Time : 3 hrs. M.M. : 300

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.

(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.
JEE (Main)-2023 : Phase-2 (11-04-2023)-Evening

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Density ($\rho$) of a body depends on the force applied ($F$), its speed ($v$) and time of motion ($t$) by the relation $\rho = KF^a v^b t^c$, where $K$ is a dimensionless constant. Then

   (1) $a = 1, b = -4, c = -2$
   (2) $a = 2, b = -4, c = -1$
   (3) $a = -1, b = -4, c = 2$
   (4) $a = 1, b = 4, c = -2$

   Answer (1)
   Sol. $[ML^{-3}] = [MLT^{-2}]^a [LT^{-1}]^b [T]^c$
   
   $a = 1,$
   $a + b = -3,$
   $\Rightarrow b = -4,$
   also $-2a - b + c = 0$
   $c = -2$

2. In which of the following process, the internal energy of gas remains constant.
    (1) Isothermal    (2) Isochoric
    (3) Isobaric      (4) Adiabatic

   Answer (1)
   Sol. $T = \text{constant } \Rightarrow U = \text{constant}$

3. A particle is projected at an angle of 30° with ground with speed 40 m/s. The speed of particle after two seconds is (use $g = 10 \text{ m/s}^2$)
   
   (1) $20\sqrt{2} \text{ m/s}$
   (2) $20\sqrt{3} \text{ m/s}$
   (3) 20 m/s
   (4) $10\sqrt{3} \text{ m/s}$

   Answer (2)
   Sol. At $t = 2$ particle is at maximum height moving with $40\cos30^\circ \text{ m/s}$.

4. Potential at the surface of a uniformly charged non-conducting sphere is $V$. Then the potential at its centre is
   
   (1) 0
   (2) $V/2$
   (3) $2V$
   (4) $3V/2$

   Answer (4)
   Sol. $V = \frac{KQ}{2R^3} (3R^2 - r^2)$ at $r = R \Rightarrow V = \frac{KQ}{R}$
   
   at $r = 0, V_0 = \frac{3KQ}{2R} = \frac{3V}{2}$

5. If $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{A} - \vec{B} = 2\hat{j}$, then find $|\vec{B}|$.
   
   (1) 3
   (2) $3\sqrt{3}$
   (3) 2
   (4) $\sqrt{3}$

   Answer (1)
   Sol. $(2\hat{i} + 3\hat{j} + 2\hat{k}) - \vec{B} = 2\hat{j}$
   
   $\vec{B} = 2\hat{i} + 3\hat{j} + 2\hat{k}$
   $|\vec{B}| = 3$

6. The resultant gate is
   
   (1) NAND
   (2) NOR
   (3) OR
   (4) AND

   Answer (4)
   Sol. $(A+B) (A\cdot B) = (A\cdot AB) + A\cdot (AB)$
   
   = $AB + AB$
   = $(AB)$

7. For the given circuit diagram, find the current $I$.

   (1) $\frac{5}{16} \text{ A}$
   (2) $\frac{5}{48} \text{ A}$
   (3) $\frac{5}{12} \text{ A}$
   (4) $\frac{1}{16} \text{ A}$

   Answer (3)
   Sol. $i_{\text{battery}} = \frac{10}{2} = 5 \text{ A}$
   
   $I = i_{\text{battery}} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} = \frac{5}{12} \text{ A}$

8. If a nucleus is divided in ratio of 1 : $2^{1/3}$, then find ratio of velocity of the parts is
    (1) 2
    (2) $2^{1/3}$
    (3) $2^{2/3}$
    (4) $2^{-1/3}$

   Answer (2)
From conservation of momentum,
\[ m_0 \vec{v}_1 + 2^{\frac{1}{3}} m_0 \vec{v}_2 = 0 \]
\[ \Rightarrow \frac{\vec{v}_1}{\vec{v}_2} = 2^{\frac{1}{3}} \]

9. If electric field \( \vec{E} \) at an instant is \( 6.6 \hat{j} \text{ N/C} \) and the EM wave is propagating along positive \( x \)-direction then \( \vec{B} \) at that instant is given by
   (1) \( 2.2 \times 10^{-8} \hat{k} \text{ T} \)  
   (2) \( -2.2 \times 10^{-8} \hat{k} \text{ T} \)  
   (3) \( -0.5 \times 10^{-8} \hat{k} \text{ T} \)  
   (4) \( 19.8 \times 10^{-8} \hat{k} \text{ T} \)

   **Answer (1)**

Sol. \[ \vec{E} = \frac{C}{\varepsilon_0} \left| \vec{B} \right| \]
\[ \left| \vec{B} \right| = \frac{6.6}{3 \times 10^8} = 2.2 \times 10^{-8} \text{ T} \]
Also \( \vec{E} \times \vec{B} = \vec{C} \)

10. Find average speed of \( \text{N}_2 \) at 27\(^\circ\)C.
   (1) 476 m/s  
   (2) 470 m/s  
   (3) 480 m/s  
   (4) 490 m/s

   **Answer (1)**

Sol. \[ v = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8 \times 8.314 \times 300}{3.14 \times 28 \times 10^{-3}}} = 476 \text{ m/s} \]

11. A charge particle is projected inside along the axis of a long solenoid, then
   (a) Path will be straight line
   (b) There is no effect of magnetic field on charge
   (c) Path will be parabolic
   (d) Path will be circular

   (1) a, d  
   (2) a, b  
   (3) b, d  
   (4) a, b, d

   **Answer (2)**

Sol. \[ \vec{F} = q \vec{v} \times \vec{B} = 0 \]

12. Six identical small liquid drops are mixed together to form a bigger drop. The terminal velocity of bigger drop if terminal velocity of small drop is 10 m/s, will be
   (1) \( 10 \times (6)^{\frac{1}{3}} \text{ m/s} \)  
   (2) \( 10 \times (6)^{\frac{2}{3}} \text{ m/s} \)  
   (3) \( 5 \times (3)^{\frac{2}{3}} \text{ m/s} \)  
   (4) \( 10 \times (6)^3 \text{ m/s} \)

   **Answer (2)**

Sol. \( R = 6 \operatorname{in} \cdot \text{r} \)

Also, \[ \frac{v_b}{v_s} = \frac{R^2}{r^2} \quad (\because v_r \propto (\text{Radius})^2) \]
\[ v_0 = 10 \times (6)^{\frac{2}{3}} \]

13. A parallel plate capacitor \( C \) connected with a battery of voltage \( V_0 \). A close gaussian surface is shown by dotted boundary as shown. The electric flux through the surface is

 Sol. \[ f = 400 \left( \frac{360}{360 + 40} \right) = 360 \text{ Hz} \]
17. Find emf induces across the faces of given cube.

Answer (1)

Sol. \( \varepsilon_{\text{ind}} = Bv \)

\[ \varepsilon_{\text{ind}} = 1(8)(0.25) \]

\[ \varepsilon_{\text{ind}} = 2 \text{ volt} \]

18.

19.

20.

**SECTION - B**

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A body is rotating with kinetic energy \( E \). If angular velocity of body is increased to three times of initial angular velocity then kinetic energy becomes \( nE \). Find \( n \).

Answer (9)

Sol. K.E. = \( \frac{1}{2} I \omega^2 = E \)

\[ E_f = \frac{1}{2} I (3\omega)^2 = 9 \times \left( \frac{1}{2} I \omega^2 \right) \]

\[ E_f = 9E \]

22. Find power delivered by \( F \) at \( t = 10 \text{ s} \). Body start from rest.

Answer (30)

Sol. \( F = 0.5 \text{ g sin}30^\circ = 0.5 \text{ a} \Rightarrow F = 0.5 + 2.5 = 3 \text{ N} \)

\[ v_{10} = u + at \Rightarrow v_{10} = 0 + 1(10) = 10 \text{ m/s} \]

\[ P_{10} = Fv = 30 \text{ w} \]

23. A ray of light is incident on a plane mirror as shown in figure. Find the deviation of ray (in degree and clockwise direction).

Answer (240)

Sol. \( \delta = 180^\circ + 60^\circ = 240^\circ \) (clockwise)

24. Proton and electrons have equal kinetic energy, the ratio of de Broglie wavelength of proton and electron is \( \frac{1}{x} \). Find \( x \). (Mass of proton = 1849 times mass of electron)

Answer (43)

Sol. \( P = \sqrt{2Km} \)

\[ \lambda_p = \frac{h}{P_p} = \sqrt{\frac{2Km_p}{m_p}} = \sqrt{\frac{m_e}{m_p}} = \frac{1}{\sqrt{1849}} = \frac{1}{43} \]

25. Energy of hydrogen in ground state is \(-13.6 \text{ eV} \). The energy of He\(^+\) in first exited state is \(-13.6x\). Find the value of \( x \).

Answer (1)

Sol. For He\(^+\)

\[ E = \frac{-13.6Z^2}{2^2} = \frac{-13.6 \times 4}{4} = -13.6 \text{ eV} \]

26.

27.

28.

29.

30.
**SECTION - A**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

**Choose the correct answer:**

1. Which of the following has minimum boiling point?
   - (1) Na
   - (2) K
   - (3) Rb
   - (4) Cs

   **Answer (4)**

   **Sol.** Cs has minimum boiling point as boiling point of alkali metals decreases down the group.

2. Which of the following has maximum number of l.p. at central atom?
   - (1) \( \text{ClO}_3^– \)
   - (2) \( \text{SF}_4 \)
   - (3) \( \text{XeF}_4 \)
   - (4) \( \text{I}_3^- \)

   **Answer (4)**

   **Sol.** From the structures of the given species, it can be clearly seen that \( \text{I}_3^- \) has maximum number of lone pairs at central atom.

3. **Statement-1:** Sulphides are converted into oxides first.
   **Statement-2:** Because oxides can be reduced easily.

   - (1) Only 1st is correct
   - (2) Only 2nd is correct
   - (3) Both are correct
   - (4) Both are incorrect

   **Answer (3)**

   **Sol.** Sulphide ores are roasted for conversion to oxides before reduction. Oxides can be easily reduced as compared to sulphides.

4. Red ppt. by Benedict solution is
   - (1) Glucose
   - (2) RNA
   - (3) DNA
   - (4) Sucrose

   **Answer (1)**

   **Sol.** Benedict solution oxidises aldoses and ketoses to gluconic acid and itself gets reduced to red ppt. of \( \text{Cu}_2\text{O} \).

   \[
   \text{Glucose + Benedict solution} \rightarrow \text{Gluconic acid} + \text{Cu}_2\text{O} \downarrow \text{(Red)}
   \]

   DNA, RNA and Sucrose do not react with Benedict solution.

5. \( \left[ \text{Fe}(\text{H}_2\text{O})_6 \right]^{3+}, \left[ \text{Fe}(\text{CN})_6 \right]^{-3} \) magnetic spin only magnetic moment is respectively

   - (1) 8.87 and 6.92
   - (2) 5.98 and 1.732
   - (3) 6.92 and 6.92
   - (4) 3.87 and 1.732

   **Answer (2)**

   **Sol.** Both complexes have \( d^8 \) configuration

   \[
   \left[ \text{Fe}(\text{H}_2\text{O})_6 \right]^{3+} \rightarrow 5 \text{ unpaired electrons} \quad \mu = \sqrt{35} \text{ B.M.}
   \]

   \[
   \left[ \text{Fe}(\text{CN})_6 \right]^{-3} \rightarrow 1 \text{ unpaired electron} \quad \mu = \sqrt{3} \text{ B. M.}
   \]

6. **Statement 1:** Nylon-6 is made by Caprolactum
   **Statement 2:** LDP is made by \( \text{TiCl}_4 \) & \( \text{Al(ET)}_3 \)

   - (1) Only 1st is correct
   - (2) Only 2nd is correct
   - (3) Both are correct
   - (4) Both are incorrect

   **Answer (1)**
Sol. TiCl₄ + Al(ET)₃ is used as a catalyst in preparation of HDP.

7. Consider the following change:

\[ \text{[NiBr}_2\text{Cl}_2]^{2-} \rightarrow [\text{PtCl}_2\text{Br}_2]^{2-} \]

During the above change, which of the following properties does not change?

1. Geometrical isomerism
2. Structure
3. Optical activity
4. Splitting energy

Answer (3)

Sol. \[ \text{[NiBr}_2\text{Cl}_2]^{2-} \rightarrow \] This complex species is tetrahedral as Br⁻ & Cl⁻ are weak field ligands.

\[ \text{[PtBr}_2\text{Cl}_2]^{2-} \rightarrow \] As Pt belongs to 5d series, this complex species is square planar.

Splitting energy will be different as central atom is different.

Both the complex species are optically inactive.

\[ \text{[NiBr}_2\text{Cl}_2]^{2-} \], being tetrahedral does not show G.I.

\[ \text{[PtBr}_2\text{Cl}_2]^{2-} \] shows two G.I.

8. \[ A \xrightarrow{K} B \]

Follows first order kinetics w.r.t. A and B, Both i.e. \( r = K[A]^1[B]^1 \)

<table>
<thead>
<tr>
<th>( r )</th>
<th>( [A] )</th>
<th>( [B] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>( X )</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>40</td>
<td>0.8</td>
<td>( Y )</td>
</tr>
</tbody>
</table>

Find out “K” and “Y”

(1) 80, 2
(2) 80, 1
(3) 80, 0.125
(4) 40, 0.125

Answer (3)

Sol. \([A] : 4 \text{ times} \Rightarrow \text{rate 4 times} \)

\( \Rightarrow X = 80 \)

9. \[ \text{OH} \quad \xrightarrow{\text{NaNO}_2, \text{HCl}} X \quad \xrightarrow{\text{HNO}_3} Y \quad \xrightarrow{(\text{NH}_4)_2\text{S}} Z \]

Compound Z is

(1) (2) (3) (4)

Answer (2)

Sol. \[ \text{OH} \quad \xrightarrow{\text{NaNO}_2, \text{HCl or HNO}_3} X \quad \xrightarrow{\text{HNO}_3} Y \quad \xrightarrow{(\text{NH}_4)_2\text{S}} Z \]

10. What is the chemical formula of freon gas?

(1) \( \text{C}_2\text{Cl}_2\text{F}_4 \)
(2) \( \text{C}_2\text{F}_2\text{H}_4 \)
(3) \( \text{CHF}_3 \)
(4) \( \text{CCl}_2\text{F}_2 \)

Answer (4)

Sol. The chemical formula of freon gas is \( \text{CCl}_2\text{F}_2 \).

11. 2 gm of x is present in 1 mole of H₂O. Find the mass % of x.

(1) 10%
(2) 20%
(3) 5%
(4) 7%

Answer (1)

Sol. Mass % of x = \( \frac{2}{20} \times 100 = 10 \)
12. **Assertion:**

\[ \text{Cl} \text{CH}_3 \xrightarrow{\text{N}_2\text{H}_4\text{KOH}} \text{Cl} \text{CH}_2\text{CH}_3 \]

**Reason:** Wolf Kirshner reduction is used for reduction of \( \text{C} \) into \( \text{CH}_2 \).

(1) Assertion and Reason both are correct and Reason is correct explanation of Assertion
(2) Assertion and Reason both are correct but the Reason is not correct explanation of Assertion
(3) Assertion and Reason both are incorrect
(4) Assertion is incorrect and reason is correct statement

**Answer (4)**

**Sol.**

Because heating in the presence of base results in elimination

13. Glucose is added in 100 gm of water. Lowering in vapor pressure is 0.2 mm Hg. Vapour pressure of pure water is 54.2 mm Hg. Then weight of glucose is

(1) 3.70 gm  (2) 4.92 gm  
(3) 6.73 gm  (4) 8.74 gm

**Answer (1)**

**Sol.**

\[
\frac{0.2}{54} = \frac{n_{\text{glucose}}}{100/18} \\
0.2 = \frac{n_{\text{glucose}}}{54/18} \\
\frac{0.2}{54} \times 100 \times 18 = 3.70 \text{ gm}
\]

14. Which of the following will not give precipitate with \( \text{AgNO}_3(\text{aq.}) \)

(1) [image]
(2) [image]
(3) [image]
(4) [image]

**Answer (2)**

**Sol.** Compounds which result in the formation of stable carbocation intermediate will give precipitate with \( \text{aq. AgNO}_3 \)

15. Least stable Hydride is

(1) \( \text{HF} \)  (2) \( \text{LiH} \)  
(3) \( \text{BeH}_2 \)  (4) \( \text{NaH} \)

**Answer (3)**

**Sol.** \( \text{BeH}_2 \) is least stable as it has significant covalent character and is an electron-deficient hydride.

16. Find the root mean square velocity for Nitrogen gas at 27ºC (in m/sec)

(1) 426  (2) 517  
(3) 327  (4) 646

**Answer (2)**

**Sol.**

\[
\nu = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 300}{28 \times 10^{-3}}} = 516.95 \\
\approx 517 \text{ m/sec}
\]
17. **Assertion (A):** Glycine react with Cl\(_2\) in the presence of red P to give optically active compound.

**Reason (R):** Compound containing two chiral centres is always optically active.

- (1) Both (A) & (R) are correct & (R) is the correct explanation of (A).
- (2) Both (A) & (R) are correct & (R) is not the correct explanation of (A).
- (3) (A) is correct, (R) is incorrect statement.
- (4) (A) & (R), both are incorrect.

**Answer:** (3)

**Sol.**

\[ H_2N\text{—CH}_2\text{—COOH} \xrightarrow{\text{RedP}} H_2N\text{—CH—COOH} \]

Contains chiral centre.

---

21. How many of the following are intrinsic properties?

- Gibbs free energy, \( E_{\text{cell}} \), Volume, Molarity

**Answer:** (02.00)

**Sol.** \( E_{\text{cell}} \) and molarity are intrinsic properties. But Gibb’s Free Energy and Volume are extrinsic properties.

22. 2-Chloro-1-butene \( \xrightarrow{\text{HCl}} \) Number of Isomeric product possible are?

(excluding rearranged products)

**Answer:** (03.00)

---

24. \( P_4 + 8\text{SOCl}_2 \rightarrow 4\text{PCl}_3 + x \text{SO}_2 + y \text{S}_2\text{Cl}_2 \)

\( x + y = \) _________

**Answer:** (06)

**Sol.**

\[ P_4 + 8\text{SOCl}_2 \rightarrow 4\text{PCl}_3 + 4\text{SO}_2 + 2\text{S}_2\text{Cl}_2 \]

\( x = 4 \)

\( y = 2 \)

\( x + y = 6 \)
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Using all he letters of the word MATHS, then rank of the word THAMS is
   (1) 101  (2) 102  (3) 103  (4) 104

Answer (3)

Sol.  

\[
\text{THAMS} \quad \begin{vmatrix}
4 & 1 & 0 & 0 & 0 \\
4 & 3 & 2 & 1 & 4
\end{vmatrix}
\]

\[
= 4 \times 4! + 1 \times 3! + 1 \\
= 96 + 6 + 1 = 103
\]

\[
\begin{vmatrix}
0 + 1 & x & x \\
x & x + \lambda & x \\
x & x & x + \lambda^2
\end{vmatrix} = 9 \times 81, \text{ then } \lambda = \frac{3}{2}
\]

are roots of

(1) \(4x^2 + 24x - 27 = 0\)

(2) \(4x^2 - 24x + 27 = 0\)

(3) \(4x^2 - 24x - 27 = 0\)

(4) \(4x^2 + 24x + 27 = 0\)

Answer (2)

Sol. Put \(x = 0\) in the given equation

\[
\begin{vmatrix}
1 & 0 & 0 \\
0 & \lambda & 0 \\
0 & 0 & \lambda^2
\end{vmatrix} = 9 \times 81
\]

\[
\Rightarrow \lambda^3 = \frac{9^6}{2^3}
\]

\[
\lambda = \frac{9}{2}
\]

\[
\Rightarrow \lambda = \frac{3}{2}
\]

\[
x^2 - \left(\frac{9}{2} + \frac{3}{2}\right)x + \frac{9 \times 3}{2} = 0
\]

\[4x^2 - 24x + 27 = 0\]

3. \(\frac{dy}{dx} + \frac{5}{x(1 + x^5)} = \frac{(1 + x^5)^2}{x^2}\). If \(y(1) = 2\), then the value of \(y(2)\) is
   (1) \(\frac{693}{128}\)  (2) \(\frac{697}{128}\)  (3) \(\frac{637}{128}\)  (4) \(\frac{627}{128}\)

Answer (1)

Sol. I.F. = \(e^{\int \frac{5}{x(1 + x^5)} \, dx} = e^{\int \frac{5x^{-6}}{(x^5 + 1)} \, dx}\)

\[
e^{-\ln(x^{-5} + 1)} = \frac{1}{x^{-5} + 1} = \frac{x^5}{x^5 + 1}
\]

\[
y \cdot \frac{x^5}{x^5 + 1} = \int \frac{(1 + x^5)^2}{x^7} \cdot \frac{x^5}{x^5 + 1} \, dx
\]

\[
= -\frac{1}{x^4 + 4} + C
\]

\[
y(1) = 2 \Rightarrow 2 \left(\frac{1}{2}\right) = -1 + \frac{1}{4} + C
\]

\[
\Rightarrow C = \frac{7}{4}
\]

Put \(x = 2\)

\[
\Rightarrow y \left(\frac{32}{33}\right) = -\frac{1}{2} + 4 + \frac{7}{4}
\]

\[
\Rightarrow y = \frac{693}{128}
\]

4. The domain of the function \(f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}\) is
   (1) \((-\infty, 3] \cup [6, \infty)\)  (2) \((-\infty, -3] \cup (2, \infty)\)
   (3) \((-\infty, 3] \cup [5, \infty)\)  (4) \((-\infty, -3] \cup [6, \infty)\)

Answer (4)

Sol. \([x]^2 - 3[x] - 10 > 0\)

\(([x] + 2)([x] - 5) > 0\)

\([x] < -2 \text{ OR } [x] > 5\)

\([x] \leq -3 \text{ OR } [x] \geq 6\)

\(x < -2 \text{ OR } x \geq 6\)

\(x \in (-\infty, -2] \cup [6, \infty)\)
5. Let mean and variance of the data 1, 2, 4, 5, \(x\), \(y\) are 5 and 10 respectively. Then mean deviation about the mean of data is

\[
\begin{align*}
(1) \quad & \frac{5}{2} \\
(2) \quad & \frac{7}{2} \\
(3) \quad & \frac{5}{6} \\
(4) \quad & \frac{7}{6}
\end{align*}
\]

Answer (1)

Sol. \(12 + x + y = 30 \Rightarrow x + y = 18\)
and \(\frac{x^2 + y^2 + 46}{6} = (5)^2 = 10\)
\[\therefore \frac{x^2 + y^2 + 46}{6} = 10 + 25\]
\[x^2 + y^2 = 164\]
\[\therefore \quad x = 10, \ y = 8\]
Now, mean deviation about mean
\[= \frac{4 + 2 + 1 + 0 + 5 + 3}{6} = \frac{15}{6} = \frac{5}{2}\]
6. If \(a + b + c + d = 11\) \((a, b, c, d > 0)\) then maximum value of \(a^5b^3c^2d\) is 3750 \(\beta\) the \(\beta\) is

\[
\begin{align*}
(1) \quad & 90 \\
(2) \quad & 115 \\
(3) \quad & 120 \\
(4) \quad & 85
\end{align*}
\]

Answer (1)

Sol. Assume numbers to be
\[
\begin{align*}
\frac{a}{5} \quad \frac{a}{5} \quad \frac{a}{5} \quad \frac{a}{5} \quad \frac{b}{5} \quad \frac{b}{5} \quad \frac{b}{5} \quad \frac{b}{5} \quad \frac{c}{3} \quad \frac{c}{3} \quad \frac{c}{3} \quad \frac{c}{3} \quad \frac{d}{2} \quad \frac{d}{2}
\end{align*}
\]
Now apply AM \(\geq\) GM
\[
\frac{a + a + a + a + b + b + b + c + c + c + c + d}{5 + 5 + 5 + 5 + 5 + 5 + 5 + 3 + 3 + 3 + 3 + 2 + 2 + 2 + 2}
\geq \frac{a^5b^3c^2d}{5^5 \cdot 3^2 \cdot 2^2}^{11}
\]
\[
\therefore \quad a^5b^3c^2d \leq 5^5 \cdot 3^2 \cdot 2^2
\]
\[\therefore \quad \beta = 90
\]
7. \(\frac{4x - 5}{2x}^{2022}\) then \(1011^{th}\) term from end is equal to \(1024\) times \(1011^{th}\) term from starting then \(|x|\) is

\[
\begin{align*}
(1) \quad & \frac{16}{7} \\
(2) \quad & \frac{16}{5} \\
(3) \quad & \frac{5}{16} \\
(4) \quad & \frac{8}{5}
\end{align*}
\]

Answer (3)

Sol. \(1011^{th}\) term from end = \(1011^{th}\) term from beginning
\[
\therefore \quad r = 1010 \quad \left(\frac{5 - 4x}{2x}\right)^{2022}
\]
\[
T_{1011} = 2^{2022} \cdot C_{1010} \left(\frac{5}{2x}\right)^{1012} \left(\frac{4x}{5}\right)^{1010}
\]
\[
1011 \text{ term from starting } \left(\frac{4x - 5}{2x}\right)^{2022}
\]
\[
T_{1011} = 2^{2022} \cdot C_{1010} \left(\frac{4x}{5}\right)^{1012} \left(\frac{5}{2x}\right)^{1010}
\]
Now,
\[
2^{2022} \cdot C_{1010} \left(\frac{5}{2x}\right)^{1012} \left(\frac{4x}{5}\right)^{1010} = 1024
\]
\[
= 2^{10}
\]
\[
\frac{25}{8x^2} = 2^6
\]
\[
x^2 = \frac{25}{2^8} = \frac{5}{2^4}
\]
8. A circle with center at \((2, 0)\) and maximum radius \(r\) is inscribed in the ellipse \(\frac{x^2}{36} + \frac{y^2}{9} = 1\). The value of \(12r^2\) is

\[
\begin{align*}
(1) \quad & 108 \\
(2) \quad & 172 \\
(3) \quad & 83 \\
(4) \quad & 92
\end{align*}
\]

Answer (4)

Sol. Equation of normal at \(P(6 \cos \theta, 3 \sin \theta)\) is
\[
(6 \sec \theta - 3 \cos \sec \theta) y = 27
\]
It passes through \((2, 0)\)
\[
12 \sec \theta = 27
\]
\[
\cos \theta = \frac{4}{9}, \ \sin \theta = \frac{\sqrt{65}}{9}
\]
\[
P \left( \frac{8 \sqrt{65}}{3}, \frac{3}{3} \right)
\]
\[
r = \sqrt{\left(\frac{8}{3} - 2\right)^2 + \left(\frac{\sqrt{65}}{3}\right)^2} = \sqrt{69} \cdot \frac{3}{3}
\]
\[
12r^2 = 12 \times \frac{69}{9} = 92
\]
9. \( f : R \rightarrow R \) be a continuous non-constant function
and \( \int_{0}^{\pi/2} f(\sin 2x) \sin x \, dx + \int_{0}^{\pi/4} f(\cos 2x) \cos x \, dx = 0 \)
then \( \alpha \) is equal to
(1) \( \sqrt{2} \) \hspace{1cm} (2) \( \sqrt{3} \) \hspace{1cm} (3) \( -\sqrt{2} \) \hspace{1cm} (4) \( -\sqrt{3} \)

**Answer (3)**

\[ \int_{0}^{\pi/2} f(\sin 2x) \sin x \, dx + \int_{0}^{\pi/4} f(\cos 2x) \cos x \, dx = 0 \]

<table>
<thead>
<tr>
<th>0</th>
<th>( t + \frac{\pi}{4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(\sin 2x) \sin \left( t + \frac{\pi}{4} \right) , dx )</td>
<td>( f(\cos 2x) \cos \left( t + \frac{\pi}{4} \right) , dx )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Here \( \int_{a}^{b} f(x) \, dx = \int_{a}^{b} f(a - x) \, dx \)

Let \( x = t + \frac{\pi}{4} \)
\[ \Rightarrow \int_{0}^{\pi/4} f(\cos 2x) \sin \left( \frac{\pi}{4} - x \right) \, dx + \int_{0}^{\pi/4} f(\cos 2t) \sin \left( t + \frac{\pi}{4} \right) \, dx \]
\[ + \alpha \int_{0}^{\pi/4} f(\cos 2x) \cos x \, dx = 0 \]

\[ \Rightarrow \int_{0}^{\pi/4} f(\cos 2x) \left[ \sin \left( \frac{\pi}{4} - x \right) + \sin \left( x + \frac{\pi}{4} \right) + \alpha \cos x \right] \, dx = 0 \]
\[ \Rightarrow \int_{0}^{\pi/4} f(\cos 2x) \left( \sqrt{2} + \alpha \right) \cos x \, dx = 0 \]
\[ \Rightarrow f(\cos 2x) \text{ and } \cos x \text{ is not zero in } \left( 0, \frac{\pi}{4} \right). \]
\[ \Rightarrow \sqrt{2} + \alpha = 0 \]
\[ \Rightarrow \alpha = -\sqrt{2}. \]

10. If the ratio of three consecutive terms is 1:3:5 in the expansion of \( (1 + x)^{n+2} \). Then sum of consecutive terms is
(1) 41 \hspace{1cm} (2) 64 \hspace{1cm} (3) 63 \hspace{1cm} (4) 43

**Answer (3)**

11. The converse of the statement \( (\neg p \land q) \Rightarrow r \) is
(1) \( r \Rightarrow (\neg p \land q) \) \hspace{1cm} (2) \( r \Rightarrow (p \lor \neg q) \) \hspace{1cm} (3) \( \neg r \Rightarrow (p \lor \neg q) \) \hspace{1cm} (4) \( \neg r \Rightarrow (\neg p \land q) \)

**Answer (1)**

12. If \( a, b, c, d \) are coplanar reactor then value of \( \left[ \begin{array}{cccc} a & b & c & d \end{array} \right] \) is
(1) \( [\vec{b} \vec{d} \vec{c}] + [\vec{a} \vec{d} \vec{b}] + [\vec{a} \vec{d} \vec{c}] \)
(2) \( [\vec{b} \vec{d} \vec{c}] + [\vec{a} \vec{b} \vec{d}] + [\vec{a} \vec{d} \vec{c}] \)
(3) \( [\vec{b} \vec{c} \vec{d}] + [\vec{a} \vec{b} \vec{d}] + [\vec{a} \vec{d} \vec{c}] \)
(4) \( [\vec{b} \vec{c} \vec{d}] + [\vec{a} \vec{b} \vec{d}] + [\vec{a} \vec{d} \vec{c}] \)

**Answer (3)**

\[ [\vec{b} \vec{a} \vec{c} \vec{d}] = 0 \]
\[ (\vec{b} \vec{a}) \cdot ((\vec{c} \vec{a}) \times (\vec{d} \vec{a})) = 0 \]
\[ (\vec{b} \vec{a}) \cdot (\vec{c} \times \vec{d} - \vec{c} 	imes \vec{a} - \vec{a} \times \vec{d}) = 0 \]
\[ \vec{b} \vec{c} \vec{d} - [\vec{b} \vec{c} \vec{a}] - [\vec{b} \vec{a} \vec{d}] - [\vec{a} \vec{c} \vec{d}] = 0 \]
\[ \Rightarrow \left[ \begin{array}{cccc} a & b & c & d \end{array} \right] = [\vec{b} \vec{c} \vec{d}] - [\vec{b} \vec{a} \vec{d}] - [\vec{a} \vec{c} \vec{d}] \]
13. \( f(x) = \begin{cases} e^{\min(x^2, ax^3)}, & x \in (0, 1) \\ e^{x-\ln x}, & x \in [1, 2) \end{cases} \) then find \( \int_0^2 xf(x) \, dx \)

(1) \( 2e - \frac{1}{2} \)  
(2) \( 2e + \frac{1}{2} \)  
(3) \( 4e - \frac{1}{2} \)  
(4) \( 4e + \frac{1}{2} \)

Answer (1)

Sol. \( f(x) = \begin{cases} e^{x^2}, & x \in (0, 1) \\ e, & x \in [1, 2) \end{cases} \)

\[
\int_0^2 xf(x) \, dx = \int_0^1 x \cdot e^{x^2} \, dx + \int_1^2 x \cdot e \, dx
\]

\( x^2 = t \)

\( 2x \, dx = dt \)

\[
= \frac{1}{2} \left[ e^t \right]_0^1 + e \left[ \frac{x^2}{2} \right]_1^2
\]

\( = \frac{1}{2} (e - 1) + \frac{3}{2} e 
\]

\( = 2e - \frac{1}{2} \)

14. The area between the curve \( y = 2x^2 + 1 \) and tangent to it at (1, 3) and \( x + y = 1 \) is

(1) \( \frac{1}{15} \)  
(2) \( \frac{1}{60} \)  
(3) \( \frac{4}{15} \)  
(4) \( \frac{8}{3} \)

Answer (3)

Sol.

Tangent at (1, 3) \( \frac{y + 3}{2} = 2x + 1 \)  
\( y = 4x - 1 \)

\( \therefore \) Area

\[
\int_0^{\sqrt{\frac{5}{14}}} \left( 2x^2 + 1 - (1 - x) \right) \, dx + \int_0^{\frac{1}{2}} \left( 2x^2 + 1 \right) \, dx - \int_0^{\frac{1}{2}} \left( 2x^2 - 4x + 2 \right) \, dx
\]

\[
= \int_0^{\sqrt{\frac{5}{14}}} \left( 2x^2 + 1 \right) \, dx + \int_0^{\frac{1}{2}} \left( 2x^2 - 4x + 2 \right) \, dx
\]

\[
= \left[ \frac{2x^3}{3} + \frac{x^2}{2} \right]_0^{\sqrt{\frac{5}{14}}} + \left[ \frac{2x^3}{3} - \frac{4x^2}{2} + 2x \right]_0^{\frac{1}{2}}
\]

\[
= \frac{92}{750} + \frac{144}{1000} = \frac{368 + 432}{3000} = \frac{800}{3000} = \frac{4}{15}
\]

15. Angle between line \( x = \frac{y - 1}{2} = z - \frac{3}{r} \) and plane \( x + 2y + 3z + 4 = 0 \) is \( \cos^{-1} \left( \frac{5}{\sqrt{14}} \right) \) then point of intersection of line and plane is

(1) \((-15, -23, -11)\)  
(2) \(\left( \frac{15}{7}, -\frac{23}{7}, \frac{11}{7} \right)\)  
(3) \((15, 23, 11)\)  
(4) \(\left( -\frac{15}{7}, -\frac{23}{7}, \frac{11}{7} \right)\)

Answer (4)

Sol. \( \sin \theta = \frac{1 + 4 + 3r}{\sqrt{14 + 5 + r^2}} \)

\( \cos^{-1} \left( \frac{5}{\sqrt{14}} \right) = \sin^{-1} \left( \frac{3}{\sqrt{14}} \right) = \sin^{-1} \left( \frac{5 + 3r}{\sqrt{14(5 + r^2)}} \right) \)

\[
\frac{3}{\sqrt{14}} = \frac{5 + 3r}{\sqrt{5 + r^2} \sqrt{14}}
\]

\[
3\sqrt{5 + r^2} = 5 + 3r
\]

\[
9(5 + r^2) = 25 + 9r^2 + 30r
\]

\( \Rightarrow \) 45 = 25 + 30r

\( \Rightarrow \) 30r = 30

\( \Rightarrow \) \( r = \frac{2}{3} \)

Let the point on line is \( P(3k, 6k + 1, 2k + 3) \)

\( 3k + 12k + 2 + 6k + 9 + 4 = 0 \)

\( \Rightarrow \) 21k = -15

\( \Rightarrow k = -\frac{5}{7} \)

\( \therefore P\left( \frac{-15}{7}, -\frac{23}{7}, \frac{11}{7} \right) \)
16. 
17. 
18. 
19. 
20. 

SECTION - B
Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If \( e^{3x} - e^{2x} - 3e^{x} - e^{x} + 1 = 0 \), then number of solutions of above equation is

Answer (2)

Sol. \( e^{3x} - e^{2x} - 3e^{x} - e^{x} + 1 = 0 \),
\[ \Rightarrow \left( e^{3x} + \frac{1}{e^{x}} \right) - \left( e^{2x} + \frac{1}{e^{2x}} \right) = 0 \]
\[ \Rightarrow \left( e^{2x} + \frac{1}{e^{x}} \right)^2 - \left( e^{2x} + \frac{1}{e^{2x}} \right) = 5 \]
\[ \Rightarrow e^{2x} + \frac{1}{e^{2x}} - 5 = 0 \]
\[ t = \frac{1 \pm \sqrt{1 + 20}}{2} \]
\[ = \frac{1 \pm \sqrt{21}}{2} \]
\[ \frac{1 - \sqrt{21}}{2} \] is rejected
\[ \therefore t = \frac{1 + \sqrt{21}}{2} \]
\[ \Rightarrow e^{2x} + \frac{1}{e^{2x}} = \frac{1 + \sqrt{21}}{2} \Rightarrow 2 \text{ values of } e^{2x} \text{ possible} \]
\[ \therefore 2 \text{ real solution} \]

22. If \( f(1) + f(2) = f(4) - 1 \) and a function from \( A \) to \( B \) is defined where \( A = \{1, 2, 3, 4, 5\} \), \( B = \{1, 2, 3, 4, 5, 6\} \). Find the numbers of function with such relation.

Answer (360)

Sol. \( f(4) = f(1) + f(2) + 1 \)
\[ \Rightarrow f(1) + f(2) + 1 \leq 6 \]
\[ f(1) + f(2) \leq 5 \]
Possible cases
1 \( \{1,2,3,4\} \rightarrow 4 \)
2 \( \{1,2,3\} \rightarrow 3 \)
3 \( \{1,2\} \rightarrow 2 \)
4 \( \{1\} \rightarrow \frac{1}{10} \)

\( f(5), f(3) \) can be filled in 6 ways
Total functions = \( 10 \times 6 \times 6 = 360 \)

23. For a biased coin, the probability of getting head is \( \frac{1}{4} \). It is tossed \( n \) times till we get head. Given a quadratic equation \( 64x^2 + 2nx + 1 = 0 \). If the probability that the quadratic equation has no real roots is \( \frac{P}{Q} \) (where \( P \) and \( Q \) are coprime), then the value of \( Q - P \) is

Answer (2187)

Sol. \( (2n)^2 - 4 \times 64 < 0 \Rightarrow n < 8 \Rightarrow n \leq 7 \)
Required probability
\[ \sum_{n=1}^{7} \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4} + \left( \frac{3}{4} \right)^{2} + \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} + \left( \frac{3}{4} \right)^{3} + \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \]
\[ = \frac{1}{4} \cdot \left( 1 - \left( \frac{3}{4} \right)^{7} \right) = \frac{4^{7} - 3^{7}}{4^{7}} = \frac{P}{Q} \]
\[ Q - P = 3^{7} = 2187 \]

24. 
25. 
26. 
27. 
28. 
29. 
30.