29/06/2022
Morning

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Memory Based
Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Fringe width in YDSE for wavelength λ = 5000 Å is 1.2 mm. Then the fringe width for light ray of wavelength λ = 6000 Å and distance between slit becomes twice is

(1) 0.52 mm (2) 0.72 mm

(3) 8.5 mm (4) 1.32 mm

Answer (2)

Sol. $\because \beta = \frac{\lambda D}{d}$

$\Rightarrow \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{d_1} \times \frac{d_2}{\lambda_2}$

$\Rightarrow \frac{1.2 \times 10^{-3}}{\beta_2} = \frac{5000}{d} \times \frac{2d}{6000} = \frac{5}{3}$

$\Rightarrow \beta_2 = \frac{3.6}{5}$ mm

= 0.72 mm

2. Water, flowing through a nozzle of cross-section area 1 cm^2, strikes a 2 kg block at rest as shown:

If water flow comes to rest immediately after striking the block, then initial acceleration of the block is

(1) 3 m/s^2 (2) 5 m/s^2

(3) 4 m/s^2 (4) 2 m/s^2

Answer (2)

Sol. $F = \frac{\Delta p}{\Delta t} = \frac{(\rho A v) \times v}{1}$

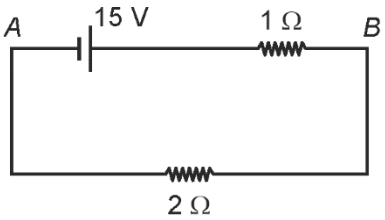
$\Rightarrow ma = \rho A v^2$

$\Rightarrow a = \frac{\rho A v^2}{m}$

$= \frac{10^3 \times 10^{-4} \times (100)}{2}$

= 5 m/s^2

3. In the following circuit the value of $V_B - V_A$ is



(1) 5 V (2) 15 V

(3) 10 V (4) 7.5 V

Answer (3)

Sol. $I = \frac{15}{2+1} = 5$ A

$\therefore V_A + 15 - 5 \times 1 = V_B$

$\Rightarrow V_B - V_A = 10$ V

4. A ball is dropped from a height of 180 m. After 2 seconds, another ball is thrown downwards with speed u so that both balls collide at a height of 100 m. Then the value of u is

(1) 10 m/s

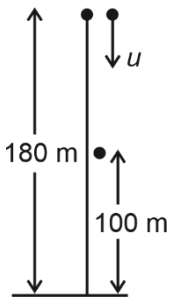
(2) 20 m/s

(3) 40 m/s

(4) 30 m/s

Answer (4)

Sol.



At t = 2 s,

$v_1 = 2 \times 10 = 20$ m/s

$v_2 = u$

$S = \frac{1}{2} \times 10 \times (2)^2$

= 20 m

$\therefore \quad 80 = \frac{1}{2} \times 10 \times t_2^2$

$\Rightarrow \quad t_2 = 4$ s

$\therefore \quad \Delta t = 2$ s,

$\Rightarrow \quad \frac{20}{(u-20)} = 2$

$\Rightarrow \quad u = 30$ m/s

5. **Assertion:** If energy of photon is less than work function, electron does not eject.

Reason: If kinetic energy of ejected electron is zero, then work function is equal to energy of photon.

(1) Both assertion and reason are true and reason is correct explanation of assertion

(2) Both assertion and reason are true but reason is not the correct explanation

(3) Assertion is true and reason is false

(4) Assertion is false and reason is true

Answer (1)

Sol. Both the statements are correct and statement-2 is correct explanation of statement-1.

6. Escape velocity on planet-1 is given as 12 m/s. If density of another planet is 4 times that of planet-1, and its radius is half of that of planet-1. Then escape velocity from planet two will be

(1) 24 m/s

(2) 12 m/s

(3) 36 m/s

(4) 6 m/s

Answer (2)

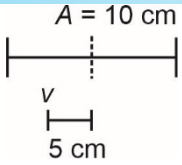
Sol. $\because \quad v_e = \sqrt{2\frac{GM}{R}} = \sqrt{\frac{2G}{R} \times \rho \times \frac{4}{3}\pi R^3}$

$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{\rho_1 R_1^2}{\rho_2 R_2^2}}$

$\Rightarrow \quad \frac{12}{v_2} = \sqrt{\frac{\rho}{4\rho} \times \frac{R^2}{\left(\frac{R^2}{4}\right)}}$

$\Rightarrow \quad v_2 = 12$ m/s

7.



A particle performing SHM with $A = 10$ cm, is given impulse at $x = 5$ cm such that velocity becomes three times. Find new amplitude

(1) 15 cm

(2) $10\sqrt{7}$ cm

(3) 12 cm

(4) $18\sqrt{3}$ cm

Answer (2)

Sol. $\because \quad v = \omega\sqrt{A^2 - x^2}$

$\Rightarrow \quad \frac{v_1}{v_2} = \frac{\sqrt{A_1^2 - x_1^2}}{\sqrt{A_2^2 - x_2^2}}$

$\Rightarrow \quad \frac{v}{3v} = \frac{\sqrt{A^2 - 25}}{\sqrt{A_2^2 - 25}}$

$\Rightarrow \quad \sqrt{A_2^2 - 25} = 3 \times \sqrt{75}$

$\Rightarrow \quad A_2^2 - 25 = 9 \times 75$

$\Rightarrow \quad A_2^2 = 700$

$\Rightarrow \quad A_2 = 10\sqrt{7}$ cm

8. Average intensity of an electromagnetic wave is $I = 0.22$ W/m^2, then magnetic field amplitude of this wave is

(1) 2.8×10^{-6} T

(2) 2.3×10^{-7} T

(3) 5.8×10^{-6} T

(4) 4.3×10^{-8} T

Answer (4)

Sol. $I = \frac{B_0^2}{2\mu_0} \times c$

$B_0 = \sqrt{\frac{0.22 \times 2 \times 4\pi \times 10^{-7}}{3 \times 10^8}}$

$\Rightarrow \quad B_0 = 4.3 \times 10^{-8}$ T

9. For two unit vectors $\hat{A}$ and $\hat{B}$, it is given that

$|\hat{A}+\hat{B}| = 2|\hat{A}-\hat{B}|$

Then the angle between the two vectors is

(1) $\sin^{-1}\left(\frac{1}{3}\right)$ (2) $\sin^{-1}\left(\frac{3}{5}\right)$

(3) $\cos^{-1}\left(\frac{1}{3}\right)$ (4) $\cos^{-1}\left(\frac{3}{5}\right)$

Answer (4)

Sol. $|\hat{A}+\hat{B}|^2 = 4|\hat{A}-\hat{B}|^2$

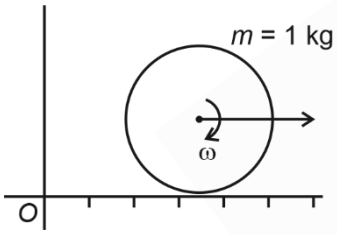
$\Rightarrow$ $1 + 1 + 2\cos\theta = 4(1 + 1 - 2\cos\theta)$

$\Rightarrow$ $2 + 2\cos\theta = 8 - 8\cos\theta$

$\Rightarrow$ $10\cos\theta = 6$

$\Rightarrow$ $\cos\theta = \frac{3}{5}$

10. A spherical shell of radius R is rolling on a level horizontal surface as shown. Then the angular momentum about the origin is



(1) $\frac{2}{3}R^2\omega$ (2) $\frac{7}{3}R^2\omega$

(3) $\frac{5}{3}R^2\omega$ (4) $\frac{1}{3}R^2\omega$

Answer (3)

Sol. $L_0 = mvR + \frac{2}{3}MR^2 \times \omega$

$= \omega R^2 + \frac{2}{3}\omega R^2$

$= \frac{5}{3}\omega R^2$

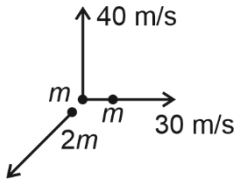
11. A particle of mass m (initially at rest) breaks into three parts in mass ratio 1 : 1 : 2. The velocities of the first two parts are 30 m/s and 40 m/s directed perpendicular to each other. Find the speed of the third part.

(1) 35 m/s (2) 40 m/s

(3) 25 m/s (4) 15 m/s

Answer (3)

Sol.



$m\sqrt{(30)^2 + (40)^2} = 2m \times v_2$

$\Rightarrow$ $50 = 2v_2$

$\Rightarrow$ $v_2 = 25$ m/s

12. If velocity is given by $v = bx^{5/2}$ then net work done in $x = 0$ to $x = 4$ m, $m = 1$ kg, $b = 0.25$ m$^{-3/2}$/s

(1) 64 J (2) 128 J

(3) 160 J (4) 32 J

Answer (4)

Sol. $v = bx^{5/2}$

$\therefore$ $v_1 = 0$, $v_2 = (0.25) \times (4)^{5/2}$

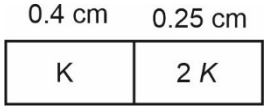
$= \frac{1}{4} \times (32) = 8$ m/s

$\therefore$ $W = \frac{1}{2}m(v_2^2 - v_1^2)$

$= \frac{1}{2} \times 1 \times (8^2 - 0^2)$

$= 32$

13. Two conducting slabs of thermal conductivity K and $2K$ are placed as shown. If cross section area of both is equal then thermal conductivity of combination is



(1) 1.72 K (2) 1.87 K

(3) 1.45 K (4) 1.23 K

Answer (4)

Sol. $\frac{0.4}{K \times A} + \frac{0.25}{2K \times A} = \frac{0.65}{K_{eq} \times A}$

$\Rightarrow$ $\frac{0.4}{K} + \frac{0.25}{2K} = \frac{0.65}{K_{eq}}$

$\Rightarrow$ $\frac{1}{K}\left(\frac{0.8 + 0.25}{2}\right) = \frac{0.65}{K_{eq}}$

$\Rightarrow$ $K_{eq} = \frac{2k \times 0.65}{(0.8 + 0.25)}$

$= 1.23$ K

14. Two coils heat same amount of water separately in 60 min and 20 min. Then time taken to heat same amount if they are connected in parallel with same source.

(1) 10 min (2) 5 min

(3) 7.5 min (4) 15 min

Answer (4)

Sol. $Q_1 = P_1 \times 60$

$Q_2 = P_2 \times 20$

$\because \quad Q_1 = Q_2 \Rightarrow P_1 \times 60 = P_2 \times 20$

And, $Q_3 = (P_1 + P_2) \times t_3$

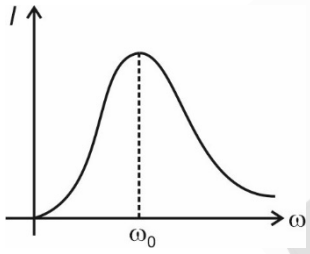
$\Rightarrow \quad P_1 \times 60 = (P_1 + P_2) \times t_3$

$\Rightarrow \quad 60 = \left(1 + \frac{P_2}{P_1}\right) \times t_3$

$\Rightarrow \quad 60 = (1 + 3) \times t_3$

$\Rightarrow \quad t_3 = 15$ min

15. For an *LCR* circuit, the graph between current and angular frequency is given. Choose the correct option.



(1) At ω_0, the circuit has impedance equal to resistance

(2) At $\omega = \omega_0$, the circuit has zero impedance

(3) In the region with $\omega < \omega_0$, the circuit behaves as inductive circuit

(4) In the region with $\omega > \omega_0$, the phase difference between source voltage and current in circuit is zero

Answer (1)

Sol. At $\omega = \omega_0$

$X_L = X_C$

$\Rightarrow Z = \sqrt{R^2 + (X_L - X_C)^2} = R$ as $X_L = X_C$ at $\omega = \omega_0$

$\Rightarrow$ The circuit is purely resistive

At $\omega < \omega_0$ as $\omega_L < \frac{1}{\omega_C} \Rightarrow X_L < X_C$

Or the circuit behaves as a capacitive one.

16. A charge is moving in a circular path in a cyclotron with radius *r*. When the kinetic energy of a particle becomes doubled then the ratio of the new radius to the original radius.

(1) $1 : \sqrt{3}$ (2) 2 : 1

(3) 1 : 4 (4) $\sqrt{2} : 1$

Answer (4)

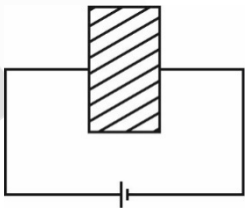
Sol. $r = \frac{mv}{qB}$

$\Rightarrow r = \frac{\sqrt{2mKE}}{qB}$

Now as the kinetic energy gets doubled so new radius r' becomes $\sqrt{2}$ times the original radius r

$\Rightarrow \frac{r'}{r} = \frac{\sqrt{2}}{1}$

17. A capacitor having a dielectric slab is connected with a source as shown. If initially dielectric constant is 10 and finally its value becomes 15. The percentage change in energy stored in the capacitor is



(1) 30% (2) 50%

(3) 20% (4) 15%

Answer (2)

Sol. $U_1 = \frac{1}{2} C_1 V^2$

$U_2 = \frac{1}{2} C_2 V^2$

$\Rightarrow \quad \frac{U_2}{U_1} = \frac{C_2}{C_1} = \frac{15}{10} = 1.5$

$\Rightarrow$ % increase = 50%

18. A rod is stretched by hanging it from top. If $l = l_1$ and $l = l_2$ when 1 kg and 2 kg is attached to it. Then the actual length *l* of rod is

(1) $2l_1 - l_2$ (2) $l_1 - 2l_2$

(3) $2l_1 - 4l_2$ (4) $l_2 - l_1$

Answer (1)

Sol. Assuming the extension is proportional to load

$\Rightarrow l_1 = c(1) + l$

And $l_2 = c(2) + l$

$\Rightarrow 2l_1 - l_2 = l$

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A particle of mass 500*mg* and charge 40 μC is thrown in a region of electric field with velocity 200 m/s directed opposite to the electric field. If strength of the electric field is 10^5 N/C, the distance travelled by the particle before it comes to rest is *x* metres. Find the value of 10*x*.

Answer (25)

Sol. $a = \frac{qE}{m}$

$= \frac{40\times10^{-6}\times10^{5}}{0.5\times10^{-3}}$

$= 4 \times 2 \times 10^3$

$= 8 \times 10^3$ m/s^2

$\therefore\ S = \frac{u^2}{2a} = \frac{(200)^2}{2\times8\times10^3}$

$= \frac{4\times10^4}{16\times10^3}$

$= \frac{1}{4}\times10 = 2.5$ m

$\therefore\quad 10x = 25$ m

22. Band width of a modulated signal is given to be 30 MHz whose 2% is used for communication. The number of channels that can be made so is equal to _____. (Each channel requires 10 kHz of space)

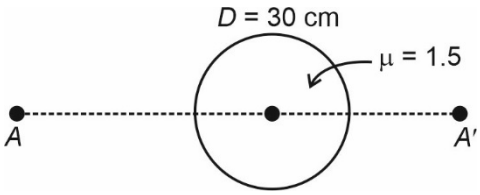
Answer (60)

Sol. Bandwidth used $= \frac{2}{100}\times30\times10^6$

$= 600$ kHz

$\therefore$ Number of channels $= \frac{600}{10} = 60$

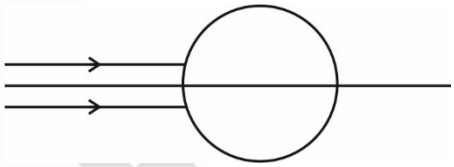
23.



The paraxial rays are incident on the sphere shown, parallel to the optical axis. The distance of the point from centre where the rays will converge is equal to *x* cm. Find value of 2*x*.

Answer (45)

Sol.



For the first surface

$\frac{1.5}{v_1} + \frac{1}{\infty} = \frac{1.5-1}{15}$

$\Rightarrow \frac{1.5}{v_1} = \frac{0.5}{15} \Rightarrow v_1 = 45$ cm

For the second surface

$\therefore\ u_2 = 45 - 30 = 15$ cm

$\therefore\ \frac{1}{u_2} + \frac{1.5}{-15} = \frac{1-1.5}{-15}$

$\Rightarrow \frac{1}{v_2} = \frac{1.5}{15} - \frac{1}{30} = \frac{1}{10} + \frac{1}{30}$

$= \frac{3+1}{30}$

$\Rightarrow u_2 = \frac{30}{40} = 7.5$ cm

$\therefore\ 2x = 2 \times (7.5 + 15) = 45$ cm

24.

25.

26.

27.

28.

29.

30.

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Arrange the following in increasing order of their covalent character.

   (1) CsCl < RbCl < NaCl < LiCl

   (2) LiCl < NaCl < RbCl < CsCl

   (3) CsCl < NaCl < LiCl < RbCl

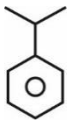
   (4) LiCl < RbCl < NaCl < CsCl

Answer (1)

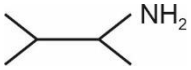
Sol. Among the chlorides of alkali metals, as the size of cation decreases, covalent character decreases. Therefore, the increasing order of covalent character of the given alkali metal chlorides is

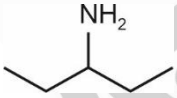
CsCl < RbCl < NaCl < LiCl

2. Which of the following has asymmetric carbon?

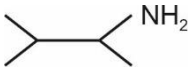
   (1) 

   (2) 

   (3) 

   (4) 

Answer (3)

Sol. 

Four valencies of carbon atom is satisfied by four different group.

3. Cyanide (CN^-) is not used in extraction of which metal

   (1) Zn (From mixture of Sulpide ore)

   (2) Au

   (3) Ag

   (4) Cu

Answer (4)

Sol. Cyanide ion is used in leaching process for Gold and Silver.

In case of an ore containing ZnS and PbS the depressant used is NaCN. It selectively prevents ZnS from coming to Froth but allows Pbs to come with Froth.

Therefore, the most appropriate option is copper.

4. Which of the following amino acids contains sulphur

   (1) Histamine

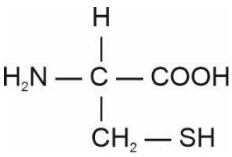
   (2) Cimetidine

   (3) Cysteine

   (4) Ranitidine

Answer (3)

Sol. Cysteine contains — CH_2 — SH group. Rest of the compounds are drugs, not amino acids



5. The configuration of element of atomic number 78 is

   (1) $[Xe]3f^{14}4d^{10}5s^0$

   (2) $[Kr]3f^{14}4d^8$

   (3) $[Xe]3f^{14}4d^{10}5s^2$

   (4) $[Xe]4f^{14}5d^96s^1$

Answer (4)

Sol. The electronic configuration of element 78, i.e., platinum is $[Xe]4f^{14}5d^96s^1$.

6. Find out the lowest wavelength of incident radiation required for Rb photoelectric effect among the following elements.

   (1) Li

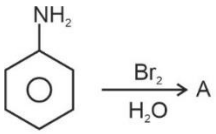
   (2) Cs

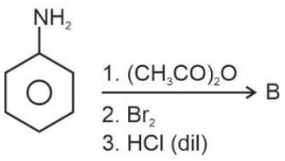
   (3) Rb

   (4) Na

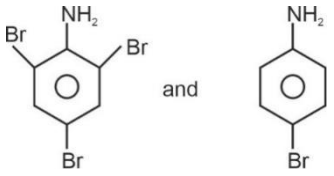
Answer (1)

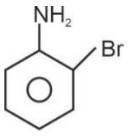
Sol. Lowest wavelength radiation is required for lithium as it has highest ionisation energy

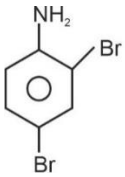
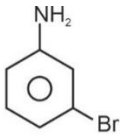
7. 

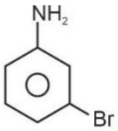


A and B are respectively

(1)  and

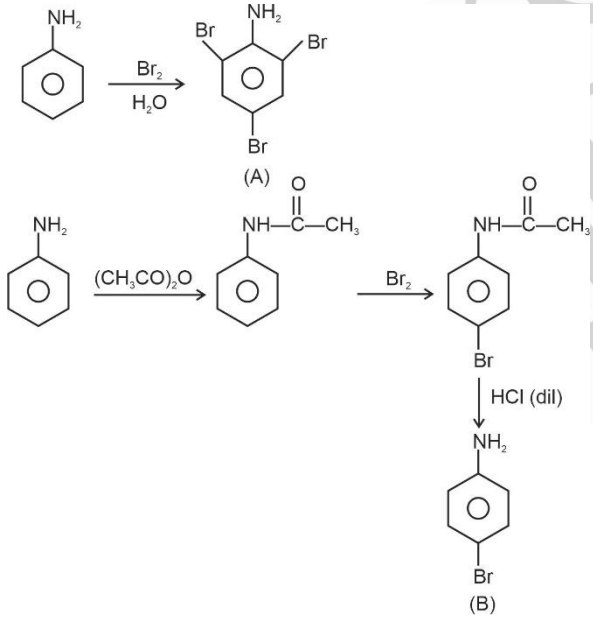
(2)  and 

(3)  and 

(4)  and 

Answer (1)

Sol.



8. Which sugar is present in DNA and RNA respectively?

(1) β-D-ribose, β-D-ribose

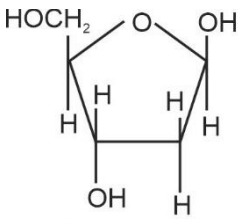
(2) β-D-ribose, β-D-2-deoxyribose

(3) β-D-2-deoxyribose, β-D-ribose

(4) β-D-2-deoxyribose, β-D-2-deoxyribose

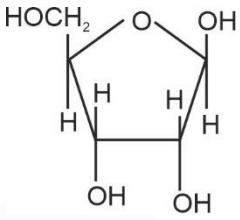
Answer (3)

Sol. the sugar present in DNA is β-D-2-deoxyribose



β-D-2-deoxyribose

The sugar present in RNA is β-D-ribose



9. **Statement 1:** Phenol is a weaker acid as compared to acetic acid.

Statement 2: Phenol is weaker acid than alcohol and water

(1) Statement 1 and 2, both are correct

(2) Statement 1 is correct but statement 2 is incorrect

(3) Statement 1 is incorrect and statement 2 is correct

(4) Statement 1 and 2, both are incorrect

Answer (3)

Sol. (Phenol)  , $pK_a = 10$

CH_3COOH , $pK_a = 4.7$

H_2O , $pK_a = 15.7$

Alcohol , $pK_a \approx 16$

10. Which of the following is present in rain water affects Taj Mahal and cause damage to the monument?

(1) H_2SO_4

(2) H_3PO_4

(3) Phenol

(4) Lactic acid

Answer (1)

Sol. The oxides of sulphur are released into air due to combustion of industrial fuel which get converted into H_2SO_4 causing acid rain. The H_2SO_4 present in acid rain reacts with marble ($CaCO_3$) of Taj Mahal causing discolouration.

$H_2SO_4 + CaCO_3 \rightarrow CaSO_4 + H_2O + CO_2$

11. Which of the following polymer can regain its shape?

(1) Nylon-6,6

(2) Buna-S

(3) Terylene

(4) Bakelite

Answer (2)

Sol. Elastomers are the polymers, which have rubber like properties and elastic in nature

Eg. Buna-S, Neoprene, Buna-N

12. $\left(P+\frac{a}{V^2}\right)(V-b)=RT, \frac{a}{b}$ will be dimensionally equal to

(1) $\frac{P}{V}$

(2) PV

(3) PV^3

(4) $\frac{P}{V^2}$

Answer (2)

Sol. The van der Waal's equation for 1 mol is

$\left(P+\frac{a}{V^2}\right)(V-b)=RT$

Units of 'a' are atm L^2 mol^{-2}

Units of 'b' are L mol^{-1}

Units of $\frac{a}{b}$ are atm L mol^{-1}

For 1 mol units of $\frac{a}{b}$ are atm L

Also the units of PV are atm L

$\therefore$ Dimensionally $\frac{a}{b}$ is equal to PV

13. Solubility of AgCl is maximum in

(1) 0.01 M HCl

(2) 0.01 M KCl

(3) Deionised water

(4) 0.01 M NH_3

Answer (4)

Sol. Since,

$AgCl(s) \rightleftharpoons Ag^+(aq) + Cl^-(aq)$

Species which promotes the reaction in forward direction, will increase solubility of AgCl.

Since, NH_3 forms complex with Ag^+. Hence, solubility of AgCl will be maximum in 0.01 M NH_3

$Ag^+(aq) + 2NH_3(aq) \rightleftharpoons [Ag(NH_3)_2]^+$

14. **Statement 1:** Photoelectric emission happens when incident energy is less than work function.

Statement 2: Kinetic energy becomes zero when incident energy becomes equal to work function.

(1) Statement 1 and 2 both are correct

(2) Statement 1 is correct but statement 2 is incorrect

(3) Statement 1 is incorrect but statement 2 is correct

(4) Statement 1 and 2 both are incorrect

Answer (3)

Sol. The emission of photoelectrons from the surface of a metal happens when the energy of incident light is higher than the work function of the metal.

$\therefore$ Statement 1 is incorrect.

Kinetic energy of photoelectrons becomes zero when the energy of incident light is just equal to the work function of the metal.

$\therefore$ Statement 2 is correct

15. Match the elements given in Column-I with their uses given in Column-II.

Column-I (Element)		Column-II (Uses)
(i) Cs	(a)	High temperature thermometer
(ii) Ga	(b)	Water proofing
(iii) B	(c)	Photoelectric cell
(iv) Si	(d)	Bullet-proof vest

(1) (i)-c, (ii)-a, (iii)-d, (iv)-b

(2) (i)-d, (ii)-c, (iii)-a, (iv)-b

(3) (i)-b, (ii)-c, (iii)-d, (iv)-a

(4) (i)-d, (ii)-a, (iii)-b, (iv)-c

Answer (1)

Sol. Caesium is used in photoelectric cell.

Gallium has high boiling point 2676 K, it is used in high temperature thermometer.

Boron fibers are used in making bullet-proof vest and light composite material for aircraft.

Silicon as silicones are used as sealant, greases, electrical insulators and for water proofing of fabrics.

16. Manganate ion disproportionates in neutral medium. Find spin only magnetic moment of the species formed by oxidation of manganate ion (in BM).

(1) 0 (2) 1.73

(3) 2.89 (4) 3.8

Answer (1)

Sol. $MnO_4^{2-} \rightarrow \underset{\substack{\text{Oxidised form}\\ d^0(Mn^{+7})}}{MnO_4^-} + \underset{\text{Reduced form}}{MnO_2}$

Since there is no unpaired electron in Mn^{+7} ion, therefore the magnetic moment of MnO_4^- is zero.

17. On reaction of white phosphorous with alkali in inert atmosphere, salt of which of the following acid is obtained?

(1) Phosphoric acid

(2) Phosphonic acid

(3) Hypophosphoric acid

(4) Orthophosphoric acid

Answer (2)

Sol. $P_4 + 3NaOH + 3H_2O \longrightarrow PH_3 + \underset{\text{(Sodium hypophosphite)}}{3NaH_2PO_2}$

Hypophosphorous acid (H_3PO_2) is also called as phosphinic acid

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Consider the following reaction

$$C_6H_{12}O_6 \xrightarrow{\text{Zymase}} A \xrightarrow{\text{NaOI}} B + CHI_3$$

The number of carbon atoms in B is

Answer (1)

Sol. $\underset{\text{Glucose}}{C_6H_{12}O_6} \xrightarrow{\text{Zymase}} \underset{(A)}{2C_2H_5OH(aq)} + 2CO_2(g)$

$\downarrow$ NaOI

$HCOO^-Na^+ + CHI_3$

22. Consider the following reaction

$Fe_2O_3 + 3CO \longrightarrow 2Fe + 3CO_2$

If 4640 g of Fe_2O_3 is allowed to react with 90 moles of CO. What is the weight (in g) of Fe produced?

Answer (3248)

Sol. $Fe_2O_3 + 3CO \longrightarrow 2Fe + 3CO_2$

(Moles) $Fe_2O_3 = \frac{4640}{160}$ = 29 moles

(Moles) CO = 90 moles

1 mole $Fe_2O_3 \longrightarrow$ 3 moles CO

29 moles $Fe_2O_3 \longrightarrow 3 \times 29$ moles CO

Hence Fe_2O_3 is the limiting reagent

Moles of Fe produced = 29 × 2 = 58

Weight of Fe produced = 58 × 56 = 3248 g

23. Given activation energy (E_a) for a chemical reaction is 23566 J. Initial temperature is 310 K and final temperature is 300 K. If the rate constant of the reaction in initial and final condition is K_1 & K_2 respectively and the value of $\frac{K_2}{K_1}$ is x × 10^{-3}. Find the value of x

[Given log(0.75) = –0.13]

Answer (750)

Sol. $\ln\frac{K_2}{K_1} = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

$$\log\frac{K_2}{K_1} = \frac{23566}{8.314\times2.303}\left[\frac{1}{310} - \frac{1}{300}\right]$$

$$= \frac{-23566}{8.314\times2.303}\left[\frac{10}{300\times310}\right]$$

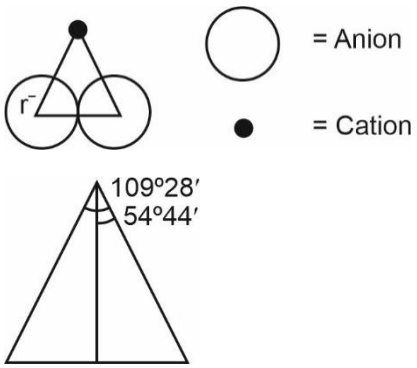
$$\simeq -0.13$$

$$\frac{K_2}{K_1} = 7.5\times10^{-1} = 750\times10^{-3}$$

24. The minimum value of radius ratio for tetrahedral void is_____

Answer (0.225)

Sol.



$$\sin(54^\circ 44') = \frac{r^-}{r^+ + r^-}$$

$$\Rightarrow \frac{r^-}{r^+ + r^-} = 0.8165$$

$$\Rightarrow \frac{r^+}{r^-} = 0.225$$

25. Find the molar mass of an ideal gas at 100 mm Hg pressure at 235°C temperature having a density of 0.46 gm/L. Use R = 0.0821 L atm mol^{-1} K^{-1} (Round off to the nearest integer)

Answer (146)

Sol. An ideal gas equation can also be written as

$$PM = dRT$$

$$\Rightarrow \quad M = \frac{dRT}{P}$$

d = 0.46 gm/L; $P = \frac{100}{760}$ atm; T = 235°C

= 508 K

$$M = \frac{0.46\times0.0821\times508\times760}{100}$$

$$= 145.81 \simeq 146 \text{ gm mol}^{-1}$$

26.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

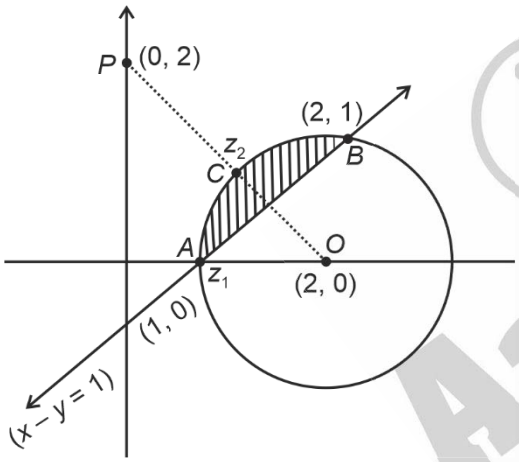
1. $\left(|z-2| \le 1,\ z(1+i)+\bar{z}(1-i) \le 2\right)$ and $|z-2i|$ attains maximum value & minimum value at z_1 & z_2 respectively. Find $|z_1|^2$ & $|z_2|^2$

(1) $7-\sqrt{2}$ (2) $10-\sqrt{2}$

(3) $14-4\sqrt{2}$ (4) $14+\sqrt{2}$

Answer (3)

Sol.



$z(1+i)+\bar{z}(1-i) \le 2$

$\Rightarrow \quad x-y \le 1 \qquad \ldots(i)$

and $|z-2| \le 1 \Rightarrow (x-2)^2+y^2 \le 1$

Maximum value = $|z_1|$

$= PA$

$= \sqrt{5}$

and minimum value = $|z_2|$

$= PO - OC$

$= 2\sqrt{2}-1 = PC$

$\therefore \quad |z_1|^2+|z_2|^2 = 5+\left(2\sqrt{2}-1\right)^2$

$= 14-4\sqrt{2}$

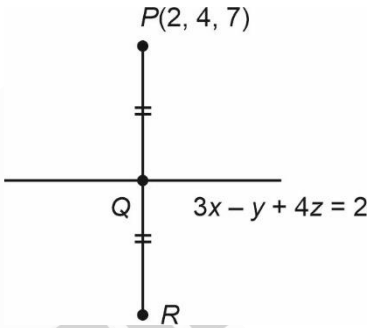
2. Find the mirror image of (2, 4, 7) in the plane $3x - y + 4z = 2$.

(1) $\left(-5, \frac{19}{3}, -\frac{7}{3}\right)$ (2) $\left(-\frac{58}{13}, \frac{80}{13}, -\frac{21}{13}\right)$

(3) $\left(-5, -\frac{19}{3}, \frac{7}{2}\right)$ (4) $\left(-5, \frac{19}{3}, \frac{7}{2}\right)$

Answer (2)

Sol. DR's of PQ are <3, –1, 4>



So, let Q as $(3\lambda + 2, -\lambda + 4, 4\lambda + 7)$

$\because$ Q lies on given plane, so

$9\lambda + 6 + \lambda - 4 + 16\lambda + 28 = 2$

$\Rightarrow \ 26\lambda = -28 \Rightarrow \lambda = -\frac{14}{13}$

So, Q is $\left(-\frac{16}{13}, \frac{66}{13}, \frac{35}{13}\right)$

Hence image of P will be

$R\left(-\frac{58}{13}, \frac{80}{13}, -\frac{21}{13}\right)$.

3. A wire of length 22 cm is given if it is cut into 2 pieces and make a square and an equilateral triangle. Then find side of triangle such that combined area of square and triangle is minimum.

(1) $\frac{22\sqrt{3}}{3+3\sqrt{3}}$ (2) $\frac{22\sqrt{3}}{4+3\sqrt{3}}$

(3) $\frac{22}{4+3\sqrt{3}}$ (4) $\frac{22\sqrt{3}}{3+4\sqrt{3}}$

Answer (2)

Sol. Let l length wire is used to form equilateral triangle.

So, total area $\Delta = \frac{\sqrt{3}}{4}\left(\frac{l}{3}\right)^2 + \left(\frac{22-l}{4}\right)^2$

$\Rightarrow \Delta = \frac{1}{12\sqrt{3}} \cdot l^2 + \frac{1}{16}(22-l)^2$

$\Rightarrow \frac{d\Delta}{dl} = \frac{1}{6\sqrt{3}} l - \frac{1}{8}(22-l) = 0$

$\Rightarrow l = \frac{66\sqrt{3}}{4+3\sqrt{3}}$

Also, $\frac{d^2\Delta}{dl^2} = \frac{1}{6\sqrt{3}} + \frac{1}{8}$ (+ve, which is case of minima)

Now, side length of triangle $= \frac{l}{3} = \frac{22\sqrt{3}}{4+3\sqrt{3}}$

4. If $|\vec{A}| = |\vec{B}| = 2$

$|\vec{A}+\vec{B}| = 2|\vec{A}-\vec{B}|$. Find the angle between $\vec{A}$ and $\vec{B}$

(1) $\sin^{-1}\left(\frac{3}{5}\right)$ (2) $\sin^{-1}\left(\frac{4}{5}\right)$

(3) $\tan^{-1}\left(\frac{1}{2}\right)$ (4) $\frac{\pi}{3}$

Answer (2)

Sol. $\because \quad |\vec{A}+\vec{B}|^2 = 4|\vec{A}-\vec{B}|^2$

$\Rightarrow 4+4+2\vec{A}\cdot\vec{B} = 4\left(4+4-2\vec{A}\cdot\vec{B}\right)$

$\Rightarrow 10\vec{A}\cdot\vec{B} = 24$

$\Rightarrow 10 \cdot 2 \cdot 2 \cdot \cos\theta = 24$

$\Rightarrow \cos\theta = \frac{3}{5}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{4}{5}\right)$

5. Domain of $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$

(1) $(-\infty, \frac{-1}{\sqrt{2}}] \cup [\frac{1}{\sqrt{2}}, \infty) \cup \{0\}$

(2) $\left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right)$

(3) $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$

(4) $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

Answer (1)

Sol. $f(x) = \cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$

For domain: $-1 \le \frac{1}{4x^2-1} \le 1$

$\Rightarrow 4x^2 - 1 \ge 1$ or $4x^2 - 1 \le -1$ and $4x^2 - 1 \ne 0$

$\therefore \quad 4x^2 \ge 2$ or $4x^2 \le 0$ and $4x^2 - 1 \ne 0$

$\therefore \quad x \in \left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$ ...(i)

For given x, $\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi} \in (0, 1) \cup \{-1\}$

Hence domain $= \left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

6. $A = [a_{ij}]_{3\times3}$ where $a_{ij} = 2^{j-i}$, where $i, j \in \{1, 2, 3\}$

then find $A^2 + A^3 + A^4 + \ldots + A^{10}$

(1) $A\left(\frac{3^{10}-1}{2}\right)$ (2) $A\left(\frac{3^{10}+1}{2}\right)$

(3) $A\left(\frac{3^{10}+3}{2}\right)$ (4) $A\left(\frac{3^{10}-3}{2}\right)$

Answer (4)

Sol. Clearly $A = \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$

So, $A^2 = 3\begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$

$\Rightarrow A^2 = 3A$, $A^3 = 3A^2 = 3^2A$ and so on

Here, $A^2 + A^3 + \ldots + A^{10} = \left(3^1 + 3^2 + \ldots 3^9\right)A$

$= \left(\frac{3^{10}-3}{2}\right)A$

7. $a_0 = a_1 = 0,\ a_{n+2} = 2a_{n+1} - a_n + 1,\ n \geq 0,$ then $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to

(1) $\frac{21}{16}$ (2) $\frac{7}{216}$

(3) $\frac{49}{216}$ (4) $\frac{16}{49}$

Answer (2)

Sol. $\because \quad \frac{a_{n+2}}{7^{n+2}} = \frac{2}{7}\cdot\frac{a_{n+1}}{7^{n+1}} - \frac{1}{49}\cdot\frac{a_n}{7^n} + \frac{1}{7^{n+2}}$

So, $\sum_{n=2}^{\infty} \frac{a_{n+2}}{7^{n+2}} = \frac{2}{7}\sum_{n=2}^{\infty} \frac{a_{n+1}}{7^{n+1}} - \frac{1}{49}\sum_{n=2}^{\infty} \frac{a_n}{7^n} + \sum_{n=2}^{\infty} \frac{1}{7^{n+2}}$

$\left(\text{Let } \sum_{n=2}^{\infty} \frac{a_n}{7^n} = S\right)$

$\Rightarrow \quad \left(S - \frac{a_3}{7^3} - \frac{a_2}{7^2}\right) = \frac{2}{7}\left(S - \frac{a_2}{7^2}\right) - \frac{1}{49}(S) + \frac{\frac{1}{7^4}}{\frac{6}{7}}$

$(\because\ a_2 = 1 \text{ and } a_3 = 3)$

$\Rightarrow \quad S\left(1 - \frac{2}{7} + \frac{1}{49}\right) = \frac{3}{7^3} + \frac{1}{7^2} - \frac{2}{7^3} + \frac{1}{6.7^3}$

$\Rightarrow \quad \frac{36S}{49} = \frac{1}{7^3}\left[8 + \frac{1}{6}\right] = \frac{1}{42}$

$\Rightarrow \quad S = \frac{7}{216}$

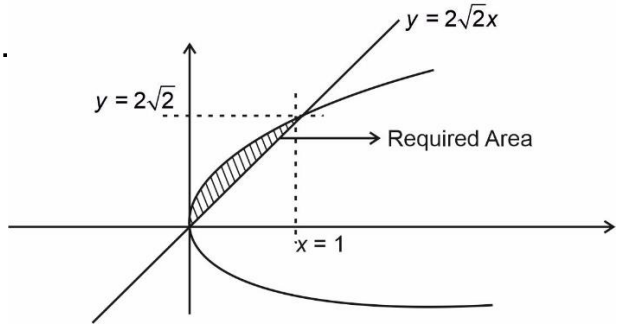
8. Find the area bounded by the curves $y^2 = 8x$ and $y = 2\sqrt{2}x$ outside the triangle formed by $y = 2\sqrt{2}x,\ y = 0,\ x = 1$.

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{2}}{3}$

(3) $\frac{2}{\sqrt{3}}$ (4) $\frac{2}{3}$

Answer (2)

Sol.



$\therefore \quad A = \int_0^1 \left(\sqrt{8x} - 2\sqrt{2}x\right)dx$

$= \frac{2\sqrt{2}x^{3/2}}{\frac{3}{2}} - 2\sqrt{2}\frac{x^2}{2}\Big|_0^1$

$= \frac{4}{3}\sqrt{2} - \sqrt{2} = \frac{\sqrt{2}}{3}$ sq. unit

9. If $(p \vee q)\ \Delta\ (p \wedge q \Rightarrow q)$ is tautology then what is Δ in the above statement?

(1) $\vee$ (2) $\wedge$

(3) $\Leftrightarrow$ (4) None of these

Answer (1)

Sol. $\because \quad (p \wedge q) \Rightarrow q \equiv \sim(p \wedge q) \vee q \equiv \sim p \vee \sim q \vee q$

Is always true

So Δ can be $\vee$ or $\Rightarrow$.

10. The probability of making a singular matrix of 2 × 2 taking elements from fist 10 prime numbers

(1) $\frac{1}{1000}$ (2) $\frac{19}{1000}$

(3) $\frac{41}{1000}$ (4) $\frac{9}{500}$

Answer (2)

Sol. If matrix is singular then

Case I : All entries are same

No. of matrices = 10

Case II : When only two prime numbers are used in matrix,

No. of such matrices = $^{10}C_2 . \lfloor 2 . \lfloor 2 = 180$

Required probability = $\frac{190}{10^4} = \frac{19}{1000}$

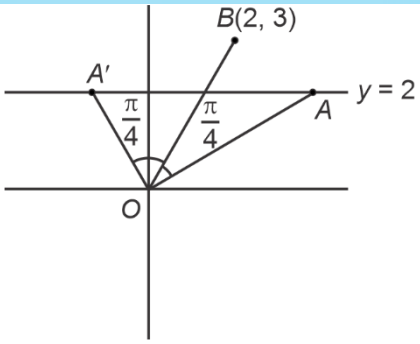
11. A line $y = 2$ contains two points A and A' such that AB and $A'B$ subtends $\frac{\pi}{4}$ at origin, then find the distance between AA' where $B(2, 3)$.

(1) $\frac{52}{5}$ (2) $\frac{48}{5}$

(3) $\frac{54}{5}$ (4) 10

Answer (1)

Sol.



If slope of OA is m, then

$$\left|\frac{m-\frac{3}{2}}{1+\frac{3}{3}m}\right| = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad m = -5 \text{ or } \frac{1}{5}$$

Therefore, A will be (10, 2) and A' will be $\left(-\frac{2}{5}, 2\right)$

So, $AA' = \frac{52}{5}$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. $x\frac{dy}{dx} - y = \sqrt{y^2+16x^2}, y(1) = 3,$ find $y(2)$.

Answer (15)

Sol. $\because \quad \frac{dy}{dx} = \frac{y+\sqrt{y^2+16x^2}}{x}$

let $y = xt, \frac{dy}{dx} = x\frac{dt}{dx} + t$

$$\Rightarrow \quad x\frac{dt}{dx} + t = t + \sqrt{t^2+16}$$

$$\Rightarrow \quad \frac{dt}{\sqrt{t^2+16}} = \frac{dx}{x}$$

$$\Rightarrow \quad \ln\left|t+\sqrt{t^2+16}\right| = \ln x + \ln c$$

$$\Rightarrow \quad \frac{y}{x} + \frac{\sqrt{y^2+16x^2}}{x} = cx$$

$\because \quad y(1) = 3 \qquad$ So $c = 8$

$$\Rightarrow \quad y + \sqrt{y^2+16x^2} = 8x^2$$

for $x = 2, \ y + \sqrt{y^2+64} = 32$

$\Rightarrow \quad y^2 + 64 = (32 - y)^2$

$\Rightarrow \quad y = 15$

22. $\int_0^5 \cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right) dx$

Answer (0)

Sol. $\int_0^5 \cos\pi\left(x - \left[\frac{x}{2}\right]\right) dx$

$$= \int_0^2 \cos(\pi x)dx + \int_2^4 \cos(\pi(x-1))\,dx + \int_4^5 \cos(\pi(x-2))\,dx$$

$$= \frac{\sin\pi x}{\pi}\Big|_0^2 + \frac{\sin(\pi(x-1))}{\pi}\Big|_2^4 + \frac{\sin(\pi(x-2))}{\pi}\Big|_4^5$$

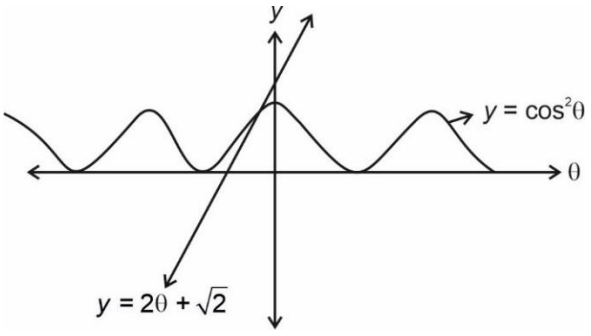
$= 0 + 0 + 0 = 0$

23. Find the number of solutions of the equation $2\theta - \cos^2\theta + \sqrt{2} = 0$.

Answer (01)

Sol. $\because \quad 2\theta - \cos^2\theta + \sqrt{2} = 0$

$\because \quad 2\theta + \sqrt{2} = \cos^2\theta$



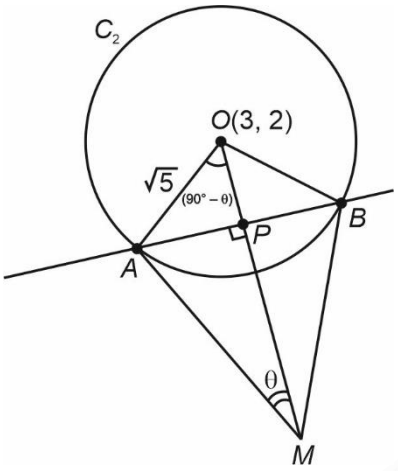
Only one point of intersection hence only one solution.

24. If a circle C_1 is $x^2 + y^2 = 2$ and other circle C_2 is $(x-3)^2 + (y-2)^2 = 5$. Tangent at $(1, -1)$ on C_1 intersect at two points on C_2 at A and B and tangent to C_2 at A and B meet at M. Then area of triangle ABM is

Answer (13.50**)**

Sol. Eqn. of AB : $x - y = 2$

Refer to diagram,



$$OP = \frac{1}{\sqrt{2}} \Rightarrow AP = \frac{3}{\sqrt{2}}$$

$\because$ $\tan(90° - \theta) = 3 = \cot\theta$

So $\sin\theta = \frac{AP}{AM} \Rightarrow AM = 3\sqrt{5}$

Now area of $\Delta ABM = \frac{1}{2} AM^2 \cdot \sin 2\theta$

$$= \left(3\sqrt{5}\right)^2 \cdot \sin\theta \cdot \cos\theta$$

$$= 45\left(\frac{3}{10}\right) = \frac{27}{2}$$

25. In the equation $x^2 + 2i - 1 = 0$. If α and β are the roots then find the value of $|\alpha^8 + \beta^8|$.

Answer (50)

Sol. Clearly $\alpha^2 = \beta^2 = 1 - 2i$

So $\alpha^8 = \beta^8 = (1 - 2i)^4 = (1 + 4i^2 - 4i)^2$

$= (-3 - 4i)^2$

$= 9 + 16i^2 + 24i$

$= -7 + 24i$

Hence $\alpha^8 + \beta^8 = -14 + 48i$

Now $\left|\alpha^8 + \beta^8\right| = \sqrt{14^2 + 48^2}$

$= 2.25$

$= 50$

26.

27.

28.

29.

30.

❑ ❑ ❑