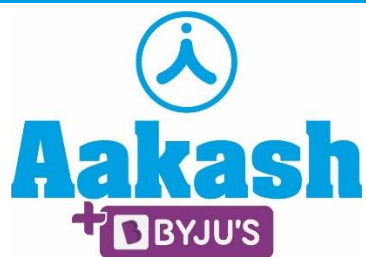


25/06/2022

Evening



Corporate Office: Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Memory Based Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
 - (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If the separation between the plates of parallel plate capacitor C is halved, its new capacitance will be

- (1) $\frac{C}{2}$ (2) $2C$
(3) $4C$ (4) $\frac{C}{4}$

Answer (2)

Sol. Capacitance of parallel plate capacitor = $\frac{\epsilon_0 A}{d}$

$$\text{If } C = \frac{\epsilon_0 A}{d}$$

$$\text{then } \frac{\epsilon_0 A}{\frac{d}{2}} = 2C$$

$$\Rightarrow \text{New capacitance} = 2C$$

2. Relationship between adiabatic coefficient (γ) and degree of freedom (f) is

- (1) $\gamma = 1 + \frac{2}{f}$ (2) $\gamma = \frac{2}{f}$
(3) $\gamma = \frac{f}{2}$ (4) $\gamma = \frac{2}{1+f}$

Answer (1)

Sol. The adiabatic constant,

$$\gamma = 1 + \frac{2}{f}$$

where, f is degree of freedom.

3. Brewster's law is valid

- (1) When reflected ray and refracted ray is perpendicular.
(2) When reflected ray and refracted ray are parallel
(3) When both are in same medium
(4) None of these

Answer (1)

Sol. According to Brewster's law, the reflected and refracted rays are mutually perpendicular.

4. A cube has surface area equal to 24 cm^2 . If temperature is changed by 10°C , then change in its volume is equal to: ($\alpha = 5 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$)

- (1) 0.12 cm^3 (2) 0.84 cm^3
(3) 0.54 cm^3 (4) 1.12 cm^3

Answer (1)

Sol. Change in volume is given by

$$\Delta V = V_0 \gamma \Delta T \text{ (where } \gamma = 3\alpha \text{)}$$

$$\Rightarrow \Delta V = [4\sqrt{4}] [15 \times 10^{-4}] [10] \text{ cm}^3$$

$$= 0.12 \text{ cm}^3$$

5. In an interference I_1 and I_2 represent the intensities of two sources. If $I_1 : I_2 = 4 : 1$, then the value of

$$\frac{I_{\max} + I_{\min}}{I_{\max} - I_{\min}} = \frac{5}{x}. \text{ The value of } x \text{ is}$$

- (1) 3 (2) 4
(3) 2 (4) 5

Answer (2)

$$\text{Sol. } \therefore I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

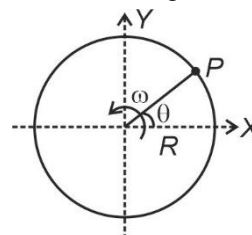
$$\text{and, } I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$$\Rightarrow \frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{4} + 1}{\sqrt{4} - 1} \right)^2 = \left(\frac{3}{1} \right)^2 = 9$$

$$\therefore \frac{I_{\max} + I_{\min}}{I_{\max} - I_{\min}} = \frac{\frac{I_{\max}}{I_{\min}} + 1}{\frac{I_{\max}}{I_{\min}} - 1}$$

$$= \frac{9 + 1}{9 - 1} = \frac{10}{8} = \frac{5}{4}$$

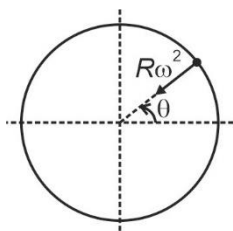
6. Find the acceleration of particle P moving on a circle as shown in the diagram



- (1) $\omega^2 R \cos \theta \hat{i} + \omega^2 R \sin \theta \hat{j}$
(2) $-\omega^2 R \cos \theta \hat{i} - \omega^2 R \sin \theta \hat{j}$
(3) $-\omega^2 R \cos \theta \hat{i} + \omega^2 R \sin \theta \hat{j}$
(4) $\omega^2 R \cos \theta \hat{i} - \omega^2 R \sin \theta \hat{j}$

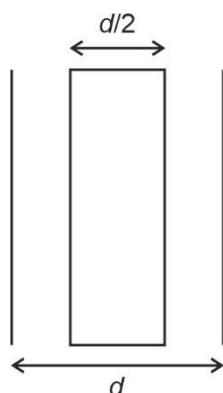
Answer (2)

Sol. We know that centripetal acceleration is directed towards the centre with magnitude $R\omega^2$:



$$\Rightarrow \vec{a} = R\omega^2 \{-\cos\theta \hat{i} - \sin\theta \hat{j}\}$$

7. A metal of thickness $\frac{d}{2}$ is inserted between plates of capacitor having capacitance C and distance between plate d . If new capacitance is C' , the value of $\frac{C}{C'}$ is



- (1) 1 : 2
(2) 2 : 1
(3) 1 : 1
(4) 2 : 3

Answer (1)

Sol. $C = \frac{\epsilon_0 A}{d}$

$$C' = \frac{\epsilon_0 A}{\left(\frac{d}{2}\right)} = \frac{2\epsilon_0 A}{d}$$

$$\therefore \frac{C}{C'} = \frac{1}{2}$$

8. For a solenoid, if number of turns are doubled and current is halved, then the new magnetic field is
- (1) Double
(2) Halfed
(3) Four times
(4) Remains same

Answer (4)

Sol. Due to a solenoid $B = \mu_0 ni$

Given that $n \rightarrow 2n$

$$i \rightarrow \frac{i}{2}$$

$$\Rightarrow B' = \mu_0 (2n) \left(\frac{i}{2}\right)$$

$$\Rightarrow B' = B$$

\Rightarrow Remains same

9. 4 objects have mass m and radius R . Moment of inertia of sphere about diameter is I_1 , that about the axis of cylinder is I_2 , about diameter of disc is I_3 , and about diameter of ring is I_4 are related as $2(I_2 + I_3) + I_4 = xI_1$. The value of x is

- (1) 3
(2) 4
(3) 5
(4) 6

Answer (3)

Sol. $I_1 = \frac{2}{5} MR^2$

$$I_2 = \frac{MR^2}{2}$$

$$I_3 = \frac{MR^2}{4}$$

$$I_4 = \frac{MR^2}{2}$$

$$\therefore 2(I_2 + I_3) + I_4 = xI_1$$

$$\Rightarrow 2\left[\frac{MR^2}{2} + \frac{MR^2}{4}\right] + \frac{MR^2}{2} = x \times \frac{2}{5} MR^2$$

$$\Rightarrow 2 \times \frac{3}{4} + \frac{1}{2} = x \times \frac{2}{5}$$

$$\Rightarrow 2 = x \times \frac{2}{5} \Rightarrow x = 5$$

10. 27 identical droplets 22 volts each coalesce together to form a bigger drop then final potential of the bigger drop is

- (1) 216
(2) 198
(3) 324
(4) 72

Answer (2)

Sol. Let q : charge on each droplet

r : radius of each droplet

Q : charge on final droplet

R : radius of final droplet

$$R = (27)^{\frac{1}{3}} r$$

and $Q = 27q$

$$\Rightarrow V_{\text{final}} = \frac{kQ}{R} = \frac{k(27q)}{(27)^{\frac{1}{3}} r} = 9 \times \frac{kq}{r}$$

$$\Rightarrow V_{\text{final}} = 9 \times 22 \text{ V} \\ = 198 \text{ V}$$

11. **Assertion (A)** : For a fix value of range R if u is same, the projectile can achieve height H_1 and H_2 , then $R = 4\sqrt{H_1 H_2}$

$$\text{Reason (R)} : H_1 \times H_2 = \frac{u^2 \sin^2 \theta}{2g} \times \frac{u^2 \cos^2 \theta}{2g}$$

- (1) (A) and (R) are true and (R) is the correct explanation of (A)
(2) (A) and (R) are true but (R) is not the correct explanation of (A)
(3) (A) is true but (R) is false
(4) (A) is false but (R) is true

Answer (1)

Sol. $\theta_1 = \theta$, $\theta_2 = 90 - \theta$

$$R = \frac{u^2 (2 \sin \theta \cos \theta)}{g}$$

$$H_1 = \frac{u^2 \sin^2 \theta}{2g}, \quad H_2 = \frac{u^2 \cos^2 \theta}{2g}$$

$$\therefore H_1 H_2 = \left(\frac{u^2 \sin \theta \cdot \cos \theta \times 2}{g} \right)^2 \times \frac{1}{16}$$

$$\Rightarrow 4\sqrt{H_1 H_2} = R$$

12. **Assertion (A)**: Magnetic susceptibility (χ) of ferro and para increase with increase in temperature.

Reason (R): In diamagnetic material, orbital magnetic moment develops opposite to the outside field.

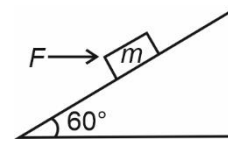
- (1) (A) and (R) both are correct and (R) is correct explanation of (A)
(2) (A) and (R) both are correct but (R) is not the correct explanation of (A)
(3) (A) is true but (R) is false
(4) (A) is false but (R) is true

Answer (2)

Sol. Magnetic susceptibility, $\chi \propto \frac{1}{T}$

$\Rightarrow \chi$ increases with decrease in temperature

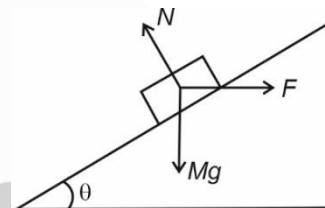
13. Find the value of force F required to maintain the box at equilibrium on smooth wedge as shown in diagram ($m = 0.5 \text{ kg}$)



- (1) 8.66
(2) 17.32
(3) 4.26
(4) 19.86

Answer (1)

Sol. Drawing the FBD:



$$\Rightarrow N \cos \theta = Mg$$

$$\& N \sin \theta = F$$

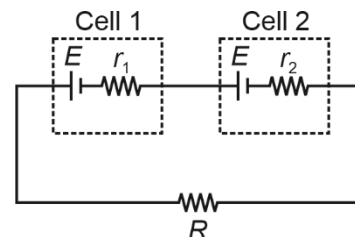
$$\Rightarrow \tan \theta = \frac{F}{Mg}$$

$$\Rightarrow F = Mg \tan \theta$$

$$\Rightarrow F = 0.5 \times 10 \times \sqrt{3}$$

$$\Rightarrow F \approx 8.66 \text{ N}$$

14. Two cells both have emf's E and internal resistance r_1 and r_2 are connected with a resistance as shown in the diagram. The value of R such that potential difference across cell 2 is 0 is



- (1) $r_2 - r_1$
(2) $2r_2 - r_1$
(3) $r_2 - 2r_1$
(4) $r_1 + r_2$

Answer (1)

Sol. $i = \frac{2\varepsilon}{r_1 + r_2 + R}$

and $e - ir_2 = 0$

$\Rightarrow \varepsilon = \frac{2\varepsilon}{r_1 + r_2 + R} \times r_2$

$\Rightarrow r_1 + r_2 + R = 2r_2$

$\Rightarrow R = r_2 - r_1$

15. 4 particles electron (e^-), neutron (n), proton (p) and alpha particle (α) have same kinetic energy if their associated de-Broglie wavelength are λ_e , λ_n , λ_p and λ_α respectively, then

(1) $\lambda_e > \lambda_n > \lambda_p > \lambda_\alpha$

(2) $\lambda_e > \lambda_p > \lambda_n > \lambda_\alpha$

(3) $\lambda_\alpha > \lambda_p > \lambda_n > \lambda_e$

(4) $\lambda_\alpha > \lambda_n > \lambda_p > \lambda_e$

Answer (2)

Sol. $\therefore \lambda = \frac{h}{\sqrt{2m(KE)}}$

$\Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$

$\therefore m_e < m_p < m_n < m_\alpha$

$\Rightarrow \lambda_e > \lambda_p > \lambda_n > \lambda_\alpha$

16. Permeability of medium is 1 and speed of light in that medium is 2×10^8 m/sec, then relative permittivity of the medium is

(1) 4

(2) $\frac{9}{4}$

(3) 9

(4) $\frac{4}{9}$

Answer (2)

Sol. $\mu = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^8}$

$\Rightarrow \mu = 1.5$

Also, $\mu = \sqrt{\varepsilon_r \mu_r}$

$\Rightarrow 1.5 = \sqrt{\varepsilon_r \times 1}$

$\Rightarrow \varepsilon_r = (1.5)^2$

$= 2.25$

17. Two satellites are revolving around a planet in a circular orbit. If v_1 is speed of a satellite in radius 800 km and v_2 is speed of satellite having radius 3200 km, then $\frac{v_1}{v_2} = x$, then value of x is

(1) 6

(2) 4

(3) 3

(4) 2

Answer (4)

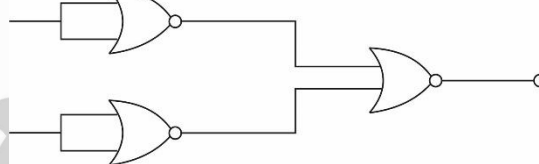
Sol. Orbital velocity, $v = \sqrt{\frac{GM}{R}}$

$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{R_2}{R_1}}$

$= \sqrt{\frac{3200}{800}}$

$= 2$

18.



The gate shown in figure behaves like

(1) AND

(2) OR

(3) NOT

(4) NOR

Answer (1)

Sol. Taking the inputs as A and B , we get output as

$= [(A + A)' + (B + B)']'$

$= (A' + B)'$

$= AB$

\Rightarrow AND gate

19. 5 kg of vessel of copper at 500°C is placed on an ice at 0°C . Assume exchange of heat between ice and vessel only, then the amount of ice melt is

($S_{\text{Copper}} = 0.39 \text{ J/gm}^\circ\text{C}$ and $L_{\text{ice}} = 335 \text{ J/gm}$)

(1) 2.9 kg

(2) 5.8 kg

(3) 1.45 kg

(4) 0.5 kg

Answer (1)

Sol. $\Delta Q = m_1 s \Delta T = m_2 \times L$

$$\Rightarrow m_2 = \frac{m_1 s \Delta T}{L} d$$

$$= \frac{5 \times 0.39 \times (500) \times 1000}{335}$$

$$= 2910 \text{ g}$$

$$= 2.9 \text{ kg}$$

20. Length of a cylindrical conductor is doubled keeping the volume constant. Percentage change in the value of its resistance across its ends to equal to

(1) 100

(2) 200

(3) 300

(4) 50

Answer (3)

Sol. We know that $R = \frac{\rho l}{A}$

$$\Rightarrow R \propto \frac{l}{A}$$

Also, since $lA = \text{constant}$

$$\Rightarrow A \propto \frac{1}{l}$$

$$\Rightarrow R \propto l^2$$

\Rightarrow % change in resistance

$$= \frac{4R_0 - R_0}{R_0} \times 100$$

$$= 300\%$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A graph is drawn between log of radius of nucleus and log of atomic mass number for various elements. The slope of the graph is equal to ____.

Answer (0.33)

Sol. We know that

$$r = r_0 A^{1/3}$$

$$\Rightarrow \frac{r}{r_0} = A^{1/3}$$

$$\Rightarrow \ln\left(\frac{r}{r_0}\right) = \frac{1}{3} \ln(A)$$

$$\Rightarrow \text{Slope} = 1/3$$

22. On a level road maximum speed of a car to take a turn of 75 m radius is 30 m/sec, then maximum speed of car to take a turn of 48 m in m/sec is ____.

Answer (24)

Sol. $\because v = \sqrt{ugr}$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}}$$

$$\Rightarrow \frac{30}{v_2} = \sqrt{\frac{75}{48}}$$

$$\Rightarrow \frac{30}{v_2} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

$$\Rightarrow v_2 = 24 \text{ m/s}$$

23.

24.

25.

26.

27.

28.

29.

30.

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Which of the following has least spin only magnetic moment?

- (1) Fe^{+3} (2) Fe^{+2}
(3) Cu^{+2} (4) Ni^{+2}

Answer (3)

Sol. Electronic configurations are,

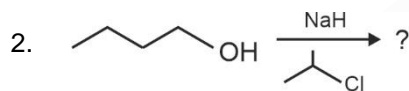
$$\text{Fe}^{+3} = [\text{Ar}] 3d^5 \quad (5 \text{ unpaired electrons})$$

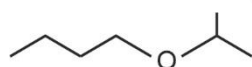
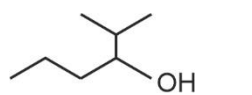
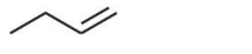

$$\text{Fe}^{+2} = [\text{Ar}] 3d^6 \quad (4 \text{ unpaired electrons})$$

$$\text{Cu}^{+2} = [\text{Ar}] 3d^9 \quad (1 \text{ unpaired electron})$$

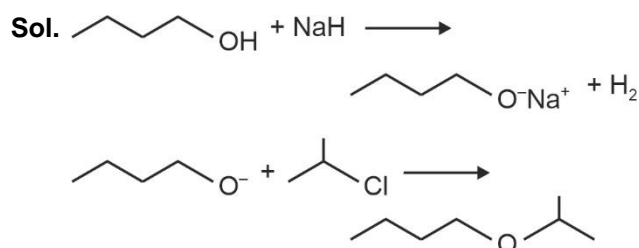
$$\text{Ni}^{+2} = [\text{Ar}] 3d^8 \quad (2 \text{ unpaired electrons})$$

\therefore Least spin only magnetic moment is of Cu^{2+} .



- (1) 
(2) 
(3) 
(4) 

Answer (1)

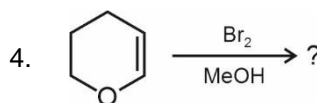
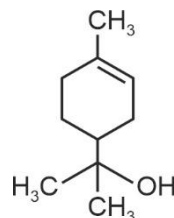


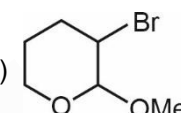
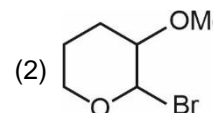
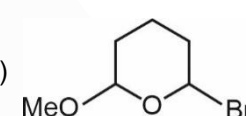
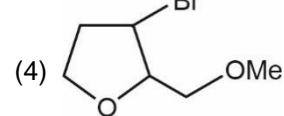
3. Dettol has two components, A has 6π electrons. What is B?

- (1) Terpineol (2) Bithionol
(3) Chloroxylonol (4)

Answer (1)

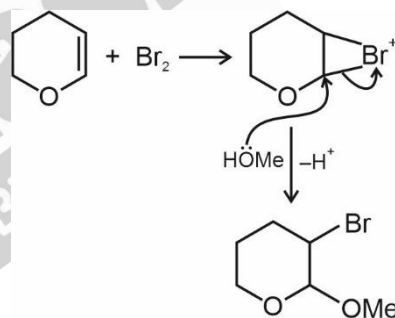
Sol. B is terpineol. Its structure is



- (1) 
(2) 
(3) 
(4) 

Answer (1)

Sol.



5. $t_{1/2}$ of a reaction and pressure of reactant is given. Find the order of reaction.

P_R^0	$t_{1/2}$
10 atm	5 min
20 atm	10 min

- (1) 1st order (2) 2nd order
(3) Zero order (4) $\frac{1}{2}$ order

Answer (3)

Sol. Since, $t_{1/2}$ of reaction is directly proportional to the initial pressure of the reactant. Hence, reaction will be zero order.

$$\text{For zero order, } t_{1/2} \propto P_0$$

6. **Assertion** : From a mixture of benzoic acid and naphthalene, benzoic acid can be separated using benzene.

Reason : Benzoic acid is soluble in hot water.

- (1) Assertion and reason both are True
- (2) Assertion is False reason is True
- (3) Assertion is False reason is False
- (4) Assertion is True reason is False

Answer (2)

Sol. Assertion : From a mixture of benzoic acid and naphthalene, benzoic acid can be separated using benzene.

Reason : Benzoic acid is soluble in hot water.

Assertion is false because both the benzoic acid and naphthalene will dissolve in benzene.

Reason is true because only benzoic acid will dissolve in hot water.

7. Correct order of electron gain enthalpy in magnitude is,

- (1) $F > Cl > Te > Po$
- (2) $Cl > F > Te > Po$
- (3) $Po > Te > Cl > F$
- (4) $Cl > Te > Po > F$

Answer (2)

Sol. Electron gain enthalpy of $Cl = -349 \text{ kJ/mole}$

$$F = -328 \text{ kJ/mole}$$

$$Po = -174 \text{ kJ/mole}$$

$$Te = -190 \text{ kJ/mole}$$

Therefore, the correct order is $Cl > F > Te > Po$

8. Arrange the following species in decreasing order to their standard reduction potential value

- (i) Cl_2/Cl^-
- (ii) F_2/F^-
- (iii) Na^+/Na
- (iv) Li^+/Li
- (v) I_2/I^-

- (1) $(ii) > (i) > (v) > (iii) > (iv)$
- (2) $(v) > (iv) > (iii) > (ii) > (i)$
- (3) $(ii) > (i) > (iv) > (v) > (iii)$
- (4) $(v) > (i) > (ii) > (iv) > (iii)$

Answer (1)

Sol. $E^\circ_{Cl_2/Cl^-} = 1.36 \text{ V}$

$$E^\circ_{F_2/F^-} = 2.87 \text{ V}$$

$$E^\circ_{Na^+/Na} = -2.71 \text{ V}$$

$$E^\circ_{Li^+/Li} = -3.05 \text{ V}$$

$$E^\circ_{I_2/I^-} = 0.54 \text{ V}$$

Hence, order of standard reduction potential is $2 > 1 > 5 > 3 > 4$.

9. The size of electron, proton, neutron and alpha particle have same value of kinetic energy. Then what is the correct order of wavelength according to de Broglie

- (1) $\lambda_p = \lambda_n = \lambda_e = \lambda_\alpha$
- (2) $\lambda_\alpha < \lambda_n < \lambda_p < \lambda_e$
- (3) $\lambda_p < \lambda_e < \lambda_\alpha < \lambda_n$
- (4) $\lambda_\alpha > \lambda_p > \lambda_e > \lambda_n$

Answer (2)

Sol. Since, $\lambda \propto \frac{1}{\sqrt{mE}}$

Kinetic energy is same for all the particles

$$(mass)_\alpha > (mass)_n > (mass)_p > (mass)_e$$

Hence order of de-Broglie wavelength is

$$\lambda_\alpha < \lambda_n < \lambda_p < \lambda_e$$

10. Which of the following is/are correct?

S-1: BOD of polluted water may be 17 ppm.

S-2: BOD represent the amount of oxygen to decompose biodegradable and non-biodegradable substances.

- (1) S-1 is correct
S-2 is correct
- (2) S-1 is correct
S-2 is not correct
- (3) S-1 is not correct
S-2 is correct
- (4) S-1 is not correct
S-2 is not correct

Answer (2)

Sol. Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

The amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water is called Biochemical Oxygen Demand (BOD).

Hence, S-1 is correct whereas S-2 is incorrect.

11. Match the following correctly.

- | | |
|----------------|--------------|
| (i) Zymase | (A) Stomach |
| (ii) Urease | (B) Yeast |
| (iii) Diastase | (C) Malt |
| (iv) Pepsin | (D) Soyabean |

- | | |
|-------------|-------------|
| (1) (i) - B | (2) (i) - B |
| (ii) - D | (ii) - A |
| (iii) - C | (iii) - C |
| (iv) - A | (iv) - D |

- | | |
|-------------|-------------|
| (3) (i) - A | (4) (i) - D |
| (ii) - B | (ii) - C |
| (iii) - C | (iii) - B |
| (iv) - D | (iv) - A |

Answer (1)

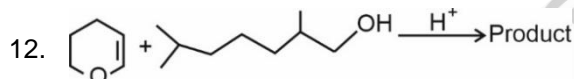
Sol. The enzyme and sources are,

Pepsin - Stomach

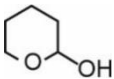
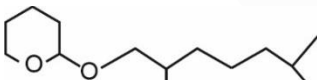
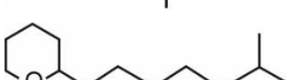

Diastase - Malt

Urease - Soyabean

Zymase - Yeast

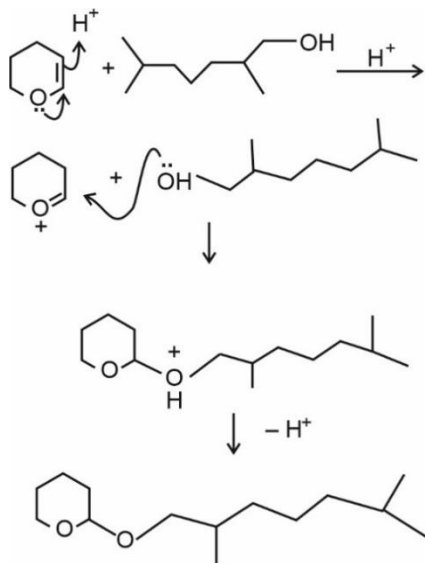


The product is

- (1) 
- (2) 
- (3) 
- (4) 

Answer (2)

Sol.



13. 0.10 M, 10 ml of acid reacts with 30 ml of 0.05 M $\text{Ca}(\text{OH})_2$ completely, then basicity of the acid is

- | | |
|-------|-------|
| (1) 1 | (2) 2 |
| (3) 3 | (4) 4 |

Answer (3)

Sol. Equivalents of acid = equivalents of base

$$0.10 \times 10 \times n = 30 \times 0.05 \times 2$$

$$n = \frac{30 \times 0.1}{1}$$

$$n = 3$$

14. Which of the following statement is correct?

- (i) H_2O is amphoteric, because act as Lewis acid as well as Lewis base.
- (ii) H_2O react like a base with NH_3 and acid with BF_3 .
- (1) (i) is correct, (ii) is not correct
- (2) (i) is not correct, (ii) is incorrect
- (3) (i) and (ii) are correct, (ii) is not the correct explanation of (i)
- (4) (i) and (ii) are correct, (ii) is correct explanation of (i)

Answer (2)

Sol. H_2O is amphoteric as per Bronsted-Lowry theory. It cannot act as a Lewis acid. Therefore, (i) is incorrect. (ii) is incorrect.

15. 0.7 gm of a solute having M_w (96) is dissolved in 42 gm of H_2O , then percentage association of solute in H_2O assuming dimerisation is, $\Delta T_f = 0.2$ and $k_f = 1.86 \text{ k kg/mole}$

- (1) 60
- (2) 80
- (3) 76
- (4) 40

Answer (3)

Sol. $\Delta T_f = i k_f m$

$$0.2 = i \times 1.86 \times \frac{0.7}{42 \times 96} \times 1000$$

$$i \approx 0.619$$

$$\approx 0.62$$

$$i = 1 - \frac{\alpha}{2}$$

$$\therefore \alpha = 0.76$$

Hence, percentage association is 76%.

16. Which of the following will absorb minimum wavelength light?

- (1) $K_4[Fe(CN)_6]$ (2) $K_4[FeCl_6]$
(3) $K_2[Fe(CN)_4(H_2O)_2]$ (4) $K_2[FeCl_4]$

Answer (1)

Sol. Wavelength of light absorbed by a complex is inversely proportional to the stability of the complex. Of the given complexes, the most stable complex is $K_4[Fe(CN)_6]$ due to 6 strong field CN^- ion ligands. Therefore, it will absorb the minimum wavelength.

17. **Statement 1** : In electrolytic refining of copper blister copper gives precious metal.

Statement 2 : In electrolysis of copper blister, anode is made of impure copper.

- (1) Statements 1 and 2 both are correct
(2) Statement 1 is correct but statement 2 is incorrect
(3) Statement 1 is incorrect but statement 2 is correct
(4) Statements 1 and 2 both are incorrect

Answer (1)

Sol. Statement 1 : In electrolytic refining of copper, blister copper gives precious metals.

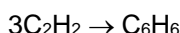
Statement 2 : In electrolysis of copper is made anode.

Statement 1 is true because blister copper contains gold and silver as impurities which settle down as anode mud in electrolytic refining of copper.

Statement 2 is true because impure copper is made the anode and pure copper is made the cathode.

18. Heat of combustion for benzene and acetylene are -3900 and -642 joule.

Then calculate heat of reaction (per mole) for following reaction.



- (1) -658 J mol^{-1} (2) $+658 \text{ J mol}^{-1}$
(3) -3000 J mol^{-1} (4) $+3400 \text{ J mol}^{-1}$

Answer (2)

Sol. Heat of Reaction

$$\begin{aligned} &= \frac{(3 \times \Delta H_{\text{combustion}} \text{ of Acetylene} - \Delta H_{\text{combustion}} \text{ of Benzene})}{3} \\ &= \frac{3 \times (-642) + (3900)}{3} \\ &= 658 \text{ J mol}^{-1} \end{aligned}$$

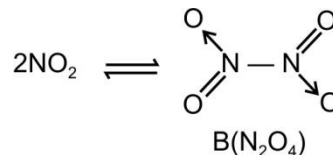
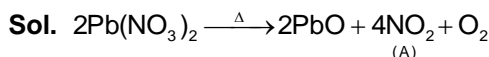
19. $Pb(NO_3)_2$ on heating gives A + PbO + O_2

A dimerizes to give B.

How many bridged oxygen atoms are present in B?

- (1) 0 (2) 1
(3) 2 (4) 3

Answer (1)



No. of bridged oxygen atoms present in B = 0

20. The threshold frequency for a metal is $1.3 \times 10^{15} \text{ Hz}$. Then minimum energy required to eject the electron from metal surface is ($h = 6.62 \times 10^{-34} \text{ Js}$)

- (1) $8.6 \times 10^{-19} \text{ J}$
(2) $7.8 \times 10^{-16} \text{ J}$
(3) $2.3 \times 10^{-19} \text{ J}$
(4) $6.4 \times 10^{-19} \text{ J}$

Answer (1)

Sol. Since, $E = h\nu$

For work function i.e. minimum energy required, will be

$$W_0 = h\nu_0$$

$$\nu_0 = 1.3 \times 10^{15} \text{ Hz}$$

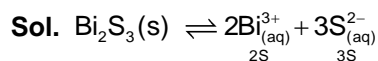
$$\begin{aligned} W_0 &= 6.62 \times 10^{-34} \times 1.3 \times 10^{15} \\ &= 8.6 \times 10^{-19} \text{ J} \end{aligned}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The solubility product (K_{sp}) of Bi_2S_3 is 1.08×10^{-73} at a certain temperature. The molar solubility of the salt at this temperature is $1 \times 10^{-x} \text{ M}$. The value of x is

Answer (15)



$$K_{\text{sp}} = [\text{Bi}^{3+}]^2 [\text{S}^{2-}]^3$$

$$K_{\text{sp}} = (2\text{S})^2 (3\text{S})^3$$

$$108\text{S}^5 = 1.08 \times 10^{-73} = 108 \times 10^{-75}$$

$$\text{S} = 1 \times 10^{-15} \text{ mol/L}$$

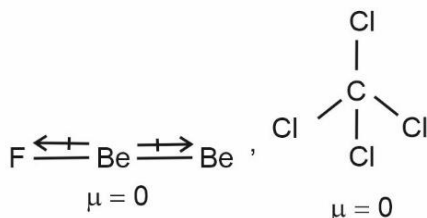
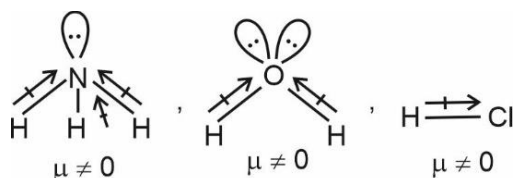
Hence $x = 15$

22. Which of the following have net dipole moment non-zero?

BeF_2 , NH_3 , H_2O , CCl_4 , HCl

Answer (3)

Sol.



23. $t_{\frac{1}{2}}$ of the reaction is 340 sec. If the initial pressure of reactant is 55.8 kPa and $t_{\frac{1}{2}}$ of the reaction is 170 sec. If the initial pressure of reactant is 27.8 kPa. Determine the order of the reaction

Answer (0)

Sol. $t_{\frac{1}{2}} \propto P_0^{1-n}$

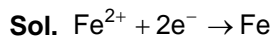
Where P_0 is initial pressure of the reactant and "n" is the order of the reaction.

On observation, $t_{\frac{1}{2}}$ gets doubled when the initial pressure is doubled.

Therefore, $n = 0$

24. During electrolysis process of FeSO_4 , 0.374 g of Fe is deposited on cathode, when 1.5 A current is passed through the solution for 'x' minutes. Then the value of x is
[Assume current efficiency as 100% and molar mass of Fe is 56 g/mol]

Answer (14.32)



2F of charge will deposit 1 mole of Fe

For, $\frac{0.374}{56} = 6.68 \times 10^{-3}$ moles

For, 6.68×10^{-3} moles,
charged required $= 2 \times 6.68 \times 10^{-3} \text{F}$

$$Q = It = \frac{1.5 \times t}{96500} \text{F}$$

Hence, $\frac{1.5 t}{96500} = 2 \times 6.68 \times 10^{-3}$

$$t = 14.32 \text{ min}$$

25. In an organic compound containing protein, amount of protein is 0.3%. Then minimum molecular weight of organic compound is
(M. wt. of protein = 75)

Answer (25000)

Sol. 0.3 g of protein is present in total mass = 100 g

1 g of protein is present in total mass $= \frac{100}{0.3}$

75 g of protein is present in total mass

$$= \frac{100}{0.3} \times 75 = 25000 \text{ g mol}^{-1}$$

26. Nitrogen gas is filled in a rigid container. At 6:00 AM, the pressure is 30 atm at 27°C , and at 3:00 PM, pressure becomes 'P' atm at 45°C . Find the value of 'P'
[Round off to the nearest integer]

Answer (32)

Sol. From $PV = nRT$

Since, no. of moles and volume is constant.

Hence, $P \propto T$

$$\frac{P_1}{T_1} = \frac{P}{T} \quad [\text{Pressure at 3:00 PM is 'P' atm and temperature 'T' K}]$$

$$\frac{30}{100} = \frac{P}{318}$$

$$P = 31.8 \text{ atm}$$

$$\text{Hence 'P' = 32}$$

27.
28.
29.
30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. $2\sin(12^\circ) - \sin(72^\circ) =$

- (1) $\frac{1-\sqrt{5}}{8}$ (2) $\frac{\sqrt{5}(1-\sqrt{3})}{4}$
(3) $\frac{\sqrt{3}(1-\sqrt{5})}{2}$ (4) $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

Answer (4)

Sol. $\sin 12^\circ + \sin 12^\circ - \sin 72^\circ$

$$= \sin 12^\circ - 2\sin 30^\circ \cdot \cos 42^\circ$$

$$= \sin 12^\circ - 2\left(\frac{1}{2}\right)\cos 42^\circ$$

$$= \sin 12^\circ - \sin 48^\circ$$

$$= -2\sin 18^\circ \cdot \cos 30^\circ$$

$$= \frac{\sqrt{3}(1-\sqrt{5})}{4}$$

2. $y = y(x)$ is the solution of the differential equation

$$2x^2 \frac{dy}{dx} - 2xy + 8y^2 = 0, y(e) = \frac{e}{3}, \text{ then } y(1) \text{ is equal}$$

to

- (1) $\frac{2}{3}$ (2) 3
(3) $\frac{3}{2}$ (4) -1

Answer (4)

Sol. Put $y = vx$, we get

$$v + x \frac{dv}{dx} = \frac{2x(vx) - 8v^2x^2}{2x^2}$$

$$\Rightarrow x \frac{dv}{dx} = -4v^2$$

$$\Rightarrow 4 \frac{dx}{x} + \frac{dv}{v^2} = 0$$

$$\Rightarrow 4 \ln x - \frac{1}{v} = c$$

$$\Rightarrow 4 \ln x - \frac{x}{y} = c$$

$$\downarrow (e, e/3)$$

$$c = 1$$

$$\Rightarrow 4 \ln x - \frac{x}{y} = 1$$

$$\Rightarrow 4 \ln 1 - \frac{1}{y} = 1$$

$$\Rightarrow y = -1$$

3. If $A = \{x \in R : |x+1| < 2\}$, $B = \{x \in R : |x-1| \geq 2\}$ then

- (1) $A \cup B = R - [1, 3]$ (2) $A \cap B = (-1, 1)$
(3) $A \cap B = (-3, -1]$ (4) $B - A = R - (-3, 1]$

Answer (3)

Sol. Clearly $A = (-3, 1)$ and $B = (-\infty, -1] \cup [3, \infty)$

$$\text{So } A \cup B = (-\infty, 1) \cup [3, \infty) = R - [1, 3]$$

$$A \cap B = (-3, -1]$$

$$\text{and } B - A = (-\infty, -3] \cup [3, \infty) = R - (-3, 3)$$

4. Find the sum $S = 1 + 2.3 + 3.3^2 + \dots + 10.3^9$

- (1) $\frac{1}{4}(19.3^{10} + 1)$ (2) $\frac{1}{4}(19.3^{10} - 1)$
(3) $\frac{1}{2}(19.3^{10} + 1)$ (4) $\frac{1}{2}(19.3^{10} - 1)$

Answer (1)

Sol. $S = 1 + 2.3 + 3.3^2 + 4.3^3 + \dots + 10.3^9$

$$\begin{aligned} 3.S &= 1.3 + 2.3^2 + \dots + 10.3^{10} \\ -2S &= (1 + 3 + 3^2 + \dots + 3^9) - 10.3^{10} \end{aligned}$$

$$\Rightarrow -2S = \frac{3^{10} - 1}{3 - 1} - 10.3^{10}$$

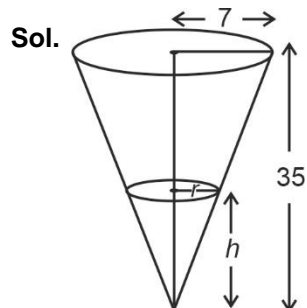
$$\Rightarrow S = 5.3^{10} - \left(\frac{3^{10} - 1}{4}\right)$$

$$= \frac{19.3^{10} + 1}{4}$$

5. Water is increasing in a right circular cone of height 35 cm and diameter of base 14 cm with rate $1 \text{ cm}^3/\text{sec}$. Then rate change of lateral surface area of cone is when height of water is 10 cm is

- (1) $\frac{\sqrt{26}}{10}$ (2) $\frac{\sqrt{26}}{5}$
 (3) $\frac{\sqrt{21}}{5}$ (4) 5

Answer (2)



$$\therefore \frac{dv}{dt} = 1 \text{ cm}^3/\text{sec}$$

$$\text{Lateral surface area} = s = \pi r \sqrt{r^2 + h^2}$$

$$\therefore \frac{h}{r} = \frac{35}{7}$$

$$\Rightarrow r = \frac{h}{5}$$

$$v = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \cdot \frac{h^3}{25}$$

$$\therefore \frac{dv}{dt} = \frac{\pi h^2}{25} \cdot \frac{dh}{dt} \quad \dots(i)$$

$$\text{and } s = \pi \cdot \frac{h}{5} \sqrt{\frac{h^2}{25} + h^2} = \frac{\pi h^2 \sqrt{26}}{25}$$

$$\frac{ds}{dt} = \frac{2h\pi \cdot \sqrt{26}}{25} \cdot \frac{dh}{dt} \quad \dots(ii)$$

$$\text{From (i) and (ii): } \frac{ds}{dt} = \frac{2h\pi \cdot \sqrt{26}}{25} \cdot \frac{25}{\pi h^2} \frac{dv}{dt}$$

$$\therefore \frac{ds}{dt} = \frac{2\sqrt{26}}{10} \quad (\text{at } h = 10)$$

$$\therefore \frac{ds}{dt} = \frac{\sqrt{26}}{5} \text{ cm}^2/\text{sec}$$

6. 2, 4, 8, 16, 32, 32 are written on the faces of a biased dice. The probability of showing up face with mark n is $\frac{1}{n}$. If dice is rolled 3 times. Find probability of sum coming as 48.

- (1) $\frac{7}{2^{12}}$ (2) $\frac{3}{2^{10}}$
 (3) $\frac{7}{2^{11}}$ (4) $\frac{13}{2^{12}}$

Answer (4)

Sol. There are only two ways to get score 48, which are (16, 16, 16) or (8, 8, 32)

$$\begin{aligned} \text{Required probability} &= \left(\frac{1}{16}\right)^3 + 3 \left(\frac{1}{8} \cdot \frac{1}{8} \cdot \frac{2}{32}\right) \\ &= \frac{13}{2^{12}} \end{aligned}$$

7. In the series $(5+x)^{500} + x(5+x)^{499} + \dots + x^{500}$, find coefficient of x^{101} .

- (1) 5^{300} (2) 5^{399}
 (3) $5^{399} {}^{501}C_{101}$ (4) $-5^{399} {}^{501}C_{101}$

Answer (3)

$$\text{Sol. } S = (5+x)^{500} + x(5+x)^{499} + \dots + x^{500}$$

Given series is G.P. of common ratio $\frac{x}{5+x}$ with 501 - terms

$$\begin{aligned} \therefore \text{Sum of G.P.} &= (5+x)^{500} \frac{\left(1 - \left(\frac{x}{5+x}\right)^{501}\right)}{\left(1 - \frac{x}{5+x}\right)} \\ &= \frac{(5+x)^{501} - x^{501}}{5} \end{aligned}$$

$$\begin{aligned} \Rightarrow \text{Coefficient of } x^{101} &= \frac{1}{5} {}^{501}C_{101} 5^{400} \\ &= {}^{501}C_{101} 5^{399} \end{aligned}$$

8. Negation of the statement $(\sim p \vee q) \Rightarrow (\sim q \wedge p)$ is equivalent to

- (1) $\sim p \Rightarrow q$ (2) $p \Rightarrow q$
 (3) $\sim q \Rightarrow p$ (4) $p \Leftrightarrow q$

Answer (2)

Sol. $(\sim p \vee q) \Rightarrow (\sim q \wedge p) \equiv (\sim p \vee q) \vee (p \wedge \sim q)$
 $\equiv (p \wedge \sim q)$

Its negation will be $\sim(p \wedge \sim q)$
 $\equiv \sim p \vee q$
 $\equiv p \Rightarrow q$

9. If $b_n = \int_0^{\pi/2} \frac{\cos^2 nx}{\sin x} dx$, then

(1) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with common difference = -2

(2) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with common difference = 2

(3) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in G.P.

(4) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in A.P. with common difference = -2

Answer (1)

Sol. $b_n = \int_0^{\pi/2} \frac{\cos^2 nx}{\sin x} dx$

$$\begin{aligned} \therefore b_n - b_{n-1} &= \int_0^{\pi/2} \frac{\cos^2 nx - \cos^2 (n-1)x}{\sin x} dx \\ &= \int_0^{\pi/2} \frac{-\sin x \cdot \sin(2n-1)x}{\sin x} dx \\ &= \left[\frac{\cos(2n-1)x}{(2n-1)} \right]_0^{\pi/2} \\ &= -\frac{1}{(2n-1)} \end{aligned}$$

Now, $\frac{1}{b_n - b_{n-1}} = -(2n-1)$

$\therefore \frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with common difference -2

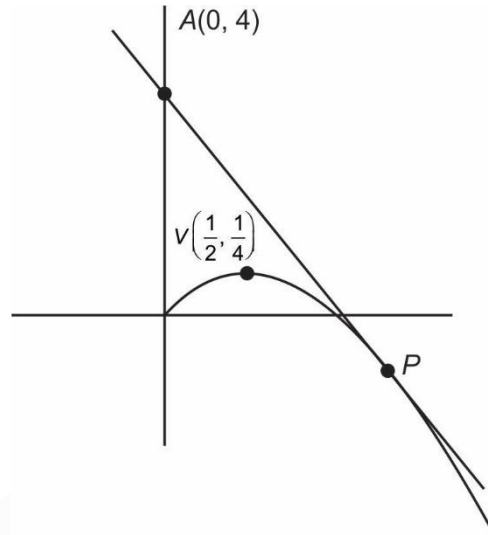
10. Equation of tangent at P on parabola $y = x - x^2$ is $y = 4 + kx$, $x > 0$ and let V be the vertex of parabola, then slope of line joining P and V is

(1) $-\frac{3}{2}$ (2) $\frac{26}{9}$

(3) $\frac{5}{2}$ (4) $\frac{23}{6}$

Answer (1)

Sol. Two tangents can be drawn from A to the given parabola whose equation is



$SS_1 = T^2$

$\Rightarrow (x^2 + y - x)4 = \left(\frac{y+4-x}{2}\right)^2$

$\Rightarrow 15x^2 - y^2 + 2xy + 8y - 8x - 16 = 0$

$\Rightarrow (5x - y + 4)(3x + y - 4) = 0$

$\therefore x > 0$ so equation of $PA : 3x + y = 4$

Solving with the parabola we get $P(2, -2)$

Slope of $PV = \frac{-2 - \frac{1}{4}}{2 - \frac{1}{2}} = -\frac{3}{2}$

11. If system of equation

$-kx + 3y - 14z = 25$

$-15x + 4y - kz = 3$

$-4x + y + 3z = 4$ is consistent, then k belongs

(1) $R - \{-11, 13\}$ (2) $R - \{-11, 11\}$

(3) $R - \{13\}$ (4) For all $k \in R$

Answer (2)

Sol. For consistent solution

$$\begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} \neq 0$$

$\therefore 121 - k^2 \neq 0$

$\therefore k \in R - \{-11, 11\}$

12. Find the value of $\tan^{-1} \left[\frac{\cos \frac{(15\pi)}{4} - 1}{\sin \frac{\pi}{4}} \right]$

- (1) $-\frac{\pi}{8}$ (2) $-\frac{4\pi}{9}$
 (3) $-\frac{5\pi}{12}$ (4) $-\frac{\pi}{4}$

Answer (1)

Sol. $\tan^{-1} \left(\frac{\cos \left(\frac{15\pi}{4} \right) - 1}{\sin \frac{\pi}{4}} \right)$

On substituting $\cos \left(\frac{15\pi}{4} \right) = \frac{1}{\sqrt{2}}$ and $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$= \tan^{-1} \left(\frac{\frac{1}{\sqrt{2}} - 1}{\frac{1}{\sqrt{2}}} \right)$$

$$= \tan^{-1} (1 - \sqrt{2})$$

$$= -\frac{\pi}{8}$$

13. α is a repeated root of $ax^2 - 2bx + 15 = 0$, ($a, b \in R$) and $x^2 - 2bx + 21 = 0$ have roots α and β then $\alpha^2 + \beta^2 =$

- (1) 58 (2) 56
 (3) 57 (4) 60

Answer (1)

Sol. $ax^2 - 2bx + 15 = 0$ will have its repeated root at $x = \frac{b}{a}$ and $D = 0$

$$\Rightarrow b^2 = 15a \quad \dots(i)$$

$$x = \frac{b}{a} \text{ also satisfies } x^2 - 2bx + 21 = 0$$

$$\text{So } \frac{b^2}{a^2} - 2b \cdot \frac{b}{a} + 21 = 0$$

$$b^2 - 2b^2a + 21a^2 = 0 \quad \dots(ii)$$

Putting $b^2 = 15a$ in (ii)

$$15a - 2 \cdot 15a \cdot a + 21a^2 = 0$$

$$\Rightarrow a = \frac{5}{3}$$

$$\alpha^2 + \beta^2 = 4b^2 - 42$$

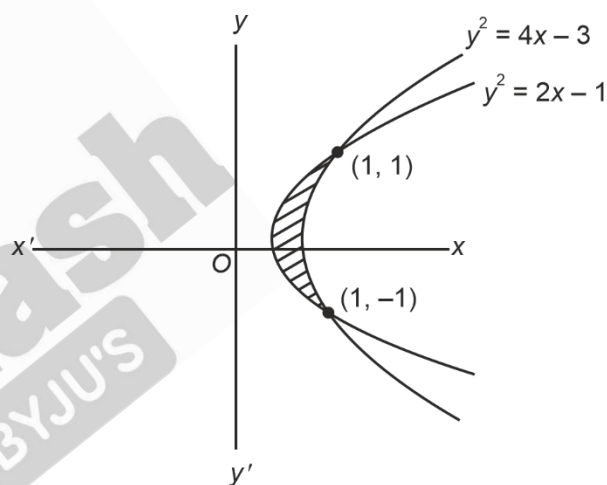
$$= 4 \cdot 15a - 42 = 58$$

14. Area of the region bounded by the curves $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is

- (1) $\frac{1}{6}$
 (2) $\frac{1}{3}$
 (3) $\frac{2}{3}$
 (4) $\frac{1}{2}$

Answer (2)

Sol.



The area of shaded region

$$= \int_{-1}^1 \left(\frac{y^2 + 3}{4} - \frac{y^2 + 1}{2} \right) dy$$

$$= \int_{-1}^1 \frac{1 - y^2}{4} dy$$

$$= \frac{1}{2} \int_0^1 (1 - y^2) dy$$

$$= \frac{1}{2} \left(y - \frac{y^3}{3} \right)_0^1$$

$$= \frac{1}{3} \text{ square unit}$$

15. If sum of first n terms of two AP's are in ratio $3n + 8 : 7n + 15$ then the ratio of their 12th terms is

- (1) $8 : 7$ (2) $7 : 16$
(3) $74 : 169$ (4) $13 : 47$

Answer (2)

Sol. Given $\frac{S_n}{S_n} = \frac{\frac{n}{2}(2a + (n-1)d)}{\frac{n}{2}[2a_1 + (n-1)d_1]} = \frac{3n+8}{7n+15}$

$$\Rightarrow \frac{a + \left(\frac{n-1}{2}\right)d}{a_1 + \left(\frac{n-1}{2}\right)d_1} = \frac{3n+8}{7n+15}$$

Put $n = 23$

$$\frac{T_{12}}{t_{12}} = \frac{77}{176} = \frac{7}{16}$$

16. The value of

$$\lim_{x \rightarrow \frac{\pi}{2}} \tan^2 x \left(\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2} \right) \text{ is}$$

- (1) $-\frac{1}{12}$ (2) $\frac{1}{18}$
(3) $\frac{1}{12}$ (4) $-\frac{1}{18}$

Answer (3)

Sol. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan^2 x (\sin^2 x - 3\sin x + 2)}{\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2}}$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan^2 x (1 - \sin x)(2 - \sin x)}{6}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin^2 x (2 - \sin x)(1 - \sin x)}{6 \cos^2 x} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{6(1 - \sin^2 x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{6(1 + \sin x)} = \frac{1}{12}$$

17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find the number of 3-digit numbers which has exactly 2 digits identical.

Answer (243)

Sol. Case I : When digit zero is repeated

$$\text{Number of numbers} = {}^9C_1 = 9$$

Case II : When digit zero is present but not repeated

$$\text{Number of numbers} = {}^9C_1 \cdot 2! = 18$$

Case III : When digit zero is not present

$$\text{Number of numbers} = 9 \cdot 8 \cdot \frac{3!}{2!} = 216$$

$$\text{Total number of numbers} = 243$$

22. Mean deviation about mean = $\frac{5(n+1)}{n}$ of data 1, 2, 3 ... n , where n is odd, then value of n is equal to

Answer (21)

Sol. Let \bar{x} be the mean of 1, 2, 3 ... n (where $n \in \text{odd}$)

$$\bar{x} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

$$\text{M.D.}(\bar{x}) =$$

$$\frac{1}{n} \left[2 \left(\left(\frac{n+1}{2} - 1 \right) + \left(\frac{n+1}{2} - 2 \right) + \dots \text{upto} \left(\frac{n-1}{2} \right) \text{terms} \right) \right]$$

$$= \frac{1}{n} \left[(n-1) + (n-3) + \dots \text{upto} \left(\frac{n-1}{2} \right) \text{terms} \right]$$

$$= \frac{1}{n} \left[\frac{n^2 - 1}{4} \right]$$

$$\therefore \frac{n^2 - 1}{4n} = \frac{5(n+1)}{n}$$

$$\Rightarrow n - 1 = 20$$

$$\Rightarrow n = 21$$

23. $12 \int_3^b \frac{dx}{(x^2-4)(x^2-1)} = \ln \frac{49}{50}$, then number of possible values of b are

Answer (4)

Sol. $12 \int_3^b \left(\frac{1}{x^2-4} - \frac{1}{x^2-1} \right) dx = \ln \left(\frac{49}{50} \right)$

$$\Rightarrow 4 \left[\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| - \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| \right]_3^b = \ln \left(\frac{49}{50} \right)$$

$$\Rightarrow \ln \left| \left(\frac{b-2}{b+2} \right)^5 \right| - 2 \ln \left| \left(\frac{b-1}{b+1} \right)^2 \right| = \ln \left(\frac{49}{50} \right)$$

$$\Rightarrow \ln \left| \frac{5 \cdot (b-2)(b+1)^2}{4 \cdot (b+2)(b+1)^2} \right| = \ln \left(\frac{49}{50} \right)$$

$$\Rightarrow \left| \frac{b^3 - 3b - 2}{b^3 - 3b + 2} \right| = \frac{98}{125}$$

$$\Rightarrow b^3 - 3b = \frac{446}{27} \text{ or } \frac{54}{223}$$

One of the cubic has 1 real root and other has 4 real roots.

\Rightarrow there will be total 4 values of b possible.

24. If sum of coefficients of positive even powers of x in the expansion of $\left(2x^3 + \frac{3}{x} \right)^{10}$ is $5^{10} - \beta \cdot 3^9$, then value of β is

Answer (83)

Sol. $T_{r+1} = {}^{10}C_r (2x^3)^{10-r} \left(\frac{3}{x} \right)^r$

$$= {}^{10}C_r 2^{10-r} 3^r x^{30-4r}$$

So we need to remove the cases when

$r = 8, 9, 10$ (as powers will be negative)

Putting $x = 1$ will give sum of all the coefficients which are possible, from which we have to remove the cases for $r = 8, 9, 10$

So required sum

$$= 5^{10} - \left({}^{10}C_8 2^2 3^8 + {}^{10}C_9 2^1 3^9 + {}^{10}C_{10} 2^0 3^{10} \right)$$

$$= 5^{10} - 83 \cdot 3^9$$

$$\beta = 83$$

25. Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has eccentricity $\frac{5}{4}$. If normal to the hyperbola at $\left(\frac{8}{\sqrt{5}}, \frac{12}{5} \right)$ is $8\sqrt{5}x + \beta y = \lambda$. Then the value of $(\lambda - \beta)$ is

Answer (85)

Sol. Normal to Hyperbola at (x_1, y_1)

$$\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 e^2$$

$$\frac{a^2 x}{\left(\frac{8}{\sqrt{5}} \right)} + \frac{b^2 y}{\frac{12}{5}} = a^2 e^2$$

$$e^2 = \frac{12}{16} \Rightarrow \frac{b^2}{a^2} = \frac{9}{16}$$

$$\frac{\sqrt{5}x}{8} + \frac{5y}{12} \cdot \frac{9}{16} = \frac{25}{16}$$

$$8\sqrt{5}x + 15y = 100$$

$$\lambda - \beta = 100 - 15$$

$$= 85$$

26.

27.

28.

29.

30.

