WBJEE - 2026

Answer Keys by

Aakash Institute, (W.B)

MATHEMATICS

	BOOKLET CODE				BOOKLET CODE					BOOKLET CODE			
Q.No.	💧	●	■	◆	Q.No.	💧	●	■	◆	Q.No.	💧	●	■	◆
01	B	A	B	B	26	D	B	A	A	51	A	B	D	C
02	B	D	C	C	27	C	A	B	C	52	D	C	B	C
03	A	C	D	A	28	B	A	B	A	53	C	C	D	A
04	C	C	C	D	29	C	D	C	B	54	B	A	B	D
05	D	C	C	A	30	B	D	C	C	55	C	B	D	C
06	D	C	B	B	31	A	D	B	A	56	D	C	A	B
07	B	B	D	B	32	D	A	C	C	57	B	D	A	B
08	A	C	C	D	33	D	C	C	C	58	D	B	C	A
09	B	A	C	C	34	C	C	A	B	59	A	D	B	D
10	A	C	C	D	35	B	B	B	D	60	A	D	C	B
11	C	A	A	A	36	A	A	A	C	61	D	A	B	C
12	A	C	A	C	37	C	B	B	B	62	B	C	A	B
13	C	A	D	A	38	A	D	C	D	63	B	B	D	A
14	C	B	D	A	39	A	C	C	C	64	C	D	C	D
15	A	D	B	B	40	C	A	A	B	65	C	A	C	D
16	D	B	C	D	41	C	D	A	B	66	D	D	A,C,D	....
17	D	C	D	C	42	B	A	B	D	67	C	A,C	A,B	D
18	C	D	D	A	43	A	C	A	A	68	A,C,D	A,B	D	C
19	A	A	D	C	44	B	B	A	A	69	D	….	B	A,C,D
20	B	D	D	A	45	B	A	B	D	70	A,C,D	A,C,D	A,C	B
21	D	C	B	B	46	B	B	D	C	71	A,C	B	….	A,C,D
22	C	B	C	C	47	C	B	A	D	72	A,C	D	A,C	A,B
23	D	C	D	B	48	C	C	A	A	73	B	A,C,D	C	A,C
24	D	B	A	D	49	A	B	A	B	74	A,B	C	D	A,C
25	C	D	C	B	50	A	A	B	C	75	…..	A,C	A,C,D	D

Aakash

Medical IIT-JEE Foundations

DATE : 24/05/2026

Code -

Aakash
Medical|IIT-JEE|Foundations

ANSWERS & HINTS
for
WBJEE - 2026
SUB : MATHEMATICS

CATEGORY - 1 (Q:1 to Q50)

(Carry 1 mark each. Only one option is correct. Negative mark : – ¼)

1. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$.

(A) $P(-1)$ is the minimum but $P(1)$ is not the maximum of P

(B) $P(-1)$ is not minimum but $P(1)$ is the maximum of P

(C) neither $P(1)$ is the minimum nor $P(1)$ is the maximum of P

(D) $P(-1)$ is the minimum and $P(1)$ is the maximum of P.

Ans : (B)

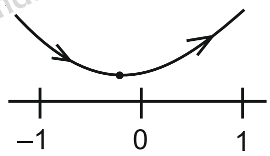
Hint : $P'(x) = 4x^3 + 3ax^2 + 2bx + c$

$P'(a) = 0 \Rightarrow C = 0$

$P'(x) = x(4x^2 + 3ax + 2b)$

Roots of $4x^2 + 3ax + 2b = 0$ are imaginary

$\Rightarrow 9a^2 - 32b < 0$ .... (1) $\Rightarrow$ $\boxed{b > 0}$



$P(x) = x^4 + ax^3 + bx^2 + d$

$P(-1) < P(1) \Rightarrow$ $\boxed{a > 0}$

$P'(x) > \forall x \in (0, 1)$, $P'(x) < 0 \quad \forall x \in [-1, 0]$

x = 0 is minima

2. If α, β are the roots of the equation $x^2 - px + q = 0$ and $\alpha > 0, \beta > 0$, then $\alpha^{\frac{1}{4}} + \beta^{\frac{1}{4}} = \left(p + 6\sqrt{p} + 4q^{\frac{1}{4}}\sqrt{p + 2\sqrt{q}}\right)^k$, where *K* is

(A) $\frac{3}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) 1

Ans : (B)

Hint : $\alpha + \beta = p_1$, $\alpha\beta = q$

$\left(\sqrt{\alpha}+\sqrt{\beta}\right)^2 = \alpha+\beta+2\sqrt{\alpha\beta} = p+2\sqrt{q} = 1\ \sqrt{\alpha}+\sqrt{\beta} = \sqrt{p+2\sqrt{q}}$

Let x = $\alpha^{1/4}+\beta^{1/4}$

$x^2 = \sqrt{\alpha}+\sqrt{\beta}+2(\alpha\beta)^{1/4} = = \sqrt{P+2\sqrt{q}}+2q^{1/4}$

Simplifying we get $k = \frac{1}{4}$

3. If $\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{2r^2}\right) = a$, then tan *a* is equal to

(A) 1 (B) 0 (C) $\sqrt{3}$ (D) $\frac{\pi}{4}$

Ans : (A)

Hint : $\sum_{r=1}^{\omega}\tan^{-1}\left(\frac{1}{2r^2}\right) = \sum_{r=1}^{\omega}\tan^{-1}\left(\frac{2}{4r^2}\right) = \sum_{r=1}^{\infty}\tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)$

$\sum_{r=1}^{\omega}\left(\tan^{-1}(2r+1)-\tan^{-1}(2r-1)\right) = \frac{\pi}{4}$

$a = \frac{\pi}{4} \Rightarrow \tan a = 1$

4. Consider a function *f*(x) which has exactly two roots at x = a. If $\lim_{x\to a}\left(\frac{\lambda f'(x)}{f(x)}-\frac{1}{x-a}\right) = m(\neq 0)$, then the value of λ ix

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans : (C)

Hint : f(x) = g(x).$(x-a)^2$

$\lim_{x\to a}\frac{(2\lambda-1)g(x)+\lambda(x-a)g'(x)}{g(x).(x-a)} = m$

for $m \neq 0 \Rightarrow 2\lambda-1 = 0, \lambda = \frac{1}{2}$

5. A vector given by $\vec{P} = f(t)\hat{i}+g(t)+\hat{k}$ moves in such a way that it is always parallel to the vector $\vec{Q} = -f''(t)\hat{i}+f'(t)\hat{j}+\hat{k}$. The magnitude of $\vec{P}$ is

(A) a linear function of time (B) a quadratic function of time

(C) a cubic function of time (D) constant

Ans : (D)

Hint : $\left|\bar{P}\right|^2 = f^2(t)+g^2(t)+1 = h(t)$

$h'(t) = 2f(t).f'(t)+2g(t).g'(t)$

$\vec{P} = \lambda\bar{Q} \Rightarrow f(t) = -\lambda f''(t)$ .... (1)

$g(t) = \lambda f'(t)$ .... (2)

$1 = \lambda$

$\Rightarrow h'(t) = 0$

$\Rightarrow$ h(t) is constant

6. The expression $\sum_{k=1}^{32}(3K+2)\left\{\sum_{r=1}^{10}\left(\sin\frac{2r\pi}{11} - i\cos\frac{2r\pi}{11}\right)\right\}^k$ represents

(A) 48 (1 + i) (B) –48 (1 – i) (C) $-\frac{48}{11}(1-i)$ (D) 48(1 – i)

Ans : (D)

Hint : $\sum_{r=0}^{10}\left(\cos\frac{2\pi r}{11}+i\sin\frac{2\pi}{11}\right) = 0$

$1+\sum_{r=1}^{10}\left(\cos\frac{2\pi r}{11}+i\sin\frac{2\pi r}{11}\right) = 0$

$= \sum_{k=1}^{32}(3k+2)(-i)^k.(-1)^k = \sum_{k=1}^{32} i^k.(3k+2)$ = 48 (1 – i)

7. θ elimination from the equation $x^2+y^2 = \frac{x\cos3\theta+y\sin3\theta}{\cos^3\theta} = \frac{y\cos3\theta - x\sin3\theta}{\sin^3\theta}$ will be

(A) $4(x^4 + y^4) = 3x + 4y$ (B) $(x^2 + y^2 + 2x)(x^2 + y^2 - x) = 2y^2$

(C) $(x^2 + y^2 - 2x)(x^2 + y^2 + x) = 9y$ (D) $x^{2/3} + y^{2/3} = 1$

Ans : (B)

Hint : Let $m = x^2 + y^2$

$m\cos^3\theta = c\cos^3\theta + y\sin^3\theta$

$m\cos^3\theta = y\cos3\theta - x\sin3\theta$

Solving $x = m(\cos^3\theta\cos3\theta - \sin^3\theta\sin3\theta)$

$y = m(\cos^2\theta\sin3\theta + \sin^3\theta\cos3\theta)$

$\Rightarrow \cos4\theta = \frac{4x-m}{3m}$ ..... (1)

$y = m\left[\left(\frac{\cos3\theta+3\cos\theta}{4}\right)\sin3\theta+\left(\frac{3\sin\theta-\sin3\theta}{4}\right)\cos3\theta\right] = \frac{3m\sin4\theta}{4}$

$\Rightarrow \sin4\theta = \frac{4y}{3m} \Rightarrow (x^2+y^2+2x)\left(x^2+y^2-x\right) = 2y^2$

8. If t_n denotes the n^{th} term of an A.P. and $t_p = \frac{1}{q}, t_q = \frac{1}{p},$ then which one of the following options is a root of the equation $(p + 2q - 3r)x^2 + (q + 2x - 3p)x + (r + 2p - 3q) = 0$?

(A) t_{pq} (B) t_p (C) t_q (D) t_{p+q}

Ans : (A)

Hint : Sum of the coefficients of the given quadratic equation is equal to zero. Hence x = 1 is a root of the equation.

$t_{pq} = t_p + (pq - r)d$

$= \frac{1}{q} + p(q-1)\left(\frac{\frac{1}{p}-\frac{1}{q}}{q-p}\right) = \frac{1}{q} + \frac{q-1}{q} = 1$

Hence option A is correct

9. Consider the sequence of numbers (1, 2, 3, ..., 13). A person choose three numbers at random from the sequence. The probability that the chosen three number form an A.P. is

(A) $\frac{21}{157}$ (B) $\frac{18}{143}$ (C) $\frac{29}{180}$ (D) $\frac{24}{163}$

Ans : (B)

Hint : {1, 2, 3, ... 13}

$1 \le d \le 6$

AP = 1 + 3 + 5 + 7 + 9 + 11 = 36

Required probability = $\frac{36}{^{13}C_3} = \frac{18}{143}$

10. If $f(x) = \frac{1+x}{1-x}$ and A is a matrix such that $A^3 = 0$, then $f(A) =$

(A) $1 + 2A + 2A^2$ (B) $1 + 2A + A^2$ (C) $1 - 2A + A^2$ (D) $1 + A + A^2$

Ans : (A)

Hint : $f(x) = \frac{1+x}{1-x} = (1+x)(1-x)^{-1} = (1 + x)(1 + x + x^2 + x^3 + ...)$

$f(x) = (1 + x + x^2 + x^3 + ...) + (x + x^2 + x^3 + ...)$

$f(x) = 1 + 2x + 2x^2 + 2x^3 + ...$

$f(A) = I + 2A + 2A^2$, (as $A^3 = 0$)

11. Which of the following statements is always true ?

(A) If $f(x)$ is decreasing, then $\frac{1}{f(x)}$ is increasing

(B) If $f(x)$ is decreasing, then $\frac{1}{f(x)}$ is also decreasing

(C) If both f and g are positive functions such that f is decreasing and g is increasing, then $\frac{f}{g}$ is a decreasing function

(D) If both f and g are positive functions such that f is increasing and g is decreasing then $\frac{f}{g}$ is a decreasing furnction

Ans : (C)

Hint : f(x) > 0, g(x) > 0, $f'(x) < 0, g'(x) > 0$

Let h(x) = $\frac{f(x)}{g(x)}$, $h'(x) = \frac{f'(x)g(x) - g'(x)\,f(x)}{g^2(x)} < 0$

$\Rightarrow$ h(x) is decreasing

12. If $0 < \alpha < \beta < \gamma < \frac{\pi}{2}$, then the equation $\frac{1}{x - \sin\alpha} + \frac{1}{x - \sin\beta} + \frac{1}{x - \sin\gamma} = 0$ has

(A) real and unequal roots
(B) imaginary roots
(C) real and equal roots
(D) rational roots

Ans : (A)

Hint : Let sinα = a, sinβ = b, sinγ = c

$$\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = \frac{(x-b)(x-c) + (x-a)(x-c) + (x-a)(x-b)}{(x-a)(x-b)(x-c)}$$

Let f(x) = (x – b) (x – c) + (x – a) (x – c) + (x – a) (x – b)

f(a) > 0

f(b) < 0

f(b) > 0

$\Rightarrow$ 2 real and unequal roots

13. On the set $\mathbb{R}$ of real numbers the relation ρ, defined by xρy (x, y ∈ $\mathbb{R}$) iff

(A) |x – y| < 2 is reflexive but neither symmetric nor transitive

(B) |x| ≥ y is reflexive and transitive but not symmetric

(C) x > |y| is transitive but neither reflexive nor symmetric

(D) x – y < 2 is reflexive and symmetric but not transitive

Ans : (C)

Hint : (A) |x – y| < 2 is symmetric

(B) |x| ≥ y is not transitive

[∵ |b| ≥ a and |c| ≥ b together do not confirm |c| ≥ a]

(C) Correct

(D) Not symmetric

[a – b < 2 does not confirm b – a < 2]

14. If $\int \frac{\operatorname{cosec}^2 x - 2010}{\cos^{2010} x}dx = -\frac{f(x)}{(g(x))^{2010}} + c$, where $f\left(\frac{\pi}{4}\right) = 1$; then the number of solutions of the equation $\frac{f(x)}{g(x)} = \{x\}$ in $[0, 2\pi]$ is/are (where $\{\cdot\}$ represents fractional part function)

(A) 3 (B) 1 (C) 0 (D) 2

Ans : (C)

Hint : $\int \frac{\operatorname{cosec}^2 x - 2010}{\cos^{2010} x}dx$

$= \int \underbrace{\operatorname{cosec}^2 x}_{2} \cdot \underbrace{\sec^{2010} x}_{1}\, dx - 2010\int \sec^{2010} x\, dx$

$= -\sec^{2010} x \cot x + \int 2010 \cdot \sec^{2009} x \cdot \sec x \tan x \cot x\, dx - 2010\int \sec^{2010} x\, dx$

$= -\sec^{2010} x \cot x + c$

$= -\frac{\cot x}{\cos^{2010} x} + c$, $f(x) = \cot x$

$g(x) = \cos x$

$\frac{f(x)}{g(x)} = \{x\} \Rightarrow \operatorname{cosec} x = \{x\}$ No solution.

15. If the locus of mid point of any normal chord of the parabola $y^2 = 4x$ is $x - \lambda = \frac{\mu}{y^2} + \frac{y^2}{\nu}$, where $\lambda, \mu, \nu \in N$, then $(\lambda + \mu + \nu)$ equals to

(A) 8 (B) 16 (C) 10 (D) 17

Ans : (A)

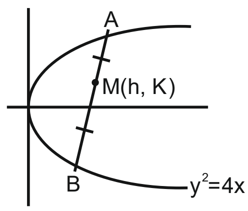
Hint : Equation of AB : $T = S_1$ $\Rightarrow ky - 2(x + h) = k^2 - 4h$

$\Rightarrow 2x - ky - 2h + k^2 = 0$ —— (1)

as its a normal : $y = -tx + 2t + t^3$

$\Rightarrow tx + y - 2t - t^3 = 0$ —— (2)

equating (1), (2) $\frac{t}{2} = \frac{1}{-k} = \frac{-2t - t^3}{-2h + k^2}$ $\Rightarrow t = \frac{-2}{k}$, $\frac{1}{2} = \frac{2 + t^2}{2h - k^2}$



$\Rightarrow 2h - k^2 = 4 + 2t^2 \Rightarrow 2h - k^2 = 4 + 2 \cdot \frac{4}{k^2}$

$\Rightarrow 2h - k^2 - 4 - \frac{8}{k^2} = 0 \Rightarrow 2h - 4 = \frac{8}{k^2} + k^2$

so locus is $x - 2 = \frac{4}{y^2} + \frac{y^2}{2}$ $\mu = 4$, $\nu = 2$, $\lambda = 2$ $\lambda + \mu + \nu = 8$

16. The true set of values of 'K' for which $\sin^{-1}\left(\frac{1}{1+\sin^2 x}\right)=\frac{K\pi}{6}$ may have a solution is

(A) $\left[\frac{1}{6},\frac{1}{2}\right]$ (B) $\left[\frac{1}{4},\frac{1}{2}\right]$ (C) [2, 4] (D) [1, 3]

Ans : (D)

Hint : $1 + 0 \le 1+ \sin^2\theta \le 1 + 1$

$$\frac{1}{2}\le\frac{1}{1+\sin^2\theta}\le 1$$

$$\sin^{-1}\frac{1}{2}\le\sin^{-1}\left(\frac{1}{1+\sin^2\theta}\right)\le\sin^{-1}1$$

$\because \sin^{-1}x$ is increasing function

$$\Rightarrow \frac{\pi}{6}\le\frac{k\pi}{6}\le\frac{\pi}{2}$$

$\Rightarrow 1\le k\le 3$

17. A mapping is selected at random from all mappings $f: A\to A,$ where set $A=\{1, 2, 3 \ldots, n\}$. If the probability that the mapping is injective is $\frac{3}{32}$, then the value of n is

(A) 8 (B) 14 (C) 3 (D) 4

Ans : (D)

Hint : Number of function = n^n

Number of injective function = n !

$$\therefore \frac{n!}{n^n}=\frac{3}{32}$$

for n = 4 $\frac{n!}{n^n}=\frac{24}{4^4}=\frac{24}{256}=\frac{3}{32}$

18. Let A = [a, ∞) denotes the domain, then $f:(a,\infty)\to B,$ which is defined by $f(x)=2x^3-3x^2+6$ will have an inverse for the smallest real value of 'a' if

(A) a = 0, B = [6, ∞) (B) a = 2, B = [10, ∞) (C) a = 1, B = [5, ∞) (D) a = –1, B = [5, ∞)

Ans : (C)

Hint : $f(x) = 2x^3 - 3x^2 + 6$

$f'(x) = 6x^2 - 6x = 6x(x - 1)$

$\therefore$ f(x) is monotonically increasing in $x\in[1,\infty)$

$\therefore$ f(x) is invertible in $x\in[1,\infty)$

$\therefore \alpha = 1 \quad f(1) = 2 - 3 + 6 = 5$

$\therefore$ for $x\in[1,\infty)$ $f(x)\in[5,\infty)$

$\therefore$ Range = Codomain of f(x) is $[5,\infty)$

19. If $a = \lim_{n\to\infty} \cos^{2n} x, (x = n\pi)$ and $b = \lim_{n\to\infty} \cos^{2n} x, (x \neq n\pi)$, then numerical value of the area of the triangle whose vertices are (a, b), (–2, 1) and (2, 1) is

(A) 2 (B) 4 (C) 1 (D) $\frac{1}{2}$

Ans : (A)

Hint : $A = \underset{n\to\alpha}{Lt} \cos^{2n} x, \; x = n\pi$

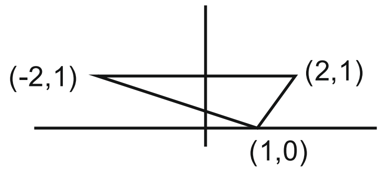
$x = n\pi \;\; \therefore \; \cos x = \pm 1 \;\; \therefore \cos^{2n} x = 1 \;\; \therefore a = 1$

if $x \neq n\pi \;\; \cos x \in [-1, 1) \;\; \therefore \cos^{2n} x \in [0, 1)$

$\underset{n\to\alpha}{Lt} \cos^{2n} x = 0$

$\therefore a = 1, b = 0$

Area of the triangle formed by (1, 0), (–2, 1), (2, 1)



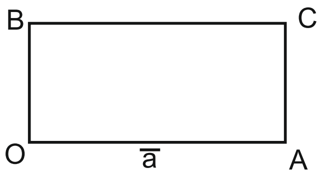
$= \frac{1}{2} \times 4 \times 1 = 2$

20. The position vectors of two adjacent sides $\overline{OA}$ and $\overline{OB}$ of a rectangle OACB are $\vec{a}$ and $\vec{b}$ respectively, where O is the origin. If $16\,|\vec{a}\times\vec{b}| = 3\left(|\vec{a}| + |\vec{b}|\right)^2$ and θ be the acute angle between the diagonals OC and AB, then the value of $\tan\left(\frac{\theta}{2}\right)$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) 1

Ans : (A)

Hint :



$\overline{a} \perp \overline{b}$

$\therefore |\overline{a}\times\overline{b}| = \left|ab\sin\frac{\pi}{2}\right| = |ab| = |\overline{a}|\cdot|\overline{b}|$

given $16ab = 3(a+b)^2$

$3a^2 - 10ab + 3b^2 = 0$

$(a - 3b)(3a - b) = 0$

$\therefore a = 3\lambda i, b = \lambda j$

We can take a = 3i, b = j angle will be same.

$\therefore \overline{OC} = 3i + j \qquad \overline{AB} = 3i - j$

$\therefore \cos\theta = \frac{(3i - j)\cdot(3i + j)}{\sqrt{10}\cdot\sqrt{10}} = \frac{4}{5}$

$$\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}=\frac{4}{5} \quad \Rightarrow \tan^2\frac{\theta}{2}=\frac{1}{9} \qquad \tan\frac{\theta}{2}=\frac{1}{3}$$

21. The point of intersection of $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}$ and $\vec{r}\times\vec{b}=\vec{a}\times\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}$ and $\vec{b}=2\hat{i}-\hat{k}$ is

(A) $3\hat{i}+2\hat{j}+\hat{k}$ (B) $\hat{i}-\hat{j}-\hat{k}$ (C) $4\hat{i}+2\hat{j}-\hat{k}$ (D) $3\hat{i}+\hat{j}-\hat{k}$

Ans : (D)

Hint : $L_1 : \bar{r}\times\bar{a}=\bar{b}\times\bar{a}$

$(\bar{r}-\bar{b})\times\bar{a}=0$

$\bar{r}=\bar{b}+\lambda\bar{a}=(2i-k)+\lambda(i+j)$

$L_2 : \bar{r}\times\bar{b}=\bar{a}\times\bar{b}$

$(\bar{r}-\bar{a})\times\bar{b}=0$

$\bar{r}=\bar{a}+\mu\bar{b}$

$=(i+j)+\mu(2i-k)$

$L_1 : \frac{x-1}{2}=\frac{y-0}{1}=\frac{z-1}{0}$

$L_2 : \frac{x-2}{1}=\frac{y-1}{0}=\frac{z}{-1}$

$\therefore$ Point of intersection is $(\alpha, 1, -1)$

from $L_1 : \frac{x-2}{1}=\frac{1-0}{1}$

$x = 3 \Rightarrow \alpha = 3$

$\therefore$ Point of intersection is $(3, 1, -1)$ or $3i + j - k$

22. Let $a_1, a_2, a_3 \ldots$ are in G.P. such that $n > m$, $a_n > a_m$ and $a_1 + a_n = 66$, $a_2 \cdot a_{n-1} = 128$. If $\sum_{r=1}^{n} a_r = 126$, then n is

(A) 11 (B) 8 (C) 6 (D) 64

Ans : (C)

Hint : $a_1 + a_1r^{n-1} = 66$

$a_2 \cdot a_{n-1} = a_1 r \cdot a_1r^{n-2} = 128$

$a_1^2 r^{n-1} = 128$

$a_1 r^{n-1} = \frac{128}{a_1}$

$$a_1 + \frac{128}{a_1} = 66$$

$$a_1^2 - 66a_1 + 128 = 0$$

$\Rightarrow a_1 = 2, 64.$

As it is increasing G.P.

$\therefore a_1 = 2 \quad \therefore 2 + 2r^{n-1} = 66 \quad \therefore r^{n-1} = 32$

$a_1 + a_2 + . . . + a_n$

$$= a_1 \cdot \frac{r^n - 1}{r - 1} = 126$$

$\Rightarrow r^n - 1 = 63\,(r - 1)$

$32r - 1 = 63r - 63$

$\Rightarrow 31r = 62 \quad \Rightarrow r = 2$

$\therefore a_1 = 2,\ r = 2$ and $r^{n-1} = 32 \quad 2^{n-1} = 32 \quad \Rightarrow n = 6$

23. The minimum length of intercept on any tangent to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ cut by the circle $x^2 + y^2 = 25$ is

(A) 6 (B) 9 (C) 11 (D) 8

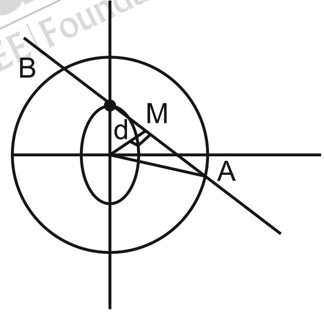
Ans : (D)

Hint : Equation of tangent

$$\frac{x\cos\theta}{2} + \frac{y\sin\theta}{3} = 1$$

$3x\cos\theta + 2y\sin\theta = 6$

distance from origin



$$d = \frac{6}{\sqrt{9\cos^2\theta + 4\sin\theta}} = \frac{6}{\sqrt{4 + 5\cos^2\theta}}$$

d is maximum then chord is minimum

$$d_{max} = \frac{6}{\sqrt{4+0}} = 3$$

Length of chord AB = 2AM = $2\sqrt{25 - d_{man}^2} = 2\sqrt{25 - 9} = 8$

24. Intercepts of the plane $\vec{r} \cdot \vec{n} = d(\neq 0)$ on the coordinate axes respectively are

(A) $\frac{\hat{i}\cdot\vec{n}}{d}, \frac{\hat{j}\cdot\vec{n}}{d}, \frac{\hat{k}\cdot\vec{n}}{d}$ (B) $\left|\frac{\hat{i}\cdot\hat{n}}{d}\right|, \left|\frac{\hat{j}\cdot\vec{n}}{d}\right|, \left|\frac{\hat{k}\cdot\vec{n}}{d}\right|$ (C) $\frac{d}{\hat{i}\cdot\hat{n}}, \frac{d}{\hat{j}\cdot\hat{n}}, \frac{d}{\hat{k}\cdot\hat{n}}$ (D) $\frac{d}{\hat{i}\cdot\vec{n}}, \frac{d}{\hat{j}\cdot\vec{n}}, \frac{d}{\hat{k}\cdot\vec{n}}$

Ans : (D)

Hint : $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$

$\therefore$ Equation of the plane

ax + by + cz = d

$$\frac{x}{d/a}+\frac{y}{d/b}+\frac{z}{d/c}=1$$

Now $\hat{i}\cdot\vec{n}=a,\ \hat{j}\cdot\vec{n}=b,\hat{k}\cdot\vec{n}=c$

$\therefore$ x intercept = $\dfrac{d}{\hat{i}\cdot\vec{n}}$

y intercept = $\dfrac{d}{\hat{j}\cdot\vec{n}}$

z intercept = $\dfrac{d}{\hat{k}\cdot\vec{n}}$

25. The general solution of the equation $\sin^{100}x - \cos^{100}x = 1$ is

(A) $\left\{2n\pi+\frac{\pi}{3}:n\in I\right\}$ (B) $\left\{n\pi+\frac{\pi}{4}:n\in I\right\}$ (C) $\left\{n\pi\pm\frac{\pi}{2}:n\in I\right\}$ (D) $\left\{2n\pi-\frac{\pi}{3}:n\in I\right\}$

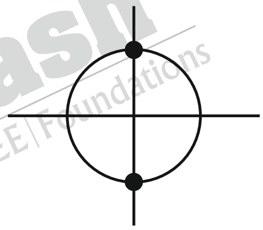
Ans : (C)

Hint : $\text{Sin}^{100}x - \text{Cos}^{100}x = 1$

$\text{Sin}^{100}x \le 1$ and $\text{Cos}^{100}x \ge 0$

$\therefore \text{Sin}^{100}x = 1,\ \text{Cos}^{100}x = 0$

$\therefore x = n\pi \pm \frac{\pi}{2}$



26. If $\vec{a}=\hat{i}+\hat{j}+\hat{k},\vec{b}=\hat{i}-\hat{j}+\hat{k},\vec{c}=\hat{i}+2\hat{j}-\hat{k}$, then the value of $\begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix}$ is equal to

(A) 64 (B) 0 (C) 14 (D) 16

Ans : (D)

Hint : $\begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix}$

$= [a\ b\ c]^2$

$$=\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 2 & -1\end{vmatrix}^2=\begin{vmatrix}1 & 0 & 0\\ 1 & -2 & 0\\ 1 & 1 & -2\end{vmatrix}^2=4^2=16$$

27. Number of elements in the range set of $f(x) = \left[\frac{x}{15}\right]\left[-\frac{15}{x}\right]$, for all $x\in(0, 90)$; (where [.] denotes the greatest integer function) is

(A) 8 (B) 7 (C) 6 (D) 5

Ans : (C)

Hint : for $x \in (0, 15)$ $f(x) = \left[\frac{x}{15}\right]\left[-\frac{15}{x}\right] = 0 \ \because 0 < \frac{x}{15} < 1$

for $x \in [15, 30)$ $f(x) = 1 \times -1 = -1$

for $x \in [30, 45)$ $f(x) = 2 \times -1 = -2$

for $x \in [45, 60)$ $f(x) = 3 \times -1 = -3$

for $x \in [60, 75)$ $f(x) = 4 \times -1 = -4$

for $x \in [75, 90)$ $f(x) = 5 \times -1 = -5$

$\therefore$ Range of f(x) for $x \in (0, 90)$ is $\{0, -1, -2, -3, -4, -5\}$

$\therefore$ Number of elements in Range of f(x) is 6.

28. Let 10 Bags $B_1, B_2,...,B_{10}$ which contains 21, 22,..., 30 different articles respectively. Then the total number of ways to bring out 10 articles from a Bag is

(A) ${}^{31}C_{20} + {}^{21}C_{10}$ (B) ${}^{31}C_{20} - {}^{21}C_{10}$ (C) ${}^{30}C_{20} - {}^{20}C_{10}$ (D) ${}^{30}C_{20} + {}^{20}C_{10}$

Ans : (B)

Hint : Number of ways is

${}^{21}C_{10} + {}^{22}C_{10} + {}^{23}C_{10} + ... + {}^{30}C_{10}$

$= {}^{21}C_{11} + {}^{21}C_{10} + {}^{22}C_{10} + ... + {}^{30}C_{10} - {}^{21}C_{11}$

$= {}^{31}C_{11} - {}^{21}C_{11} = {}^{31}C_{20} - {}^{21}C_{10}$

29. Let domain and range of f(x) and g(x) is $[0, \infty)$. If f(x) is an increasing function, g(x) is a decreasing function, h(x) = f{g(x)}, h(0) = 0 and $p(x) = h(x^3 - 2x^2 + 2x) - h(4)$, then for all $x \in (0, 2)$

(A) $p(x) = -3$ (B) $p(x) = 0$ (C) $0 < p(x) < -h(4)$ (D) $0 \le p(x) \le -h(4)$

Ans : (C)

Hint : f(x) is increasing, g(x) is decreasing

$\therefore \frac{d}{dx}(f(g(x))) = f'(g(x))\cdot g'(x) < 0$

$\therefore$ h(x) = f(g(x)) is decreasing

$y = x^3 - 2x^2 + 2x$

$\frac{dy}{dx} = 3x^2 - 4x + 2 > 0 \ \forall x \in R.$

$\therefore$ y is increasing in (0, 2)

y (0) = 0, y (2) = 4

$h(y(2)) < h(y(x)) < h(y(0))$

$h(4) < h(y(x)) < h(0)$

$0 < h(y(x)) - h(4) < h(0) - h(4)$

$0 < p(x) < -h(4) \ \because h(0) = 0$

30. Consider the following ellipse :

$\frac{x^2}{f(K^2+2K+5)}+\frac{y^2}{f(K+11)}=1$, where f(x) is a positive decreasing function. Then the value (values) of K for which the major axis coincides with x-axis is

(A) K = –5 (B) $K \in (-3, 2)$ (C) $K \in (-7, -5)$ (D) K = 2

Ans : (B)

Hint : $\frac{x^2}{f(K^2+2K+5)}+\frac{y^2}{f(K+11)}=1$

as major axis is along x axis

$\therefore f(K^2 + 2K + 5) > f(K + 11)$

$\therefore K^2 + 2K + 5 < K + 11 \because$ f(x) is decreasing

$\Rightarrow K^2 + K - 6 < 0 \Rightarrow (K + 3)(K - 2) < 0$

$K \in (-3, 2)$

31. The solution of the differential equation $2x^2y\frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$, given $y(1) = \sqrt{\frac{\pi}{2}}$ is

(A) $\sin(x^2y^2) = e^{x-1}$ (B) $\sin(x^2y^2) = e^{2(x-1)}$

(C) $\cos\left(\frac{\pi}{2}+x^2y^2\right)+x=0$ (D) $\sin(x^2y^2) = 1$

Ans : (A)

Hint : $2x^2y\left(\frac{dy}{dx}\right)=\tan\left(x^2y^2\right)-2xy^2$

$\Rightarrow \left(2x^2y\left(\frac{dy}{dx}\right)+2xy^2\right)=\tan\left(x^2y^2\right)$

$\Rightarrow d(x^2y^2) = \tan(x^2y^2)dx$

$\Rightarrow \int\frac{d\left(x^2y^2\right)}{\tan\left(x^2y^2\right)}=\int dx$

$\Rightarrow \ell n\left|\sin\left(x^2y^2\right)\right|=x+c$

$\Rightarrow \sin(x^2y^2) = e^{x+c}$

$\Rightarrow$ As $y(1) = \sqrt{\frac{\pi}{2}}$; $\sin\left(\frac{\pi}{2}\right) = e^{1+c}$

$\Rightarrow 1 = e^{1+c}$; c = –1

$\Rightarrow$ So $\sin(x^2y^2) = e^{x-1}$

32. $\int \frac{\left(\sqrt[3]{x+\sqrt{2-x^2}}\right)\left(\sqrt[6]{1-x\sqrt{2-x^2}}\right)}{\sqrt[3]{1-x^2}}dx;\ \left(x\in(0,1)\right)=$

(A) $2^{\frac{1}{12}}x+c$ (B) $2^{\frac{3}{4}}x+c$ (C) $2^{\frac{1}{3}}x+c$ (D) $2^{\frac{1}{6}}x+c$

Ans : (D)

Hint : $\int \frac{\left(\sqrt[3]{x+\sqrt{2-x^2}}\right)\left(\sqrt[6]{1-x\sqrt{2-x^2}}\right)}{\sqrt[3]{1-x^2}}dx \qquad x\in(0,\,1)$

$\sqrt[6]{1-x\sqrt{2-x^2}}=2^{-\frac{1}{6}}\cdot\sqrt[6]{\left(-x+\sqrt{2-x^2}\right)^2}=2^{-\frac{1}{6}}\cdot\left(\sqrt[3]{-x+\sqrt{2-x^2}}\right)$

$N^r=2^{-\frac{1}{6}}\int\frac{\sqrt[3]{-x^2+2-x^2}}{2\sqrt{1-x^2}}dx=2^{\left(-\frac{1}{6}+\frac{1}{3}\right)}\int\frac{\sqrt[3]{1-x^2}}{\sqrt[3]{1-x^2}}dx$

$=2^{\frac{1}{3}-\frac{1}{6}}\cdot x+c=2^{\frac{1}{6}}x+c$

33. Consider the function y = f(x) defined implicitly by the equation $y^3-3y+x=0$ on the interval $(-\infty,-2)\cup(2,\infty)$. The area of the region bounded by the curve y = f(x), the x-axis and the lines x = a, x = b, where $-\infty<a<b<-2$ is

(A) $\int_a^b\frac{x\,dx}{3\left(\left(f(x)\right)^2-1\right)}-bf(b)+af(a)$

(B) $\int_a^b\frac{x\,dx}{3\left(\left(f(x)\right)^2-1\right)}+bf(b)-af(a)$

(C) $-\int_a^b\frac{x\,dx}{3\left(\left(f(x)\right)^2-1\right)}-bf(b)+af(a)$

(D) $-\int_a^b\frac{x\,dx}{3\left(\left(f(x)\right)^2-1\right)}+bf(b)-af(a)$

Ans : (D)

Hint : $y^3-3y+x=0,\ g(x)=f^{-1}(x)$

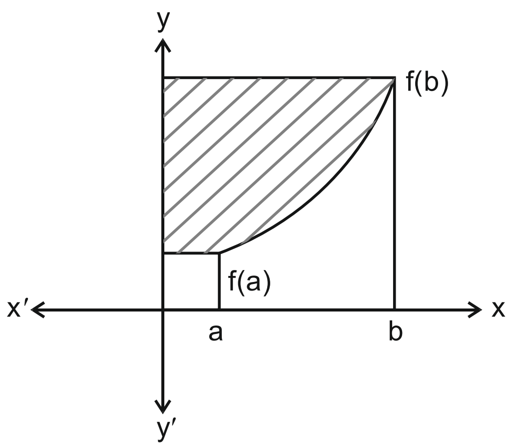
gof(x) = x

$g'(f(x))f'(x)=1;\ g'(f(x))=\frac{1}{f'(x)}$

$\therefore g'(f(x))=\frac{1}{3\left(f^2(x)-1\right)}$

$\int_a^b f(x)dx+\int_a^b\frac{x\,dx}{3\left(f^2(x)-1\right)}$

$=bf(b)-af(a)$



$$\int_a^b f(x)dx = \left[-\int_a^b \frac{x\,dx}{3\left(f^2(x)-1\right)} + b\,f(b) - a\,f(a)\right]$$

34. The total number of polynomials of the form $x^3 + ax^2 + bx + c$ which is divisible by $x^2 + 1$, where $a,b,c \in \{1,2,3,....,10\}$ is

(A) 120 (B) 45 (C) 10 (D) 15

Ans : (C)

Hint : $x^3 + ax^2 + bx + c$

$x(x^2 + b) + a(x^2 + \frac{c}{a})$

$\therefore$ b = 1 and $\left(\frac{c}{a}\right) = 1$

So a = c. Hence total 10 polynomials can be made.

35. The term independent of x in the expansion of $\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{15}$ is equal to

(A) 5105 (B) 5005 (C) 1365 (D) 105

Ans : (B)

Hint : $\left(\frac{x+1}{\left(x^{2/3} - x^{1/3} + 1\right)} - \frac{(x-1)}{\left(x - x^{1/2}\right)}\right)^{15}$

$\left(\left(x^{1/3}+1\right) - (x)^{-1/2}\left(\sqrt{x}+1\right)\right)^{15}$

$\left(\left(x^{1/3}+1\right) - 1 - \frac{1}{\sqrt{x}}\right)^{15} = \left(x^{1/3} - \frac{1}{\sqrt{x}}\right)^{15}$

${}^{15}Cr.(x)^{\frac{15-r}{3}}.\left(\frac{1}{x^{r/2}}\right)$

$\frac{15-r}{3} = \frac{r}{2}$ $30 - 2r = 3r$

$5r = 30$ $\boxed{r = 6}$

So 7th term :

$\left({}^{15}C_6\right) = 5005$

36. For a real number y, consider (y) denotes the greatest integer less than or equal to y. If $f(x) = \frac{\tan(\pi[x-\pi])}{1+[x]^2}$, then

(A) $f'(x)$ exists for all x (B) $f'(x)$ does not exist (C) $f'(1) = \frac{\pi}{4}$ (D) $f'(1) = -\frac{\pi}{4}$

Ans : (A)

Hint : $f(x) = \frac{\tan(\pi[x-\pi])}{1+[x]^2}$

So $f(x) = 0$ Hence f'(x) exists for all $x \in \mathbb{R}$

As integral multiple of π.

37. If $\int_0^1 \left(\sum_{r=1}^{2013} \frac{x}{x^2 + r^2} \right) \left(\prod_{r=1}^{2013} \left(x^2 + r^2 \right) \right) dx = \frac{1}{2} \left[\left(\prod_{r=1}^{2013} (1 + r^2 \right) - K^2 \right]$, then K is

(A) $\frac{2013(2014)(4027)}{6}$ (B) $(2013)^{2013}$ (C) (2013)! (D) $(2013!)^2$

Ans : (C)

Hint : $\int_0^1 \left(\sum_{r=1}^{2013} \frac{x}{x^2 + r^2} \right) . \prod_{r=1}^{2013} \left(x^2 + r^2 \right) dx$

$$\frac{1}{2} \left[\sum_{r=1}^{2013} \left(\frac{2x}{\left(x^2 + r^2 \right)} \right) \prod_{r=1}^{2013} \left(x^2 + r^2 \right) \right]$$

$$\left(\frac{d}{dx} \right) \left[\left(x^2 + 1 \right) . \left(x^2 + 2^2 \right) . \left(x^2 + 3^2 \right) ...... \left(x^2 + (2013)^2 \right) \right]$$

$$\frac{1}{2} \int_0^1 \frac{d}{dx} \left[\left(x^2 + 1 \right) \left(x^2 + 2^2 \right) \left(x^2 + 3^2 \right) ..... \left(x^2 + (2013)^2 \right) dx \right]$$

$$= \frac{1}{2} \left[\prod_{r=1}^{2013} \left(1 + r^2 \right) - (2013!)^2 \right]$$

Then the value of K = (2013!)

38. The least positive value of 'a' for which the equation $\int_0^x \left(t^2 - 8t + 13 \right) dt = x \sin \frac{a}{x}$ has a solution is

(A) 3π (B) 4π (C) π (D) 2π

Ans : (A)

Hint : $\int_0^x \left(t^2 - 8t + 13 \right) dt = x \sin \left(\frac{a}{x} \right)$

$$= \left[\frac{x^3}{3} - 8 \frac{x^2}{2} + 13x \right] = x \sin \left(\frac{a}{x} \right)$$

$$= x \neq 0 \left(\frac{x^2}{3} - 4x + 13 \right) = \sin \left(\frac{a}{x} \right)$$

$$= \left(x^2 - 12x + 39 \right) = 3 \sin \left(\frac{a}{x} \right)$$

$$= (x - 6)^2 + 3 = 3 \sin \left(\frac{a}{x} \right)$$

So x = 6 $\boxed{a = 3\pi}$

39. Let all the points on the curve $x^2 + y^2 - 10x = 0$ are reflected about the line $y = x + 3$. If the locus of the reflected points is in the form $x^2 + y^2 + gx + fy + c = 0$, then the value of $(g + f + c)$ is

(A) 38 (B) –28 (C) 28 (D) –38

Ans : (A)

Hint : $x^2 + y^2 - 10x + 25 = 25$

$(x - 5)^2 + y^2 = 5^2$

Any point on it $(5 + 5\cos\theta; 5\sin\theta)$ $\boxed{x - y + 3 = 0}$

Let the reflected point be . (α,β)

$$\frac{\alpha - 5 - 5\cos\theta}{1} = \frac{\beta - 5\sin\theta}{-1} = \frac{-2(5 + 5\cos\theta - 5\sin\theta + 3)}{(1+1)}$$

$$\frac{\alpha - 5 - 5\cos\theta}{1} = \frac{\beta - 5\sin\theta}{-1} = (-5\cos\theta + 5\sin\theta - 8)$$

$\alpha = -3 + 5\sin\theta$ $\beta - 5\sin\theta = 5\cos\theta - 5\sin\theta + 8$

$\frac{(\alpha + 3)}{5} = \sin\theta$ .... (1) $\left(\frac{\beta - 8}{5}\right) = \cos\theta$ ..... (2)

from (1) and (2)

$$\left(\frac{\alpha - 3}{5}\right)^2 + \left(\frac{\beta - 8}{5}\right)^2 = 1$$

$x^2 + y^2 + 6x - 16y + 64 + 9 - 25 = 0$

$x^2 + y^2 + 6x - 16y + 48 = 0$

$g = 6$, $f = -16$, $c = 48$

$(g + f + c) = (48 + 6 - 16) = 38$

40. The equation $|x + 1|^{\log_{(x+1)}\left(3 + 2x - x^2\right)} = (x - 3)|x|$ has

(A) no solution (B) two solutions (C) unique solution (D) infinite no. of solutions

Ans : (C)

Hint : $|x + 1|^{\log_{(x+1)}\left(3 + 2x - x^2\right)} = (x - 3)|x|$

If $x > -1$ then only the equation is valid.

$(3 + 2x - x^2) = (x - 3)|x|$

If $x \in (-1, 0)$ then

$= 3 + 2x - x^2 = -x^2 + 3x$

$= x = 3$ Not accepted

If $x \geq 0$ then

$= 3 + 2x - x^2 = x^2 - 3x$

$= 2x^2 - 5x - 3 = 0$

$= 2x^2 - 6x + x - 3 = 0$

$= (2x + 1)(x - 3) = 0$ $x = \frac{-1}{2}, 3$ But $x = \frac{-1}{2}$ not accepted so only one soln.

41. If the domain of f(x) is (0,1), then the domain of $y = f(e^x) + f(\ln|x|)$ is

(A) $\left(-1,-\frac{1}{e}\right)$ (B) $\left(\frac{1}{e},1\right)$ (C) $(-e,-1)$ (D) $(-e,-1)\cup(1,e)$

Ans : (C)

Hint : Domain of f(x) is (0,1)

$f(e^x) + f(\ln|x|)$

As, $0 < e^x < 1$ $\quad$ $0 < \ln|x| < 1$

$\therefore -\infty < x < 0$ ...... (1) $\quad$ $1 < |x| < e$

$x \in (-e,-1)\cup(1, e)$ .... (2)

from (1) and (2) are get domain of the f(x) is (–e, –1)

42. The number of 3-digit numbers we of the form xyz with x < y, z < y and $x \neq 0$ is

(A) 284 (B) 240 (C) 44 (D) 270

Ans : (B)

Hint : x < y : z < y $\quad$ xyz

Number of options

$$\left.\begin{array}{l|ll} y=9 & x\to 8 & y\to 9 \\ y=8 & x=7 & y=8 \\ y=7 & x=6 & y=7 \\ y=6 & x=5 & y=6 \\ y=5 & x=4 & y=5 \\ y=4 & x=3 & y=4 \\ y=3 & x=2 & y=3 \\ y=2 & x=1 & y=2 \\ y=1 & & \end{array}\right\}\text{Total number} = 240$$

43. Suppose A is denoted the set of all numbers between 1 and 700 which are divisible by 3 and let B is denoted the set of all numbers between 1 and 300 which are divisible by 7. If C = {(a,b)| $a \in A$, $b \in B$, $a \neq b$ and a + b = even number}, then order of C is

(A) 4879 (B) 4789 (C) 6789 (D) 9876

Ans : (A)

Hint : When both 'a' and 'b' even then the number of cases

$$\left[\frac{700}{6}\right]\times\left[\frac{300}{14}\right] = 116\times 21 = 2436$$

When both 'a' and 'b' odd then the total number of cases

$$\left(\left[\frac{700}{3}\right]-116\right)\times\left(\left[\frac{300}{7}\right]-21\right)$$

= (117) × (21) = 2457

Now the cases when a = b

So $\left[\frac{300}{21}\right] = 14$

So total number of elements in C

$= (2436 + 2457 - 14) = 4879$

44. Let us define the power of a matrix A as the maximum $m \in Z^{+}$ such that $A^m = I$. For two matrices A and B if $A^5 = I$ and $ABA^{-1} = B^2$, then the power of the matrix B is between

(A) 20 and 24 (B) 28 and 32 (C) 36 and 40 (D) 4 and 8

Ans : (B)

Hint : $A^5 = I$. $(ABA^{-1}) = B^2$

$(ABA^{-1}\,ABA^{-1}) = B^4$

$(AB^2A^{-1}) = B^4$

$A(ABA^{-1})\,A^{-1} = B^4$

$(A^k BA^{-k}) = B^{2k}$

from induction put k = 5 we get that

$A^5BA^{-5} = B^{2^5}$

$\therefore$ $B = b^{32}$ Power of B = 31

45. If for two real numbers a, b with $|a| \le 1$ and $|b| \le 1$,

$\frac{1}{3}+\frac{\sin^{-1}a+\sin^{-1}b}{4}+\frac{(\sin^{-1}a+\sin^{-1}b)^2}{16}+\frac{(\sin^{-1}a+\sin^{-1}b)^3}{64}+\cdots=\frac{2(8-3\pi)}{3(16+3\pi)}$, then the value of $\sin^{-1}\left(a\sqrt{1-b^2}+b\sqrt{1-a^2}\right)$ is

(A) $\frac{2(32+15\pi)}{3\pi-8}$ (B) $\frac{-\pi}{4}$ (C) $-\frac{3\pi}{4}$ (D) $\frac{1}{3}+\frac{\pi}{4}$

Ans : (B)

Hint : $\frac{1}{3}+\frac{(\sin^{-1}a+\sin^{-1}b)}{4}+\frac{(\sin^{-1}a+\sin^{-1}b)^2}{16}+\frac{(\sin^{-1}a+\sin^{-1}b)^3}{64}+\cdots=\frac{2(8-3\pi)}{3(16+3\pi)}$

$$1+\left(\frac{\sin^{-1}a+\sin^{-1}b}{4}\right)+\left(\frac{\sin^{-1}a+\sin^{-1}b}{4}\right)^2+\left(\frac{\sin^{-1}a+\sin^{-1}b}{4}\right)^3+\cdots\alpha-\frac{2}{3}=\frac{2(8-3\pi)}{3(16+3\pi)}$$

$$1+\left(\frac{\sin^{-1}a+\sin^{-1}b}{4}\right)+\left(\frac{\sin^{-1}a+\sin^{-1}b}{4}\right)^2+\cdots\alpha=\left[\frac{2(8-3\pi)}{3(16+3\pi)}+\frac{2}{3}\right]$$

$$\frac{1}{1-\frac{\sin^{-1}a+\sin^{-1}b}{4}}=\frac{2}{3}\left[\frac{8-3\pi+16+3\pi}{16+3\pi}\right]$$

$$\frac{4}{4-(\sin^{-1}a+\sin^{-1}b)}=\frac{2}{3}\frac{24}{(16+3\pi)}$$

$$4-\left[\sin^{-1}(a)+\sin^{-1}b\right]=\frac{16+3\pi}{4}$$

$$-\left[\sin^{-1}(a)+\sin^{-1}(b)\right]=\frac{3\pi}{4}$$

$$\sin^{-1}(a)+\sin^{-1}(b)=\left(\frac{-3\pi}{4}\right)$$

$$\sin^{-1}\left(a\sqrt{1-b^2}+b\sqrt{1-a^2}\right)=-\frac{3\pi}{4}$$

The value of the expression is $\sin^{-1}\left(a\sqrt{1-b^2}+b\sqrt{1-a^2}\right)=-\frac{\pi}{4}$

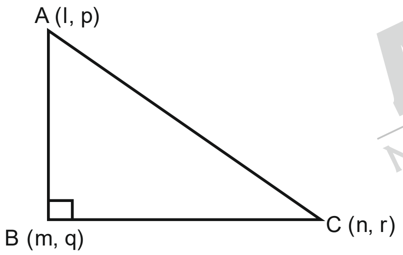
46. Let det $A=\begin{vmatrix} l & m & n \\ p & q & r \\ l & l & l \end{vmatrix}$

If $(l-m)^2+(p-q)^2=9$, $(m-n)^2+(q-r)^2=16$, $(n-l)^2+(r-p)^2=25$, then the value of $(\det A)^2$ is

(A) 169 (B) 144 (C) 121 (D) 100

Ans : (B)

Hint :



$$AB=\sqrt{(l-m)^2+(p-q)^2}=3$$

$$BC=\sqrt{(m-n)^2+(q-r)^2}=4$$

$$AC=\sqrt{(n-l)^2+(r-p)^2}=5$$

or, $(\Delta ABC)=\frac{1}{2}\times 3\times 4=6$

or, $2(ar\Delta ABC)=12$

$|\det A|^2=12^2=144$

47. Let f: (0,1) $\to$ (0,1) be a differentiable function such that $f'(x) \neq 0\ \forall x \in (0,1)$ and $f\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$. Suppose for all x,

$$\lim_{t\to x}\frac{\int_0^t \sqrt{1-(f(s))^2}\,ds-\int_0^x \sqrt{1-(f(s))^2}\,ds}{f(t)-f(x)}=f(x).$$ Then the value of $f\left(\frac{1}{4}\right)$ belongs to

(A) $\left\{\sqrt{7},\sqrt{6}\right\}$ (B) $\left\{\frac{\sqrt{7}}{2},\frac{\sqrt{15}}{2}\right\}$ (C) $\left\{\frac{\sqrt{7}}{4},\frac{\sqrt{15}}{4}\right\}$ (D) $\left\{\frac{\sqrt{7}}{3},\frac{\sqrt{15}}{3}\right\}$

Ans : (C)

Hint : $$\lim_{t\to x}\frac{\int_0^t \sqrt{1-(f(s))^2}\,ds-\int_0^x \sqrt{1-(f(s))^2}\,ds}{f(t)-f(x)}=f(x)$$

$$\lim_{t\to x}\frac{\sqrt{1-f(t)^2}}{f'(t)}=f(x) \qquad [\div \text{form}]$$

$$\Rightarrow \sqrt{1-(f(x))^2}=f'(x)f(x)$$

$$\Rightarrow \sqrt{1-y^2}=\frac{dy}{dx}y$$

$$\Rightarrow dx=\frac{y\,dy}{\sqrt{1-y^2}}$$

$$\Rightarrow x=-\sqrt{1-y^2}+c$$

for $x=\frac{1}{2},\ y=\frac{\sqrt{3}}{2}$

$$\Rightarrow \frac{1}{2}=-\sqrt{1-\frac{1}{4}}+c$$

$$\Rightarrow \boxed{c=1}$$

$$\therefore \sqrt{1-y^2}+x=1$$

for $x=\frac{1}{4}$

$$\sqrt{1-y^2}+\frac{1}{4}=1$$

$$\Rightarrow \sqrt{1-y^2}=\frac{3}{4}$$

$\Rightarrow y^2 = 1 - \frac{9}{16} = \frac{7}{16}$

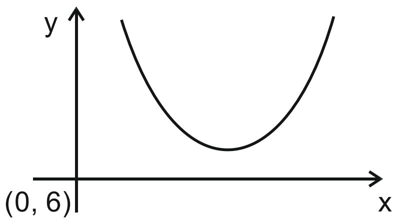
$y = \pm\frac{\sqrt{7}}{4}$

48. If 'a' is an integer lying in [–5, 30], then the probability that the graph of $y = x^2 + 2(a + 4)x - 5a + 64$ lies above the x-axis is

(A) $\frac{1}{6}$ (B) $\frac{7}{36}$ (C) $\frac{2}{9}$ (D) $\frac{3}{5}$

Ans : (C)

Hint : $y = x^2 + 2(a + 4)x - 5a + 64$

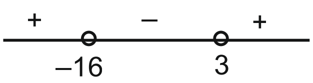


D < 0 (should be)

$\Rightarrow 4(a + 4)^2 - 4(64 - 5a) < 0$

$\Rightarrow a^2 + 8a + 16 + 5a - 64 < 0$

$\Rightarrow a^2 + 13a - 48 < 0$



$a \in (-16, 3)$

n(s) number of integers = 36 = n(s)

n(E) = 8

$\therefore P(E) = \frac{8}{36} = \frac{2}{9}$

49. Consider a square ABCD of diagonal length 2a. The square is folded along the diagonal AC so that the plane of ΔABC is perpendicular to the plane of ΔADC. In this case the shortest distance between AB and CD is

(A) $\frac{2a}{\sqrt{3}}$ (B) $\frac{a}{2\sqrt{3}}$ (C) $\frac{a}{\sqrt{3}}$ (D) $\frac{\sqrt{3}a}{2}$

Ans : (A)

Hint : A = (–a, 0, 0), C = (a, 0, 0)

B be y axis

A(–a, 0, 0) B = (0, a, 0)

$\vec{u} = (B - A) = (a, a, 0) \times (1, 1, 0)$

Equation of line CD $\rightarrow$ (a, 0, 0) and D (0, 0, a)

D R of CD (–1, 0, 1)

$$(\text{S.D.}) = \frac{(\vec{r}_1 - \vec{r}_2)\times(\vec{u}-\vec{v})}{|\vec{u}\times\vec{v}|}$$

$(r_1 - r_2) = (c–a) = (2a, 0, 0)$

$$\text{S.D.} = \frac{\begin{vmatrix} 2a & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}}{\sqrt{1^2+1^2+1^2}} = \left|\frac{2a}{\sqrt{3}}\right|$$

50. If $\int \frac{(1-x^2)}{\sqrt{x}\sqrt{(1+x^2)^3}} = \alpha \frac{x^\beta}{(1+x^2)^\gamma} + C$; $\alpha, \beta, \gamma \in \mathbb{R}$ and C is constant of integration, then $\alpha : \beta : \gamma$ will be

(A) 4 : 1 : 1 (B) $2:2:\frac{1}{2}$ (C) $\frac{1}{6}:2:\frac{1}{2}$ (D) $1:2:\frac{1}{2}$

Ans : (A)

Hint : $\int \frac{(1-x^2)dx}{\sqrt{x}\sqrt{(1+x^2)^3}} = \alpha \frac{x\beta}{(1+x^2)^\gamma} + c$

$$I = \int \frac{(1-x^2)dx}{\sqrt{x}\sqrt{x^3}\sqrt{\left(\frac{1}{x}+x\right)^3}} = \int \frac{\left(\frac{1}{x^2}-1\right)dx}{\left(\frac{1}{x}+x\right)^3}$$

Let $\frac{1}{x} + x = t^2$

$$\left(-\frac{1}{x^2}+1\right)dx = 2tdt$$

$$\therefore I = \int \frac{-2t\,dt}{t^3} = 2\int \frac{dt}{t^2}$$

$$= \frac{2}{t} = \frac{2}{\sqrt{x+\frac{1}{x}}}$$

$$\therefore I = 2\frac{\sqrt{x}}{\sqrt{x^2+1}}$$

$\alpha = 2,\ \beta = \frac{1}{2},\ \gamma = \frac{1}{2}$

$\alpha : \beta : \gamma = 4 : 1 : 1$

CATEGORY - 2 (Q.51 to 65)

(Carry 2 marks each. Only one option is correct. Negative marks : –½)

51. Let $\vec{a}=(x,y,z)$ be the vector with $|\vec{a}|=2\sqrt{3}$, which makes equal angles with the vector $\vec{b}=(y,-2z,3x)$ and $\vec{c}=(2z,3x,-y)$ and is perpendicular to the vector $\vec{d}=(1,-1,2)$. If the angle between $\vec{a}$ and the unit vector $\hat{j}$ is obtuse, then $\vec{a}$ is

(A) (2, –2, –2) (B) (–2, –2, 2) (C) (–2, 2, –2) (D) (2, –2, 2)

Ans : (A)

Hint : $|a|=2\sqrt{3}$

a = (x, y, z) b = (y, –2z, 3x) c = (2z, 3x, –y)

d = (1, –1, 2)

a.d = 0

x – y + 2z = 0 -----------(1)

Since, |b| = |c| = $\sqrt{y^2+9x^2+4z^2}$

Therefore,

a.b = a.c

xy – 2yz + 3xz = 2zx + 3xy – zy

0 = 2xy + yz – zx

zx – yz = 2xy

z(x–y) = 2xy

$x-y=\frac{2xy}{z}$

From....(1)

$\frac{2xy}{z}+2z=0$

xy = $-z^2$

Again put, y = x + 2z

x(x + 2z) = $-z^2$

$x^2 + 2zx + z^2 = 0$

$(x + z)^2 = 0$

$\Rightarrow x = -z$

–z – y + 2z = 0

y = z

Hence, a = (–z, z, z)

|a| = $2\sqrt{3}$

$\sqrt{z^2+z^2+z^2}=2\sqrt{3}$

$3z^2 = 12$

z = ±2

If z = 2

(–2, +2, +2)

If z = –2

(2, –2, –2)

52. Let $A_1, A_2, ..., A_6$ are six sets, each with four elements and $B_1, B_2, ..., B_n$ are n sets, each with two elements. Let $S = A_1 \cup A_2 \cup .... \cup A_6 = B_1 \cup B_2 \cup ... \cup B_n$.

Given that each element of S belongs to exactly four of the A's and to exactly three of the B's. Then n is

(A) 12 (B) 24 (C) 6 (D) 9

Ans : (D)

Hint : $A_1 \rightarrow 6$ sets $\rightarrow 4$ elements

$B_1 \rightarrow n$ sets $\rightarrow 2$ elements

$$\left[\frac{24}{4}\right] = \left[\frac{2a}{3}\right]$$

$\therefore$ (2a) = 3 × 6

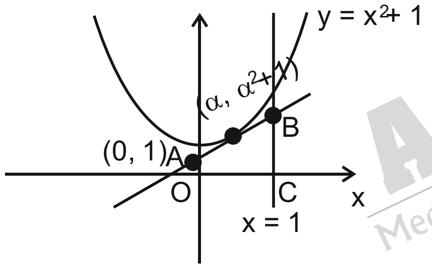
$\therefore$ a = 9

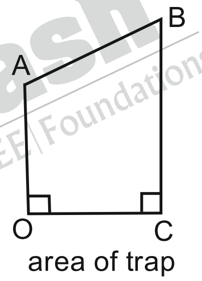
53. A figure is bounded by the curves $y = x^2 + 1$, $y = 0$, $x = 0$ and $x = 1$. The point at which a tangent should be drawn to the curve $y = x^2 + 1$ for it to cut off trapezium of the greatest area from the figure is

(A) (1, 2) (B) (–1, 2) (C) $\left(\frac{1}{2}, \frac{5}{4}\right)$ (D) (0, 1)

Ans : (C)

Hint :





Let point be $(\alpha, \alpha^2 + 1)$

$$\left.\frac{dy}{dx}\right|_{x=\alpha} = 2\alpha$$

Equation of tangent: $y - (\alpha^2+1) = 2\alpha(x-\alpha)$

$y = 2ax - \alpha^2 + 1$

$$ar(trap) = \frac{1}{2}(OA + BC) \times OC = \frac{1}{2}(OA + BC)$$

$$= \frac{1}{2}(2\alpha - \alpha^2 + 1 - \alpha^2 + 1)$$

$f(A) = \alpha - \alpha^2 + 1$

for maximum $\alpha = \frac{1}{2}$

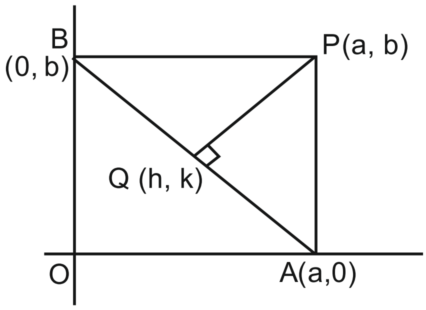
$\therefore$ point $\left(\frac{1}{2}, \frac{5}{4}\right)$

54. The ends A, B of a straight line segment of constant length c slide upon the fixed rectangular axes OX, OY respectively. If the rectangle OAPB completed, then the locus of the foot of perpendicular drawn from P to AB is

(A) $x^2 + y^2 = c^2$ (B) $x^{2/3} + y^{2/3} = c^{2/3}$ (C) $\sqrt{x}+\sqrt{y}=\sqrt{c}$ (D) $xy = c^2$

Ans : (B)

Hint :



$AB^2 = C^2$

$OA^2 + OB^2 = C^2$

$\Rightarrow a^2 + b^2 = c^2$ --------(i)

$m_{AB} \times m_{PQ} = -1$

$\Rightarrow \frac{0-b}{a-0}\cdot\frac{k-b}{h-a}=-1$

$\Rightarrow ah - bk = a^2 + b^2$ --------- (ii)

$\because \frac{h}{a}+\frac{k}{b}=1$ --------(iiii)

Solve (ii) & (iii)

$h^{2/3} + k^{2/3} = c^{2/3}$

55. Let 1 lies between the roots of the equation $y^2 - my + 1 = 0$ and [x] denotes the greatest integer function. Then the value of $\left[\left(\frac{4|x|}{x^2+16}\right)^m\right]$ is

(A) 5 (B) 4 (C) 0 (D) 1

Ans : (C)

Hint : Let, $f(y) = y^2 - my + 1$

$\because$ 1 lies between roots

$f(1) < 0$

$1 - m + 1 < 0 \quad \Rightarrow \boxed{m > 2}$

$\because$ A.M $\geq$ G.M.

$\frac{|x|^2+16}{2} \geq 4|x|$

$$\Rightarrow 0 \le \frac{4|x|}{x^2+16} \le \frac{1}{2}$$

$$\Rightarrow 0 < \frac{4|x|}{|x|^2+16} < 1$$

$$\therefore \frac{4|x|}{|x|^2+16} = 0$$

56. Let f(x) be a twice differentiable function in [1, 3] and f(1) = f(3). Further if $|f''(x)| \le 2$, then for all x in [1, 3]

(A) $|f'(x)| \ge 4$ (B) $|f'(x)| \le -1$ (C) $|f'(x)| > 2$ (D) $|f'(x)| < 4$

Ans : (D)

Hint : $\because$ f(x) is twice differentiable in [1, 3] and f(1) and f(3)

$\therefore$ According to Roll's theorem

$c \in (1, 3)$

Such that $f'(c)=0$

Now according to L.M. V.T

$\frac{f'(x)-f'(c)}{x-c} = f''(d)$ where $d \in (1, 3)$

$\because f'(c) = 0$

$f'(x) = f''(d)(x-c)$

$\because f''(x) \le 2$

and $x, c \in (1, 3)$

$\therefore |x-c| < 2$

$\therefore |f''(x).(x-c)| < 2 . 2 = 4$

$\therefore |f'(x)| < 4$

57. The quantities $a_1, a_2, a_3, \ldots$ form an infinite decreasing G.P. If $a_1 = 1$, then the common ratio of the progression for which the expression $6a_5 - 16a_4 - 3a_3 + 12a_2$ is at a maximum is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $-\frac{1}{4}$

Ans : (B)

Hint : $6a_5 - 16a_4 - 3a_3 + 12a_2$

$f(r) = 6r^4 - 16r^3 - 3r^2 + 12r$

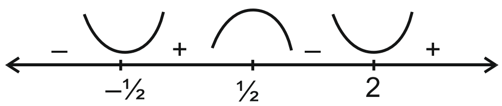
$f'(r) = 24r^3 - 48r^2 - 6r + 12$

$f'(r) = 6(4r^3 - 8r^2 - r + 2)$

for maximum and minimum

$f'(r) = 0 \Rightarrow 4r^2(r-2) - 1(r-2) = 0$

$\Rightarrow r = 2, r = \pm ½$



r = ½ is a point of maximum

$\therefore$ f(r) will be maximum at r = $\frac{1}{2}$

58. If f be a real valued function defined for all real numbers x such that for some fixed a > 0, it satisfies $f(x+a)=\frac{1}{2}+\sqrt{f(x)-\left(f(x)\right)^2}\,\forall x$, then f(x) is periodic with period

(A) a (B) 4a (C) $\frac{a}{2}$ (D) 2a

Ans : (D)

Hint : f(x+a) = $\frac{1}{2}+\sqrt{f(x)-(f(x)^2}\,\forall x$

$(f(x+a)-\frac{1}{2})^2 = f(x)-(f(x))^2 = -\left[\left(f(x)\right)^2-2.\frac{f(x)}{2}+\frac{1}{4}-\frac{1}{4}\right] = -\left[\left(f(x)-\frac{1}{2}\right)^2-\frac{1}{4}\right]$

$\left(f(x+a)-\frac{1}{2}\right)^2+\left(f(x)-\frac{1}{2}\right)^2=\frac{1}{4}$ ....(1)

Put x = x + a

$\left(f(x+2a)-\frac{1}{2}\right)^2+\left(f(x+a)-\frac{1}{2}\right)^2=\frac{1}{4}$ .....(2)

(2) – (1), we get

$\left(f(x+2a)-\frac{1}{2}\right)^2-\left(f(x)-\frac{1}{2}\right)^2=0$

f(x + 2a) = f(x)

Hence period is 2a

Definition of period f(x + T) = f(x) $\forall$ x where T is the smallest positive value.

59. Four natural numbers selected at random are multiplied together, then the probability that the digit in the unit's place in the product be 1, 3, 7 or 9 is

(A) $\frac{16}{625}$ (B) $\frac{18}{625}$ (C) $\frac{4}{625}$ (D) $\frac{5}{625}$

Ans : (A)

Hint : Required probability is $\frac{\left({}^4C_1\right)^4}{10^4}=\frac{2^8}{10^4}=\frac{16}{625}$

60. Let f(x) be a real valued function which is monotonic and differentiable. Then for any reals a and b, $\int_{f(a)}^{f(b)} 2x\{b - f^{-1}(x)\}dx =$

(A) $\int_a^b \left(f^2(x) - f^2(a)\right)dx$ (B) $\int_a^b \left(f(x) - f(a)\right)^2 dx$ (C) $\int_a^b \left(bf^2(x) - af^2(a)\right)dx$ (D) $bf^2(b) + f^{-1}(a)$

Ans : (A)

Hint : $\int_{f(a)}^{f(b)} 2x\{b - f^{-1}(x)\}dx$

Let x = f(t)

dx = f′(t) dt

$\int_a^b 2f(t)\{b - t\}f'(t)dt$

$\int_a^b 2f(t)f'(t)(b - t)dt$

by using integration by part

$\left[(b - t)(f(t))^2\right]_a^b + \int_a^b (f(t))^2 dt$

$-(b - a)(f(a))^2 + \int_a^b (f(t))^2 dt$

$-(f(a))^2 \int_a^b dt + \int_a^b (f(t))^2 dt$

$\int_a^b \{-(f(a))^2 + (f(t))^2\}dt$

$\int_a^b \{(f(x))^2 - (f(a))^2\}dx$

61. Tangent at a point P_1 (other than (0, 0)) on the curve $y = x^3$ meets the curve again at P_2. The tangent at P_2 meets the curve at P_3 and so on. Then the abscissae of $P_1, P_2, P_3, ..., P_n$ form

(A) an A.P. with common difference 1

(B) an H.P. with common difference $\frac{1}{2}$

(C) a G.P. with common ratio 2

(D) a G.P. with common ratio (–2)

Ans : (D)

Hint : Let (a, a^3) be any point on the curve $y = x^3$

Tangent Equation at $P(a, a^3)$

$(y - a^3) = 3a^2(x - a)$ $\left.\frac{dy}{dx}\right|_{(a, a^3)} = 3a^2$

$y = 3a^2(x - a) + a^3$

Since tangent meets again the curve

$x^3 = 3a^2 (x - a) + a^3$

$x^3 - a^3 = 3a^2 (x - a)$

$(x - a) (x^2 + ax + a^2 - 3a^2) = 0$

$\therefore x = a, x = -2a$

$\therefore P_2 (-2a, (-2a)^3)$

$y - (-2a)^3 = 12a^2 (x + 2a)$

$x^3 + 8a^3 = 12a^2 (x + 2a)$

$(x + 2a) (x^2 - 2ax + 4a^2) = 12 a^2 (x + 2a)$

$\therefore x = -2a$ $\quad$ $x^2 - 2ax - 8a^2 = 0$

$\Rightarrow x = -4a$

$\therefore P_2 (-2a, 4a)$

and so on

62. The equation $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is represented by x_1 and x_2 respectively, the GCD of x_1, x_2 will be

(A) 3 (B) 1 (C) p (D) 2

Ans : (B)

Hint : $x^3 + 5x^2 + px + q = 0$ ....(I)

$x^3 + 7x^2 + px + r = 0$ ....(II)

roots of equation (1) be α, β, x_1 and roots of equation (II) be α, β, x_2

$\because \alpha\beta + \beta x_1 + \alpha x_1 = P$ ....(III) [from (I)]

$\alpha\beta + \beta x_2 + \alpha x_2 = P$ ....(IV) [from (II)]

$\therefore$ from (III) and (IV)

$(\alpha + \beta)x_1 = (\alpha + \beta)x_2$

$\because x_1 \neq x_2$

$\therefore \alpha + \beta = 0$

from (I)

$\alpha + \beta + x_1 = -5$

$x_1 = -5$

Similarly $x_2 = -7$

G.C.D 1

63. Let a, b, c be non-zero real numbers, such that $\int_0^{1'}(1+\cos^8 x)(ax^2+bx+c)dx = \int_0^{2'}(1+\cos^8 x)(ax^2+bx+c)dx$, then $ax^2 + bx + c = 0$ has

(A) no solution in (0, 2) (B) at least one root in (1, 2) (C) two imaginary roots (D) two roots in (0, 2)

Ans : (B)

Hint : $\int_0^{1}(1+\cos^8 x)(ax^2+bx+c)dx = \int_0^{2}(1+\cos^8 x)(ax^2+bx+c)dx$

$f(x) = \int_0^{x}(1+\cos^8 t)(at^2+bt+c)dt$

$f(1) = 0, f(2) = 0$

According Rolle's theorem there will be at least one root of $f'(x)$ in (1, 2)

$\because f'(x) = (1+\cos^8 x)(ax^2 + bx + c) = 0 \Rightarrow ax^2 + bx + c = 0 \quad [\because 1 + \cos^8 x \neq 0]$

64. Let Z_1, Z_2 be the roots of the equation $Z^2 + pZ + q = 0$, where the coefficients p and q may be complex numbers and also let A, B represent Z_1, Z_2 respectively in the complex plane. If $\angle AOB = \alpha \neq 0$ and OA = OB, where O is the origin, then the value of $\frac{p^2}{q}\sec^2\frac{\alpha}{2}$ will be

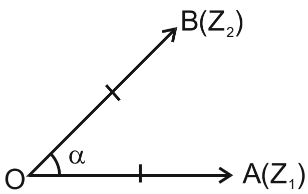
(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) 4 (D) 1

Ans : (C)

Hint : $Z^2 + pZ + q = 0$

$Z_1 + Z_2 = -p$

$Z_1 . Z_2 = q$



$$\frac{Z_1}{Z_2} = e^{i\alpha} = \cos\alpha + i\sin\alpha$$

$$\frac{Z_2}{Z_1} = e^{-i\alpha} = \cos\alpha - i\sin\alpha$$

$$\because \frac{p^2}{q} = \frac{(Z_1+Z_2)^2}{Z_1Z_2} = \frac{Z_1}{Z_2} + \frac{Z_2}{Z_1} + 2$$

$$= 2\cos\alpha + 2$$

$$\frac{p^2}{q} = 4\cos^2\frac{\alpha}{2}$$

$\Rightarrow \frac{p^2}{q}\sec^2\frac{\alpha}{2} = 4$

65. Let g(x) = ax + b, where a < 0 and g is defined from [1, 3] onto [0, 2]. Then the value of $\cot(\cos^{-1}(|\sin x| + |\cos x|) + \sin^{-1}(-|\cos x| - |\sin x|))$ is equal to

(A) g(2) + g(3) (B) g(2) (C) g(3) (D) g(1) + g(2)

Ans : (C)

Hint : g(x) = ax + b

g′(x) = a < 0

Therefore, g is decreasing function

$\therefore g(1) = 2$

g(3) = 0

a + b = 2 ....(1)

3a + b = 0 ....(2)

(2) – (1), we get

2a = – 2

a = – 1

b = 3

$\therefore g(x) = -x + 3$

Given that,

$\cot(\cos^{-1}(|\sin x| + |\cos x|) + \sin^{-1}(-|\cos x| - |\sin x|)$

Take x = 0

$= \cot(\cos^{-1}(1) + \sin^{-1}(-1))$

$= \cot(\cos^{-1}1 - \sin^{-1}1)$

$= \cot(\cos^{-1}\cos 0 - \sin^{-1}\sin \pi/2)$

$= \cot(0 - \frac{\pi}{2})$

$= \cot\left(-\frac{\pi}{2}\right)$

= 0

Since,

g (3) = 0

Hence (C) is correct Ans

CATEGORY - 3 (Q66 to Q75)

(Carry 2 marks each. One or more options are correct. No negative marks)

66. If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r$ and $a_k = 1 \forall k \ge 1$, then the value of $\frac{b_n}{^{2n+1}C_{n+1}}$ is

(A) $\frac{1}{2}$ (B) 2 (C) $\frac{1}{4}$ (D) 1

Ans : (D)

Hint : $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r$ $a_k = 1 \forall k \ge 1$ $\frac{bn}{^{2n+1}C_{n+1}}$

Let y = x – 3 y + 1 = x – 2

$\sum_{r=0}^{2n} a_r(y+1)^r = \sum_{r=0}^{2n} b_r\ y^r$ $a_k = 1 \forall k \ge 1$

$a_1 = a_2 = ... a_{2n} = 1$

LHS = $a_0\,(y+1)^\circ + a_1(y+1)^1 + a_2(y+1)^2 + a_3(y+1)^3 + ...+ a_{2n}\,(y+1)^{2n}$

$a_0 + \sum_{r=1}^{2n}(y+1)^r$

coefficient of y^n in $(y+1)^r = {}^rC_n$

b_n is coefficient of y^n on RHS

$b_n = \sum_{r=n}^{2n} {}^rC_n = {}^nC_n + {}^{n+1}C_n + {}^{n+2}C_n + ... + {}^{2n}C_n$

$b_n = {}^{2n+1}C_{n+1}$

$\frac{b_n}{^{2n+1}C_{n+1}} = 1$

67. If f(x) is differentiable for all $x \in \mathbb{R}$ and satisfies the relation

$x = \lim_{n\to\infty} \frac{\left[l^2(f(x))^x\right]+\left[2^2(f(x))^x\right]+...+\left[n^2(f(x))^x\right]}{n^3}$ where [.] denotes the greatest integer function, then f′(x) =

(A) $\frac{l}{3x^2}\log x$ (B) $3x^{\frac{1}{x}}(l-\log 3x)$ (C) $(3x)^{\frac{1}{x}}\left[\frac{l-\log 3x}{x^2}\right]$ (D) $(3x)^{\frac{1}{x}}\left[\frac{\log 3x-l}{x^2}\right]$

Ans : (C)

Hint : $x \lim_{x\to\infty} \frac{\left[1^2(f(x))^x\right]+\left[2^2(f(x))^x\right]+.....+\left[n^2(f(x))^x\right]}{n^3}$

Using $x - 1 < [x] \le x$

$\left(i^2(f(x))^x - 1\right) < \left[i^2(f(x))^x\right] \le i^2(f(x))^x$

$$\sum_{i=1}^{n}\frac{i^2(f(x))^x-1}{n^3}<\sum_{i=1}^{n}\frac{\left[i^2(f(x))^x\right]}{n^3}\le\sum_{i=1}^{n}\frac{i^2(f(x))^x}{n^3}$$

$$\lim_{n\to\infty}\frac{(f(x))^x}{n^3}\sum_{i=1}^{n}i^2-n\le x\le\lim_{n\to\infty}\frac{(f(x))^x}{n^3}\sum i^2$$

$$\lim_{n\to\infty}\frac{(f(x))^x}{n^3}\frac{n(n+1)(2n+1)}{6}-\frac{n}{n^3}\le x\le\lim_{n\to\infty}\frac{(f(x))^x}{n^3}\times\frac{n(n+1)(2n+1)}{6}$$

$$\lim_{n\to\infty}\frac{(f(x))^x\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}-\frac{n}{n^3}\le x\le\lim_{n\to\infty}\frac{(f(x))^x}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$\frac{f(x)^x}{6}\times 2\le\frac{f(x)^x}{3}$$

$$x=\frac{(f(x))^x}{3}$$

$(f(x))^x = 3x$

$$\boxed{f(x)=(3x)^{\frac{1}{x}}}$$

$$\log(f(x))=\frac{1}{x}\log(3x)$$

$$\frac{1}{(f(x))}=\frac{1}{x}\log(3x)$$

$$\frac{f'(x)}{f(x)}=\frac{1}{x^2}-\frac{\log(3x)}{x^2}$$

$$f'(x)=f(x)\frac{(1-\log(3x))}{x^2}$$

$$f'(x)=\frac{(3x)^{\frac{1}{x}}(1-\log(3x))}{x^2}$$

68. If a differentiable function satisfies

$(x–y)\, f(x+y) – (x+y)\, f(x–y) = 2\,(x^2y – y^3)\ \forall\, x, y \in \mathbb{R}$ and f(I) = 2, then

(A) f(x) must be a polynomial function

(B) f(3) = 13

(C) f(3) = 12

(D) f(0) = 0

Ans : (A. C, D)

Hint : (x–y) f(x+y) – (x+y) f(x–y) = 2(x2y –y3) $\forall x, y \in \mathbb{R}$

$$\frac{(x-y)f(x+y)-(x+y)f(x-y)}{(x+y)(x-y)}=\frac{2y(x+y)(x-y)}{(x+y)(x-y)}$$

$$\frac{f(x+y)}{(x+y)}-\frac{f(x-y)}{(x-y)}=2y$$

Let x + y = u

x – y = v

2x = u + v

$2y = u - v$

$$\frac{f(u)}{u} - \frac{f(v)}{v} = u - v$$

$$\frac{f(u)}{u} - u = \frac{f(v)}{v} - v = \text{constant}$$

$$\frac{f(x)}{x} - x = c$$

$f(x) = x^2 + cx$ $f(1) = 2$ $2 = 1+c$ $\boxed{C = 1}$

$f(x) = x^2 + x \rightarrow$ Polynomial function

$f(3) = 9 + 3 = 12$ $f(0) = 0$

69. Let f(x) > 0 for all $x \in \mathbb{R}$ and f(x) is bounded. If $\lim\limits_{r=\infty}\sum\limits_{r-1}^{n} a^{r-1}\int\limits_{(r-1)a}^{ra} \frac{f(x)dx}{f(x)+f(2ra-a-x)} = \frac{3}{5}$ where 0 < a < 1, then the value(s) of a is/are

(A) $\frac{5}{11}$ (B) $\frac{7}{11}$ (C) $\frac{1}{11}$ (D) $\frac{6}{11}$

Ans : (D)

Hint : $f(x) > 0 \ \forall x \in \mathbb{R}$

$$\lim_{n\to\infty}\sum_{r=1}^{n} a^{r-1}\underbrace{\int_{(r-1)a}^{ra} \frac{f(x) \quad dx}{f(x)+f(2ra-a-x)}}_{\text{By King's Rule}} = \frac{3}{5}$$

$$\lim_{n\to\infty}\sum_{r=1}^{n} a^{r-1}\frac{(ra-(r-1)a)}{2} = \frac{3}{5}$$

$$\lim_{n\to\infty}\sum_{r=1}^{n} a^{r-1}\frac{a}{2} = \frac{3}{5} \quad \frac{a}{2}\lim_{n\to\infty}\sum_{r=1}^{n} a^{r-1}\frac{3}{5} \quad (0 < a < 1)$$

$$\frac{a}{2}\times\left(1+a+a^2+......\infty\right) = \frac{3}{5}$$

$$\frac{a}{2}\times\frac{1}{1-a} = \frac{3}{5}$$

$5a = 6(1-a)$ $5a = 6 - 6a$ $11a = 6$ $\boxed{a = \frac{6}{11}}$

70. Consider the curve $x = 1 - 3t^2$, $y = t - 3t^3$. The tangent to the curve at the point t is the point t is inclined at an angle ϕ to OX and the tangent at P(–2, 2) meets the curve again at Q. Then

(A) the curve is symmetrical about x-axis

(B) the curve is symmetrical about y-axis

(C) $3t = \tan\phi + \sec\phi$

(D) tangents at P and Q are at right angle

Ans : (A,C,D)

Hint : $x = 1 - 3t^2$ $\quad y = t - 3t^3$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1-9t^2}{-6t^2} \quad \frac{dy}{dx} = \frac{9t^2-1}{6t} \quad \tan\phi = \frac{9t^2-1}{6t}, \sec\phi = \sqrt{1+\frac{(9t^2+1)^2}{(6t)^2}}$$

$$\tan\phi + \sec\phi = \frac{9t^2-1}{6t} + \frac{9t^2+1}{6t} \quad \frac{2\times 9t}{6t} = 3t \quad \boxed{\tan\phi + \sec\phi = 3t}$$

$x = 1 - 3t^2$ $\quad$ $y = t - 3t^3$

Replace $t \to -t$ $\quad$ Replace $t \to -t$

$x' = 1 - 3t^2 = x$ $\quad$ $y' = -(t - 3t^3)$ $\quad y' = -y'$

Curve is symmetrical about x –axis

At $t = \frac{2}{3}$

$$\text{Slope at Q} = \frac{9\times\frac{1}{4}-1}{6\times\frac{2}{3}} = \frac{3}{4} \qquad \frac{dy}{dx}\Big|_{\text{at Q}} = \frac{3}{4}$$

$$\frac{dy}{dx}\Big|_{atp} = \frac{9(-1)^2-1}{6(-1)} = \frac{-8}{6} = \frac{-4}{3} \qquad \therefore \frac{dy}{dx}\Big|_P \times \frac{dy}{dx}\Big|_Q = -1$$

71. If $f(x) = x(1331x^2 - 3630x + 3300)$, then for $a = \cos^2(\tan^{-1}(\sin(\cot^{-1} 3)))$

(A) $f(a+1) = 2331$

(B) $f'(a) = 11$

(C) $\lim\limits_{r\to a} f(x) = 1000$

(D) $\int_0^a (f(x) - 1000)dx = \frac{2500}{11}$

Ans : (A, C)

Hint : $f(x) = x(1331x^2 - 3630x + 3300)$

$f(x) = x(113x^2 - 11^2\times 30x + 300\times 11)$

$(11x - 10)^3 = (11x)^3 - 10^3 - 3\times 11x \times 10(11x - 10)$

$= (11x)^3 - 3\times 11^2\times 10x + 3\times 11x \times 10^2 - 10^3$

$(11x-10)^3 = f(x) - 10^3$

$f(x) = (11x-10)^3 + 10^3$

$a = \cos^2(\tan^{-1}(\sin(\cot^{-1}(3))))$

$$= \cos^2\left(\tan^{-1}\left(\sin\left(\sin^{-1}\frac{1}{\sqrt{10}}\right)\right)\right)$$

$$a = \cos^2\left(\tan^{-1}\left(\frac{1}{\sqrt{10}}\right)\right)$$

$$\left(\cos\left(\tan^{-1}\left(\frac{1}{\sqrt{10}}\right)\right)\right)^2$$

$$\left(\cos\ \cos^{-1}\left(\frac{\sqrt{10}}{\sqrt{11}}\right)\right)^2$$

$$\boxed{a = \frac{10}{11}}$$

$$f(a+1) = f\left(\frac{10}{11}+1\right) = f\left(\frac{21}{11}\right) = \left(11\times\frac{21}{11}-10\right)^3 + 1000 = 11^3 + 1000 = 1331 + 1000 = f(a+1) = 2331$$

$$f'(x) = 3(11x-10)^2 \times 11$$

$$f'(a) = f'\left(\frac{10}{11}\right) = 3\times 0 = 0 \quad \lim_{x\to a} f(x) = \lim_{x\to\frac{10}{11}} (11x-10)^3 + 1000 = 1000$$

$$\int_0^a (f(x)-1000)\,dx = \int_0^{\frac{10}{11}} (11x-10)^3\,dx$$

$$= \left[\frac{(11x-10)^4}{11\times 4}\right]_0^{\frac{10}{11}} = \frac{0-(-10)^4}{11\times 4} = \frac{-10000}{11\times 4} = \frac{-2500}{11}$$

72. Let $\bar{r} = \sin x(\bar{a}\times\bar{b}) + \cos y(\bar{b}\times\bar{c}) + 2(\bar{c}\times\bar{a})$, where $\bar{a}$, $\bar{b}$ and $\bar{c}$ are three non-coplanar vectors. It is given that $\bar{r}$ is perpendicular to $(\bar{a}+\bar{b}+\bar{c})$. Then the possible value(s) of $(x^2 + y^2)$ is/are

(A) $\frac{5\pi^2}{4}$ (B) $\frac{35\pi^2}{4}$ (C) $\frac{37\pi^2}{4}$ (D) $\frac{\pi^2}{4}$

Ans : (A, C)

Hint : $\vec{r} = \sin x\left(\vec{a}\times\vec{b}\right) + \cos y\left(\vec{b}\times\vec{c}\right) + 2\left(\vec{c}\times\vec{a}\right)$

$$\vec{r}\cdot\left(\vec{a}+\vec{b}+\vec{c}\right) = 0$$

$$\vec{r}\cdot\vec{a} = \cos y\left[\vec{a}\,\vec{b}\,\vec{c}\right]$$

$\vec{r}.\vec{b} = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

$\vec{r}.\vec{c} = \sin x\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

$(\cos y + 2 + \sin x)\left[\vec{a}\,\vec{b}\,\vec{c}\right] = 0$

$\left[\vec{a}\,\vec{b}\,\vec{c}\right] \neq 0$ ($\because \vec{a}\,\vec{b}$ are non-coplanar vectors)

$\cos y + 2 + \sin x = 0$

$\sin x \in [-1, 1]$

$\cos y \in [-1, 1]$

Only Possibility when $\cos y = -1$ and $\sin x = -1$

$y = (2n+1)\pi$

$x = (4k-1)\dfrac{\pi}{2}$

If $y = \pi,\ x = \dfrac{3\pi}{2}$

$x^2 + y^2 = \pi^2 + \dfrac{9\pi^2}{4} = \dfrac{13\pi^2}{4}$

If $x = -\dfrac{\pi}{2}, y = \pi$

$x^2 + y^2 = \dfrac{\pi^2}{4} + \pi^2 = \dfrac{5\pi^2}{4}$ (Option A)

If $x = -\dfrac{\pi}{2}$ & $y = 3\pi$

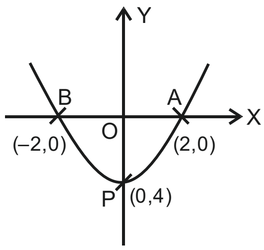
$x^2 + y^2 = \dfrac{\pi^2}{4} + 9\pi^2 = \dfrac{37\pi^2}{4}$

73. The parabola $y = 4 - x^2$ has vertex P. It intersects x-axis at A and B. If the parabola is translated from its initial position to a new position by moving its vertex along the line $y = x + 4$, so that it intersects x-axis at B and C, then the abscissa of C will be

(A) 12 (B) 8 (C) 6 (D) $\dfrac{7}{3}$

Ans : (B)

Hint : $y = 4 - x^2$



Coordinates of A & B

A (–2, 0) & B(+2, 0)

Vertex P(0, 4)

P is moving along line y = x+4

Let New vertex P′ (h, h+4) on y = x+4

$y - (h+4) = -(x-h)^2$

Passes through (2, 0) $\therefore$ h = 5

New equa. $y = 9 - (x-5)^2$

x – intercept at x = 2 & x = 8

$\therefore$ abscissa of c = 8

74. If A_1, A_2, A_3 ...., A_{1006} be independent events such that $P(A_t) = \frac{1}{2i}, (i = 1, 2, ..., 1006)$ and the probability that none of the events occurs be $\frac{\alpha!}{2^{\alpha}(\beta!)^2}$, then

(A) β is of the form 4k + 2, k $\in$ I

(B) $\alpha = 2\beta$

(C) β is of the form 4k + I, k $\in$ I

(D) β is a prime number

Ans : (A, B)

Hint : P(Ai) = $\frac{1}{2i}$ Independent condition

$P(A_i^C) = 1 - \frac{1}{2i} = \frac{2i-1}{2i}$ $P(A \cap B) = P(A).P(B)$

Since all one indedependent, then probabicity of that none of the events occurs be

$P(A^C_1 \cap A^C_2 \cap .... \cap A^C_{1006}) = P(A^C_1).\ P(A^C_2) .... P(A^C_{1006})$

$$= \frac{1.3.5....(2012-1)}{2.4.6.....(2012)} = \frac{(2012)!}{(2.4.6.....2012)^2} = \frac{(2012)!}{(2^{1006}1006!)^2} = \frac{(2012)!}{2^{2012}(1006!)^2}$$

$\therefore$ $\alpha = 2012$ $\beta = 1006$

(B) $\alpha = 2\beta$

(A) β is of the to form 4k+2, K $\in$ I

75. If $\left(4^{\sec^2\alpha}\right)x^2+2x+\left(\beta^2-\beta+\frac{1}{2}\right)=0$ has real roots, then the value/values of $(\cos\alpha + \cos^{-1}\beta)$ is/are

(A) $1+\frac{x}{3}$, if n is even (B) $-1-\frac{x}{3}$, if n is odd (C) $-1+\frac{x}{3}$, if n is odd (D) $-1+\frac{x}{3}$, if n is even

Ans : ()

Hint : $\left(4^{\sec^2\alpha}\right)x^2+2x+\left(\beta^2-\beta+\frac{1}{2}\right)=0$

for Real Roots $b^2 - 4ac \geq 0$

$4^{\sec^2\alpha}\left(\beta^2-\beta+\frac{1}{2}\right)\leq 1$ ---- (1)

Min value of $\sec^2\alpha$ is 1, i.e. $\boxed{\sec^2\alpha \geq 1}$

Now $\beta^2-\beta+\frac{1}{2}\geq\frac{1}{4}$

To satisfy Equation (1) $\sec^2\alpha$ must be equal to 1 and $\left(\beta^2-\beta+\frac{1}{2}\right)$ is $\frac{1}{4}$

Hence $\sec^2\alpha = 1 \Rightarrow a = 0, \pm\pi, \pm 2\pi \Rightarrow \cos\alpha = 1$ or -1

❑ ❑ ❑

WBJEE - 2026

Answer Keys by

Aakash Institute, (W.B)

PHYSICS & CHEMISTRY

BOOKLET CODE					BOOKLET CODE					BOOKLET CODE					BOOKLET CODE				
Q.No.	■	●	◆	💧	Q.No.	■	●	◆	💧	Q.No.	■	●	◆	💧	Q.No.	■	●	◆	💧
01	A	A	D	A	21	A	B	D	C	41	B	A	B	C	61	A	B	B	B
02	B	D	C	B	22	C	D	C	*	42	D	D	B	A	62	D	D	A	D
03	C	D	A	A	23	D	C	D	A	43	C	D	C	C	63	A	C	C	D
04	C	C	B	C	24	D	C	B	C	44	C	D	A	C	64	C	D	D	A
05	C	B	B	D	25	C	**	A	A	45	B	D	D	A	65	A	D	C	C
06	D	D	C	D	26	A	A	C	D	46	B	D	D	D	66	D	C	B	D
07	A	A	A	A	27	C	A	D	B	47	D	A	C	C	67	D	C	D	B
08	B	A	C	D	28	C	A	A	A	48	D	A	D	D	68	A	C	C	D
09	C	*	C	C	29	C	D	C	C	49	D	B	C	D	69	C	D	B	B
10	*	C	D	A	30	A	B	A	C	50	B	B	A	C	70	C	B	A	D
11	B	C	**	B	31	C	B	C	C	51	D	C	D	D	71	B	A	B	D
12	D	B	C	**	32	B	C	C	C	52	D	B	D	B	72	A	D	B	A
13	B	C	C	C	33	C	C	B	C	53	C	A	A	B	73	B	B	A	A
14	B	C	B	C	34	C	C	C	C	54	D	A	D	B	74	D	B	A	B
15	A	D	B	B	35	C	C	C	B	55	D	D	D	D	75	A	A	D	B
16	**	A	C	D	36	A,B,D	A,B,C	B,C	A,D	56	B	B	D	D	76	A,B,C	B,C,D	A,C	A,B,D
17	A	B	A	C	37	A,B,C	A,D	A,B,D	A,B,C	57	D	C	D	C	77	B,C,D	A,C	A,B,D	A,B.C,D
18	D	C	D	C	38	A,D	A,B,D	A,B,C	B,C	58	A	D	B	A	78	A,B,C,D	A,B,D	A,B,C	A,C
19	D	C	*	B	39	A,B,C	B,C	A,D	A,B,C	59	B	D	C	A	79	A,C	A,B,C	A,B,C,D	B,C,D
20	C	C	A	D	40	B,C	A,B,C	A,B,C	A,B,D	60	C	C	D	D	80	A,B,D	A,B,C,D	B,C,D	A,B,C

Aakash
Medical|IIT-JEE|Foundations

* Incomplete question
** None of the given option correct

DATE : 24/05/2026

Code - [redacted]

Aakash
Medical|IIT-JEE|Foundations

ANSWERS & HINTS
for
WBJEE - 2026
SUB : PHYSICS & CHEMISTRY

PHYSICS

CATEGORY - 1 (Q1 to Q30)

(Carry 1 mark each. Only one option is correct. Negative mark : – ¼)

1. Radiation of wavelength λ is incident on a photocell. The fastest emitted electron has speed *v.* If the wavelength is changed to $\frac{3\lambda}{4}$, then the speed of the fastest emitted electron will be –

(A) greater than $v\sqrt{\frac{4}{3}}$ (B) less than $v\sqrt{\frac{4}{3}}$

(C) equal to $v\sqrt{\frac{4}{3}}$ (D) equal to $v\sqrt{\frac{3}{4}}$

Ans : (A)

Hint : $K = \frac{hc}{\lambda} - \phi$, $K' = \frac{4hc}{3\lambda} - \phi$

$K' = \frac{4}{3}(K + \phi) - \phi$, $K' = \frac{4}{3}K + \frac{\phi}{3}$

$K' > \frac{4K}{3}$, $\frac{1}{2}mv_0^2 > \frac{4}{3}\frac{1}{2}mv^2$

$v_0 > \sqrt{\frac{4}{3}}v$

2. The equation of a transverse wave is $y = y_0 \sin 2\pi(ft - \frac{x}{\lambda})$. If the maximum particle velocity be four times that of wave velocity then

(A) $\lambda = \frac{\pi y_0}{4}$ (B) $\lambda = \frac{\pi y_0}{2}$ (C) $\lambda = \pi y_0$ (D) $\lambda = 2\pi y_0$

Ans : (B)

Hint : $(V_p)_{max} = 4V_{wave}$

$y_0 \times 2\pi f = 4 \times f\lambda \Rightarrow \lambda = \frac{\pi y_0}{2}$

3. A person has a minimum distance of distinct vision of 50 cm. The power of lens required to read a book at a distance of 25 cm is

(A) 3 D (B) 1 D

(C) 2 D (D) 5 D

Ans : (C)

Hint : $\frac{1}{f}=\frac{1}{v}-\frac{1}{u} = \frac{1}{-0.5}-\frac{1}{-0.25}=2D$

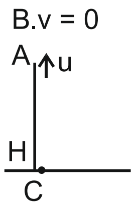
4. From a tower of height *H*, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground is *n* times that taken by it to reach the highest point of its path. The relation between *H, u* and *n* is

(A) $2gH = n^2u^2$ (B) $gH = (n-2)^2u^3$

(C) $2gH = nu^2(n-2)$ (D) $2gH = u^2(n-2)^2$

Ans : (C)

Hint :



$t_{A\to B\to C} = nt_{A\to B}$

$\frac{u}{g}+\sqrt{\frac{2\left(H+\frac{u^2}{2g}\right)}{g}}=n\frac{u}{g}$, $2gh = nu^2(n-2)$

5. A resistor of resistance 'R' draws power 'P' when connected to an AC source. If an inductance is now placed in series with *R*, such that the impedance of the circuit becomes 'Z', the power drawn will be

(A) $P\left(\frac{R}{Z}\right)$ (B) $P\left(\frac{R}{Z}\right)^3$

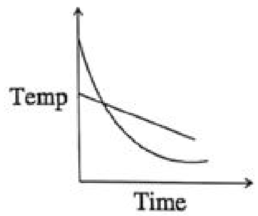
(C) $P\left(\frac{R}{Z}\right)^2$ (D) $P\sqrt{\frac{Z}{R}}$

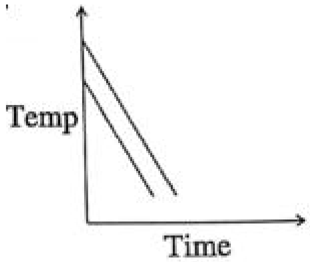
Ans : (C)

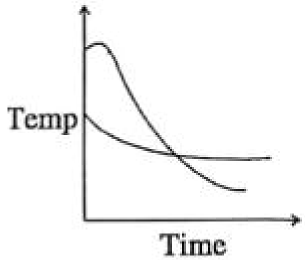
Hint : $P=\frac{\varepsilon^2_{rms}}{R}$, $P'=\frac{\varepsilon^2_{rms}}{Z}\frac{R}{Z}$

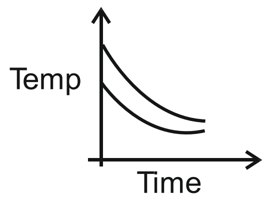
$\frac{P'}{P}=\left(\frac{R}{Z}\right)^2 \Rightarrow P'=\left(\frac{R}{Z}\right)^2 P$

6. Two identical metal bars are heated in two different temperature and allowed to cool in the same surroundings. Which one of the following figures correctly shows their cooling curves?

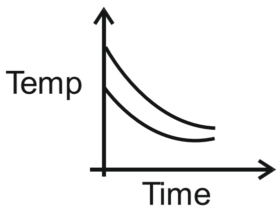
(A) 

(B) 

(C) 

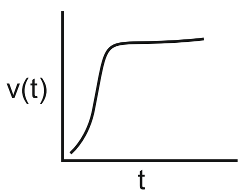
(D) 

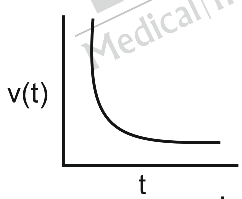
Ans : (D)

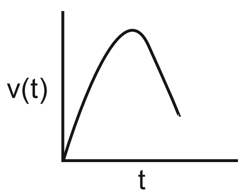
Hint : 

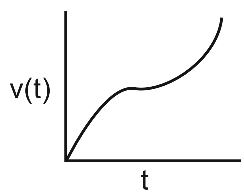
T decreases exponentially

7. Which one of the following graphs represents the velocity-time (v – t) graph of a small spherical body falling in a viscous liquid?

(A) 

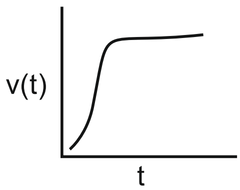
(B) 

(C) 

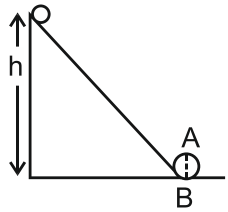
(D) 

Ans : (A)

Hint : Speed increases & becomes constant



8. A body initially at rest and sliding along a frictionless track from a height 'h' (as shown in figure) just completes a vertical circle of diameter AB = d. The height 'h' is equal to



(A) $\frac{3}{2}d$ (B) $\frac{5}{4}d$ (C) $\frac{7}{5}d$ (D) $\frac{d}{2}$

Ans : (B)

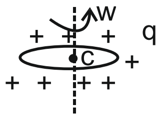
Hint : $V_B = \sqrt{2gh} = \sqrt{5g\frac{d}{2}} \Rightarrow h = \frac{5d}{4}$

9. There is a ring of radius r having linear charge density λ and rotating with a uniform angular velocity ω. The magnitude of the magnetic field produced by this ring at its own centre would be (μ_o = permeability of air)

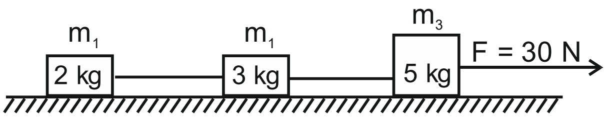
(A) $\frac{\lambda\omega^2}{2-\mu_0}$ (B) $\frac{\mu_0\lambda^2\omega}{\sqrt{2}}$ (C) $\frac{\mu_0\lambda\omega}{2}$ (D) $\frac{\mu_0\lambda}{2\omega^2}$

Ans : (C)

Hint : $q = \lambda 2\pi r$, $I = \frac{qw}{2\pi}$, $B_c = \frac{\mu_0 I}{2r} = \frac{\mu_0\lambda\omega}{2}$



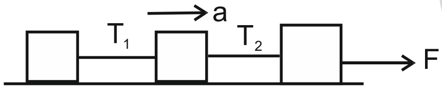
10. Theree block of masses m_1 = 2 kg, m_2 = 3 kg and m_3 = 5 kg are placed on a horizontal frictionless surface and a force of 30N pulls the system as shown below. The value of tension *T* will be.



(A) 15 N (B) 30 N (C) 6 N (D) 10 N

Ans : (A,C)*

Hint :



$a = \frac{30}{10} = 3 m/s^2$, $T_1 = 2a = 2 \times 3 = 6$ N

$T_2 = 5a = 5 \times 3 = 15$ N

11. A radioactive element $^{242}_{92}X$ emits two α-particle, one electrons and two positrons. The transformed nucleus is represented by $^{234}_{P}Y$. The value of P is

(A) 85 (B) 87 (C) 92 (D) 96

Ans : (B)

Hint : $^{242}_{92}x \rightarrow 2\,^4_2He + ^{0}_{-1}e + 2\,^{0}_{+1}e + ^{234}_{p}Y$

92 = 2 × 2 – 1 + 2 × 1 + P

$\Rightarrow$ P = 87

12. Density and volume of a body are given as (20 ± 4) gm/cm³ and (10 ± 1) cm³ respectively. The absolute error in measurement of mass is

(A) 20 gm (B) 30 gm (C) 45 gm (D) 60 gm

Ans : (D)

Hint : m = vρ = 10 × 20 = 200

$$\frac{\Delta m}{m} = \frac{\Delta v}{v} + \frac{\Delta \rho}{\rho}$$

$$\Rightarrow \frac{\Delta m}{200} = \frac{1}{10} + \frac{4}{20} \Rightarrow \Delta m = 60$$

13. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $|\vec{a}| = 1, |\vec{b}| = 2$ and $|\vec{c}| = 4$ along with $\vec{a} + \vec{b} + \vec{c} = 0$. Then, the value of $4\vec{a}\cdot\vec{b} + 3\vec{b}\cdot\vec{c} + 3\vec{c}\cdot\vec{a}$ will be

(A) 27 (B) –26 (C) –68 (D) –34

Ans : (B)

Hint : $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{b} + \vec{c} = -\vec{a}$

$(\vec{b} + \vec{c})\cdot(\vec{b} + \vec{c}) = (-\vec{a})\cdot(-\vec{a})$

$\Rightarrow b^2 + c^2 + 2\vec{b}\cdot\vec{c} = \vec{a}$

$\Rightarrow 2^2 + 4^2 + 2(\vec{b}\cdot\vec{c}) = 1^2 \quad \Rightarrow \vec{b}\cdot\vec{c} = \frac{-19}{2}$

Similarly $\vec{c}\cdot\vec{a} = \frac{-13}{12}$

$\vec{a}\cdot\vec{b} = \frac{11}{12}$

$\therefore 4\vec{a}\cdot\vec{b} + 3\vec{b}\cdot\vec{c} + 3\vec{c}\cdot\vec{a} = -26$

Note: With the given values of $|\vec{a}| = 1, |\vec{b}| = 2$ and $|\vec{c}| = 4$, $\vec{a} + \vec{b} + \vec{c}$ cannot be zero so question is not correct. But most appropriate answer is option (B)

14. Beyond what distance, the ray optics is sufficiently valid when the aperature is 6 mm wide and the wavelength is 6000 Å?

(A) 50 m (B) 60 m (C) 40 m (D) 10 m

Ans : (B)

Hint : Actually Ray Optics is valid within Fresnel's distance (Z_f), so word beyond is not correct in question.

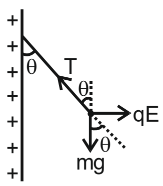
$$Z_f = \frac{a^2}{\lambda} = \frac{(6\times10^{-3})}{6000\times10^{-10}} = 60\text{m}$$

15. A simple pendulum of length l has a bob of mass m, with a charge q. On it a vertical sheet of charge, with surface charge density 'σ' passes through the point of suspension. At equilibrium, if the string makes an angle θ with the vertical , then

(A) $\tan\theta = \frac{\sigma q}{2\varepsilon_0 mg}$ (B) $\tan\theta = \frac{\sigma q}{\varepsilon_0 mg}$ (C) $\cot\theta = \frac{\sigma q}{2\varepsilon_0 mg}$ (D) $\cot\theta = \frac{\sigma q}{\varepsilon_0 mg}$

Ans : (A)

Hint :

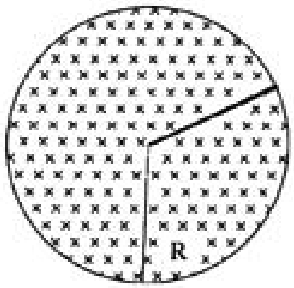


$T\cos\theta = mg$

$T\sin\theta = qE$

$\therefore \tan\theta = \frac{qE}{mg} = \frac{q\sigma}{2mg\varepsilon_0}$

16. A uniform but time varying magnetic field is present in a circular region of radius 'R'. The magnetic field is perpendicular and into the plane of loop and the magnitude of field is increasing at constant rate α. There is a straight conducting rod of length 2R placed as shown in figure. The magnitude of induced emf across the rod is.



(A) $\pi R^2 \alpha$ (B) $\frac{1}{2}\pi R^2 \alpha$ (C) $\frac{1}{\sqrt{2}} R^2 \alpha$ (D) $\frac{1}{4}\pi R^2 \alpha$

Ans : ()

Hint : Rod is not shown in figure. (Bonus)

17. A circular coil, carrying current, has radius R. The distance from the centre of the coil on the axis where the magnetic induction will be $\frac{1}{27}$ th of its value at the centre of the coil is

(A) $2\sqrt{2}R$ (B) $3\sqrt{2}R$ (C) 3R (D) $2\sqrt{3}R$

Ans : (A)

Hint : $\frac{\mu_0 i R^2}{2\left(x^2+R^2\right)^{\frac{3}{2}}} = \frac{1}{27}\frac{\mu_0 i}{2R}$

$x = 2\sqrt{2}\ R$

18. Two spherical soap bubbles of radii r_1 and r_2 in vaccum coalesce under isothermal condition. The newly formed bubble has a radius (r) given by

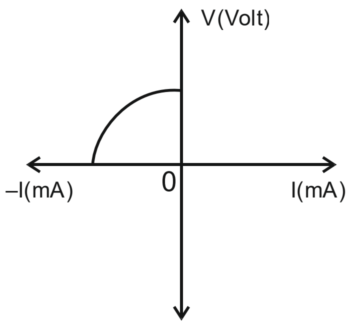
(A) $r_1 + r_2$ (B) $\frac{r_1+r_2}{2}$ (C) $\frac{r_1 r_2}{r_1+r_2}$ (D) $\sqrt{r_1^2+r_2^2}$

Ans : (D)

Hint : In isothermal condition.

$$4\pi r_1^2 + 4\pi r_2^2 = 4\pi r^2 \Rightarrow r = \sqrt{r_1^2+r_2^2}$$

19. The I - V characteristics graph shown below is exhibited by

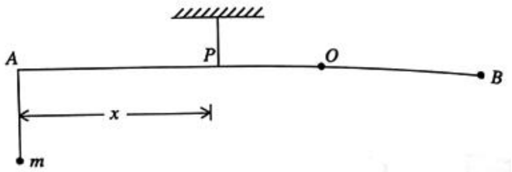


(A) LED (B) Zener diode (C) Photodiode (D) Solar cell

Ans : (D)

Hint : The given I – V graph is of solar cell.

20. A uniform rod AB is suspended from a point P, at a variable distance x, from A, as shown in figure. To make the rod horizontal, a mass ‘m’ is suspended from its end A. Which set of variables will give a straight line when they are plotted?



(A) m, x^2 (B) $m, \frac{1}{x^2}$ (C) $m, \frac{1}{x}$ (D) m, x

Ans : (C)

Hint : Taking torque about ‘P’ and equating it to zero

$$mgx = Mg\left(\frac{L}{2} - x\right)$$

$$m = \frac{ML}{2x} - M$$

Hence m vs $\frac{1}{x}$ is straight line.

21. The magnetic moment of an iron bar is M. It is now bent in such a way that it forms an arc section of a circle subtending an angle of 60° at the centre. The magnetic moment of the arc section is

(A) $\frac{3M}{\pi}$ (B) $\frac{4M}{\pi}$ (C) $\frac{M}{\pi}$ (D) $\frac{2M}{\pi}$

Ans : (A)

Hint : $m\ell = M = \frac{\pi R}{3}$

$$M = \frac{3M}{\pi}$$

22. Consider a fuse wire of length l and radius r. The time of heating (t) for passing the maximum current will depend on

(A) $t \propto r^2 l$ (B) $t \propto r^3 l^2$ (C) $t \propto r^4 l^0$ (D) $t \propto r^2 l^3$

Ans : (C)

Hint : $H = I_{max}^2 Rt \propto m$

$$I_{max}^2 \frac{\rho l}{\pi r^2} t \propto \rho \pi r^2 l$$

$t \propto r^4$

23. A square of side L lies in the x – y plane, where the magnetic field is given by $B = B_0\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ where B_0 is constant. The magnetic flux passing through the square is

(A) $5 B_0L^2$ (B) $2 B_0L^2$ (C) $3 B_0L^2$ (D) $4 B_0L^2$

Ans : (D)

Hint : f = B.A

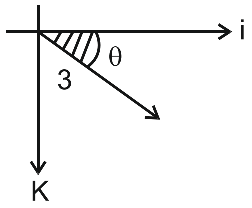
$= 4B_0L^2$

24. If a vector $\vec{v} = 3\hat{i}$ is rotated in the x – z plane by an angle θ with respect to x-axis in the clockwise direction, then for an observer at +y axis the vector will be

(A) $3\sin\theta\hat{i}$ (B) $3\cos\theta\hat{i}$ (C) $3\sin\theta\hat{i} + 3\cos\theta\hat{k}$ (D) $3\cos\theta\hat{i} + 3\sin\theta\hat{k}$

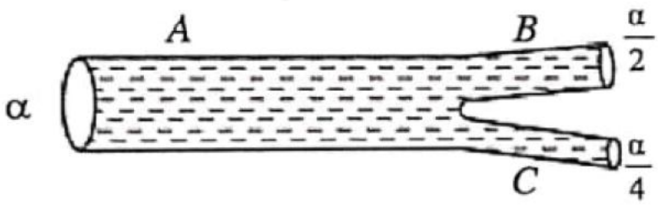
Ans : (D)

Hint :



$3\cos\theta\hat{i} + 3\sin\theta\hat{k}$

25. A pipe A is connected with other pipes B and C as shown in the figure. The areas of cross-section of A, B and C are respectively α, $\frac{\alpha}{2}$ and $\frac{\alpha}{4}$. If the velocities of flow of water through A and B are 10 m/sec and 6 m/sec, respectively, then velocity of flow, V_c along C is



(A) 21 m/sec (B) 12 m/sec (C) 28 m/sec (D) 18 m/sec

Ans : (C)

Hint : $\alpha 10 = \frac{\alpha}{2} \times 6 + \frac{\alpha}{4} \times v$

$\Rightarrow$ v = 28 m/sec.

26. A plano-convex lens fits exactly into a plano-concave lens. Their plane surfaces are parallel to each other. If lenses are made of different materials of refractive indices μ_1 and μ_2 and R is the radius of curvature of the curved surface of the lenses, then the focal length of the combination is

(A) $\frac{R}{(\mu_1 - \mu_2)}$ (B) $\frac{2R}{(\mu_1 - \mu_2)}$

(C) $\frac{R}{2(\mu_1 + \mu_2)}$ (D) $\frac{R}{2(\mu_1 - \mu_2)}$

Ans : (A)

Hint : $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

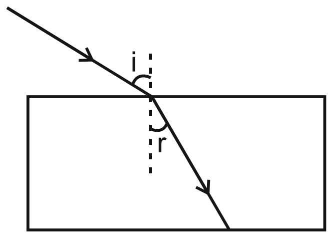
$\Rightarrow \left(f = \frac{R}{\mu_1 - \mu_2} \right)$

27. A ray of light travelling in air is incident on one face of a parallel glass slab of thickness t a refractive index μ at an angle of incidence i. Total time spent by the ray inside the slab is

(A) $\dfrac{\mu^2 t}{c\sqrt{1-\mu^2 \sin i}}$ (B) $\dfrac{\mu t}{c\sqrt{\mu^2 - \sin^2 i}}$ (C) $\dfrac{\mu^2 t}{c\sqrt{\mu^2 - \sin^2 i}}$ (D) $\dfrac{t}{c\sqrt{\mu^2 - \sin^2 i}}$

Ans : (C)

Hint :



$$\text{time} = \frac{t}{\cos r} \times \frac{\mu}{c}$$

$$\Rightarrow \text{time} = \frac{\mu^2 t}{c\sqrt{\mu^2 - \sin^2 i}}$$

28. The velocity v of a particle at time t is given by $v = at + \dfrac{b}{t+c}$, where a, b and c are constants. The dimension of a, b and c are, respectively

(A) LT^2, LT, L (B) L, LT, T^2 (C) LT^{-2}, L, T (D) L^2, T, LT^2

Ans : (C)

Hint : [c] = [T]

[a] = [LT^{-2}]

[b] = [L]

29. The inputs to a digital circuit are as shown below. The output Y is

(A) $A + B + \bar{C}$ (B) $(A + B)\bar{C}$ (C) $\bar{A} + \bar{B} + \bar{C}$ (D) $\bar{A} + \bar{B} + C$

Ans : (C)

Hint : $Y = \overline{A \cdot B} + \bar{C} \Rightarrow Y = \bar{A} + \bar{B} + \bar{C}$

30. A body of density 'ρ' is dropped slowly on the surface of a lake of depth d. If the density of the water be 'ρ'' ($\rho' < \rho$) then the time taken by the body to reach the bottom of the lake is

(A) $\left[\dfrac{2d\rho}{g(\rho-\rho')}\right]^{\frac{1}{2}}$ (B) $\left[\dfrac{2gd}{\rho(\rho-\rho')}\right]^{\frac{1}{2}}$ (C) $\left[\dfrac{2d\rho'}{\rho g(\rho-\rho')}\right]^{\frac{1}{2}}$ (D) $\left[\dfrac{g(\rho-\rho')}{2d\rho}\right]^{\frac{1}{2}}$

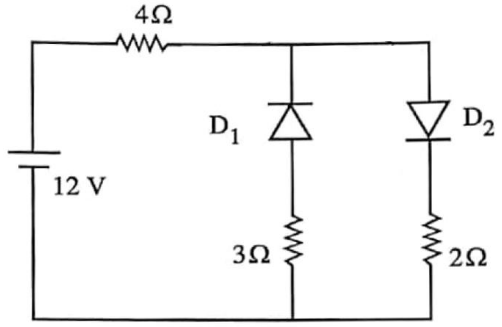
Ans : (A)

Hint : $\text{time} = \sqrt{\dfrac{2d\,v\rho}{v(\rho-\rho')g}} = \left[\dfrac{2d\rho}{g(\rho-\rho')}\right]^{\frac{1}{2}}$

Category 2 (Q. 31 to 35)

(Carry 2 marks each. Only one option is correct. Negative marks – ½)

31. The circuit has two oppositely connected ideal diodes in parallel as shown in the figure. What is the current flowing in the circuit?



(A) 1.33A (B) 1.71A (C) 2.00A (D) 2.31A

Ans : (C)

Hint : D_1 is reversed biased, D_2 is forward biased

so $i = \frac{12}{4+2} = 2A$

32. 2 moles of an ideal gas with $\frac{C_p}{C_v} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_p}{C_v} = \frac{4}{3}$. The value of $\frac{C_p}{C_v}$ for the mixture is

(A) 1.5 (B) 1.42 (C) 1.48 (D) 1.6

Ans : (B)

Hint : $\gamma_{eq} = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1C_{v_1} + n_2C_{v_2}}$

$\gamma_{eq} = 1.42$

33. The de-Broglie wavelength of an electron in 4th orbit is (where r = radius of the 1st orbit)

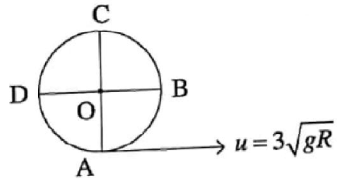
(A) $2\pi r$ (B) $4\pi r$ (C) $8\pi r$ (D) $16\pi r$

Ans : (C)

Hint : $4\lambda = 2\pi r_4$ & $r_4 = 16r$

$\lambda = 8\pi r$

34. A particle of mass m is suspended from a point O by a string of length R. It is given a velocity $u = 3\sqrt{gR}$ at the bottom. The difference in tension at point B and at the point C is



(A) 6 mg (B) 4 mg (C) 3 mg (D) 8 mg

Ans : (C)

Hint : $T_B = \frac{m}{R}\left(u^2 - 2gR\right)$

$T_C + mg = \frac{m}{R}(u^2 - 2g \times 2R) \Rightarrow T_B - T_C = 3mg$

35. An electromagnetic wave, whose wave normal makes an angle of 45° with the vertical, is travelling in air and strikes a horizontal liquid surface. While travelling through the liquid, it gets deviated by 15°. If the speed of electromagnetic wave in air is 3×10^8 m/s, then the speed of electromagnetic wave in the liquid will be

(A) $\frac{\sqrt{2}}{3}\times10^8$ m/s (B) 1.5×10^8 m/s (C) 2.1×10^8 m/s (D) 2.5×10^8 m/s

Ans : (C)

Hint : i = 45° $\delta = 15°$ $\Rightarrow$ r = 30°

sin i = μsin r

$V = \frac{c}{\mu} = 2.1 \times 10^8$ m/s

Category 3 (Q36 to 40)

(Carry 2 marks each. One or more options are correct. No negative marks)

36. For Boolean variables A and B, $A \oplus B = A\bar{B} + \bar{A}B$. Then, which of the following statements is/are correct?

(A) $1\oplus A = \bar{A}$ (B) $A \oplus A = 0$ (C) $0 \oplus A = 0$ (D) $A \oplus \bar{A} = 1$

Ans : (A, B, D)

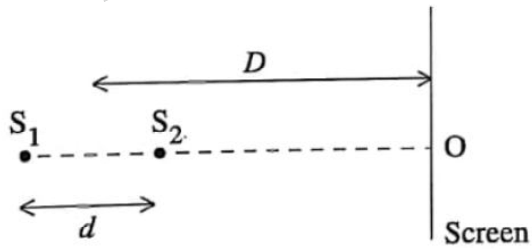
Hint : (A) $1\oplus A = 1\bar{A} + \bar{1}A = \bar{A} + 0 = \bar{A}$

(B) $A \oplus A = \bar{A}.A + A + \bar{A} = 0$

(C) $0 \oplus A = \bar{0}.A + \bar{A}.0 = A$

(D) $A \oplus \bar{A} = A.\bar{\bar{A}} + \bar{A}.\bar{A} = A + \bar{A} = 1$

37. Two points of monochromatic and coherent sources of light of wavelength λ each, are placed as shown in figure. The initial phase difference between the sources is zero , (D>>d). Mark the correct statement(s)



(A) If $d = \frac{7\lambda}{2}$, O will be a minima

(B) If d = λ, only one maxima can be observed on the screen

(C) If d = 4.8λ, then total 5 minima would be there on the screen

(D) If d = λ, the intensity at O would be minimum

Ans : (A, B, C)

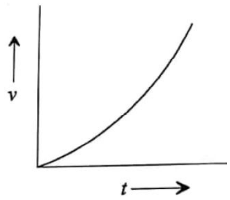
Hint : If $d = \frac{7\lambda}{2}$ Δx at O Is (2n +1) $\frac{\lambda}{2}$ hence minima.

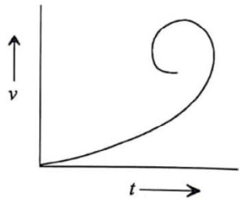
If d = λ only maxima at O

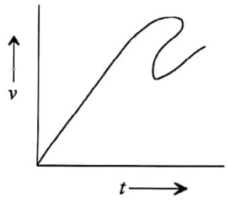
If d = 4.8λ ; total 5 circular minima

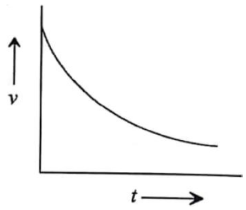
If d = λ maxima at O.

38. Which of the velocity-time (v – t) graph(s) can possibly represent one-dimensional motion of a particle?

(A)


(B)


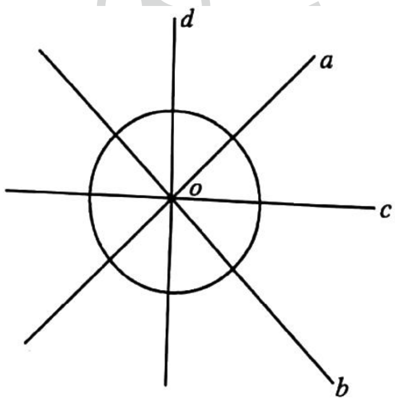
(C)


(D)


Ans : (A, D)

Hint : We cannot have more than one velocity at a given time instant and hence option B and C are incorrect.

39. The moment of inertia of a thin disc about axes a, b, c, d are I_1, I_2, I_3 and I_4 respectively, as shown in figure. If the moment of inertia about an axis passing through the centre and perpendicular to the plane of the disc is I then,



(A) $I = I_1 + I_2$

(B) $I = I_3 + I_4$

(C) $I = I_1 + I_3$

(D) $I = I_1 + I_2 + I_3 + I_4$

Ans : (A, B, C)

Hint : $I_1 = I_2 = I_3 = I_4$ and $I_{perpendicular}$ = sum of any two

40. The displacement current flows through a capacitor when the voltage across its plates

(A) becomes zero

(B) is increasing with time

(C) is decreasing with time

(D) attains a constant value

Ans : (B, C)

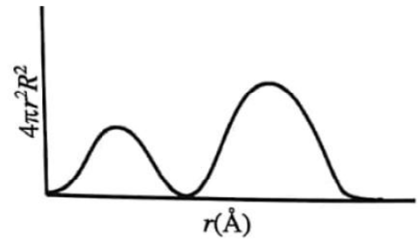
Hint : Displacement current is there when electric field in changing with time.

CHEMISTRY

CATEGORY - 1 (Q 41 to 70)

(Carry 1 mark each. Only one option is correct. Negative marks: –¼)

41. The plot of radial probability density ($4\pi r^2R^2$) against r for an electron in np orbital of a many electron atom is given below :



The value of n is

(A) 2 (B) 3 (C) 4 (D) 5

Ans : (B)

Hint : Number of radial node : $n - \ell - 1$

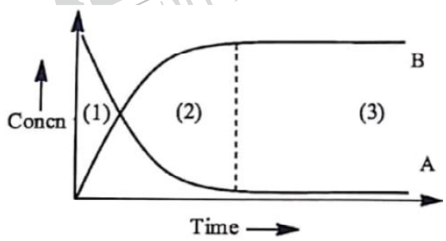
42. In a first order reaction the concentration of reactant decreases from 400 moles lit^{-1} to 50 moles lit^{-1} in 7.5×10^3s. The rate constant of the reaction is (approximately)

(A) $1 \times 10^{-2}\ s^{-1}$ (B) $2.5 \times 10^{-3}\ s^{-1}$ (C) $1 \times 10^{-5}\ s^{-1}$ (D) $2.77 \times 10^{-4}\ s^{-1}$

Ans : (D)

Hint : $K = \frac{1}{t}\ln\frac{a_0}{a_t}$

43. For the reaction $A \rightleftharpoons B$, variation of concentration is plotted against time as shown below.



Which of the following statements is true ?

(A) Region (1) indicates equilibrium
(B) Region (2) indicates equilibrium
(C) Region (3) indicates equilibrium
(D) Both the Regions (2) and (3) indicate equilibrium

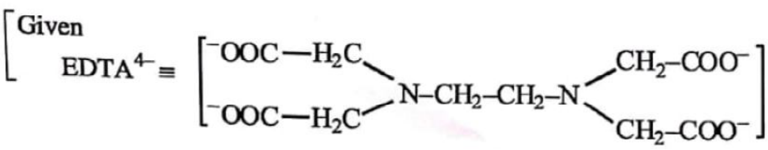
Ans : (C)

44. Peroxide ion is

(A) Paramagnetic (B) Ferromagnetic (C) Diamagnetic (D) Antiferromagnetic

Ans : (C)

45. The calculated magnetic moment for low spin $[Ru(EDTA)]^-$ is



(A) 2.73 BM (B) 1.73 BM (C) 3.23 BM (D) 0.00 BM

Ans : (B)

Hint : Ru^{3+} (d^5) : Low spin complex

46. A compound contains two types of atoms A and B. Its crystal structure is a cubic lattice with 'A' atoms at the corner of the unit cells and 'B' atoms at the body centres. The simplest formula of the compound will be

(A) A_2B (B) AB (C) AB_2 (D) AB_3

Ans : (B)

47. Glucose is added to 1 litre of water to such an extent that $\Delta T_f/K_f$ equals to $\frac{1}{1000}$. The weight of glucose added is

(A) 180 gm (B) 18 gm (C) 1.8 gm (D) 0.18 gm

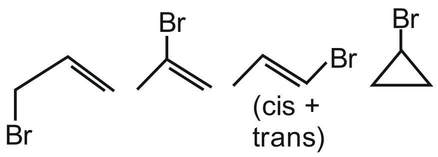
Ans : (D)

Hint : $\Delta T_f = K_f \times m$

48. How many isomers can a compound with molecular formula C_3H_5Br have?

(A) 2 (B) 3 (C) 4 (D) 5

Ans : (D)

Hint : 

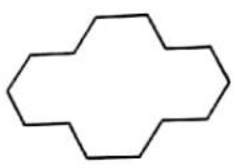
49. The correct order of conductivity of 0.001 (M) separate aqueous solutions of $[Pt(NH_3)_6]Cl_4$ (i); $[Cr(NH_3)_6]Cl_3$ (ii); $[Co(NH_3)_4Cl_2]Cl$ (iii) and K_2PtCl_6 (iv) each containing octahedral complex species is

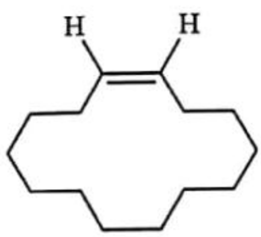
(A) (i) < (ii) < (iii) < (iv) (B) (i) < (ii) < (iv) < (iii) (C) (i) < (iv) < (iii) < (ii) (D) (iii) < (iv) < (ii) < (i)

Ans : (D)

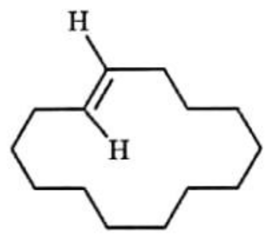
50. In the following reaction sequence, the product Y is

$$Br(CH_2)_{12} - C \equiv CH \xrightarrow{NaNH_2} X \xrightarrow[\text{catalyst}]{H_2,\text{Lindler}} Y$$

(A) 

(B) 

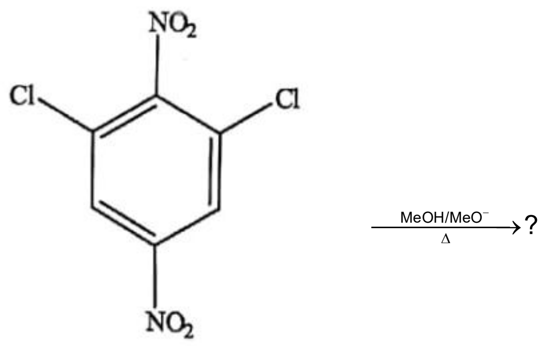
(C) $Br(CH_2)_{12}—CH=CH_2$

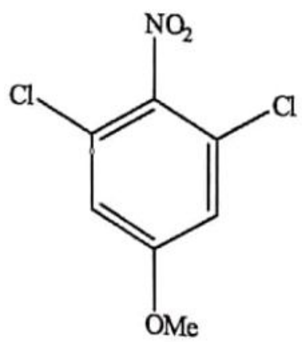
(D) 

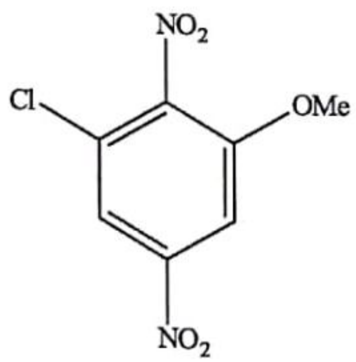
Ans : (B)

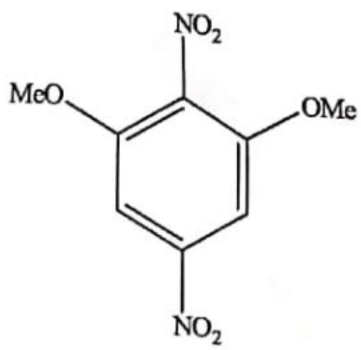
Hint : Cyclisation followed by cis hydrogenation in presence of Lindlar catalyst

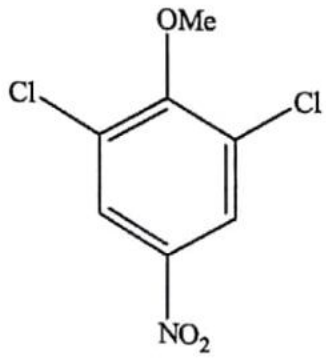
51. The major product in the following reaction is



(A)


(B)


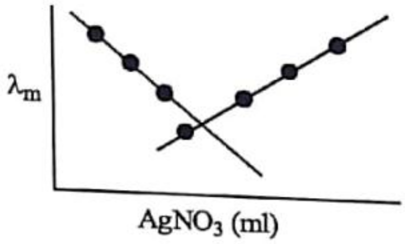
(C)


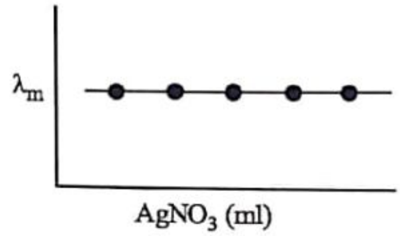
(D)


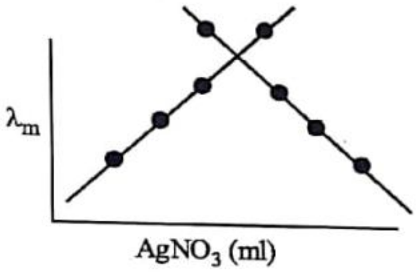
Ans : (D)

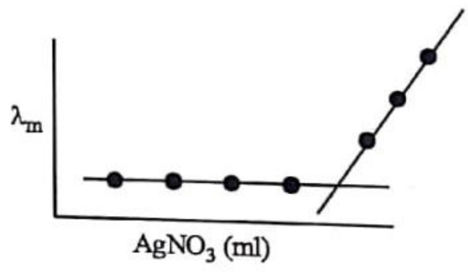
Hint : Carbanion leading to formation of compound given in option (D) is more stable. The alternative carbanion is less stable due to Steric inhibition of Resonance.

52. In a conductance experiment, aqueous $AgNO_3$ solution is added to aqueous KCl solution gradually and simultaneously the molar conductivity (λ_m) is measured. The correct plot of λ_m versus volume of $AgNO_3$ solution is

(A) 

(B) 

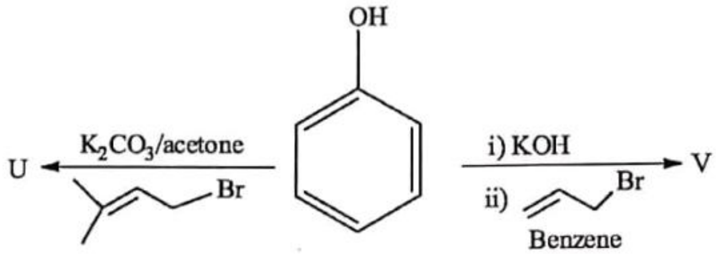
(C) 

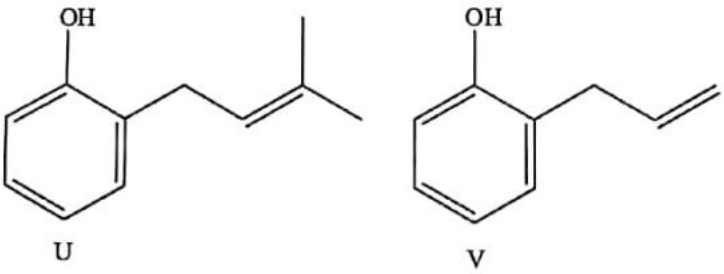
(D) 

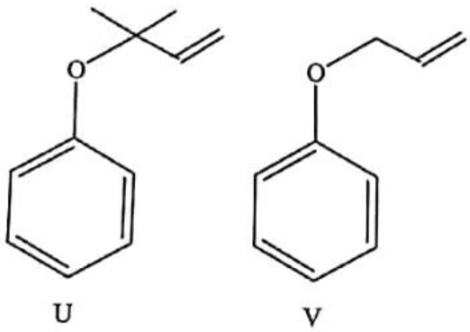
Ans : (D)

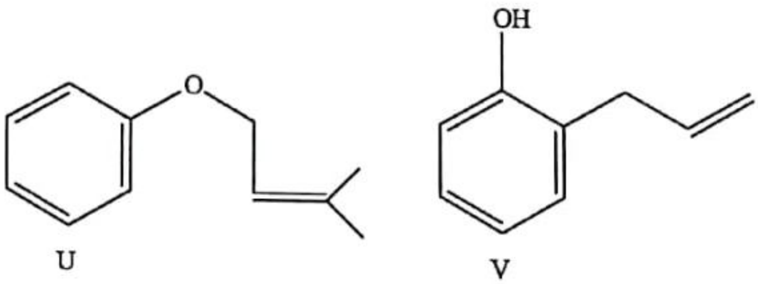
Hint : $AgNO_3 + KCl \rightarrow AgCl\downarrow + KNO_3$ ionic conductance of $Cl^- \simeq NO_3^-$

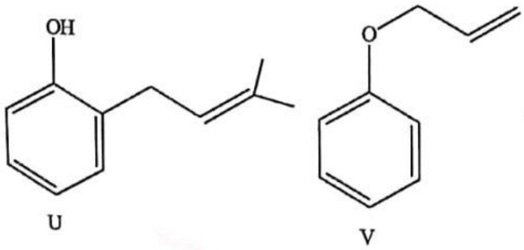
53. The major products U and V in the following reaction are



(A)


(B)


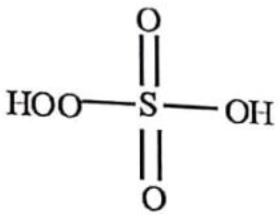
(C)


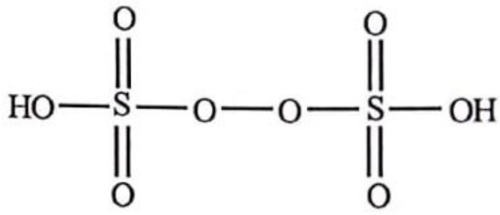
(D)


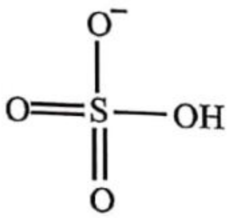
Ans : (C)

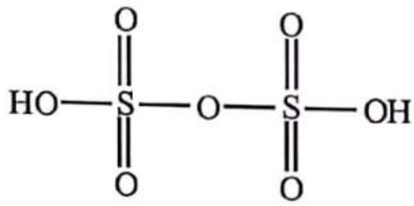
Hint : In acetone medium, K^+ ion is solvated hence phenoxide gives O-alkylation. In benzene medium, potasium phenoxide doesn't ionise, hence c-alkylation takes place.

54. Which of the following is the structure of pyrosulphuric acid?

(A) 

(B) 

(C) 

(D) 

Ans : (D)

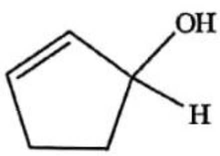
55. The mass of an electron is 9.1×10^{-31} kg. If its K.E. is 3.0×10^{-25}J, its wavelength is (approximately)

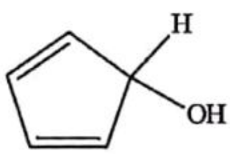
(A) 250nm (B) 990nm (C) 400nm (D) 850nm

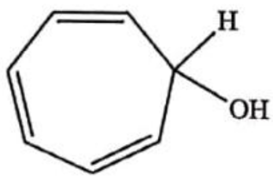
Ans : (D)

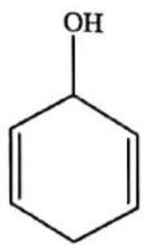
Hint : $\lambda = \frac{h}{\sqrt{2mKE}}$

56. Which one of the following does not lose water even in conc. H_2SO_4?


1


2


3


4

(A) 1 (B) 2 (C) 3 (D) 4

Ans : (B)

Hint : on losing H_2O, 2 gives anti aromatic ion

57. The van't Hoff Factor (i) for a dilute aqueous solution of Na_2SO_4 is

(A) $1-\alpha$ (B) $1-2\alpha$ (C) $1+\alpha$ (D) $1+2\alpha$

Ans : (D)

Hint : $\alpha = \frac{i-1}{n-1}$

58. In which of the following species, sp^3d^2 hybridisation is not associated?

(A) XeF_6 (B) BrF_6^+ (C) IF_5 (D) XeF_4

Ans : (A)

Hint : XeF_6 is sp^3d^3 hybridised

59. Which one of the following cations gives a chocolate brown precipitate upon addition of aqueous solution of $K_4[Fe(CN)_6]$?

(A) Fe^{3+} (B) Cu^{2+} (C) Zn^{2+} (D) Ca^{2+}

Ans : (B)

60. Borazole is prepared by heating the product isolated by reacting

(A) boron with dinitrogen (B) diborane with ammonium nitrate

(C) diborane with ammonia (D) boron with ammonia

Ans : (C)

61. Three engines A, B and C take steam at 130°C and reject it at 20°C, 40°C and 50°C respectively. The most efficient engine will be

(A) A

(B) B

(C) C

(D) All the three engines will be equally efficient

Ans : (A)

Hint : Lower the temperature of the sink, higher is the efficiency

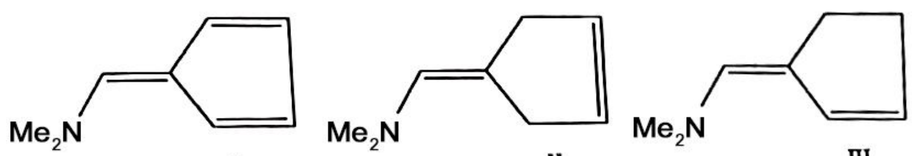
62. A buffer solution contains 100 ml of 0.01 (M) CH_3COOH and 200 ml of 0.02 (M) CH_3COONa. 700 ml of water is added subsequently to the buffer solution. The pH before and after dilution are [given, pK_a = 4.74; log 2 = 0.301]

(A) 5.04, 5.04 (B) 5.04, 0.504 (C) 5.04, 1.54 (D) 5.34, 5.34

Ans : (D)

Hint : $pH = pK_a + \log\frac{[Salt]}{[Acid]}$, On dilution, pH remains same

63. The increasing order of basicity of the following compounds is



I II III

(A) I < III < II (B) III < I < II (C) II < I < III (D) II < III < I

Ans : (A)

Hint : In I, delocalisation of lone pair of N is maximum due to formation of cyclopentadienyl anion. So its basicity is minimum.

64. A compound (X) when treated with $CuSO_4$ solution yields a brown precipitate. On adding hypo solution the precipitate turns white. The compound (X) is

(A) KBr (B) K_2CrO_3 (C) KI (D) K_3PO_4

Ans : (C)

Hint : $KI \xrightarrow{CuSO_4} I_2(\text{brown}) \xrightarrow{Na_2S_2O_3} \text{white precipitate}$

65. Among N_2O, ClF_2^-, SO_2 and I_3^+, the species having the linear structures are

(A) N_2O, ClF_2^- (B) ClF_2^-, I_3^+ (C) I_3^+, SO_2 (D) N_2O, SO_2

Ans : (A)

66. The van der Waal's equation : $\left(P+\frac{a}{4V^2}\right)\left(V-\frac{b}{2}\right)=\frac{RT}{2}$ is valid for

(A) 1 mole of an ideal gas (B) 2 moles of a real gas (C) $\frac{1}{2}$ mole of an ideal gas (D) $\frac{1}{2}$ mole of a real gas

Ans : (D)

Hint : $\left(P+a\frac{n^2}{V^2}\right)(V-nb)=nRT$

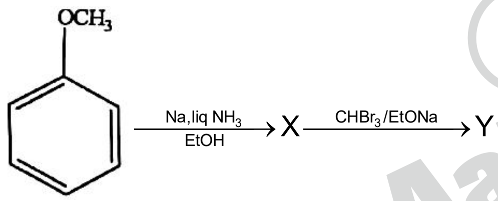
67. Rank the following anions in order of decreasing nucleophilicity in a polar protic solvent (most $\rightarrow$ least nucleophilic)

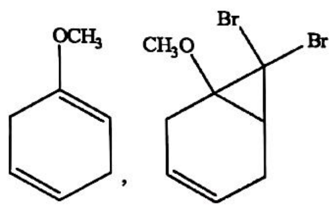
$CH_3CH_2CH_2{-}O^-$ (1) $CH_3CH_2CH_2{-}S^-$ (2) $CH_3CH_2{-}C(=O){-}O^-$ (3)

(A) 3 > 2 > 1 (B) 2 > 3 > 1 (C) 1 > 3 > 2 (D) 2 > 1 > 3

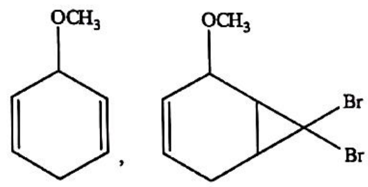
Ans : (D)

68. The products X and Y in the following reaction sequence are

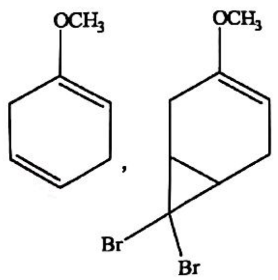


(A)


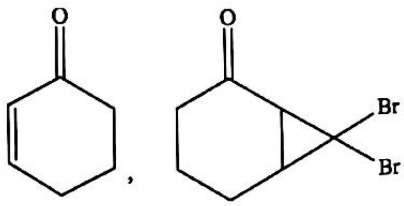
X Y

(B)


X Y

(C)


X Y

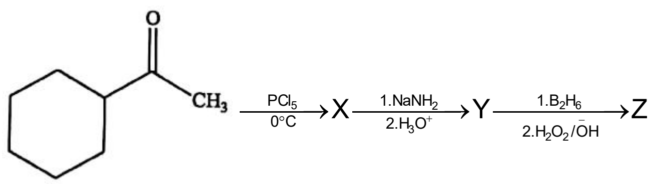
(D)


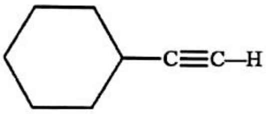
X Y

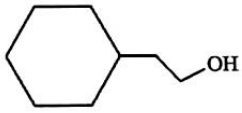
Ans : (A)

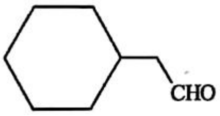
Hint : Birch reduction, followed by carbene attack at more e^-- rich site

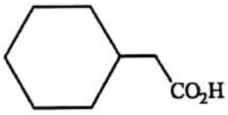
69. In the following sequence of reactions, what is the end product 'Z'?



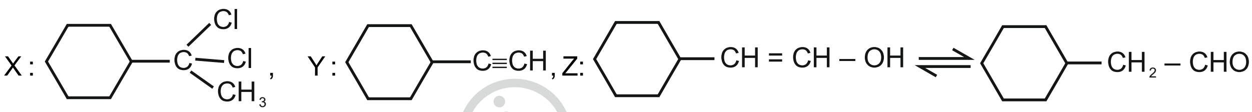
(A) 

(B) 

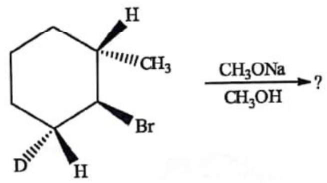
(C) 

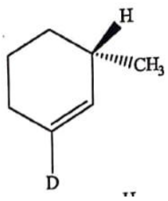
(D) 

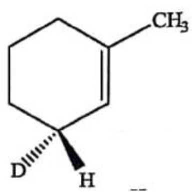
Ans : (C)

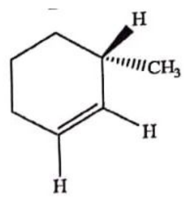
Hint : 

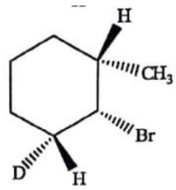
70. Indicate the major product of the following reaction :



(A) 

(B) 

(C) 

(D) 

Ans : (C)

Hint : E2 elimination

Category 2 (Q 71 to Q 75)

(Carry 2 marks each. Only one option is correct. Negative marks :– ½)

71. $RO–CH_2–C≡CH \xrightarrow{X} \xrightarrow{Y} RO–CH_2–C≡C–CH_2CH_2–Br$

To carry out the above conversion X and Y are respectively

(A) $NaNH_2$, $Br–CH_2–CH_2–Cl$

(B) $NaNH_2$, $I–CH_2–CH_2–Br$

(C) $NaNH_2$, $F–CH_2–CH_2–Br$

(D) $NaNH_2$, $CH_2 = CH–Br$

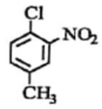
Ans : (B)

Hint : First step is acid-base reaction; second step is nucleophilic substitution

72. For the metal complex $[Co(NH_3)_5SO_4]Br$, coordination number, oxidation number, number of d-electrons and number of unpaired d-electrons are respectively

(A) 6, 3, 6, 0 (B) 7, 2, 6, 2 (C) 6, 2, 6, 0 (D) 6, 2, 7, 0

Ans : (A)

73. The IUPAC name of  is

(A) 1-Chloro-2-nitro-4-methylbenzene (B) 1-Chloro-4-methyl-2-nitrobenzene

(C) 2-Chloro-1-nitro-5-methylbenzene (D) m-Nitro-p-chlorotoluene

Ans : (B)

74. A 5.0 cm^3 solution of H_2O_2 liberates 1.27 g of iodine from an acidified KI solution. The percentage strength of H_2O_2 is close to

(A) 11.2 (B) 5.8 (C) 1.9 (D) 3.4

Ans : (D)

Hint: No. of moles of I_2 : $\frac{1.27}{254} = 0.005$. $H_2O_2 + 2KI \rightarrow 2KOH + I_2$

No. of moles of H_2O_2 = 0.005 (in 5 mL). Hence 3.4% (w/v)

75. An organic compound undergoes first order decomposition. The time taken for its decomposition to $\frac{1}{8}$ th and $\frac{1}{10}$ th of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively.The value of $\left[\frac{t_{1/8}}{t_{1/10}}\right]$ is [Given $\log_{10}2 = 0.3$]

(A) 0.9 (B) 0.6 (C) 0.3 (D) 0.5

Ans : (A)

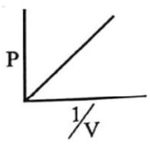
Hint : $K = \frac{2.303}{t_{\frac{1}{8}}}\log\frac{a_0}{a_0/8}$

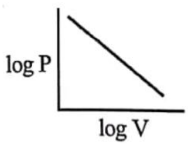
$$K = \frac{2.303}{t_{\frac{1}{10}}}\log\frac{a_0}{a_0/10}$$

Category 3 (Q76 to Q80)

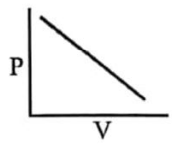
(Carry 2 marks each. One or more options are correct. No negative marks)

76. Which of the following plot(s) is/are correct representation(s) of Boyle's Law?

(A) 

(B) 

(C) 

(D) 

Ans : (A, B, C)

Hint : PV = Constant

77. Which of the following have tetrahedral structure ?

(A) $[Ni(CN)_4]^{2-}$ (B) $[Ni(CO)_4]$ (C) $[NiCl_4]^{2-}$ (D) CrO_4^{2-}

Ans : (B, C, D)

Hint : CN^- is a strong field ligand - hence $Ni(CN)_4^{2-}$ is square planar

78. 1 mole of an ideal gas undergoes the following processes :

Process A → Isothermal expansion at 400K from volume V_1 to volume V_2, such that $V_2 = 4V_1$

Process B → Adiabatic expansion from volume V_1 to volume V_2, such that $V_2 = 4V_1$

(A) Work done by gas in Process A is greater than in Process B

(B) Final temperature in Process B is less than 400K

(C) Change in internal energy is 0 in Process A but non-zero in Process B

(D) Heat absorbed by the gas is positive in Process A but zero in Process B

Ans : (A, B, C, D)

Hint : Adiabatic expansion leads to cooling, magnitude of $W_{iso} > W_{adi}$

79. Which of the following statement(s) is/are correct?

(A) Strach is composed of repeating α–D glucose units

(B) Nylon-6 is an addition polymer whereas nylon–6,6 is a condensation polymer

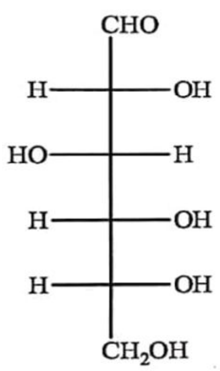
(C) Isoprene is the monomer unit of natural rubber

(D) Bakelite is obtained from reaction between phenol and acetaldehyde

Ans : (A, C)

Hint : Fact

80. Which of the following statement(s) is/are correct about the given compound ?



(A) It exhibits ring-chain tautomerism

(B) It forms osazone with phenylthydrazine

(C) It gives eight (8) stereoisomers

(D) It responds to Tollen's reagent

Ans : (A, B, D)

❑ ❑ ❑