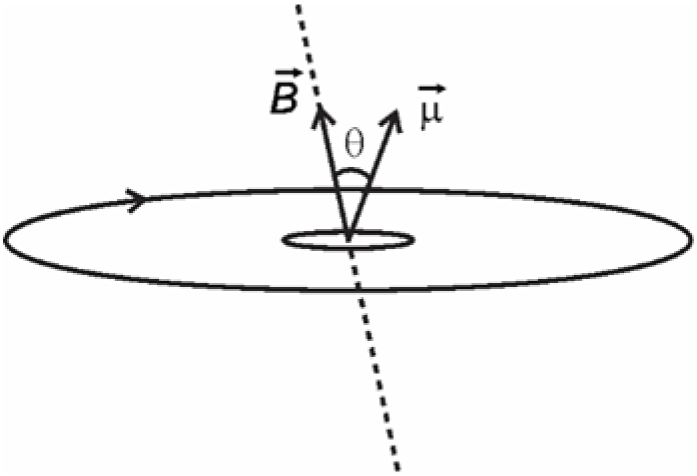
$i = \frac{CV}{t} = \frac{20\times10^{-6}\times200\times3}{10\times10^{-2}} = 6$ mA

(4) **Answer :** (B)

Hint:

$\tau = -\mu B\sin\theta$

Solution:



$\tau = -\mu B\sin\theta$

$\tau = -\mu B\theta = -(\pi r^2 i)\frac{\mu_0 I}{2R}\theta$

$t = \frac{\tau}{4} = \frac{1}{4}2\pi\sqrt{\frac{I}{c}}$

$t = \frac{\pi}{2}\sqrt{\frac{mR}{\mu_0\pi Ii}}$

$\Rightarrow\ t = \sqrt{\frac{\pi mR}{4\mu_0 Ii}}$

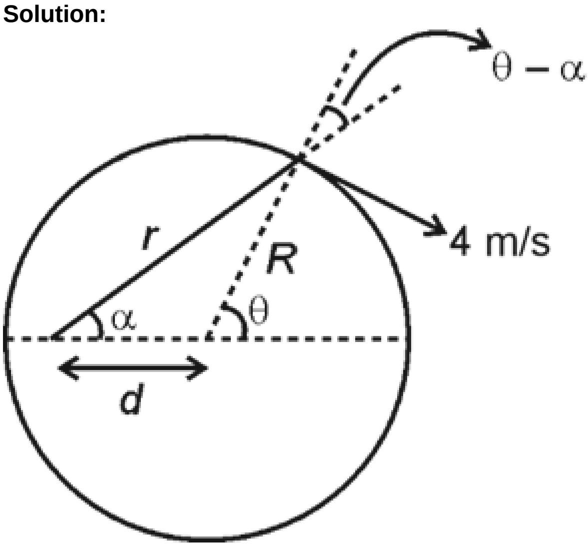
Section-II

(5) **Answer :** (A,C)

Hint:

$\frac{dr}{dt} = 4\sin(\theta-\alpha)$

Solution:



$\frac{dr}{dt} = 4\sin(\theta-\alpha)$

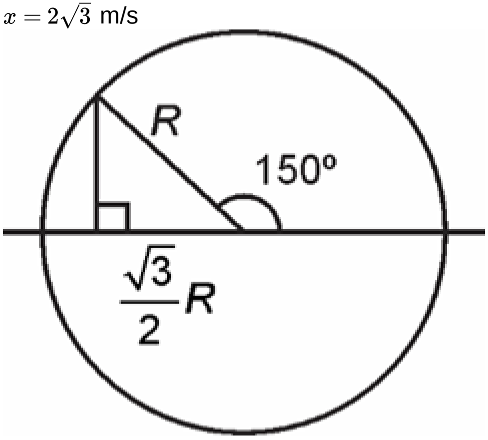
$\frac{R}{\sin\alpha} = \frac{d}{\sin(\theta-\alpha)}$

$\sin(\theta-\alpha) = \frac{3\sqrt{3}}{6}\sin\alpha$

$\frac{dr}{dt} = 2\sqrt{3}\sin\alpha$

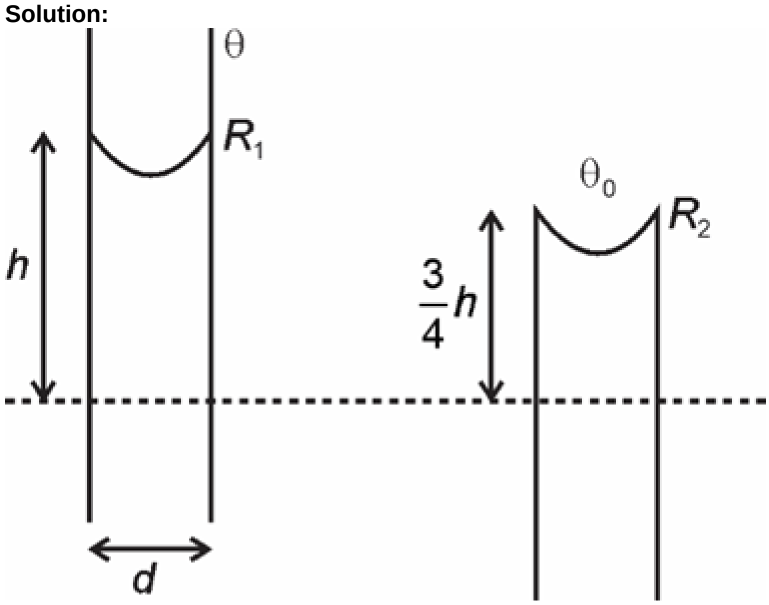
$\left(\frac{dr}{dt}\right)$ is maximum when α = 90º

$x = 2\sqrt{3}$ m/s



$t_0 = \frac{5\pi}{6\times\left(\frac{4}{6}\right)} = \frac{5\pi}{4}$

(6) **Answer :** (A,C)
Hint:
$P = P_0 - \frac{s}{R} + \rho gh$
Solution:



$P = P_0 - \frac{s}{R_1} + \rho gh$ $\frac{s}{R_1} = \rho gh$ $\frac{d}{2R_1} = \cos\theta$ $\frac{s2\cos\theta}{d} = \rho gh$ $h = \frac{2s\cos\theta}{\rho gd}$	$P_0 = P_0 - \frac{s}{R_2} + \frac{\rho g3h}{4}$ $\frac{s}{R_2} = \frac{3}{4}\rho gh$ $\frac{2s\cos\theta_0}{d} = \frac{3}{4}\rho gh$ $\frac{2s\cos\theta_0}{d} = \frac{3}{4}\cdot\frac{2s\cos\theta}{d}$ $\cos\theta_0 = \frac{3}{4}\cos\theta$ $= \frac{3}{4}\times\frac{\sqrt{3}}{2}$ $= \frac{3\sqrt{3}}{8}$

(7) **Answer :** (A,C)
Hint:
$\frac{dN}{dt} = R - \lambda N$
Solution:
$\overset{R}{\rightarrow} N \overset{\lambda}{\rightarrow}$

$\frac{dN}{dt} = R - \lambda N$

$$\int_0^N \frac{dN}{R-\lambda N} = \int_{t=0}^{t} dt$$

$N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$

Activity $A = \lambda N = R(1 - e^{-\lambda t})$

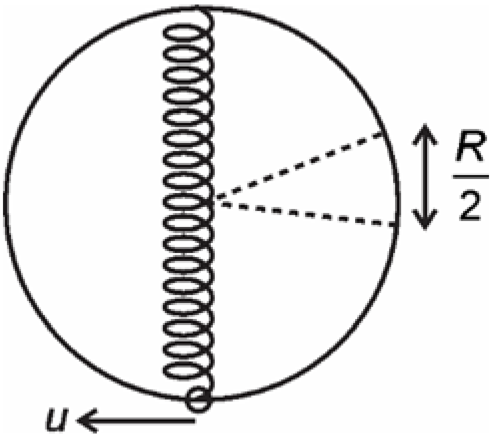
Section-III

(8) **Answer :** 16

Hint:

Work energy theorem

Solution:



$kR = mg + \frac{mv^2}{R}$

$\frac{1}{2}mv^2 = mg\frac{3R}{2} - \frac{1}{2}kR^2$

$k = \frac{2mg}{R}$

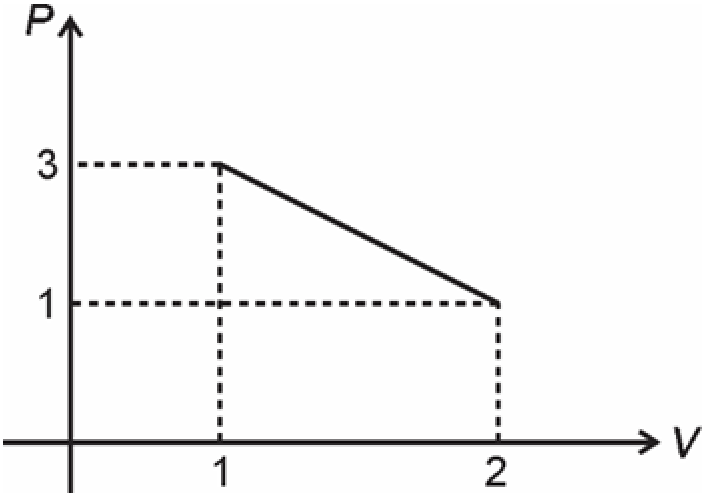
k = 16 N/m

(9) **Answer :** 25

Hint:

Work = Area of PV graph

Solution:



$W = \left[\frac{1}{2} \times 2 \times 1 + 1\right] \times 10^5 \times 10^{-2} = 200$ J

$T_i = \frac{3 \times 10^5 \times 10^{-3}}{\frac{1}{5} \times \frac{25}{3}}$ = 180 K $\quad\quad \tau_f = 120^\circ$

$\Delta U = \frac{1}{5} \times \frac{5}{2} \cdot \frac{25}{3} \times (-60) = -250$

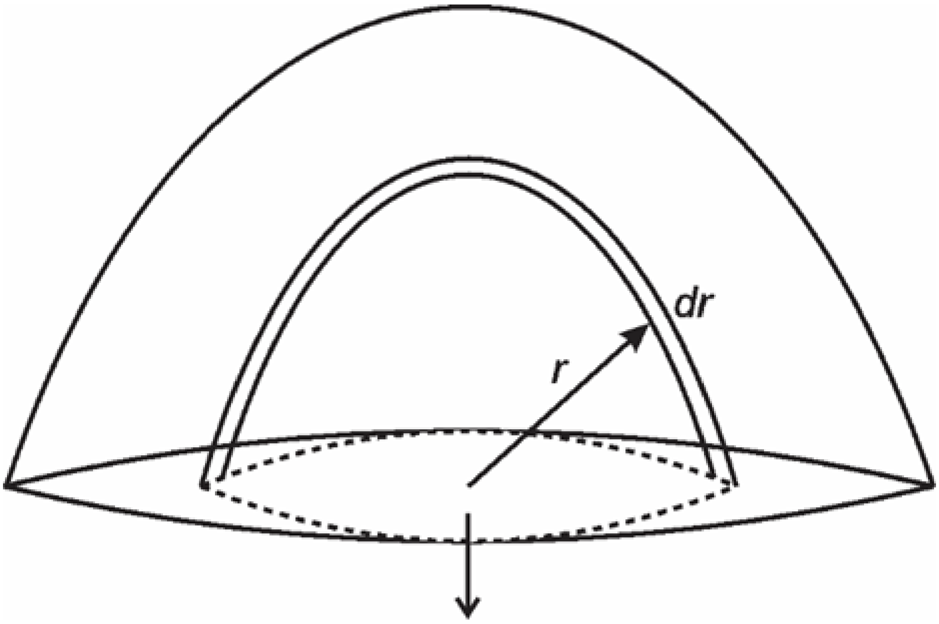
Q = –50

$-50 = \frac{1}{5} \times C \times (-60)$

$C_0 = \frac{25}{6}$ ⇒ 6C_0 = 25

(10) **Answer :** 11

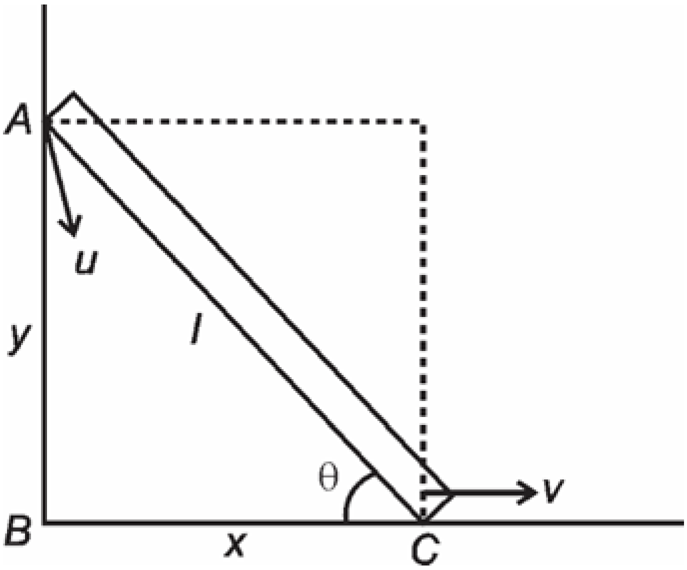
Hint:
$E=\frac{\sigma}{4\varepsilon_0}$
Solution:



$dE=\frac{\sigma}{4\varepsilon_0}=\frac{\rho\cdot dr}{4\varepsilon_0}$

$E=\int_R^{2R}\frac{kr}{4\varepsilon_0 R}dr=\frac{3kR}{8\varepsilon_0}$

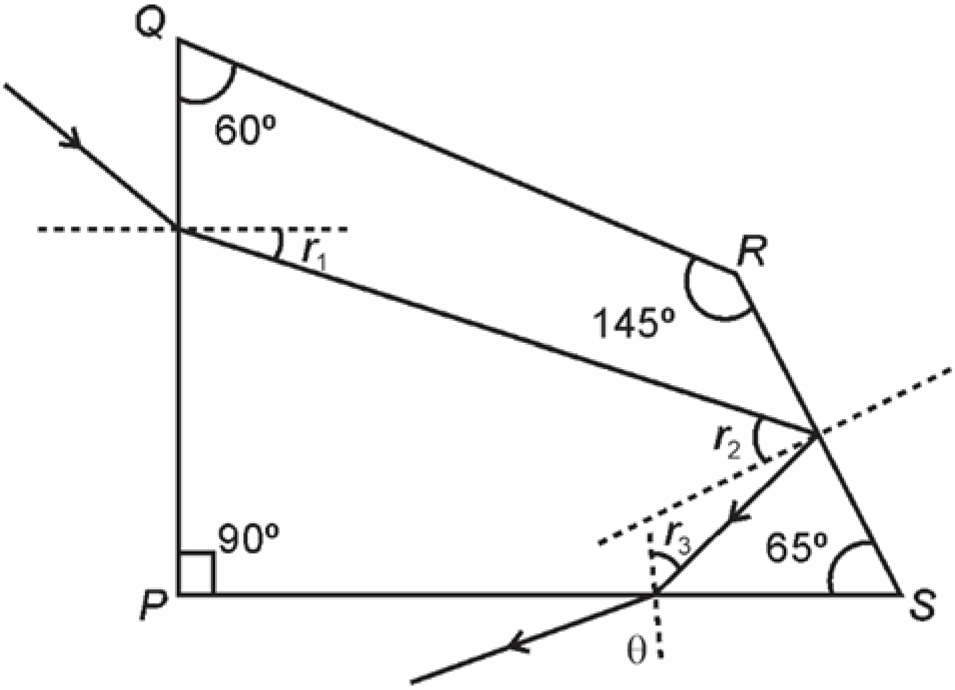
(11) **Answer :** 3
Hint:
$\omega=\frac{v}{l\sin\theta}$
Solution:



$u\sin\theta=v\cos\theta$
$u=v\cot\theta$
$\omega=\frac{v}{l\sin\theta}=2$
Consider loop *ABC*
$A=\frac{1}{2}xyB$
EMF = $-\frac{B}{2}\frac{d(xy)}{dt}$ = $-\frac{B}{2}(vy-xu)$
$=-\frac{B}{2}\left(v\cdot l\sin\theta-l\cos\theta\cdot v\cot\theta\right)$
$=\frac{Bvl\cos2\theta}{2\sin\theta}$ = $\frac{3\times4\times\frac{1}{2}\times\frac{1}{2}}{2\times\frac{1}{2}}$ = 3 volt

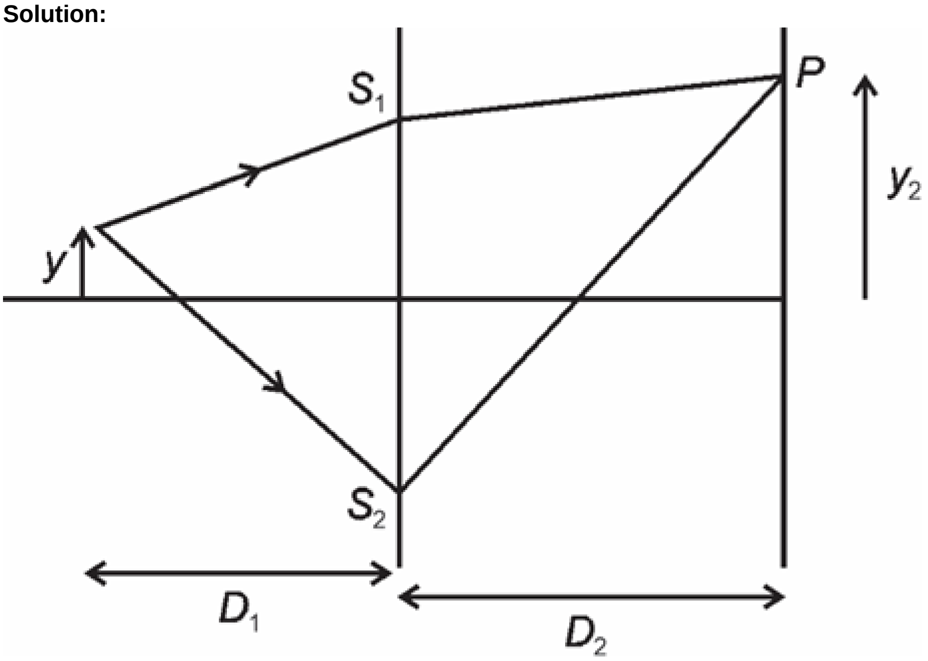
(12) **Answer :** 13

Hint:
$r_2 > \theta_c$ so *TiR* at *RS*
Solution:



At face *PQ*
$\sin 60^{\circ} = \sqrt{3}\sin r_1$
$r_1 = 30^{\circ}$
$\theta_c = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
$r_2 - r_1 = 180 - (90 + 65)$
$r_2 = 25^{\circ} + 30^{\circ}$
$r_2 = 55^{\circ}$
$r_2 > \theta_c$ so *TiR* at *RS*
$r_3 = 10^{\circ} \qquad \sin\theta = \sqrt{3}\sin 10^{\circ} \Rightarrow \ \theta = \sin^{-1}\left(\sqrt{3}\sin 10^{\circ}\right)$

(13) **Answer :** 12
Hint:
$\frac{y}{D} + \frac{y_1}{D_1} = 0$
Solution:



At point *P*
$\Delta x = 0$
$\frac{y}{D_1} + \frac{y_2}{D_2} = 0$

$y_2 = -\frac{D_2}{D_1}y$

$y_2 = -2 \cdot 4 \cdot \sin\left(15t + \frac{\pi}{6}\right)$

V_{max} = 2 × 4 × 15 mm/s
= 120 mm/s
= 12 cm/s

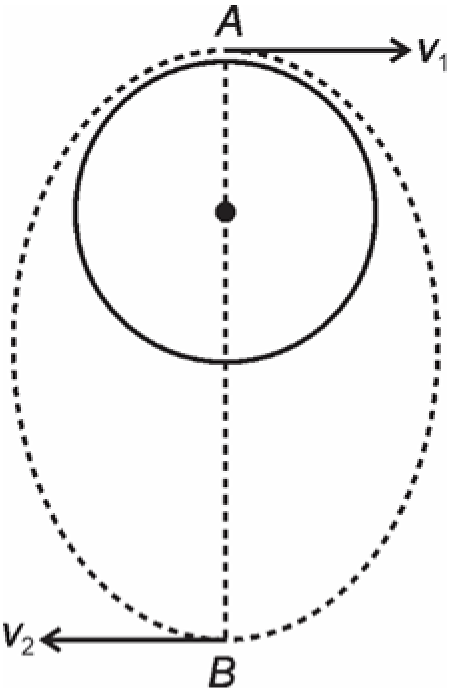
Section-IV

(14) Answer : 15.00

Hint:

$TE = -\frac{GMm}{2a}$

Solution:



$E_A = -\frac{GMm}{R} + \frac{1}{2}m \cdot \frac{5}{3}GM = -\frac{GMm}{6R}$

In elliptical path

$\tau E = -\frac{GMm}{r_1+r_2} = -\frac{GMm}{6R}$

$r_1 + r_2 = 6R$

Semi-major axis *a* = 3*R*

$r_1 = R_1,\ r_2 = 5R$

$-\frac{GMm}{6R} = \frac{1}{2}mV_2^2 - \frac{GMm}{5R}$

$\sqrt{\frac{GM}{15r}} = v_2$

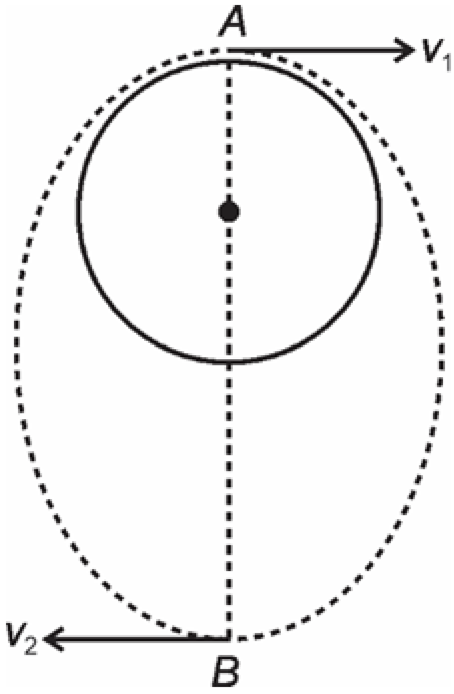
$\tau = 2\pi\sqrt{\frac{27R^3}{GM}}$

(15) Answer : 27.00

Hint:

$TE = -\frac{GMm}{2a}$

Solution:



$E_A = -\frac{GMm}{R} + \frac{1}{2}m \cdot \frac{5}{3}GM = -\frac{GMm}{6R}$

In elliptical path

$\tau E = -\frac{GMm}{r_1+r_2} = -\frac{GMm}{6R}$

$r_1 + r_2 = 6R$

Semi-major axis *a* = 3*R*

$r_1 = R_1,\ r_2 = 5R$

$-\frac{GMm}{6R} = \frac{1}{2}mV_2^2 - \frac{GMm}{5R}$

$\sqrt{\frac{GM}{15r}} = v_2$

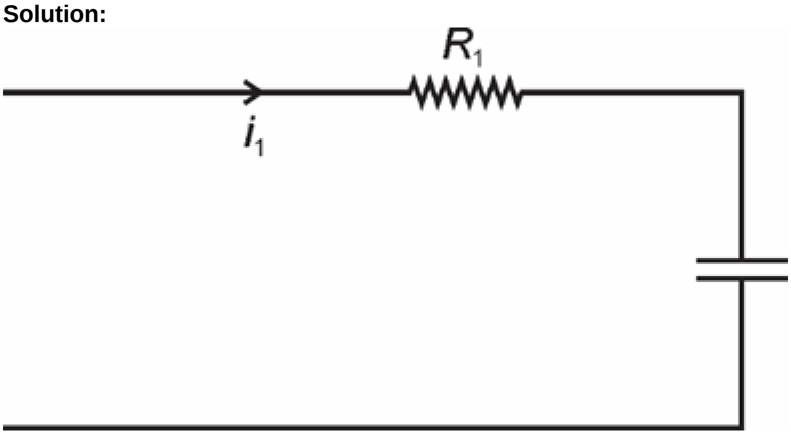
$\tau = 2\pi\sqrt{\frac{27R^3}{GM}}$

(16) **Answer :** 02.25

Hint:

$q = q_0\left(1 - e^{-\frac{t}{\tau}}\right)$

Solution:



$R_{\text{eff}} = \frac{R}{2} + R = \frac{3R}{2}$

$\tau = \frac{3RC}{2} = \frac{3}{2} \times 100 \times 60 \times 10^{-6} = 9$ ms

$q = \frac{CV}{2}\left(1 - e^{-\frac{t}{\tau}}\right)$

$q = 3\left(1 - e^{-\frac{t}{\tau}}\right)$ mc

$q = 3\left(1 - \frac{1}{4}\right) = 2.25$

$i_1 = \frac{dq}{dt} = \frac{3}{\tau}e^{-\frac{t}{\tau}}$ mA

$$H=\int i_1^2 R dt=\frac{9R}{\tau^2}\int\limits_0^\infty e^{-\frac{2t}{\tau}}dt\times10^{-6}$$

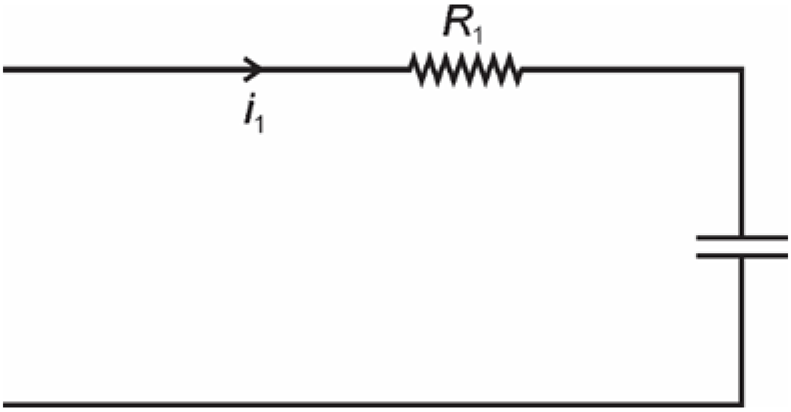
$=\frac{9R\times10^{-6}}{2\tau}$ = $\frac{9\times100\times10^{-6}}{2\times9\times10^{-3}}$ = 50 mJ

(17) Answer : 50.00

Hint:

$q=q_0\left(1-e^{-\frac{t}{\tau}}\right)$

Solution:



$R_{\text{eff}}=\frac{R}{2}+R=\frac{3R}{2}$

$\tau=\frac{3RC}{2}=\frac{3}{2}\times100\times60\times10^{-6}=9$ ms

$q=\frac{CV}{2}\left(1-e^{-\frac{t}{\tau}}\right)$

$q=3\left(1-e^{-\frac{t}{\tau}}\right)$ mc

$q=3\left(1-\frac{1}{4}\right)=2.25$

$i_1=\frac{dq}{dt}=\frac{3}{\tau}e^{-\frac{t}{\tau}}$ mA

$$H=\int i_1^2 R dt=\frac{9R}{\tau^2}\int\limits_0^\infty e^{-\frac{2t}{\tau}}dt\times10^{-6}$$

$=\frac{9R\times10^{-6}}{2\tau}$ = $\frac{9\times100\times10^{-6}}{2\times9\times10^{-3}}$ = 50 mJ

CHEMISTRY

Section-I

(18) Answer : (C)

Hint:

$\ln k=\ln A-\frac{E_a}{R}\times\frac{1}{T}$

Solution:

$\ln k=\ln A-\frac{E_a}{RT}$ ⇒ $\frac{d\ln k}{dT}=\frac{E_a}{R}\times\frac{1}{T^2}$

Now, $\frac{d\ln k}{dT}=\beta^3\times\frac{1}{T}+\frac{1}{\gamma T^2}$

Equating $E_a=RT^2\left[\frac{\beta^3}{T}+\frac{1}{\gamma T^2}\right]$ = $RT\left(\beta^3\right)+\frac{R}{\gamma}$

(19) Answer : (B)

Hint:

Pyroxene contains three oxygen atoms per silicon atom.

Solution:

In pyroxene there is total of 3 oxygen atoms per silicon atom, so simplest repeating unit is $(SiO_3)_n^{2n-}$

(20) Answer : (B)

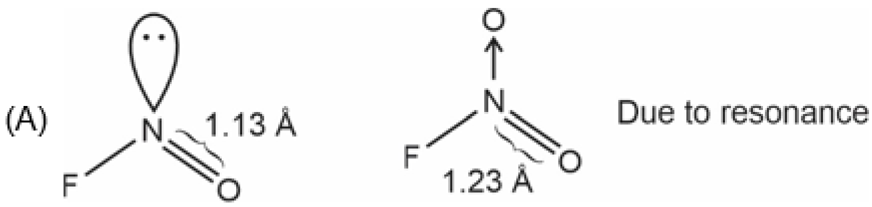
Hint:

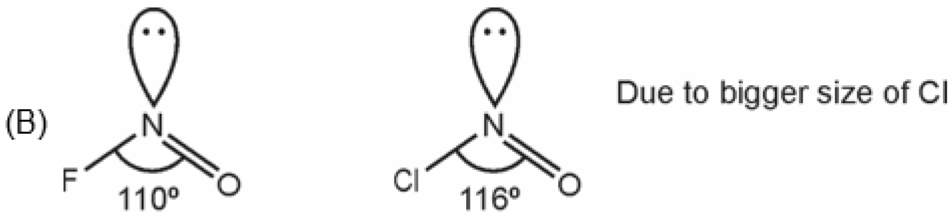
In DNA, adenine combines with thymine and guanine combines with cytosine.

Solution:
In DNA A = T
G ≡ C
And A% + T% + C% + G% = 100 (G = 23%)
2x = 100 – 46 ⇒ x = 27%
So, number of adenine residue = 27% of 5000 kb
$= 0.27 \times 5000 \times 10^3$ b
$= 2.7 \times 5 \times 10^5$
$= 1.35 \times 10^6$ b

(21) Answer : (C)
Hint:
C – O stretching frequency ∝ C – O bond energy.
Solution:
C – O stretching frequency ∝ C – O bond energy or bond order.
Bond order $\propto \frac{1}{\text{Amount of M}\rightarrow\text{CO back bonding}}$
Correct sequence is
$[HBI_2 \leftarrow CO] > CO > Mn(CO)_5 > [Tc(CO)_5]^-$

Section-II

(22) Answer : (B,C,D)
Hint:
$ClSO_3H$ converted to $ClNO_2$ when treated with HNO_3.
Solution:




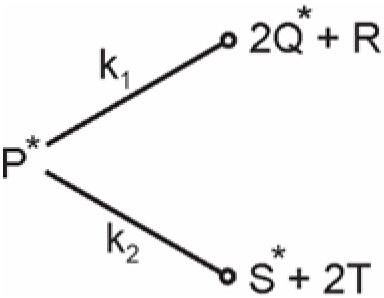
(C) $N_2O_4 + 2\,CoF_3(s) \xrightarrow{300^\circ C} 2\,FNO_2 + 2\,CoF_2(s)$

(D) $ClSO_3\,H + HNO_3(\text{anhydrous}) \xrightarrow{0^\circ C} ClNO_2 + H_2\,SO_4$

(23) Answer : (A,B)
Hint:
A compound must follow Huckel's rule to show aromaticity.
Solution:
(A) Enol form is less stable so $K_{eq} > 1$.
(B) Correct dipole moment order is
$\underset{0.37}{Ph-Me} < \underset{0.58}{Ph-Et} < \underset{0.65}{PhCH(CH_3)_2} < \underset{0.7D}{PhC(CH_3)_3}$
(C) Is an aromatic compound.
(D) Ferrocene contains Fe^{2+} ion.

(24) Answer : (B,C)
Hint:
For first order reaction $k = \frac{1}{t}\ln\frac{A_0}{A}$, here A_0 is initial concentration, A is final concentration.
Solution:
Number of moles of P = 2
Overall rate constant = $k_1 + k_2 = 0.0693 + 0.1386 = 3 \times 0.0693$ hrs^{-1}
Overall $t_{1/2} = \frac{0.693}{3\times 0.0693}$ hour $= \frac{10}{3} \times 60$ min
= 200 minutes

= 3 hours 20 minutes

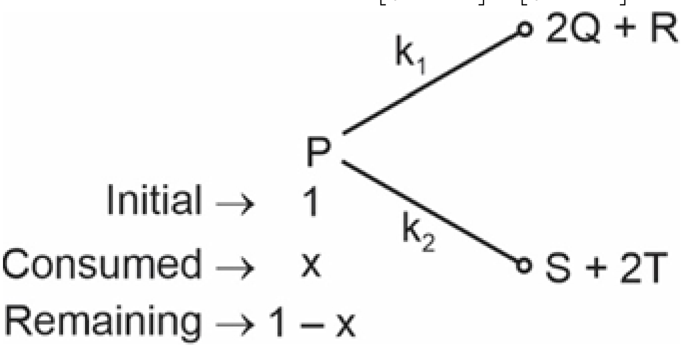


After 3 hours 20 minutes, number of mole of P left = 1

Number of moles of Q formed = $2 \times \frac{0.0693}{3 \times 0.0693} \times 1$ = $\frac{2}{3}$

Number of moles of S formed = $1 \times \frac{2 \times 0.0693}{3 \times 0.0693} \times 1$ = $\frac{2}{3}$

Total optical rotation = $[1 \times +60^{\circ}] + \left[\frac{2}{3} \times +30^{\circ}\right] + \left[\frac{2}{3} \times -90^{\circ}\right] = +20^{\circ}$



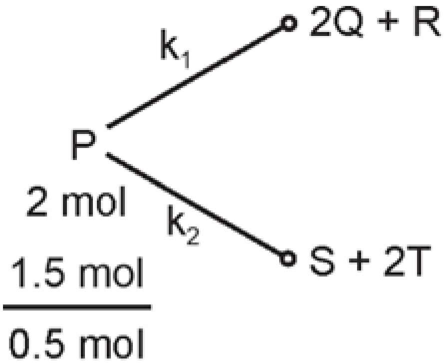
Q formed = $2 \times \frac{k_1}{k} x = 2 \times \frac{1}{3} x$

R formed = $1 \times \frac{k_1}{k} x = \frac{1}{3} x$

S formed = $1 \times \frac{k_2}{k} x = \frac{2}{3} x$

T formed = $2 \times \frac{k_2}{k} x = 2 \times \frac{2}{3} x$

Fraction of S = $\frac{\frac{2}{3}x}{\frac{9}{3}x} = \frac{2}{9} = 0.22$



Now, moles of Q formed = $2 \times \frac{k_1}{k} \times 1.5 = 2 \times \frac{1}{3} \times \frac{3}{2} = 1$

Moles of S formed = $1 \times \frac{k_2}{k} \times 1.5 = 1 \times \frac{2}{3} \times \frac{3}{2} = 1$

Moles of P leftover = 0.5

So, net optical rotation = [0.5 × +60º] + [1 × 30º] + [1 × –90º]

30º + 30º – 90º = –30º (laevorotatory)

Section-III

(25) **Answer :** 2

Hint:

$\frac{\mathrm{x}}{\mathrm{m}} = \mathrm{k}(\mathrm{P})^{\frac{1}{\mathrm{n}}}$; k and n are Freundlich constants.

Solution:

$\log \frac{x}{m} = \log k + \frac{1}{n} \log P$ $\qquad$ $\frac{1}{n} = \tan\theta = 1$

or, $\frac{x}{m} = k(P)^{1/n}$ $\qquad$ $\log k = 2$, k = 100

$\frac{x}{2} = 100 \times (0.01)^1$

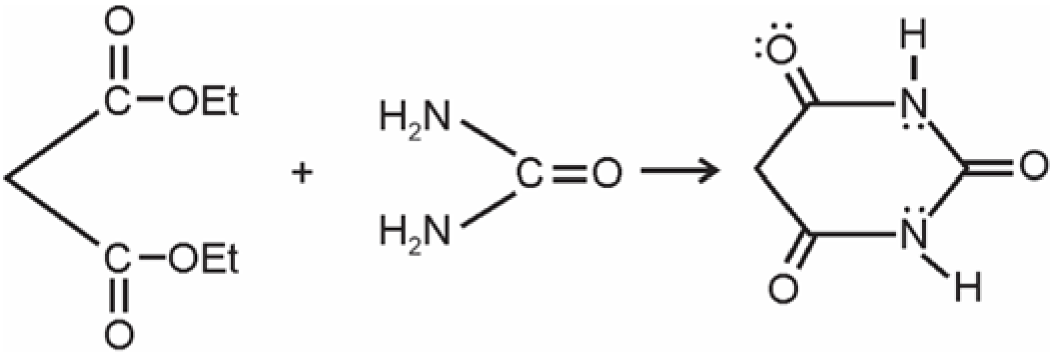
$\boxed{x = 2\text{ g}}$

(26) **Answer :** 19
Hint:
As pH increases, solution becomes more alkaline.
Solution:
At low pH (pH = 1) cationic species P (2 positive charges) is formed. At high pH (pH = 12), anionic species Q is formed.
x = 1
y = 12
z = 7

(27) **Answer :** 4
Solution:
Magnesite, Dolomite, Calamine and Cerussite are carbonate ores

(28) **Answer :** 10
Hint:
I_2 is more soluble in ether than water.
Solution:
Let x mg I_2 is present in 42 mL water at equilibrium.
⇒ 9 – x mg I_2 has transformed to 4.2 mL THF.
So, $\frac{9-x}{4.2} = 890\left(\frac{x}{42}\right)$
x = 0.1 mg = 1 × 10^{-4} g

(29) **Answer :** 2
Hint:
$[Fe(EDTA)]^{2-}$ does not show geometric isomerism.
Solution:
(1) $[Fe(EDTA)]^{2-}$ p = 0
q = 2
(2) $[Mo(acac)_2en]Br$ r = 0
s = 2
(3) K[PtBrCl(gly)] t = 2
u = 0
(p + q) + (r + s) – (t + u) = (0 + 2) + (0 + 2) – (2 + 0) = 2

(30) **Answer :** 10
Hint:
Esters react with urea to give amides.
Solution:


x = 2
y = 8

Section-IV

(31) **Answer :** 70.00
Hint:
$\Delta H = \Delta U + (\Delta n_g)\ RT$
Solution:
$|q_{water}| = m_{water}\ c_{water}\ \Delta T$
= 420 × 1 × 4.184 × 1.37 J
= 2.407 kJ
$|q_{bomb\ calorimeter}|$ = 1.93 × 1.37 = 2.644 kJ
$|q_{combined}|$ = 2.407 + 2.644 = 5.051

$$\left|\Delta U\right| = \frac{\left|q_{combined}\right|}{\frac{0.173}{24}} = 700.73\ \frac{kJ}{mol}$$

$\frac{x}{10} = \frac{700.73}{10} \cong 70.07$

$Mg\ (s) + \frac{1}{2}O_2(g) \rightarrow MgO\ (s) \qquad \Delta n = -\frac{1}{2}$

ΔH = (ΔU) + (Δn) R × 298

= $-700.73 + \left(-\frac{1}{2}\right) \times 8.314 \times 10^{-3} \times 298$

= $701.96\ \frac{kJ}{mol}$

y = 701.96 × 2 = 1403.94 kJ

$\frac{y}{100} = 14.03$

(32) **Answer :** 14.00

Hint:

$\Delta H = \Delta U + (\Delta n_g)\ RT$

Solution:

$|q_{water}| = m_{water}\ c_{water}\ \Delta T$

= 420 × 1 × 4.184 × 1.37 J

= 2.407 kJ

$|q_{bomb\ calorimeter}|$ = 1.93 × 1.37 = 2.644 kJ

$|q_{combined}|$ = 2.407 + 2.644 = 5.051

$$\left|\Delta U\right| = \frac{\left|q_{combined}\right|}{\frac{0.173}{24}} = 700.73\ \frac{kJ}{mol}$$

$\frac{x}{10} = \frac{700.73}{10} \cong 70.07$

$Mg\ (s) + \frac{1}{2}O_2(g) \rightarrow MgO\ (s) \qquad \Delta n = -\frac{1}{2}$

ΔH = (ΔU) + (Δn) R × 298

= $-700.73 + \left(-\frac{1}{2}\right) \times 8.314 \times 10^{-3} \times 298$

= $701.96\ \frac{kJ}{mol}$

y = 701.96 × 2 = 1403.94 kJ

$\frac{y}{100} = 14.03$

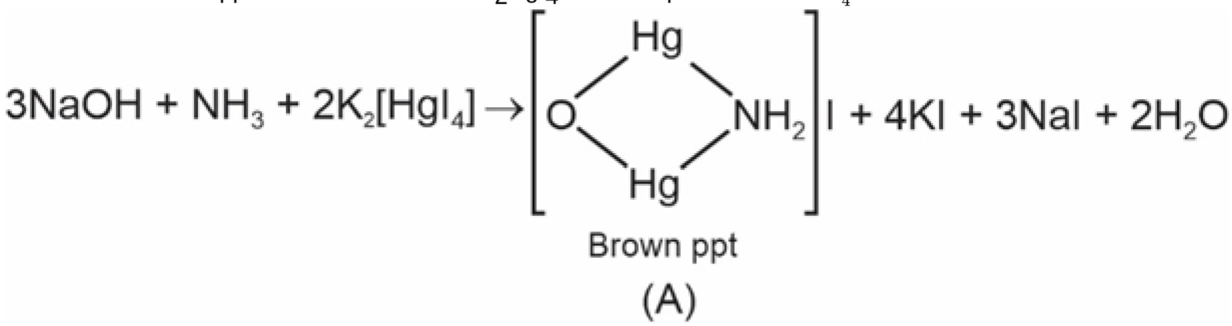
(33) **Answer :** 04.00

Hint:

Ammonium ions give a brown precipitate with K_2HgI_4

Solution:

Formation of brown ppt on reaction with alk. K_2HgI_4 indicates presence of NH_4^+.



$$3\,Fe^{2+} + 2K_3\left[Fe\,(CN)_6\right] \rightarrow \underset{(B)}{Fe_3\,[Fe\,(CN)_6]_2} + 6K^+$$

Appearance of 'B' confirm presence of Fe^{2+}.

$$SO_4^{2-} + BaCl_2 \rightarrow \underset{(C)}{BaSO_4} + 2\,Cl^-$$

Appearance of 'C' confirms presence of SO_4^{2-} ions, so 'X' is $FeSO_4 \cdot (NH_4)_2\,SO_4 \cdot 6H_2O$.

Total positive or negative charge on X is 4.

$E = \frac{M}{4} \qquad \boxed{Z = 4}$

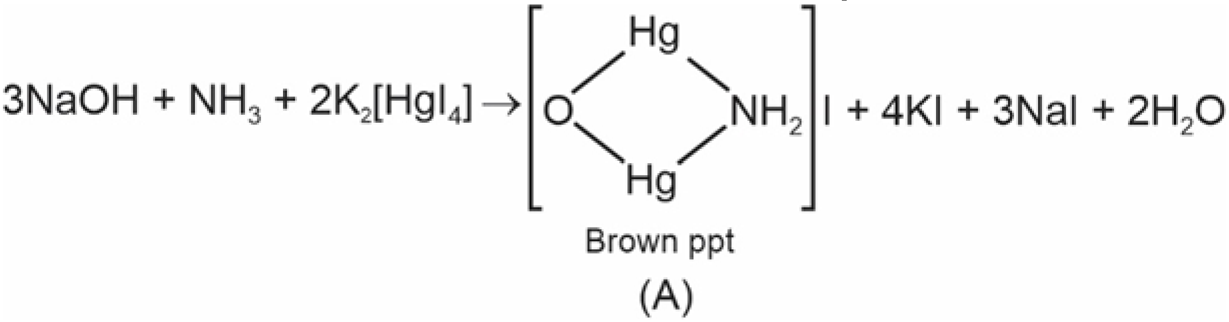
(34) **Answer :** 05.00

Hint:

Ammonium ions give a brown precipitate with K_2HgI_4

Solution:

Formation of brown ppt on reaction with alk. K_2HgI_4 indicates presence of NH_4^+



$3\,Fe^{2+} + 2K_3\left[Fe\,(CN)_6\right] \rightarrow \underset{(B)}{Fe_3\,[Fe\,(CN)_6]_2} + 6K^+$

Appearance of 'B' confirm presence of Fe^{2+}.

$SO_4^{2-} + BaCl_2 \rightarrow \underset{(C)}{BaSO_4} + 2\,Cl^-$

Appearance of 'C' confirms presence of SO_4^{2-} ions, so 'X' is $FeSO_4 \cdot (NH_4)_2\,SO_4 \cdot 6H_2O$.

Number of Fe atoms per molecule of B = 5

MATHEMATICS

Section-I

(35) **Answer :** (A)

Hint:

$0 < x < \frac{\pi}{2} \Rightarrow \frac{\sin x}{x}$ is decreasing and $\sin x < x < \tan x$

Solution:

$0 < x < \frac{\pi}{2} \Rightarrow \frac{\sin x}{x}$ is decreasing and $\sin x < x < \tan x$

$\Rightarrow \frac{\sin(\sin x)}{\sin x} > \frac{\sin x}{x} > \frac{\sin(\tan x)}{\tan x}$

$\Rightarrow I_1 > I_2 > I_3$

(36) **Answer :** (D)

Hint:

Let $|X| = \frac{|4x-3y|}{5}$ and $|Y| = \frac{|3x+4y|}{5}$

Solution:

⇒ Given equation is $[|X|] + [|Y|] = 3$

symmetric about *X*-axis and *Y*-axis.

For X, Y ≥ 0

⇒ area $= 4$ sq. units

⇒ Total area enclosed $= 4 \times 4 = 16$ sq. units.

(37) **Answer :** (B)

Hint:

The key idea is to pair up the terms $\left[\frac{2025}{-x}\right]$ and $\left[\frac{2025}{x}\right]$.

Solution:

The key idea is to pair up the terms $\left[\frac{2025}{-x}\right]$ and $\left[\frac{2025}{x}\right]$. There are 1000 such pairs and one lone term, $\left[\frac{2025}{1000.5}\right] = 2$.

Thus, $\sum_{j=-1000}^{1000}\left[\frac{2025}{j+0.5}\right] = 2 + \sum_{x\in\{0.5,\,1.5,\,\ldots,\,999.5\}}\left(\left[\frac{2025}{x}\right] + \left[\frac{2025}{-x}\right]\right)$

We note that $[a] + [-a] = \begin{cases} 0 & \text{if } a \text{ is an integer} \\ -1 & \text{otherwise} \end{cases}$

Therefore, $\left[\frac{2025}{x}\right] + \left[\frac{2025}{-x}\right] = \begin{cases} 0 & \text{if } 2x \text{ divides } 4050 \\ -1 & \text{otherwise} \end{cases}$

As x ranges in the set $\{0.5,\ 1.5,\ 2.5,\ \ldots,\ 999.5\}$, $2x$ ranges in the set $\{1,\ 3,\ 5,\ \ldots,\ 1999\}$. This set includes all 15 odd divisors of 4050 except for 2025. Thus, there are 14 values of x or which $\left[\frac{2025}{x}\right]+\left[\frac{2025}{-x}\right]$ evaluates to 0 , and the remaining $1000-14=986$ values of x make it evaluate to –1 . Therefore,

$\sum_{j=-1000}^{1000}\left[\frac{2025}{j+0.5}\right]=2+\sum_{x\in\left\{0.5,\,1.5,\,\ldots,\,999.5\right\}}\left(\left[\frac{2025}{x}\right]+\left[\frac{2025}{-x}\right]\right)$

$=2+986\cdot(-1)=-984$

(38) **Answer :** (D)

Hint:

$\det(A)=\det(BCD)$

$\det(A)=\det(B)\det(C)\det(D)$

Solution:

$P=\begin{bmatrix}1&1&1\\0&2&2\\0&0&3\end{bmatrix}$ and $Q=\begin{bmatrix}2&x&x\\0&4&0\\x&x&6\end{bmatrix}$

So $6P^{-1}=\begin{bmatrix}6&-3&0\\0&3&-2\\0&0&2\end{bmatrix}$

$\because\ R=PQP^{-1}\Rightarrow \text{det}\,(R)=\text{det}\,(P)\cdot\text{det}\,(Q)\cdot\text{det}\,\left(P^{-1}\right)$

$\Rightarrow \text{det}\,(R)=\text{det}\,(Q)=\text{det}\begin{bmatrix}2&x&x\\0&4&0\\x&x&6\end{bmatrix}$

$\Rightarrow \text{det}\,(R)=\text{det}\begin{bmatrix}2&x&x\\0&4&0\\x&x&5\end{bmatrix}+\text{det}\begin{bmatrix}2&x&0\\0&4&0\\x&x&1\end{bmatrix}$

$\Rightarrow \text{det}\,(R)=\text{det}\begin{bmatrix}2&x&x\\0&4&0\\x&x&5\end{bmatrix}+8$

Section-II

(39) **Answer :** (A,B,D)

Hint:

$m=\frac{y_2-y_1}{x_2-x_1}\ \Rightarrow\ (y_2-y_1)=m\,(x_2-x_1)$

Solution:

The equation of the line through $P\,(x,\ y)$ making an angle with the x -axis which is supplementary to the angle made by the tangent at $P\,(x,\ y)$ is

$Y-y=-\frac{dy}{dx}\,(X-x)$

Where it meets the *x*-axis

$Y=0,\ X=x+\frac{y}{\frac{dy}{dx}}\Rightarrow OA=x+\frac{y}{\frac{dy}{dx}}$

The line through $P\,(x,\ y)$ and perpendicular to (1) is

$Y-y=\frac{dx}{dy}\,(X-x)$

Where it meets the *y*-axis

$X=0,\ Y=y-\frac{x}{\frac{dy}{dx}}\Rightarrow OB=y-\frac{x}{\frac{dy}{dx}}$

Since $OA=OB$,

$x+\frac{y}{\frac{dy}{dx}}=y-\frac{x}{\frac{dy}{dx}}$ or $(y-x)=\frac{y+x}{\frac{dy}{dx}}$ or $\frac{dy}{dx}=\frac{y+x}{y-x}$.

Writing $y=vx$ this equation becomes

$v+x\frac{dv}{dx}=\frac{1+v}{v-1}$

$x\frac{dv}{dx}=\frac{1+2v-v^2}{v-1}$

$\frac{(1-v)dv}{1+2v-v^2}+\frac{dx}{x}=0$

$\Rightarrow \ln\left(1+2v-v^2\right)+\ln x^2=$ constant or $x^2+2xy-y^2=c$

Since the curve passes through the focus of parabola $x^2-4xy+4y^2-32x+4y+16=0$ which can be written as

$(x-2y-4)^{\,2}=12\,(2x+y)$

Which the focus $(2, \ -1)$
Hence, $c=-1$.
Hence the required curve is $x^2-y^2+2xy+1=0$.

(40) **Answer :** (A,B,C)
Hint:

Use the function $f(x)=\left(\cot^{-1}(x)\right)^2+\frac{2}{\sqrt{x^2+1}}$

Solution:

Use the function $f(x)=\left(\cot^{-1}(x)\right)^2+\frac{2}{\sqrt{x^2+1}}$

$\cot^{-1}\left(\frac{1}{\pi}\right)=\tan^{-1}\pi, \ \cot^{-1}(\pi)=\tan^{-1}\left(\frac{1}{\pi}\right)$

So, $I_1\approx\frac{9.8696}{16}+1.414\approx2.031$
$I_2\approx1.593+1.906\approx3.499$
$I_3\approx0.095+0.607\approx0.702$

(41) **Answer :** (A,B,C)
Hint:
If $f''(x)>0 \Rightarrow f'(x)$ is increasing $\forall x\in R$.
Solution:

$g(x)=2f\left(\frac{x^2}{2}\right)+f\left(6-x^2\right)$

$g'(x)=f'\left(\frac{x^2}{2}\right)2x+f'\left(6-x^2\right)\cdot(-2x)=2x\left(f'\left(\frac{x^2}{2}\right)-f'\left(6-x^2\right)\right)$

We have been given that $f''(x)>0$
$\Rightarrow f'(x)$ is increasing for all real values of x.
Let $\frac{x^2}{2}>6-x^2\Rightarrow x^2>4$
$\Rightarrow x\in(-\infty, \ -2)\cup(2, \ \infty)$
$\Rightarrow f'\left(\frac{x^2}{2}\right)>f'\left(6-x^2\right) \ \forall \ x\in(-\infty, \ -2)\cup(2, \ \infty)$
also, $\frac{x^2}{2}<6-x^2\Rightarrow x\in(-2,2)$
$\Rightarrow f'\left(\frac{x^2}{2}\right)<f\left(6-x^2\right)\forall(-2, \ 2)$
Now let us check the sign of $g'(x)$.
From the sign scheme we get,
$g'(x)<0 \ \forall x\in(-\infty, \ -2)\cup(0, \ 2)$
$g'(x)>0 \ \forall x\in(-2, \ 0)\cup(2, \ \infty)$
$g'(x)=0, \ x=-2, \ 0, \ 2$
Thus $g(x)$ is monotonically decreasing in $(-\infty, \ -2)\cup(0, \ 2)$ and monotonically increasing in $(-2, \ 0)\cup(2, \ \infty)$.
Clearly $x=-2,2$ are the points of local minima of $g(x)$ whereas $x=0$ is the point of local maxima of $g(x)$

Section-III

(42) **Answer :** 3
Hint:
$a_n=a_1+(n-1)d$ for A.P.
$a_n=ar^{n-1}$ for G.P.
Solution:
From $a_1^2+a_2^2+a_3^2=d^2+4d^2+9d^2=14d^2$, $b_1+b_2+b_3=d^2+d^2q+d^2q^2$
$\frac{a_1^2+a_2^2+a_3^2}{b_1+b_2+b_3}=\frac{14}{1+q+q^2}\Rightarrow1+q+q^2=\frac{14}{m}$
for some positive integer m. Hence,
$q=-\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{14}{m}-1}=-\frac{1}{2}+\sqrt{\frac{56-3m}{4m}}$
Now $0<q<1\Rightarrow5\le m\le13$. Note also that, $\frac{56-3m}{4m}$ is a square of a rational number, so $m=8$ and $q=\frac{1}{2}$

(43) **Answer :** 5
Hint:
Volume of tetrahedron $V=\frac{1}{6}\left[\bar{a}\bar{b}\bar{c}\right]$
Solution:
Volume of tetrahedron $V=\frac{1}{6}\left[\bar{a}\bar{b}\bar{c}\right]$

Here $[\bar{a}\bar{b}\bar{c}]^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix}$

$= \begin{vmatrix} 4 & 3 & 4\sqrt{3} \\ 3 & 9 & 6\sqrt{2} \\ 4\sqrt{3} & 6\sqrt{2} & 16 \end{vmatrix}$

$= 4\left[144-72\right]+3[24\sqrt{6}-48]+4\sqrt{3}\left[18\sqrt{2}-36\sqrt{3}\right]$

$= 288+72\sqrt{6}-144+72\sqrt{6}-432$

$= 144\sqrt{6}-288 = 144\left(\sqrt{6}-2\right)$

$V = \frac{1}{6}\times 12\sqrt{\left(\sqrt{6}-2\right)} = 2\sqrt{\sqrt{6}-2}$

Since $V = \frac{1}{3}\times$ base area $\times$ height

$\Rightarrow$ Height $= \frac{6\sqrt{\sqrt{6}-2}}{2} = 3\sqrt{\left(\sqrt{6}-2\right)}$

(44) Answer : 18

Hint:

Let *a* = *x*, *b* = 3*y* and *c* = 2*z*

Apply A.M. – G.M. rule i.e., $(a+b)^2+(b+c)^2 \geq 2(a+b)(b+c)$

Solution:

We first complete the square and rewrite our equation as

$(x+3y)^2+(3y+2z)^2$

We then substitute $a=x, b=3y$, and $c=2z$ to minimize the equivalent sum

$(a+b)^2+(b+c)^2$

under the condition $abc=6xyz=4$. Applying AM-GM gives us

$(a+b)^2+(b+c)^2 \geq 2(a+b)(b+c)$

We can apply AM-GM again to $a+b$ and $b+c$ individually via

$a+b = \frac{a}{2}+\frac{a}{2}+b \geq 3\sqrt[3]{\frac{a^2b}{4}}$

$b+c = b+\frac{c}{2}+\frac{c}{2} \geq 3\sqrt[3]{\frac{bc^2}{4}}$

to get $(a+b)^2+(b+c)^2 \geq 2(a+b)(b+c) \geq 18\sqrt[3]{\frac{(abc)^2}{16}} = 18$

Note that equality holds when $a=2b=c$, so $a=c=2$, and $b=1$, or equivalently when $x=2,\ y=\frac{1}{3}$, and $z=1$

(45) Answer : 9

Hint:

Equation of a line $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

Solution:

Equations of L_1 are $\frac{x-7}{-3}=\frac{y-6}{2}=\frac{z-2}{4}=r_1$

so that any point on it is $[7-3r_1,\ 6+2r_1,\ 2+4r_1]$. Let this point be C. Equations of line L_2 are $\frac{x-5}{2}=\frac{y-3}{1}=\frac{z-4}{3}=r_2$

so that any point on it is $[5+2r_2,\ 3+r_2,\ 4+3r_2]$. Let this point be D. Now, C and D lie on line L_3 with d.r. $(2,\ -2,\ -1)$.

Hence $\frac{2-3r_1-2r_2}{2}=\frac{3+2r_1-r_2}{-2}=\frac{-2+4r_1-3r_2}{-1}$

or $r_1+3r_2=5$ and $6r_1-5r_2=7 \Rightarrow r_1=2,\ r_2=1$.

Hence the points C and D are $(1,\ 10,\ 10)$ and $(7,\ 4,\ 7)$

$\Rightarrow CD=\sqrt{36+36+9}=\sqrt{81}=9$

(46) Answer : 0

Hint:

$f(x)=f'(x)\times f'''(x)$ is satisfied by only the polynomial of degree 4 .

Solution:

$f(x)=f'(x)\times f'''(x)$ is satisfied by only the polynomial of degree 4 .

Since $f(x)=0$ satisfies $x=1,\ 2,\ 3$ only. It is clear one of the root is twice repeated.

$\Rightarrow f'(1)f'(2)f'(3)=0$

(47) Answer : 36

Hint:

Find *m* by counting ordered pairs (*a*, *b*) with $1 \le a,\ b \le 60$ and *b* = product of two prime ≥ 3.
And *n* by counting $(a,\ b) \in A^2$ such that $2(a-b)^2 + 3(a-b) \in \{0,\ 1, 2, 3, 4\}$ then compute $\left[\frac{m}{n}\right]$

Solution:

b can take its values as $9,\ 15,\ 21,\ 33,\ 39,\ 51,\ 57,\ 25,\ 35,\ 55,\ 49$
b can take these 11 values
and *a* can take any of 60 values
So, number of elements in $R_1 = 60 \times 11 = 660$
$A = \{1,\ 2,\ 3,\ ...10\}$
$B = \{0,\ 1,\ 2,\ 3,\ 4\}$
$R_2 = \left\{(a,\ b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\right\}$
Now $2(a-b)^2 + 3(a-b) = (a-b)(2(a-b)+3)$
$\Rightarrow a = b$ or $a - b = -2$
When $a = b \Rightarrow 10$ order pairs
When $a - b = -2 \Rightarrow 8$ order pairs
Total $= 18$

Section-IV

(48) **Answer :** 01.00

Hint:

$|z - z_0| \ = \ r$
Centre z_0 and radius *r*

Solution:

$|z - (-1+i)| = 1$
$\Rightarrow |z + 1 - i| = 1$
Also $w = \frac{z+i}{1-i}$
$\Rightarrow (1-i)\mathrm{w} = \mathrm{z} + \mathrm{i} \Rightarrow (1-\mathrm{i})\mathrm{w} - \mathrm{i} = \mathrm{z}$
$\Rightarrow |(1-\mathrm{i})\mathrm{w} - \mathrm{i} + 1 - \mathrm{i}| = |\mathrm{z} + 1 - \mathrm{i}|$
$\Rightarrow |1-\mathrm{i}|\left|\mathrm{w} + \frac{1-2\mathrm{i}}{1-\mathrm{i}}\right| = 1 \Rightarrow \left|\mathrm{w} + \frac{(1-2\mathrm{i})(1+\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})}\right| = \frac{1}{\sqrt{2}}$
$\Rightarrow \left|\mathrm{w} + \frac{3-\mathrm{i}}{2}\right| = \frac{1}{\sqrt{2}} \Rightarrow \left|\mathrm{w} - \frac{-3+\mathrm{i}}{2}\right| = \frac{1}{\sqrt{2}}$
$\Rightarrow$ The locus of w is a circle centered at $\left(-\frac{3}{2}, \frac{1}{2}\right)$ and radius $\frac{1}{\sqrt{2}}$.

(49) **Answer :** 02.00

Hint:

$|z - z_0| \ = \ r$
Centre z_0 and radius *r*

Solution:

$|z - (-1+i)| = 1$
$\Rightarrow |z + 1 - i| = 1$
Also $w = \frac{z+i}{1-i}$
$\Rightarrow (1-i)\mathrm{w} = \mathrm{z} + \mathrm{i} \Rightarrow (1-\mathrm{i})\mathrm{w} - \mathrm{i} = \mathrm{z}$
$\Rightarrow |(1-\mathrm{i})\mathrm{w} - \mathrm{i} + 1 - \mathrm{i}| = |\mathrm{z} + 1 - \mathrm{i}|$
$\Rightarrow |1-\mathrm{i}|\left|\mathrm{w} + \frac{1-2\mathrm{i}}{1-\mathrm{i}}\right| = 1 \Rightarrow \left|\mathrm{w} + \frac{(1-2\mathrm{i})(1+\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})}\right| = \frac{1}{\sqrt{2}}$
$\Rightarrow \left|\mathrm{w} + \frac{3-\mathrm{i}}{2}\right| = \frac{1}{\sqrt{2}} \Rightarrow \left|\mathrm{w} - \frac{-3+\mathrm{i}}{2}\right| = \frac{1}{\sqrt{2}}$
$\Rightarrow$ The locus of w is a circle centered at $\left(-\frac{3}{2}, \frac{1}{2}\right)$ and radius $\frac{1}{\sqrt{2}}$.

(50) **Answer :** 10.00

Hint:

Put *x* = 1 to get sum of coefficients.
Unit digit of 3^n follows a cycle of 4 : (3, 9, 7, 1) ⇒ value of *b*.

Solution:

$s = (1 + 1 + 1^2)^{20} = 3^{20}$
Units digit of 3^{20} is *b* = 1
Probability = $\frac{20}{{}^{41}C_2} = \frac{1}{41}$

$a = \frac{1-p}{10p} = \frac{1-\frac{1}{41}}{\frac{10}{41}} = 4$

Now, $\left(\sqrt{a-b}+b\right)^6 = \left(\sqrt{3}+1\right)^6 = I + F$, where $0 \le F < I$

Let $\left(\sqrt{3}-1\right)^6 = G$ where $0 \le G < I$

$\therefore\ I + F + G = \left(\sqrt{3}+1\right)^6 + \left(\sqrt{3}-1\right)^6$

$= 2\left\{{}^6C_0\left(\sqrt{3}\right)^6 + {}^6C_2\left(\sqrt{3}\right)^4 + {}^6C_4\left(\sqrt{3}\right)^2 + {}^6C_6\right\}$

$I = 416 - (F+G)$

but $0 \le F + G < 2$ and F + G has to be an integer

∴ I = 416 – 1 = 415

Sum of digits of I = 4 + 1 + 5 = 10

(51) **Answer :** 95.00

Hint:

General term in the expansion of $(1+x+x^2)$ is $\frac{20!}{r!s!t!}(x)^s(x)^{2t}$

Solution:

General term in the expansion of $(1+x+x^2)$ is $\frac{20!}{r!s!t!}(x)^s(x)^{2t}$

Where $r + s + t = 20$

For coefficient of x^3, $s + 2t = 3 \Rightarrow t = 0, s = 3, r = 17$

or $t = 1, s = 1, r = 18$

∴ Coefficient of $x^3 = \frac{20!}{17!3!} + \frac{20!}{18!1!} = \frac{4 \cdot 20!}{18!}$

$\therefore\ \frac{\lambda}{16} = \frac{1520}{16} = 95$