Time: 3 hrs.  M.M.: 300

IMPORTANT INSTRUCTIONS:

1. The test is of **3 hours** duration.

2. The Test Booklet consists of 90 questions. The maximum marks are 300.

3. There are three parts in the question paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

   (i) **Section-A**: This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

   (ii) **Section-B**: This section contains 10 questions. In Section-B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Following circuit contains diodes with forward bias having resistance 25 Ω and reverse bias having infinite resistance. The ratio of \( \frac{I_2}{I_1} \) is equal to

   \( (1) \ 1 \quad (2) \ 2 \quad (3) \ 2 \quad (4) \ 4 \)

   Answer (2)

   Sol. \( I_2 = I_4 \)

   \[ I_3 = 0 \]

   and \( I_2 + I_4 + I_3 = I_1 \)

   \[ \Rightarrow I_2 = \frac{I_1}{2} \]

2. An object moves \( x \) distance with speed \( v_1 \) and next \( x \) distance with speed \( v_2 \). The average velocity \( v \) is related to \( v_1 \) and \( v_2 \) as

   \( (1) \ v = \left( \frac{v_1 + v_2}{2} \right) \quad (2) \ \frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2} \)

   \( (3) \ v = \left( \frac{2v_1v_2}{v_1 + v_2} \right) \quad (4) \ v = \left( \frac{v_1 - v_2}{2} \right) \)

   Answer (3)

   Sol. \( v = \frac{2x}{x + x} = \frac{2v_1v_2}{v_1 + v_2} \)

3. An infinitely-long conductor has a current 14 A flowing as shown:

   Find magnetic field at centre C.

   \( (1) \ 88 \ \mu T \quad (2) \ 44 \ \mu T \quad (3) \ 10 \ \mu T \quad (4) \ 120 \ \mu T \)

   Answer (2)

   Sol. \( B = \frac{\mu_0 i}{4R} \)

   \[ = \frac{4\pi \times 10^{-7} \times 14}{4 \times 0.1} \text{T} \]

   \[ = \frac{22}{7} \times 10^{-7} \times 140 \text{T} \]

   \[ = 44 \ \mu T \]

4. A point object (O) is placed on the principal axis of a system of two lenses as shown. Find the distance between the final image and object.

   \( (1) \ 45 \text{ cm} \quad (2) \ 40 \text{ cm} \quad (3) \ 55 \text{ cm} \quad (4) \ 50 \text{ cm} \)

   Answer (1)

   Sol. From lens (1)

   \[ \frac{1}{v} - \frac{1}{(-6)} = \frac{1}{2} \Rightarrow v = 3 \text{ cm} \]

   From lens (2)

   \[ \frac{1}{v'} - \frac{1}{(-18)} = \frac{1}{9} \Rightarrow v' = +18 \text{ cm} \]

   Distance between O and I = 6 + 21 + 18 = 45 cm

5. Based on given graph between stopping potential and frequency of irradiation, work function of metal is equal to

   \( (1) \ 1 \text{ eV} \quad (2) \ 3 \text{ eV} \quad (3) \ 2 \text{ eV} \quad (4) \ 4 \text{ eV} \)

   Answer (3)

   Sol. \( eV_s = \hbar v - \phi \)

   On extrapolating the graph, the graph cuts the \( y \)-axis at \(-2\)

   \[ \Rightarrow \text{at} \ v = 0, \ V_s = -2 \text{ V} \]

   \[ \Rightarrow \phi = 2 \text{ eV} \]
6. **Assertion (A):** Fan spins even after switch is in OFF.
   **Reason (R):** Fan in rotation has rotational inertia.
   (1) A is correct and R is correct explanation of A
   (2) A and R both are correct but R is not correct explanation of A.
   (3) A is correct and R is incorrect
   (4) Both A and R are incorrect

**Answer (1)**

7. **Assertion (A):** Fan spins even after switch is in OFF.
   **Reason (R):** Fan in rotation has rotational inertia.
   (1) A is correct and R is correct explanation of A
   (2) A and R both are correct but R is not correct explanation of A.
   (3) A is correct and R is incorrect
   (4) Both A and R are incorrect

**Answer (1)**

8. **Assertion (A):** Fan spins even after switch is in OFF.
   **Reason (R):** Fan in rotation has rotational inertia.
   (1) A is correct and R is correct explanation of A
   (2) A and R both are correct but R is not correct explanation of A.
   (3) A is correct and R is incorrect
   (4) Both A and R are incorrect

**Answer (1)**

9. **Assertion (A):** Fan spins even after switch is in OFF.
   **Reason (R):** Fan in rotation has rotational inertia.
   (1) A is correct and R is correct explanation of A
   (2) A and R both are correct but R is not correct explanation of A.
   (3) A is correct and R is incorrect
   (4) Both A and R are incorrect

**Answer (1)**

10. **Assertion (A):** Fan spins even after switch is in OFF.
    **Reason (R):** Fan in rotation has rotational inertia.
    (1) A is correct and R is correct explanation of A
    (2) A and R both are correct but R is not correct explanation of A.
    (3) A is correct and R is incorrect
    (4) Both A and R are incorrect

**Answer (1)**

11. **Assertion (A):** Fan spins even after switch is in OFF.
    **Reason (R):** Fan in rotation has rotational inertia.
    (1) A is correct and R is correct explanation of A
    (2) A and R both are correct but R is not correct explanation of A.
    (3) A is correct and R is incorrect
    (4) Both A and R are incorrect

**Answer (1)**

12. **Assertion (A):** Fan spins even after switch is in OFF.
    **Reason (R):** Fan in rotation has rotational inertia.
    (1) A is correct and R is correct explanation of A
    (2) A and R both are correct but R is not correct explanation of A.
    (3) A is correct and R is incorrect
    (4) Both A and R are incorrect

**Answer (1)**

13. **Assertion (A):** Fan spins even after switch is in OFF.
    **Reason (R):** Fan in rotation has rotational inertia.
    (1) A is correct and R is correct explanation of A
    (2) A and R both are correct but R is not correct explanation of A.
    (3) A is correct and R is incorrect
    (4) Both A and R are incorrect

**Answer (1)**
14. Two projectiles $A$ and $B$ are projected from the same point on ground with same speed of projection as shown. Find the ratio of maximum height attained by $A$ to that of $B$.

\[
\begin{align*}
\text{(1)} & \quad 3 : 1 \\
\text{(2)} & \quad 1 : 3 \\
\text{(3)} & \quad \sqrt{3} : 1 \\
\text{(4)} & \quad \sqrt{3} : 2
\end{align*}
\]

Answer (1)

\[
\begin{align*}
\text{Sol.} & \quad \frac{h_A}{h_B} = \frac{\frac{u^2 \sin^2 60^\circ}{2g}}{\frac{u^2 \sin^2 30^\circ}{2g}} = \frac{3}{1}
\end{align*}
\]

15. Force acting on a particle at origin is $\vec{F} = -c\hat{k}$. The torque of this force about ($-2$, $3$, $0$) is $-c(A\hat{i} + B\hat{j})$.

If $\frac{A}{B} = \frac{x}{2}$ find $x$.

\[
\begin{align*}
\text{(1)} & \quad \frac{2}{3} \\
\text{(2)} & \quad 3 \\
\text{(3)} & \quad 1.5 \\
\text{(4)} & \quad 1
\end{align*}
\]

Answer (3)

\[
\begin{align*}
\text{Sol.} & \quad \vec{r} = \vec{r} \times \vec{F} \\
& \quad = (2\hat{i} + 3\hat{j}) + (-c\hat{k}) \\
& \quad = 2c\hat{i} + 3c\hat{i} \\
& \quad = c(2\hat{i} + 3\hat{j}) \\
& \quad \Rightarrow \quad \frac{A}{B} = 1.5
\end{align*}
\]

16. If time period for one revolution by satellite near the earth’s surface is $T$. Then the time period of revolution of satellite at a height equal to radius of earth will be

\[
\begin{align*}
\text{(1)} & \quad \sqrt{3}T \\
\text{(2)} & \quad \sqrt{2}T \\
\text{(3)} & \quad \sqrt{4}T \\
\text{(4)} & \quad \sqrt{3}T
\end{align*}
\]

Answer (1)

\[
\begin{align*}
\text{Sol.} & \quad T^2 \propto r^3 \\
& \quad \left(\frac{T'}{T}\right)^2 = \left(\frac{R}{2R}\right)^3 \\
& \quad T' = \sqrt{3}T
\end{align*}
\]

17. In a mixture 0.5 moles of $O_2$ and 4 moles of Ne gas are taken at temperature $T$. The internal energy of the system is equal to

\[
\begin{align*}
\text{(1)} & \quad \frac{13}{2}RT \\
\text{(2)} & \quad \frac{11}{4}RT \\
\text{(3)} & \quad \frac{29}{4}RT \\
\text{(4)} & \quad \frac{13}{4}RT
\end{align*}
\]

Answer (3)

\[
\begin{align*}
\text{Sol.} & \quad U = U_{O_2} + U_{Ne} \\
& \quad = \frac{5}{2}n_{O_2}RT + \frac{3}{2}n_{Ne}RT \\
& \quad = \left(\frac{5}{4} + \frac{24}{4}\right)RT
\end{align*}
\]

18. Assertion (A): Acceleration due to gravity is minimum at equator.

Reason (R): Rotation of earth influences acceleration.

\[
\begin{align*}
\text{(1)} & \quad A \text{ is correct, R is correct explanation of A} \\
\text{(2)} & \quad A \text{ is correct, R is incorrect explanation of A} \\
\text{(3)} & \quad A \text{ is correct and R is incorrect} \\
\text{(4)} & \quad \text{Both A and R are incorrect}
\end{align*}
\]

Answer (1)

\[
\begin{align*}
\text{Sol.} & \quad \text{Acceleration due to gravity depends on the rotation of earth and distance from the centre.}
\end{align*}
\]
19. An infinite line charge of charge density \( \lambda \) has a charged particle \((-q, m)\) revolving about it in a circle of radius \( r \). Find the orbital speed.

\[
\begin{align*}
1. & \quad \sqrt{\frac{\lambda q}{4\pi\varepsilon_0 m}} \\
2. & \quad \sqrt{\frac{\lambda q}{2\pi\varepsilon_0 m}} \\
3. & \quad \sqrt{\frac{4\lambda a}{\pi\varepsilon_0 m}} \\
4. & \quad \sqrt{\frac{\lambda a}{4\pi\varepsilon_0 m}}
\end{align*}
\]

Answer (4)

Sol. 
\[
E = \frac{\lambda}{2\pi\varepsilon_0 r}
\]

\[
\Rightarrow \frac{\lambda q}{2\pi\varepsilon_0 r} = \frac{mv^2}{r}
\]

\[
\Rightarrow v = \sqrt{\frac{\lambda q}{2\pi\varepsilon_0 m}}
\]

20. A gas is compressed adiabatically. Which of the following statements is not correct?

1. Internal energy is constant
2. Temperature increases
3. \(|\text{Work done}| = |\text{Change in internal energy}|\)
4. Heat is not supplied to the system

Answer (1)

Sol. \( \Delta Q = \Delta U + W \)

For adiabatic, \( \Delta Q = 0 \) \( \ldots(1) \)

Also, since compression \( \Rightarrow T \)

\( \Rightarrow U \) increases.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Frictional force acts on the lift of mass 1400 kg is 2000 N. If lift moves with constant velocity of 3 m/s in upward direction, the power (in kW) of motor is (take \( g = 10 \text{ m/s}^2 \))

Answer (48)

Sol.

\[
P = F \cdot V
\]

\[
= 16000 \times 3
\]

\[
= 48000 \text{ watt} = 48 \text{ kW}
\]

22. If the angular frequency of the given motion is \( k\omega \), find the value of \( k \)

\[
y = \sin(\omega t) + \cos \omega t
\]

Answer (1)

Sol. 
\[
y = \sqrt{2} \left[ \sin(\omega t) \cos \left(\frac{\pi}{4}\right) + \cos(\omega t) \sin \left(\frac{\pi}{4}\right) \right]
\]

\[
= \sqrt{2} \sin \left(\omega t + \frac{\pi}{4}\right)
\]

Angular frequency = \( \omega \)

23.
24.
25.
26.
27.
28.
29.
30.
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

1. Delicate balance of CO\(_2\) and O\(_2\) is not disturbed by
   (1) Deforestation  (2) Photosynthesis  
   (3) Burning of coal  (4) Burning of petroleum
   Choose the correct answer:
   Answer (2)

2. The increasing order of metallic character
   (1) Be > Ca > K  (2) K > Ca > Be  
   (3) Ca > K > Be  (4) K > Be > Ca
   Answer (2)

3. Relation between radius of a lattice (r) and edge length (a) of an FCC unit cell is __________.
   (1) \( r = \frac{a}{2} \)  
   (2) \( r = \frac{\sqrt{2}a}{4} \)  
   (3) \( r = \frac{\sqrt{2}a}{2} \)  
   (4) \( r = \frac{\sqrt{3}a}{4} \)
   Answer (3) 

4. Sol. Deforestation & burning of fossil fuels increase CO\(_2\) level and disturb the balance in the atmosphere.

5. Which of the following options correctly represent the structure of Buna-\(S\) ?
   (1) \( \text{CH}_2 - 
   (2) \( \text{CH}_2 - \text{CH} = \text{CH} - \text{CH}_2 \) \(_n\)  
   (3) \( \text{H}_2 \text{C} - \text{CH} = \text{CH} - \text{CH}_2 \) \(_n\)  
   (4) \( \text{CH}_2 - \text{CH} = \text{CH} - \text{CH}_2 \) \(_n\)
   Answer (1) 

6. Buna-\(S\) is formed by polymerisation of 1, 3 – butadiene & styrene
   \( \text{CH}_2 \) \(_2\) + \( \text{CH}_2 = \text{CH} - \text{CH} = \text{CH}_2 \) \(_n\)

7. Consider, a mixture of 2 moles of oxygen, 4 moles of Neon gas.
   Neglect any vibrational degree of freedom.
   Calculate the total internal energy of system
   (Assuming \( E = 0 \) at \( T = 0 \) K)
   (1) 5RT  (2) 11RT  
   (3) 6RT  (4) 7RT
   Answer (2)
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Sol. \[ E = (2) \left( \frac{5R}{2} \right) (T) + (4) \left( \frac{3R}{2} \right) (T) \]
\[ = 11 RT \]

8. Which of the following is the correct hydride affinity order of carbocations?

(a) \[ \text{CH}_3 \quad \text{CH} - \quad \text{C} - \quad \text{CH}_2 \]
(b) \[ \text{C}_6\text{H}_5 - \quad \text{C} - \quad \text{C}_6\text{H}_5 \]
(c) \[ \text{CH}_3 \quad \text{CH} - \quad \text{CH}_3 \]
(d) \[ \text{CH}_3 \quad \text{C} - \quad \text{CH}_3 \]

Answer (1)
Sol. The correct hydride affinity order of carbocations will be decided by the stability of carbocation. Higher the stability of carbocation, lower will be hydride affinity.

\[ \therefore \text{Correct hydride affinity order of carbocations is (c) < (b) < (d) < (a)} \]

9. Water of crystallization in Soda ash and washing soda is respectively.

(1) 0,10
(2) 10,0
(3) 0,0
(4) 0,1

Answer (1)
Sol. Soda ash is \( \text{Na}_2\text{CO}_3 \)
Washing soda is \( \text{Na}_2\text{CO}_3.10\text{H}_2\text{O} \).
Therefore correct answer is 0,10.

10. Order of acidic strength of

Answer (1)
Sol. Correct order is

11. What process is used to make soap from fat?

(1) Saponification
(2) Electrolysis
(3) Solvay process
(4) Haber process

Answer (1)
Sol.

\[ \text{CH}_2 - \quad \text{O} - \quad \text{C} - \quad \text{C}_17\text{H}_{35} \]
\[ \text{CH} - \quad \text{O} - \quad \text{C} - \quad \text{C}_17\text{H}_{35} + 3\text{NaOH} \]
\[ \text{CH}_2 - \quad \text{O} - \quad \text{C} - \quad \text{C}_17\text{H}_{35} \]
\[ 3\text{C}_17\text{H}_{35}\text{COONa} + \text{CH}_2 - \quad \text{OH} \]
\[ \text{CH}_2 - \quad \text{OH} \]
12. **Assertion:** Higher energy is required for the conversion of Mg to Mg\(^{2-}\) than that for Mg to Mg\(^{-}\).

**Reason:** Mg\(^{2-}\) has very small size and more charge.

(1) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion
(2) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion
(3) Assertion is correct but Reason is incorrect
(4) Assertion is incorrect but Reason is correct

**Answer (3)**

**Sol.** Since Mg\(^{2-}\) has higher charge density than Mg\(^{-}\), then interelectronic repulsion will be higher in case of Mg\(^{2-}\) as compared to Mg\(^{-}\).

Hence, higher energy is required for the conversion of Mg to Mg\(^{2-}\) than that of Mg to Mg\(^{-}\).  

13. An unknown organic compound is heated with fuming HNO\(_3\). The reaction mixture is treated with aq BaCl\(_2\) solution which gives white precipitate. Identify the unknown organic compound.

(1) Phenylalanine (2) Proline (3) Cysteine (4) Valine

**Answer (3)**

**Sol.** The unknown organic compound contains S-atom which gets oxidised by fuming HNO\(_3\) to SO\(_4^{2-}\) ions. Addition of aq BaCl\(_2\) gives white precipitate of BaSO\(_4\). Among the given compounds only cysteine has S-atom.

14. Following two columns are provided

<table>
<thead>
<tr>
<th>Column-I (Complex)</th>
<th>Column-II (CFSE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. [Ti(H(_2)O(_6))](^{2+})</td>
<td>(i) −1.2 (\Delta_0)</td>
</tr>
<tr>
<td>b. [V(H(_2)O(_6))](^{2+})</td>
<td>(ii) −0.6 (\Delta_0)</td>
</tr>
<tr>
<td>c. [Mn(H(_2)O(_6))](^{3+})</td>
<td>(iii) 0</td>
</tr>
<tr>
<td>d. [Fe(H(_2)O(_6))](^{3+})</td>
<td>(iv) −0.8 (\Delta_0)</td>
</tr>
</tbody>
</table>

(1) a(iv); b(i); c(ii); d(iii) (2) a(i); b(ii); c(iv); d(iii)
(3) a(iv); b(iii); c(i); d(ii) (4) a(i); b(ii); c(iii); d(iv)

**Answer (1)**

**Sol.** CFSE = \(-\frac{2}{5} \Delta_0(t_{2g} \text{ electrons}) + \frac{3}{5} \Delta_0 (e_9 \text{ electrons})\)

15. 16. 17. 18. 19. 20.

**SECTION - B**

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, −00.33, −00.30, 30.27, −27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. For a metal ion, \(\mu = 4.9\) B.M. Find out number of unpaired electrons

**Answer (04.00)**

**Sol.** \(\sqrt{(n)(n + 2)} = 4.92\)

\(n(n + 2) = 24\)

\(n = 4\)

22. Find out difference in oxidation state of Xe in completely Hydrolysed form of XeF\(_4\) and XeF\(_6\)

**Answer (00.00)**

**Sol.** XeF\(_6\) + 3H\(_2\)O \(\xrightarrow{\text{Complete Hydrolysis}}\) XeO\(_3\) + 6HF  
XeF\(_4\) + H\(_2\)O \(\xrightarrow{\text{Complete Hydrolysis}}\) XeO\(_3\) + Xe + O\(_2\) + HF
23. NH₃, NO, N₂, F₂, CO, CO₂, H₂O, and XeF₄
   Fill the number of above molecules having only two lone pair of electrons.
   **Answer (3)**
   **Sol.** These are N₂, CO and H₂O.

24. How many electrons are gained by MnO₄⁻ in strongly alkaline medium?
   **Answer (1)**
   **Sol.** MnO₄⁻ gains one electron to form MnO₄²⁻ in strongly alkaline medium.

25. Consider a reaction at equilibrium
   \[ A(g) \rightleftharpoons 2B(g) + C(g) \]
   If final pressure at equilibrium is 1 atm & \( k_p = \frac{1}{27} \),
   then % dissociation of A will be (consider \( 1 - \alpha \approx 1 \))
   (nearest integer)
   **Answer (21)**
   **Sol.**
   \[
   k_p = \frac{4P^2 \alpha^3}{P(1 - \alpha)} = \frac{4P^2 \alpha^3}{1 - \alpha} \\
   k_p = \frac{4P^2 \alpha^3}{(1 - \alpha)(1 + 2\alpha)^2} \\
   \frac{1}{27} = \frac{4P^2 \alpha^3}{1} = \frac{1}{108} \\
   \alpha^3 = \frac{1}{108} \\
   \alpha = \left( \frac{1}{108} \right)^{\frac{1}{3}} \times 100 \approx 4.762 \\
   \alpha \approx 21
   \]

26. 0.02 M CH₃COOH has specific conductance, \( K = 5 \times 10^{-5} \) S cm⁻¹. Also given limiting molar conductance of CH₃COOH is 400 S cm² mol⁻¹.
   Therefore, \( K_a \) for CH₃COOH is \( \_\_\_\_ \times 10^{-7} \) M
   **Answer (8)**
   **Sol.**
   \[
   \Lambda_m = \frac{K \times 1000}{M} = \frac{5 \times 10^{-5} \times 10^3}{2 \times 10^{-2}} = 2.5 \\
   \alpha = \frac{\Lambda_m}{\Lambda_m^*} = \frac{2.5}{400} \\
   \therefore K = \frac{C \alpha^2}{1 - \alpha} \\
   = \frac{0.02 \times \left( \frac{2.5}{400} \right)^2}{1 - \frac{2.5}{400}} \\
   = \frac{7.8125 \times 10^{-7}}{0.99375} = 7.861 \times 10^{-7} \\
   K_a = 8 \times 10^{-7} \text{ M}
   \]

27. For a first-order reaction, if the value of \( t_{1/2} \) is T,
   then the value of \( t_{7/8} \) will be \( \_\_\_\_ T \).
   **Answer (3)**
   **Sol.** \( t_{7/8} \) means 3 half lives.
   \[ \therefore t_{7/8} = 3T \]

28. Number of endothermic reactions among following
   (a) 2HCl(g) \( \rightarrow \) H₂(g) + Cl₂(g)
   (b) H₂O(l) \( \rightarrow \) H₂O(g)
   (c) C(s) + O₂(g) \( \rightarrow \) CO₂(g)
   (d) Dissolution of NH₄Cl
   (e) I₂(g) \( \rightarrow \) 2I(g)
   **Answer (04)**
   **Sol.** Burning of carbon is exothermic, all other are endothermic.

29. 30.
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Let a circle \( x^2 + y^2 = 16 \) and line passing through (1, 2) cuts the curve at A and B then the locus of the mid-point of AB is
   (1) \( x^2 + y^2 + x + y = 0 \) (2) \( x^2 + y^2 - x + 2y = 0 \)
   (3) \( x^2 + y^2 - x - 2y = 0 \) (4) \( x^2 + y^2 + x + 2y = 0 \)

Answer (3)

Sol.

Let \( P(x, y) \) be the mid-point of \( AB \)
Then \( T = S_i \)
\[ x_1^2 + y_1^2 - 16 = xx_1 + yy_1 - 16 \]
\[ \Rightarrow xx_1 + yy_1 = x_1^2 + y_1^2 \] ... (i)
\[ \therefore \text{ (i) passes through (1, 2)} \]
\[ \therefore x_1 + 2y_1 = x_1^2 + y_1^2 \]
\[ \therefore \text{ required locus } \]
\[ x^2 + y^2 - x - 2y = 0 \]

2. Consider \( f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right) \), domain of \( f(x) \) is
   \[ \{\alpha, \beta\} \cup [\gamma, \delta] \], then the value of \( |3\alpha + 10\beta + 5\gamma + 21\delta| \) is
   (1) 22 (2) 23
   (3) 21 (4) 19

Answer (3)

Sol.

\[ \frac{2x}{5x+3} \geq 1 \text{ OR } \frac{2x}{5x+3} \leq -1 \]
\[ \Rightarrow \frac{2x}{5x+3} - 1 \geq 0 \]
\[ \Rightarrow x = 0 \text{ or } y = -1 \]

\[ \Rightarrow -\frac{3x-3}{5x+3} \geq 0 \]
\[ \Rightarrow \frac{x+1}{5x+3} \leq 0 \quad \Rightarrow x \in \left[-1, -\frac{3}{5}\right] \]
\[ \Rightarrow \frac{2x}{5x+3} + 1 \leq 0 \]
\[ \Rightarrow \frac{7x+3}{5x+3} \leq 0 \quad \Rightarrow x \in \left[-\frac{3}{5}, \frac{-3}{7}\right] \]

\[ \alpha = -1, \beta = -\frac{3}{5}, \gamma = -\frac{3}{5}, \delta = -\frac{3}{7} \]
\[ |3\alpha + 10\beta + 5\gamma + 21\delta| = |3 - 6 - 3 - 9| = 21 \]

3. 8 persons has to travel from A to B in 3 allotted cars. If a car can carry maximum 3 persons. Then find the number of ways they can travel.

(1) 1880 (2) 1800
(3) 1680 (4) 1600

Answer (3)

Sol. \( C_1 \cdot C_2 \cdot C_3 \rightarrow \)
\[ \text{Total } 8! \text{ groups} \]
\[ \text{So, they can travel in } \frac{8!}{3!3!2!2!} \times 31 \text{ ways} \]
\[ = 1680 \]

4. If \( \frac{z + i}{4z + zi} \) is purely real \( \Delta z = x + iy \text{ (x, y } \in R) \) then
   one of the possibility is
   (1) \( x \neq 0, y \neq -1 \)
   (2) \( x \neq 0, y = -1 \)
   (3) \( x = -1, y = 1 \)
   (4) \( x = 1, y \neq -1 \)

Answer (2)

Sol. \( \text{Im } \left( \frac{x + iy + 1}{2x + iy + 1} \cdot 2x - i(y + 1) \right) \)
\[ = \frac{2x(y + 1) - x(y + 1)}{4x^2 + (y + 1)^2} \]
\[ = \frac{x(y + 1)}{4x^2 + (y + 1)^2} = 0 \]
\[ \Rightarrow x = 0 \text{ or } y = -1 \]
5. If \[ \int \left( \frac{x^{2e}}{e} + \frac{e^{2x}}{x} \right) \ln x \, dx = \alpha \left( \frac{x^{2e}}{e} \right) + \beta \left( \frac{e^{2x}}{x} \right) + c \]

where \( c \) is constant of integration, then

(1) \( \alpha + \beta = 0 \)  
(2) \( \alpha + \beta = 1 \)  
(3) \( \alpha \beta = \frac{1}{2} \)  
(4) \( \alpha \beta = \frac{1}{4} \)

**Answer (1)**

**Sol.** Let \( \left( \frac{x^{2e}}{e} \right) = t \)

\[ 2x(\ln x - 1) = \ln t \]

\[ \int \left[ 2(\ln x - 1) + 2x \left( \frac{1}{x} \right) \right] \, dx = \int \frac{1}{t} \, dt \]

\[ \ln x = \frac{1}{2t} \, dt \]

\[ l = \int \left( t + \frac{1}{t} \right) \times \frac{1}{2t} \, dt = \frac{1}{2} \int \left( 1 + \frac{1}{t^2} \right) \, dt \]

\[ = \frac{1}{2} \left( t - \frac{1}{t} \right) + c \]

\[ = \frac{1}{2} \left( x^{2e} \right) - \left( \frac{e^{2x}}{x} \right) + c \]

\[ \Rightarrow \alpha = \frac{1}{2}, \beta = \frac{-1}{2} \]

6. If a dice is thrown \( n \) times and probability of getting 7 times odd is equal to 9 times even. Then

\( P(2\text{ even}) = \frac{K}{2^{15}} \) then \( K \) is

(1) 58  
(2) 60  
(3) 48  
(4) 65

**Answer (2)**

**Sol.** \( P(\text{getting odd 7 times}) = P(\text{getting even 9 times}) \)

\[ nC_7 \left( \frac{1}{2} \right)^7 \left( \frac{1}{2} \right)^{n-7} = nC_9 \left( \frac{1}{2} \right)^9 \left( \frac{1}{2} \right)^{n-9} \]

\[ nC_7 = nC_9 \]

\[ n = 9 + 7 = 16 \]

\[ P(2\text{ times even}) = ^{16}C_2 \left( \frac{1}{2} \right)^{14} \left( \frac{1}{2} \right)^2 \]

\[ = \frac{^{16}C_2}{2^{16}} = \frac{16!}{2!14!} \times \frac{1}{2^{16}} \]

7. If \( \int_{0}^{2} (f(x) + x^2) \, dx = \frac{4}{3} t^3 \), then \( f(x) \) is

(1) \( x^2 - 2\sqrt{x} \)  
(2) \( x^2 + 2\sqrt{x} \)  
(3) \( x^2 \)  
(4) \( -x^2 + 2\sqrt{x} \)

**Answer (4)**

**Sol.** \( (f(t^2) + t^4)2t = 4t^2 \)

\[ f(t^2) + t^4 = 2t \]

\[ f(x^2) = -x^4 + 2x \]

Let \( x^2 = u \)

\[ f(u) = -u^2 + 2\sqrt{u} \]

8. The equation of conic is \( 19x^2 + 15y^2 = 285 \). A concentric circle with radius 4 units is given then angle of common tangent made by minor axis of ellipse is

(1) \( \frac{\pi}{3} \)  
(2) \( \frac{\pi}{2} \)  
(3) \( \frac{\pi}{6} \)  
(4) \( \frac{\pi}{4} \)

**Answer (1)**

**Sol.**

\[ x^2 + y^2 = 16 \]

\[ \frac{x^2}{16} + \frac{y^2}{19} = 1 \]
19x^2 + 15y^2 = 285
\Rightarrow \frac{x^2}{15} + \frac{y^2}{19} = 1

Let equation of tangent of ellipse be
\[ y = mx \pm \sqrt{15m^2 + 19} \] ...(i)

If it is tangent to \[ x^2 + y^2 = 16 \] then
\[ \frac{\sqrt{15m^2 + 19}}{\sqrt{m^2 + 1}} = 4 \]

\[ 15m^2 + 19 = 16m^2 + 16 \]
\[ m^2 = 3 \]

\[ m = \pm \sqrt{3} \]

\[ \therefore \text{ angle made by the tangent by minor axis (x- axis) will be } \frac{\pi}{3} \]

9. \[ y = f(x) \] is a quadratic function passing through \((-1, 0)\) and tangent to it at \((1, 1)\) is \( y = x \). Find \( x \) intercept by normal at point \((\alpha, \alpha + 1)\), \( \alpha > 0 \)

(1) 7  (2) -7  (3) 5  (4) -5

Answer (1)

Sol. Let \( f(x) = ax^2 + bx + (b - a) \)

\[ f(1) = 2a + b + 1 \] and \( f(1) = 1 = 2b \)

\[ b = \frac{1}{2}, \quad a = \frac{1}{4} \]

\[ -\frac{dx}{dy} = \frac{-1}{2ax + b} = \frac{-2}{x + 1} \]

and \( \alpha + 1 = \frac{1}{4} \alpha^2 + \frac{1}{2} \alpha + \frac{1}{4} \)

\[ 4(\alpha + 1) = (\alpha + 1)^2 \]

\[ \Rightarrow \alpha + 1 = 4 \text{ OR } \alpha = 3 \]

Slope of normal \[ = \frac{-2}{4} = \frac{-1}{2} \]

Equation of normal
\[ y - 4 = \frac{-1}{2}(x - 3) \]
\[ 2y - 4 = -x + 3 \]
\[ 2y + x = 7 \]
23. In $\triangle ABC$, $P$ is circumcentre, $\Delta Q$ is orthocentre, then $PA + PB + PC$ is

(1) $2PQ$   (2) $PQ$   (3) $3PQ$   (4) $\frac{1}{2}PQ$

Answer (2)

Sol.

Let $P$ be origin then

$PA + PB + PC = \bar{a} + \bar{b} + \bar{c}$

$PD = \frac{\bar{b} + \bar{c}}{2}$

$PQ = 2PD$

$= \bar{b} + \bar{c}$

$PQ = \bar{a} + (\bar{b} + \bar{c})$

$= \bar{a} + \bar{b} + \bar{c}$

24. Let $S = \left\{ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]: g^{\tan^2 x} + g^{\tan^2 x} = 10 \right\}$ and

$\beta = \sum_{x \in S} \left( \frac{x}{3} \right)$. Then $\frac{1}{7}(\beta - 14)^2$ is

Answer (28)

25. The coefficient of $x$ and $x^2$ in $(1 + x)^p (1 - x)^q$ are 4 and $-5$, then $2p + 3q$ is

Answer (63)

Sol. $(1 + x)^p (1 - x)^q = (1 + px + \frac{p(p-1)}{2} x^2 + ...) (1 - qx + q \frac{(q-1)}{2} x^2 + ...)$

$\therefore$ Coefficient of $x = p - q = 4 \quad \text{(i)}$

Coefficient of $x^2 = \frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2}$

$= -5$

$\therefore$ Coefficient of $x^2 = q(q-1) - pq + \frac{p(p-1)}{2}$

$\Rightarrow (p - q)^2 - (p + q) = -10$ \quad \text{(ii)}

By (i) and (ii) $p = 15$, $q = 11$

So, $2p + 3q = 30 + 33 = 63$

26. Let $\alpha$ be the remainder $(22)^{2022} + (2022)^{22}$ is divided by 3 and $\beta$ be the remainder when the same is divided by 7 then $\alpha^2 + \beta^2$ is

Answer (05)
Sol. \((22)^{2022} + (2022)^{22}\)

For \(\alpha\)
\[
(21+1)^{2022} + (2022)^{22}
\]
\[
= (3k_1 + 1)
\]

For \(\beta\)
\[
(21+1)^{2022} + (2023 - 1)^{22}
\]
\[
= (7\lambda + 1) + (7\mu + 1)
\]
\[
= 7k_2 + 2
\]
So, \(\alpha = 1\), \(\beta = 2\)
\[
\alpha^2 + \beta^2 = 5
\]
27. If area bounded by region \((x, y) \mid |x^2 - 2| \leq y \leq x\) is \(A\), then \(6A + 16\sqrt{2}\) is

Answer (27)

Sol.

\[
\int_{-2}^{2} x - (-x^2 - 2) \, dx + \int_{-2}^{2} (x - (x^2 - 2)) \, dx
\]
\[
= \sqrt{2} \left[ x - \left( \frac{x^3}{3} + \frac{x^2}{2} - 2x \right) \right]_{-2}^{2} + \left[ \frac{x^3}{3} + \frac{x^2}{2} + 2x \right]_{-2}^{2}
\]
\[
= \left( 2\sqrt{2} + 1 - 2\sqrt{2} \right) - \left( \frac{1}{3} + \frac{1}{2} - 2 \right)
\]
\[
+ \left( \frac{8}{3} + 2 + 4 \right) - \left( -\frac{2\sqrt{2}}{3} + 1 + 2\sqrt{2} \right)
\]
\[
\therefore \ 6A + 16\sqrt{2} = 27 - 16\sqrt{2} + 16\sqrt{2}
\]
\[
= 27
\]