Date: 10/03/2023

Question Paper Code

T23 511

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Time: 2½ Hrs.

Class-X

Max. Marks: 80

MATHEMATICS

(ICSE 2022-23)

Answers & Solutions

GENERAL INSTRUCTIONS

(i) You will **not** be allowed to write during first 15 minutes.

(ii) Attempt **all** questions from **Section A** and any four questions from **Section B.**

(iii) **Omission of essential working will result in loss of marks.**

(iv) The intended marks for questions or parts of questions are given in brackets [ ].

(v) **Mathematical tables and graph papers are provided.**

SECTION-A (40 Marks)

*(Attempt **all** questions from this Section.)*

Choose the correct answers to the questions from the given options. **[15×1=15]**

(Do not copy the questions, write the correct answer only.)

1. (i) If $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$, the value of x and y respectively are

(a) 1, –2 (b) –2, 1

(c) 1, 2 (d) –2, –1

Answer (a) **[1]**

Sol. $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x \\ 4y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$

$\Rightarrow 2x = 2$ and $4y = -8$

$\Rightarrow x = 1$ and $y = -2$

Option (a) is correct

(ii) If $x - 2$ is a factor of $x^3 - kx - 12$, then the value of k is

(a) 3 (b) 2

(c) –2 (d) –3

Answer (c) **[1]**

Sol. $x^3 - kx - 12$

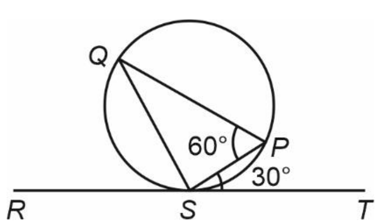
$p(2) = (2)^3 - 2k - 12 = 0$

$\Rightarrow 8 - 2k - 12 = 0$

$\Rightarrow k = -2$

Option (c) is correct

(iii) In the given diagram RT is a tangent touching the circle at S. If $\angle PST = 30°$ and $\angle SPQ = 60°$ then $\angle PSQ$ is equal to



(a) 40° (b) 30°

(c) 60° (d) 90°

Answer (d) **[1]**

Sol. $\angle SQP = \angle PST = 30°$

$\therefore$ $\angle PSQ = 180° - (60° + 30°)$

$\Rightarrow$ $\angle PSQ = 90°$

Option (d) is correct

(iv) A letter is chosen at random from all the letters of the English alphabets. The probability that the letter chosen is a vowel, is

(a) $\frac{4}{26}$ (b) $\frac{5}{26}$

(c) $\frac{21}{26}$ (d) $\frac{5}{24}$

Answer (b) **[1]**

Sol. Number of vowels in English Alphabet = 5

Total number of letters in English Alphabet = 26

$\therefore$ Required probability $= \frac{5}{26}$

(v) If 3 is a root of the quadratic equation $x^2 - px + 3 = 0$ then p is equal to

(a) 4 (b) 3

(c) 5 (d) 2

Answer (a) **[1]**

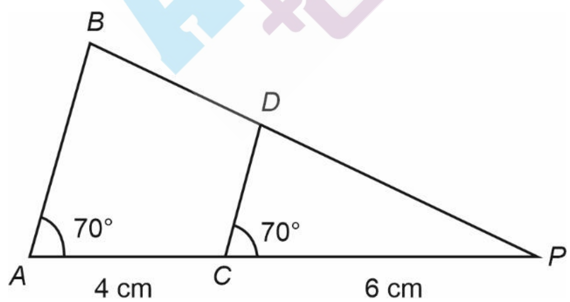
Sol. $3^2 - p(3) + 3 = 0$

$\Rightarrow$ $9 - 3p + 3 = 0$

$\Rightarrow$ $3p = 12$

$\Rightarrow$ $p = 4$

(vi) In the given figure $\angle BAP = \angle DCP = 70°$, $PC = 6$ cm and $CA = 4$ cm, then $PD : DB$ is :



(a) 5 : 3 (b) 3 : 5

(c) 3 : 2 (d) 2 : 3

Answer (c) **[1]**

Sol. In ΔPAB,

$\frac{CP}{AC} = \frac{DP}{BD}$ [By BPT]

$\Rightarrow$ $\frac{6}{4} = \frac{PD}{DB}$

$\Rightarrow$ $PD : DB = 3 : 2$

(vii) The printed price of an article is ₹3080. If the rate of GST is 10% then the GST charged is:

(a) ₹154 (b) ₹308

(c) ₹30.80 (d) ₹15.40

Answer (b) **[1]**

Sol. Printed price = ₹3080

GST rate = 10%

GST charged $= \dfrac{3080 \times 10}{100} = ₹308$

(viii) (1 + sinA) (1 – sinA) is equal to:

(a) cosec^2A (b) $\sin^2A$

(c) $\sec^2A$ (d) $\cos^2A$

Answer (d) **[1]**

Sol. (1 + sinA) (1 – sinA)

$\Rightarrow$ $1 - \sin^2A$

$\Rightarrow$ $\cos^2A$ $[\because \sin^2A + \cos^2A = 1]$

(ix) The coordinates of the vertices of ΔABC are respectively (–4, –2), (6, 2) and (4, 6). The centroid G of ΔABC is :

(a) (2, 2) (b) (2, 3)

(c) (3, 3) (d) (0, –1)

Answer (a) **[1]**

Sol. $A(-4, -2)$, $B(6, 2)$, $C(4, 6)$

Centroid $(G) = \left(\dfrac{-4+6+4}{3}, \dfrac{-2+2+6}{3}\right) = (2, 2)$

(x) The n^{th} term of an Arithmetic progression (A.P.) is $2n + 5$. The 10^{th} term is:

(a) 7 (b) 15

(c) 25 (d) 45

Answer (c) **[1]**

Sol. $a_{10} = 2(10) + 5 = 25$

(xi) The mean proportional between 4 and 9 is:

(a) 4 (b) 6

(c) 9 (d) 36

Answer (b) **[1]**

Sol. $b = \sqrt{ac}$

$\Rightarrow$ $b = \sqrt{4 \times 9} = 6$

(xii) Which of the following cannot be determined graphically for a grouped frequency distribution?

(a) Median (b) Mode

(c) Quartiles (d) Mean

Answer (d) **[1]**

Sol. Mean cannot be determined graphically for a grouped frequency distribution.

(xiii) Volume of a cylinder of height 3 cm is 48π. Radius of the cylinder is:

(a) 48 cm (b) 16 cm

(c) 4 cm (d) 24 cm

Answer (c) **[1]**

Sol. Volume of cylinder = $\pi r^2 h$

$48\pi = \pi r^2 \times 3$

$r^2 = 16$

$r = 4$ cm

(xiv) Naveen deposits ₹800 every month in a recurring deposit account for 6 months. If he receives ₹4884 at the time of maturity, then the interest he earns is:

(a) ₹84 (b) ₹42

(c) ₹24 (d) ₹284

Answer (a) **[1]**

Sol. Interest = ₹4884 – ₹4800

= ₹84

(xv) The solution set for the inequation $2x + 4 \le 14$, $x \in W$ is:

(a) {1, 2, 3, 4, 5} (b) {0, 1, 2, 3, 4, 5}

(c) {1, 2, 3, 4} (d) {0, 1, 2, 3, 4}

Answer (b) **[1]**

Sol. $2x + 4 \le 14$, $x \in W$

$2x \le 10$

$x \le 5$

$x = \{0, 1, 2, 3, 4, 5\}$

2. (i) Find the value of 'a' if $x – a$ is a factor of the polynomial $3x^3 + x^2 – ax – 81$. **[4]**

(ii) Salman deposits ₹ 1000 every month in a recurring deposit account for 2 years. If he receives ₹ 26000 on maturity, find: **[4]**

(a) the total interest Salman earns.

(b) the rate of interest.

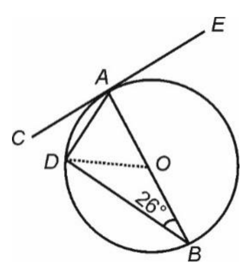
(iii) In the given figure O, is the centre of the circle. CE is a tangent to the circle at A.
If $\angle ABD = 26°$, then find: **[4]**

(a) $\angle BDA$

(b) $\angle BAD$

(c) $\angle CAD$

(d) $\angle ODB$



Sol. (i) If $(x – a)$ is a factor of the polynomial

$p(x) = 3x^3 + x^2 – ax – 81$, then

$p(a) = 0$ [Using factor theorem] **[1]**

$\Rightarrow$ $3(a)^3 + a^2 - a(a) - 81 = 0$ **[½]**

$\Rightarrow$ $3a^3 - 81 = 0$ **[1]**

$\Rightarrow$ $a^3 = 27$ **[1]**

$\Rightarrow$ $a = 3$ **[½]**

(ii) Given, P = ₹ 1000, n = 2 years = 24 months, M.V. = ₹ 26000

Let rate of interest be r% p.a. and interest earned be I,

(a) M.V. = $Pn + I$ **[½]**

$\Rightarrow$ 26000 = 1000 × 24 + I **[½]**

$\Rightarrow$ I = 26000 – 24000 **[½]**

= ₹2000 **[½]**

(b) Now,

$$I = 2000 = \frac{P \times n \times (n+1) \times r}{2 \times 12 \times 100}$$ **[½]**

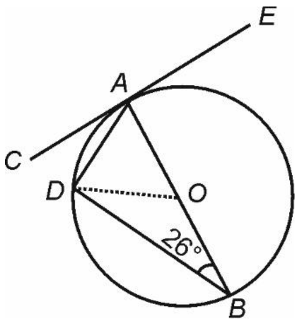
$$\Rightarrow \quad 2000 = \frac{1000 \times 24 \times 25 \times r}{2400}$$ **[½]**

$$\Rightarrow \quad r = \frac{2000 \times 2400}{24 \times 25 \times 1000}$$ **[½]**

$\Rightarrow$ r = 8% p.a. **[½]**

Hence, the total interest Salman earns is ₹ 2000 and the rate of interest is 8% p.a.

(iii)



(a) $\angle BDA = 90°$ [Angle in semicircle] **[1]**

(b) $\angle BAD = 180° - \angle BDA - \angle ABD$ [Angle sum property of triangle] **[½]**

$= 180° - 90° - 26°$

$= 64°$ **[½]**

(c) $\angle CAD = \angle OAC - \angle BAD$

$= 90° - 64°$ [$\because OA \perp EC$] **[½]**

$= 26°$ **[½]**

(d) $\angle ODB = \angle OBD = 26°$ [$\because OB = OD$ = radii of circle] **[1]**

3. (i) Solve the following quadratic equation : **[4]**

$x^2 + 4x - 8 = 0$

Give your answer correct to one decimal place.

(*Use mathematical tables if necessary.*)

(ii) Prove the following identity : **[4]**

$(\sin^2\theta - 1)(\tan^2\theta + 1) + 1 = 0$

(iii) Use **graph sheet** to answer this question. Take 2 cm = 1 unit along both the axes. **[5]**

(a) Plot A, B, C where $A(0, 4)$, $B(1, 1)$ and $C(4, 0)$.

(b) Reflect A and B on the x-axis and name them as E and D respectively.

(c) Reflect B through the origin and name it F. Write down the coordinates of F.

(d) Reflect B and C on the y-axis and name them as H and G respectively.

(e) Join points A, B, C, D, E, F, G, H and A in order and name the closed figure formed.

Sol. (i) $x^2 + 4x - 8 = 0$

Comparing $x^2 + 4x - 8 = 0$ with $ax^2 + bx + c = 0$,

we get

$a = 1$, $b = 4$ and $c = -8$ **[½]**

and so, $x = \dfrac{-4 \pm \sqrt{4^2 - 4\times1\times(-8)}}{2\times1}$ $\left[\because\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$ **[½]**

$= \dfrac{-4 \pm \sqrt{16+32}}{2}$ **[½]**

$= \dfrac{-4 \pm \sqrt{48}}{2}$ **[½]**

$= \dfrac{-4 \pm 4\sqrt{3}}{2}$ **[½]**

$= -2 \pm 2\sqrt{3}$ **[½]**

$= -2 + 2\sqrt{3}$ or $-2 - 2\sqrt{3}$

$= -2 + 2\times1.7$ or $-2 - 2\times1.7$ **[½]**

$= -2 + 3.4$ or $-2 - 3.4$

$= 1.4$ or -5.4 **[½]**

(ii) LHS = $(\sin^2\theta - 1)(\tan^2\theta + 1) + 1$

$\because$ $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta - 1 = -\cos^2\theta$ and $1 + \tan^2\theta = \sec^2\theta$ **[1]**

LHS = $-\cos^2\theta \cdot \sec^2\theta + 1$ **[½]**

$= -\cos^2\theta \cdot \dfrac{1}{\cos^2\theta} + 1$ $\left[\because\ \sec\theta = \dfrac{1}{\cos\theta}\right]$ **[1]**

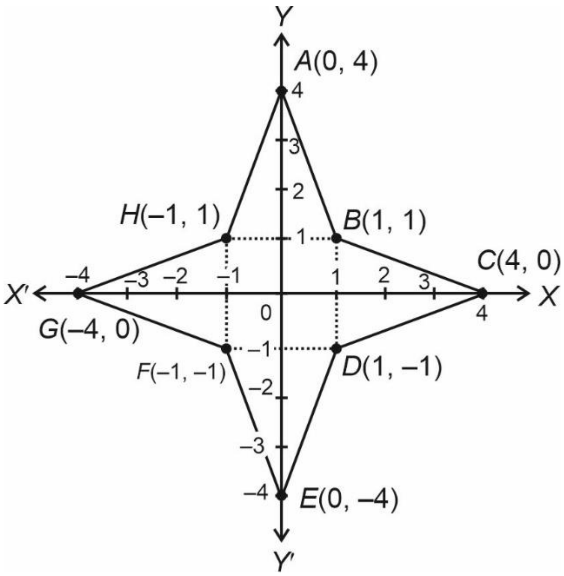
$= -1 + 1$ **[½]**

$= 0$ = RHS **[½]**

$\therefore$ LHS = RHS **[½]**

Hence, proved.

(iii)



[4]

(c) Coordinates of F = (–1, –1). [½]

(e) Closed figure formed is concave octagon. [½]

SECTION-B (40 Marks)

*(Attempt **any four** questions from this Section.)*

4. (i) If $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. [3]

Find $A(B + C) - 14I$

(ii) ABC is a triangle whose vertices are A(1, –1), B(0, 4) and C(–6, 4). [3]

D is the midpoint of BC. Find the:

(a) coordinates of D.

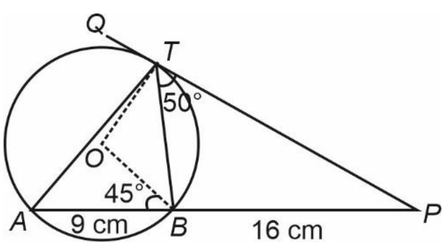
(b) equation of the median AD.

(iii) In the given figure, O is the centre of the circle. PQ is a tangent to the circle at T. [4]

Chord AB produced meets the tangent at P.

AB = 9 cm, BP = 16 cm, $\angle PTB$ = 50°

$\angle OBA$ = 45°



Find:

(a) length of PT

(b) $\angle BAT$

(c) $\angle BOT$

(d) $\angle ABT$

Sol. (i) $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix}$.

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A(B + C) - 14I$

$= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1+4 & 2+1 \\ 2+1 & 4+5 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **[1]**

$= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$ **[½]**

$= \begin{bmatrix} 5+9 & 3+27 \\ 10+12 & 6+36 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$ **[½]**

$= \begin{bmatrix} 14 & 30 \\ 22 & 42 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$ **[½]**

$= \begin{bmatrix} 0 & 30 \\ 22 & 28 \end{bmatrix}$ **[½]**

(ii) Given $A(1, -1)$, $B(0, 4)$ and $C(-6, 4)$

(a) Coordinates of D = mid point of BC [Given]

mid point of $BC = \left(\dfrac{0-6}{2}, \dfrac{4+4}{2}\right)$ **[½]**

$= (-3, 4)$ **[½]**

(b) Given, $A(1, -1)$, $D(-3, 4)$

Equation of the median AD is given by,

$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ **[½]**

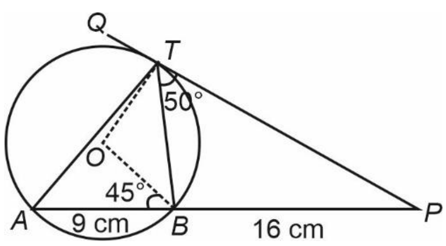
$y - (-1) = \dfrac{4-(-1)}{-3-1}(x-1)$ **[½]**

$y + 1 = \dfrac{5}{-4}(x-1)$ **[½]**

$-4y - 4 = 5x - 5$ **[½]**

$5x + 4y - 1 = 0$

(iii) (a)



$\because \quad PA \times PB = PT^2$

$\Rightarrow \quad (16 + 9)16 = PT^2$

⇒ $PT^2 = 400$

⇒ $PT = 20$ cm **[1]**

∴ The length of PT is 20 cm.

(b) $\angle BAT = \angle PTB = 50°$ [Angles in the alternate segment] **[1]**

(c) $\angle BAT = \frac{1}{2}\angle BOT$ [∵ Angle subtended by an arc of a circle at the centre is twice the angle subtended by the same arc at a point on the circumference]

⇒ $\angle BOT = 100°$ **[1]**

(d) In ΔBOT

$\angle OTB + \angle OBT + \angle BOT = 180°$

⇒ $2\angle OBT = 180° - 100°$ [∵ $\angle OTB = \angle OBT$]

⇒ $\angle OBT = 40°$ **[½]**

∴ $\angle ABT = \angle ABO + \angle OBT$

$= 45° + 40°$

$= 85°$

∴ $\angle ABT = 85°$ **[½]**

5. (i) Mrs. Arora bought the following articles from a departmental store: **[3]**

S.No.	Item	Price	Rate of GST	Discount
1.	Hair oil	₹ 1200	18%	₹ 100
2.	Cashew nuts	₹ 600	12%	–

Find the:

(a) Total GST paid.

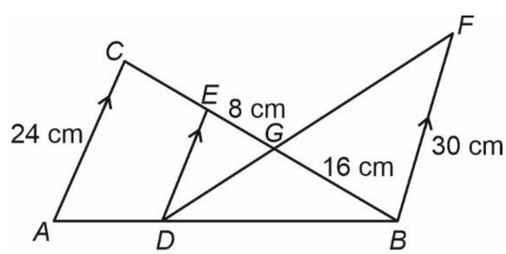
(b) Total bill amount including GST.

(ii) Solve the following inequation. Write down the solution set and represent it on the real number line. **[3]**

$-5(x - 9) \geq 17 - 9x > x + 2, x \in R$

(iii) In the given figure, $AC \parallel DE \parallel BF$. **[4]**

If $AC = 24$ cm, $EG = 8$ cm, $GB = 16$ cm, $BF = 30$ cm.



(a) Prove $\Delta GED \sim \Delta GBF$

(b) Find DE

(c) $DB : AB$

Sol. (i) (a) Total GST paid **[1]**

S. No.	Item	Price	Rate of GST	Discount	Price after discount	GST paid
1.	Hair oil	₹1200	18%	₹100	₹1100	$1100\times\frac{18}{100}=$₹198
2.	Cashew nuts	₹600	12%	–	₹600	$600\times\frac{12}{100}=$₹72
						Total GST paid = 198 + 72 = ₹270

(b) Total bill amount

1. Hair oil: ₹1100 + ₹198 = ₹1298 **[1]**

2. Cashew nuts : ₹600 + ₹72 = ₹672

Total bill amount = ₹1298 + ₹672

= ₹1,970 **[1]**

(ii) $-5(x-9) \geq 17-9x > x+2,\ x \in R$

$\Rightarrow \quad -5x+45 \geq 17-9x > x+2\ ,\ x \in R$

Now, $-5x+45 \geq 17-9x$

$\Rightarrow \quad 9x-5x \geq 17-45$

$\Rightarrow \quad 4x \geq -28$

$x \geq -7$ ...(i) **[1]**

Again, $17-9x > x+2$

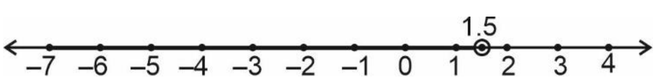
$\Rightarrow \quad 17-2 > 9x+x$

$\Rightarrow \quad 15 > 10x$

$\Rightarrow \quad x < \frac{15}{10}$

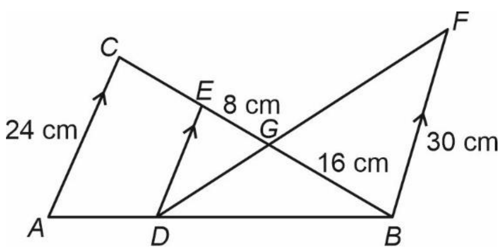
$\Rightarrow \quad x < 1.5$ ...(ii) **[1]**

Hence, solution set is $\{x : -7 \leq x < 1.5\}$



[1]

(iii)



(a) In ΔGED and ΔGBF,

$\angle EGD = \angle FGB$ [Vertically opposite angles]

$\angle EDG = \angle GFB$ [Alternate interior angles]

$\therefore \quad \Delta GED \sim \Delta GBF$ [By AA criterion] **[1]**

(b) $\dfrac{EG}{DE} = \dfrac{GB}{BF}$ [$\because \Delta GED \sim \Delta GBF$]

$\Rightarrow \quad \dfrac{8}{DE} = \dfrac{16}{30}$

$\Rightarrow \quad DE = \dfrac{30 \times 8}{16}$

$\Rightarrow \quad DE = 15$ cm **[1]**

(c) In ΔDBE and ΔABC

$\angle BAC = \angle BDE$ [$\because DE \parallel AC$]

$\angle ACB = \angle DEB$ [$\because DE \parallel AC$]

$\Delta DBE \sim \Delta ABC$ [BY AA criterion] **[1]**

$\Rightarrow \quad \dfrac{DB}{AB} = \dfrac{DE}{AC}$

$\Rightarrow \quad \dfrac{DB}{AB} = \dfrac{15}{24}$

$\Rightarrow \quad \dfrac{DB}{AB} = \dfrac{5}{8}$ **[1]**

6. (i) The following distribution gives the daily wages of 60 workers of a factory. **[3]**

Daily income in ₹	Number of workers (*f*)
200 – 300	6
300 – 400	10
400 – 500	14
500 – 600	16
600 – 700	10
700 – 800	4

Use graph paper to answer this question.

Take 2 cm = ₹100 along one axis and 2 cm = 2 workers along the other axis.

Draw a histogram and hence find the mode of the given distribution.

(ii) The 5^{th} term and the 9^{th} term of an Arithmetic Progression are 4 and –12 respectively. **[3]**

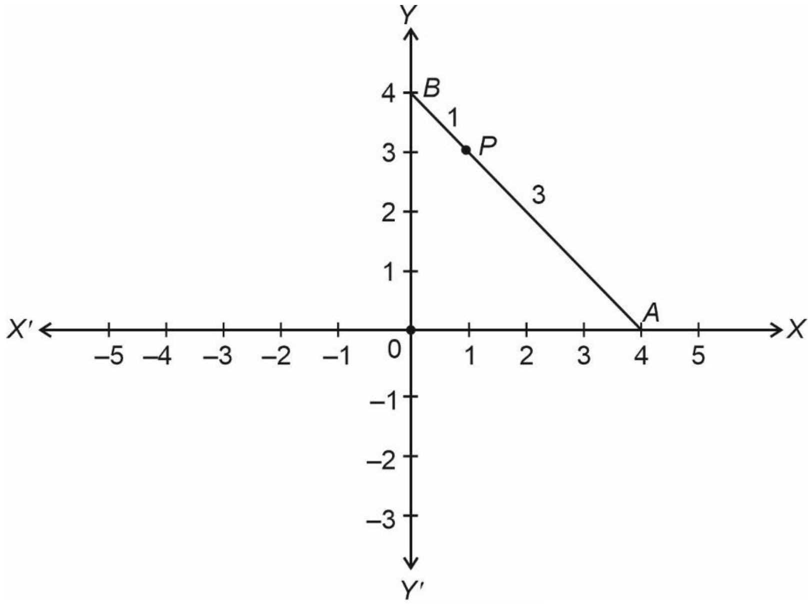
Find :

(a) the first term

(b) common difference

(c) sum of 16 terms of the AP.

(iii) A and B are two points on the x-axis and y-axis respectively. **[4]**

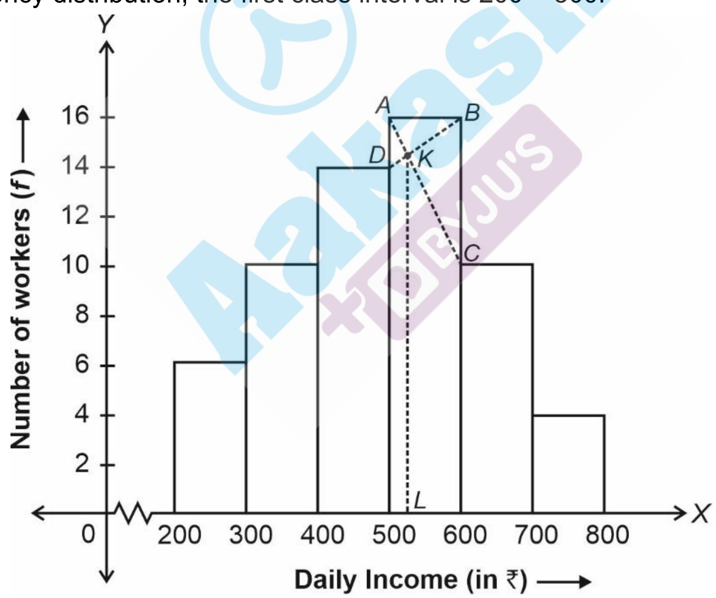


(a) Write down the coordinates of A and B.

(b) P is a point on AB such that $AP : PB = 3 : 1$. Using section formula find the coordinates of point P.

(c) Find the equation of a line passing through P and perpendicular to AB.

Sol. (i) In the given frequency distribution, the first class interval is 200 – 300.



[2]

Here, L = 525

∴ Mode = 525 **[1]**

(ii) 5^{th} term, $a_5 = 4$

9^{th} term, $a_9 = -12$

(a) $a_5 = a + 4d$, $a_9 = a + 8d$

$a + 4d = 4$ ...(i)

$a + 8d = -12$ ...(ii)

From (i) and (ii), we get

$a = 20$ and $d = -4$ **[1]**

∴ The first term of the AP is 20

(b) The common difference of the given AP is –4 **[1]**

(c) $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ **[½]**

$\Rightarrow \quad S_{16} = \frac{16}{2}\left[2\times 20+(16-1)(-4)\right]$

$= 8\left[40-60\right]$

$= -160$ **[½]**

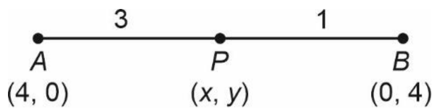
(iii) (a) Coordinates of A = (4, 0) **[½]**

Coordinates of B = (0, 4) **[½]**

(b) $AP : PB = 3 : 1$

Let coordinates of P be (x, y)

Using section formula



$x = \frac{3(0)+1(4)}{3+1}$ **[½]**

$\Rightarrow \quad x = \frac{4}{4} = 1$

$y = \frac{3(4)+1(0)}{3+1}$ **[½]**

$\Rightarrow \quad y = \frac{12}{4} = 3$

$\therefore$ Hence, the coordinates of P are (1, 3)

(c) Slope of $AB = \frac{4-0}{0-4} = -1$ **[½]**

Slope of line perpendicular to AB = 1

Equation of line passing through P and perpendicular to AB is given by

$(y - y_1) = m(x - x_1)$ **[½]**

$(y - 3) = 1\,(x - 1)$

$\Rightarrow \quad y - 3 = x - 1$

$\Rightarrow \quad x - y + 2 = 0$ **[1]**

7. (i) A bag contains 25 cards, numbered through 1 to 25. A card is drawn at random. What is the probability that the number on the card drawn is : **[3]**

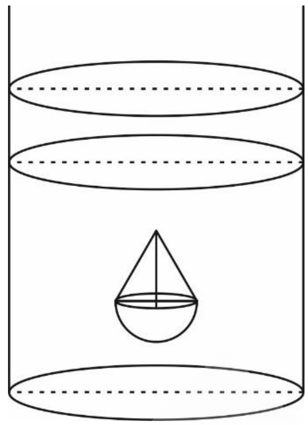
(a) multiple of 5

(b) a perfect square

(c) a prime number?

(ii) A man covers a distance of 100 km, travelling with a uniform speed of x km/hr. Had the speed been 5 km/hr more it would have taken 1 hour less. Find x the original speed. **[3]**

(iii) A solid is in the shape of a hemisphere of radius 7 cm, surmounted by a cone of height 4 cm. The solid is immersed completely in a cylindrical container filled with water to a certain height. If the radius of the cylinder is 14 cm, find the rise in the water level. **[4]**



Sol. (i) (a) Total number of elementary events, $n(S) = 25$

The numbers which are multiple of 5 are 5, 10, 15, 20 and 25

Number of favourable events, $n(E) = 5$ **[½]**

$\therefore$ Required probability = $\frac{5}{25} = \frac{1}{5}$ **[½]**

(b) The numbers which are perfect square are 1, 4, 9, 16 and 25 **[½]**

Favourable number of events, n $(E) = 5$

$\therefore$ Required probability = $\frac{5}{25} = \frac{1}{5}$ **[½]**

(c) The numbers which are prime are 2, 3, 5, 7, 11, 13, 17, 19, 23 **[½]**

Favourable number of events, n$(E) = 9$

$\therefore$ Required probability = $\frac{9}{25}$ **[½]**

(ii) Original speed of man = x km/hr

Original time to cover 100 km $= \frac{100}{x}$ hr **[½]**

When the speed is increased by 5 km/hr, then increased speed = $(x + 5)$ km/hr

Time to cover 100 km with increased speed $= \frac{100}{x+5}$ hr **[½]**

Now, according to question

$$\frac{100}{x} - \frac{100}{x+5} = 1$$

$$\Rightarrow \quad \frac{100[x+5-x]}{x(x+5)} = 1$$

$\Rightarrow \quad x^2 + 5x - 500 = 0$ **[½]**

$\Rightarrow \quad x^2 + 25x - 20x - 500 = 0$

$\Rightarrow \quad (x + 25)(x - 20) = 0$ **[½]**

$\Rightarrow \quad x = 20, x = -25$ (reject)

$\therefore \quad x = 20$ km/hr **[1]**

(iii) Volume of solid = Volume of cone + volume of hemisphere

$$= \frac{\pi}{3}(7)^2 4 + \frac{2}{3}\pi(7)^3$$ **[1]**

$$= \frac{\pi}{3}(7)^2\left[4 + (2 \times 7)\right]$$

$$= \frac{22}{7} \times 7^2 \times \frac{18}{3}$$

$$= 6 \times 22 \times 7 = 924 \text{ cm}^3$$ **[1]**

Let rise in water level of cylinder be H cm

Rise of volume in cylinder = Volume of solid immersed in cylinder

$\pi R^2 H = 924$

$\frac{22}{7} \times 14^2 \times H = 924$ **[1]**

$H = \frac{924 \times 7}{22 \times 14^2}$

$H = 1.5$ cm

Hence, rise in water level is 1.5 cm **[1]**

8. (i) The following table gives the marks scored by a set of students in an examination. Calculate the mean of the distribution by using the short cut method. **[3]**

Marks	**Number of students (*f*)**
0 – 10	3
10 – 20	8
20 – 30	14
30 – 40	9
40 – 50	4
50 – 60	2

(ii) What number must be added to each of the numbers 4, 6, 8, 11 in order to get the four numbers in proportion? **[3]**

(iii) Using ruler and compass construct a triangle ABC in which AB = 6 cm, $\angle BAC$ = 120° and AC = 5 cm. Construct a circle passing through A, B and C. Measure and write down the radius of the circle. **[4]**

Sol. (i)

Marks	Number of students (f_i)	Class marks (x_i)	$d_i = x_i - 25$	$f_i d_i$
0 – 10	3	5	–20	–60
10 – 20	8	15	–10	–80
20 – 30	14	25 = A	0	0
30 – 40	9	35	10	90
40 – 50	4	45	20	80
50 – 60	2	55	30	60
Total	**N = 40**			**90**

[1]

Hence, by assumed mean method

$\vec{x} = A + \frac{1}{N}\sum_{i=1}^{6} f_i d_i$ [1]

$= 25 + \frac{90}{40}$

= 25 + 2.25

= 27.25 [1]

(ii) Let x be the number that to be added

$\Rightarrow \frac{4+x}{6+x} = \frac{8+x}{11+x}$ [1]

By applying componendo and dividendo, we get

$\Rightarrow \frac{4+x+6+x}{4+x-(6+x)} = \frac{8+x+11+x}{8+x-(11+x)}$ [½]

$\Rightarrow \frac{10+2x}{-2} = \frac{19+2x}{-3}$ [½]

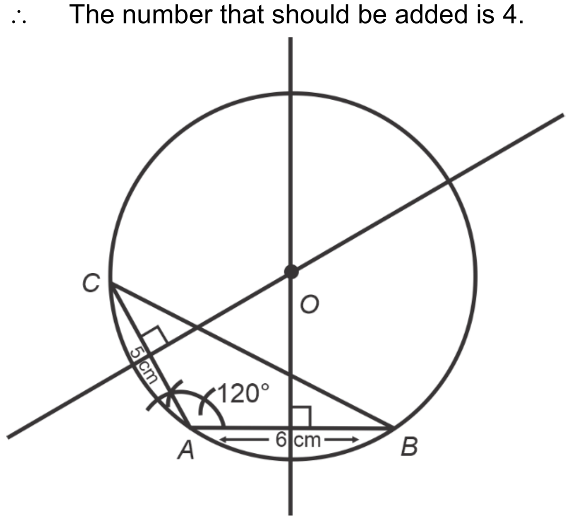
$\Rightarrow$ 30 + 6x = 38 + 4x

$\Rightarrow$ 2x = 8

$\Rightarrow$ x = 4 [½]

$\therefore$ The number that should be added is 4. [½]

(iii)



[3]

Steps of Construction:

(i) Draw ΔABC with AB = 6 cm, AC = 5 cm, and $\angle BAC$ = 120°.

(ii) Draw the perpendicular bisectors of any two sides of the triangle.

Let the perpendicular bisectors of AC and AB intersect each other at O.

(iii) Taking O as centre and radius OA (or OB, or OC), draw a circle.

The circle so obtained is the required circle.

The radius of the circle is approximately 5.5 cm. **[1]**

9. (i) Using Componendo and Dividendo solve for x: **[3]**

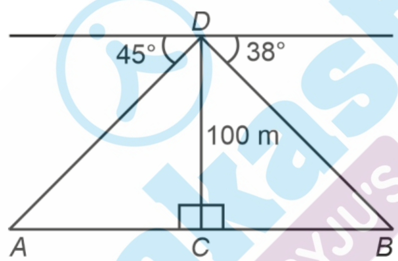
$$\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+2}-\sqrt{2x-1}}=3$$

(ii) Which term of the Arithmetic Progression (A.P.) 15, 30, 45, 60… is 300? **[3]**

Hence find the sum of all the terms of the Arithmetic Progression (A.P.)

(iii) From the top of a tower 100 m high a man observes the angles of depression of two ships **A** and **B**, on opposite sides of the tower as 45° and 38° respectively. If the foot of the tower and the ships are in the same horizontal line find the distance between the two ships **A** and **B** to the nearest metre. **[4]**

(*Use Mathematical Tables for this question.*)



Sol. (i) Given,

$$\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+2}-\sqrt{2x-1}}=\frac{3}{1}$$

Using Componendo and Dividendo, we get

$$\frac{\sqrt{2x+2}+\sqrt{2x-1}+\sqrt{2x+2}-\sqrt{2x-1}}{\sqrt{2x+2}+\sqrt{2x-1}-\sqrt{2x+2}+\sqrt{2x-1}}=\frac{3+1}{3-1}$$ **[1]**

$\Rightarrow \dfrac{2\sqrt{2x+2}}{2\sqrt{2x-1}}=\dfrac{4}{2}$ **[½]**

$\Rightarrow \dfrac{\sqrt{2x+2}}{\sqrt{2x-1}}=2$ **[½]**

Squaring both sides, we get

$\Rightarrow \dfrac{2x+2}{2x-1}=4$ **[½]**

$\Rightarrow$ $2x + 2 = 8x - 4$

$\Rightarrow$ $6x = 6$

$\Rightarrow$ $x = 1$ **[½]**

(ii) Given A.P.,

15, 30, 45, ….., 300

Here,

First term, $a = 15$

Common difference, $d = 30 - 15 = 15$ **[½]**

Let the n^{th} term be 300.

$\therefore \quad a_n = 300$

$\Rightarrow \quad a + (n - 1)d = 300$ **[½]**

$\Rightarrow \quad 15 + (n - 1)15 = 300$

$\Rightarrow \quad 15(1 + n - 1) = 300$

$\Rightarrow \quad n = \frac{300}{15} = 20$ **[½]**

$\therefore \quad 20^{th}$ term is 300. **[½]**

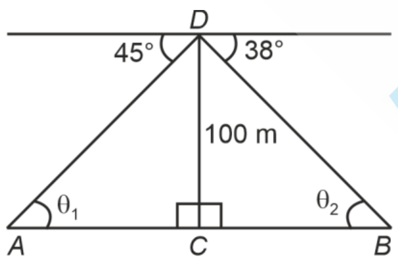
Also,

$S_{20} = \frac{20}{2}(15 + 300)$ $\left[\because S_n = \frac{n}{2}(a + a_n)\right]$ **[½]**

$\Rightarrow \quad S_{20} = 10 \times 315$

$= 3150$ **[½]**

(iii)



Here, $CD = 100$ m

Let, $\angle DAC = \theta_1 = 45°$ [Alternate interior angles]

$\angle DBC = \theta_2 = 38°$ [Alternate interior angles] **[½]**

In ΔACD,

$\tan\theta_1 = \frac{CD}{AC}$ **[½]**

$\Rightarrow \quad \tan 45° = \frac{100}{AC}$ **[½]**

$\Rightarrow \quad 1 = \frac{100}{AC}$

$\therefore \quad AC = 100$ m ...(i) **[½]**

In ΔBCD,

$\tan\theta_2 = \frac{CD}{CB}$ **[½]**

$\Rightarrow \tan 38° = \frac{100}{CB}$ **[½]**

$\Rightarrow CB = \frac{100}{\tan 38°}$

$\Rightarrow CB = 100 \times \cot 38°$

$= 100 \times 1.28$

$= 128$ m ...(ii) **[½]**

$\therefore AC + CB = 100 + 128$ [From (i) and (ii)]

$= 228$ m **[½]**

Hence, distance between the two ships A and B to the nearest metre is 228 m.

10. (i) Factorize completely using factor theorem: **[4]**

$2x^3 - x^2 - 13x - 6$

(ii) Use **graph paper** to answer this question. **[6]**

During a medical checkup of 60 students in a school, weights were recorded as follows:

Weight (in kg)	Number of students
28 – 30	2
30 – 32	4
32 – 34	10
34 – 36	13
36 – 38	15
38 – 40	9
40 – 42	5
42 – 44	2

Taking 2 cm = 2 kg along one axis and 2 cm = 10 students along the other axis draw an ogive. Use your graph to find the:

(a) median

(b) upper quartile

(c) number of students whose weight is above 37 kg

Sol. (i) $p(x) = 2x^3 - x^2 - 13x - 6$

Factors of 6 are ±1, ±2, ±3 and ±6 **[½]**

$p(1) = 2 - 1 - 13 - 6 = -18$

$p(-1) = -2 - 1 + 13 - 6 = 4$

$p(2) = 16 - 4 - 26 - 6 = -20$

$p(-2) = -16 - 4 + 26 - 6 = 0$ **[½]**

$\therefore$ $x + 2$ is a factor of $p(x)$

$$
\begin{array}{r|l|l}
x+2 & 2x^3 - x^2 - 13x - 6 & 2x^2 - 5x - 3 \\
 & 2x^3 + 4x^2 & \\
 & - \quad - & \\
\hline
 & -5x^2 - 13x - 6 & \\
 & -5x^2 - 10x & \\
 & + \quad + & \\
\hline
 & -3x - 6 & \\
 & -3x - 6 & \\
 & + \quad + & \\
\hline
 & 0 &
\end{array}
$$

[2]

$\Rightarrow$ $p(x) = (x + 2)(2x^2 - 5x - 3)$

$= (x + 2)(2x^2 - 6x + x - 3)$ **[½]**

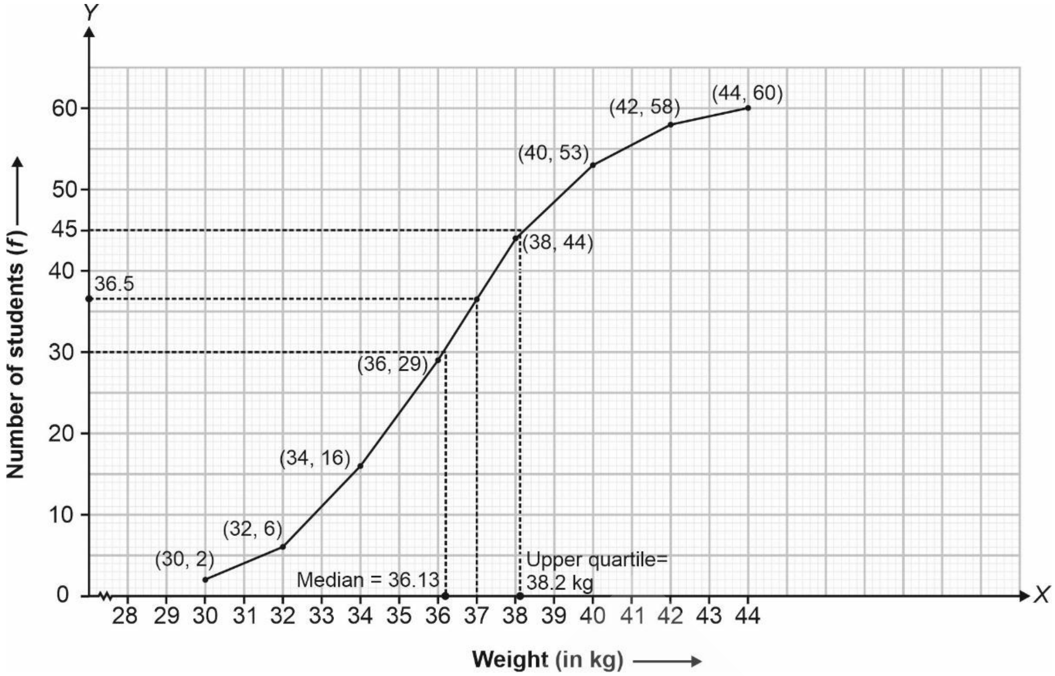
$= (x + 2)[2x(x - 3) + 1(x - 3)]$

$= (x + 2)(x - 3)(2x + 1)$ **[½]**

(ii)

Weight (in kg)	Number of students	Cummulative frequency
28 – 30	2	2
30 – 32	4	6
32 – 34	10	16
34 – 36	13	29
36 – 38	15	44
38 – 40	9	53
40 – 42	5	58
42 – 44	2	60

[1]



[2]

(a) Median = 36.13 kg **[1]**

(b) Upper quartile = 38.2 kgs

(c) Number of students whose weight is above 37 kg **[1]**

= 60 – 36.5

= 23.5 **[1]**

≃ 23

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