

Date: 21/03/2023



Question Paper Code

30/4/2

SET-2

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Time: 3 Hrs.

Class-X

Max. Marks: 80

MATHEMATICS (Standard) - Theory
(CBSE 2022-23)
Answers & Solution

GENERAL INSTRUCTIONS

Read the following instructions very carefully and follow them :

- (i) This question paper contains **38** questions. **All** questions are compulsory.
- (ii) Question paper is divided into **FIVE** sections – **Section A, B, C, D** and **E**.
- (iii) In **section A** – question number **1** to **18** are multiple choice questions (MCQs) and question number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **section B** – question number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.
- (v) In **section C** – question number **26** to **31** are Short Answer (SA) type questions carrying **3** marks each.
- (vi) In **section D** – question number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.
- (vii) In **section E** – question number **36** to **38** are **case based integrated units** of assessment questions carrying **4** marks each. Internal choice is provided in **2** marks question in each case-study.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section **B**, 2 questions in Section **C**, 2 questions in Section **D** and 3 questions in Section **E**.
- (ix) Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.
- (x) Use of calculators is **NOT allowed**.

SECTION-A

Section – A consists of Multiple Choice type questions of 1 mark each.

1. Which of the following is true for all values of θ ($0^\circ \leq \theta \leq 90^\circ$)? [1]

- (a) $\cos^2\theta - \sin^2\theta = 1$ (b) $\operatorname{cosec}^2\theta - \sec^2\theta = 1$
(c) $\sec^2\theta - \tan^2\theta = 1$ (d) $\cot^2\theta - \tan^2\theta = 1$

Answer (c) [1]

Sol. $\sec^2\theta - \tan^2\theta = 1$

2. If $k + 2$, $4k - 6$ and $3k - 2$ are three consecutive terms of an A.P., then the value of k is [1]

- (a) 3 (b) -3
(c) 4 (d) -4

Answer (a) [1]

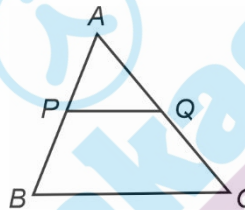
Sol. $(4k - 6) - (k + 2) = (3k - 2) - (4k - 6)$

$$\Rightarrow 3k - 8 = -k + 4$$

$$\Rightarrow 4k = 12$$

$$\Rightarrow k = 3$$

3. In $\triangle ABC$, $PQ \parallel BC$. If $PB = 6$ cm, $AP = 4$ cm, $AQ = 8$ cm, find the length of AC . [1]



- (a) 12 cm (b) 20 cm
(c) 6 cm (d) 14 cm

Answer (b) [1]

Sol. $\frac{AP}{AB} = \frac{AQ}{AC}$ [By BPT]

$$\Rightarrow \frac{4}{10} = \frac{8}{AC}$$

$$\therefore AC = 20 \text{ cm}$$

4. The ratio of HCF to LCM of the least composite number and the least prime number is [1]

- (a) 1 : 2 (b) 2 : 1
(c) 1 : 1 (d) 1 : 3

Answer (a) [1]

Sol. Least composite number is 4

and least prime number is 2

$$\Rightarrow \frac{\text{HCF}(2, 4)}{\text{LCM}(2, 4)} = \frac{2}{4} = \frac{1}{2}$$

5. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that the card drawn is not an ace is [1]

- (a) $\frac{1}{13}$ (b) $\frac{9}{13}$
(c) $\frac{4}{13}$ (d) $\frac{12}{13}$

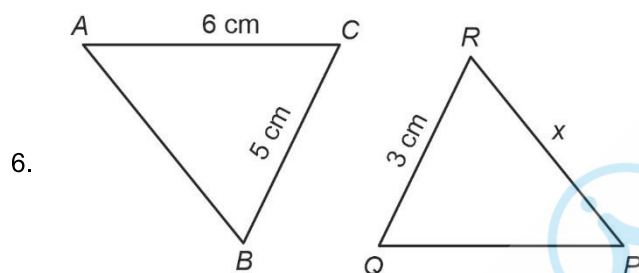
Answer (d)

[1]

Sol. Probability (not Ace) = $1 - P(\text{Ace})$

$$= 1 - \frac{4}{52}$$

$$= \frac{12}{13}$$



In the given figure, $\triangle ABC \sim \triangle QPR$. If $AC = 6$ cm, $BC = 5$ cm, $QR = 3$ cm and $PR = x$; then the value of x is [1]

- (a) 3.6 cm (b) 2.5 cm
(c) 10 cm (d) 3.2 cm

Answer (b)

[1]

Sol. $\triangle ABC \sim \triangle QPR$

$$\Rightarrow \frac{AC}{BC} = \frac{QR}{PR}$$

$$\Rightarrow \frac{6}{5} = \frac{3}{x}$$

$$\Rightarrow x = 2.5 \text{ cm}$$

7. The roots of the equation $x^2 + 3x - 10 = 0$ are : [1]

- (a) 2, -5 (b) -2, 5
(c) 2, 5 (d) -2, -5

Answer (a)

[1]

Sol. $x^2 + 3x - 10 = 0$

$$\therefore (x + 5)(x - 2) = 0$$

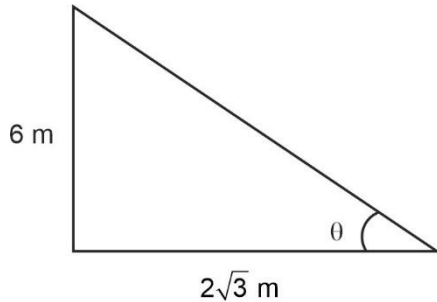
$$\Rightarrow x = -5, 2$$

8. If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then sun's elevation is [1]

- (a) 60° (b) 45°
(c) 30° (d) 90°

Answer (a) [1]

Sol.



$$\tan \theta = \frac{6}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \theta = 60^\circ$$

9. The distance of the point $(-6, 8)$ from origin is [1]

- (a) 6 (b) -6
(c) 8 (d) 10

Answer (d) [1]

Sol. Required distance $= \sqrt{(-6)^2 + (8)^2}$
 $= \sqrt{100}$
 $= 10$ units

10. What is the area of a semi-circle of diameter ' d '? [1]

- (a) $\frac{1}{16}\pi d^2$ (b) $\frac{1}{4}\pi d^2$
(c) $\frac{1}{8}\pi d^2$ (d) $\frac{1}{2}\pi d^2$

Answer (c) [1]

Sol. area of semicircle $= \frac{\pi \left(\frac{d}{2}\right)^2}{2} = \frac{1}{8}\pi d^2$

11. For the following distribution : [1]

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The sum of lower limits of median class and modal class is

- (a) 15 (b) 25
(c) 30 (d) 35

Answer (b) [1]

Sol. Median class = 10-15

Modal class = 15-20

\therefore Required sum = 10 + 15 = 25

12. The length of tangent drawn to a circle of radius 9 cm from a point 41 cm from the centre is

[1]

- (a) 40 cm
- (b) 9 cm
- (c) 41 cm
- (d) 50 cm

Answer (a)

[1]

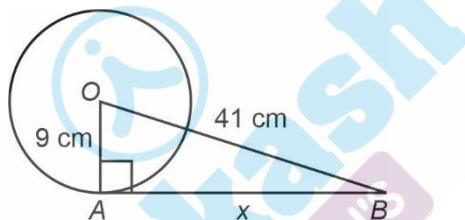
Sol. In $\triangle OAB$,

$$OA^2 + AB^2 = OB^2$$

$$\Rightarrow 9^2 + x^2 = 41^2$$

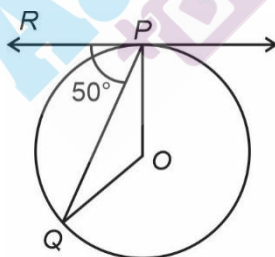
$$\Rightarrow x^2 = 41^2 - 9^2 = 1600$$

$$\Rightarrow x = 40 \text{ cm}$$



13. In the given figure, O is the centre of the circle and PQ is the chord. If the tangent PR at P makes an angle of 50° with PQ, then the measure of $\angle POQ$ is

[1]



- (a) 50°
- (b) 40°
- (c) 100°
- (d) 130°

Answer (c)

[1]

Sol. $\angle OPQ = \angle OPR - \angle QPR$

$$= 90^\circ - 50^\circ = 40^\circ$$

Also, $\angle OPQ = \angle OQP = 40^\circ$

$$\therefore \angle POQ = 180^\circ - 40^\circ - 40^\circ$$

$$= 100^\circ$$

14. A bag contains 5 red balls and n green balls. If the probability of drawing a green ball is three times that of a red ball, then the value of n is [1]

- (a) 18 (b) 15
(c) 10 (d) 20

Answer (b)

[1]

Sol. $P(\text{green ball}) = 3P(\text{red ball})$

$$\Rightarrow \frac{n}{5+n} = 3 \times \frac{5}{5+n}$$

$$\Rightarrow n = 15$$

15. If α, β are zeroes of the polynomial $x^2 - 1$, then the value of $(\alpha + \beta)$ is [1]

- (a) 2 (b) 1
(c) -1 (d) 0

Answer (d)

[1]

Sol. $\alpha + \beta = 0$ $\left[\because \alpha + \beta = \frac{-b}{a} \right]$

16. If α, β are the zeroes of the polynomial $p(x) = 4x^2 - 3x - 7$, then $\left(\frac{1}{\alpha} + \frac{1}{\beta} \right)$ is equal to: [1]

- (a) $\frac{7}{3}$ (b) $\frac{-7}{3}$
(c) $\frac{3}{7}$ (d) $\frac{-3}{7}$

Answer (d)

[1]

Sol. $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$$\therefore \alpha + \beta = \frac{3}{4}, \quad \alpha\beta = \frac{-7}{4}$$

$$\Rightarrow \frac{\alpha + \beta}{\alpha\beta} = \frac{3 \times 4}{4(-7)} = \frac{-3}{7}$$

17. The pair of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are: [1]

- (a) intersecting (b) parallel
(c) coincident (d) either intersecting or parallel

Answer (c)

[1]

Sol. $2x - 5y - 6 = 0$

$$-6x + 15y + 18 = 0$$

$$\frac{1}{3} = \frac{-1}{3} = \frac{-1}{3}$$

18. The distance of the point $(-1, 7)$ from x-axis is:

[1]

- (a) -1
- (b) 7
- (c) 6
- (d) $\sqrt{50}$

Answer (b)

[1]

Sol. Distance of point $(-1, 7)$ from x-axis is 7.

DIRECTIONS : In the question number **19** and **20**, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option out of the following :

19. **Assertion (A)** : a, b, c are in A.P. if and only if $2b = a + c$.

[1]

Reason (R) : The sum of first n odd natural numbers is n^2 .

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
- (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A)
- (c) Assertion (A) is true but Reason (R) is false
- (d) Assertion (A) is false but Reason (R) is true

Answer (b)

[1]

Sol. A : $b - a = c - b$

$$\Rightarrow 2b = a + c$$

R : Sum of first n odd natural numbers $= n^2$

20. **Assertion (A)** : The probability that a leap year has 53 Sundays is $\frac{2}{7}$.

[1]

Reason (R) : The probability that a non-leap year has 53 Sundays is $\frac{5}{7}$.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
- (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A)
- (c) Assertion (A) is true but Reason (R) is false
- (d) Assertion (A) is false but Reason (R) is true

Answer (c)

[1]

Sol. $P(\text{leap year having 53 Sundays}) = \frac{2}{7}$

$$P(\text{getting 53 Sundays in a non-leap year}) = \frac{1}{7}$$

SECTION-B

Section – B consists of Very Short Answer (VSA) type questions of 2 marks each.

21. (A) Evaluate : $\frac{5}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cot^2 45^\circ + 2\sin^2 90^\circ$ [2]

OR

(B) If θ is an acute angle and $\sin\theta = \cos\theta$, find the value of $\tan^2\theta + \cot^2\theta - 2$. [2]

Sol. (A) $\frac{5}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cot^2 45^\circ + 2\sin^2 90^\circ$
 $= \frac{5}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - (1)^2 + 2(1)^2$ [1/2]

$= \frac{5}{3} + \frac{4}{3} - 1 + 2$ [1/2]

$= 3 - 1 + 2$ [1/2]

$= 4$ [1/2]

OR

(B) $\sin\theta = \cos\theta$
 $\Rightarrow \frac{\sin\theta}{\cos\theta} = \tan\theta = 1$ [1/2]

$\Rightarrow \theta = 45^\circ$ [1/2]

$\therefore \tan^2 45^\circ + \cot^2 45^\circ - 2 = 1^2 + 1^2 - 2$
 $= 0$ [1/2]

22. If a fair coin is tossed twice, find the probability of getting 'atmost one head'. [2]

Sol. Total possible outcomes are HH, HT, TH, TT [1/2]

And favourable outcomes are HT, TH, TT [1/2]

\therefore Required probability $= \frac{3}{4}$ [1]

23. Two numbers are in the ratio 2 : 3 and their LCM is 180. What is the HCF of these numbers? [2]

Sol. Let the numbers be $2x$ and $3x$ [1/2]

Given,

LCM = 180

Clearly, HCF = x [1/2]

\therefore LCM (a, b) \times HCF (a, b) = $a \times b$

$\Rightarrow 180 \times x = 2x \times 3x$ [1/2]

$\Rightarrow x^2 - 30x = 0$

$\Rightarrow x(x - 30) = 0$

$\therefore x = 0$ or $x = 30$

$\therefore x = 0$ is not possible as HCF can't be 0

\therefore HCF = $x = 30$ [1/2]

24. (A) Find the sum and product of the roots of the quadratic equation $2x^2 - 9x + 4 = 0$. [2]

OR

(B) Find the discriminant of the quadratic equation $4x^2 - 5 = 0$ and hence comment on the nature of roots of the equation. [2]

Sol. (A) \therefore For $ax^2 + bx + c = 0$,

$$\text{Sum of roots} = \frac{-b}{a} \quad [1/2]$$

$$\text{Product of roots} = \frac{c}{a} \quad [1/2]$$

$$\Rightarrow \text{For } 2x^2 - 9x + 4 = 0$$

$$\therefore \text{Sum of roots} = \frac{-(-9)}{2} = \frac{9}{2} \quad [1/2]$$

$$\text{Product of roots} = \frac{4}{2} = 2 \quad [1/2]$$

OR

(B) For $ax^2 + bx + c = 0$, $D = b^2 - 4ac$

$$\therefore \text{For } 4x^2 - 5 = 0$$

$$D = (0)^2 - 4(4)(-5) \quad [1/2]$$

$$= 80 \quad [1/2]$$

$$\therefore D > 0 \quad [1/2]$$

$$\therefore \text{Roots are real and distinct.} \quad [1/2]$$

25. If one zero of the polynomial $p(x) = 6x^2 + 37x - (k - 2)$ is reciprocal of the other, then find the value of k . [2]

Sol. $p(x) = 6x^2 + 37x - (k - 2)$

Let α, β be the zeroes of $p(x)$

$$\therefore \beta = \frac{1}{\alpha} \quad [\text{Given condition}]$$

$$\therefore \alpha\beta = 1 \quad \dots(i) \quad [1/2]$$

$$\text{Also, } \alpha\beta = \frac{-(k-2)}{6} \quad \dots(ii) \quad [1/2]$$

From (i) and (ii),

$$\frac{-(k-2)}{6} = 1 \quad [1/2]$$

$$\Rightarrow 2 - k = 6$$

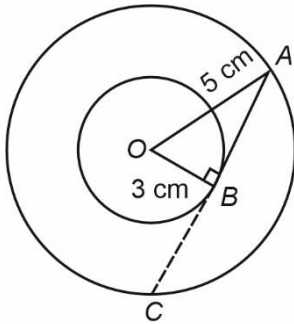
$$\Rightarrow k = -4 \quad [1/2]$$

SECTION-C

Section – C consists of Short Answer (SA) type questions of 3 marks each.

26. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. [3]

Sol. $\angle OBA = 90^\circ$ [A tangent to the circle is perpendicular to the radius through the point of contact] [½]



$$OA^2 = OB^2 + AB^2 \quad [½]$$

$$\Rightarrow (5)^2 = (3)^2 + AB^2$$

$$\Rightarrow AB^2 = 25 - 9 \quad [½]$$

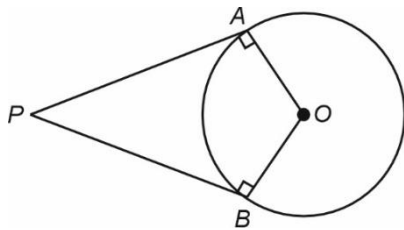
$$\Rightarrow AB = 4 \quad [½]$$

$$\Rightarrow \text{Length of chord } AC = 2AB = 2(4) = 8 \text{ cm} \quad [½]$$

\therefore The length of chord of the larger circle which touches the smaller circle is 8 cm. [½]

27. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre. [3]

Sol. Given : A circle with centre O. PA and PB are tangents to the circle at A and B.



To prove : $\angle APB + \angle AOB = 180^\circ$ [½]

Proof : We know that radius is perpendicular to tangent at point of contact. [½]

$$\Rightarrow OA \perp PA \text{ and } OB \perp PB$$

$$\Rightarrow \angle PAO = \angle PBO = 90^\circ \quad [½]$$

In quadrilateral PBOA,

$$\angle APB + \angle PBO + \angle BOA + \angle OAP = 360^\circ \quad [\text{Angle sum property of quadrilateral}]$$

$$\Rightarrow \angle APB + 90^\circ + \angle BOA + 90^\circ = 360^\circ \quad [½]$$

$$\therefore \angle APB + \angle BOA = 360^\circ - 180^\circ = 180^\circ \quad [½]$$

Hence, proved. [½]

28. Find the value of 'p' for which the quadratic equation [3]

$px(x - 2) + 6 = 0$ has two equal real roots.

Sol. For equal real roots, [½]

Discriminant, $D = 0$

Here, equation is

$$px(x - 2) + 6 = 0$$

$$\Rightarrow px^2 - 2px + 6 = 0 \quad [½]$$

$$\therefore D = b^2 - 4ac \text{ for } ax^2 + bx + c = 0$$

Here,

$$D = (-2p)^2 - 4(p)(6) \quad [½]$$

$$\Rightarrow 0 = 4p^2 - 24p \quad [½]$$

$$\Rightarrow 0 = 4p(p - 6)$$

$$\therefore p = 0 \text{ or } p = 6 \quad [½]$$

But $p \neq 0$ as coefficient of x^2 should be non-zero.

$$\therefore p = 6 \quad [½]$$

29. (A) The sum of first 15 terms of an A.P. is 750 and its first term is 15. Find its 20th term. [3]

OR

(B) Rohan repays his total loan of ₹ 1,18,000 by paying every month starting with the first instalment of ₹ 1,000. If he increases the instalment by ₹ 100 every month, what amount will be paid by him in the 30th instalment? What amount of loan has he paid after 30th instalment? [3]

Sol. (A) **Given :** $a = 15$, $S_{15} = 750$

To find : a_{20}

Let d and n be common difference and number of terms respectively

$$\therefore S_n = \frac{n}{2}(2a + (n - 1)d) \quad [½]$$

$$\Rightarrow S_{15} = \frac{15}{2}(2(15) + (15 - 1)d) \quad [½]$$

$$\Rightarrow 750 = \frac{15}{2}(30 + 14d)$$

$$\Rightarrow \frac{750 \times 2}{15} = 30 + 14d$$

$$\Rightarrow 14d + 30 = 50 \times 2$$

$$\Rightarrow 14d = 100 - 30$$

$$\Rightarrow 14d = 70 \quad [½]$$

$$\therefore d = 5 \quad [½]$$

$$\text{Also, } a_n = a + (n - 1)d \quad [1/2]$$

$$\begin{aligned} \Rightarrow a_{20} &= 15 + (20 - 1)(5) \\ &= 15 + 19(5) \\ &= 15 + 95 \\ &= 110 \end{aligned}$$

[1/2]

OR

(B) Instalments to be paid by Rohan

1000, 1100, 1200, ...

\therefore This sequence is an A.P

Here, $a = 1000$ and $d = 100$

[1/2]

To find : a_{30} and S_{30}

$$\therefore a_n = a + (n - 1)d$$

[1/2]

$$\begin{aligned} \Rightarrow a_{30} &= 1000 + (30 - 1)(100) \\ &= 1000 + 29(100) \\ &= 1000 + 2900 \\ &= 3900 \end{aligned}$$

[1/2]

$$\text{Also, } S_n = \frac{n}{2}(a + a_n)$$

[1/2]

$$\Rightarrow S_{30} = \frac{30}{2}(1000 + 3900)$$

$$\begin{aligned} \Rightarrow S_{30} &= 15 \times 4900 \\ &= 73500 \end{aligned}$$

[1/2]

\therefore In the 30th instalment, he will pay ₹3900 and he has paid ₹73500 after 30 instalments.

[1/2]

30. Prove that $\sqrt{3}$ is an irrational number.

[3]

Sol. Let us assume $\sqrt{3}$ is rational number

So there exists co-prime integers p and q , $q \neq 0$ such that $\sqrt{3} = \frac{p}{q}$

[1/2]

Squaring both sides,

$$3 = \frac{p^2}{q^2}$$

$$p^2 = 3q^2 \quad \dots(i)$$

[1/2]

$$\Rightarrow 3 \text{ is a factor of } p^2$$

[1/2]

$$\Rightarrow 3 \text{ is a factor of } p$$

[1/2]

$$\Rightarrow p = 3m, \text{ where } m \text{ is an integer}$$

$$\Rightarrow p^2 = 9m^2 \quad \dots(ii) \quad [\text{Squaring both sides}]$$

From equation (i) and (ii), we get

$$\Rightarrow 3q^2 = 9m^2$$

$$\Rightarrow q^2 = 3m^2 \quad [1/2]$$

$$\Rightarrow 3 \text{ is a factors of } q^2$$

$$\Rightarrow 3 \text{ is a factor of } q \text{ also} \quad [1/2]$$

So both p and q have 3 as their common factor, which contradicts the fact that p and q are co-prime

So are assumption is wrong [1/2]

Hence $\sqrt{3}$ is irrational

31. (A) Prove that $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$ [3]

OR

(B) Prove that $\sec A (1 - \sin A)(\sec A + \tan A) = 1$ [3]

Sol. (A) To prove : $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

$$\text{LHS} = \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$

Dividing numerator and denominator by $\cos^3 A$, we get

$$\text{LHS} = \frac{\frac{\sin A}{\cos A \cdot \cos^2 A} - \frac{2\sin^3 A}{\cos^3 A}}{\frac{2\cos^3 A}{\cos^3 A} - \frac{\cos A}{\cos^3 A}} \quad [1]$$

$$= \frac{\sec^2 A \tan A - 2\tan^3 A}{2 - \sec^2 A} \quad [1/2]$$

$$= \frac{\tan A(\sec^2 A - 2\tan^2 A)}{2 - 1 - \tan^2 A} \quad \left[\because \sec^2 A - \tan^2 A = 1 \right] \quad [1/2]$$

$$= \frac{\tan A(1 - \tan^2 A)}{1 - \tan^2 A} \quad [1/2]$$

$$= \tan A = \text{RHS}$$

Hence, proved. [1/2]

OR

(B) To prove : $\sec A(1 - \sin A)(\sec A + \tan A) = 1$

$$\text{LHS} = \sec A(1 - \sin A)(\sec A + \tan A)$$

$$= \sec A(1 - \sin A) \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) \quad [1]$$

$$= \frac{\sec A}{\cos A} (1 - \sin A)(1 + \sin A)$$

$$= \frac{1}{\cos^2 A} (1 - \sin^2 A) \quad [1]$$

$$= \frac{1}{\cos^2 A} \cdot \cos^2 A \quad \left[\because \sin^2 A + \cos^2 A = 1 \right]$$

$$= 1 = \text{RHS} \quad [1]$$

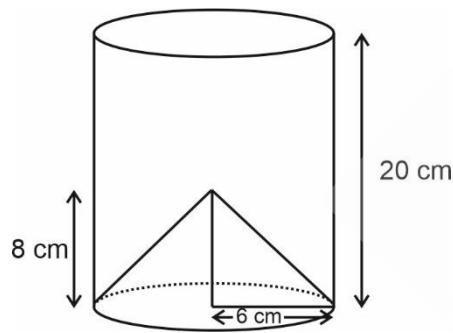
Hence, proved

SECTION-D

Section – D consists of Long Answer (LA) type questions of 5 marks each.

32. From a solid cylinder of height 20 cm and diameter 12 cm, a conical cavity of height 8 cm and radius 6 cm is hallowed out. Find the total surface area of the remaining solid. [5]

Sol. Surface area of remaining solid = Total surface area of cylinder – Area of base + curved surface area of cone



[1]

Height of cylinder(H) = 20 cm, Height of cone(h) = 8 cm, Radius of base of cylinder(r) = 6 cm and Radius of base of cone(r) = 6 cm [1/2]

$$\Rightarrow \text{Surface Area of Remaining solid} = 2\pi rH + 2\pi r^2 - \pi r^2 + \pi r l \quad [1]$$

where l = slant height of cone

$$= 2 \times \frac{22}{7} \times 6 \times 20 + \frac{22}{7} \times 6 \times 6 + \frac{22}{7} \times 6 \times 10 \quad \left[\because l = \sqrt{6^2 + 8^2} = 10 \text{ cm} \right] \quad [1]$$

$$= \frac{22}{7} \times 6 [40 + 6 + 10] \quad [1/2]$$

$$= \frac{22}{7} \times 6 \times 56 \quad [1/2]$$

$$= 1056 \text{ cm}^2 \quad [1/2]$$

33. The monthly expenditure on milk in 200 families of a Housing Society is given below :

Monthly Expenditure (in ₹)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of families	24	40	33	x	30	22	16	7

Find the value of x and also, find the median and mean expenditure on milk.

[5]

Sol.	Monthly Expenditure (in ₹)	Number of families (f_i)	Class mark (x_i)	$d_i = x_i - A$	$f_i d_i$	Cumulative frequency
	1000 - 1500	24	1250	-1500	-36000	24
	1500 - 2000	40	1750	-1000	-40000	64
	2000 - 2500	33	2250	-500	-16500	97
	2500 - 3000	$x = 28$	$2750 = A$	0	0	125
	3000 - 3500	30	3250	500	15000	155
	3500 - 4000	22	3750	1000	22000	177
	4000 - 4500	16	4250	1500	24000	193
	4500 - 5000	7	4750	2000	14000	200
	Total	200			-17500	

[2]

Here,

$$24 + 40 + 33 + x + 30 + 22 + 16 + 7 = 200$$

$$\Rightarrow x + 172 = 200$$

$$\Rightarrow x = 28$$

[1/2]

Now,

$$\text{Mean, } \bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$$

[1/2]

$$= 2750 + \frac{(-17500)}{200}$$

$$= 2750 - 87.5$$

$$= 2662.5$$

[1/2]

$$\text{Also, } \frac{N}{2} = \frac{200}{2} = 100$$

$$\therefore \text{Median class} = 2500 - 3000$$

[1/2]

Here,

$$l = 2500$$

$$cf = 97$$

$$f = 28$$

$$h = 500$$

$$\therefore \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h \quad [1/2]$$

$$= 2500 + \frac{100 - 97}{28} \times 500$$

$$= 2500 + \frac{375}{7}$$

$$= 2500 + 53.57$$

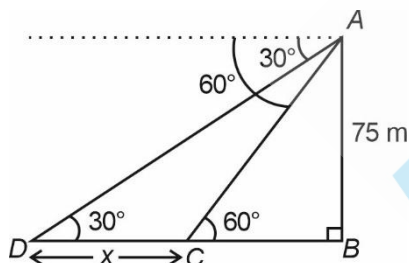
$$= 2553.57 \text{ (approx.)} \quad [1/2]$$

34. (A) A straight highway leads to the foot of a tower. A man standing on the top of the 75 m high tower observes two cars at angles of depression of 30° and 60° , which are approaching the foot of the tower. If one car is exactly behind the other on the same side of the tower, find the distance between the two cars. (use $\sqrt{3} = 1.73$) [5]

OR

- (B) From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30° . Determine the height of the tower. [5]

Sol. (A)



In $\triangle ABC$:

$$\tan 60^\circ = \frac{AB}{BC} \quad [2]$$

$$\Rightarrow BC = \frac{75}{\sqrt{3}} \quad \dots(i)$$

In $\triangle ABD$:

$$\tan 30^\circ = \frac{AB}{BC + DC} \quad [1]$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BC + x}$$

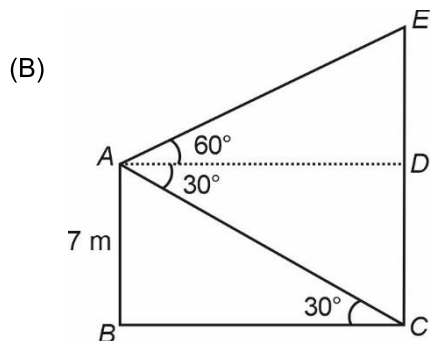
$$\Rightarrow BC + x = 75\sqrt{3} \quad [1]$$

$$\Rightarrow x = 75\sqrt{3} - \frac{75}{\sqrt{3}} \quad (\text{from (i)})$$

$$\Rightarrow x = \frac{75 \times 2}{\sqrt{3}} = \frac{75 \times 2 \times \sqrt{3}}{3}$$

$$\Rightarrow x = 86.5 \text{ m} \quad [1]$$

OR



Let AB be the building of height 7 m and EC be tower.

A is the point from where angle of elevation of the top of tower is 60° and angle of depression of its foot is 30° . [1]

$$EC = DE + CD$$

Also, $CD = AB = 7 \text{ m}$ and $BC = AD$ [1/2]

To find : height of tower EC

In $\triangle ABC$

$$\tan 30^\circ = \frac{AB}{BC} \quad [1/2]$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{7}{BC}$$

$$\Rightarrow BC = 7\sqrt{3} \quad [1/2]$$

In $\triangle ADE$

$$\tan 60^\circ = \frac{DE}{AD} \quad [1/2]$$

$$\Rightarrow \sqrt{3} = \frac{DE}{7\sqrt{3}} \quad [BC = AD \text{ and } BC = 7\sqrt{3}] \quad [1/2]$$

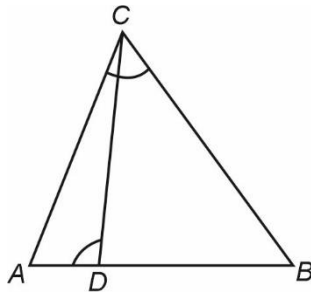
$$\Rightarrow DE = 7\sqrt{3} \times \sqrt{3} = 21$$

$$\text{Now, } EC = DE + CD = 21 + 7 \quad [1/2]$$

$$= 28 \text{ m}$$

\therefore Height of cable tower is 28 m. [1]

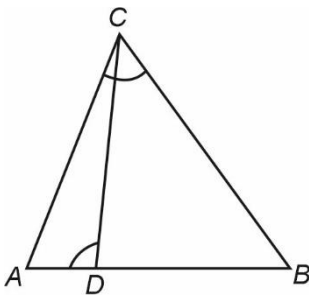
35. (A) In the given figure, $\angle ADC = \angle BCA$; prove that $\triangle ACB \sim \triangle ADC$. Hence find BD if $AC = 8$ cm and $AD = 3$ cm. [5]



OR

- (B) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio. [5]

Sol. (A)



In $\triangle ACB$ and $\triangle ADC$

$$\angle CAB = \angle CAD$$

[Common]

[1/2]

$$\angle ADC = \angle BCA$$

[Given]

[1/2]

$$\therefore \triangle ACB \sim \triangle ADC$$

[By AA similarity criterion]

[1]

$$\frac{AC}{AD} = \frac{BC}{CD} = \frac{AB}{AC}$$

[1/2]

$$\Rightarrow AC \times AC = AD \times AB$$

[1/2]

$$\Rightarrow 8 \times 8 = 3(AB)$$

[1/2]

$$\Rightarrow AB = \frac{64}{3}$$

[1/2]

$$\therefore BD = AB - AD$$

[1/2]

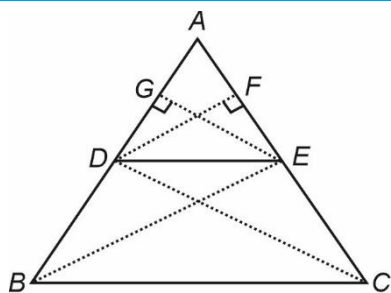
$$= \frac{64}{3} - 3$$

$$= \frac{55}{3}$$

[1/2]

$$\therefore BD = \frac{55}{3} \text{ cm}$$

(B)



Given : $\triangle ABC$, in which DE is drawn parallel to BC

[1/2]

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join CD and BE . Draw $DF \perp AE$ and $EG \perp AD$

[1/2]

Proof : $\text{ar}(\triangle ADE) = \frac{1}{2} \times AD \times EG$... (i)

[1/2]

$\text{ar}(\triangle BDE) = \frac{1}{2} \times BD \times EG$... (ii)

Dividing (i) by (ii), we get

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \times AD \times EG}{\frac{1}{2} \times BD \times EG} = \frac{AD}{BD} \quad \dots \text{(iii)} \quad [1/2]$$

Similarly,

$$\text{ar}(\triangle ADE) = \frac{1}{2} \times DF \times AE \quad [1/2]$$

$$\text{ar}(\triangle CDE) = \frac{1}{2} \times CE \times DF \quad [1/2]$$

$$\Rightarrow \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{\frac{1}{2} \times DF \times AE}{\frac{1}{2} \times DF \times CE} = \frac{AE}{CE} \quad \dots \text{(iv)} \quad [1/2]$$

$\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$ [\because Triangles on the same base and between the same parallel lines are equal in area] [1/2]

$$\Rightarrow \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} \quad [1/2]$$

\therefore From (iii) & (iv), we get

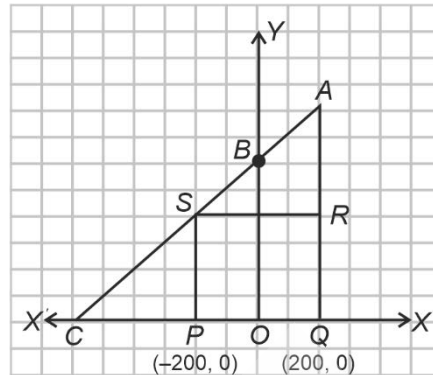
$$\frac{AD}{DB} = \frac{AE}{EC} \quad [1/2]$$

Hence proved.

SECTION-E

Section – E consists of three Case Study Based questions of 4 marks each.

36. Jagdish has a field which is in the shape of a right angled triangle AQC . He wants to leave a space in the form of a square $PQRS$ inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O .



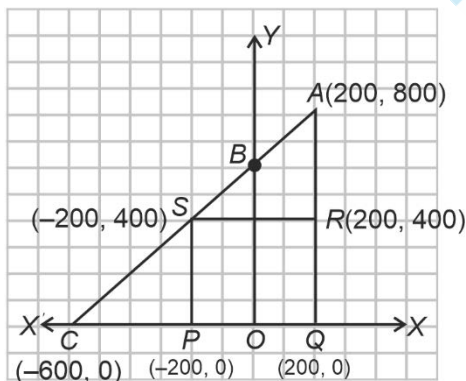
Based on the above information, answer the following questions:

- (i) Taking O as origin, coordinates of P are $(-200, 0)$ and of Q are $(200, 0)$. $PQRS$ being a square, what are the coordinates of R and S ? [1]
 - (ii) (a) What is the area of square $PQRS$? [2]
- OR**
- (b) What is the length of diagonal PR in square $PQRS$? [2]
 - (iii) If S divides CA in the ratio $K : 1$, what is the value of K , where point A is $(200, 800)$? [1]

Sol. (i) Coordinates of R are $(200, 400)$ [1½]

[\because Abscissa of R = Abscissa of Q and side of square $PQRS$ = 400 units]

Similarly, coordinates of S are $(-200, 400)$ [½]



- (ii) (a) Area of square $PQRS$ = PQ^2 sq. units

$$= \left(\sqrt{(200 + 200)^2 + (0 - 0)^2} \right)^2 \text{ sq. units} \quad [1]$$

$$= 160000 \text{ sq. units} \quad [1]$$

OR

$$\begin{aligned} \text{(b) Length of diagonal } PR &= \sqrt{PQ^2 + QR^2} & [1] \\ &= \sqrt{400^2 + 400^2} \\ &= 400\sqrt{2} \text{ units} & [1] \end{aligned}$$

(iii) Here,

⇒ Coordinates of C are $(-600, 0)$,

Coordinates of A are $(200, 800)$.

And Coordinates of S are $(-200, 400)$

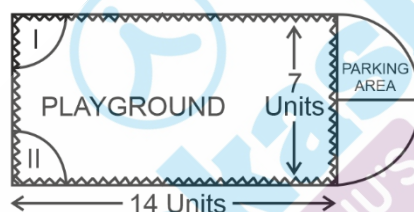
$$\Rightarrow (-200, 400) = \left(\frac{200K + (-600)}{K+1}, \frac{800K + 0}{K+1} \right) \quad \text{[Using section formula]} \quad [1/2]$$

$$\Rightarrow 400K + 400 = 800K + 0 \quad \text{[Comparing y-coordinates]}$$

$$\Rightarrow 400K = 400 \quad [1/2]$$

$$\Rightarrow K = 1$$

37. Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking.



After survey, it was decided to build rectangular playground, with a semi-circular area allotted for parking at one end of the playground. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats.

Based on the above information, answer the following questions:

- (i) What is the total perimeter of the parking area? [1]
 (ii) (a) What is the total area of parking and the two quadrants? [2]

OR

- (b) What is the ratio of area of playground to the area of parking area? [2]
 (iii) Find the cost of fencing the playground and parking area at the rate of ₹2 per unit. [1]

Sol. (i) Here,

$$\text{Radius of parking, } r = \frac{7}{2} = 3.5 \text{ units}$$

$$\therefore \text{ Perimeter of parking area} = 2r + \pi r \quad [1/2]$$

$$= 2 \times \frac{7}{2} + \frac{22}{7} \times \frac{7}{2}$$

$$= 7 + 11$$

$$= 18 \text{ units} \quad [1/2]$$

(ii) (a) Radius of one quadrant $r' = 2$ units

Radius of parking area, $r = \frac{7}{2}$ units

$$\therefore \text{ Required area} = \frac{\pi r^2}{2} + 2 \times \frac{\pi r'^2}{4} \quad [1/2]$$

$$= \frac{\pi}{2} (r^2 + r'^2)$$

$$= \frac{22}{7 \times 2} \left(\left(\frac{7}{2} \right)^2 + (2)^2 \right) \quad [1/2]$$

$$= \frac{11}{7} \left(\frac{49}{4} + 4 \right)$$

$$= \frac{11 \times 65}{28} \quad [1/2]$$

$$= 25.54 \text{ square units (approx.)} \quad [1/2]$$

OR

(b) Area of playground = $\ell \times b$

$$= 14 \times 7$$

$$= 98 \text{ square units} \quad [1/2]$$

$$\text{Area of parking area} = \frac{\pi r^2}{2}$$

$$= \frac{\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}}{2}$$

$$= \frac{77}{4} \text{ square units} \quad [1/2]$$

$$\therefore \text{ Required ratio} = \frac{98}{\frac{77}{4}} \quad [1/2]$$

$$= \frac{56}{11}$$

$$= 56 : 11 \quad [1/2]$$

(iii) Length of fencing required = $2\ell + b + \pi r$

$$= 2(14) + 7 + \frac{22}{7} \times \frac{7}{2}$$

$$= 28 + 7 + 11$$

$$= 46 \text{ units} \quad [1/2]$$

$$\therefore \text{ Cost of Fencing} = 46 \times 2 = ₹92. \quad [1/2]$$

38. Two schools 'P' and 'Q' decided to award prizes to their students for two games of Hockey ₹x per student and Cricket ₹y per student. School 'P' decided to award a total of ₹9,500 for the two games to 5 and 4 students respectively; while school 'Q' decided to award ₹7,370 for the two games to 4 and 3 students respectively.



Based on the above information, answer the following questions:

(i) Represent the following information algebraically (in terms of x and y). [1]

(ii) (a) What is the prize amount for hockey? [2]

OR

(b) Prize amount on which game is more and by how much? [2]

(iii) What will be the total prize amount if there are 2 students each from two games? [1]

Sol. (i) $5x + 4y = 9500$... (i)

$4x + 3y = 7370$... (ii) [1]

(ii) (a) Multiplying (i) by 3 and (ii) by 4; we get

$15x + 12y = 28,500$... (iii)

$16x + 12y = 29,480$... (iv) [1]

Subtracting (iii) from (iv)

$16x + 12y = 29,480$

$15x + 12y = 28,500$

$(-)\quad (-)\quad (-)$ [1]

$x = 980$

Prize amount for Hockey = ₹980 per student

OR

(b) Multiplying (i) by 3 and (ii) by 4; we get

$15x + 12y = 28,500$... (iii)

$16x + 12y = 29,480$... (iv) [1/2]

Subtracting (iii) from (iv)

$$16x + 12y = 29,480$$

$$15x + 12y = 28,500$$

$$\begin{array}{r} (-) \quad (-) \quad (-) \\ \hline \end{array}$$

$$x = ₹980$$

[½]

Putting $x = 980$ in (i) :-

$$5(980) + 4y = 9500$$

$$\Rightarrow y = ₹1150$$

[½]

\therefore Prize amount of Hockey = ₹980 per student

Prize amount of Cricket = ₹1150 per student

\Rightarrow Prize amount per student of Cricket is greater by ₹170

[½]

(iii) Total prize amount if there are 2 students each from 2 games = $2(x + y)$

$$= 2(980 + 1150)$$

[½]

$$= ₹4,260$$

[½]

