Memory Based Answers & Solutions for
JEE (Main)-2023 (Online) Phase-2
(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each part (subject) has two sections.

   (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.

   (ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Identify the logic operation of following circuit.

   (1) AND  (2) OR  (3) NOR  (4) NAND

Answer (2)

Sol.

   \[
   \begin{array}{c|cccc}
   A & B & P & Q & Y \\
   \hline
   1 & 0 & 0 & 0 & 1 \\
   0 & 1 & 0 & 0 & 1 \\
   1 & 1 & 0 & 0 & 1 \\
   0 & 0 & 1 & 1 & 0 \\
   \end{array}
   \]

   OR gate

2. Force acting on a particle moving along x-axis is given by \( \vec{F} = (2 + 3x) \hat{i} \). The work done by this force from \( x = 0 \) to \( x = 4 \) m is

   (1) 16 J  (2) 32 J  (3) 4 J  (4) 8 J

Answer (2)

Sol. \( W = \int_0^4 (2 + 3x) \hat{i} \cdot dx \hat{i} = 2x + \frac{3}{2} x^2 \bigg|_0^4 = 32 \) J

3. If half life of a radioactive nuclide A is equal to average life of another radioactive nuclide B. Find the ratio of decay constant of A to that of B.

   (1) \( \ln 2 : 1 \)  (2) \( 1 : \ln 2 \)  (3) \( 2 : \ln 2 \)  (4) \( \ln 2 : 2 \)

Answer (1)

Sol. \( t_{1/2} = (t_{\text{mean}})_B \)

\[
\ln(2) = \frac{1}{\lambda_A} = \frac{1}{\lambda_B}
\]

\( \Rightarrow \frac{\lambda_A}{\lambda_B} = \ln 2 \)

4. Variation of magnetic field through a coil of area \( 4 \) m\(^2\) is shown in figure. What is the emf induced in the coil (in mV)?

   (1) 8  (2) 16  (3) 4  (4) 2

Answer (1)

Sol. From given figure, \( \frac{dB}{dt} = 2 \) mT/sec

\[
\varepsilon_{\text{ind}} = |A| \frac{dB}{dt} = 4 \times 2 \text{ mV} = 8 \text{ mV}
\]

5. The characteristics of two coils is given below

<table>
<thead>
<tr>
<th>Coil-A</th>
<th>Coil-B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>( r_A = 10 ) cm</td>
</tr>
<tr>
<td>Number of turns</td>
<td>( N_A )</td>
</tr>
<tr>
<td>Current</td>
<td>( I_A )</td>
</tr>
</tbody>
</table>

If magnetic moment of both coil A and B are equal, then choose the correct relation

   (1) \( 2N_A I_A = N_B I_B \)  (2) \( N_A I_A = N_B I_B \)  (3) \( N_A I_A = 4N_B I_B \)  (4) \( N_A I_A = 2N_B I_B \)

Answer (3)

Sol. \( \mu = NI \Rightarrow N_A I_A r_A^2 = N_B I_B r_B^2 \)

\( \Rightarrow N_A I_A = N_B I_B \times 4 \)

\( \Rightarrow N_A I_A = 4N_B I_B \)
6. The variation of impedance \( Z \) with angular frequency \( \omega \) for two electrical elements is shown in the graph given. If \( X_L, X_C \) and \( R \) are inductive reactance, capacitive reactance and resistance respectively, then

\[
Z \propto \omega
\]

(1) A is resistor, B is inductor

(2) A is inductor, B is capacitor

(3) A is inductor, B is resistor

(4) A is capacitor, B is inductor

Answer (2)

Sol. \( X_L \propto \omega \), \( X_C \propto \frac{1}{\omega} \), \( R \) is independent of \( \omega \)

7. Find the current flowing in 3Ω resistor in the given circuit.

\[
\text{Current } (i) = \frac{12}{10} = 1.2 \text{ A}
\]

\[
\therefore \text{Current } (i) = \frac{6 \times \frac{12}{9}}{10} = 0.8 \text{ A}
\]

(1) 0.4 A

(2) 0.2 A

(3) 0.8 A

(4) 0.6 A

Answer (3)

Sol. \( \because \) Current \( i \) through equivalent battery

8. Velocity of particle moving along a straight line is shown in figure. The distance and displacement travelled by the body is

\[
\text{Distance} = 250 \text{ m}
\]

\[
\text{Displacement} = 150 \text{ m}
\]

(1) 150 m and 250 m

(2) 250 m and 250 m

(3) 150 m and 150 m

(4) 50 m and 150 m

Answer (1)

Sol. Refractive index of medium, \( \mu = \frac{1}{\sin \theta_c} \rightarrow \mu = \sqrt{2} \)

\[
\therefore \text{Light speed} = \frac{c}{\mu} = \frac{3 \times 10^8}{\sqrt{2}} \text{ m/s}
\]

10. In moving coil galvanometer if number of turns increases by 25%, then change in voltage sensitivity is

(1) Zero

(2) 1%

(3) 25%

(4) 50%

Answer (1)

Sol. Voltage sensitivity = \( \left( \frac{NAB}{KR} \right) \times \left( \frac{N}{R} \right) \)

\[
\frac{N}{R} \text{ remains same.}
\]
11. A fixed charge $P$ and another free charge $Q$ having same mass and charge are shown in the diagram find the maximum height ($h$) attained by charge $Q$ in equilibrium state on smooth inclined plane if $q = 2 \mu C$, $\theta = 30^\circ$, $m = 20 \text{ g}$

![Diagram of charges](image)

(1) 0.1 m  (2) 0.3 m  (3) 0.4 m  (4) 0.5 m

**Answer (2)**

**Sol.**

\[ F = mg \sin \theta \]

\[ Kq^2/l^2 = mg \sin \theta \]

\[ l = \sqrt{\frac{Kq^2}{mg \sin \theta}} = 0.6 \text{ m} \]

Also, $h = l \sin \theta = 0.3 \text{ m}$

12. If a planet 'A' has density $\rho$ and radius $r$, planet 'B' has density $\frac{\rho}{3}$ and radius $4r$. Then, find ratio of their acceleration due to gravity at their surface.

(1) 3 : 4  (2) 4 : 3  (3) 1 : 3  (4) 2 : 3

**Answer (1)**

**Sol.**

\[ g \propto \rho R \]

\[ \therefore \frac{g_A}{g_A} = \frac{\rho r}{\frac{\rho}{3} \times 4r} = \frac{3}{4} \]

13. A $2 \mu F$ capacitor is charged with potential $V$ and energy stored in capacitor is $E_1$. Now the capacitor is disconnected with battery and connected with another identical capacitor in parallel. Now the energy stored in capacitor is $E_2$. Find $\frac{E_1}{E_2}$

(1) 2  (2) 4  (3) 5  (4) 6

**Answer (4)**
16. The correct order of root mean square speed \((v_{rms})\) for Ne, Cl\(_2\) and OF\(_6\) at same temperature is

(1) \((v_{rms})_{Ne} < (v_{rms})_{Cl_2} < (v_{rms})_{OF_6}\)

(2) \((v_{rms})_{Cl_2} < (v_{rms})_{Ne} < (v_{rms})_{OF_6}\)

(3) \((v_{rms})_{OF_6} < (v_{rms})_{Cl_2} < (v_{rms})_{Ne}\)

(4) \((v_{rms})_{OF_6} < (v_{rms})_{Ne} < (v_{rms})_{Cl_2}\)

Answer (3)

Sol. \(v_{rms} \propto \frac{1}{\sqrt{M}}\)

also, \(M_{OF_6} > M_{Cl_2} > M_{Ne}\)

\(\therefore (v_{rms})_{OF_6} < (v_{rms})_{Cl_2} < (v_{rms})_{Ne}\)

17. Two identical bulbs are first connected in series then in parallel. Find the ratio of power consumed in two cases.

(1) 1 : 1

(2) 1 : 4

(3) 4 : 1

(4) 1 : 2

Answer (2)

Sol. \(P_1 = \frac{v^2}{2R}; P_2 = \frac{v^2}{R} = 2\frac{v^2}{R}\)

\(\frac{P_1}{P_2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\)

18. Statement-I: Light year, parsec and AU are units for measuring distance.

Statement-II: (1 light year) > (1 parsec) > 1 AU

(1) Both statements I and II are correct

(2) Statement I is correct, statement II is incorrect

(3) Both statements I and II are incorrect

(4) Statement I is incorrect, statement II is correct

Answer (2)

Sol. \(1 \text{ parsec} > 1 \text{ light year} > 1 \text{ AU}\)

19. For a particle undergoing linear SHM, the graph showing the variation of kinetic energy \((K)\) with position \((x)\) of particle is

Answer (4)

Sol. \(K = \frac{1}{2} mo^2 (A^2 - x^2)\)

\(K\) vs \(x\) will be a parabola.

20. A scale read melting point of ice –15°F and boiling point 65°F. The, find 95°F temperature in fahrenheit.

(1) 428 F

(2) 280 F

(3) 350 F

(4) 210 F

Answer (2)

Sol. \(\frac{95 - (-15)}{F - 32} = \frac{65 - (-15)}{180}\)

\(\Rightarrow \frac{110 \times 180}{80} + 32 = 279.5\)
SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Equation of progressive wave is
\[ y = A \sin(160t - 0.5x). \]
Let the speed of wave is 10x, find x.

Answer (32)

Sol. From given equation,
\[ \omega = 160 \text{ and } k = 0.5 \]
\[ \therefore \text{ Speed of wave, } \]
\[ v = \frac{\omega}{k} = \frac{160}{0.5} = 320 \text{ m/s} \]

22. A machine gun is firing 10 g bullets with speed 250 m/s. To keep machine gun in position 125 N force is required. Find no. of bullets fired per second.

Answer (50)

Sol. \[ F = n \cdot mv \Rightarrow n = \frac{125}{10 \times 10^{-3} \times 250} = 50 \]

23. A particle is projected at an angle of 30° with horizontal. Height of particle at 3 s and 5 s are same. Find the speed of projection in m/s. \((g = 10 \text{ m/s}^2)\)

Answer (80)
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:
1. In a container at a constant temperature, arrange the RMS velocity of following
   Ne, Cl₂, UF₆
   (1) Ne > Cl₂ > UF₆  (2) Cl₂ > UF₆ > Ne  
   (3) UF₆ > Ne > Cl₂  (4) UF₆ > Cl₂ > Ne

Answer (1)

Sol. \[ \text{U}_{\text{rms}} = \sqrt{\frac{3 \text{RT}}{M_w}} \]
\[ M_w \uparrow \text{U}_{\text{rms}} \downarrow \]

2. Which of the following is correct order of first ionisation energy for
   Li, Be, C, B, N, O, F
   (1) B > N > O > Li > Be > F > C
   (2) N > F > O > C > B > Be > Li
   (3) F > O > N > C > B > Be > Li
   (4) F > N > O > C > Be > B > Li

Answer (4)

Sol. Option (4) is correct order
F > N > O > C > Be > B > Li

3. Match the columns

<table>
<thead>
<tr>
<th>Column-I</th>
<th>Column-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ClO₂</td>
<td>(1) Linear</td>
</tr>
<tr>
<td>(B) N₅</td>
<td>(2) Tetrahedral</td>
</tr>
<tr>
<td>(C) NH₄⁺</td>
<td>(3) Bent</td>
</tr>
<tr>
<td>(D) SF₄</td>
<td>(4) See-Saw</td>
</tr>
</tbody>
</table>

(1) A → 1; B → 2; C → 3; D → 4
(2) A → 3; B → 1; C → 2; D → 4
(3) A → 4; B → 2; C → 1; D → 3
(4) A → 3; B → 2; C → 1; D → 4

Answer (2)

Sol. The reactivity of an aromatic compound towards electrophilic aromatic substitution (EAS) is decided by the kind of substituents bonded to it. Any substituent that increases the electron density of benzene makes it more reactive towards EAS. In compound(A), the O-atom directly bonded to benzene ring increases the electron density by +R. In compound(B), the carbonyl group decreases the electron density by -R. In compound(C), the electron density of benzene increases by +I effect.

∴ The correct increasing order towards EAS is
B < C < A

4. Increasing order of electrophilic aromatic substitution reaction

(A) \[ \text{OH} \quad \text{OH} \quad \text{OH} \]
(B) \[ \text{OH} \quad \text{OH} \quad \text{OH} \]
(C) \[ \text{OH} \quad \text{OH} \quad \text{OH} \]

(1) A < B < C  (2) B < C < A  (3) C < B < A  (4) B < A < C

Answer (2)

5. o-phenylenediamine \[ \text{HNO}_2 \rightarrow \]

(1) \[ \text{NH}_3 \]
(2) \[ \text{NH}_2 \]
(3) \[ \text{N}_2^+ \]
(4) \[ \text{N}_2^- \]

Answer (2)
6. Consider the reaction sequence

\[
\begin{align*}
\text{R} & \quad \xrightarrow{\text{kMnO}_4} \quad (A) \\
\text{R} & \quad \xrightarrow{(i) \text{NH}_2 - \text{NH}_2/\text{KOH}} \quad (B) \\
\end{align*}
\]

B is :

(1) \[ \text{R} \quad \text{C} \quad \text{CH} \]

(2) \[ \text{R} \quad \text{CH} \quad \text{OH} \]

(3) \[ \text{R} \quad \text{C} \quad \text{OH} \]

(4) \[ \text{R} \quad \text{C} \quad \text{OH} \]

Answer (4)

Sol.

7. Which of the following does not contain ambidentate ligand

(1) \( C_2O_4^{2-} \), \( H_2O \)

(2) \( \text{EDTA}^{-4} \), \( \text{NO}_2^- \)

(3) \( \text{NO}_2^- \), \( \text{SCN}^- \)

(4) \( \text{SCN}^- \), \( \text{CN}^- \)

Answer (1)

Sol. \( C_2O_4^{2-} \) is bidentate (chelate) ligand and \( H_2O \) is simple monodentate ligand.

8. Which of the following can be represented as meridional isomer

(1) \( \text{[Pt(NH}_3)_2\text{Cl}_3]^+ \)

(2) \( \text{[Pt(en)_3]^+} \)

(3) \( \text{[Pt(en)_2\text{Cl}_3]^2+} \)

(4) \( \text{[Pt(en)_2(NH}_3)_2]^4+} \)

Answer (1)

Sol. \( \text{[M}_3\text{b}_3 \) exist as fac. & mer isomer.

9. When ethene reacts with Ziegler–Natta and at 6–7 atm pressure it gives.

(1) Low density polythene

(2) Polyacrylonitrile

(3) Polyamide

(4) High density polythene

Answer (4)

Sol. Addition polymerisation of ethene at pressure 6–7 atm in the pressure of Ziegler–Natta catalyst gives high density polythene.

10. Identify the correct statement about the compound \( \text{GaAlCl}_4 \).

(1) Chlorine atom is bonded to both Ga and Al

(2) Ga is cationic part and less electronegative than Al

(3) Chlorine atom forms co-ordinate bond with Ga

(4) Chlorine atom is bonded to Al

Answer (4)

Sol. The structure of \( \text{GaAlCl}_4 \) is

Chlorine atoms is bonded to Al only. Gallium is the cationic part but more electronegative than Al.

11. Match the column

<table>
<thead>
<tr>
<th>Column - I</th>
<th>Column - II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) K(^+) Ions</td>
<td>(P) Thermonuclear reactions</td>
</tr>
<tr>
<td>(B) KCl</td>
<td>(Q) Sodium-potassium pump</td>
</tr>
<tr>
<td>(C) Mg</td>
<td>(R) Absorbent of ( \text{CO}_2 )</td>
</tr>
<tr>
<td>(D) KOH</td>
<td>(S) Fertiliser</td>
</tr>
</tbody>
</table>

(1) \( \text{A} \rightarrow \text{Q}; \text{B} \rightarrow \text{S}; \text{C} \rightarrow \text{R}; \text{D} \rightarrow \text{P} \)

(2) \( \text{A} \rightarrow \text{P}; \text{B} \rightarrow \text{Q}; \text{C} \rightarrow \text{S}; \text{D} \rightarrow \text{R} \)

(3) \( \text{A} \rightarrow \text{Q}; \text{B} \rightarrow \text{S}; \text{C} \rightarrow \text{P}; \text{D} \rightarrow \text{R} \)

(4) \( \text{A} \rightarrow \text{P}; \text{B} \rightarrow \text{Q}; \text{C} \rightarrow \text{R}; \text{D} \rightarrow \text{S} \)

Answer (3)
Sol. K⁺ : Sodium-potassium pump
      KCl : Fertiliser
      Mg : Used in thermonuclear reactions
      KOH : Absorber of CO₂

12. Which type of copper is formed by the following reactions?

   \[
   \begin{align*}
   2\text{Cu}_2\text{S} + 3\text{O}_2 &\rightarrow 2\text{Cu}_2\text{O} + 2\text{SO}_2 \\
   2\text{Cu}_2\text{O} + \text{Cu}_2\text{S} &\rightarrow 6\text{Cu} + \text{SO}_2
   \end{align*}
   \]

   (1) Blister copper
   (2) Copper crisp
   (3) Reduced copper
   (4) Copper slag

Answer (1)

Sol. The solidified copper obtained has blistered appearance due to the evolution of SO₂ and so it is called blister copper.

13. To 25 ml of 1 M AgNO₃, 1.05 M KI is added dropwise. In the colloidal sol formed, fixed and diffused layer consists of respectively:

   \[(\text{AgNO}_3 \text{ is in excess})\]
   (1) I⁻ and NO₃⁻
   (2) Ag⁺ and NO₃⁻
   (3) Ag⁺ and K⁺
   (4) K⁺ and Ag⁺

Answer (2)

Sol. Fixed layer → Ag⁺

Diffused layer → NO₃⁻

14. L-isomer of a tetrose (A) gives Schiff's test having two chiral carbons.

\[
\text{A} \xrightarrow{\text{Conc. HNO}_3} \text{X} \quad \text{(Optically inactive)}
\]

A is:

15. **Statement I (S I)**: A water sample having BOD = 4 ppm is of good quality.

**Statement II (S II)**: If the concentration of Zn and NO₃⁻ each is 5 ppm, then water is of good quality.

   (1) Both S I and S II are correct
   (2) S I is incorrect and S II is correct
   (3) S I is correct and S II is incorrect
   (4) Neither S I nor S II is correct

Answer (1)

Sol. Clean water has BOD less than 5 ppm

The maximum concentration of Zn = 5 ppm

Concentration of NO₃⁻ = 50 ppm

(as per international standards of drinking water)
16. 
17. 
18. 
19. 
20. 

SECTION - B 
Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find spin only magnetic moment ratio for complexes [Cr(CN)]\(^{3-}\) and [Cr(H\(_2\)O)]\(^{3+}\)

Answer (1)

Sol. Spin only magnet magnetic for [Cr(CN)]\(^{3-}\) (d\(^3\)) = 15 B.M.
For [Cr(H\(_2\)O)]\(^{3+}\) (d\(^3\)) = \(\sqrt{15}\) B.M.
So ratio is 1 : 1

22. 25% of 250 g sugar solution and 40% of 500 g sugar solution are mixed then find out the mass percentage of sugar in solution

Answer (35.00)

Sol. Mass of Sugar = (.25 \times 250) + (.40 \times 500)
= 262.5 gm

\[
\% \left( \frac{w}{w} \right) = \frac{262.5}{750} \times 100 = 35\%
\]

23. \(\text{CH}_3\text{MgBr}\)

\[(x \text{ mol}) \quad \quad \quad \quad \quad \quad (y \text{ mol})\]

Value of \(\frac{x}{y}\) when A & B completely react will be:

Answer (1)

Sol.

Hence value \(\frac{x}{y} = 1\)

24. Find the number of atoms per unit cell if edge length = 408 pm, density = 3 g/cm\(^3\), Molecular mass = 40 (Nearest integer)

Answer (3)

Sol. \(Z = \frac{3}{40} \times 6 \times 0.1 \times (4.08)^3 = 3.056 = 3\)

25. Given

Electrode | Potential |
--- | --- |
Pb\(^{2+}/\text{Pb}\) | M |
Pb\(^{4+}/\text{Pb}\) | N |
Pb\(^{2+}/\text{Pb}^{4+}\) | ? |

Value of Pb\(^{2+}/\text{Pb}^{4+}\) is M–x N, then value of x is

Answer (2)

Sol. \(E^{-}_{\text{Pb}^{2+}/\text{Pb}^{4+}} = \frac{4E^{-}_{\text{Pb}^{2+}/\text{Pb}} - 2E^{-}_{\text{Pb}^{4+}/\text{Pb}}}{2} = \frac{4 \times N - 2M}{2} = 2N - M\)

\(\therefore E^{-}_{\text{Pb}^{2+}/\text{Pb}^{4+}} = M - 2N\)

Hence x = 2
3. For a given function $f(x) = \left[x^2 - x\right] + |x|$

   (1) $f(x)$ is continuous at $x = 0, x = 1$
   (2) $f(x)$ is continuous and differentiable at $x = 0$ and $x = 1$
   (3) $f(x)$ is continuous but non-differentiable at $x = 0, 1$
   (4) $f(x)$ is continuous at $x = 1$ but discontinuous at $x = 0$

   **Answer (4)**

   \[
   f(x) = \begin{cases} 
   x + 1 & -0.5 < x < 0 \\
   0 & x = 0 \\
   -1 + x & 0 < x < 1 \\
   0 & x = 1 \\
   x - 1 & 1 < x < 1.5 
   \end{cases} 
   \]

   - At $x = 0^+$, $f(x) = -1$
   - At $x = 0^-$, $f(x) = 1$
   - $f(x)$ is discontinuous at $x = 0$
   - At $x = 1^+$, $f(x) = 0$
   - At $x = 1^-$, $f(x) = 0$
   - $f(1) = 0$
   - $f(x)$ is continuous at $x = 1$

4. Two complex numbers $w_1$ and $w_2$, given by $w_1 = 3 + 5i$ and $w_2 = 5 + 4i$ are both rotated by $90^\circ$ with respect to origin anti-clockwise and clock-wise respectively to get the new complex numbers $w_3$ and $w_4$. The principal argument of $w_3 - w_4$ is

   (1) $-\pi - \tan^{-1} \frac{8}{9}$
   (2) $-\pi - \tan^{-1} \frac{33}{5}$
   (3) $\pi - \tan^{-1} \frac{8}{9}$
   (4) $\pi - \tan^{-1} \frac{33}{5}$

   **Answer (3)**

   \[
   \begin{align*}
   w_3 &= i w_1 = i(3 + 5i) = -5 + 3i \\
   w_4 &= -i w_2 = -(5 + 4i) = 4 - 5i \\
   w_3 - w_4 &= -9 + 8i \\
   \arg (w_3 - w_4) &= \pi - \tan^{-1} \frac{8}{9}
   \end{align*}
   \]

   ($\therefore w_3 - w_4 \in \text{II Quadrant}$)
5. Let \(a\) and \(b\) are roots of \(x^2 - 7x - 1 = 0\). The value of \(\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}\) is 

(1) 29  
(2) 49  
(3) 53  
(4) 51  

Answer (4)  

\[
\frac{a^{17}(a^4 + 1) + b^{17}(b^4 + 1)}{a^{19} + b^{19}}
\]

Sol.  
\[
\alpha^2 - 1 = 7\alpha  
\Rightarrow \alpha^4 + 1 = 51\alpha^2 < a  
\Rightarrow \frac{51\alpha^{19} + 51b^{19}}{a^{19} + b^{19}} = 51
\]

6. Find the area bounded by \(x^2 + (y - 2)^2 \leq 4\) and \(y^2 \leq 2x\) 

(1) \(\pi + \frac{4}{3}\)  
(2) \(\pi - \frac{4}{3}\)  
(3) \(2\pi + \frac{8}{3}\)  
(4) \(2\pi - \frac{8}{3}\)  

Answer (2) 

Sol.  
\[
\begin{align*}
\text{On solving the curves, } (0, 0) \text{ and } (2, 2) \text{ will be the point of intersection} \\
\text{Required area } &= \int_{0}^{2} \left[ \sqrt{4 - (y - 2)^2} - \frac{y^2}{2} \right] dy \\
&= \left[ \frac{1}{2}(y - 2)\sqrt{4 - (y - 2)^2} + \frac{4}{2}\sin^{-1}\left(\frac{y - 2}{2}\right) - \frac{y^3}{6} \right]_{0}^{2} \\
&= -\frac{8}{6} \left[ \frac{1}{2}(-2) \times 0 - \pi \right] = \pi - \frac{4}{3}
\end{align*}
\]

7. The number of solutions of \(\cos^4 \theta - 2\cos^2 \theta + 3\sin^6 \theta + 1 = 0\) in \([0, 2\pi]\) is 

(1) 1  
(2) 2  
(3) 3  
(4) 4  

Answer (3)  

Sol. \((1 - \cos^2 \theta)^2 + \sin^2 \theta = 0\)  
\Rightarrow \sin^4 \theta + \sin^2 \theta = 0  
\Rightarrow \sin^2 \theta(\sin^2 \theta + 1) = 0  
\Rightarrow \sin^2 \theta = 0  
\theta = 0, \pi, 2\pi  
\therefore \text{Three solutions in } [0, 2\pi]\)

8. A rectangle is drawn by lines \(x = 0, x = 2, y = 0\) and \(y = 5\). Points \(A\) and \(B\) lie on coordinate axes. If line \(AB\) divides the area of rectangle in 4 : 1, then the locus of mid-point of \(AB\) is 

(1) Circle  
(2) Hyperbola  
(3) Ellipse  
(4) Straight line  

Answer (2)  

Sol. 
\[
A(2h, 0), B(0, 2k)
\]

Area of \(\triangle OAB = 8\)  
\[
\frac{1}{2} \times 2h \times 2k = 8
\]

\(hk = 4\)  
Locus is \(xy = 4\)
9. Let awards in event A is 48 and awards in event B is 25 and awards in event C is 18 and also \( n(A \cup B \cup C) = 60 \), \( n(A \cap B \cap C) = 5 \), then how many got exactly two awards is

(1) 21 (2) 25 (3) 24 (4) 23

Answer (1)

Sol.

\[
\therefore \text{Number of persons who get exactly two awards} = n(A) + n(B) + n(C) - n(A \cup B \cup C) - \frac{1}{2}n(A \cap B \cap C) = 48 + 25 + 18 - 60 - 10 = 21
\]

10. Let \( p \) : I have fever

\( q \) : I do not take medicine

\( r \) : I take rest

If I have fever then I take medicine and I take rest is equivalent to

(1) \((\neg p \vee \neg q) \land (\neg p \vee r)\)

(2) \((\neg p \vee q) \land (\neg p \vee \neg r)\)

(3) \((\neg p \vee q) \land (\neg p \vee r)\)

(4) \((\neg p \vee q) \land (\neg p \vee \neg r)\)

Answer (3)

Sol. Given statement is equivalent to

\[ p \rightarrow (q \land r) \]

\[ = \neg p \lor (q \land r) \]

\[ = (\neg p \lor q) \land (\neg p \lor r) \]

11. Let \( A \) be a 2 \times 2 matrix such that \( A^T = \alpha A + 1 \) and \( |A^2 + 2A| = 4 \), then a possible value of \( \alpha \) is

(1) 1

(2) -1

(3) 3

(4) 0

Answer (2)

Sol. Let \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \)

\( A^T = \alpha A + 1 \)

\[
\Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} \alpha a + 1 & \alpha b \\ \alpha c & \alpha d + 1 \end{bmatrix}
\]

\[ a = \alpha c + 1 \Rightarrow a = \frac{1}{1-\alpha} \quad \ldots(i) \]

\[ b = \alpha c \quad \ldots(ii) \]

\[ c = \alpha b \quad \ldots(iii) \]

(ii) and (iii) \( c = 0 \) or \( \alpha = \pm 1 \)(\( \alpha \neq 1 \))

\[ \therefore c = 0 \text{ or } \alpha = -1 \]

Also, \( d = \alpha d + 1 \Rightarrow d = \frac{1}{1-\alpha} \)

Case (I)

\[ c = 0 \Rightarrow b = 0 \]

\[ |A^2 + 2A| = 4 \quad \Rightarrow \quad |A| \cdot |A + 2I| = 4 \]

\[ \left( \frac{1}{1-\alpha} \right)^2 \left( \frac{3-2\alpha}{1-\alpha} \right)^2 = 4 \]

\[ \frac{3-2\alpha}{(1-\alpha)^2} = \pm 2 \]

\[ \therefore \alpha = \frac{1 \pm \sqrt{3}}{2} \]

Case (II)

\[ \alpha = -1 \]

12. Let \( \hat{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k} \) and \( 0 < |a_i| < 1, \; i = \{1, 2, 3\} \)

Statement-A : \( |\hat{a}| \geq \max \{|a_1|, |a_2|, |a_3|\} \)

Statement-B : \( |\hat{a}| < 3 \max \{|a_1|, |a_2|, |a_3|\} \)

(1) Both A and B are true

(2) Both A and B are false

(3) A is true, B is false

(4) A is false, B is true

Answer (1)

Sol. \[ |\hat{a}|^2 = \sqrt{a_1^2 + a_2^2 + a_3^2} \geq \max \{|a_1|, |a_2|, |a_3|\} \]

\[ |\hat{a}|^2 = \sqrt{a_1^2 + a_2^2 + a_3^2} < 3 \max \{|a_1|, |a_2|, |a_3|\} \]
13. Solution of differential equation is 
\((1 - x^2 y^2) \, dx + y \, dy\) if \(y(2) = 4\), then 
\[\frac{5y(5) + 1}{5y(5) - 1} = k\] 
then \(k\) is

\[\begin{align*}
(1) \quad & \frac{49}{81} e^2 \\
(2) \quad & \frac{81}{49} e^2 \\
(3) \quad & e^2 \\
(4) \quad & \frac{9}{7} e^2
\end{align*}\]

**Answer (2)**

**Sol.** 
\((1 - x^2 y^2) \, dx = x \, dy + y \, dx\)

\[\int dx = \int \frac{d\left(xy\right)}{1 - x^2 y^2}\]

\[x = \frac{1}{2} \log \left| \frac{1 + xy}{1 - xy} \right| + c\]

\[\therefore \quad y(2) = 4\]

\[2 = \frac{1}{2} \log \left| 1 + 4 \times 2 \right| + c\]

\[4 = \log \left| \frac{9}{7} \right| + c\]

\[c = 4 - \log \frac{9}{7}\]

\[x = \frac{1}{2} \log \left| \frac{1 + xy}{xy - 1} \right| + 4 - \log \frac{9}{7}\]

For \(y(5)\)

\[5 = \frac{1}{2} \log \left| \frac{1 + 5y}{5y - 1} \right| + 4 - \log \frac{9}{7}\]

\[2 \left( 1 + \log \frac{9}{7} \right) = \log \left( \frac{1 + 5y}{5y - 1} \right)\]

\[\frac{5y + 1}{5y - 1} = e^{\left( 1 + \log \frac{9}{7} \right)} \Rightarrow \frac{81}{49} e^2\]

14. 
15. 
16. 
17. 
18. 
19. 
20.

---

**SECTION - B**

**Numerical Value Type Questions:*** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. 5 boys with allotted roll numbers and seat numbers are seated in such a way that no one sits on the allotted seat. The number of such seating arrangements is

**Answer (44)**

**Sol.** Number of seating arrangements

\[5! - \frac{1}{1!} 1! 2! 3! 4! 5! \]

\[= 120 - 120 + 60 - 20 + 5 - 1\]

\[= 44\]

22. 

\[(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)\] 

Number of triplets \((p, q, r)\) such that it is true, is ________.

**Answer (7)**

**Sol.** 

\[(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)\]

\[\Rightarrow p \vee (q \wedge r) \Rightarrow (q \vee r)\]

This is always true if \(p \vee (q \wedge r)\) is F.

\[\Rightarrow (p, q, r) = (F, F, F) \vee (F, F, T) \vee (F, T, F) \vee (T, T, T)\]

This is true if \(p \vee (q \wedge r)\) is T and \((q \vee r)\) is T

\[\Rightarrow (p, q, r) = (T, T, F) \vee (T, F, T) \vee (T, T, T) \vee (F, T, T)\]

\[\therefore \quad 7 \text{ triplets are possible}\]

23. The number of rational terms in the expansion of 

\[\left(3^{\frac{3}{4}} + 5^{\frac{3}{2}}\right)^{60}\]

**Answer (16)**

**Sol.** 

\[T_{r+1} = \binom{60}{r} 3^{\frac{3}{4} \cdot 60-r} 5^{\frac{3}{2} \cdot r}\]

\[= \binom{60}{r} 3^{180-3r} 4^{5r/2}\]

Total rational terms = 16
24. Consider the plane \(2x + y - 3z = 6\). If \((\alpha, \beta, \gamma)\) is the image of point \((2, 3, 5)\) in the given plane, then \(\alpha + \beta + \gamma = \underline{\text{______}}\)

**Answer (10)**

\[
\begin{align*}
\frac{x - 2}{2} &= \frac{y - 3}{1} = \frac{z - 5}{-3} = -2
\end{align*}
\]

\(\Rightarrow x = 6, y = 5, z = -1\)

\(\therefore \alpha = 6, \beta = 5, \gamma = -1\)

\(\alpha + \beta + \gamma = 6 + 5 - 1 = 10\)

25. The mean of coefficients of \(x, x^2, \ldots, x^7\) in the binomial expansion of \((2 + x)^8\) is

**Answer (2736)**

\[
\begin{align*}
\text{Sol.} \quad \text{Sum of coefficients} &= 9C_1 \cdot 2^8 + 9C_2 \cdot 2^7 + \ldots + 9C_7 \cdot 2^2 \\
&= (2 + 1)^9 - 9C_0 \cdot 2^8 - 9C_8 \cdot 2 - 9C_9 \\
&= 3^9 - 2^9 - 19 = 19683 - 572 - 19 = 19152 \\
\text{Mean} &= \frac{\text{Sum}}{7} = \frac{19152}{7} = 2736
\end{align*}
\]

26. Consider two sets \(A\) and \(B\). Set \(A\) has 5 elements whose mean & variance are 5 and 8 respectively. Set \(B\) has also 5 elements whose mean & variance are 12 & 20 respectively. A new set \(C\) is formed by subtracting 3 from each element of set \(A\) and by adding 2 to each element of set \(B\). The sum of mean & variance of the set \(C\) is _______.

**Answer (58)**

\[
\begin{align*}
\overline{x}_c &= \text{mean of } c = \frac{(5 - 3) + (12 + 2)}{2} = 8 \\
\sigma^2_{12} &= \text{variance of } c = \frac{n_1(\sigma^2_1 + d^2_1) + n_2(\sigma^2_2 + d^2_2)}{n_1 + n_2} \\
d_1 &= \overline{x}_{12} - \overline{x}_1 \\
d_2 &= \overline{x}_{12} - \overline{x}_2 \\
n_1 = 5, \sigma^2_1 = 8, d_1 = 8 - 2 = 6 \\
n_2 = 5, \sigma^2_2 = 20, d_2 = 8 - 14 = -6 \\
\sigma^2_{12} &= \frac{5(8 + 36) + 5(20 + 36)}{10} = 50 \\
\therefore \quad \sigma^2_{12} + \overline{x}_c &= 50 + 8 = 58
\end{align*}
\]

27. If \(S = 109 + \frac{108}{5} + \frac{107}{5^2} + \ldots + \frac{2}{5^{107}} + \frac{1}{5^{108}}\) then the value of \(165 - \left(25\right)^{54}\) is

**Answer (2175)**

\[
\begin{align*}
\text{Sol.} \quad S &= 109 + \frac{108}{5} + \frac{107}{5^2} + \ldots + \frac{2}{5^{107}} + \frac{1}{5^{108}} \\
\therefore \quad \frac{4}{5}S &= 109 - \left[\frac{1}{5} + \frac{1}{5^2} + \ldots + \frac{1}{5^{108}} + \frac{1}{5^{109}}\right] \\
&= 109 - \left[\frac{\frac{1}{5} - \left(\frac{1}{5}\right)^{109}}{\frac{1}{5} - 1}\right] \\
&= \frac{5}{4} \left[109 - \frac{1}{4} - \left(\frac{1}{5}\right)^{109}\right] \\
&= \frac{5}{4} \left[109 - \frac{1}{4} - \left(\frac{1}{5}\right)^{109}\right] \\
\therefore \quad S &= \frac{5}{4} \left[109 - \frac{1}{4} - \left(\frac{1}{5}\right)^{109}\right] \\
165 &= 20 \times 109 - 5 + 5^{-108} \\
165 - \left(25\right)^{54} &= 2180 - 5 \\
&= 2175
\end{align*}
\]

28. If \(\log_{\frac{7}{2}} \left(\frac{x + 7}{2x + 3}\right)^2 \geq 0\), then total number of integral solution(s) is/are_____.

**Answer (8)**

\[
\begin{align*}
\text{Sol.} \quad \frac{x + \frac{7}{2}}{2} > 0 &\Rightarrow x > -\frac{7}{2} \quad \ldots (i) \\
x + \frac{7}{2} &\neq 1 \Rightarrow x \neq -\frac{5}{2} \quad \ldots (ii) \\
\frac{x + 7}{2x + 3} &\neq 0 \\
&\Rightarrow x \neq -7, -\frac{3}{2} \quad \ldots (iii) \\
\text{log}_{\frac{7}{2}} \left(\frac{x + 7}{2x + 3}\right)^2 &\geq 0
\end{align*}
\]
\[
\Rightarrow \left( \frac{x+7}{2x+3} \right)^2 \geq 1
\]
\[
\Rightarrow \left( \frac{x+7}{2x+3} - 1 \right) \left( \frac{x+7}{2x+3} + 1 \right) \geq 0
\]
\[
\Rightarrow \left( \frac{-x+4}{2x+3} \right) \left( \frac{3x+10}{2x+3} \right) \geq 0
\]
\[
\Rightarrow \frac{(x-4)(3x+10)}{(2x+3)^2} \leq 0
\]
\[
\Rightarrow \frac{10}{3} \leq x \leq 4
\]