25/06/2022
Morning

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Memory Based
Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

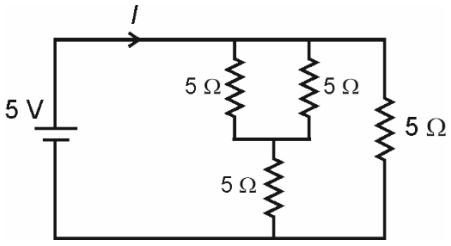
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. In the following circuit, the current through the cell is

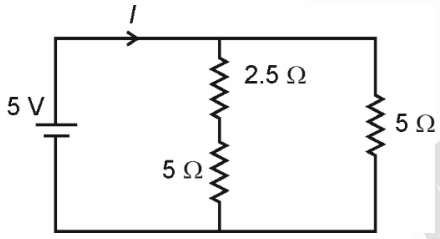


(1) $\frac{3}{4}$ A (2) $\frac{5}{3}$ A

(3) $\frac{5}{4}$ A (4) $\frac{4}{5}$ A

Answer (2)

Sol.



$\therefore \quad R_{eq} = \frac{5 \times 7.5}{12.5} = 3\ \Omega$

$\therefore \quad I = \frac{5}{3}$ A

2. Electric field due to a charged sheet at a distance of l and $4l$ at points A and B is (surface charge density of sheet is σ).

(1) $E_A = \frac{\sigma}{\varepsilon_0}, E_B = \frac{\sigma}{2\varepsilon_0}$

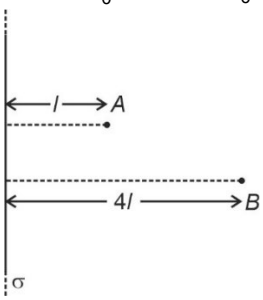
(2) $E_A = E_B = \frac{\sigma}{2\varepsilon_0}$

(3) $E_A = E_B = \frac{\sigma}{\varepsilon_0}$

(4) $E_A = \frac{2\sigma}{\varepsilon_0}, E_B = \frac{\sigma}{\varepsilon_0}$

Answer (2)

Sol. $E_A = \frac{\sigma}{2\varepsilon_0}, \quad E_B = \frac{\sigma}{2\varepsilon_0}$



$\therefore \quad E_A = E_B = \frac{\sigma}{2\varepsilon_0}$

3. If $\hat{A}$ and $\hat{B}$ are unit vectors and θ is angle between them, then choose the correct option.

(1) $\left|\hat{A}-\hat{B}\right| = \left|\hat{A}+\hat{B}\right| \tan\left(\frac{\theta}{2}\right)$

(2) $\left|\hat{A}+\hat{B}\right| = \left|\hat{A}-\hat{B}\right| \tan\frac{\theta}{2}$

(3) $\left|\hat{A}-\hat{B}\right| = \left|\hat{A}+\hat{B}\right| \cos\frac{\theta}{2}$

(4) $\left|\hat{A}+\hat{B}\right| = \left|\hat{A}-\hat{B}\right| \cos\frac{\theta}{2}$

Answer (1)

Sol. $\left|\hat{A}+\hat{B}\right| = 2\cos\left(\frac{\theta}{2}\right)$

$\left|\hat{A}-\hat{B}\right| = 2\sin\left(\frac{\theta}{2}\right)$

$\therefore \quad \frac{\left|\hat{A}-\hat{B}\right|}{\left|\hat{A}+\hat{B}\right|} = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \tan\left(\frac{\theta}{2}\right)$

$\Rightarrow \quad \left|\hat{A}-\hat{B}\right| = \left|\hat{A}+\hat{B}\right| \tan\left(\frac{\theta}{2}\right)$

4. Find the ratio of speed of electron in He^+ 3rd orbit and H^+ 3rd orbit

(1) 1 : 1 (2) 1 : 2

(3) 2 : 1 (4) 4 : 1

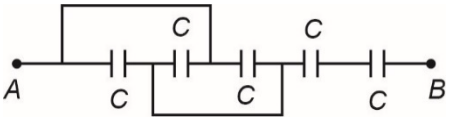
Answer (3)

Sol. $v \propto \frac{Z}{n}$

$\Rightarrow \quad \frac{v_{He^+}}{v_H} = \frac{\frac{2}{3}}{\frac{1}{3}} = 2:1$

$\Rightarrow \quad \frac{v_{He^+}}{v_H} = \frac{2}{1}$

5. Find the equivalent capacitance between A and B. ($C = 8\ \mu F$)

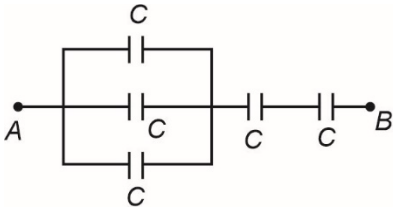


(1) $\frac{21}{8}\ \mu F$ (2) $\frac{27}{4}\ \mu F$

(3) $\frac{24}{7}\ \mu F$ (4) $\frac{29}{7}\ \mu F$

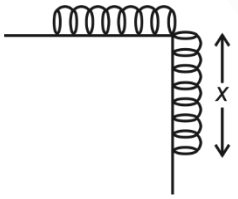
Answer (3)

Sol. Equivalent circuit is as shown



$$C_{eq} = \frac{3C \times \frac{C}{2}}{3C + \frac{C}{2}} = \frac{24 \times 4}{28} = \frac{24}{7}\mu F$$

6. In the following figure x length is hanging from the table. For what maximum value of x the chain will not slip



$\mu = 0.5$, $L = 6$ m

(1) 3 m (2) 4 m

(3) 2 m (4) 1 m

Answer (3)

Sol. $(\lambda x) \times g = \mu \times (L - x) \times g \times \lambda$

$\Rightarrow x = 0.5\ (6 - x)$

$\Rightarrow x = 3 - 0.5\ x$

$\Rightarrow 1.5\ x = 3$

$\Rightarrow x = 2$ m

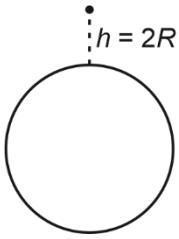
7. If on the surface of earth, the gravitational acceleration is g_0, then the value of gravitational acceleration at height of $2R$ is (R is the radius of earth)

(1) $\frac{g_0}{4}$ (2) $\frac{g_0}{2}$

(3) $\frac{g_0}{3}$ (4) $\frac{g_0}{9}$

Answer (4)

Sol.



$g_0 = \frac{GM}{R^2}$

$$g_0 = \frac{GM}{R^2}$$

$$g = \frac{GM}{(R+h)^2}$$

$$= \frac{GM}{(R+2R)^2}$$

$$= \frac{GM}{9R^2}$$

$$= \frac{g_0}{9}$$

8. A particle starts moving in the influence of force $\vec{F} = \left(10\hat{i} + 5\hat{j}\right)$ N, if mass of particle is 0.1 kg then its displacement $(\vec{s})$ in $t = 2$ sec is $\vec{s} = a\hat{i} + b\hat{j}$ then the value of $\frac{a}{b}$ is

(1) 1 (2) 2

(3) 3 (4) 4

Answer (2)

Sol. $\vec{F} = 10\hat{i} + 5\hat{j}$

$$\therefore \vec{a} = \frac{10\hat{i} + 5\hat{j}}{0.1} = 100\hat{i} + 50\hat{j}$$

$$\therefore \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$= 0 + \frac{1}{2} \times \left(100\hat{i} + 50\hat{j}\right) \times 4$$

$$= 200\hat{i} + 100\hat{j}$$

$\therefore a = 200$; $b = 100$

$$\therefore \frac{a}{b} = \frac{200}{100} = 2$$

9. For O_2 ratio of rms speed of molecule and most probable speed of molecule is

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{2}}{3}$

(3) $\sqrt{\frac{3}{2}}$ (4) $\sqrt{\frac{2}{3}}$

Answer (3)

Sol. $\because u_{rms} = \sqrt{\frac{3RT}{M}}$

$$v_{mps} = \sqrt{\frac{2RT}{M}}$$

$$\therefore \frac{u_{rms}}{v_{mps}} = \sqrt{\frac{3}{2}}$$

10. 5 MHz frequency is transmitted by
    (1) Coaxial
    (2) Optical fibres
    (3) Twisted copper wire
    (4) None of these

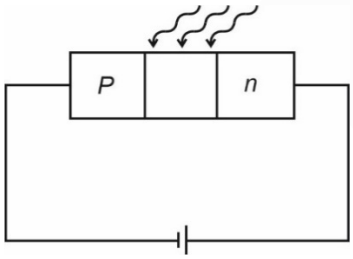
Answer (3)

Sol. 5 MHz frequency is transmitted through twisted copper wire.

11. Photodiode is reverse biased for which of the following reason?
    (1) To increase the sensitivity
    (2) To increase the current flow
    (3) To decrease depletion width
    (4) To decrease the potential barrier

Answer (1)

Sol.



Photodiode is reversed biased to facilitate collection of photons (detection) through large depletion region. Current sensitivity is increased in reversed biased photodiodes.

12. $\frac{A^2B^3}{C^4} = D$

Find the maximum percentage error in D.

(1) $\left(\frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}\right) \times 100$

(2) $\frac{2\Delta A}{A} - \frac{3\Delta B}{B} + \frac{4\Delta C}{C}$

(3) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} - \frac{4\Delta C}{C}$

(4) $\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$

Answer (1)

Sol. According to the given relation,

$$D = \frac{A^2B^3}{C^4}$$

$$\Rightarrow \frac{\Delta D}{D} = 2\cdot\frac{\Delta A}{A} + 3\cdot\frac{\Delta B}{B} + 4\cdot\frac{\Delta C}{C}$$

$\Rightarrow$ Maximum percentage error is D is

$$\frac{\Delta D}{D} \times 100 = \left(\frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}\right) \times 100$$

13. Choose the correct option matching entries of column 1 and column 2.

	Column 1		**Column 2**
(i)	AC-generator	(a)	Detects current
(ii)	Transformer	(b)	Changes AC voltage
(iii)	Metal detector	(c)	Identify the resonance in circuit
(iv)	Galvanometer	(d)	Converts mechanical energy into electrical energy

(1) (i) $\rightarrow$ (a), (ii) $\rightarrow$ (c), (iii) $\rightarrow$ (d), (iv) $\rightarrow$ (b)
(2) (i) $\rightarrow$ (d), (ii) $\rightarrow$ (b), (iii) $\rightarrow$ (c), (iv) $\rightarrow$ (a)
(3) (i) $\rightarrow$ (d), (ii) $\rightarrow$ (b), (iii) $\rightarrow$ (a), (iv) $\rightarrow$ (c)
(4) (i) $\rightarrow$ (a), (ii) $\rightarrow$ (c), (iii) $\rightarrow$ (b), (iv) $\rightarrow$ (d)

Answer (2)

Sol. AC generator is used to convert mechanical energy into electrical energy.

Transformer is used to change AC voltages.

Metal detector is used to detect resonance in circuit.

Galvanometer is used to detect current.

14. Terminal velocity of rain drop of radius r depends on
    (1) $r^{1/2}$ (2) $r^{3/2}$
    (3) r^2 (4) r

Answer (3)

Sol. Terminal velocity $V_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$

$$\Rightarrow V_T \propto r^2$$

15. If, $I_1 = 9I$, $I_2 = I$ at point P and phase difference is $\frac{\pi}{2}$. At point Q, phase diff. is π, find the difference between the intensity of waves at P and Q.

(1) $9I$ (2) $6I$

(3) $8I$ (4) $10I$

Answer (2)

Sol. We know that

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\phi)$$

$$I_P = 9I + I + 6I\cos\left(\frac{\pi}{2}\right) = 10I$$

and $I_Q = 9I + I + 6I\cos(\pi) = 4I$

$\Rightarrow I_P - I_Q = 6I$

16. Dielectric constant of material is 4 and relative permeability is 1, then find critical angle for the refraction with the air.

(1) 10° (2) 20°

(3) 30° (4) 60°

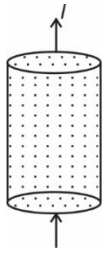
Answer (3)

Sol. $\mu = \sqrt{\varepsilon_r \mu_r} = 2$

$$C = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

17. Current flows through a cross-section of radius R as shown:

The value of this current is i. For radial distance $r < R$, magnetic field depends on r as



(1) $B \propto r^1$ (2) $B \propto r^2$

(3) $B \propto \frac{1}{r}$ (4) $B \propto r^0$

Answer (1)

Sol. Applying Ampere's circuital law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enc}$$

$\Rightarrow$ For $r < R$,

$$B(2\pi r) = \mu_0\left[\frac{I}{\pi R^2}\times\pi r^2\right]$$

$\Rightarrow B \propto r^1$

18. A teacher uses 3 times the reading of shunted galvanometer for an experiment

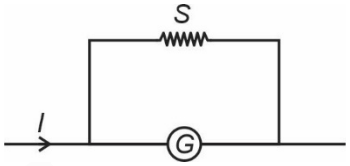
(1) Shunt resistance is double the resistance of galvanometer

(2) Shunt resistance is half of the resistance of galvanometer

(3) Shunt resistance has the value equal to that of galvanometer

(4) None of these

Answer (2)



$$I_G = \frac{r_s}{r_s + r_G}\times I$$

$$\Rightarrow \frac{I}{3} = \frac{r_s}{r_s + r_G}\times I$$

$\Rightarrow r_s + r_G = 3r_s$

$$\Rightarrow r_s = \frac{r_G}{2}$$

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21.

22.

23.

24.

25.

26.

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Strongest oxidising agent is,

   (1) Mn^{+3}

   (2) Ti^{+3}

   (3) Fe^{+3}

   (4) Cr^{+3}

Answer (1)

Sol. Of the four metal ions given the strongest oxidising agent is Mn^{3+} because its standard reduction potential is higher than those of others.

$E^{o}_{Mn^{3+}/Mn^{2+}} = +1.57\ V$; $E^{o}_{Ti^{3+}/Ti^{2+}} = -0.37\ V$

$E^{o}_{Fe^{3+}/Fe^{2+}} = +0.77\ V$; $E^{o}_{Cr^{3+}/Cr^{2+}} = -0.41\ V$

2. Product formed on reaction of AgCl with aq. NH_3

   (1) $[Ag(NH_3)_4]Cl$

   (2) $[Ag(NH_3)_2Cl_2]$

   (3) $[Ag(NH_3)_2]Cl$

   (4) $[Ag(NH_3)Cl]$

Answer (3)

Sol. AgCl dissolves in aq. NH_3 to form a complex $[Ag(NH_3)_2]Cl$

$AgCl(s) + 2NH_3(aq) \rightarrow [Ag(NH_3)_2]Cl\ (aq)$

3. Find intermediate in the following reaction.

$$R - CONH_2 + Br_2 \xrightarrow{NaOH} R - NH_2$$

   (1) R-CN

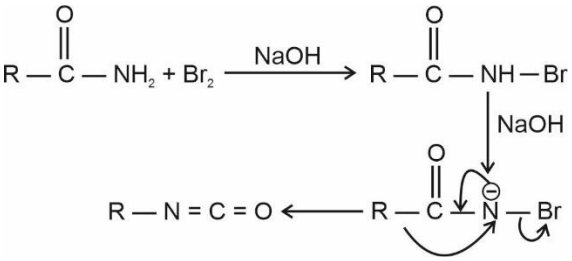
   (2) R-NC

   (3) R-NCO

   (4) R-COOH

Answer (3)

Sol.



4. Which of the following are isoelectronic species?

   (1) HF and H_2O

   (2) CH_4 and SF_6

   (3) O_2 and O_3

   (4) H_2 and F_2

Answer (1)

Sol. Isoelectronic means having same number of electrons.

Total number of electrons in HF = 10

Total number of electrons in H_2O = 10

Hence, HF and H_2O are isoelectronic

5. In which of the following bond order increases?

   (1) $CO \rightarrow CO^+$

   (2) $N_2 \rightarrow N_2^+$

   (3) $H_2 \rightarrow H_2^+$

   (4) $O_2 \rightarrow O_2^+$

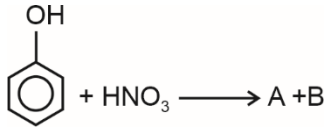
Answer (1, 4)

Sol. $\underset{B.O.=3}{CO} \rightarrow \underset{B.O.=3.5}{CO^+}$

$\underset{B.O.=3.0}{N_2} \rightarrow \underset{B.O.=2.5}{N_2^+}$

$\underset{B.O.=1.0}{H_2} \rightarrow \underset{B.O.=0.5}{H_2^+}$

$\underset{B.O.=2.0}{O_2} \rightarrow \underset{B.O.=2.5}{O_2^+}$

6. 

A & B can be separated by

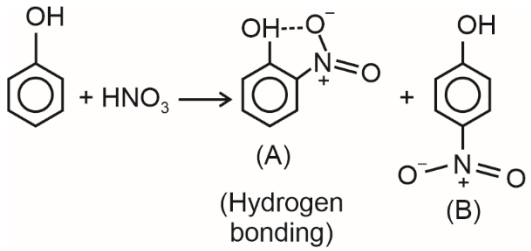
(1) Chromatography

(2) Fractional distillation

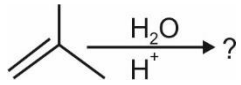
(3) Mass spectrometry

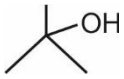
(4) NMR

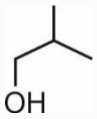
Answer (2)

Sol. 

Boiling point of A is lower than that of B, so A and B can be separated by fractional distillation.

7. 

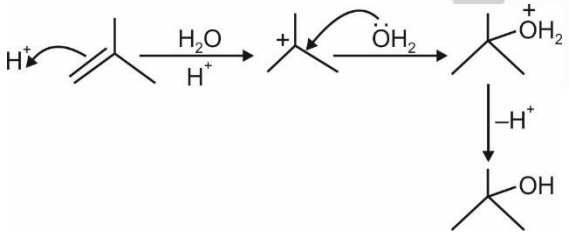
(1) 

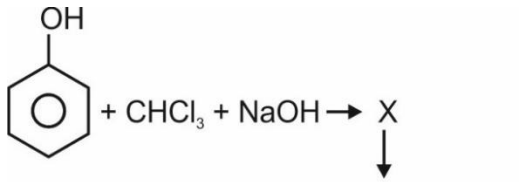
(2) 

(3) 

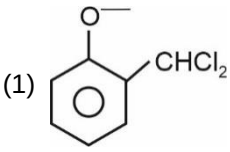
(4) 

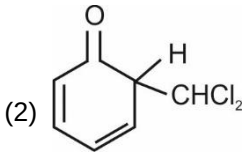
Answer (1)

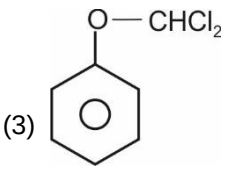
Sol. 

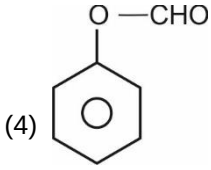
8. 

The intermediate X is

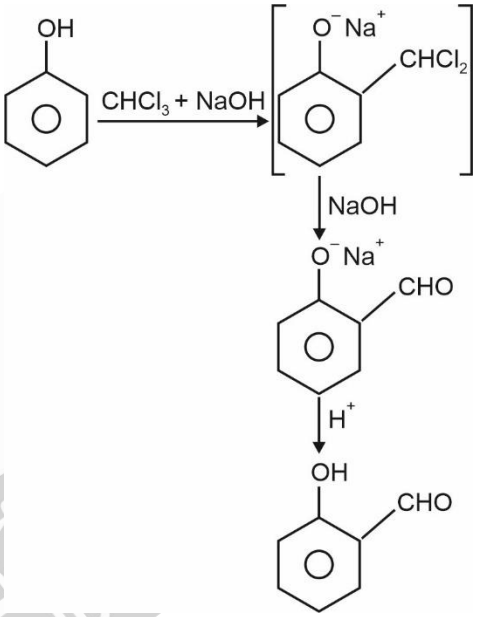
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

9. Which of the following is a natural sweetner?

(1) Bithionol

(2) Sucralose

(3) Alitame

(4) Lactose

Answer (4)

Sol. The lactose found in plain milk is considered as natural sugar.

10. Why micelle is not formed when concentration of micelle is less?

(1) Micelle is not formed as the concentration is less than CMC

(2) Less hydrophilic

(3) Amount of soap is less

(4) None of these

Answer (1)

Sol. If concentration is less, or on dilution, the associated colloid *i.e*. micelles revert back to individual ions as concentration becomes less than CMC.

11. Electron deficient species is,

(1) B_2H_6

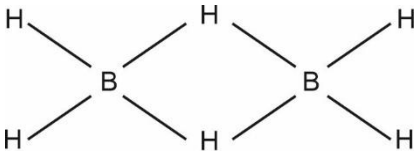
(2) LiH

(3) CCl_4

(4) PH_3

Answer (1)

Sol.



B_2H_6 is electron deficient as it has (3c – 2e) bond.

12. Effect of Eutrophication is

(1) BOD of water decreases

(2) Biodiversity of living organism decreases

(3) Oxygen concentration of water increases

(4) None of these

Answer (2)

Sol. The process in which nutrient enriched water bodies support a dense plant population, which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity is known as eutrophication.

13. Which among the following has highest ionic mobility in solution?

(1) Be^{2+}

(2) Mg^{2+}

(3) Ca^{2+}

(4) Sr^{2+}

Answer (4)

Sol. Among the given cations, ionic mobility of Sr^{2+} ion in water will be highest due to lower charge density.

14. Complex "X" is formed by leaching of Au. After reaction with Zn, complex "Y" is formed. Identify X and Y

(1) $[Au(CN)_2]^-$, $[Zn(CN)_4]^{2-}$

(2) $[Au(CN)_4]^-$, $[Zn(CN)_4]^{2-}$

(3) $[Au(CN)_3]$, $[Zn(CN)_4]^{2-}$

(4) None of these

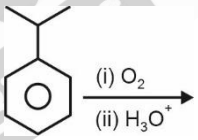
Answer (1)

Sol. $4Au(s) + 8CN^-(aq) + 2H_2O(aq) + O_2(g)$

$$\longrightarrow 4\left[Au(CN)_2\right]^-(aq) + 4OH^-(aq)$$

$2[Au(CN)_2]^-(aq) + Zn(s)$

$$\longrightarrow 2Au(s) + \left(Zn(CN)_4\right)^{2-}(aq)$$

15. 

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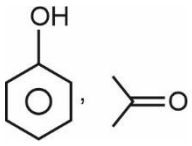
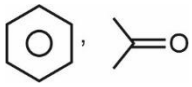
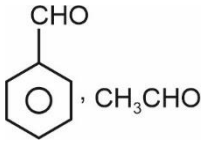
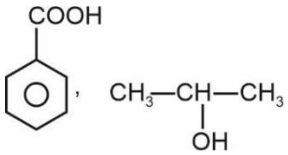
(1) 

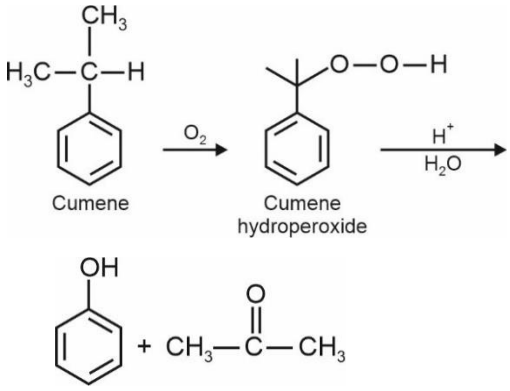
(2) 

(3) 

(4) 

Answer (1)

Sol.



16. Which of the following is incorrect about Tyndall Effect?

(1) Density difference must be minimum

(2) λ difference must be maximum

(3) Refractive index difference must be maximum

(4) None of these

Answer (3)

Sol. Tyndall effect is observed only when

(i) Diameter of the dispersed particle is not much smaller than the wavelength of the light used.

(ii) The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. For He^+ and H in 3rd orbit ratio of de Broglie wavelength is?

Answer ($\frac{1}{2}$)

Sol. $\frac{\lambda_{He^+}}{\lambda_H} = \frac{h}{\frac{mV_{He^+}}{h}} = \frac{V_H}{V_{He^+}} = \frac{Z_H/(3)}{Z_{He^+}/(3)} = \frac{1}{2} = 0.5$

22. Density of NaCl is 43.1 g/cc. Find the distance between Na^+ and Cl^- ions?

Answer (1.04)

Sol. Density of NaCl = 43.1 g/cc

NaCl has FCC unit cell with 4 NaCl units per unit cell. If "a" is the edge length, then density (d) of NaCl is given by,

$$d = \frac{4 \times M_{NaCl}}{N_A (a)^3}$$

$$(a)^3 = \frac{4 \times 58.5}{6 \times 10^{23} \times 43.1} = 9.049 \times 10^{-24}$$

$$a = 2.08 \times 10^{-8} \text{ cm}$$

Distance between Na^+ and Cl^- ion = 1.04 Å

23. 0.01 m moles, 1 L H_2SO_4 solution is given 50% of this solution is taken out and 500 ml water is added to make 1L solution. Further, 0.01 m moles H_2SO_4 is added to this solution. Find out the final m moles H_2SO_4 in terms of 10^{-3} m mol.

Answer (15)

Sol. In 1 L H_2SO_4 solution, m moles of H_2SO_4 = 0.01

50% of this solution contains 0.005 m moles H_2SO_4.

After addition of 500 ml water, m moles of H_2SO_4 (Solute) remains constant.

Since, 0.01 m moles of H_2SO_4 is added.

Total m moles of H_2SO_4 = 0.01 + .0005 m moles

= 0.015 m moles

= 15 × 10^{-3} m moles.

24.

25.

26.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}$, where (a, b, c) are the sides of ΔABC, then find $\frac{R}{r}$ (where R is circumradius and r is inradius)

(1) $\frac{5}{2}$ (2) 3

(3) 1 (4) $\frac{1}{2}$

Answer (1)

Sol. Let $a + b = 7k$, $b + c = 8k$ and $c + a = 9k$

So, $a = 4k$, $b = 3k$ and $c = 5k$ and $s = 6k$

$\because \quad \frac{r}{R}=4\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$

$$=4\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-a)(s-c)}{ac}}\sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$=4\frac{(s-a)(s-b)(s-c)}{abc}$$

$$=\frac{4\cdot(2k)(3k)(k)}{(4k)(3k)(5k)}=\frac{2}{5}$$

$\Rightarrow \quad \frac{R}{r}=\frac{5}{2}=2.5$

2. Circle C touches the line $L_1 = 4x – 3y + k_1 = 0$, $L_2 = 4x – 3y + k_2 = 0$, $k_1k_2 \in R$. If a line passing through the centre of circle intersects L_1 at (–1, 2) and L_2 at (3, –6) then equation of circle is

(1) $x^2 + y^2 – 2x + 4y – 11 = 0$

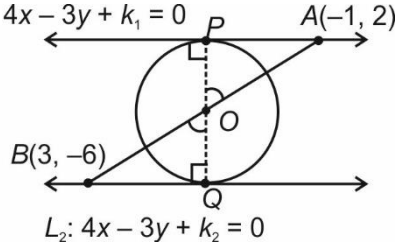
(2) $x^2 + y^2 + 2x – 4y – 11 = 0$

(3) $x^2 + y^2 – 2x + 6y – 11 = 0$

(4) $x^2 + y^2 – 2x – 4y + 11 = 0$

Answer (1)

Sol. L_1: $4x – 3y + k_1 = 0$



$\because$ (–1, 2) lies on line $L_1 = 0$

$\therefore \quad k_1 = 10$

and (3, –6) lies on line $L_2 = 0$

$\therefore \quad k_2 = –30$

Distance between lines L_1 and L_2

$$=\frac{|k_1-k_2|}{\sqrt{4^2+(-3)^2}}=8$$

$\therefore$ Radius of circle = 4 units

Centre of circle = midpoint of AB = (1, –2)

$\therefore$ Equation of circle is $(x – 1)^2 + (y + 2)^2 = 4^2$

$\therefore \quad x^2 + y^2 – 2x + 4y – 11 = 0$

3. If $g(0,\infty)\to R$ is a differentiable function and

$$\int\left[x\left(\frac{\cos x-\sin x}{e^x+1}\right)+g(x)\left(\frac{e^x+1-xe^x}{\left(e^x+1\right)^2}\right)\right]dx=\frac{xg(x)}{e^x+1}+c$$

for all $x > 0$, here c = constant then

(1) g is decreasing in $(0, \pi/4)$

(2) g is increasing in $\left(0,\frac{\pi}{4}\right)$

(3) $g + g'$ is increasing in $\left(0,\frac{\pi}{2}\right)$

(4) $g – g'$ is decreasing in $\left(0,\frac{\pi}{2}\right)$

Answer (2)

Sol. Differentiating w.r.t. x, we get

$$\frac{x(\cos x-\sin x)}{1+e^x}+\frac{g(x)}{1+e^x}-\frac{xe^xg(x)}{\left(1+e^x\right)^2}$$

$$=\frac{g(x)+xg'(x)}{1+e^x}-\frac{xe^xg(x)}{\left(1+e^x\right)^2}$$

Clearly $g'(x)=\cos x-\sin x=\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$

$\because\quad g'(x)>0\ \forall\ x\in\left(0,\frac{\pi}{4}\right)\Rightarrow g$ is increasing

also $g''(x)=-(\sin x+\cos x)=-\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

$g'(x)+g''(x)=-2\sin x<0\ \forall\ x\in\left(0,\frac{\pi}{2}\right)$

So $g(x)+g'(x)$ is decreasing in $\left(0,\frac{\pi}{2}\right)$

Similarly, $g'(x)-g''(x)=2\cos x>0\ \forall\ x\in\left(0,\frac{\pi}{2}\right)$

So $g(x)-g'(x)$ is increasing in $\left(0,\frac{\pi}{2}\right)$.

4. Let $f(x)$ be a polynomial function such that $f(x)+f'(x)+f''(x)=x^5+64$. Value of $\lim\limits_{x\to1}\frac{f(x)}{x-1}$ is

(1) –15 (2) 15

(3) 60 (4) –60

Answer (1)

Sol. $\because\quad f(x)+f'(x)+f''(x)=x^5+64$

let $f(x)=x^5+ax^4+bx^3+cx^2+dx+e$

So $x^5+(a+5)x^4+(20+4a+b)x^3+(12a+3b+c)x^2$

$+(2c+d)x+(2c+d+e)=x^5+64$

Clearly $a=-5,\ b=0,\ c=60,\ d=-120,\ e=64$

Hence $f(x)=x^5-5x^4+60x^2-120x+64$

$$\Rightarrow\quad \lim_{x\to1}\frac{f(x)}{x-1}=\lim_{x\to1}\frac{(x-1)\left(x^4-4x^3-4x^2+56x-64\right)}{x-1}$$

$=-15$

5. If $y=y(x)$ be the solution of given equation $y^2\,dx+(x^2-xy+y^2)\,dy=0$ and this curve also passes through (1, 1). Line $y=\sqrt{3}x$ intersects it at $\left(\alpha,\sqrt{3}\alpha\right)$ then find the value of $\log_e\left(\sqrt{3}x\right)$.

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{12}$

Answer (4)

Sol. $y^2\,dx+(x^2-xy+y^2)\,dy=0$

$$\frac{dy}{dx}=\frac{-y^2}{x^2-xy+y^2}$$

Let $y=vx$

$$\Rightarrow\quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\Rightarrow\quad v+x\frac{dv}{dx}=\frac{-v^2}{1-v+v^2}$$

$$\Rightarrow\quad \frac{1-v+v^2}{v(1+v^2)}dv=-\int\frac{1}{x}dx$$

$$\Rightarrow\quad \int\frac{1}{v}dv-\int\frac{1}{1+v^2}dv=-\int\frac{1}{x}dx$$

$\Rightarrow\quad \ln v+\ln x=\tan^{-1}v+c$

$\Rightarrow\quad \ln y=\tan^{-1}\left(\frac{y}{x}\right)+c$

As this curve passes through (1, 1)

$\therefore\quad c=\frac{-\pi}{4}$

$\therefore$ Equation of curve is

$$\ln y=\tan^{-1}\left(\frac{y}{x}\right)-\frac{\pi}{4}$$

$$\Rightarrow\quad \log_e(\sqrt{3}\alpha)=\frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$$

6. Let $f:R\to R$, $g:R\to R$ be two function defined by $f(x)=\log_e(x^2+1)-e^{-x}+1$ & $g(x)=\frac{1-2e^{2x}}{e^x}$. Range of α, if the inequality $f\left(g\frac{(\alpha-1)^2}{3}\right)>f\left(g\left(\alpha-\frac{5}{3}\right)\right)$ holds is

(1) $\alpha\in(0,2)$ (2) $\alpha\in(2,3)$

(3) $\alpha\in(3,4)$ (4) $\alpha\in R$

Answer (2)

Sol. $f(x) = \ln(x^2 + 1) - e^{-x} + 1$

$\Rightarrow f'(x) = \frac{1}{1+x^2} + e^{-x} > 0 \ \forall x \in R$

So $f(x)$ is increasing

$g(x) = e^{-x} - 2e^{x}$

$\Rightarrow g'(x) = -e^{-x} - 2e^{x} < 0 \ \forall x \in R$

So $g(x)$ is decreasing

Clearly $f(g(x))$ will be a decreasing function.

So $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$

$\Rightarrow \frac{(\alpha-1)^2}{3} < \alpha - \frac{5}{3}$

$\Rightarrow \alpha^2 - 5\alpha + 6 < 0$

$\Rightarrow \alpha \in (2, 3)$

7. If E_1 & E_2 are two conditional probability events such that $P\left(\frac{E_1}{E_2}\right) = \frac{1}{2}$, $P\left(\frac{E_2}{E_1}\right) = \frac{3}{4}$, $P(E_1 \cap E_2) = \frac{1}{8}$, then

(1) $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

(2) $P(E_1 \cap E_2') = P(E_1') \cdot P(E_2)$

(3) $P(E_1' \cap E_2') = P(E_1) \cdot P(E_2)$

(4) $P(E_1 \cap E_2') = P(E_1) \cdot P(E_2)$

Answer (4)

Sol. $P\left(\frac{E_1}{E_2}\right) = \frac{1}{2}$, $P\left(\frac{E_2}{E_1}\right) = \frac{3}{4}$ and $P(E_1 \cap E_2) = \frac{1}{8}$

$\because P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$

$\Rightarrow P(E_2) = \frac{1}{4}$

Similarly, $P(E_1) = \frac{1}{6}$

Now $P(E_1 \cup E_2) = \frac{1}{4} + \frac{1}{6} - \frac{1}{8} = \frac{7}{24}$

So, $P(E_1' \cap E_2') = 1 - P(E_1 \cup E_2) = \frac{17}{24}$

Now $P(E_1' \cap E_2) = P(E_2) - P(E_1 \cap E_2) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$

and $P(E_1 \cap E_2') = P(E_1) - P(E_1 \cap E_2) = \frac{1}{6} - \frac{1}{8} = \frac{1}{24}$

$= P(E_1) \cdot (E_2)$

8. $\int_0^{\pi} \frac{\sin x \cdot e^{\cos x}}{\left(1+\cos^2 x\right)\left(e^{\cos x} + e^{-\cos x}\right)} dx$ equals

(1) $\frac{\pi}{2}$ (2) π

(3) $\frac{\pi}{4}$ (4) None of these

Answer (3)

Sol. Let $I = \int_0^{\pi} \frac{\sin x \cdot e^{\cos x}}{\left(1+\cos^2 x\right)\left(e^{\cos x} + e^{-\cos x}\right)} dx$ ...(1)

Using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$I = \int_0^{\pi} \frac{\sin x \cdot e^{-\cos x}}{\left(1+\cos^2 x\right)\left(e^{\cos x} + e^{-\cos x}\right)} dx$ ...(2)

Adding (1) and (2) we get

$2I = \int_0^{\pi} \frac{\sin x \, dx}{1+\cos^2 x}$

let $\cos x = t$

$\sin x dx = -dt$

$\Rightarrow 2I = -\int_1^{-1} \frac{dt}{1+t^2} = \tan^{-1} t \Big|_{-1}^{1} = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

9. Let $f(x) = x^3 + x - 5$ and $f(g(x)) = x$, find $g'(63)$

(1) $\frac{1}{49}$ (2) $\frac{1}{48}$

(3) $\frac{1}{17}$ (4) $\frac{1}{16}$

Answer (1)

Sol. $f(x) = x^3 + x - 5$

$f(g(x)) = x$

As f and g are inverse to each other.

$\therefore \ g(f(x)) = x$

$\therefore \ g'(f(x)) \times f'(x) = 1$

Put $x = 4$

$g'(63) = \frac{1}{(3x^2+1)_{\text{at } x=4}} = \frac{1}{49}$

10. $y = y(x)$ be the solution $\to (x+1)y' - y = e^{3x}(x+1)^2$, $y(0) = \frac{1}{3}$, then $y\left(-\frac{4}{3}\right)$ for the curve $y = y(x)$ is

(1) $\frac{1}{9e^4}$ (2) $-\frac{1}{9e^4}$

(3) $\frac{e^4}{9}$ (4) $-\frac{e^4}{9}$

Answer (2)

Sol. Differential equation

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2, \quad y(0) = \frac{1}{3}$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{(x+1)}y = e^{3x}(x+1) \quad ....(i)$$

$$\text{I.F} = e^{\int -\frac{1}{(x+1)}dx} = \frac{1}{x+1}$$

$\therefore$ Solution of D.E

$$\frac{1}{x+1}\cdot y(x) = \int e^{3x}(x+1)\frac{1}{(x+1)}dx + c$$

$$\Rightarrow y(x) = \frac{(x+1)}{3}e^{3x} + c(x+1)$$

Now, $y(0) = \frac{1}{3} \Rightarrow c = 0$

$$\therefore y\left(-\frac{4}{3}\right) = -\frac{1}{9e^4}.$$

11. If $\vec{a}$ and $\vec{b}$ are unit vectors and acute angle between them is θ, then

(1) $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|\tan\left(\frac{\theta}{2}\right)$

(2) $|\vec{a}-\vec{b}| = |\vec{a}+\vec{b}|\tan\frac{\theta}{2}$

(3) $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

(4) $|\vec{a}-\vec{b}| = |\vec{a}+\vec{b}|\tan\theta$

Answer (2)

Sol. $|\vec{a}| = |\vec{b}| = 1$

$$\therefore |\vec{a}+\vec{b}|^2 = 1+1+2\cos\theta$$

and $|\vec{a}-\vec{b}|^2 = 2-2\cos\theta$

$$\therefore \frac{|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \tan\frac{\theta}{2}$$

12. Consider :

$P_1 : \sim(p \to \sim q)$

$P_2 : (p \wedge \sim q) \wedge ((\sim p) \vee q)$

If $p \to ((\sim p) \vee q)$ is false then

(1) P_1 is true, P_2 is false

(2) P_1 is false, P_2 is true

(3) Both P_1 and P_2 are true

(4) Both P_1 and P_2 are false

Answer (4)

Sol. $P_1 : \sim(p \to \sim q)$

$P_2 : (p \wedge \sim q) \wedge ((\sim p) \vee q)$

If $p \to ((\sim p) \vee q)$ is false

Then p is true and $\sim p \vee q$ is false

$\therefore$ q is false

$\Rightarrow$ p_1 is false and p_2 is also false

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Let $f : N \to R$ be a function, $f(x+y) = 2f(x)\cdot f(y)$ for natural numbers x and y, if $f(1) = 2$ and $\sum_{k=1}^{10} f(a+k) = \frac{512}{3}(2^{20} - 1)$, then the value of '$a$' is ____.

Answer (04)

Sol. $f(x+y) = 2f(x)f(y), \; x, y \in N$ ...(i)

and $f(1) = 2$

$f(2) = 2^3$

$f(3) = 2^5$

$\vdots$

Now, $\sum_{k=1}^{10} f(a+k) = 2f(a)\sum_{k=1}^{10} f(k) = \frac{512}{3}(2^{20}-1)$

$= 2f(a)\left[\frac{2(2^{20}-1)}{3}\right] = \frac{512}{3}(2^{20}-1)$

$\Rightarrow$ $f(a) = 128$

$\Rightarrow$ $\boxed{a = 4}$

22. If $\frac{1}{2\cdot 3^{10}} + \frac{1}{2^2\cdot 3^9} + \ldots\ldots + \frac{1}{2^{10}\cdot 3} = \frac{k}{2^{10}\cdot 3^{10}}$, then find the remainder when k is divisible by 6.

Answer (5)

Sol. Given series

$$\frac{1}{2\cdot 3^{10}} + \frac{1}{2^2\cdot 3^9} + \frac{1}{2^3\cdot 3^8} + \ldots\ldots + \frac{1}{2^{10}\cdot 3} = \frac{k}{2^{10}\cdot 3^{10}}$$

Series in LHS is a G.P with common ratio $\frac{3}{2}$

and $T_n = \frac{1}{2^{10}\cdot 3} = \frac{1}{2\cdot 3^{10}}\left(\frac{3}{2}\right)^{n-1}$

$\Rightarrow$ $n = 10$

$\therefore$ First term of G.P $= \frac{1}{2\cdot 3^{10}}$,

Number of terms = 10

and common ratio $= \frac{3}{2}$

Hence sum is $= \frac{\frac{1}{2\cdot 3^{10}}\left(1-\left(\frac{3}{2}\right)^{10}\right)}{1-\frac{3}{2}} = \frac{\left(3^{10}-2^{10}\right)}{2^{10}\cdot 3^{10}}$

$\therefore$ $k = 3^{10} - 2^{10}$

Remainder when $3^{10} - 2^{10}$ divided by 6 is

(Remainder when 3^{10} divided by 6) – (Remainder when 2^{10} divided by 6)

= 3 – 4 = –1 (i.e., 5)

23. For $f(x) = x^3 + 3x^2 + 2x + 9$, point of inflection is $P(\alpha, \beta)$ calculate $(\alpha^2 + \beta)/5$.

Answer (2)

Sol. $\because$ $f'(x) = 3x^2 + 6x + 2$, $f''(x) = 6x + 6 = 0 \Rightarrow x = -1$

And $f(-1) = 9$

So $\alpha = -1$ and $\beta = 9$

Clearly $\frac{\alpha^2+\beta}{5} = 2$

24. Let $y = m_1x + c_1$, $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of $x^2 + y^2 = 2$ & parabola $y^2 = x$ then find the value of $|8m_1m_2|$.

Answer (0.24)

Sol. Given curves: $y^2 = x$ and $x^2 + y^2 = 2$

Let the tangent of parabola

$y = mx + \frac{1}{4m}$ if it touches to circle

then $\left|\frac{\frac{1}{4m}}{\sqrt{1+m^2}}\right| = \sqrt{2}$

$\Rightarrow$ $32m^4 + 32m^2 - 1 = 0$

$\therefore$ $m^2 = \frac{-32 \pm \sqrt{(32)^2 + 4\times 32}}{64}$

$\therefore$ $|m_1m_2| = \frac{-4+3\sqrt{2}}{8}$

$\therefore$ $8|m_1m_2| = 0.24$

25. Let the series $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}$.....100 terms and S is the sum of this series, then find the value of $[S]$. (where $[\cdot]$ is greatest integer function)

Answer (98)

Sol. Let $S = \frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + ...100 \text{ terms}$

$\Rightarrow$ $S = \sum_{r=1}^{100}\left(\frac{3^r - 2^r}{3^r}\right) = \sum_{r=1}^{100} 1 - \sum_{r=1}^{100}\left(\frac{2}{3}\right)^r$

$$= 100 - \frac{\frac{2}{3}\left(1-\left(\frac{2}{3}\right)^{100}\right)}{\frac{1}{3}}$$

$$= 98 + 2\left(\frac{2}{3}\right)^{100}$$

So $[S] = 98$

26.

27.

28.

29.

30.

❑ ❑ ❑