DATE : 27/05/2019

Aakash

Medical | IIT-JEE | Foundations

(Divisions of Aakash Educational Services Limited)

Regd. Office : Aakash Tower, 8, Pusa Road, New Delhi-110005

Ph.: 011-47623456 Fax : 011-47623472

Time : 3 hrs. | Max. Marks: 186

Questions & Answers

for

JEE (Advanced)-2019

PAPER - 2

PART-I : PHYSICS

SECTION - 1 (Maximum Marks : 32)

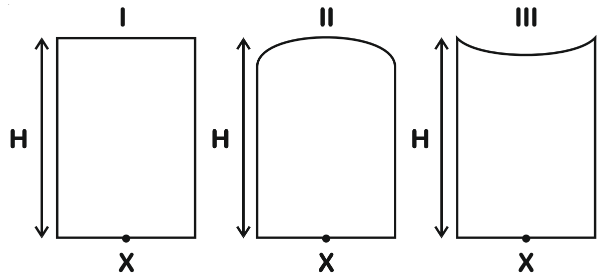
- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

- For example : in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e., the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 mark.

1. Three glass cylinders of equal height H = 30 cm and same refractive index n = 1.5 are placed on a horizontal surface as shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3 m). If H_1, H_2, and H_3 are the apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are:

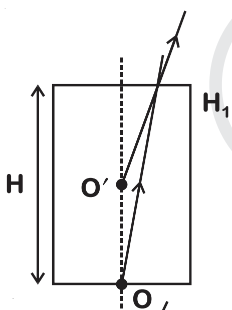


(A) $0.8 \text{ cm} < (H_2 - H_1) < 0.9 \text{ cm}$ (B) $H_2 > H_3$

(C) $H_3 > H_1$ (D) $H_2 > H_1$

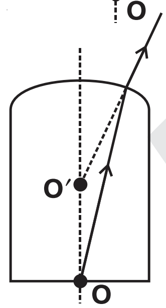
Answer (B, D)

Sol. Case I



$$H_1 = \frac{H}{\mu} = \frac{30 \times 2}{3} = 20 \text{ cm (below)}$$

Case II

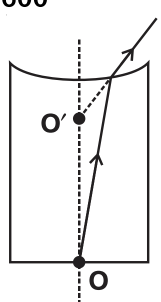


$$\frac{1}{V} + \frac{3}{2H} = \frac{1-1.5}{-R}$$

$$\Rightarrow \frac{1}{V} + \frac{1}{20} = \frac{1}{2 \times 300} \quad \Rightarrow \frac{1}{V} = \frac{1}{600} - \frac{1}{20}$$

$$\Rightarrow \frac{1}{V} = \frac{-29}{600} \qquad \therefore\ H_2 = \frac{600}{29} = 20.68 \text{ cm (below)}$$

Case III



$$\frac{1}{V} + \frac{3}{2H} = \frac{-1}{2 \times 300} \quad \Rightarrow \frac{1}{V} = \frac{-1}{600} - \frac{1}{20}$$

$$\Rightarrow \frac{1}{V} = \frac{-31}{600} \quad \Rightarrow H_3 = \frac{600}{31} = 19.35 \text{ cm}$$

$H_2 - H_1 \simeq 20.68 - 20 \approx 0.68$

$H_2 > H_1$ and $H_2 > H_3$

But $H_1 > H_3$

2. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical ?

[g is the acceleration due to gravity]

(A) The angular acceleration of the rod will be $\frac{2g}{L}$

(B) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

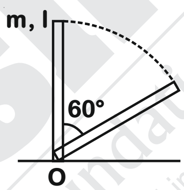
(C) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(D) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

Answer (B, C, D)

Sol. By applying conservation of energy (As the friction is acting at the point of no slipping)

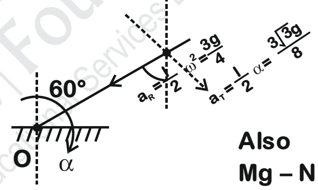
$$\frac{Mgl}{2}(1-\cos 60^\circ)=\frac{1}{2}I_0\omega^2$$



$$\therefore\ \omega=\sqrt{\frac{3g}{2l}}\quad \left(I_0=\frac{Ml^2}{3}\right)$$

Also $\tau_0=Mg\frac{l}{2}\sin 60^\circ=I_0\alpha$

$$\therefore\ \alpha=\frac{3\sqrt{3}g}{4l}$$



Also

$Mg-N=Ma_y$

$N=\frac{Mg}{16}$

For CM of the rod

$$a_y=a_T\sin 60^\circ+a_R\cos 60^\circ$$

$$=\frac{l}{2}\alpha\sin 60^\circ+\frac{\omega^2 l}{2}\cos 60^\circ$$

$$=\frac{\sqrt{3}}{4}\left[\frac{3\sqrt{3}g}{4}\right]+\frac{3g}{2l}\times\frac{l}{2}\times\frac{1}{2}$$

$$=\left(\frac{9}{16}+\frac{3}{8}\right)g=\frac{15g}{16}$$

3. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state n = 1 to the state n = 4. Immediately after that the electron jumps to n = m state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e, respectively. If $\lambda_a/\lambda_e=\frac{1}{5}$, which of the option(s) is/are correct?

[Use hc = 1242 eV nm; 1 nm = 10^{-9} m, h and c are Planck's constant and speed of light, respectively]

(A) λ_e = 418 nm

(B) $\Delta p_a/\Delta p_e=\frac{1}{2}$

(C) The ratio of kinetic energy of the electron in the state n = m to the state n = 1 is $\frac{1}{4}$

(D) m = 2

Answer (C, D)

Sol. $\frac{hc}{\lambda_a} = (E_4 - E_1)$ and $\frac{hc}{\lambda_e} = (E_4 - E_m)$

$\Delta p_a = \frac{(E_4 - E_1)}{c}$ and $\Delta p_e = \frac{(E_4 - E_m)}{c}$

$\Rightarrow \frac{\lambda_a}{\lambda_e} = \frac{(E_4 - E_m)}{E_4 - E_1} = \frac{1}{5} = \frac{\frac{1}{m^2} - \frac{1}{16}}{\frac{15}{16}}$

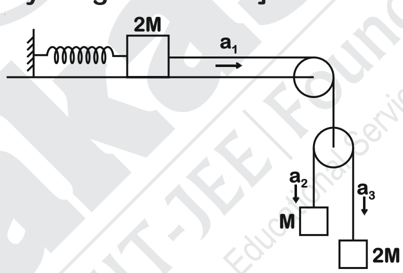
$\Rightarrow \frac{15}{16 \times 5} = \frac{1}{m^2} - \frac{1}{16} \Rightarrow \frac{1}{m^2} = \frac{1}{4} \Rightarrow m = 2$

$\frac{\Delta p_a}{\Delta p_e} = \frac{(E_4 - E_1)}{(E_4 - E_m)} = \frac{15 \times 16}{16 \times 3} = 5$

$\frac{hc}{\lambda_e} = (13.6\text{ eV}) \times \frac{3}{16} \Rightarrow \frac{1242 \times 16}{3 \times 13.6}$ nm = $\lambda_e \cong 487$ nm

4. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in the figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct?

[g is the acceleration due to gravity. Neglect friction]



(A) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

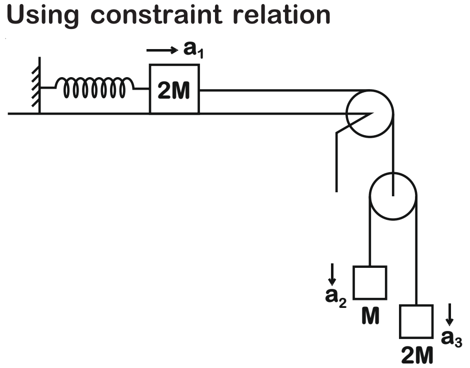
(B) $x_0 = \frac{4\,Mg}{k}$

(C) $a_2 - a_1 = a_1 - a_3$

(D) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

Answer (C)

Sol. Using constraint relation



$2a_1 = a_2 + a_3$

$a_1 - a_3 = a_2 - a_1$

Also

$2mg - T = 2ma_3$ ...(1)

$mg - T = ma_2$ ...(2)

and $2T - kx = 2ma_1$ ...(3)

By solving the above equation $T = \frac{4mg}{7} + \frac{2kx}{7}$ and $a_1 = \frac{4g}{7} - \frac{3kx}{14m}$

For $x = \frac{x_0}{4} = \frac{4mg}{3k}$

$a_1 = \frac{4g}{7} - \frac{2g}{7} = \frac{2g}{7}$

for the oscillation of mass '2m' (at mean position)

$a_1 = 0 \Rightarrow x = \frac{8mg}{3k}$ (Amplitude)

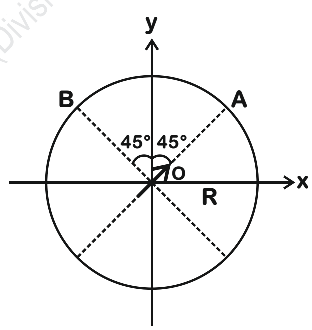
$\therefore \quad x_0 = 2A = \frac{16mg}{3k}$ also $\omega = \sqrt{\frac{3k}{14m}}$

Also $v_{\text{at } x=\frac{x_0}{2}\text{(mean)}} = A\omega$

$$= \frac{x_0}{2}\sqrt{\frac{3k}{14m}} = \frac{8mg}{3k}\sqrt{\frac{3k}{14m}}$$

5. An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are:

(ϵ_0 is permittivity of free space. R >> dipole size)



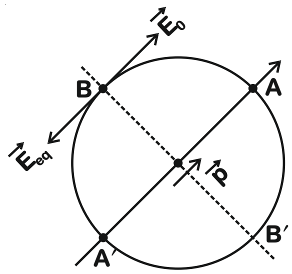
(A) Total electric field at point B is $\vec{E}_B = 0$

(B) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$

(C) $R = \left(\frac{p_0}{4\pi\epsilon_0 E_0}\right)^{1/3}$

(D) The magnitude of total electric field on any two points of the circle will be same.

Answer (A, C)

Sol. 

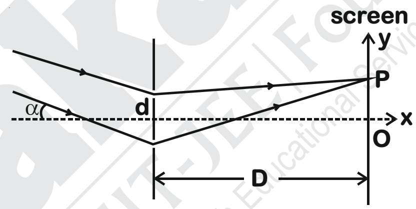
$E_0 \rightarrow$ external field in the direction of $\vec{p} = \frac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$ for an equipotential circle of radius r point B is the point (equitorial) of the dipole.

So $\frac{kp_0}{R^3} = E_0$

$\therefore \quad R = \left(\frac{kp_0}{E_0}\right)^{1/3} = \left(\frac{p_0}{4\pi\epsilon_0 E_0}\right)^{1/3}$ & $\vec{E}_A = \frac{2k\vec{p}}{R^3} + \vec{E}_0 = 3\vec{E}_0$

Also $E_B = 0$

6. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1 m. A parallel beam of light of wavelength 600 nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct?



(A) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P.

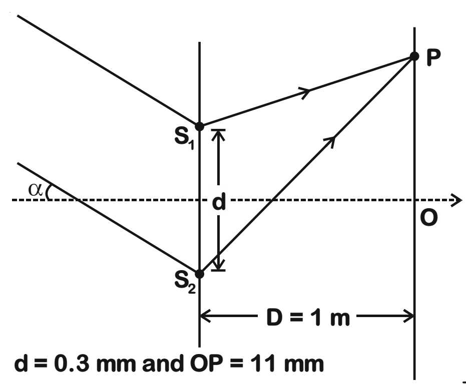
(B) Fringe spacing depends on α.

(C) For α = 0, there will be constructive interference at point P.

(D) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.

Answer (A)

Sol. Total geometric path difference for point P is



$$\Delta x = d\sin\alpha + \frac{yd}{D}$$

For option (A)

$\alpha = \frac{0.36}{\pi}$ degree, then Δx = 3900 nm

or $(2n-1)\frac{\lambda}{2} = \Delta x$ (for destructive interference)

$\therefore \quad n = 7$

For option (D)

If $\alpha = \frac{0.36}{\pi}$ degree

Then $\Delta x_0 = d\alpha = 600$ nm

or $\Delta x_0 = n\lambda$ (for n = 1)

(Constructive interference)

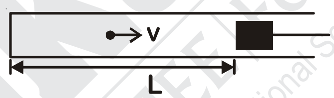
For option (C)

$\Delta x_0 = 0$ and $\frac{yd}{D} = 3300\,\text{nm}$

as ($\alpha = 0$) (destructive interference at P)

Fringe width in all the above case remain unchanged.

7. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L}v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct?



(A) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

(B) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

(C) After each collision with the piston, the particle speed increases by 2V

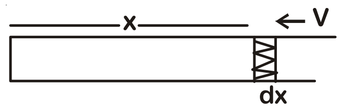
(D) The rate at which the particle strikes the piston $\frac{v}{L}$

Answer (B, C)

Sol. Initial velocity of particle is $v = v_0$

And distance of piston $x = L_0$

dx is (–ve)



Vdt = –dx(distance moved by piston)

$\therefore \quad dt = -\left(\frac{dx}{V}\right)$

Collision time for 'm' $\Rightarrow \quad dt' = \frac{2x}{V}$

No. of collision for sec $\Rightarrow$ $n=\frac{V}{2n}$ $\therefore$ Total number of collision is dt is $N=-\left(\frac{V}{2x}\right)\frac{dx}{V}$

Change in speed per collision is 2V

$\therefore$ Total change in speed for 'dx' shifting is $dv=\frac{-vdx}{2xV}\cdot 2V$

$\Rightarrow |dV|=\frac{Vdx}{x}$ At $x=L$, $|dV|=\frac{v_0 dL}{L}$

Now, $\frac{dV}{V}=-dx$ $\therefore$ $\ln\left(\frac{V'}{V_0}\right)=\ln 2$ $\Rightarrow$ $V'=2V_0$

$\therefore$ KE increases by 4 times.

8. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0, and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are, (Given $2^{1.2}=2.3$; $2^{3.2}=9.2$; R is gas constant)

(A) The work |W| done during the process is $13RT_0$

(B) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$

(C) Adiabatic constant of the gas mixture is 1.6

(D) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$

Answer (A, B, C)

Sol. $$\gamma_{mixture}=\frac{5\times\frac{5R}{2}+\frac{7R}{2}}{5\times\frac{3R}{2}+1\times\frac{5R}{2}}=\frac{8}{5}=1.6$$

For adiabatic process

PV^{γ} = const.

$$\therefore\quad P=P_0\left(\frac{V_0}{V}\right)^{\gamma}=(4)^{8/5}\cdot P_0=9.2\,P_0$$

$$\therefore\quad W=\frac{P_0V_0-9.2\,P_0\frac{V_0}{4}}{\gamma-1}=\frac{-13P_0V_0}{6}$$

As we know $P_0V_0=6RT_0$

$\therefore$ $W=-13RT_0$

Final temp after compression

$$T=T_0\left(\frac{PV}{P_0V_0}\right)=2.3\,T_0$$

Also, $$(C_V)_{mixture}=\frac{n_1C_{V_1}+n_2C_{V_2}}{n_1+n_2}=\frac{5R}{3}$$

$\therefore$ Average KE of gas $=U_f=nC_VT$

$$=6\times\frac{5R}{3}\times 2.3T_0$$

$=23RT_0$

SECTION - 2 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO decimal places**.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered.

Zero Marks : 0 In all other cases.

1. Suppose a ${}^{226}_{88}\text{Ra}$ nucleus at rest and in ground state undergoes α-decay to a ${}^{222}_{86}\text{Rn}$ nucleus in its excited state. The kinetic energy of the emitted α particle is found to be 4.44 MeV. ${}^{222}_{86}\text{Rn}$ nucleus then goes to its ground state by γ-decay. The energy of the emitted γ photon is _______ keV.

[Given: atomic mass of ${}^{226}_{88}\text{Ra}$ = 226.005 u, atomic mass of ${}^{222}_{86}\text{Rn}$ = 222.000 u, atomic mass of α particle = 4.000 u, 1 u = 931 MeV/c^2, c is speed of the light]

Answer (135.00)

Sol. Δm = [226.005 – 222 – 4]

= 0.005 amu

$Q = \Delta mc^2$

= 931.5 × 0.005 = 4.655 MeV

Since momentum is conserved, kinetic energy is in inverse ratio of masses.

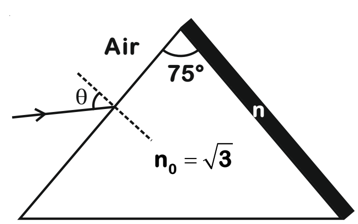
$K_T = 4.44 + K_{Rn}$

$K_{Rn} = \frac{4.44 \times 4}{222}$ = 0.08 MeV

γ-photon = 4.655 – 4.520

= 0.135 MeV = 135 keV

2. A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $n_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \leq 60°$. The value of n^2 is _______.

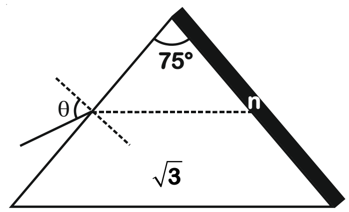


Answer (1.50)

Sol. For TIR at other face.

$\sin\theta'_c = \frac{n}{\sqrt{3}}$

Snell law at first surface



$\sin\theta = \sqrt{3}\sin(75 - \theta'_c)$

$\Rightarrow$ For $\theta = 60°$

$\therefore \quad \frac{\sqrt{3}}{2} = \sqrt{3}\sin(75 - \theta'_c)$

$\Rightarrow \quad \sin(75 - \theta'_c) = \frac{1}{2}$

$\therefore \quad \theta'_c = 45°$

$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{n}{\sqrt{3}}$

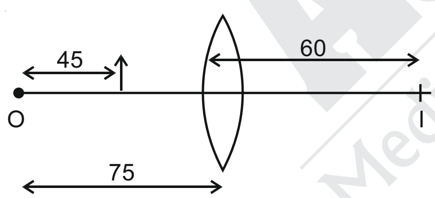
$\Rightarrow \quad n = \sqrt{\frac{3}{2}}$

$\therefore \quad n^2 = \frac{3}{2} = 1.5$

3. An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is _______.

Answer (1.38/1.39)

Sol.



$|u| = x_2 - x_1 = 30$ cm

$\Delta u = 0.25 + 0.25 = 0.50$

$|v| = 60$ cm

$\Delta v = 0.05$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$f = 20$ cm

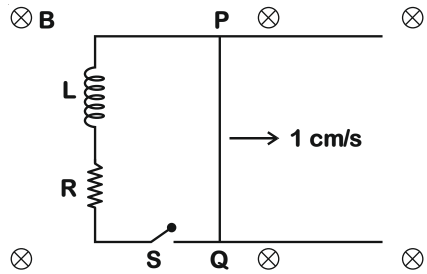
$\frac{dv}{v^2} + \frac{du}{u^2} = \frac{df}{f^2}$

$400 \times \frac{1}{2}\left[\frac{1}{3600} + \frac{1}{900}\right] = df$

% error = 1.38 or 1.39

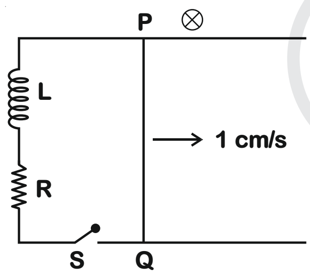
4. A 10 cm long perfectly conducting wire PQ is moving with a velocity 1 cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1 mH and a resistance R = 1 Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1 T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3}$ A, where the value of x is ______.

[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed. Given: $e^{-1} = 0.37$, where e is base of the natural logarithm]



Answer (0.63)

Sol.



B = 1T, ℓ = 10 cm, v = 1 cm/s

$$\varepsilon = vB\ell = \frac{1}{100} \times 1 \times \frac{1}{10} = 1 \times 10^{-3} \text{ volt}$$

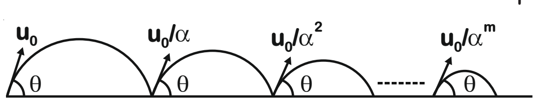
$$\tau = \frac{L}{R} = \frac{1 \times 10^{-3}}{1} = 10^{-3} \text{ sec}$$

$\therefore$ $i = I_0(1 - e^{-t/\tau})$

$$\Rightarrow \quad i = \frac{10^{-3}}{1}(1 - e^{-1})$$

$\Rightarrow$ $i = 10^{-3}(1 - 0.37) = 0.63$ mA

5. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is ________.



Answer (4.00)

Sol. $$< V > = \frac{\Sigma R}{\Sigma T}$$

$$\Sigma R = \frac{2}{g}u_0^2(\sin\theta\cos\theta)\left[1+\frac{1}{\alpha^2}+\frac{1}{\alpha^4}+...\right]$$

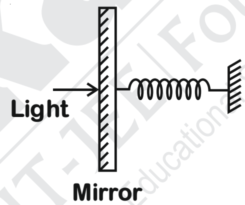
$$\Sigma T = \frac{2u_0}{g}\sin\theta\left[1+\frac{1}{\alpha}+\frac{1}{\alpha^2}+...\right]$$

$$0.8\,V_1 = \frac{u_0\cos\theta\left[\dfrac{1}{1-\dfrac{1}{\alpha^2}}\right]}{u_0\dfrac{1}{1-\dfrac{1}{\alpha}}}$$

α = 4.00

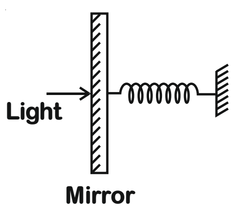
6. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24}\ m^{-2}$ with h as Planck's constant, N photons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1 μm. If the value of N is $x \times 10^{12}$, then the value of x is ______.

[Consider the spring as massless]



Answer (1.00)

Sol.



Momentum transferred on mirror

$$\Delta p = \frac{2Nh}{\lambda}$$

$$\text{Given } \Omega = \frac{h\times 10^{24}}{4\pi m}$$

So the speed, Mirror would acquire

$$V_0 = \frac{2Nh}{\lambda M}$$

Also as the system oscillate in SHM

$V_0 = A\Omega$ (A = 1 μm)

$$\frac{2Nh}{\lambda M} = A\Omega \Rightarrow \frac{2Nh}{\lambda M} = \frac{A \cdot h \times 10^{24}}{4\pi M}$$

$$\Rightarrow N = \frac{10^{18} \times 8\pi \times 10^{-6}}{4\pi \times 2} = 1 \times 10^{12}$$

$\therefore$ x = 1

SECTION - 3 (Maximum Marks : 12)

- This section contains TWO (02) List-Match sets.
- Each List-Match set has TWO (02) Multiple Choice Questions.
- Each List-Match set has two lists: List-I and List-II.
- List-I has Four entries (I), (II), (III) and (IV) and List-II has Six entries (P), (Q), (R), (S), (T) and (U).
- Four options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : –1 In all other cases.

1. Answer the following by appropriately matching the lists based on the information given in the paragraph

A musical instrument is made using four different metal strings. 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while List-II lists the magnitude of some quantity.

List-I	List-II
(I) String-1 (μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

(A) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ Q

(B) I $\rightarrow$ Q, II $\rightarrow$ P, III $\rightarrow$ R, IV $\rightarrow$ T

(C) I $\rightarrow$ Q, II $\rightarrow$ S, III $\rightarrow$ R, IV $\rightarrow$ P

(D) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ T, IV $\rightarrow$ S

Answer (A)

Sol. (1) $f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

(2) $f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu}}$

(3) $f_2 = \frac{1}{2L_0}\sqrt{\frac{T_0}{3\mu}}$

(4) $f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}}$

For all highest fundamental is when length is L_0

2. Answer the following by appropriately matching the lists based on the information given in the paragraph

A musical instrument is made using four different metal strings. 1, 2, 3 and 4 with mass per unit length μ, 2μ, and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while List-II lists the magnitude of some quantity.

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(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

The length of the strings 1, 2, 3 and 4 are kept fixed at $L_0, \frac{3L_0}{2}, \frac{5L_0}{4}$ and $\frac{7L_0}{4}$ respectively. Strings 1, 2, 3 and 4 are vibrated at their 1st, 3rd, 5th, and 14th harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be,

(A) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ R, IV $\rightarrow$ T

(B) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ T, IV $\rightarrow$ U

(C) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ T, IV $\rightarrow$ U

(D) I $\rightarrow$ T, II $\rightarrow$ Q, III $\rightarrow$ R, IV $\rightarrow$ U

Answer (B)

Sol. (1) $f_0 = \frac{1}{2L_0} \times \sqrt{\frac{T_1}{\mu}} = \frac{1}{2L}\sqrt{\frac{T_0}{\mu}}$

(2) $f_0 = \frac{3 \times 2}{2 \times 3L_0}\sqrt{\frac{T_2}{2\mu}} \Rightarrow T_2 = \frac{T_0}{2}$

(3) $f_0 = \frac{5 \times 2}{5L_0}\sqrt{\frac{T_3}{3\mu}} \Rightarrow T_3 = \frac{3T_0}{16}$

(4) $f_0 = \frac{14 \times 4}{2 \times 7L_0}\sqrt{\frac{T_4}{4\mu}} \Rightarrow T_4 = \frac{T_0}{16}$

I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ T, IV $\rightarrow$ U

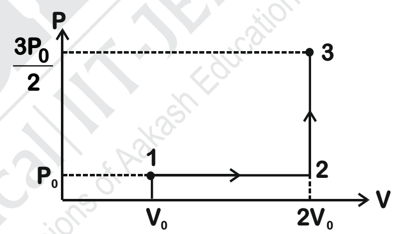
3. Answer the following by appropriately matching the lists based on the information given in the paragraph

In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I	List-II
(I) Work done by the system in process $1 \to 2 \to 3$	(P) $\frac{1}{3}RT_0\ln 2$
(II) Change in internal energy in process $1 \to 2 \to 3$	(Q) $\frac{1}{3}RT_0$
(III) Heat absorbed by the system in process $1 \to 2 \to 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \to 2$	(S) $\frac{4}{3}RT_0$
	(T) $\frac{1}{3}RT_0(3+\ln 2)$
	(U) $\frac{5}{6}RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in figure in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(A) $I \to Q, II \to R, III \to S, IV \to U$

(B) $I \to Q, II \to S, III \to R, IV \to U$

(C) $I \to S, II \to R, III \to Q, IV \to T$

(D) $I \to Q, II \to R, III \to P, IV \to U$

Answer (A)

Sol. (I) $W_{1-2-3} = P_0V_0 = \frac{1}{3}RT_0$

(II) $\Delta U = \frac{3}{2}\times\left[\frac{3P_0}{2}\times 2V_0 - P_0V_0\right] = RT_0$

(III) $\Delta Q = \Delta U + W$

$= \frac{4}{3}RT_0$

(IV) $\Delta Q = \frac{1}{3}RT_0 + \frac{3}{2}\times(2P_0V_0 - P_0V_0)$

$= \frac{5}{6}RT_0$

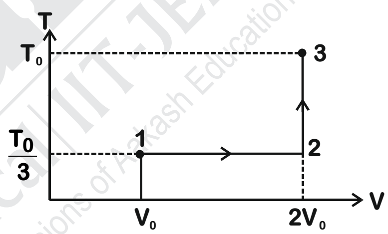
4. Answer the following by appropriately matching the lists based on the information given in the paragraph

In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I		List-II	
(I)	Work done by the system in process $1 \to 2 \to 3$	(P)	$\frac{1}{3}RT_0\ln 2$
(II)	Change in internal energy in process $1 \to 2 \to 3$	(Q)	$\frac{1}{3}RT_0$
(III)	Heat absorbed by the system in process $1 \to 2 \to 3$	(R)	RT_0
(IV)	Heat absorbed by the system in process $1 \to 2$	(S)	$\frac{4}{3}RT_0$
		(T)	$\frac{1}{3}RT_0(3+\ln 2)$
		(U)	$\frac{5}{6}RT_0$

If the process on one mole of monatomic ideal gas is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(A) $I \to P$, $II \to T$, $III \to Q$, $IV \to T$

(B) $I \to P$, $II \to R$, $III \to T$, $IV \to P$

(C) $I \to S$, $II \to T$, $III \to Q$, $IV \to U$

(D) $I \to P$, $II \to R$, $III \to T$, $IV \to S$

Answer (B)

Sol. (I) $W_{1-2-3} = \frac{RT_0}{3}\ln\left(\frac{V_2}{V_1}\right) + 0 = \frac{RT_0}{3}\ln 2$

(II) $\Delta U = \frac{3}{2}R\left(T_0 - \frac{T_0}{3}\right) = RT_0$

(III) $\Delta Q = \frac{RT_0}{3}\ln 2 + RT_0$

(IV) $\Delta Q = \frac{RT_0}{3}\ln(2)$

$I \to P$, $II \to R$, $III \to T$, $IV \to P$

PART-II : CHEMISTRY

SECTION - 1 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

- For example : in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 mark;

choosing ONLY (B) will get +1 mark;

choosing ONLY (D) will get +1 mark;

choosing no option (i.e., the question is unanswered) will get 0 marks; and

choosing any other combination of options will get –1 mark.

1. The ground state energy of hydrogen atom is –13.6 eV. Consider an electronic state ψ of He^+ whose energy, azimuthal quantum number and magnetic quantum number are –3.4 eV, 2 and 0, respectively. Which of the following statement(s) is(are) true for the state ψ?

(A) It is a 4d state

(B) The nuclear charge experienced by the electron in this state is less than 2e, where e is the magnitude of the electronic charge

(C) It has 2 angular nodes

(D) It has 3 radial nodes

Answer (A, C)

Sol. $E_n = -13.6 \times \frac{Z^2}{n^2} eV$

$\therefore \quad 3.4\ eV = -13.6 \times \frac{2^2}{n^2}$

$\Rightarrow$ n = 4

n = 4, l = 2, m = 0 belongs to 4d orbital

no. of angular nodes in an orbital = value of l

no. of radial node in 4d = 1

Since it is a unielectronic species, there will be no shielding and the nuclear charge felt will be 2e.

2. Consider the following reactions (unbalanced)

$Zn + \text{hot conc. } H_2SO_4 \longrightarrow G + R + X$

$Zn + \text{conc. } NaOH \longrightarrow T + Q$

$G + H_2S + NH_4OH \longrightarrow Z \text{ (a precipitate)} + X + Y$

Choose the correct option(s)

(A) Z is dirty white in colour

(B) Bond order of Q is 1 in its ground state

(C) R is a V-shaped molecule

(D) The oxidation state of Zn in T is + 1

Answer (A, B, C)

Sol. $\underset{}{Zn} + \underset{\text{conc.}}{H_2SO_4} \longrightarrow \underset{(G)}{ZnSO_4} + \underset{(R)}{SO_2} + \underset{(X)}{H_2O}$

$Zn + \underset{\text{conc.}}{NaOH} \longrightarrow \underset{(T)}{Na_2[ZnO_2]} + \underset{(Q)}{H_2}$

$\underset{(G)}{ZnSO_4} + \underbrace{H_2S + NH_4OH}_{\text{group-IV reagent}} \longrightarrow \underset{(Z)}{ZnS\downarrow} + \underset{(Y)}{(NH_4)_2SO_4} + \underset{(X)}{H_2O}$

SO_2 (R) is V-shaped.

ZnS (Z) is dirty white gelatinous ppt.

Bond order in H_2 (Q) is 1.

3. The cyanide process of gold extraction involves leaching out gold from its ore with CN^- in the presence of Q in water to form R. Subsequently, R is treated with T to obtain Au and Z. Choose the correct option(s)

(A) Q is O_2 (B) R is $[Au(CN)_4]^-$ (C) Z is $[Zn(CN)_4]^{2-}$ (D) T is Zn

Answer (A, C, D)

Sol. Extraction of gold is done by leaching with dil. solution of NaCN in the presence of air (O_2).

$$Au + NaCN + \underset{(Q)}{O_2} \longrightarrow \underset{(R)}{[Au(CN)_2]^-} + OH^-$$

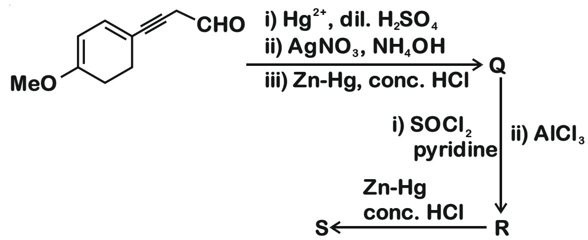
$$\underset{(R)}{[Au(CN)_2]^-} + \underset{(T)}{Zn} \longrightarrow \underset{(Z)}{[Zn(CN)_4]^{2-}} + \underset{(S)}{Au}$$

R is $[Au(CN)_2]^-$.

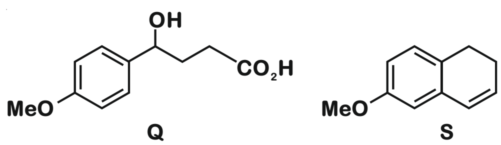
Q is O_2.

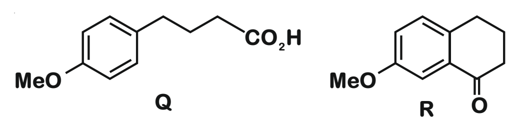
Z is $[Zn(CN)_4]^{2-}$.

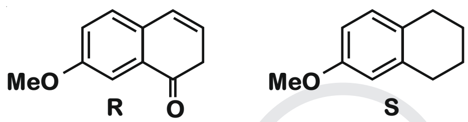
4. Choose the correct option(s) for the following reaction sequence

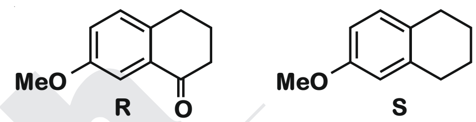


Consider Q, R and S as major products

(A) 

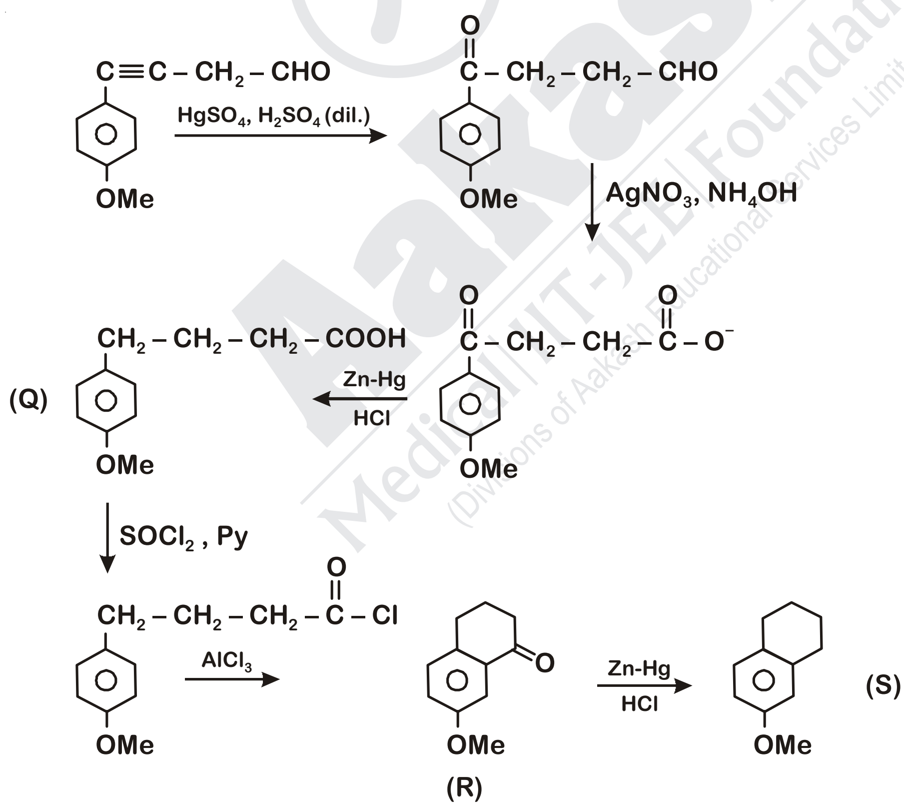
(B) 

(C) 

(D) 

Answer (B, D)

Sol.



5. Choose the correct option(s) from the following

(A) Nylon-6 has amide linkages

(B) Teflon is prepared by heating tetrafluoroethene in presence of a persulphate catalyst at high pressure

(C) Cellulose has only α-D-glucose units that are joined by glycosidic linkages

(D) Natural rubber is polyisoprene containing trans alkene units

Answer (A, B)

Sol. (A) Nylon-6 is a polyamide formed by heating caprolactam with water at 533-543 K.

(B) Teflon is polytetrafluoroethene which is an addition polymer catalysed by persulphate.

(C) Cellulose consists of β-D-Glucose units which are joined by C_1 — C_4 glycosidic linkage.

(D) Natural rubber is cis-polyisoprene.

6. With reference to aqua regia, choose the correct option(s).

(A) The yellow colour of aqua regia is due to the presence of NOCl and Cl_2

(B) Aqua regia is prepared by mixing conc. HCl and conc. HNO_3 in 3 : 1 (v/v) ratio

(C) Reaction of gold with aqua regia produces an anion having Au in +3 oxidation state

(D) Reaction of gold with aqua regia produces NO_2 in the absence of air

Answer (A, B, C)

Sol. (A) $HNO_3 + 3HCl \longrightarrow NOCl + Cl_2 + 2H_2O$

The nitrosyl chloride and chlorine impart yellow color to aqua regia.

(B) Aqua regia is a 3 : 1 mixture of conc. HCl and conc. HNO_3.

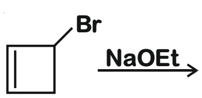
(C) Gold dissolves in aqua regia to form $[AuCl_4]^-$.

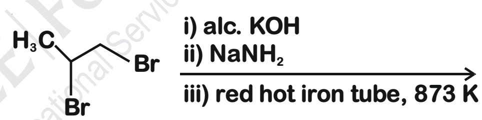
$$Au + 4H^+ + NO_3^- + 4Cl^- \longrightarrow [AuCl_4]^- + NO + 2H_2O$$

(D) Air oxidises NO to NO_2.

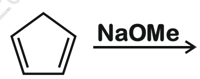
Hence in absence of air, possibility of formation of NO_2 is less.

7. Choose the correct option(s) that give(s) an aromatic compound as the major product

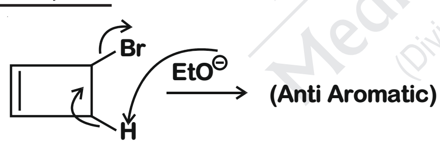
(A) 

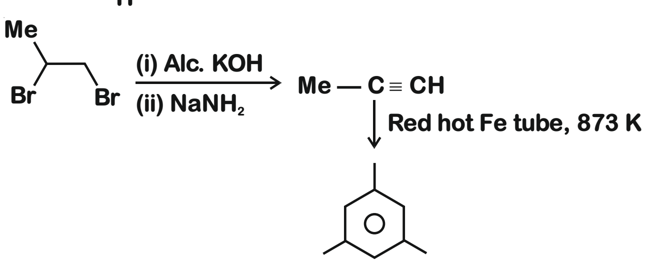
(B) 

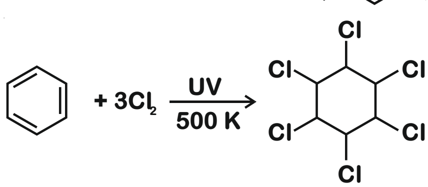
(C) 

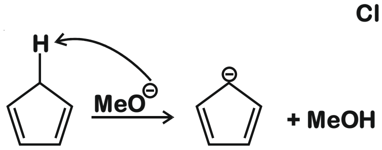
(D) 

Answer (B, D)

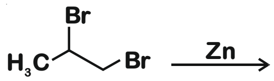
Sol. (A) 

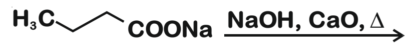
(B) 

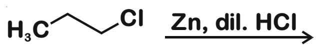
(C) 

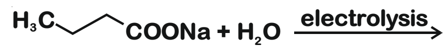
(D) 

8. Which of the following reactions produce(s) propane as a major product?

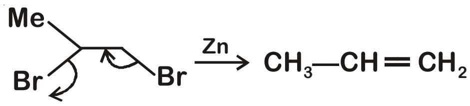
(A) 

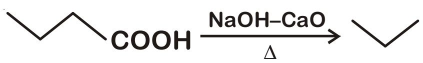
(B) 

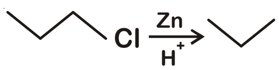
(C) 

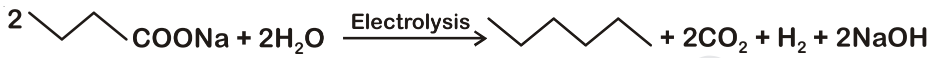
(D) 

Answer (B, C)

Sol. (A) 

(B) 

(C) 

(D) 

SECTION - 2 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

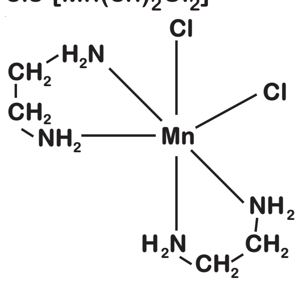
Full Marks	:	+3	If ONLY the correct numerical value is entered.
Zero Marks	:	0	In all other cases.

1. Total number of cis N–Mn–Cl bond angles (that is, Mn–N and Mn–Cl bonds in cis positions) present in a molecule of cis-$[Mn(en)_2Cl_2]$ complex is ________

(en = $NH_2CH_2CH_2NH_2$)

Answer (6)

Sol. cis-$[Mn(en)_2Cl_2]$



No. of N-Mn-Cl bonds (in which N-Mn bond is cis to Mn-Cl bonds) = 6
(Two N-Mn-Cl bonds have N-Mn bond trans to Mn-Cl bonds)

2. The amount of water produced (in g) in the oxidation of 1 mole of rhombic sulphur by conc. HNO_3 to a compound with the highest oxidation state of sulphur is ___

(Given data : Molar mass of water = 18 g mol^{-1})

Answer (288)

Sol. Conc. HNO_3 oxidises rhombic sulphur (S_8) to H_2SO_4 and itself gets reduced to NO_2.

$$\overset{0}{S_8} + 48H\overset{+5}{N}O_3 \rightarrow 8H_2\overset{+6}{S}O_4 + 48\overset{+4}{N}O_2 + 16H_2O$$

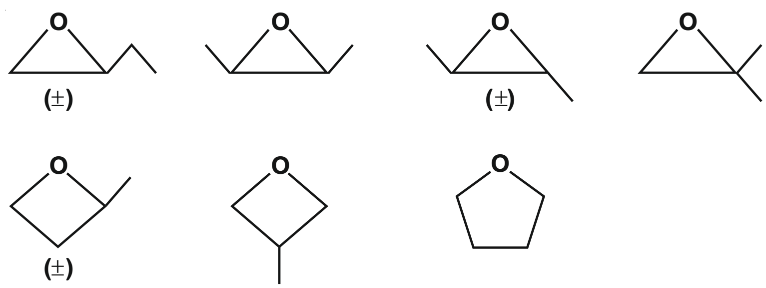
1 mole of S_8 gives 16 moles of H_2O.

Mass of H_2O = 16 × 18 = 288 gm

3. Total number of isomers, considering both structural and stereoisomers, of cyclic ethers with the molecular formula C_4H_8O is ____

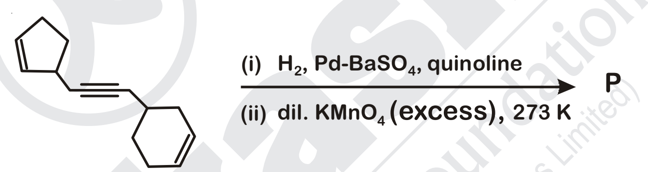
Answer (10)

Sol. Cyclic ethers, including stereoisomers of formula C_4H_8O are



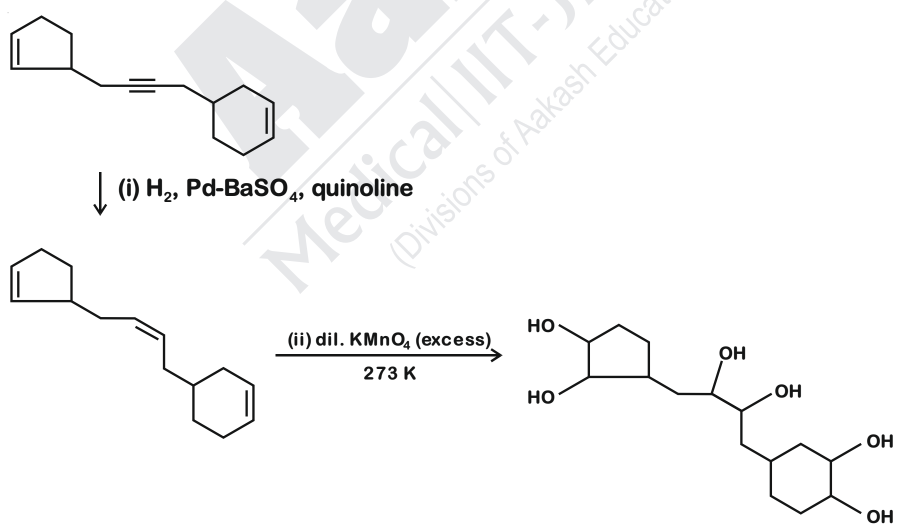
Total number of cyclic ethers = 10

4. Total number of hydroxyl groups present in a molecule of the major product P is ____



Answer (6)

Sol.



5. The decomposition reaction

$2N_2O_5(g) \xrightarrow{\Delta} 2N_2O_4(g) + O_2(g)$ is started in a closed cylinder under isothermal isochoric condition at an initial pressure of 1 atm. After $Y \times 10^3$ s, the pressure inside the cylinder is found to be 1.45 atm. If the rate constant of the reaction is $5 \times 10^{-4}\ s^{-1}$, assuming ideal gas behavior, the value of Y is ________

Answer (2.3)

Sol. $2N_2O_5 \xrightarrow{\Delta} 2N_2O_4 + O_2\ ;\ K = 5\times10^{-4}\ sec^{-1}$

	$2N_2O_5$	$2N_2O_4$	O_2
Initial pressure (in atm)	1	–	–
Pressure after time t sec	1 – 2x	2x	x

$\therefore$ 1 – 2x + 2x + x = 1.45 atm

$\Rightarrow$ x = 0.45 atm

For 1st order reaction (taking into account of reaction stoichiometry)

$$2Kt = \ln\frac{a_0}{a_0 - x}$$

$\because$ $a_0 \propto 1$ and $a_0 - x \propto 0.1$

$$\Rightarrow t = \frac{1}{2\times5\times10^{-4}}\ln\frac{1}{0.1} = 2.3\times10^3\ sec$$

6. The mole fraction of urea in an aqueous urea solution containing 900 g of water is 0.05. If the density of the solution is 1.2 g cm^{-3}, the molarity of urea solution is ________

(Given data: Molar masses of urea and water are 60 g mol^{-1} and 18 g mol^{-1}, respectively)

Answer (2.98)

Sol. $\chi_{urea} = 0.05 = \dfrac{n_{urea}}{n_{water} + n_{urea}}$

$$0.05 = \frac{n_{urea}}{\frac{900}{18} + n_{urea}}$$

$\Rightarrow$ n_{urea} = 2.63 mol

Mass of urea = 2.63 × 60 g/mol Molar Mass (urea) = 60 g/mol

= 157.8 g

Mass of Solution = 157.8 + 900 = 1057.8 g

Volume of Solution = $\dfrac{1057.8}{1.2}$ = 881.5 ml

Molarity of Solution = $\dfrac{2.63}{881.5}\times1000$ = 2.98 M

SECTION - 3 (Maximum Marks : 12)

- This section contains TWO (02) List-Match sets.
- Each List-Match set has TWO (02) Multiple Choice Questions.
- Each List-Match set has two lists: List-I and List-II.
- List-I has Four entries (I), (II), (III) and (IV) and List-II has Six entries (P), (Q), (R), (S), (T) and (U).
- Four options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the option corresponding to the correct combination is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

1. Answer the following by appropriately matching the lists based on the information given in the paragraph

Consider the Bohr's model of a one-electron atom where the electron moves around the nucleus. In the following, List-I contains some quantities for the n^{th} orbit of the atom and List-II contains options showing how they depend on n.

List-I	List-II
(I) Radius of the n^{th} orbit	(P) $\propto n^{-2}$
(II) Angular momentum of the electron in the n^{th} orbit	(Q) $\propto n^{-1}$
(III) Kinetic energy of the electron in the n^{th} orbit	(R) $\propto n^{0}$
(IV) Potential energy of the electron in the n^{th} orbit	(S) $\propto n^{1}$
	(T) $\propto n^{2}$
	(U) $\propto n^{1/2}$

Which of the following options has the correct combination considering List-I and List-II?

(A) (III), (P)
(B) (IV), (U)
(C) (III), (S)
(D) (IV), (Q)

Answer (A)

Sol. Kinetic energy $\propto \frac{1}{n^2}$

Potential energy $\propto \frac{1}{n^2}$

$\therefore$ Correct match : (III), P ; (IV), P

2. Answer the following by appropriately matching the lists based on the information given in the paragraph

Consider the Bohr's model of a one-electron atom where the electron moves around the nucleus. In the following, List-I contains some quantities for the n^{th} orbit of the atom and List-II contains options showing how they depend on n.

List-I	List-II
(I) Radius of the n^{th} orbit	(P) $\propto n^{-2}$
(II) Angular momentum of the electron in the n^{th} orbit	(Q) $\propto n^{-1}$
(III) Kinetic energy of the electron in the n^{th} orbit	(R) $\propto n^{0}$
(IV) Potential energy of the electron in the n^{th} orbit	(S) $\propto n^{1}$
	(T) $\propto n^{2}$
	(U) $\propto n^{1/2}$

Which of the following options has the correct combination considering List-I and List-II?

(A) (II), (R)

(B) (I), (P)

(C) (II), (Q)

(D) (I), (T)

Answer (D)

Sol. In Bohr's model

Radius $\propto n^2$

Angular momentum $\propto n$

Kinetic energy $\propto \frac{1}{n^2}$

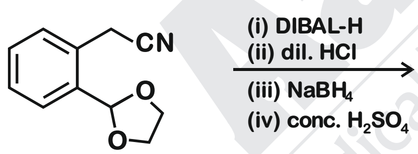
Potential energy $\propto \frac{1}{n^2}$

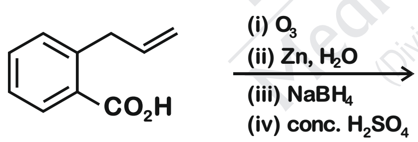
$\therefore$ Correct match : (I), (T) ; (II), (S)

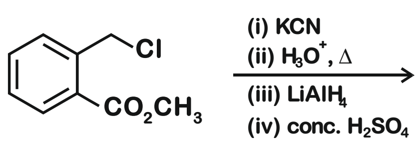
3. Answer the following by appropriately matching the lists based on the information given in the paragraph

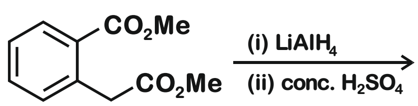
List-I includes starting materials and reagents of selected chemical reactions. List-II gives structures of compounds that may be formed as intermediate products and/ or final products from the reactions of List-I.

List-I

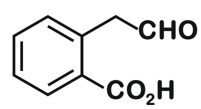
(I) 

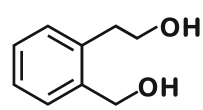
(II) 

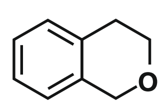
(III) 

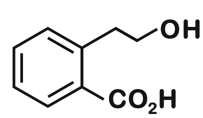
(IV) 

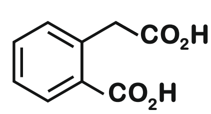
List-II

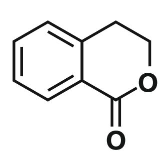
(P) 

(Q) 

(R) 

(S) 

(T) 

(U) 

Which of the following options has correct combination considering List-I & List-II?

(A) (IV), (Q), (R)

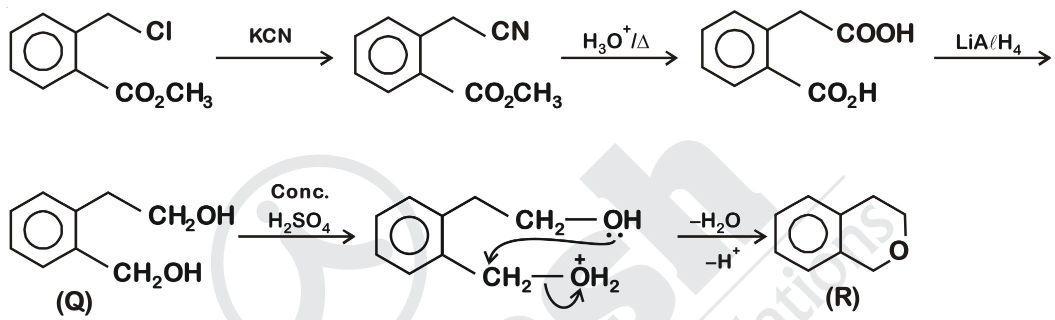
(B) (III), (T), (U)

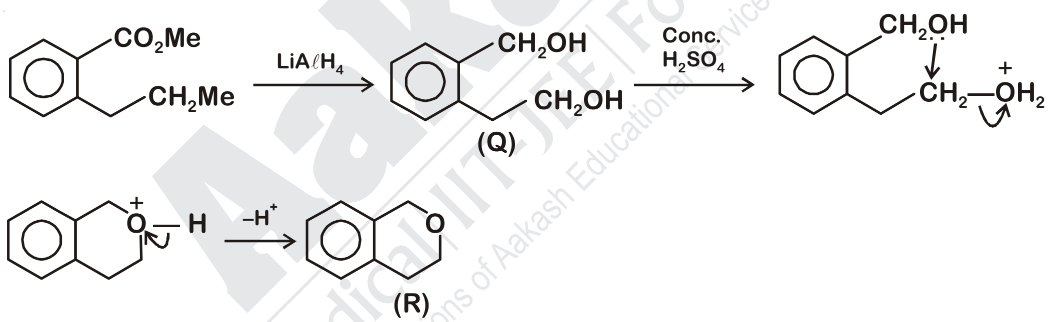
(C) (IV), (Q), (U)

(D) (III), (S), (R)

Answer (A)

Sol.

(III) 

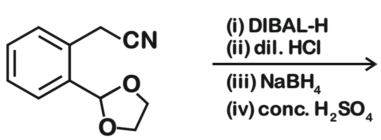
(IV) 

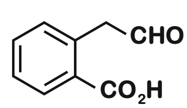
(III), (Q), (R) ; (IV), (Q), (R)

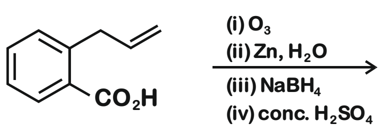
4. Answer the following by appropriately matching the lists based on the information given in the paragraph

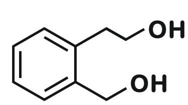
List-I includes starting materials and reagents of selected chemical reactions. List-II gives structures of compounds that may be formed as intermediate products and/or final products from the reactions of List-I.

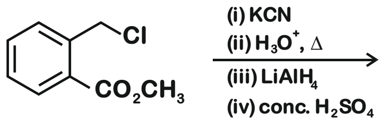
List-I | **List-II**

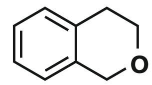
(I) 

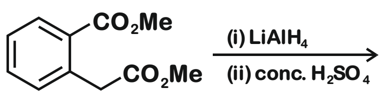
(P) 

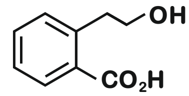
(II) 

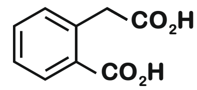
(Q) 

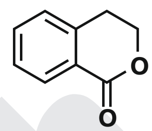
(III) 

(R) 

(IV) 

(S) 

(T) 

(U) 

Which of the following options has correct combination considering List-I and List-II?

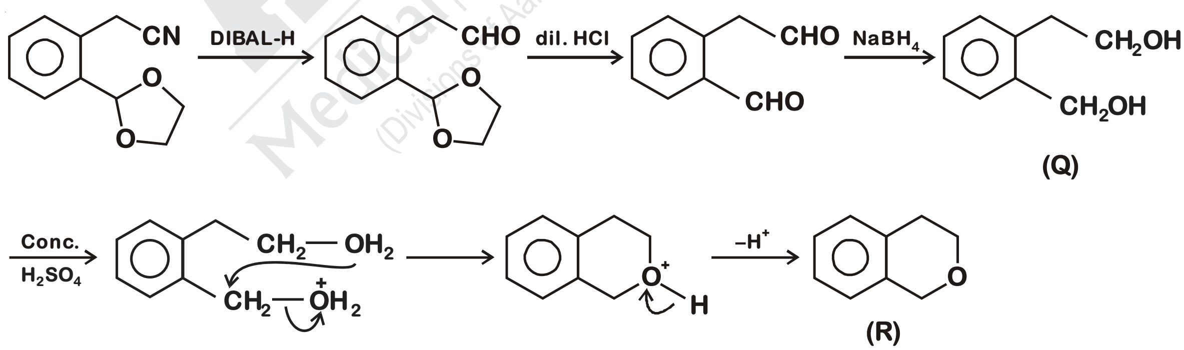
(A) (I), (Q), (T), (U)

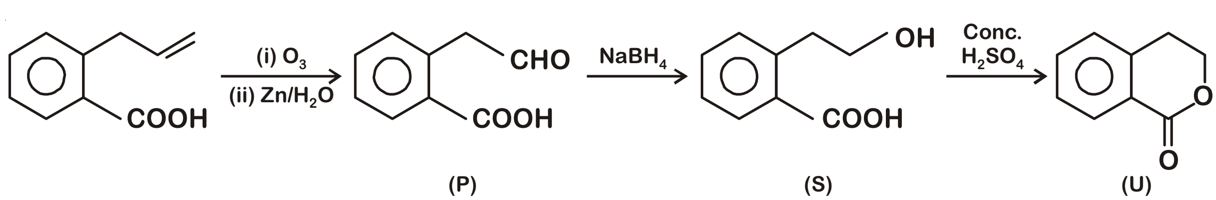
(B) (II), (P), (S), (U)

(C) (I), (S), (Q), (R)

(D) (II), (P), (S), (T)

Answer (B)

Sol. (I) 

(II) 

(I), (Q), (R); (II), (P), (S), (U)

PART-III : MATHEMATICS

SECTION - 1 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

- For example : in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e., the question is unanswered) will get 0 marks; and
choosing any other combination of options will get –1 mark.

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a function. We say that f has

PROPERTY 1 if $\lim_{h\to 0} \frac{f(h)-f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim_{h\to 0} \frac{f(h)-f(0)}{h^2}$ exists and is finite.

Then which of the following options is/are correct?

(A) $f(x) = x^{2/3}$ has PROPERTY 1

(B) $f(x) = |x|$ has PROPERTY 1

(C) $f(x) = x|x|$ has PROPERTY 2

(D) $f(x) = \sin x$ has PROPERTY 2

Answer (A, B)

Sol. (A) $f(x) = x^{2/3}$

$$\lim_{h\to 0} \frac{h^{2/3} - 0}{\sqrt{|h|}} = \lim_{h\to 0} |h|^{1/6} = 0$$

(B) $f(x) = |x|$

$$\lim_{h\to 0} \frac{|h| - 0}{\sqrt{|h|}} = \lim_{h\to 0} \sqrt{|h|} = 0$$

(C) $f(x) = x|x|$

$$\lim_{h\to 0} \frac{h(h) - 0}{h^2} = \lim_{h\to 0} \frac{|h|}{h} = \begin{cases} 1 & \text{if } h \to 0^+ \\ -1 & \text{if } h \to 0^- \end{cases}$$

So, $\lim_{h\to 0} \frac{f(h) - f(0)}{h}$ does not exist.

(D) $f(x) = \sin x$

$$\lim_{h\to 0} \frac{\sin h - 0}{h^2} = \lim_{h\to 0} \frac{1}{h} \cdot \frac{\sin h}{h} = \text{does not exist.}$$

2. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = (x - 1)(x - 2)(x - 5)$. Define $F(x) = \int_0^x f(t)\,dt,\ x > 0$.

Then which of the following options is/are correct?

(A) F has a local maximum at x = 2

(B) F has a local minimum at x = 1

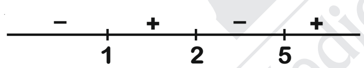
(C) $F(x) \neq 0$ for all $x \in (0, 5)$

(D) F has two local maxima and one local minimum in $(0, \infty)$

Answer (A, B, C)

Sol. $F(x) = \int_0^x f(t)\,dt$

$\Rightarrow F'(x) = f(x) = (x - 1)(x - 2)(x - 5)$



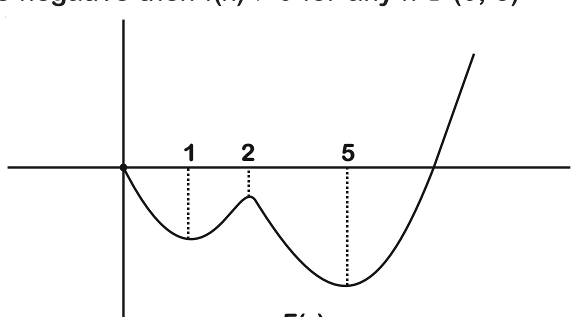
F(x) has maxima at x = 2 and minima at x = 1 and x = 5

$$F(2) = \int_0^2 [x^3 - 8x^2 + 17x - 10]\,dx = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{17x^2}{2} - 10x\Big|_0^2$$

$$= 4 - \frac{64}{3} + 34 - 20$$

$$= 38 - \frac{124}{3} = -\frac{10}{3}$$

If maximum value of f(x) is negative then $f(x) \neq 0$ for any $x \in (0, 5)$



y = F(x)

3. Let $x \in \mathbb{R}$ and let

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} \text{ and } R = PQP^{-1}.$$

Then which of the following options is/are correct?

(A) $\det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$, for all $x \in \mathbb{R}$

(B) For x = 1, there exists a unit vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ for which $R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(C) For x = 0, if $R\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$, then a + b = 5

(D) There exists a real number x such that PQ = QP

Answer (A, C)

Sol. $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix}$

So $6P^{-1} = \begin{bmatrix} 6 & -3 & 0 \\ 0 & 3 & -2 \\ 0 & 0 & 2 \end{bmatrix}$

(A) $\because$ $R = PQP^{-1}$

$\Rightarrow$ det(R) = det(P) · det(Q) · det(P^{-1})

$\Rightarrow$ det(R) = det(Q) $= \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix}$

$\Rightarrow \det(R) = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + \det \begin{bmatrix} 2 & x & 0 \\ 0 & 4 & 0 \\ x & x & 1 \end{bmatrix}$

$\Rightarrow \det(R) = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$

(B) $R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ must have non trivial solutions of (α, β, γ)

So det(R) = 0

$\Rightarrow$ $4(12 - x^2) = 0$

$\Rightarrow$ $x = \pm 2\sqrt{3}$

(C) $PQP^{-1}\begin{bmatrix}1\\a\\b\end{bmatrix}=\begin{bmatrix}6\\6a\\6b\end{bmatrix}$

$$\Rightarrow \frac{1}{6}\begin{bmatrix}1&1&1\\0&2&2\\0&0&3\end{bmatrix}\begin{bmatrix}2&0&0\\0&4&0\\0&0&6\end{bmatrix}\begin{bmatrix}6&-3&0\\0&3&-2\\0&0&2\end{bmatrix}\begin{bmatrix}1\\a\\b\end{bmatrix}=\begin{bmatrix}6\\6a\\6b\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}12&6&4\\0&24&8\\0&0&36\end{bmatrix}\begin{bmatrix}1\\a\\b\end{bmatrix}=\begin{bmatrix}36\\36a\\36b\end{bmatrix}$$

$\Rightarrow$ 12 + 6a + 4b = 36 and 24a + 8b = 36a

$\Rightarrow$ a = 2 and b = 3

$\Rightarrow$ a + b = 5

(D) PQ = QP

$$\begin{bmatrix}1&1&1\\0&2&2\\0&0&3\end{bmatrix}\begin{bmatrix}2&x&x\\0&4&0\\x&x&6\end{bmatrix}=\begin{bmatrix}2&x&x\\0&4&0\\x&x&6\end{bmatrix}\begin{bmatrix}1&1&1\\0&2&2\\0&0&3\end{bmatrix}$$

On both sides a_{12} elements are not equal for any value of x. So there exists no value of x for which PQ = QP.

4. Let $f(x)=\dfrac{\sin \pi x}{x^2}$, x > 0.

Let $x_1 < x_2 < x_3 < \dots < x_n < \dots$ be all the points of local maximum of f and $y_1 < y_2 < y_3 < \dots < y_n < \dots$ be all the points of local minimum of f.

Then which of the following options is/are correct?

(A) $x_1 < y_1$

(B) $x_{n+1} - x_n > 2$ for every n

(C) $|x_n - y_n| > 1$ for every n

(D) $x_n \in \left(2n, 2n+\dfrac{1}{2}\right)$ for every n

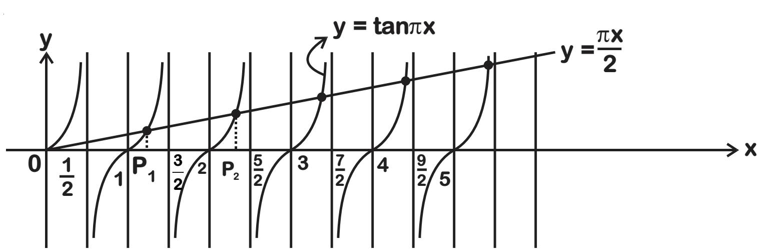
Answer (B, C, D)

Sol. $f'(x)=\dfrac{2x\cos \pi x\left(\dfrac{\pi x}{2}-\tan \pi x\right)}{x^4}$

for maxima/minima $f'(x) = 0$

$\Rightarrow \cos \pi x = 0$ or $\tan \pi x = \dfrac{\pi}{2}x$

$\because \cos \pi x \neq 0 \quad \Rightarrow \quad \tan \pi x = \dfrac{\pi}{2}x$



$f'(P_1^-) < 0,\ f'(P_1^+) > 0 \quad \Rightarrow \quad x = P_1 \in \left(1, \frac{3}{2}\right)$ is point of local minima

$f'(P_2^-) > 0,\ f'(P_2^+) < 0 \quad \Rightarrow \quad x = P_2 \in \left(2, \frac{5}{2}\right)$ is point of local maxima.

From the graph, it is clear $\frac{5}{2} - x_1 > \frac{9}{2} - x_2 > \frac{13}{2} - x_3 > \frac{17}{2} - x_4 \ldots$

$\Rightarrow \quad x_{n+1} - x_n > 2\ \forall\ n$

$\frac{3}{2} - y_1 > \frac{5}{2} - x_1 > \frac{7}{2} - y_2 > \frac{9}{2} - x_2 > \ldots$

$\Rightarrow \quad |x_n - y_n| > 1\ \forall\ n$

$\because \quad x_1 \in \left(2, \frac{5}{2}\right), x_2 \in \left(4, \frac{9}{2}\right), x_3 \left(6, \frac{13}{2}\right) \ldots$

$\Rightarrow \quad x_n \in \left(2n, 2n + \frac{1}{2}\right) \forall\ n$

5. For $a \in \mathbb{R}$, $|a| > 1$, let

$$\lim_{n\to\infty}\left(\frac{1+\sqrt[3]{2}+\ldots+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+\ldots+\frac{1}{(an+n)^2}\right)}\right) = 54.$$

Then the possible value(s) of a is/are

(A) 7 (B) –6 (C) 8 (D) –9

Answer (C, D)

Sol. $$\lim_{n\to\infty}\frac{\sum_{r=1}^{n} r^{1/3}}{n^{7/3}\sum_{r=1}^{n}\frac{1}{(na+r)^2}} = \lim_{n\to\infty}\frac{\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{1/3}\frac{1}{n}}{\sum_{r=1}^{n}\frac{1}{\left(a+\frac{r}{n}\right)^2}\frac{1}{n}}$$

$$= \frac{\int_0^1 x^{1/3}dx}{\int_0^1 \frac{dy}{(x+a)^2}} = 54$$

$$\Rightarrow \quad \frac{3\left.\frac{x^{4/3}}{4}\right|_0^1}{\left.\frac{-1}{(x+a)}\right|_0^1} = 54$$

$$\Rightarrow \quad \frac{3}{4\times 54} = \frac{1}{a} - \frac{1}{1+a}$$

$$\Rightarrow \quad \frac{1}{a(a+1)} = \frac{1}{72}$$

$\Rightarrow \quad a^2 + a - 72 = (a + 9)(a - 8) = 0$

$a = 8$ or -9

6. For non-negative integers n, let

$$f(n)=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0, \pi]$, which of the following options is/are correct?

(A) $\lim_{n\to\infty} f(n)=\frac{1}{2}$

(B) $f(4)=\frac{\sqrt{3}}{2}$

(C) $\sin(7\cos^{-1} f(5)) = 0$

(D) If $\alpha = \tan(\cos^{-1}f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$

Answer (B, C, D)

Sol. $$f(n)=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\cdot\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}=\frac{\sum_{k=0}^{n}\left(\cos\left(\frac{\pi}{n+2}\right)-\cos\left(\frac{2k+3}{n+2}\pi\right)\right)}{\sum_{k=0}^{n}\left(1-\cos\left(\frac{2k+2}{n+2}\pi\right)\right)}$$

$$\Rightarrow f(n)=\frac{(n+1)\cos\left(\frac{\pi}{n+2}\right)-\frac{\sin\left(\frac{n+1}{n+2}\pi\right)}{\sin\left(\frac{\pi}{n+2}\right)}\cos\left(\frac{n+3}{n+2}\pi\right)}{(n+1)-\frac{\sin\left(\frac{n+1}{n+2}\pi\right)}{\sin\left(\frac{\pi}{n+2}\right)}\cos(\pi)}$$

$$\Rightarrow f(n)=\frac{(n+1)\cos\left(\frac{\pi}{n+2}\right)+\cos\left(\frac{\pi}{n+2}\right)}{(n+1)+1}=\cos\left(\frac{\pi}{n+2}\right)$$

(A) $\lim_{n\to\infty} f(n)=\lim_{n\to\infty}\cos\left(\frac{\pi}{n+2}\right)=1$

(B) $f(4)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

(C) $f(5)=\cos\frac{\pi}{7}$

So, $\sin\left(7\cos^{-1}\left(\cos\frac{\pi}{7}\right)\right)=\sin\pi=0$

(D) $\because\ f(6)=\cos\left(\frac{\pi}{8}\right)$; then $\alpha=\tan\left(\frac{\pi}{8}\right)=\sqrt{2}-1$

Clearly $\alpha^2 + 2\alpha = 1$

7. Three lines

$L_1: \quad \vec{r}=\lambda\hat{i},\ \lambda\in\mathbb{R},$

$L_2: \quad \vec{r}=\hat{k}+\mu\hat{j},\ \mu\in\mathbb{R}$ and

$L_3: \quad \vec{r}=\hat{i}+\hat{j}+\nu\hat{k},\ \nu\in\mathbb{R}$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear?

(A) $\hat{k}+\hat{j}$

(B) $\hat{k}$

(C) $\hat{k}+\frac{1}{2}\hat{j}$

(D) $\hat{k}-\frac{1}{2}\hat{j}$

Answer (C, D)

Sol. Let $P \equiv (\lambda, 0, 0)$, $Q(0, \mu, 1)$, $R(1, 1, \nu)$

For collinearity $\overrightarrow{PQ} = k\,\overrightarrow{PR}$

$$\frac{\lambda - 0}{\lambda - 1} = \frac{0-\mu}{0-1} = \frac{0-1}{0-\nu}$$

$$\Rightarrow \mu = \frac{1}{\nu}, \quad \lambda = \lambda\mu - \mu$$

$$\Rightarrow \nu = \frac{1}{\mu}, \quad \lambda = \frac{\mu}{\mu - 1}$$

$$\Rightarrow \mu \neq 0, \quad \mu \neq 1$$

$$\Rightarrow Q \neq \hat{k}, \quad Q \neq \hat{k} + \hat{j}$$

8. Let $P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and

$$X = \sum_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$$

where P_k^T denotes the transpose of the matrix P_k. Then which of the following options is/are correct?

(A) The sum of diagonal entries of X is 18

(B) If $X\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $\alpha = 30$

(C) X is a symmetric matrix

(D) $X - 30I$ is an invertible matrix

Answer (A, B, C)

Sol. $\because$ $P_1^T = P_1$, $P_2^T = P_2$, $P_3^T = P_3$, $P_4^T = P_5$, $P_5^T = P_4$ and $P_6^T = P_6$

$$X = \sum_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T; \text{ let } \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} = Q; \text{ here } Q^T = Q$$

(A) $\mathrm{Tr}(X) = \sum_{i=1}^{6} \mathrm{Tr}\left(P_i Q P_i^T\right)$ $\quad(\because \mathrm{Tr}(AB) = \mathrm{Tr}(BA))$

$= \sum_{i=1}^{6} \mathrm{Tr}\left(Q P_i^T \cdot P_i\right)$ $\quad(\because P_i$'s are orthogonal matrices)

$= \sum_{i=1}^{6} \mathrm{Tr}(QI)$

$= \sum_{i=1}^{6} \mathrm{Tr}(Q)$

$= 6 \cdot \mathrm{Tr}(Q)$

$= 6 \times 3$

$= 18$

(B) Let $R = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $XR = \sum_{i=1}^{6} P_i Q P_i^T \cdot R = \sum_{i=1}^{6} P_i Q R = \sum_{i=1}^{6} P_i \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 30 \\ 30 \\ 30 \end{bmatrix} = 30 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(C) $X = P_1QP_1 + P_2QP_2 + P_3QP_3 + P_4QP_5 + P_5QP_4 + P_6QP_6$

$X^T = P_1^TQP_1^T + P_2^TQP_2^T + P_3^TQP_3^T + P_5^TQP_4^T + P_4^TQP_5^T + P_6^TQP_6^T$

$X^T = P_1QP_1 + P_2QP_2 + P_3QP_3 + P_4QP_5 + P_5QP_4 + P_6QP_6 = X$

So X is a symmetric matrix.

(D) $X \cdot R = 30\,R$

$\Rightarrow (X - 30I)R = O$

So $|X - 30I| = 0$

So $X - 30I$ is not invertible

SECTION - 2 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct numerical value is entered.
Zero Marks	:	0	In all other cases.

---

1. Let |X| denote the number of elements in a set X. Let S = {1, 2, 3, 4, 5, 6} be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A, B) such that $1 \le |B| < |A|$, equals ____

Answer (422)

Sol. A and B are independent events

$\because P(A \cap B) = P(A).\,P(B)$

$\Rightarrow \dfrac{|A \cap B|}{|S|} = \dfrac{|A|}{|S|} \cdot \dfrac{|B|}{|S|}$

$\Rightarrow |S|\,|A \cap B| = |A|.\,|B|$

$\Rightarrow 6.|A \cap B| = |A|.\,|B|$ ...(i)

$\because |A| > |B| \ge 1$

So, possible values of |A| can be 2, 3, 4 or 6

$|A| \ne 5$; because $|A| = 5$ cannot satisfy the equation (i))

Case-1 :

If |A| = 2 then |B| = 1 (It is also not satifying equation (i))

Case-2 :

If |A| = 3 then |B| = 2 and $|A \cap B| = 1$

Number of ways = $^6C_3 . ^3C_1 . ^3C_1 = 180$

Case-3 :

If |A| = 4 then |B| = 3 and $|A \cap B| = 2$

Number of ways = $^6C_4 . ^4C_2 . ^2C_1 = 180$

Case-4 :

If |A| = 6 then |B| may be any number between 1 to 5

Number of ways = $^6C_6 . [\,^6C_1 + ^6C_2 + ... + ^6C_5\,] = 62$

Total number of ways = 180 + 180 + 62

= 422

2. **The value of the integral**

$$\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5}d\theta$$

equals ____

Answer (0.5)

Sol. $$I = \int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\sin\theta}+\sqrt{\cos\theta}\right)^5} \quad ...(i)$$

$$I = \int_0^{\pi/2} \frac{3\sqrt{\sin\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5} \quad ...(ii)$$ [Using property-III f(x) = f(a + b – x)]

Now, add (i) + (ii)

$$2I = \int_0^{\pi/2} \frac{3\left(\sqrt{\sin\theta}+\sqrt{\cos\theta}\right)}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5}d\theta$$

$$\frac{2I}{3} = \int_0^{\pi/2} \left(\frac{1}{\sqrt{\sin\theta}+\sqrt{\cos\theta}}\right)^4 d\theta$$

$$\frac{2I}{3} = \int_0^{\pi/2} \frac{\sec^2\theta}{\left(1+\sqrt{\tan\theta}\right)^4}d\theta$$

Let $\tan\theta = U^2$

$\sec^2\theta d\theta = 2UdU$ $\qquad \theta \to 0 \quad U \to 0$

$\theta \to \frac{\pi}{2} \quad U \to \infty$

Now, $$\frac{2I}{3} = \int_0^{\infty} \frac{2UdU}{(1+U)^4}$$

$$\frac{2}{3}I = \int_0^{\infty} 2.\frac{(U+1-1)}{(1+U)^4}dU$$

$$\frac{I}{3} = \int_0^{\infty} \left[\frac{1}{(U+1)^3} - \frac{1}{(U+1)^4}\right]dU$$

$$\frac{I}{3} = \left[\frac{1}{-2(U+1)^2} + \frac{1}{3(U+1)^3}\right]_0^{\infty}$$

$$\frac{I}{3} = +\frac{1}{6}$$

$$I = \frac{1}{2}$$

$$\boxed{I = 0.5}$$

3. Suppose $\det\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k k^2 \\ \sum_{k=0}^{n} {}^nC_k k & \sum_{k=0}^{n} {}^nC_k 3^k \end{bmatrix} = 0$ holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^nC_k}{k+1}$ equals___

Answer (6.20)

Sol. $$\begin{vmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k k^2 \\ \sum_{k=0}^{n} {}^nC_k k & \sum_{k=0}^{n} {}^nC_k 3^k \end{vmatrix} = \begin{vmatrix} \frac{n(n+1)}{2} & n(n+1)2^{n-2} \\ n.2^{n-1} & 4^n \end{vmatrix} = 0$$

$$\Rightarrow \frac{n(n+1)}{2}\cdot 2^{2n} = n^2(n+1)\cdot 2^{2n-3} \Rightarrow n = 4$$

Now, $$\sum_{k=0}^{4} \frac{{}^4C_k}{k+1} = \frac{1}{5}\sum_{k=0}^{4} {}^5C_{k+1} = \frac{1}{5}[2^5 - 1] = \frac{31}{5}$$

$$= 6.20$$

4. The value of

$$\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10} \sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$$ in the interval $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$ equals ___

Answer (0)

Sol. $$\sum_{k=0}^{10} \frac{1}{\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right).\cos\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)} = \sum_{k=0}^{10} \frac{\sin\left[\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)-\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\right]}{\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right).\cos\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)}$$

$$= \sum_{k=0}^{10}\left[\tan\left(\frac{7\pi}{12}+(k+1)\frac{\pi}{2}\right)-\tan\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\right]$$

$$= \tan\left(\frac{7\pi}{12}+\frac{11\pi}{2}\right)-\tan\left(\frac{7\pi}{12}\right)$$

$$= \tan\left(\frac{\pi}{12}\right)+\cot\left(\frac{\pi}{12}\right)$$

$$= \frac{1}{\sin\frac{\pi}{12}.\cos\frac{\pi}{12}} = \frac{2}{\sin\frac{\pi}{6}} = 4$$

So, $$\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\cdot\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right) = \sec^{-1}(1)$$

$$= 0$$

5. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}, \alpha, \beta \in \mathbb{R}$. If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b})).\vec{c}$ equals _____

Answer (18)

Sol. Given $\dfrac{(\vec{a}+\vec{b})}{(|\vec{a}+\vec{b}|)} \cdot \vec{c} = 3\sqrt{2}$ $\because$ $\vec{a}+\vec{b} = 3\hat{i} + 3\hat{j}$

So $(\vec{a}+\vec{b}) \cdot (\alpha\vec{a} + \beta\vec{b}) = 18$ $|\vec{a}+\vec{b}| = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$\Rightarrow$ $6\alpha + 6\beta + 3\alpha + 3\beta = 18$ $|\vec{a}| = \sqrt{6}, \; |\vec{b}| = \sqrt{6}$

$\boxed{\alpha + \beta = 2}$ $\vec{a} \cdot \vec{b} = 3$

Now minimum value of $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$

$= (\alpha\vec{a} + \beta\vec{b} - (\vec{a} \times \vec{b}))(\alpha\vec{a} + \beta\vec{b})$

$= \alpha^2(6) + \alpha\beta(3) - 0 + \alpha\beta(3) + \beta^2(6) - 0$

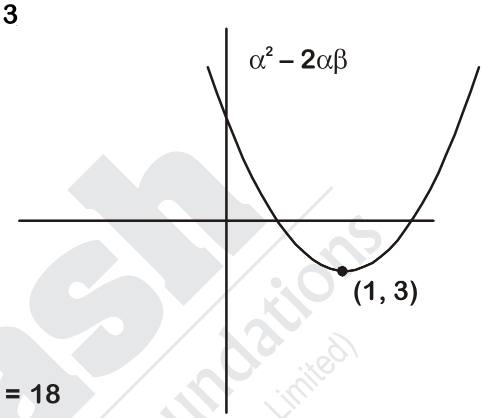
$= 6(\alpha^2 + \beta^2 + \alpha\beta)$

$= 6\left[(\alpha+\beta)^2 - \alpha\beta\right]$

$= 6[4 - \alpha(2 - \alpha)]$

$= 6(4 - 2\alpha + \alpha^2)$

Minimum value of $6(4 - 2\alpha + \alpha^2) = 6(4 - 2 + 1) = 18$



6. Five persons A, B, C, D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is _____

Answer (30)

Sol. Given that no two persons sitting adjacent, have hats of same colour. So hats of all colours must be used. Also hats of different colours cannot be used in 1 + 1 + 3 combination because any three hats cannot be of same colour.

Therefore only combination left is 2 + 2 + 1.

There are a total of 3 cases of selecting hats which are

2R + 2B + 1G or 2B + 2G + 1R or 2G + 2R + 1B

To distribute these hats first we select a person in ${}^5C_1$ ways and distribute that hat which is one of it's colour. Then for remaining four persons there are two ways of distributing hats of alternate colours. So total ways will be equal to $3 \times {}^5C_1 \times 2 = 30$

SECTION - 3 (Maximum Marks : 12)

- This section contains TWO (02) List-Match sets.
- Each List-Match set has TWO (02) Multiple Choice Questions.
- Each List-Match set has two lists: List-I and List-II.
- List-I has Four entries (I), (II), (III) and (IV) and List-II has Six entries (P), (Q), (R), (S), (T) and (U).
- Four options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the option corresponding to the correct combination is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e., the question is unanswered);
Negative Marks	:	–1	In all other cases.

1. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let f(x) = sin(πcosx) and g(x) = cos(2πsinx) be two functions defined for x > 0. Define the following sets whose elements are written in the increasing order :

X = {x : f(x) = 0}, Y = {x : f′(x) = 0},

Z = {x : g(x) = 0}, W = {x : g′(x) = 0}.

List-I contains the sets X, Y, Z and W. List-II contains some information regarding these sets.

List-I	List-II
(I) X	(P) $\supseteq\left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) NOT an arithmetic progression
(IV) W	(S) $\supseteq\left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
	(T) $\supseteq\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(U) $\supseteq\left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination?

(A) (II), (R), (S)

(B) (I), (Q), (U)

(C) (II), (Q), (T)

(D) (I), (P), (R)

Answer (C)

2. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let f(x) = sin(πcosx) and g(x) = cos(2πsinx) be two functions defined for x > 0. Define the following sets whose elements are written in the increasing order:

X = {x : f(x) = 0}, Y = {x : f′(x) = 0},

Z = {x : g(x) = 0}, W = {x : g′(x) = 0}.

List-I contains the sets X, Y, Z and W. List-II contains some information regarding these sets.

List-I	List-II
(I) X	(P) $\supseteq\left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) NOT an arithmetic progression
(IV) W	(S) $\supseteq\left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
	(T) $\supseteq\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(U) $\supseteq\left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination?

(A) (IV), (P), (R), (S)

(B) (IV), (Q), (T)

(C) (III), (R), (U)

(D) (III), (P), (Q), (U)

Answer (A)

Combine solution for (Q.1 & Q.2)

Sol. $f(x) = \sin(\pi\cos x)$, $x > 0$

$g(x) = \cos(2\pi\sin x)$, $x > 0$

$f'(x) = -\pi\sin x\cos(\pi\cos x)$

$g'(x) = -2\pi\cos x\sin(2\pi\sin x)$

Now,

$X = \{x : f(x) = 0\}$

so $\sin(\pi\cos x) = 0$

$\pi\cos x = n\pi$, $n \in Z$

$\cos x = n$

$\cos x = -1, 0, 1$

$\cos x = \pm 1 \Rightarrow x = n\pi$

$\cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$

$\Rightarrow$ (I) $\rightarrow$ P, Q

$Y = \{x : f'(x) = 0\}$

so $-\pi\sin x\cos(\pi\cos x) = 0$

$\sin x = 0$ or $\pi\cos x = (2n+1)\frac{\pi}{2}$, $n \in Z$

$x = n\pi$, $n \in Z$ $\qquad \cos x = (2n+1)\frac{1}{2}$, $n \in Z$

$\cos x = \pm\frac{1}{2}$

$x = 2n\pi \pm \frac{\pi}{3}$ or $x = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow$ (II) $\rightarrow$ Q, T

$Z = \{x : g(x) = 0\}$

so $\cos(2\pi\sin x) = 0$

$2\pi\sin x = (2n+1)\frac{\pi}{2}$, $n \in Z$

$\sin x = \frac{(2n+1)}{4}$ so $\sin x = \frac{1}{4}, \frac{3}{4}, \frac{-1}{4}, \frac{-3}{4}$

$\Rightarrow$ (III) $\rightarrow$ R

$W = \{x : g'(x) = 0\}$

$-2\pi\cos x\sin(2\pi\sin x) = 0$

$\cos x = 0$

$\Rightarrow x = (2n+1)\frac{\pi}{2}$, $n \in Z$

or $\sin(2\pi\sin x) = 0$

$\Rightarrow 2\pi\sin x = n\pi$, $n \in Z$

$\sin x = \frac{n}{2}$ so $\sin x = 0, \frac{1}{2}, 1, \frac{-1}{2}, -1$

$x = n\pi,\ x = n\pi + (-1)^n\left(\pm\frac{\pi}{6}\right),\ x = (2n+1)\frac{\pi}{2}$

$\Rightarrow$ (IV) $\rightarrow$ P, R, S

3. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x - 3)^2 + (y - 4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x - h)^2 + (y - k)^2 = r^2$ satisfies the following conditions :

(i) centre of C_3 is collinear with the centres of C_1 and C_2,

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N.

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expressions given in the List-I whose values are given in List-II below :

List-I	List-II
(I) $2h + k$	(P) 6
(II) $\frac{\text{Length of ZW}}{\text{Length of XY}}$	(Q) $\sqrt{6}$
(III) $\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R) $\frac{5}{4}$
(IV) α	(S) $\frac{21}{5}$
	(T) $2\sqrt{6}$
	(U) $\frac{10}{3}$

Which of the following is the only CORRECT combination?

(A) (I), (S)

(B) (II), (Q)

(C) (II), (T)

(D) (I), (U)

Answer (B)

4. Answer the following by appropriately matching the lists based on the information given in the paragraph.

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There are some expressions given in the List-I whose values are given in List-II below :

	List-I		List-II
(I)	$2h + k$	(P)	6
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(III)	$\dfrac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R)	$\dfrac{5}{4}$
(IV)	α	(S)	$\dfrac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\dfrac{10}{3}$

Which of the following is the only INCORRECT combination?

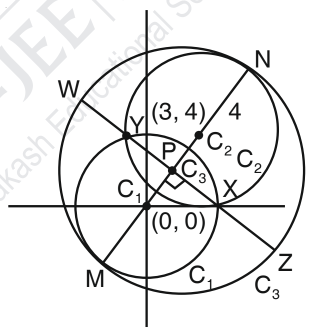
(A) (III), (R)
(B) (IV), (U)
(C) (IV), (S)
(D) (I), (P)

Answer (C)

Combine solution for (Q.3 & Q.4)

Sol. Given centres of C_1, C_2 + C_3 are co-linear

$$\begin{vmatrix} 0 & 0 & 1 \\ 3 & 4 & 1 \\ h & k & 1 \end{vmatrix} = 0$$



$\boxed{3k = 4h}$ ...(i)

& MN is diameter of C_3

So, $MN = 3 + \sqrt{(3-0) + (4-0)^2} + 4$

$= 12$

So $\boxed{r = 6}$ ...(ii)

Given C_3 touches C_1 at M $C_1(0, 0)$

So, $|C_1C_3| = |r - 3|$ $C_3(h, k)$

$\Rightarrow h^2 + k^2 = 9$ ...(iii)

From equation (i) + (iii)

$$h = \pm\frac{9}{5} \;\&\; k = \pm\frac{12}{5}$$

So centre of C_3 is $\left(\dfrac{9}{5}, \dfrac{12}{5}\right)$

Now equation of XY is

$\Rightarrow C_1 - C_2 = 0$

$\Rightarrow 6x + 8y = 18$

$\boxed{3x + 4y = 9}$ ...(iv)

Now $C_1P = \frac{9}{5}$ (distance from origin to equation of XY)

Now $PY^2 = C_1Y^2 - C_1P^2 = 9 - \frac{81}{25} = \frac{144}{25}$

$XY = 2PY = 2\times\frac{12}{5} = \frac{24}{5}$

Similarly equation of ZW is $3x + 4y = 9$

So length of perpendicular from C_3 to $ZW = \left|\frac{3\left(\frac{9}{5}\right)+4\left(\frac{12}{5}\right)-9}{5}\right|$

$= \frac{6}{5}$

Now $ZW = 2\sqrt{(6)^2 - \left(\frac{6}{5}\right)^2} = \frac{24\sqrt{6}}{5}$

(I) $2h + k = 2\times\frac{9}{5}+\frac{12}{5} = \frac{30}{5} = 6$ $\boxed{(I)\to P}$

(II) $\frac{\text{Length of ZW}}{\text{Length of XY}} = \sqrt{6}$ $\boxed{(II)\to Q}$

(III) $\frac{\text{Area of }\Delta MZN}{\text{Area of }\Delta ZMW} = \frac{\frac{1}{2}\times MN\times PZ}{\frac{1}{2}\times ZW\times MP} = \frac{\frac{1}{2}\times(2)(6)\times\frac{1}{2}(ZW)}{\frac{1}{2}(ZW)\times(MC_1 + C_1P)}$

$= \frac{6}{\left(3+\frac{9}{5}\right)} = \frac{6\times 5}{24} = \frac{5}{4}$ $\boxed{(III)\to R}$

(IV) Tangent at M, is also tangent to parabola $x^2 = 8\alpha y$

So slope of tangent at $M = -\left(\frac{1}{4/3}\right) = -\frac{3}{4}$

So equation of tangent at M to C_1 is

$y = mx \pm a\sqrt{1+m^2}$ where $a = 3$, $m = -\frac{3}{4}$

$y = -\frac{3}{4}x - 3\sqrt{1+\frac{9}{16}}$ ($\because$ y-intercept is –ve)

$\Rightarrow$ $\boxed{4y + 3x + 15 = 0}$

which is tangent to $x^2 = 8\alpha y$

So equation of tangent to $x^2 = 8\alpha y$ is $y = mx - 2\alpha m^2$

So by compare, so

$\boxed{\alpha = \frac{10}{3}}$ $\boxed{(IV)\to U}$

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