17/08/2022
Slot-1

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## Answers \& Solutions

Time : 45 min .


## CUET UG-2022

(Mathematics)

## IMPORTANT INSTRUCTIONS:

1. The test is of 45 Minutes duration.
2. The test contains is divided into two sections.
a. Section A contains 15 questions which will be compulsory for all candidates.
b. Section B will have 35 questions out of which 25 questions need to be attempted.
3. Marking Scheme of the test:
a. Correct answer or the most appropriate answer: Five marks (+5)
b. Any incorrect option marked will be given minus one mark ( -1 ).

## Choose the correct answer :

## Question ID: 481291

If $x=2 a t, y=a t^{2}$, then $\frac{d^{2} y}{d x^{2}}$ is
(A) 1
(B) $\frac{1}{2 a}$
(C) $t$
(D) 0

## Answer (B)

Sol. $x=2 a t, y=a t^{2}$
So, $\frac{d x}{d t}=2 a, \frac{d y}{d t}=2 a t$
$\therefore \quad \frac{d y}{d x}=t$
$\therefore \quad \frac{d^{2} y}{d x^{2}}=\frac{d}{d t}\left(\frac{d y}{d x}\right) \cdot \frac{d t}{d x}$

$$
=1 \cdot \frac{1}{2 a}=\frac{1}{2 a}
$$

## Question ID: 481292

A die is thrown once. If $E$ is the event that the number appearing is a multiple of 3 ' and $F$ be the event 'the number appearing is even', then the incorrect option is
(A) $P(E)=\frac{1}{3}$
(B) $P(F)=\frac{1}{2}$
(C) $P(E \cap F)=\frac{1}{6}$
(D) $E$ and $F$ are dependent events

Answer (D)
Sol. $P(E)=\frac{2}{6}=\frac{1}{3}$
$P(F)=\frac{3}{6}=\frac{1}{2}$
$P(E \cap F)=\frac{1}{6}$
Now, $P(E) \cdot P(F)=\frac{1}{2} \cdot \frac{1}{3}=\frac{1}{6}$
$\therefore \quad P(E \cap F)=P(E) \cdot P(F)$
$\therefore \quad E$ and $F$ are independent events

Question ID: 481293
Ten eggs are drawn successively with replacement from a lot containing 10\% defective eggs. The probability that there is at least one defective egg is
(A) $\frac{10^{10}-9^{10}}{10^{10}}$
(B) $\frac{9^{10}-10^{10}}{10^{10}}$
(C) $\frac{10^{9}-9^{10}}{10^{10}}$
(D) $\frac{10^{10}+9^{10}}{10^{10}}$

## Answer (A)

Sol. $P(E)=1-\left(\frac{9}{10}\right)^{10}$
$=\frac{10^{10}-9^{10}}{10^{10}}$
Question ID: 481294
If $m$ is the degree and $n$ is the order of the given differential equation
$\frac{x^{3}\left(\frac{d^{3} y}{d x^{3}}\right)^{2}+2 x^{2}\left(\frac{d^{2} y}{d x^{2}}\right)^{3}}{(x+1)^{5}}=\left(3 x-\frac{d^{2} y}{d x^{2}}\right)^{4}$
(A) $m-n=2$
(B) $m+n=5$
(C) $m=4, n=3$
(D) Order $(n)$ is 3 but degree $(m)$ is not defined

Answer (B)
Sol. $\frac{x^{3}\left(\frac{d^{3} y}{d x^{3}}\right)^{2}+2 x^{2}\left(\frac{d^{2} y}{d x^{2}}\right)^{3}}{(x+1)^{5}}=\left(3 x-\frac{d^{2} y}{d x^{2}}\right)^{4}$
$\Rightarrow \quad x^{3}\left(\frac{d^{3} y}{d x^{3}}\right)^{2}+2 x^{2}\left(\frac{d^{2} y}{d x^{2}}\right)^{3}-(x+1)^{5}\left(3 x-\frac{d^{2} y}{d x^{2}}\right)^{4}=0$
$\therefore \quad \operatorname{Order}(n)=3$
Degree $(m)=2$

## Question ID: 481295

The differential equation representing the family of curves $y=m(x-d)$ where $m$ and $d$ are arbitrary constants, is
(A) $\frac{d y}{d x}=0$
(B) $\frac{d^{2} y}{d x^{2}}=0$
(C) $x \frac{d^{2} y}{d x^{2}}+y=0$
(D) $x \frac{d^{2} y}{d x^{2}}-y=0$

## Answer (B)

Sol. $y=m(x-d)$
$\frac{d y}{d x}=m$
$\frac{d^{2} y}{d x^{2}}=0$, this is the required differential equation

## Question ID: 481296

$\int_{1}^{2} \frac{d x}{x\left(x^{4}+1\right)}=?$
(A) $\log \left(\frac{32}{17}\right)$
(B) $\log \left(\frac{16}{17}\right)$
(C) $\frac{1}{4} \log \left(\frac{16}{17}\right)$
(D) $\frac{1}{4} \log \left(\frac{32}{17}\right)$

## Answer (D)

Sol. $I=\int_{1}^{2} \frac{d x}{x\left(x^{4}+1\right)}=\int_{1}^{2} \frac{d x}{x^{5}\left(1+x^{-4}\right)}$
Let $1+x^{-4}=t$

$\therefore \quad I=\int_{2}^{17 / 16} \frac{-1}{4 t} d t=\frac{-1}{4}[\ln t]_{2}^{17 / 16}$
$=\frac{-1}{4}\left[\ln \frac{17}{16}-\ln 2\right]$
$=\frac{1}{4} \ln \left[\frac{32}{17}\right]$

## Question ID: 481297

If the area of the region in first quadrant, bounded by the curve
$y^{2}=9 x, x=2, x=4$ and the $x$-axis is $a+b \sqrt{2}$, then the value of $a+b$ is
(A) 16
(B) 12
(C) 20
(D) 8

Answer (B)
Sol. Given curves : $y^{2}=9 x, x=2, x=4$ and $y=0$

$\therefore$ Required area $=\int_{2}^{4} \sqrt{9 x} d x=3\left[\frac{x^{3 / 2}}{\frac{3}{2}}\right]_{2}^{4}$
$=2[8-2 \sqrt{2}]$

$$
=16-4 \sqrt{2}
$$

$$
\therefore a+b=12
$$

Question ID: 481298
If $\left[\begin{array}{c}x+y+z \\ x+z \\ y+z\end{array}\right]=\left[\begin{array}{c}11 \\ 6 \\ 8\end{array}\right]$, then the value of $x+2 y-3 z$ is
(A) 5
(B) 4
(C) 3
(D) 7

Answer (B)
Sol. $\left[\begin{array}{c}x+y+z \\ x+z \\ y+z\end{array}\right]=\left[\begin{array}{c}11 \\ 6 \\ 8\end{array}\right]$
$\therefore \quad x+y+z=11$

By (i) \& (ii) $y=5$
By (ii) \& (i) $x=3$
$\therefore \quad z=3$
So, $x+2 y-32=4$

Question ID: 481299
If $x=3 t^{2}+5 t+6$ and $y=-4 t^{3}-2 t^{2}+5 t+7, t \neq \frac{-5}{6}$ then the value of $\frac{d y}{d x}$ is
(A) $-2 t+1$
(B) $\frac{-12 t^{2}-4 t-5}{6 t+5}$
(C) $\frac{-4 t^{3}-2 t^{2}+5 t+7}{3 t^{2}+5 t+6}$
(D) $\frac{-4 t^{3}-2 t^{2}+5 t+7}{6 t+5}$

## Answer (B)

Sol. $x=3 t^{2}+5 t+6, y=-4 t^{3}-2 t^{2}+5 t+7$
So, $\frac{d x}{d t}=6 t+5, \frac{d y}{d t}=-12 t^{2}-4 t+5$
$\therefore \quad \frac{d y}{d x}=\frac{-12 t^{2}-4 t+5}{6 t+5}$

## Question ID: 4812910

The interval in which the function $f$ given by $f(x)=x^{2}-4 x+6$ is strictly increasing is
(A) $(-\infty, 2)$
(B) $(-\infty,-2)$
(C) $(2, \infty)$
(D) $(-2, \infty)$

## Answer (C)

Sol. $f(x)=x^{2}-4 x+6$
for increasing, $f(x)>0$

$$
\begin{aligned}
& \Rightarrow \quad 2 x-4>0 \\
& \Rightarrow \quad x>2
\end{aligned}
$$

Question ID: 4812911
If $y=\log _{e}\left(\frac{2 x}{1-x}\right)$, then $\frac{d^{2} y}{d x^{2}}$ at $x=\frac{1}{2}$ is
(A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) 0
(D) $\frac{3}{5}$

## Answer (C)

Sol. $y=\log _{e}\left(\frac{2 x}{1-x}\right)$
So, $\frac{d y}{d x}=\frac{1}{\frac{2 x}{1-x}} \cdot \frac{d}{d x}\left(\frac{2 x}{1-x}\right)$

$$
=\frac{1-x}{2 x} \frac{(1-x) 2+2 x}{(1-x)^{2}}
$$

$$
\begin{aligned}
& \quad=\frac{2}{2 x(1-x)}=\frac{1}{1-x}+\frac{1}{x} \\
& \therefore \quad \frac{d^{2} y}{d x^{2}}=\frac{1}{(1-x)^{2}}-\frac{1}{x^{2}} \\
& \text { at } x=\frac{1}{2}, \frac{d^{2} y}{d x^{2}}=4-4=0
\end{aligned}
$$

## Question ID: 4812912

If $a, b, c$ are mutually unequal real numbers, then
the value of $\frac{\left|\begin{array}{lll}1 & a & a^{3} \\ 1 & b & b^{3} \\ 1 & c & c^{3}\end{array}\right|}{\left|\begin{array}{lll}1 & a & a^{2} \\ 1 & b & b^{2} \\ 1 & c & c^{2}\end{array}\right|}=$
(A) $-(a+b+c)$
(B) $a+b+c$
(C) $a^{2}+b^{2}+c^{2}$
(D) $a^{3}+b^{3}+c^{3}$

## Answer (B)

Sol.

| $\frac{\left\|\begin{array}{lll}1 & a & a^{3} \\ 1 & b & b^{3} \\ 1 & c & c^{3}\end{array}\right\|}{\left\|\begin{array}{ccc}1 & a & a^{2} \\ 1 & b & b^{2} \\ 1 & c & c^{2}\end{array}\right\|}=\frac{\left\|\begin{array}{ccc}1 & a & a^{3} \\ 0 & b-a & b^{3}-a^{3} \\ 0 & c-a & c^{3}-a^{3}\end{array}\right\|}{\left\|\begin{array}{ccc}1 & a & a^{2} \\ 0 & b-a & b^{2}-a^{2} \\ 0 & c-a & c^{2}-a^{2}\end{array}\right\|}$$=\frac{(b-a)(c-a)\left\|\begin{array}{ccc} 1 & a & a^{3} \\ 0 & 1 & b^{2}+a^{2}+a b \\ 0 & 1 & c^{2}+a^{2}+a c \end{array}\right\|}{(b-a)(c-a)\left\|\begin{array}{ccc} 1 & a & a^{2} \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{array}\right\|} \begin{array}{ll}  & =\frac{c^{2}+a c-b^{2}-a b}{(c-b)} \\ & =a+b+c+a \end{array}$ |  |  |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |

## Question ID: 4812913

If $A=\left[\begin{array}{ll}1 & 1 \\ 1 & 1\end{array}\right]$ and $n \in N$ (where $N$ is the set of natural numbers), then $A^{n}$ is equal to
(A) $n A$
(B) $2 n A$
(C) $2^{n-1} A$
(D) $2^{n} A$

Answer (C)

Sol. $A=\left[\begin{array}{ll}1 & 1 \\ 1 & 1\end{array}\right]$

$$
\begin{aligned}
& A^{2}=\left[\begin{array}{ll}
1 & 1 \\
1 & 1
\end{array}\right]\left[\begin{array}{ll}
1 & 1 \\
1 & 1
\end{array}\right]=\left[\begin{array}{ll}
2 & 2 \\
2 & 2
\end{array}\right]=2 A \\
& A^{3}=\left[\begin{array}{ll}
1 & 1 \\
1 & 1
\end{array}\right]\left[\begin{array}{ll}
2 & 2 \\
2 & 2
\end{array}\right]=\left[\begin{array}{ll}
4 & 4 \\
4 & 4
\end{array}\right]=2^{2} A \\
& A^{4}=\left[\begin{array}{ll}
1 & 1 \\
1 & 1
\end{array}\right]\left[\begin{array}{ll}
4 & 4 \\
4 & 4
\end{array}\right]=\left[\begin{array}{ll}
8 & 8 \\
8 & 8
\end{array}\right]=2^{3} A
\end{aligned}
$$

$$
\vdots
$$

$$
A^{n}=2^{n-1} A
$$

## Question ID: 4812914

If $y=\frac{1}{\sqrt{1+x^{2}}-x}$, then the value of $\left(1+x^{2}\right)^{\frac{3}{2}} \cdot \frac{d^{2} y}{d x^{2}}$ is
(A) $x$
(B) $x^{2}-1$
(C) $\sqrt{1+x^{2}}-1$
(D) 1

## Answer (D)

Sol. $y=\frac{1}{\sqrt{1+x^{2}}-x} \times \frac{\sqrt{1+x^{2}}+x}{\sqrt{1+x^{2}}+x}$

$$
=\frac{\sqrt{1+x^{2}}+x}{1}
$$

$\therefore \quad \frac{d y}{d x}=\frac{1}{2 \sqrt{1+x^{2}}} \times 2 x+1=\frac{y}{\sqrt{1+x^{2}}}$
$\therefore \quad \frac{d^{2} y}{d x^{2}}=\frac{\sqrt{1+x^{2}} \frac{d y}{d x}-y \frac{1}{2 \sqrt{1+x^{2}}} \cdot 2 x}{\left(1+x^{2}\right)}$
$\Rightarrow\left(1+x^{2}\right)^{\frac{3}{2}} \frac{d^{2} y}{d x^{2}}=y\left(\sqrt{1+x^{2}}-x\right)=1$

## Question ID: 4812915

The equation of the tangent to the curve $y=x^{2}-2 x+7$, which is parallel to the line $2 x-y+9=0$, is
(A) $2 x-y+3=0$
(B) $2 x-y+6=0$
(C) $2 x-y+1=0$
(D) $2 x-y+4=0$

Answer (A)

Sol. $y=x^{2}-2 x+7$
$\frac{d y}{d x}=2 x-2$, given slope $(m)=2$
Let $\left(x_{1}, y_{1}\right)$ be the point of tangency
So, $2 x_{1}-2=2$
$x_{1}=2, y_{1}=7$
Equation of tangent, will be

$$
\begin{aligned}
& y-7=2(x-2) \\
& \Rightarrow \quad 2 x-y+3=0
\end{aligned}
$$

## Question ID: 4812951

Consider the non-empty set consisting of children in a family and a relation $R$ defined as $a R b$ if $a$ is brother of $b$, Then $R$ is
(A) Symmetric but not transitive
(B) Transitive but not symmetric
(C) Neither symmetric nor transitive
(D) Both symmetric and transitive

## Answer (B)

Sol. $a R b$ if $a$ is brother of $b$
(i) It can't be symmetric because

Let $a$ (male) is the brother of $b$ (female) then $b$ will be sister of $a$.
(ii) It will be transitive

If $(a, b)(b, c) \in R$
Then $a, b$ must be male
So, $(a, c) \in R$

## Question ID: 4812952

The relation $R$ in the set $\{1,2,3\}$ given by $R=\{(1,1),(2,2),(3,3),(1,2),(2,3)\}$ is
(A) Reflexive only
(B) Reflexive and symmetry relation
(C) Transitive only
(D) Equivalence relation

## Answer (A)

Sol. $R=\{(1,1),(2,2),(3,3),(1,2),(2,3)\}$
and $A=\{1,2,3\}$
(i) $\because(a, a) \in R \forall a \in A$
$\therefore \quad R$ is Reflexive
(ii) $\because(1,2) \in R$ but $(2,1) \notin R$
$\therefore \quad R$ is not symmetric
(iii) $\because(1,2)(2,3) \in R$ but $(1,3) \notin R$
$\therefore \quad R$ is not transitive

## Question ID: 4812953

If $f: R-\{-1\} \rightarrow R-\{1\}$ be a function defined by $f(x)=\frac{x-1}{x+1}$, then
A. $f$ is one-one but not onto
B. $f$ is onto but not one-one
C. $f$ is one-one and onto
D. $f^{-1}(x)=\frac{x+1}{x-1}$
E. $(f o f)(x)=-\frac{1}{x} ; x \neq 0,-1$

Choose the correct answer from the options given below
(A) A, D, E only
(B) C, D only
(C) B, E only
(D) C, E only

## Answer (D)

Sol. $f: R-\{-1\} \rightarrow R-\{1\}$
and $f(x)=\frac{x-1}{x+1}=\frac{(x+1)}{x+1}-\frac{2}{x+1}=1-\frac{2}{x+1}$
$f^{\prime}(x)=\frac{2}{(x+1)^{2}}>0 \quad \therefore \quad f$ is one-one
and $f(x)=1-\frac{2}{x+1} \in R-\{1\}$
$\therefore \quad f$ is onto also and $y=\frac{x-1}{x+1}$
$\Rightarrow \quad \frac{y+1}{y-1}=\frac{x}{-1}$
$\therefore \quad f^{-1}(x)=\frac{x+1}{1-x}$
and $f \circ f(x)=\frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}=-\frac{2}{2 x}$

$$
=-\frac{1}{x}
$$

## Question ID: 4812954

The domain of the function $\cos ^{-1}(2 x-1)$ is
(A) $[0,1]$
(B) $[-1,1]$
(C) $(-1,1)$
(D) $[0, \pi]$

## Answer (A)

Sol. $f(x)=\cos ^{-1}(2 x-1)$
$\therefore$ Degree for domain

$$
\begin{aligned}
& -1 \leq 2 x-1 \leq 1 \\
& 0 \leq 2 x \leq 2 \\
& 0 \leq x \leq 1 \\
\therefore & D_{f} \equiv[0,1]
\end{aligned}
$$

## Question ID: 4812955

$\tan ^{-1}\left(\frac{x}{y}\right)-\tan ^{-1}\left(\frac{x-y}{x+y}\right)=$
(A) $\frac{\pi}{6}$
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{2}$

## Answer (C)

Sol. Let $\tan ^{-1}\left(\frac{x}{y}\right)-\tan ^{-1}\left(\frac{x-y}{x+y}\right)=t$

$$
\begin{aligned}
& \therefore \quad \tan t=\frac{\frac{x}{y}-\left(\frac{x-y}{x+y}\right)}{1+\frac{x}{y} \cdot \frac{x-y}{x+y}}=\frac{x^{2}+x y-x y+y^{2}}{x y+y^{2}+x^{2}-x y} \\
& \quad=1 \\
& \therefore \quad t=\frac{\pi}{4}
\end{aligned}
$$

## Question ID: 4812956

If the matrix $A=\left[\begin{array}{ccc}3 & 2 a & -5 \\ -4 & 0 & b \\ -5 & 3 & 7\end{array}\right]$ is symmetric then the value of $(a+b)$ is
(A) 1
(B) 5
(C) 3
(D) 4

## Answer (A)

Sol. $A=\left[\begin{array}{ccc}3 & 2 a & -5 \\ -4 & 0 & b \\ -5 & 3 & 7\end{array}\right]$

$$
A^{T}=\left[\begin{array}{ccc}
3 & -4 & -5 \\
2 a & 0 & 3 \\
-5 & b & 7
\end{array}\right]
$$

Since $A$ is symmetric

$$
\begin{aligned}
\therefore & A=A^{T} \\
\Rightarrow & 2 a=-4 \text { and } b=3 \\
& a=-2
\end{aligned}
$$

So, $a+b=1$

## Question ID: 4812957

If $A$ is square matrix of size 4 and $|A|=6$. If $\mid \operatorname{Adj}$. $\left(\operatorname{Adj} .(3 A) \mid=2^{a} .3^{b}\right.$, then value of $a+b$ is
(A) 24
(B) 54
(C) 72
(D) 216

## Answer (B)

Sol. $\because|\operatorname{Adj} .(\operatorname{Adj} A)|=|A|^{(n-1)^{2}}$

$$
\begin{aligned}
\therefore \quad|\operatorname{Adj}(\operatorname{Adj} 3 A j)| & =|3 A|^{3^{2}} \\
& =\left(3^{4}|A|\right)^{9} \\
& =3^{36} 6^{9} \\
& =3^{45} 2^{9}
\end{aligned}
$$

$\therefore \quad a+b=54$

## Question ID: 4812958

The value of $x$ for which $\left|\begin{array}{ll}3 & x \\ x & 1\end{array}\right|=\left|\begin{array}{ll}3 & 2 \\ 4 & 1\end{array}\right|$, is
(A) 2
(B) $\pm 2 \sqrt{2}$
(C) 4
(D) $\pm 2 \sqrt{3}$

Answer (B)

Sol. $\left|\begin{array}{ll}3 & x \\ x & 1\end{array}\right|=\left|\begin{array}{ll}3 & 2 \\ 4 & 1\end{array}\right|$

$$
\begin{aligned}
& \Rightarrow \quad 3-x^{2}=3-8 \\
& \Rightarrow \quad x^{2}=8 \\
& \Rightarrow \quad x= \pm 2 \sqrt{2}
\end{aligned}
$$

## Question ID: 4812959

If $y=\left(\frac{1}{x}\right)^{x}$, then $\frac{d^{2} y}{d x^{2}}=$
(A) $x^{-x}(1+\log x)^{2}-x^{-(x+1)}$
(B) $x^{-x}(1+\log x)^{2}-x^{-(x-1)}$
(C) $x^{-x}(1+\log x)^{-2}-x^{-(x+1)}$
(D) $x^{-x}(1+\log x)^{-1}+x^{-(x-1)}$

## Answer (A)

Sol. $y=\left(\frac{1}{x}\right)^{x}$

$$
y=x^{-x}
$$

$$
\therefore \quad \log y=-x \log x
$$

$$
\Rightarrow \frac{1}{y} \frac{d y}{d x}=\frac{-x}{x}+\log x(-1)=-1-\log x
$$

$$
\Rightarrow \frac{d y}{d x}=-y(1+\log x)
$$

$$
\therefore \quad \frac{d^{2} y}{d x^{2}}=-y\left[\frac{1}{x}\right]-(1+\log x) \frac{d y}{d x}
$$

$$
=-x^{-x-1}+x^{-x}(1+\log x)^{2}=x^{-x}(1+\log x)^{2}-x^{-(x+1)}
$$

## Question ID: 4812960

If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}) \cdot(\vec{x}+\vec{a})=8$ then $|\vec{x}|$ is :
(A) 2
(B) 3
(C) $\pm 3$
(D) 5

Answer (B)

Sol. $(\vec{x}-\vec{a}) \cdot(\vec{x}+\vec{a})=8$

$$
\begin{aligned}
& \Rightarrow|\vec{x}|^{2}-1=8 \\
& \therefore|\vec{x}|=3
\end{aligned}
$$

## *Question ID: 4812961

$$
\left|\begin{array}{ccc}
0 & \sin 2 \alpha & -\cos ^{2} \alpha \\
-\sin ^{2} \alpha & 0 & \sin \alpha \sin \beta \\
-\cos \alpha \sin \beta & 2 \sin ^{2} \beta & 0
\end{array}\right|=
$$

(A) 0
(B) -1
(C) Independent of $\alpha$
(D) Independent of $\beta$

Answer (A, C, D)
Sol. $\left|\begin{array}{ccc}0 & \sin 2 \alpha & -\cos ^{2} \alpha \\ -\sin ^{2} \alpha & 0 & \sin \alpha \sin \beta \\ -\cos \alpha \sin \beta & 2 \sin ^{2} \beta & 0\end{array}\right|$
$=-\sin 2 \alpha\left(\sin \alpha \cos \alpha \sin ^{2} \beta\right)$
$-\cos ^{2} \alpha\left(-2 \sin ^{2} \alpha \sin ^{2} \beta\right)$
$=-2 \sin ^{2} \alpha \cos ^{2} \alpha \sin ^{2} \beta+12 \cos ^{2} \alpha \sin ^{2} \alpha \sin ^{2} \beta$
$=0$

## Question ID: 4812962

$\int \frac{2}{x^{4}-1} d x=$
(A) $\log \left|\frac{x^{2}-1}{x^{2}+1}\right|+C$
(B) $2 \tan ^{-1}\left(x^{2}\right)+C$
(C) $\frac{1}{2} \log \left|\frac{x-1}{x+1}\right|-\tan ^{-1} x+C$
(D) $\tan ^{-1} x+\frac{1}{2} \log \left|\frac{x+1}{x-1}\right|+C$

## Answer (C)

Sol. $\int \frac{2}{x^{4}-1} d x=\int \frac{2}{\left(x^{2}-1\right)\left(x^{2}+1\right)} d x$
$=\int\left(\frac{1}{x^{2}-1}-\frac{1}{\left(x^{2}+1\right)}\right) d x$
$=\frac{1}{2} \ln \left|\frac{x-1}{x+1}\right|-\tan ^{-1} x+C$

Question ID: 4812963
The acute angle between the diagonals $O G$ and $A D$ of the cuboid (shown in the figure) is

(A) $\cos ^{-1}\left(\frac{1}{3}\right)$
(B) $\cos ^{-1}\left(-\frac{1}{3}\right)$
(C) $\cos ^{-1}\left(\frac{7}{9}\right)$
(D) $\cos ^{-1}\left(\frac{1}{9}\right)$

## Answer (A)

Sol. $\overrightarrow{O G}=(2 \hat{i}+2 \hat{j}+\hat{k})$

$$
\begin{aligned}
\overrightarrow{A D}=\overrightarrow{O D}-\overrightarrow{O A} & =(2 \hat{i}+\hat{k})-(2 \hat{j}) \\
& =2 \hat{i}-2 \hat{j}+\hat{k}
\end{aligned}
$$

Let $\theta$ be the angle between $\overrightarrow{O G}$ and $\overrightarrow{A D}$

$$
\begin{aligned}
& \cos \theta=\frac{4-4+1}{\sqrt{3} \cdot \sqrt{3}}=\frac{1}{3} \\
& \therefore \quad \theta=\cos ^{-1}\left(\frac{1}{3}\right)
\end{aligned}
$$

## Question ID: 4812964

In the following figure $A B C D E F$ is a regular hexagon. If $\overrightarrow{A B}=\vec{a}$ and $\overrightarrow{B C}=\vec{b}$ then $\overrightarrow{C D}$ in terms of $\vec{a}$ and $\vec{b}$ is :

(A) $\vec{a}+\vec{b}$
(B) $\vec{a}-\vec{b}$
(C) $\vec{b}-\vec{a}$
(D) $3 \vec{b}-\vec{a}$

Answer (C)

Sol.

$\therefore \quad \overrightarrow{C D}=\overrightarrow{C A}+\overrightarrow{A D}$

$$
=-(\vec{a}+\vec{b})+2 \vec{b}=\vec{b}-\vec{a}
$$

## Question ID: 4812965

The integrating factor of the differential equation $x \frac{d y}{d x}+y-x+x y \cot x=0,(x \neq 0)$ is :
(A) $x \sin x$
(B) $x \cos x$
(C) $x$
(D) $\sin x$

## Answer (A)

Sol. $x \frac{d y}{d x}+y-x+x y \cot x=0(x \neq 0)$

$$
\begin{array}{ll}
\Rightarrow & \frac{d y}{d x}+\left(\frac{1}{x}+\cot x\right) y=1 \\
\therefore & \text { I.F }=e^{\int\left(\frac{1}{x}+\cot x\right) d x}=e^{\ln x+\ln \sin x} \\
& =x \sin x
\end{array}
$$

## Question ID: 4812966

$\int_{\frac{1}{4}}^{1} \frac{d x}{\sqrt{-x^{2}-2 x+3}}=$
(A) $\sin ^{-1}\left(\frac{1}{4}\right)$
(B) $\sin ^{-1}\left(\frac{3}{4}\right)$
(C) $\sin ^{-1}\left(\frac{5}{8}\right)$
(D) $\cos ^{-1}\left(\frac{5}{8}\right)$

## Answer (D)

Sol. $I=\int_{\frac{1}{4}}^{1} \frac{d x}{\sqrt{-x^{2}-2 x+3}}=\int_{\frac{1}{4}}^{1} \frac{d x}{\sqrt{4-(x+1)^{2}}}$
$=\left(\sin ^{-1} \frac{x+1}{2}\right)_{\frac{1}{4}}^{1}$
$=\frac{\pi}{2}-\sin ^{-1}\left(\frac{5}{8}\right)$
$=\cos ^{-1}\left(\frac{5}{8}\right)$

## Question ID: 4812967

A random variable $X$ has the following probability distribution

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P(X=x)$ | 0 | $k$ | $2 k$ | $2 k$ | $3 k$ | $k^{2}$ | $2 k^{2}$ | $7 k^{2}+k$ |

The value of $P(0<X<5)$ is
(A) $\frac{1}{5}$
(B) $\frac{2}{5}$
(C) $\frac{4}{5}$
(D) $\frac{3}{5}$

## Answer (C)

Sol. $P(0<X<5)=P(x=1)+P(x=2)+P(x=3)+P(x$ =4)
and

$$
\begin{aligned}
& k+2 k+2 k+3 k+k^{2}+2 k^{2}+7 k^{2}+k=1 \\
& 10 k^{2}+9 k=1 \\
& \Rightarrow \quad 10 k^{2}+10 k-k-1=0 \\
& \Rightarrow \quad(k+1)(10 k-1)=0 \\
& \quad k=\frac{1}{10} \text { or }(-1) x \\
& \therefore \quad P(0<x<5)=k+2 k+2 k+3 k \\
& \quad=8 k=\frac{8}{10}=\frac{4}{5}
\end{aligned}
$$

## Question ID: 4812968

Two independent events $A$ and $B$ are such that $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{3}$, the $P(A \mid B)=$
(A) $\frac{1}{3}$
(B) $\frac{1}{2}$
(C) $\frac{2}{3}$
(D) 1

## Answer (B)

Sol. $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$

$$
\therefore \quad P(A \mid B)=\frac{P(A \cap B)}{P(B)}=\frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{1}{3}}=\frac{1}{2}
$$

## Question ID: 4812969

Probability that $A$ speaks truth is $\frac{4}{5}$. He tosses a coin and reports that a head appears. The probability that actually there was a head, is
(A) $\frac{4}{5}$
(B) $\frac{1}{2}$
(C) $\frac{1}{5}$
(D) $\frac{2}{5}$

## Answer (A)

Sol. $P(T)=\frac{4}{5}$
So, $P\left(\frac{T}{H}\right)=\frac{P(H \cap T)}{P(H)}$

$$
=\frac{P(T) P\left(\frac{H}{T}\right)}{P(T) P\left(\frac{H}{T}\right)+P(F)\left(\frac{H}{F}\right)}
$$

$$
=\frac{\frac{4}{5} \times \frac{1}{2}}{\frac{4}{5} \times \frac{1}{2}+\frac{1}{5} \times \frac{1}{2}}=\frac{4}{5}
$$

## Question ID: 4812970

The expectation of a number obtained when throwing a die having 1 written on three faces, 2 on two faces and 5 on one face is given by
(A) 2
(B) $\frac{3}{2}$
(C) $\frac{71}{30}$
(D) 6

## Answer (A)

Sol. Given: $P(1)=\frac{1}{2}, P(2)=\frac{1}{3}, P(5)=\frac{1}{6}$

$$
\begin{aligned}
\therefore \quad & E=1 \cdot \frac{1}{2}+2 \cdot \frac{1}{3}+5 \cdot \frac{1}{6} \\
& =\frac{3+4+5}{6}=\frac{12}{6}=2
\end{aligned}
$$

## *Question ID: 4812971

If $\int \frac{d x}{\sqrt{x+2}-\sqrt{x+1}}=\frac{2}{3}\left[(\lambda+1)^{\frac{3}{2}}-\lambda^{\frac{3}{2}}\right]+C$, then the value of $\lambda$ is
(A) $x-1$
(B) $x$
(C) $x+1$
(D) $\frac{1}{x}$

## Answer (*)

As per given situation integral of L.H.S can not be express in R.H.S for any $\lambda$.

Sol. $I=\int \frac{d x}{\sqrt{x+2}-\sqrt{x+1}}=\int(\sqrt{x+2}+\sqrt{x+1}) d x$

$$
=\frac{2}{3}(x+2)^{3 / 2}+\frac{2}{3}(x+1)^{3 / 2}+C
$$

## Question ID : 4812972

$\int_{0}^{\pi} \sin ^{3} x \cdot \cos ^{2} x . d x=$
(A) $-\frac{4}{15}$
(B) $\frac{2}{15}$
(C) 0
(D) $\frac{4}{15}$

## Answer (D)

Sol. $I=\int_{0}^{\pi} \sin ^{3} x \cos ^{2} x d x$
$I=2 \int_{0}^{\pi / 2} \sin ^{3} x \cos ^{2} x d x$
$=2 \cdot \frac{(3-1)(3-2)(2-1)}{5 \cdot 3 \cdot 1}=\frac{4}{15}$

## Question ID : 4812973

The distance of the point $(3,-2,1)$ from the plane $2 x-y+2 z+3=0$ is :
(A) $\frac{3}{13}$
(B) $\frac{13}{3}$
(C) $\frac{14}{3}$
(D) $\frac{3}{14}$

## Answer (B)

Sol. $P: 2 x-y+2 z+3=0$
$P_{1} \equiv(3,-2,1)$
$\therefore \quad d=\frac{6+2+2+3}{\sqrt{4+1+4}}=\frac{13}{3}$

## Question ID : 4812974

The maximum value of the function $z=3 x+3 y$, subject to the constraints
$x+2 y \leq 30,2 x+y \leq 50, x \geq 0, y \geq 0$ is :
(A) 75
(B) 90
(C) 80
(D) 45

Answer (C)
Sol. $z=3 x+3 y$
So, corner points of feasible region are
$O(0,0), A(25,0), B(0,15)$ and $C\left(\frac{70}{3}, \frac{10}{3}\right)$


| Value of $z$ | $A$ |  |
| :--- | :--- | :--- | :--- |
| $=3 x+3 y$ | $A \times 25=75$ | $\begin{array}{l}B \\ 3 \times 15=45\end{array}\left\|\begin{array}{c}C \\ 80\end{array}\right\| \begin{array}{l}O \\ 0\end{array}, ~$ |

$\therefore$ Maximum value of $z=80$

Question ID : 4812975
Let $R$ be the relation in the set $A=\{a, b, c, d\}$ given by $R=\{(a, a),(b, b)(c, c),(a, b),(b, a),(c, d),(d$, d), $(d, c)\}$
(A) $R$ is reflexive and symmetric but not transitive
(B) $R$ is reflexive and transitive but not symmetric
(C) $R$ is symmetric and transitive but not reflexive
(D) $R$ is an equivalence relation

Answer (D)
Sol. $A=\{a, b, c, d\}$
$R=\{(a, a)(b, b)(c, c),(a, b)(b, a)(c, d)(d, d)(d, c)\}$
(i) $\because(a, a) \in R \forall a \in A$
(ii) If $(x, y) \in R \Rightarrow(y, x) \in R \therefore R$ is symmetric
(iii) $R$ is transitive also
$\therefore \quad R$ is an equivalence relation.

## Passage :

Three pizza outlets $A, B$ and $C$ sell three types of pizza namely cheese pizza, veg pizza and paneer pizza. In a day, $A$ can sell 40 cheese pizza, 30 veg pizza and 20 paneer pizza; $B$ can sell 20 cheese pizza, 40 veg pizza and 60 paneer pizza; $C$ can sell 60 cheese pizza, 20 veg pizza and 30 paneer pizza. If the revenue generated in a day by $A$ is $₹ 6000$, by $B$ is ₹ 9000 and by $C$ is $₹ 7000$. If $x$ denotes selling price of cheese pizza, $y$ is selling price of veg pizza and $z$ be the selling price of Paneer pizza then based on this information, answer the following question:

## Question ID : 4812976

The revenue generated by three outlets $A, B$ and $C$ are :
(A) 6000
(B) 22000
(C) 16000
(D) 15000

Answer (B)

$$
\begin{aligned}
& \text { Outlet } A \\
& 40 x+30 y+20 z=6000
\end{aligned}
$$

Sol.

Total revenue generated by $A, B$ and $C$ are :
$6000+9000+7000=₹ 22000$

## Question ID : 4812977

The matrix representation of the above problem is :
(A) $\left[\begin{array}{lll}4 & 2 & 6 \\ 3 & 4 & 2 \\ 2 & 6 & 3\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{l}600 \\ 900 \\ 700\end{array}\right]$
(B) $\left[\begin{array}{lll}4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{l}600 \\ 900 \\ 700\end{array}\right]$
(C) $\left[\begin{array}{lll}4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{l}600 \\ 450 \\ 700\end{array}\right]$
(D) $\left[\begin{array}{lll}4 & 2 & 6 \\ 3 & 4 & 2 \\ 2 & 6 & 3\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{l}6000 \\ 9000 \\ 7000\end{array}\right]$

## Answer (C)

Sol. Equation will be
$4 x+3 y+2 z=600$
$x+2 y+3 z=450$
$6 x+2 y+3 z=700$
$\left[\begin{array}{lll}4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{l}600 \\ 450 \\ 700\end{array}\right]$

## Question ID: 4812978

The price of a cheese pizza is :
(A) ₹50
(B) ₹80
(C) ₹500
(D) ₹800

## Answer (A)

Sol. Price of cheese pizza $=x$
i.e. ₹50

## Question ID: 4812979

The price of a paneer pizza is :
(A) ₹50
(B) ₹60
(C) ₹65
(D) ₹80

## Answer (D)

Sol. Price of paneer pizza $=z$
i.e. ₹80

## Question ID: 4812980

If the cost price of a cheese pizza is ₹30, a veg pizza is ₹50 and a paneer pizza is ₹50, what is the profit of outlet $A$ in a day?
(A) ₹ 6300
(B) ₹ 3300
(C) ₹2300
(D) ₹18300

## Answer (C)

Sol. CP of 40 cheese $+30 \mathrm{veg}+20$ paneer

$$
\begin{aligned}
& =40 \times 30+50 \times 30+20 \times 50 \\
& =1200+1500+1000 \\
& =₹ 3700
\end{aligned}
$$

SP of 40 cheese $+30 \mathrm{veg}+20$ paneer
= ₹ 6000
$\therefore \quad$ Profit $=6000-3700$
= ₹2300

## Passage:

A rectangle of length ' $x$ ' and breadth ' $y$ ' is inscribed in a semi-circle of fixed radius ' $r$ ' as shown in the figure given below.


Based on the above information answer the following question:

## Question ID: 4812981

Area $A(\theta), 0<\theta<\frac{\pi}{2}$ of the rectangle $A B C D$, is
(A) $r^{2} \sin \theta$
(B) $r^{2} \sin 2 \theta$
(C) $r^{2} \cos 2 \theta$
(D) $r^{2} \cos \theta$

Answer (B)

Sol.


Now,

$$
\begin{aligned}
& x=2 r \cos \theta \\
& y=r \sin \theta
\end{aligned}
$$

Area $(A B C D)=2 r^{2} \cos \theta \sin \theta$

$$
=r^{2} \sin 2 \theta
$$

## Question ID: 4812982

The value of $\theta$, for which $A^{\prime}(\theta)=0$ is
(A) $\pi$
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{3}$

## Answer (C)

Sol. $A^{\prime}(\theta)=0$

$$
\begin{aligned}
& A=r^{2} \sin 2 \theta \\
& A^{\prime}=2 r^{2} \cos 2 \theta=0 \\
& \therefore 2 \theta=(2 n+1) \frac{\pi}{2} \\
& \quad \theta=(2 n+1) \frac{\pi}{4} \\
& \therefore \theta=\frac{\pi}{4}
\end{aligned}
$$

## Question ID: 4812983

Dimensions $x, y$ of the rectangle $A B C D$, when area is maximum are:
(A) $r \frac{\sqrt{3}}{2}, \frac{2 r}{\sqrt{3}}$
(B) $r \sqrt{2}, \frac{r}{\sqrt{2}}$
(C) $\frac{r}{\sqrt{2}}, \sqrt{2} r$
(D) $r, \frac{r}{\sqrt{2}}$

## Answer (B)

Sol. If area is max.

$$
\begin{array}{rl}
A^{\prime}=0 \\
\Rightarrow & \theta=\frac{\pi}{4} \\
\therefore & x=2 r \cos \theta \\
& y=r \sin \theta \\
x & =\frac{2 r}{\sqrt{2}} \\
x & y=\frac{r}{\sqrt{2}} \\
& y=\frac{r}{\sqrt{2}}
\end{array}
$$

## Question ID: 4812984

Maximum area of the Rectangle is:
(A) $2 r^{2}$
(B) $3 r^{2}$
(C) $r^{2}$
(D) $4 r^{2}$

Answer (C)
Sol. Areamax. $\Rightarrow r^{2} \sin 2 \theta$ is max.

$$
\text { Area }=r^{2}
$$

## Question ID: 4812985

Perimeter of rectangle when its area is maximum is:
(A) $\frac{8 \sqrt{3} r}{3}$
(B) $4 r$
(C) $\frac{7 \sqrt{3} r}{3}$
(D) $3 \sqrt{2} r$

## Answer (D)

Sol. Max. perimeter $=2(I+b)$

$$
\begin{aligned}
& =2\left(r \sqrt{2}+\frac{r}{\sqrt{2}}\right) \\
& =2 r\left[\frac{2+1}{\sqrt{2}}\right] \\
& =3 \sqrt{2} r
\end{aligned}
$$

