26/06/2022
Evening

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Memory Based
Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Dimension of mutual inductance are

(1) $ML^2T^{-2}A^{-1}$ (2) $ML^2T^{-2}A^{-2}$

(3) $ML^2T^{-1}A^{-1}$ (4) $ML^2T^{-3}A^{-2}$

Answer (2)

Sol. $\because\ U = \frac{1}{2}Mi^2$, M is mutual induction

$$\Rightarrow [M] = \frac{[U]}{[i^2]} = \frac{ML^2T^{-2}}{A^2}$$

$= [ML^2T^{-2}A^{-2}]$

2. Find the ratio of rotational kinetic energy and total kinetic energy of a rolling solid sphere?

(1) $\frac{7}{5}$ (2) $\frac{2}{5}$

(3) $\frac{2}{7}$ (4) $\frac{5}{7}$

Answer (3)

Sol. Under rolling condition ($v = \omega R$)

Translational $KE = \frac{1}{2}mv^2$

Rotational $KE = = \frac{1}{2}\times\frac{2}{5}mR^2\left(\frac{v}{R}\right)^2 = \frac{1}{5}mv^2$

Total $KE = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$

So, $\frac{\text{Rotational } KE}{\text{Total } KE} = \frac{\frac{1}{5}mv^2}{\frac{7}{10}mv^2} = \frac{2}{7}$

3. Arrange the EM waves according to increasing order of wavelength

(1) $\lambda_{gamma} < \lambda_{x\text{-}ray} < \lambda_{microwave} < \lambda_{visible}$

(2) $\lambda_{gamma} < \lambda_{x\text{-}ray} < \lambda_{visible} < \lambda_{microwave}$

(3) $\lambda_{x\text{-}ray} < \lambda_{microwave} < \lambda_{gamma} < \lambda_{visible}$

(4) $\lambda_{microwave} < \lambda_{visible} < \lambda_{x\text{-}ray} < \lambda_{gamma}$

Answer (2)

Sol. Energy relation of EMW:

$E_{gamma} > E_{x\text{-}ray} > E_{visible} > E_{microwave}$

$\Rightarrow \lambda_{gamma} < \lambda_{x\text{-}ray} < \lambda_{visible} < \lambda_{microwave}$

4. What is the formula for Reynold's number?

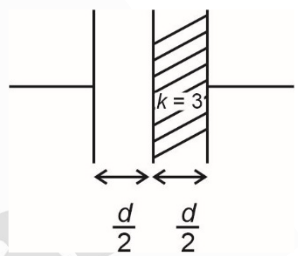
(1) $\frac{\rho vd}{\eta}$ (2) $\frac{\rho v}{\eta d}$

(3) $\frac{\rho\eta}{dv}$ (4) $\frac{\rho}{\eta dv}$

Answer (1)

Sol. We know the relation involving Reynold's number:

$$R = \frac{\rho vd}{\eta}$$

5. 

The shown capacitor has its capacitance equal to 4μF without dielectric. After insertion of dielectric as shown the new capacitance of capacitor will be (in μF)

(1) 6 (2) 8

(3) 5 (4) 3

Answer (1)

Sol. $C = \frac{\varepsilon_0 A}{d} = 4\ \mu F$ (given)

When dielectric is filled,

$$C' = \frac{\varepsilon_0 A}{\frac{d}{2} + \frac{d}{2k}}$$

$$= \frac{\varepsilon_0 A}{\frac{d}{2} + \frac{d}{6}} = \frac{\varepsilon_0 A \times 6}{d \times (4)}$$

$$= \frac{3}{2}\frac{\varepsilon_0 A}{d}$$

$$= \frac{3}{2} \times 4$$

$= 6\ \mu F$

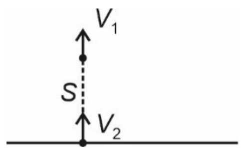
6. Two objects are thrown in the air in upward direction. One at $t = 0$ and other after two seconds. Both objects were thrown with speed = 50 m/s. At what time both objects will meet?

(1) 6 (2) 7

(3) 8 (4) 10

Answer (1)

Sol. At $t = 2$ s,



$V_1 = 50 - 10 \times 2$

$= 30$ m/s upward

$V_2 = 50$ m/s

$$S = 50 \times 2 - \frac{1}{2} \times 10 \times 2^2$$

$= 100 - 20 = 80$ m

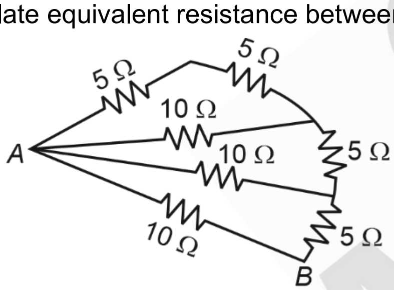
$\therefore \quad V_{rel} = (50 - 30) = 20$ m/s

$a_{rel} = g - g = 0$

$S_{rel} = 80$ m

$\therefore \quad \Delta t = \frac{80}{20} = 4\,\text{s}, \quad \therefore\ t_f = 2 + 4 = 6$ s

7. Calculate equivalent resistance between A and B.

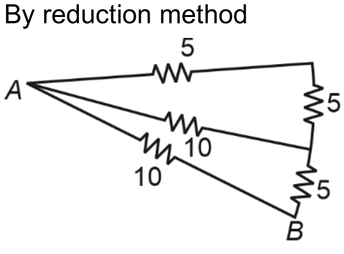


(1) 10 Ω (2) 5 Ω

(3) 15 Ω (4) 2.5 Ω

Answer (2)

Sol. By reduction method



$$R_{AB} = \frac{10}{2} = 5\ \text{W}$$

8. 64 drops of radius 0.02 mm combine to form a single drop. Each drop has charge 50 μC. Find ratio of surface charge densities of bigger to smaller drop.

(1) 1 : 4 (2) 8 : 1

(3) 1 : 8 (4) 4 : 1

Answer (4)

Sol. Let radius of bigger drop is R then

$$\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi\,(0.02)^3$$

$\Rightarrow \quad R = 0.08$ mm

total charge on bigger drop

$q = 64 \times 50\ \mu$C

$= 3200\ \mu$C

So, surface charge density $\sigma_b = \dfrac{q}{4\pi R^2}$

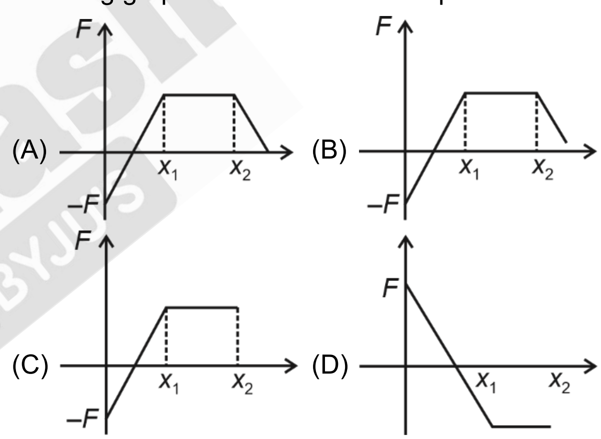
$$\sigma_b = \frac{3200}{4\pi(0.08)^2}$$

and surface charge density of smaller drop

$$\sigma_s = \frac{50}{4\pi(0.02)^2}$$

So, $\dfrac{\sigma_b}{\sigma_s} = \dfrac{3200}{50} \times \dfrac{(0.02)^2}{(0.08)^2} = 4:1$

9. Find the descending order of work done among the following graphs between force and position.



(1) B > C > A > D (2) B > C > D > A

(3) C > B > A > D (4) C > A > B > D

Answer (1)

Sol. From the graphs given,

Work done in (B) is maximum and work done in (D) is minimum.

So, descending order is as below:

B > C > A > D

10. Work done in rotating a magnetic dipole of dipole moment $M = 14 \times 10^{-5}$ Am^2 in a uniform magnetic field $B = 2 \times 10^5$ tesla by and angle of $\theta = 60$ from $\theta = 0°$ (Initially dipole is aligned with the field) is

(1) 7 J (2) 14 J

(3) 28 J (4) 21 J

Answer (2)

Sol. $W_{ext} = \Delta U$

$= U_f - U_i$

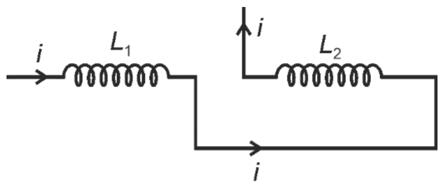
$= -\mu B\cos\theta_2 + \mu B\cos\theta_1$

$= \mu B(\cos\theta_1 - \cos\theta_2)$

$= 14 \times 10^{-5} \times 2 \times 10^5(\cos 0° - \cos 60°)$

$= 14 \times 2 \times \left(\frac{1}{2}\right) = 14$ J

11. If mutual inductance of the circuit is M then total inductance is



(1) $L_1 - L_2 + M$ (2) $L_1 + L_2 + M$

(3) $L_1 + L_2 - M$ (4) $L_2 - L_1 - M$

Answer (3)

Sol. Two inductors are coupled with opposing currents.

$\therefore\ L_{eff} = L_1 + L_2 - M$

12.

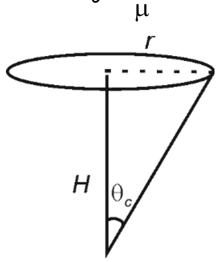
13. A light source is placed at the bottom of container filled with a liquid upto height H. If refractive index of liquid is μ then find the maximum radius of circle on the surface of liquid after which light will not emerge out.

(1) $\frac{H}{\sqrt{\mu^2 - 1}}$ (2) $\frac{H}{(\mu^2 - 1)}$

(3) $\frac{H}{2\sqrt{\mu^2 - 1}}$ (4) $\frac{H}{2(\mu^2 - 1)}$

Answer (1)

Sol. $\because\ \sin\theta_c = \frac{1}{\mu}$



and, $r = H\tan\theta_c$

$= H \times \frac{1}{\sqrt{\mu^2 - 1}}$

$= \frac{H}{\sqrt{\mu^2 - 1}}$

14. A conducting rod of length l = 1 m is rotating with angular velocity ω = 5 rad/sec in a uniform magnetic field B = 0.2 tesla about its one end. If field is perpendicular to plane of rotation then emf induced across the end is

(1) 5 V (2) 1.25 V

(3) 0.5 V (4) 2.5 V

Answer (3)

Sol. EMF induced across the ends of the rotating rod is given by

$\varepsilon = \frac{1}{2}B\omega l^2$

$= \frac{1}{2} \times 0.2 \times 5 \times 1^2 = 0.5$ volts

15. A source of light (wavelength = 670 nm) is moving away from earth. An observer on earth observes wavelength of 670.4 nm. Find the speed of the source with respect to earth.

(1) 1.8×10^5 m/s (2) 3.6×10^5 m/s

(3) 0.6×10^5 m/s (4) 10^5 m/s

Answer (1)

Sol. Using doppler's effect of light,

$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

$\Rightarrow \frac{670.4 - 670}{670} = \frac{v}{3 \times 10^8}$

$\Rightarrow v = 3 \times 10^8 \times \left(\frac{0.4}{670}\right) = 1.8 \times 10^5$ m/s

16. A light of wavelength 4000Å is incident on a metal, due to which electrons ejected which are left in a uniform magnetic field $B = 2mT$ perpendicularly. If maximum radius created by the electrons are r = 2 mm then work function of metal will be

(1) 2.8 eV (2) 1.7 eV

(3) 2.1 eV (4) 1.3 eV

Answer (2)

Sol. $\therefore\ r = \frac{mv}{qB}$

$\Rightarrow mv = eBr$

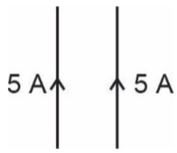
$\therefore\ (\text{KE})_{max} = \frac{(mv)^2}{2m} = \frac{e^2B^2r^2}{2m} = 1.4$ eV

$\because\ \text{KE}_{max} = \frac{hc}{\lambda} - \phi$

$\Rightarrow 1.4 = \frac{1242}{400} - \phi$

$\Rightarrow \phi = 1.7$ eV

17. Force between wires is 10^{-6} N and length of wires is 10 cm. Find distance between them.



(1) 1 m
(2) 0.5 m
(3) 2 m
(4) 0.25 m

Answer (2)

Sol. Force between parallel wires,

$$F = \frac{\mu_0 i_1 i_2}{2\pi d} \times l$$

$$\Rightarrow \quad 10^{-6} = \frac{10^{-7} \times 2 \times 5 \times 5}{d} \times 0.1$$

$$\Rightarrow \quad d = 0.5 \text{ m}$$

18.

19.

20.

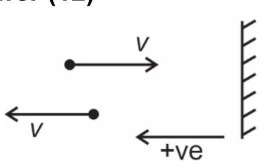
SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Mass of ball is 0.4 kg and initial speed is 15 m/s. It is hit with a bat and returned with same speed find impulse.

Answer (12)

Sol.



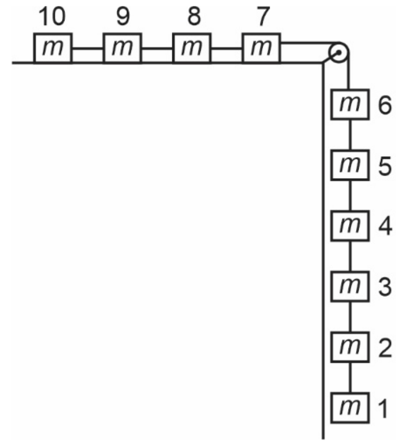
$p_i = -mv = -6$ kgm/sec.

$p_f = mv = +6$ kgm/sec.

So, impulse $I = \Delta p = p_f - p_i$

$= 12$ kg m/sec.

22. Find the tension between 7th and 8th blocks connected by strings as shown in figure below.



Answer (36)

Sol. Acceleration of blocks, $a = \frac{6mg}{10m}$

$= \frac{3g}{5}$

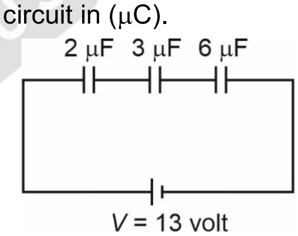
$= \frac{3 \times 10}{5}$

$= 6 \text{ m/s}^2$

$\therefore \quad T_{78} = (3m) \times a$

$= 3 \times 2 \times 6 = 36$ N

23. Find the value of charge flown through battery in the given circuit in (μC).



Answer (13)

Sol. $\frac{1}{C_{net}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

$C_{net} = 1\ \mu$F

So, charge flown $q = CV$

$= 1\ \mu\text{F} \times 13$ V

$= 13\ \mu$C

24.

25.

26.

27.

28.

29.

30.

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Which of the following water soluble vitamin cannot be excreted easily?

(1) B_1 (2) B_2

(3) B_6 (4) B_{12}

Answer (4)

Sol. Water soluble vitamins are readily excreted except vitamin B_{12}.

2. A nucleus has 2 types of radioactive decays. The half life of 1st is 3.0 hours, and for the 2nd is 4.5 hours. Calculate the correct half life of the nucleus.

(1) 0.56 hours

(2) 3.75 hours

(3) 2.23 hours

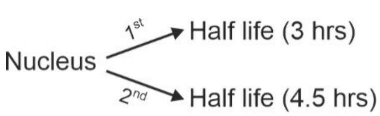
(4) 1.80 hours

Answer (4)

Sol. $\frac{1}{t_{1/2}} = \frac{1}{3} + \frac{1}{4.5}$

$$\frac{1}{t_{1/2}} = \frac{7.5}{3 \times 4.5}$$

$$t_{1/2} = \frac{9}{5} = 1.8 \text{ hr}$$



3. Select the nitrogen atom having odd number electron.

(1) N_2O_5

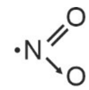
(2) NO_2

(3) N_2O

(4) N_2O_4

Answer (2)

Sol. The oxide of nitrogen having odd number of electrons is NO_2.



4. Which of the following element is most likely to deviate from +3 oxidation state?

(1) La

(2) Ce

(3) Lu

(4) Gd

Answer (2)

Sol. Ce can show +4 oxidation state which is favoured by its noble gas configuration.

Electronic configuration of Ce = $4f^1\ 5d^1\ 6s^2$

5. Which of the following s-block element does not give flame test?

(1) Be

(2) Na

(3) Li

(4) Rb

Answer (1)

Sol. Be does not give flame colouration in Bunsen burner flame as its electrons are tightly held so cannot be excited.

6. Sum of radial nodes and angular nodes in 4*s*?

(1) 1

(2) 3

(3) 2

(4) 4

Answer (2)

Sol. Radial node = n – l –1

Angular node = l

So, total node = n – 1

= 4 – 1 = 3

7. Toluene can be easily converted into benzaldehyde by which of the following reagents?

(1) CO, HCl, Anhy $AlCl_3$

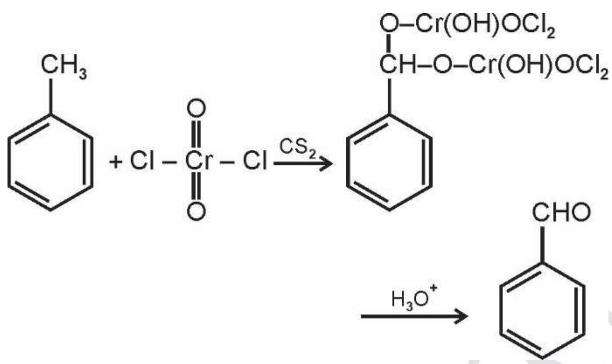
(2) Acetic acid, CS_2

(3) (i) CS_2, chromyl chloride, (ii) H_3O^+

(4) H_2, Pd, $BaSO_4$

Answer (3)

Sol.



8. In an electrochemical cell $E^\circ_{A^{+2}/A} = -0.33V$, $E^\circ_{B/B^{+2}} = 0.50\ V$. Find ΔG°.

(1) –0.20 F

(2) –0.34 F

(3) –0.02 F

(4) –0.04 F

Answer (2)

Sol. $B|B^{2+}||A^{2+}|A$

$E^\circ_{cell} = E^\circ_{A^{2+}/A} - E^\circ_{B^{2+}/B}$

$= -0.33 - (-0.50) = +0.17\ V$

$\Delta G^\circ = -nFE^\circ_{cell}$

$= -2 \times F \times 0.17 = -0.34\ F$

9. Which of the following is Metalloid?

(1) Bi

(2) Sc

(3) Te

(4) Hg

Answer (3)

Sol. Metalloids are – B, Si, Ge, As, Sb, Te, Po. So the correct choice is Te.

10. Match the following :

(i)	Invertase	(a)	Starch to maltose
(ii)	Zymase	(b)	Maltose to glucose
(iii)	Maltase	(c)	Sugar to ethanol
(iv)	Diastase	(d)	Inversion of cane Sugar

(1) (i)-(d), (ii)-(c), (iii)-(b), (iv)-(a)

(2) (i)-(c), (ii)-(d), (iii)-(a), (iv)-(b)

(3) (i)-(b), (ii)-(a), (iii)-(d), (iv)-(c)

(4) (i)-(d), (ii)-(a), (iii)-(c), (iv)-(b)

Answer (1)

Sol.

ENZYME		CONVERSION
Invertase	–	Inversion of cane sugar
Zymase	–	Sugar to ethanol
Maltase	–	Maltose to glucose
Diastase	–	Starch to maltose

11. A solid A_xB_y has CCP structure. A forms CCP and B is present in all the octahedral voids. If atom 'A' are removed from two opposite faces then x will be

(1) 1

(2) 3

(3) 8

(4) 6

Answer (2)

Sol. A_xB_y

A atoms per unit cell $\rightarrow$ 4

B atoms per unit cell $\rightarrow$ 4 (Octahedral voids)

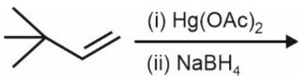
When atoms A are removed from two opposite faces,

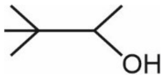
A removed $= 2 \times \frac{1}{2} = 1$

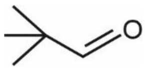
A left = 4 – 1 = 3

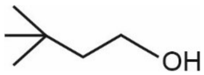
Formula becomes A_3B_4

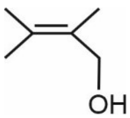
Hence, x = 3

12. 

(1) 

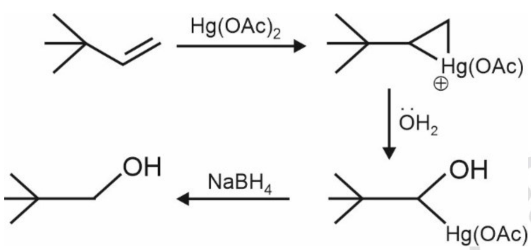
(2) 

(3) 

(4) 

Answer (1)

Sol.



13. Boiling of hard water produces

(1) $CaCO_3$ and $Mg(OH)_2$

(2) $Ca(OH)_2$ and $MgCO_3$

(3) $CaCO_3$ and $MgCO_3$

(4) $Ca(OH)_2$ and $Mg(OH)_2$

Answer (1)

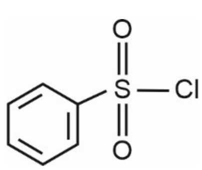
Sol. During boiling $Mg(HCO_3)_2$ is converted to insoluble $Mg(OH)_2$

$$Mg(HCO_3)_2 \xrightarrow{\text{Heating}} Mg(OH)_2 \downarrow + 2CO_2$$

$$Ca(HCO_3)_2 \xrightarrow{\text{Heating}} CaCO_3 \downarrow + H_2O + CO_2$$

$Ca(HCO_3)_2$ changes to insoluble $CaCO_3$.

14. Which of the following is not correct about this compound



(1) It is Hinsberg reagent

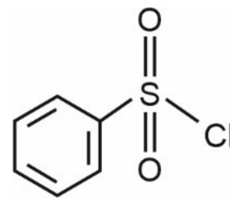
(2) It is used to differentiate between 1°, 2° and 3° amine

(3) After reaction with 2° amine it is soluble in NaOH

(4) After reaction with 1° amine it is soluble in NaOH

Answer (3)

Sol. Benzene sulfonyl chloride, as shown below,



is a Hinsberg reagent. It is used to differentiate 1°, 2° and 3° amines. The product obtained by the reaction of 2° amine with Hinsberg reagent is not soluble in aq NaOH but the product obtained from 1° amine is soluble in aq NaOH.

15. Which water sample is most polluted?

(A) BOD = 5

(B) BOD = 8

(C) BOD = 10

(D) BOD = 14

(1) Sample A

(2) Sample B

(3) Sample C

(4) Sample D

Answer (4)

Sol. The BOD value is most commonly expressed in milligrams of oxygen consumed per litre of sample during 5 days of incubation at 20°C. Higher the BOD value more the polluted water.

16. BOD of A = 3

B = 18

C = 27

D = 8

Then which water bodies given above have very high concentration of organic matter.

(1) A and D

(2) C and D

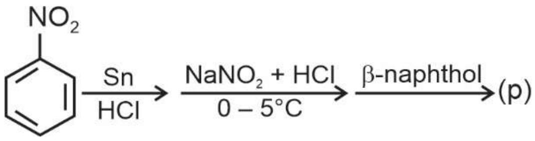
(3) B and C

(4) A and C

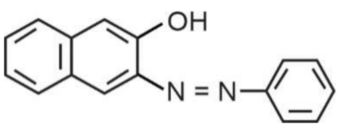
Answer (3)

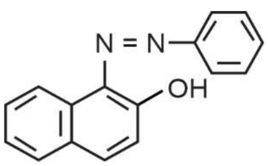
Sol. BOD is the amount of oxygen required by bacteria to break down the organic matter present in a certain volume of sample of water.

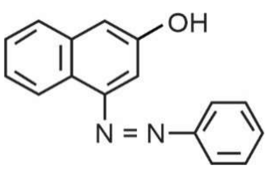
Those water bodies which have high concentration of organic matter require large amount of oxygen for breaking down biologically. Hence B and C will have highest BOD.

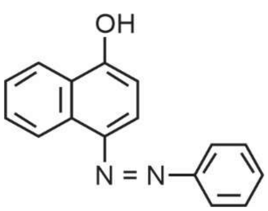
17. 

Identify the major product (p) formed in the above sequence of reactions.

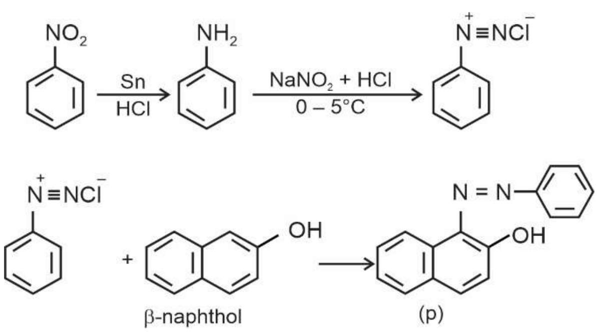
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

18. Which of the following is not a synthetic detergent

(1) Sodium lauryl sulphate

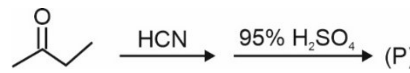
(2) Sodium dodecyl benzene sulphonate

(3) Cetyl trimethyl ammonium bromide

(4) Sodium stearate

Answer (4)

Sol. Synthetic detergents: Sodium lauryl sulphate, sodium dodecyl benzene sulphonate and cetyl trimethyl ammonium bromide.

19. 

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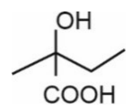
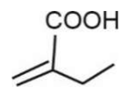
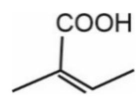
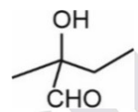
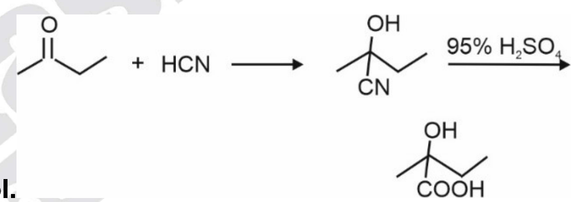
(1) 

(2) 

(3) 

(4) 

Answer (1)



Sol.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. 6.100 gm of CNG is supplied with 208 g of oxygen. CO_2 and H_2O is produced with a lot of heat. How much CO_2 is produced? (Consider CNG as methane) [Round off to nearest integer]

Answer (17)

Sol. $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$ + heat

Moles of $CH_4 = \frac{6.1}{16}$

Moles of $O_2 = \frac{208}{32}$

CH_4 is limiting reagent.

So, moles of CO_2 formed $= \frac{6.1}{16}$

Weight of CO_2 formed $= \frac{6.1}{16} \times 44 = 16.775$

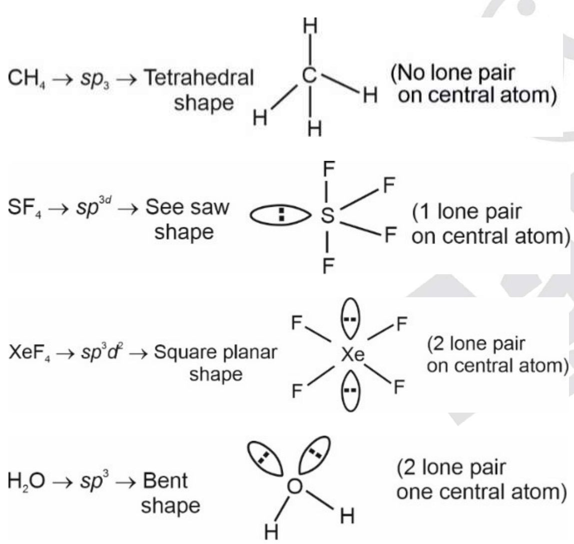
≈ 17 g

22. Number of molecules having two lone pairs on the central atom among the following is

CH_4, SF_4, XeF_4, H_2O

Answer (2)

Sol.



23. Number of electrons in t_{2g} orbital of compound formed by reacting $[Co(H_2O)_6]^{2+}$ with excess NH_3 is

Answer (6)

Sol. When $[Co(H_2O)_6]^{2+}$ is treated with excess of NH_3, all the water molecules will be replaced by NH_3 molecules. Since NH_3 is a strong field ligand, the substitution reaction will the accompanied with oxidation of Co^{2+} to Co^{3+}

$[Co(H_2O)_6]^{2+} + 6NH_3 \rightarrow [Co(NH_3)_6]^{3+} + 6H_2O + e$

Electronic configuration of $[Co(NH_3)_6]^{3+}$: $t_{2g}^6 e_g^0$

24. A metal is irradiated with light of wavelength 6640 Å and its stopping potential is 0.4 V. The threshold frequency (ν_0) of the metal is 3.55 × 10^x Hz. The value of x is

Answer (14)

Sol. $\frac{hc}{\lambda} = h\nu_0 + qV$

$$\nu_0 = \frac{\frac{hc}{\lambda} - qV}{h}$$

$$= \frac{\frac{6.626 \times 10^{-34} \times 3 \times 10^8}{6640 \times 10^{-10}} - 1.6 \times 10^{-19}(0.4)}{6.626 \times 10^{-34}}$$

= 3.55 × 10^{14} Hz

25. Find the osmotic pressure (in atm) of a mixture in which 2 g of protein (having molar mass 6 kg) is present in 2 ml solution at 27°C.

[Round off to the nearest integer]

Answer (04)

Sol. Osmotic pressure (π) = iCRT

$= 1 \times \frac{2}{6000} \times \frac{1000}{2} \times 0.0821 \times 300$

$\simeq 4$ atm

26. Consider the following reaction at 383 K

$H_2O(l) \rightleftharpoons H_2O(g)$

Given, ΔH = 41.1 kJ/mol

R = 8.314 JK^{-1} mol^{-1}

The value of ΔU (in kJ/mol) is

(Round off to the nearest integer]

Answer (38)

Sol. $H_2O(l) \rightleftharpoons H_2O(g)$

Since, $\Delta H = \Delta U + \Delta n_g RT$

$\Delta n_g = 1$

$\Delta U = \Delta H - \Delta n_g RT = 41.1 \times 10^3 - (1)(8.314)(383)$

$\simeq 38$ kJ / mol

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Inverse trigonometric functions have principal value.

Find $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$

(1) 0
(2) $\frac{\pi}{3}$
(3) $\frac{\pi}{6}$
(4) $\frac{\pi}{2}$ or $\frac{\pi}{4}$

Answer (2)

Sol. $\cos^{-1}\left(\frac{3}{10}\left(\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)+\frac{2}{5}\left(\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)\right)$

$=\cos^{-1}\left(\frac{3}{10}\cdot\frac{3}{5}+\frac{2}{5}\cdot\frac{4}{5}\right)$

$=\cos^{-1}\left(\frac{1}{2}\right)$

$=\frac{\pi}{3}$

2. $\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to

(1) $\frac{1}{4}$
(2) $\frac{1}{12}$
(3) $\frac{1}{6}$
(4) $\frac{1}{8}$

Answer (3)

Sol. $\lim\limits_{x\to 0}\frac{2\sin\left(\frac{\sin x+x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$

$=\lim\limits_{x\to 0}\frac{2\cdot\frac{\sin x+x}{2}\cdot\frac{x-\sin x}{2}}{x^4}$

$=\frac{1}{2}\lim\limits_{x\to 0}\frac{\sin x+x}{x}\cdot\frac{x-\sin x}{x^3}$

$=\frac{1}{2}\cdot 2\cdot\lim\limits_{x\to 0}\frac{x-\left(x-\frac{x^3}{3!}\right)}{x^3}$

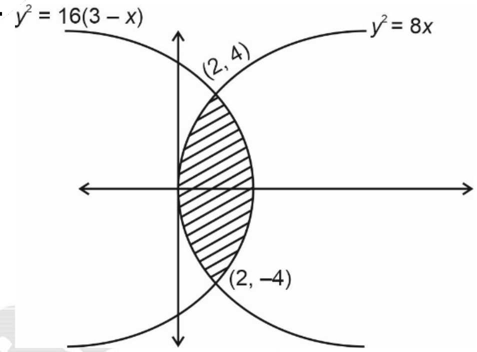
$=\frac{1}{2}\cdot 2\cdot\frac{1}{6}=\frac{1}{6}$

3. Find the area bounded between curve $y^2 = 8x$ & $y^2 = 16(3 - x)$.

(1) 16
(2) 8
(3) 32
(4) 64

Answer (1)

Sol.



The area of region bounded by two given curves

$=\int_{-4}^{4}\left(3-\frac{y^2}{16}-\frac{y^2}{8}\right)dy$

$=\int_{-4}^{4}\left(3-\frac{3y^2}{16}\right)dy$

$=2\int_{0}^{4}\left(3-\frac{3y^2}{16}\right)dy$

$=2\left[3y-\frac{y^3}{16}\right]_0^4$

$= 2(12 - 4)$

$= 16$ square units

4. The value of 16sin20°.sin40°.sin80° is

(1) $2\sqrt{3}$
(2) 3
(3) $\sqrt{3}$
(4) $4\sqrt{3}$

Answer (1)

Sol. $\because\quad \sin\theta.\sin\left(\frac{\pi}{3}-\theta\right).\sin\left(\frac{\pi}{3}+\theta\right)=\frac{1}{4}\sin 3\theta$

$\therefore\quad$ 16sin20°.sin40°.sin80°

$= 16.\frac{1}{4}\sin 60°$

$= 4.\frac{\sqrt{3}}{2}$

$= 2\sqrt{3}$

5. $\frac{dy}{dx}+\frac{2y}{x}=e^x$ be a differential equation & $y(x)$ be a solution satisfying $y(1) = 0$, then $z(x) = x^2 \cdot y(x) - e^x$ is

(1) $z(x) = e^x(x^2 + 2x + 1) - e$

(2) $z(x) = e^x(x - 1)^2 - e$

(3) $z(x) = e^x(x^2 + 1) - e$

(4) $z(x) = e^x(x^2 + 1) + e$

Answer (2)

Sol. Given differential equation,

$$\frac{dy}{dx}+\frac{2y}{x}=e^x \quad \ldots(i)$$

$$IF = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$$

$\therefore$ Solution of differential equation

$y(x) \cdot x^2 = \int e^x \cdot x^2\, dx$

$= x^2e^x - \int 2x e^x dx$

$= x^2e^x - 2xe^x + 2e^x + c$

Now, $y(1) = 0 \Rightarrow 0 = e - 2e + 2e + c$

$\Rightarrow c = -e$

Now, $z(x) = x^2e^x - 2xe^x + 2e^x - e - e^x$

$= e^x(x^2 - 2x + 1) - e$

$= e^x(x - 1)^2 - e$

6. $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,dx = g(x) + c$, then find the value of $g\left(\frac{1}{2}\right)$.

(1) $\ln\left(2-\sqrt{3}\right)-\frac{\pi}{6}$ (2) $\ln\left(2+\sqrt{3}\right)-\frac{\pi}{3}$

(3) $\ln\left(2+\sqrt{3}\right)-\frac{\pi}{6}$ (4) $\ln\left(2-\sqrt{3}\right)-\frac{\pi}{3}$

Answer (1)

Sol. $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,dx$ let $x = \sin\theta$, $dx = \cos\theta d\theta$

$$= \int \frac{1}{\sin\theta}\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}\cdot\cos\theta\, d\theta$$

$$= \int \frac{1-\sin\theta}{\sin\theta\cdot\cos\theta}\cos\theta\, d\theta$$

$$= \int (\text{cosec}\theta - 1)d\theta$$

$= \ln(\text{cosec}\theta - \cot\theta) - \theta + c$

$$= \ln\left(\frac{1-\sqrt{1-x^2}}{x}\right) - \sin^{-1}x + c$$

So $g(x) = \ln\left(\frac{1-\sqrt{1-x^2}}{x}\right) - \sin^{-1}x$

$$\Rightarrow g\left(\frac{1}{2}\right) = \ln\left(2-\sqrt{3}\right)-\frac{\pi}{6}$$

7. If $A = \sum_{n=1}^{\infty}\frac{1}{\left(3+(-1)^n\right)^n}$ and $B = \sum_{n=1}^{\infty}\frac{(-1)^n}{\left(3+(-1)\right)^n}$, then value of $\frac{A}{B}$ is

(1) $\frac{-11}{9}$ (2) $\frac{-11}{3}$

(3) $\frac{-11}{6}$ (4) -11

Answer (1)

Sol. $A = \sum_{n=1}^{\infty}\frac{1}{\left(3+(-1)^n\right)^n} = \frac{1}{2}+\frac{1}{4^2}+\frac{1}{2^3}+\frac{1}{4^4}+\frac{1}{2^5}+\ldots$

It is sum of two G.P.s i.e

$$\left(\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}+\ldots\right)+\left(\frac{1}{4^2}+\frac{1}{4^4}+\ldots\right)$$

$$\Rightarrow \left(\frac{\frac{1}{2}}{1-\frac{1}{4}}\right)+\frac{1}{16}\left(\frac{1}{1-\frac{1}{16}}\right) = \frac{2}{3}+\frac{1}{15} = \frac{11}{15} = A$$

Similarly

$$B = \sum_{n=1}^{\infty}\frac{(-1)^n}{\left(3+(-1)^n\right)^n} = \frac{-1}{2}+\frac{1}{4^2}-\frac{1}{2^3}+\frac{1}{4^4}-\ldots$$

$$\Rightarrow B = \frac{-2}{3}+\frac{1}{15} = \frac{-9}{15}$$

$$\therefore \frac{A}{B} = \frac{-11}{9}$$

8. If function $f(x) = x - 1$ and $g(x) = \dfrac{x^2}{x^2+1}$, then fog is

(1) One-one and onto

(2) One-one but not onto

(3) Onto but not one-one

(4) Neither one-one nor onto

Answer (4)

Sol. $\text{fog}(x) = f(g(x))$

$= g(x) - 1$

$$= \frac{x^2}{x^2+1} - 1 = \frac{-1}{x^2+1}$$

fog(x) is even function so many-one

If $h(x) = \dfrac{-1}{x^2+1}$

$\infty > x^2 + 1 \geq 1$

$$0 < \frac{1}{x^2+1} \leq 1$$

$$0 > \frac{-1}{x^2+1} \geq -1$$

So clearly $h(x)$ is into

$*$ $f : R \to R$ must be given

9. The sides of a cuboid are $2x$, $4x$, $5x$. There is a closed hemisphere of radius r. The sum of their surface area is a constant K. What is the ratio of $x : r$ such that the sum of their volume is maximum?

(1) $\dfrac{19}{45}$ (2) $\dfrac{45}{19}$

(3) $\dfrac{19}{24}$ (4) $\dfrac{24}{7}$

Answer (1)

Sol. Let sum of surface are $S = 3\pi r^2 + 76x^2$

$\because$ S is constant so $\dfrac{dS}{dx} = 0$

$$\Rightarrow 6\pi r \frac{dr}{dx} + 2.76x = 0$$

$$\Rightarrow \frac{dr}{dx} = -\frac{76x}{3\pi r} \quad .....(i)$$

Now total volume $V = \dfrac{2}{3}\pi r^3 + 40x^3$

For maximum volume $\dfrac{dV}{dx} = 0$

$$\Rightarrow 2\pi r^2 \frac{dr}{dx} + 120x^2 = 0 \quad .... (ii)$$

$$\Rightarrow 2\pi r^2 \left(-\frac{76x}{3\pi r}\right) + 120x^2 = 0$$

$$\Rightarrow 8x\left[-\frac{19r}{3} + 15x\right] = 0$$

$$\Rightarrow \frac{x}{r} = \frac{19}{45}$$

10. If the given system $\alpha x + y + z = 5$, $x + 2y + 4z = 4$ and $x + 3y + 5z = \beta$ has infinitely many solutions, then α, β is

(1) 0, 9 (2) – 1, – 3

(3) – 1, 3 (4) 1, – 3

Answer (1)

Sol. For infinite solution

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 5 \end{vmatrix} = 0 \Rightarrow \alpha = 0$$

Plane P_3 should be member of family of plane passing through line of intersection of $P_1 = 0$ and $P_2 = 0$ i.e.

$x + 2y + 4z + \lambda(y + z) = 4 + 5\lambda$

For it to represent $P_3 = 0 \Rightarrow \lambda = 1$

$x + 3y + 5z = 9$

$\beta = 9$

11. Consider the function

$f(x) = \min\{1, 1 + x\sin x\}$, $x \in [0, \pi]$

(1) Continuous and differentiable in $[0, \pi]$

(2) Discontinuous at $x = \dfrac{\pi}{2}$

(3) Continuous but not differentiable at $x = \dfrac{\pi}{2}$

(4) Discontinuous at two points in $[0, \pi]$

Answer (1)

Sol. $\therefore$ $f(x) = \min\{1, 1 + x\sin x\}$, $x \in [0, \pi]$

$\because$ if $x \in [0, \pi]$ then $1 + x\sin x \geq 1$.

$\therefore$ $f(x) = 1$, $x \in [0, \pi]$

$\therefore$ $f(x)$ is continuous and differentiable everywhere in $x \in [0, \pi]$

12. Find the eccentricity of locus of mid-point of a line segment joining (4, 3) to each point on ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$.

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{2}$

(3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{\sqrt{2}}$

Answer (4)

Sol. Let $A(4, 3)$ and a point $B\left(2\cos\theta, \sqrt{2}\sin\theta\right)$ on given ellipse.

Assume mid-point of AB at $M(h, k)$

So $2h = 4 + 2\cos\theta \Rightarrow \cos\theta = h - 2$

and $2k=3+\sqrt{2}\sin\theta \Rightarrow \sin\theta=\frac{2k-3}{\sqrt{2}}$

hence $(h-2)^2+\left(\frac{2k-3}{\sqrt{2}}\right)^2=1$

Required locus is

$$\frac{(x-2)^2}{1}+\frac{\left(y-\frac{3}{2}\right)^2}{\frac{1}{2}}=1$$

Its eccentricity $e=\sqrt{1-\frac{\frac{1}{2}}{1}}=\frac{1}{\sqrt{2}}$

13. $\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{2-x^2}{\left(2+x^2\right)\sqrt{4+x^4}}dx$ equals

(1) 0 (2) $\frac{1}{12}$

(3) 3 (4) $\frac{1}{8}$

Answer (3)

Sol. $I=\int_0^{\sqrt{2}}\frac{2-x^2}{\left(2+x^2\right)\sqrt{4+x^4}}dx$

Putting $x=\sqrt{2}t$

$$I=\int_0^{\frac{1}{\sqrt{2}}}\frac{2\left(1-t^2\right)}{2\left(1+t^2\right)2\sqrt{1+t^4}}dt$$

$$=\frac{1}{\sqrt{2}}\int_0^1\frac{1-t^2}{\left(1+t^2\right)\sqrt{1+t^4}}dt$$

$$=\frac{1}{\sqrt{2}}\int_0^1\frac{\left(\frac{1}{t^2}-1\right)dt}{\left(t+\frac{1}{t}\right)\sqrt{\left(t+\frac{1}{t}\right)^2-2}}$$

Let $t+\frac{1}{t}=z$

$$=\frac{1}{\sqrt{2}}\int_{\infty}^{2}\frac{-dz}{z\sqrt{z^2-2}}$$

$$=\frac{1}{2}\sec^{-1}\left(\frac{z}{\sqrt{2}}\right)\Bigg|_2^{\infty}$$

$$=\frac{\pi}{8}$$

So, $\frac{24}{\pi}I=3$

14. Let a_1, a_2, a_3, ... are in G.P. such that $a_2 + a_4 = 2a_3 + 1$, $a_2 + a_3 = 2a_4$. Then the value of $a_2 + a_3 + 2a_4$ is

(1) $\frac{4}{27}$ (2) $\frac{4}{9}$

(3) $\frac{3}{14}$ (4) $\frac{3}{19}$

Answer (2)

Sol. Let r be the common ratio of G.P.

A/Q $a_1r + a_1r^3 = 2a_1r^2 + 1$ ...(i) and

$a_1r + a_1r^2 = 2a_1r^3$ ...(ii)

$\Rightarrow 1 + r = 2r^2$

$\Rightarrow 2r^2 - r - 1 = 0$

$\Rightarrow r=1, -\frac{1}{2}$

$r\neq 1$ as it does not satisfy (i)

$\therefore r=-\frac{1}{2}$

Now, $a_1\left(-\frac{1}{2}\right)+a_1\left(-\frac{1}{2}\right)^3=2a_1\left(-\frac{1}{2}\right)^2+1$

$\Rightarrow -\frac{1}{2}a_1-\frac{1}{8}a_1-\frac{1}{2}a_1=1$

$\Rightarrow -\frac{9}{8}a_1=1 \Rightarrow a_1=-\frac{8}{9}$

Now, $a_2 + a_3 = 2a_4$

$= 4a_4$

$=4a_1r^3=4\left(-\frac{8}{9}\right)\left(-\frac{1}{8}\right)=\frac{4}{9}$

15. If slope of common tangent of circle $x^2 + y^2 = 12$ and ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ is m in then $12m^2$ is

(1) 16 (2) 3

(3) 4 (4) $\frac{4}{3}$

Answer (1)

Sol. Tangent to circle

$$y = mx \pm 2\sqrt{3}\sqrt{1+m^2} \quad \ldots(1)$$

Tangent to ellipse

$$y = mx \pm \sqrt{9m^2 + 16} \quad \ldots(2)$$

As both are same

$$9m^2 + 16 = 12m^2 + 12$$

$$\Rightarrow \quad m^2 = \frac{4}{3}$$

$$\Rightarrow \quad 12m^2 = 16$$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. $\left[{}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \ldots.. + {}^{60}C_{20}\right] = \frac{m}{n}\ {}^{60}C_{20}$ where m and n are co-prime, then $m + n$ is equal to

Answer (102)

Sol. ${}^{40}C_{40} + {}^{41}C_{40} + {}^{42}C_{40} + \ldots. + {}^{60}C_{40} = {}^{61}C_{41}$

$$= \frac{61}{41}\ {}^{60}C_{40}$$

So $m = 61$ and $n = 41$

$\Rightarrow \quad m + n = 102$

22. If L is tangent to hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ and L_2 is a straight line passing through (0, 0) and perpendicular to L_1. If the locus of point of intersection of L_1 and L_2 is $(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$, then $\alpha + \beta$ is equal to

Answer (05)

Sol. Hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Equation of tangent $L_1 : y = mx \pm \sqrt{9m^2 - 4} \quad \ldots(i)$

then $L_2 : y = \frac{-1}{m}x \quad \ldots(ii)$

for locus of point of intersection, eliminate m

$$y = \frac{-x^2}{y} \pm \sqrt{\frac{9x^2}{y^2} - 4}$$

$(x^2 + y^2)^2 = 9x^2 - 4y^2$

$\therefore \quad \alpha = 9, \beta = -4$

$\therefore \quad \alpha + \beta = 5$

23. If p and q are real numbers and $p + q = 3$, $p^4 + q^4 = 369$. Find $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2}$

Answer (4)

Sol. $\because \quad (p + q)^4 = p^4 + q^4 + 4pq(p^2 + q^2) + 6p^2q^2$

$\Rightarrow \quad 81 = 369 + 4pq(9 - 2pq) + 6p^2q^2$

$\Rightarrow \quad p^2q^2 - 18pq - 144 = 0$

$\Rightarrow \quad pq = -6$ or 24

when $pq = 24$ and $p + q = 3$, then p and q will be imaginary

So $pq = -6$

$$\text{Now } \left(\frac{1}{p} + \frac{1}{q}\right)^{-2} = \left(\frac{pq}{p+q}\right)^2 = \left(\frac{-6}{3}\right)^2 = 4$$

24. If $Z^2 + Z + 1 = 0$, $Z \in C$ $\left|\sum_{K=1}^{15}\left(Z^K + \frac{1}{Z^K}\right)^2\right|$ is equal to

Answer (30)

Sol. $Z = w$ and w^2, i.e., roots of given equation

$$\sum_{K=1}^{15}\left(Z^K + \frac{1}{Z^K}\right)^2 = 5(1+1)^2 + 10\left(w + w^2\right)^2$$

$$= 20 + 10$$

$$= 30$$

$$\text{So } \left|\sum_{K=1}^{15}\left(Z^K + \frac{1}{Z^K}\right)^2\right| = 30$$

25. A six digit number is formed randomly using digits 1 and 8. If the probability of this number is divisible by 21 is P, then $96P$ is equal to

Answer (33)

Sol. $\because$ 1 and 8 are of the type $3\lambda + 1$ and $3\lambda - 1$.

So equal number of digits 1 and 8 are being used to form a number divisible by 3. (Or all the digits are either 1 or 8).

In all cases what even number is formed using six 1's or six 8's or three 1's + three 8's, that number can be splited into sum of multiples of 1001, 1008, 8001 or 8008 (all of them are divisible by 7). So, all the numbers thus formed will be divisible by 21.

Ex: $811188 = 8001 \times 10^2 + 1008 \times 10^1 + 1008$

Number of such numbers $= \dfrac{\underline{|6}}{\underline{|3}\,\underline{|3}} + 2 = 22$

Required probability $P = \dfrac{22}{2^6} = \dfrac{11}{32}$.

26. The mean of 50 observations is 15 and Standard deviation is 2. However one observation was wrongly recorded. The sum of the correct and incorrect observation is 70. If the mean of the corrected set of observations is 16, then the variance is

Answer (43)

Sol. $\dfrac{\sum x_i}{50} = 15 \Rightarrow \sum x_i = 750$

$\dfrac{\sum x_i^2}{50} - (15)^2 = 2^2 \Rightarrow \sum x_i^2 = 11450$

$\sum x_{i(\text{new})} = 50 \times 16 = 800$

So, if wrong observation was 'a' then corrected one is 'a + 50'

$a + (a + 50) = 70 \Rightarrow a = 10$

Correct observation = 60

$\sum x_{i(\text{new})}^2 = 11450 - 100 + 3600 = 14950$

$\text{variance}_{(\text{new})} = \dfrac{14950}{50} - (16)^2 = 299 - 256 = 43$

27. Find number of three digit numbers n such that GCD (n, 36) = 2.

Answer (112)

Sol. Total number of three digit numbers = 900

Number of even numbers = 450

Number of numbers divisible by 4 out of 450 even numbers = 225

Number of numbers divisible by 3 out of 450 even numbers = 150

Number of numbers divisible by 12 out of 450 even numbers = 37

Number of possible values of

n = 450 – 225 – 150 + 37

= 112

28.

29.

30.

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