IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

   (i) **Section-A**: This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

   (ii) **Section-B**: This section contains 10 questions. In Section-B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.
**SECTION - A**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

1. If the height of the tower used for L.O.S is increased by 21% then percentage change in range is
   (1) 10% (2) 21% (3) 19% (4) 42%

   **Answer (1)**
   **Sol.**
   \[ l' = \sqrt{2Rh} \]
   \[ l' = \sqrt{2R(1.21h)} = 1.1\sqrt{2Rh} \]
   \[ \frac{l' - l}{l} = 0.1 = 10\% \]

2. Select the correct graph showing the difference \( d \) between total energy and potential energy of a particle in linear SHM with position \( x \) of the particle \( (x = 0 \) is the mean position)

   \( (1) \)
   \( (2) \)
   \( (3) \)
   \( (4) \)

   **Answer (1)**
   **Sol.**
   \[ d = T.E - P.E \]
   \[ = K.E \quad (\because \text{Total energy is conserved}) \]
   \[ = \frac{1}{2} m_0^2 (A^2 - x^2) \]

3. A dipole of charge 0.01 C and separation 0.4 mm, is placed in an electric field of strength 10 dyne/C. Find the maximum torque exerted on the dipole in the field.
   (1) \( 4 \times 10^{-9} \) Nm  (2) \( 2 \times 10^{-10} \) Nm
   (3) \( 4 \times 10^{-10} \) Nm  (4) \( 2 \times 10^{-9} \) Nm

   **Answer (3)**
   **Sol.**
   \[ P = 0.01 \times 0.4 \times 10^{-3} = 4 \times 10^{-6} \ \text{cm} \]
   \[ E = 10 \times 10^{-5} \ \text{N} \]
   \[ \tau = |\vec{P} \times \vec{E}| = 4 \times 10^{-6} \times 10 \times 10^{-5} = 4 \times 10^{-10} \ \text{Nm} \]

4. Two bodies having same linear momentum have ratio of kinetic energy as 16 : 9. Find the ratio of masses of these bodies.
   (1) \( \frac{9}{16} \)  (2) \( \frac{4}{3} \)
   (3) \( \frac{3}{4} \)  (4) \( \frac{16}{9} \)

   **Answer (1)**
   **Sol.**
   \[ \frac{P^2}{2m} = K \]
   \[ \Rightarrow \frac{K_1}{K_2} = \left( \frac{m_2}{m_1} \right) = \frac{16}{9} = \left( \frac{m_2}{m_1} \right) \]

5. What is the centre of gravity of semi-circular disc of radius \( R \)?
   (1) \( \frac{2R}{\pi} \)  (2) \( \frac{4R}{3\pi} \)
   (3) \( \frac{R}{2} \)  (4) \( \frac{3R}{8} \)

   **Answer (2)**
   **Sol.**
   \[ y_{cm} = \int_{0}^{R} \frac{dmy}{M} = \int_{0}^{\frac{\pi R^2}{2}} \frac{m}{M} r^2 dr = \frac{4}{\pi R^2} \int_{0}^{R} r^2 dr = \frac{4R}{3\pi} \]

6. The work function for two metals are 9 eV and 4.5 eV. Find the approx. difference between their threshold wavelengths. (Use \( hC = 1240 \text{ eV-nm} \))
   (1) 138 nm  (2) 130 nm
   (3) 112 nm  (4) 145 nm

   **Answer (1)**
   **Sol.**
   \[ \Delta \lambda = \left[ \frac{1240}{4.5} - \frac{1240}{9} \right] \text{nm} \]

7. In the given figure, find the speed of bird as seen by fish.
10. For the given radioactive decay
\[ {}^{298\text{y}}_{94}X \rightarrow {}^{294\text{y}}_{92}Y + {}^{4\text{a}}_{\alpha} + Q\text{-value}, \]

binding energy per nucleon of \( X \), \( Y \) and \( \alpha \) are \( a \), \( b \) and \( c \). The \( Q\)-value is equal to

(1) \( (294b + 4c - 298a) \)
(2) \( (92b + 2c - 94a) \)
(3) \( (294b + 4c + 298a) \)
(4) \( (92b + 2c + 94a) \)

Answer (1)

Sol. \( Q\)-value = (B.E.\(_\text{product}\) – (B.E.)\(_\text{reaction}\)

11. Energy of He\(^+\) in 2\(^{nd}\) orbit is \(-13.6\) eV then energy of Be\(^{++}\) in \( n = 4 \).

(1) \(-3.4\) eV
(2) \(-27.2\) eV
(3) \(-13.6\) eV
(4) \(-54.4\) eV

Answer (3)

Sol. \( E = -13.6 \frac{2^2}{n^2} \) eV

For He\(^+\) (\( Z = 2, n = 2 \)), \( E = -13.6 \left( \frac{2^2}{2^2} \right) \) eV
\[ = -13.6 \text{ eV} \]

For Be\(^{++}\) (\( Z = 4, n = 4 \)), \( E = -13.6 \left( \frac{4^2}{4^2} \right) \) eV
\[ = -13.6 \text{ eV} \]

12. A line charge of linear charge density \( \lambda \) and a large non-conducting sheet of charge density \( \sigma \) are placed parallel to each other as shown. Find ratio of electric field at \( A \) to that at \( B \).

Answer (2)

Sol. \( \frac{E_A}{E_B} = \frac{\lambda}{\lambda + 4\sigma} \cdot \frac{3\pi}{2\pi} = \frac{3\pi}{2\pi} \cdot \frac{\lambda}{\lambda + 4\sigma} \)

(1) \( \frac{3(\lambda - 3\sigma)}{4(\lambda + 4\sigma)} \)
(2) \( \frac{4(\lambda - 3\sigma)}{3(\lambda - 4\sigma)} \)
(3) \( \frac{2(\lambda - 4\sigma)}{3(\lambda - 3\sigma)} \)
(4) \( \frac{3(\lambda - 4\sigma)}{2(\lambda - 3\sigma)} \)
13. Which of the following show time varying magnetic field?
   (1) Linearly varying
   (2) Permanent magnet
   (3) Antenna signal
   (4) Constant electric field

   Answer (3)

   Sol. Antenna signal carries sinusoidal EM wave where \( E \) and \( B \) both varying with time.

14. Find the apparent depth of bottom of beaker shown in figure, filled with water and oil.

   ![Beaker Diagram]

   (1) \( \frac{5H}{8} \)  
   (2) \( \frac{4H}{5} \)  
   (3) \( \frac{3H}{4} \)  
   (4) \( \frac{7H}{8} \)

   Answer (1)

   Sol. \( d_{app} = \frac{H}{2} \div \frac{H}{2} = \frac{5H}{8} \)

15. If a particle is moving in a uniform circular motion of radius 1 m, is having velocity \( 3\hat{j} \) m/s at point \( B \). What are the velocity (\( v \)) and acceleration (\( a \)) at diametrically opposite point \( A \).

   (1) \( v_A = 3\hat{j} \) m/s  
   \( a_A = -9\hat{i} \) m/s²

   (2) \( v_A = -3\hat{j} \) m/s  
   \( a_A = 9\hat{i} \) m/s²

   (3) \( v_A = -3\hat{i} \) m/s  
   \( a_A = +9\hat{j} \) m/s²

   (4) \( v_A = +3\hat{i} \) m/s  
   \( a_A = +9\hat{j} \) m/s²

   Answer (2)

   Sol. \( v_A = -3\hat{j} \) m/s  

   \( a_A = -9\hat{i} \) m/s²

16. The input signal is given below for the circuit

   ![Circuit Diagram]

   Input graph of A and B is

   Pick the correct output graph for the circuit.

   (1) Output
   (2) Output
   (3) Output
   (4) Output

   Answer (1)

   Sol. \( \overrightarrow{A \cdot B} = \overrightarrow{A} + \overrightarrow{B} = (A + B) = OR \text{ gate} \)
17. Find the displacement of point A on the top of the disc rolling without slipping on horizontal surface with angular speed $\omega$, in half rotation.

18. A point $R$ is at $\left(\frac{5}{8}, \frac{3}{8}, \frac{1}{8}\right)$ and a plane mirror is placed on $xy$ plane such that normal to the plane mirror from $R$ intersect at point $P$ on mirror. Find distance of image formed by the mirror and object.

19. 

20. 

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If a wire of resistance $R$ is connected across $V_0$, then power is $P_0$. The wire is cut into two equal parts and connected with $V_0$ individually, then sum of power dissipated is $P_1$ then $\frac{P_0}{P_1} = \frac{1}{x}$ find the value of $x$.

22. A particle is performing SHM having position $x = A\cos(30^\circ)$, and $A = 40$ cm. If its kinetic energy at this position is 200 J. The value of force constant in $\left(\frac{kN}{m}\right)$ is
23. Solid sphere rolls on horizontal plane. Ratio of angular momentum about COM to total energy is \( \frac{\pi}{22} \). Find \( \omega = ? \)

Answer (4)

\[
\begin{align*}
L &= I_{\text{COM}} \omega \\
K &= \frac{1}{2} I_{\text{COM}} \omega^2 + \frac{1}{2} Mv_0^2 \\
L &= \frac{2}{5} MR^2 \frac{v_0}{R} \\
K &= \frac{1}{2} \left( \frac{2}{5} MR^2 \right) \frac{v_0^2}{R^2} + \frac{1}{2} Mv_0^2 \\
L &= \frac{2MRv_0}{5} \\
K &= \frac{7}{10} Mv_0^2
\end{align*}
\]

Ratio \( \frac{L}{K} = \frac{4R}{7v_0} = \frac{\pi}{22} \Rightarrow \omega = \frac{4}{7} \times \frac{22}{7} = 4 \)

24. If \( m = 5 \pm 0.2 \) and \( v = 20 \pm 0.4 \), calculate error in measurement of K.E.

Answer (8)

\[
\text{Sol.} \quad \frac{\% \text{ error in } m}{5} \times 100 = 4\% \\
\frac{\% \text{ error in } v}{20} \times 100 = 2\% \\
\frac{\% \text{ error in } \frac{1}{2}mv^2}{m} = \% \text{ error in } m + 2 \times \% \text{ error in } v \\
= 4 + 2 \times 2 = 8\%
\]

25. Water is flowing inside the conical type tube having ratio of area of cross-section 6 : 1. If the speed of water outlet through smaller area is 60 m/s, then the pressure difference across these two cross-section is \( x \times 10^4 \) Pa, find the value of \( x \). (Assume incompressible fluid, density of water = 1000 kg/m³)

Answer (175)

\[
\text{Sol.} \quad A_1v_1 = A_2v_2 \\
\Rightarrow 6v_1 = 60 \\
v_1 = 10 \text{ m/s} \\
\Delta P = \frac{1}{2} \rho (v_2^2 - v_1^2) \\
= \frac{1}{2} \times 1000 (3600 - 100) \\
= 3500 \times \frac{1}{2} \times 1000 \\
= 175 \times 10^4
\]

26. Train \( A \) of length \( l \) is moving with speed 108 km/hr. Another train \( B \) of length 4\( l \) is moving parallel to train \( A \) with speed 72 km/hr. They both move through a tunnel of length 60\( l \) and train \( B \) takes 35 s more time than train \( A \) to pass through the tunnel, if they enter the tunnel simultaneously. Find the length (in m) of tunnel.

Answer (1575)

\[
\text{Sol.} \quad t_A = \frac{2l + 60l}{30}, \quad t_B = \frac{8l + 60l}{20} \\
\text{Also, } t_B - t_A = 35 \\
\frac{68l}{20} - \frac{62l}{30} = 35 \\
\frac{105}{4} = l \\
\therefore \text{Length of tunnel} = 60 \left( \frac{105}{4} \right) = 1575 \text{ m}
\]

27. 28. 29. 30.
**CHEMISTRY**

**SECTION - A**

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. In which of the following free radical helps in depletion of ozone layer?
   (1) \( \text{NO} \)
   (2) \( \text{Cl} \)
   (3) \( \text{OH} \)
   (4) \( \text{CH}_3 \)
   Answer (2)
   Sol. \( \text{O}_2 \xrightarrow{\text{UV}} \text{O} + \text{O} \)
   \( \text{O}_2 + \text{O} \rightarrow \text{O}_3 \)
   \( \text{CF}_2\text{Cl}_2 \xrightarrow{\text{UV}} \text{Cl} + \text{CF}_2\text{Cl} \)
   \( \text{Cl} + \text{O}_3 \rightarrow \text{ClO} + \text{O}_2 \)
   \( \text{ClO} + \text{O} \rightarrow \text{Cl} + \text{O}_2 \)

2. In which of the following option the species changes from paramagnetic to diamagnetic and bond order increases?
   (1) \( \text{N}_2 \rightarrow \text{N}_2^+ \)
   (2) \( \text{O}_2 \rightarrow \text{O}_2^{2-} \)
   (3) \( \text{NO} \rightarrow \text{NO}^+ \)
   (4) \( \text{O}_2 \rightarrow \text{O}_2^+ \)
   Answer (3)
   Sol. \( \text{NO} \) is paramagnetic with BO = 2.5 \( \text{NO}^+ \) is diamagnetic with BO = 3.0

3. What happens when lyophilic sol is added to lyophobic sol?
   (1) Prevention from coagulation
   (2) Precipitation
   (3) Emulsion
   (4) Electrophoresis
   Answer (1)
   Sol. On addition of lyophilic sol to lyophobic sol, prevention from coagulation takes place in the sense that more amount of electrolyte is needed to cause coagulation of same colloidal sol.

4. What is the major product formed in the following reaction
   \( \text{CH}_3 \xrightarrow{\text{Anhy. AlCl}_3, \text{HCl, } \Delta} \text{CH}_3-\text{(CH}_2)_4-\text{CH}_3 \)
   (1) \( \text{CH}_3-\text{C}_\text{---CH}_2-\text{CH}_3 \)
   (2) \( \text{CH}_3-\text{CH}---\text{CH}---\text{CH}_3 \)
   (3) \( \text{CH}_3-\text{CH}---(\text{CH}_2)_2---\text{CH}_3 \)
   (4) \( \text{CH}_3-\text{CH}---(\text{CH}_3)_2---\text{CH}_3 \)
   Answer (3)
   Sol. \( n \)-Alkanes on heating in this presence of anhydrous \( \text{AlCl}_3 \) and hydrogen chloride gas isomerise to branched chain alkanes. The major product has one methyl side chain.

5. Which one of the following shows incorrect method of refining?
   (1) Zinc : Liqueation
   (2) Copper : Electrolysis
   (3) Titanium : Van Arkel Method
   (4) Nickel : Mond's Process
   Answer (1)
   Sol. Zinc is refined by distillation, method used for metals having low boiling point.

6. Consider a reaction
   \( \text{P} \)
   Which of the following conclusions is correct about the product P?
(1) Both ring will be 5-membered
(2) Both ring will be 6-membered
(3) One ring is 5-membered and other is 6-membered
(4) One of the rings is 4-membered

Answer (2)

Sol. Among the given compounds, the following compound has the highest dipole moment because both the +ve and –ve ends acquire aromaticity.

9. Which one of the following compounds has the highest dipole moment?

(1) \[ \text{\\} \]
(2) \[ \text{\\} \]
(3) \[ \text{\\} \]
(4) \[ \text{\\} \]

Answer (2)

Sol. Among the given compounds, the following compound has the highest dipole moment because both the +ve and –ve ends acquire aromaticity.

10. Identify the product formed in the following reaction

Answer (3)

Sol. ClF₅ is a colourless liquid with square pyramidal structure. Hence the correct option is (4).

[Reference : NCERT]
12. Match the Column-I and Column-II

<table>
<thead>
<tr>
<th>Column-I</th>
<th>Column-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Nylon-6</td>
<td>1 Caprolactum</td>
</tr>
<tr>
<td>B Natural rubber</td>
<td>2 Chloroprene</td>
</tr>
<tr>
<td>C Vulcanized rubber</td>
<td>3 Isoprene</td>
</tr>
<tr>
<td>D Neoprene</td>
<td>4 Sulfur containing rubber</td>
</tr>
</tbody>
</table>

(1) A → 1; B → 3; C → 4; D → 2
(2) A → 1; B → 2; C → 4; D → 3
(3) A → 4; B → 3; C → 1; D → 2
(4) A → 2; B → 3; C → 4; D → 1

**Answer (1)**

**Sol.** Nylon-6 – Caprolactum
Natural rubber – Isoprene
Vulcanized rubber – Sulfur containing rubber
Neoprene – Chloroprene

13. Consider a reaction.
Be(OH)_2 + Sr(OH)_2 → Product
Incorrect statement regarding the product is
(1) Be is tetrahedrally bonded in the product
(2) Be forms cationic part
(3) It is an acid-base reaction
(4) Be(OH)_2 acts as a Lewis acid

**Answer (2)**

**Sol.** Be(OH)_2 + Sr(OH)_2 → Sr^2+[BeOH]^4^2^-
As Be is present in the anionic part, option (2) is incorrect.

14. Following two columns are given

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Troposphere</td>
<td>(p) From 10 to 50 km from sea level</td>
</tr>
<tr>
<td>(b) Stratosphere</td>
<td>(q) Upto 10 km from sea level</td>
</tr>
<tr>
<td>(c) Mesosphere</td>
<td>(R) From 85 km to 100 km from sea level</td>
</tr>
<tr>
<td>(d) Thermosphere</td>
<td>(s) From 50 km to 85 km from sea level</td>
</tr>
</tbody>
</table>

(1) a(p); b(q); c(r); d(s)
(2) a(r); b(s); c(p); d(q)
(3) a(q); b(p); c(s); d(r)
(4) a(s); b(r); c(p); d(q)

15. Energy of first Bohr orbit E_1 is –2.18 × 10^{-18} J, then find energy of third Bohr orbit for hydrogen

(1) 3E_1
(2) 9E_1
(3) \frac{E_1}{9}
(4) \frac{E_1}{27}

**Answer (3)**

**Sol.** 
\[ E_3 = \frac{2.18 \times 10^{-18}}{n^2} \]
\[ E_3 = \frac{E_1}{(3)^2} = \frac{E_1}{9} \]

16. Which one of the following is the best method for the removed of hardness of water?

(1) Boiling
(2) Treatment with washing soda
(3) Permutit process
(4) Synthetic resin method

**Answer (4)**

**Sol.** Boiling of hard water removes temporary harness only. Treatment with washing soda as well as permutit process are useful in removing Ca^{2+} and Mg^{2+} ions from hard water but do not remove Cl^- and SO_4^{2-} ions. Synthetic resin method enables us to remove Ca^{2+} and Mg^{2+} ions as well as Cl^- and SO_4^{2-} ions. Therefore, synthetic resin method is the best method to remove hardness of water.

17. Glyceraldehyde \( \text{HCN} \) A + B
\( \text{H}_3\text{O}^- \)
\( \text{HNO}_3 \)

Then select the correct option about the product A and B

(1) Both are optically active
(2) Both are optically inactive
(3) One is optically active and another is optically inactive
(4) None of these

**Answer (3)**
18. Consider the following reactions

(I) \[
\text{Br} \quad \text{C}_2\text{H}_5\text{O}^- \rightarrow \quad \text{OC}_2\text{H}_5
\]

(II) \[
\text{Br} \quad \text{C}_2\text{H}_5\text{OH} \rightarrow \quad \text{OC}_2\text{H}_5
\]

and identify the correct statement.

(1) In reaction (I), mechanism is \( S_{n1} \) and the product is \( \text{OC}_2\text{H}_5 \)

(2) In reaction (II), mechanism is \( S_{n1} \) and the product is \( \text{OC}_2\text{H}_5 \)

(3) In reaction (I), mechanism is \( S_{n2} \) and the product is \( \text{OC}_2\text{H}_5 \)

(4) In reaction (II), mechanism is \( S_{n2} \) and the product is \( \text{OC}_2\text{H}_5 \)

Answer (2)

19. Identify the major product formed in the following reaction

Answer (3)

20. **SECTION - B**

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
21. Radius of 2\textsuperscript{nd} orbit of He\textsuperscript{+} is \( r_0 \). Radius of 4\textsuperscript{th} orbit of Be\textsuperscript{3+} is \( x r_0 \).

Find \( x \).

**Answer (02.00)**

\[
\text{Sol.} \quad r_0 = 0.529 \times \frac{(4)}{2} \\
= 0.529 \times 2 \text{ Å} \\
r_{13}^{13} = 0.529 \times \frac{(4)^2}{4} \\
= 0.529 \times 4 \\
= 2r_0 \\
\Rightarrow \quad x = 2
\]

22. An organic compound on combustion gives 0.22 g of CO\textsubscript{2} and 0.126 g H\textsubscript{2}O. If the percentage of C in given organic compound is 40%, the % of H will be?

**Answer (9.34)**

\[
\text{Sol.} \quad C\% = \frac{12 \times 0.22}{44 \times \text{weight of sample}} \times 100 \\
40 = \frac{12 \times 0.22}{44 \times \text{weight of sample}} \times 100 \\
\therefore \quad \text{weight of sample} = \frac{6}{40} = 0.15 \text{ g} \\
\therefore \quad H\% = \frac{2 \times 0.126}{18 \times 0.15} \times 100 \\
= 9.33\%
\]

23. For the 1\textsuperscript{st} order reaction, the ratio of \( t_{50\%} \) to \( t_{87.5\%} \) will be:

**Answer (3)**

\[
\text{Sol.} \quad \frac{t_{87.5\%}}{t_{50\%}} = 3
\]

24. If \( \left(1 + \frac{1}{x}\right)^{1/2} v_{av} = v_{rms} \), then \( x \) is (Nearest integer)

**Answer (6)**
MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:
1. Find the sum of series
   \[2 \times 2^2 - 2 \times 3^2 + 2 \times 4^2 \ldots \ldots \ldots \ldots \ldots \text{(20 terms)}\]
   (1) 462   (2) –462   (3) 460   (4) –460

   Answer (4)
   Sol.
   \[S = 2 \left[2^2 - 3^2 + 4^2 \ldots \ldots + 20^2 - 21^2\right]\]
   \[= 2 \left[(2^2 + 4^2 + 6^2 + \ldots + 20^2) - (3^2 + 5^2 + \ldots + 21^2)\right]\]
   \[= 2 \left[2^2(1^2 + 2^2 + 3^2 + \ldots + 10^2) - (2^2 + 3^2 + 4^2 + \ldots + 21^2)\right]\]
   \[= 2 \left[\frac{8 \times 10 \times 11 \times 21}{6} - \frac{21 \times 22 \times 43}{6} + 1\right]\]
   \[= 2[3080 - 3311 + 1]\]
   \[= 2[-230]\]
   \[= -460\]

2. The number of seven digit numbers made using 1, 2, 3, 4 whose sum of digits is 12 is
   (1) 413   (2) 311   (3) 308   (4) 393

   Answer (1)
   Sol.
   \[
   \begin{align*}
   1 & 1 & 1 & 1 & 1 & 4 & 3 \\
   1 & 1 & 1 & 1 & 3 & 3 & 2 \\
   2 & 2 & 2 & 3 & 1 & 1 & 1 \\
   2 & 2 & 2 & 2 & 1 & 1 & 1 \\
   1 & 1 & 1 & 4 & 2 & 2 & 2 \\
   \end{align*}
   \]
   \[= \frac{7!}{5!} = 42\]
   \[= \frac{7!}{4!2!} = 105\]
   \[= \frac{7!}{3!1!} = 140\]
   \[= \frac{7!}{5!1!} = 21\]
   \[= \frac{7!}{4!2!} = 105\]
   \[\text{Total} = 413\]

3. If \(\frac{dy}{dx} = y + 7\) & \(y(0) = 0\), then the value of \(y(1) =\)
   (1) \(7(e – 1)\)   (2) \(2(e – 1)\)
   (3) \(7e\)   (4) None of these

   Answer (1)
   Sol.
   \[\frac{dy}{y + 7} = dx\]
   \[\Rightarrow \log|y + 7| = x + c\]
   \[y(0) = 0\]
   \[\Rightarrow c = \log 7\]
   \[\log|y + 7| = x + \log 7\]
   \[\Rightarrow |y + 7| = ye^x\]
   \[\Rightarrow y = 7(e – 1)\]

4. If \(g(x) = \sqrt{x + 1}\) and \(f(g(x)) = 3 - \sqrt{x + 1}\) then value of \(f(0) =\)
   (1) 2   (2) 3
   (3) 4   (4) \(-4\)

   Answer (2)
   Sol. For \(f(0)\), put \(x = -1\) in \(f(g(x))\)
   \[\Rightarrow f(g(-1)) = 3 + \sqrt{-1 + 1}\]
   \[\Rightarrow f(0) = 3\]

5. If \(3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10\), then \(f(3) =\)
   (1) 2   (2) \(-3\)
   (3) \(-4\)   (4) None of these

   Answer (2)
   Sol.
   \[3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10 \ldots (i)\]
   \[x \rightarrow \frac{1}{x}\]
   \[3f\left(\frac{1}{x}\right) + 2f(x) = x - 10 \ldots (ii)\]
3(i) \(-2(ii)\)

\[9f(x) - 4f(x) = \frac{3}{x} - 30 - 2x + 20\]

\[\Rightarrow 5f(x) = \frac{3}{x} - 2x - 10\]

\[x = 3\]

\[5f(3) = 1 - 6 - 10 = -15\]

\[f(3) = -3\]

6. Find area bounded by the curves \(y = \max\{\sin x, \cos x\}\) and \(x\)-axis between \(x = -\pi\) and \(x = \pi\)

\((1)\) \(2 + \sqrt{2}\)

\((2)\) \(\sqrt{2}\)

\((3)\) \(1 + \sqrt{2}\)

\((4)\) \(2\sqrt{2}\)

Answer (4)

Sol.

\[
\int_{-\pi}^{\pi} \sin x \, dx + \int_{-\frac{3\pi}{4}}^{\pi/4} \cos x \, dx + \int_{-3\pi/4}^{\pi/4} \sin x \, dx
\]

\[
= -\cos x \left|_{-\pi}^{\pi} \right. + \sin x \left|_{-3\pi/4}^{\pi/4} \right. - \cos x \left|_{-3\pi/4}^{\pi/4} \right.
\]

\[
= \left( \frac{1}{\sqrt{2}} - 1 \right) + \left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) + \left( 1 + \frac{1}{\sqrt{2}} \right)
\]

\[
= 2\sqrt{2}
\]

7. The negation of \(((A \cap (B \cup C)) \Rightarrow (B \cap C)) \Rightarrow A\) is equivalent to

\((1)\) \(~(A \cup B)\)

\((2)\) \(~A\)

\((3)\) \(A\)

\((4)\) None of these

Answer (2)

Sol. Let \(P = (((A \cap (B \cup C)) \Rightarrow (B \cap C)) \Rightarrow A\)

\(\sim P = (((A \cap (B \cup C)) \Rightarrow (B \cap C)) \Rightarrow A\)

\(\sim A\)

\(\sim P = \sim A\)

8. \(f(x) = x - 2\sin x \cos x + \frac{1}{3}\sin 3x, \ x \in [0, \pi]\) then maximum value of \(f(x)\) is

\((1)\) \(\frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3}\)

\((2)\) \(0\)

\((3)\) \(\frac{5\pi}{6} + \frac{\sqrt{3}}{2} - \frac{1}{3}\)

\((4)\) \(\frac{5\pi}{6}\)

Answer (1)

Sol. \(f(x) = x - 2\sin 2x + \frac{1}{3}\sin 3x\)

\(f'(x) = 1 - 2\cos 2x + \cos 3x = 0\)

\(x = \frac{5\pi}{6}, \frac{\pi}{6}\)

\(\therefore f''(x) = 4\sin 2x - 3\sin 3x\)

\(f''(x)\) at \(x = \frac{5\pi}{6}\) is < 0

\(\Rightarrow \left( \frac{5\pi}{6} \right)\) is point of maxima

\(f'\left( \frac{5\pi}{6} \right) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2} - \frac{1}{3}\)

9. If \(\vec{a} = 2\hat{i} + 3\hat{j} + 5\hat{k}\)

\(\vec{b} = 3\hat{i} + 3\hat{j} + 7\hat{k}\)

\(\vec{c} = 7\hat{i} + 8\hat{j} + 9\hat{k}\)

If \(\vec{a} \times \vec{b} = \vec{c} + \vec{d}\) then \(|\vec{d}|\) is

\((1)\) \(\sqrt{174}\)

\((2)\) \(8\sqrt{2}\)

\((3)\) \(\sqrt{168}\)

\((4)\) \(5\sqrt{5}\)

Answer (1)

Sol. \(\vec{a} \times \vec{b} = 6\hat{i} + \hat{j} - 3\hat{k}\)

\(\vec{d} = -\hat{i} - 7\hat{j} - 12\hat{k}\)

\(|\vec{d}| = \sqrt{174}|\)
10. **Statement 1:** \( \lim_{x \to \infty} \frac{1}{n^2} (1 + 2 + 3 + \ldots + n) = \frac{1}{4} \)

**Statement 2:** \( \lim_{x \to \infty} \frac{1}{6^n} (1 + 15^{n^2} + \ldots + n^{15}) = \frac{1}{16} \)

(1) Statement 1 and statement 2 both are correct
(2) Only statement 1 is correct
(3) Only statement 2 is correct
(4) Both statement are incorrect

**Answer (3)**

**Sol.** Statement 1: \( \lim_{x \to \infty} \frac{1}{n^2} (1 + 2 + 3 + \ldots + n) \)

\[ \lim_{x \to \infty} \frac{n(n+1)}{2n^2} = \frac{1}{2} \]

Statement 2: \( \sum_{n=1}^{\infty} \frac{r^n}{n^{n+1}} = \frac{1}{n+1} \)

Hence \( n = 15 \)

\( \therefore \) \( \sum_{n=1}^{15} \frac{r^n}{n^16} = \frac{1}{16} \)

11. If \( \sin^{-1} \left( \frac{x+1}{\sqrt{x^2 + 2x + 2}} \right) - \sin^{-1} \left( \frac{x}{\sqrt{x^2 + 1}} \right) = \frac{\pi}{4} \) (For \( x \geq 0 \)) then \( \sin \left( \left( x^2 + x + 5 \right) \frac{\pi}{2} \right) - \cos \left( \left( x^2 + x + 5 \right) \pi \right) \) is

(1) 0
(2) 2
(3) 1
(4) None of these

**Answer (2)**

**Sol.** \( \tan^{-1} (x+1) - \tan^{-1} x = \frac{\pi}{4} \)

\( \tan^{-1} \left( \frac{x+1 - x}{1 + x(x+1)} \right) = \frac{\pi}{4} \)

\( 1 = 1 + x(x+1) \)

(1) \( x = 0 \) or \(-1\). rejected

\( \therefore x = 0 \)

\( \sin \left( \left( x^2 + x + 5 \right) \frac{\pi}{2} \right) - \cos \left( \left( x^2 + x + 5 \right) \pi \right) \)

\( \sin \frac{5\pi}{2} - \cos 5\pi = 1 - (-1) = 2 \)

12. \( \frac{dy}{dx} = 6e^x + e^{2x} + e^{3x} \)

then \( y(2) - y(0) \) is

(1) \( e^2 + \frac{6e^4}{4} - \frac{e^6}{3} + \frac{15}{6} \)
(2) \( 6e^2 + \frac{e^4}{3} + \frac{e^6}{2} - \frac{15}{6} \)
(3) \( 6e^2 + \frac{e^4}{2} + \frac{e^6}{3} - \frac{15}{6} \)
(4) \( e^2 + \frac{e^4}{2} + \frac{e^6}{3} - \frac{15}{6} \)

**Answer (3)**

**Sol.** \( y(2) - y(0) = \int_0^2 (6e^x + e^{2x} + e^{3x})dx \)

\( = 6e^x + \frac{e^{2x}}{2} + \frac{e^{3x}}{3} \bigg|_0^2 \)

\( = 6e^2 + \frac{e^4}{2} + \frac{e^6}{3} - \frac{15}{6} \)

13. The integral \( \int_0^\infty \frac{6}{e^{3x} + 6e^{2x} + 11e^x + 6} \) is

(1) \( \ln 32 \)
(2) \( \ln 27 \)
(3) \( \ln \frac{32}{27} \)
(4) None of these

**Answer (3)**

**Sol.** \( \int_0^\infty \frac{6}{e^{x} + 2 + e^{2x} + e^{3x}}dx \)

\( = 6 \int_0^\infty \left( \frac{1/2}{e^x+1} - \frac{1/2}{e^x+2} + \frac{1/2}{e^x+3} \right)dx \)

\( = 6 \int_0^\infty \frac{1}{e^x+1}dx - 6 \int_0^\infty \frac{1}{e^x+2}dx + 6 \int_0^\infty \frac{1}{e^x+3}dx \)

\( = 3 \left[ \ln (1+e^{-x}) \right]^\infty_0 + 6 \left[ \frac{1}{2} \ln (1+2e^{-x}) \right]^\infty_0 - \frac{3}{3} \left[ \ln (1+3e^{-x}) \right]^\infty_0 \)

\( = 3 \ln 2 - 3 \ln 3 + \ln 4 \)

\( = 3 \ln \frac{2}{3} + \ln 4 \)

\( = \ln \frac{32}{27} \)
14. Plane $P_3$ is passing through $(1, 1, 1)$ and point of intersection of $P_1$ and $P_2$ where $P_1 : 2x - y + z = 5$ and $P_2 : x + 3y + 3z + 2 = 0$, then distance of $(1, 1, 10)$ from $P_3$ is

(1) $\frac{53}{85}$
(2) $\sqrt{85}$
(3) $\frac{54}{\sqrt{85}}$
(4) $53$

Answer (3)

Sol. $P_1 + \lambda P_2 = 0$

$(2x - y + z - 5) + \lambda(x + 3y + 3z + 2) = 0$

Passing though $(1, 1, 1)$

$(2 - 1 + 1 - 5) + \lambda(1 + 3 + 3 + 2) = 0$

$-3 + \lambda(9) = 0$

$\lambda = \frac{1}{3}$

$P_3 = 3(2x - y + z - 5) + (x + 3y + 3z + 2) = 0$

$7x + 6z = 13$

Distance of $(1, 1, 10)$ is

$\frac{|7 \times 1 + 6 \times 10 - 13|}{\sqrt{7^2 + 6^2}} = \frac{7 + 60 - 13}{\sqrt{49 + 36}}$

$= \frac{54}{\sqrt{85}}$ unit

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Let the number of matrices of order $3 \times 3$ are possible using the digits {0, 1, 2, 3, …….., 10} is $m^n$, then $(m + n)$ is, (where $m$ is a prime number)

Answer (20)

Sol. So, we have 9 places and 11 numbers

$\therefore$ Number of matrices = $11^9$

$\therefore m + n = 20$.

22. Remainder when $2^{2022}$ is divided by 15 is equal to

Answer (4)

Sol. $(2^8)^{252} \cdot 2^6$

$= (255 + 1)^{252} \cdot 64$

$= (\binom{252}{0}(255)^{252} + \binom{252}{1}(255)^{251} + \ldots + \binom{252}{252}(255)^{0}) \cdot 64$

$= 64 (15k + 1)$

Remainder = 4

23. Let 10 APs are there whose first terms are $(1, 2, 3, \ldots, 10)$ respectively & common differences are $(1, 3, 5, \ldots)$ respectively and $S_i$ denotes sum of 10 terms of $i^{th}$ A.P then $\sum_{i=1}^{10} S_i$ is

Answer (5050)

Sol. $S_i = \frac{10}{2}[2i + (10 - 1)(2i - 1)]$

$= 5[2i + 18i - 9]$

$= 5[20i - 9]$

$\therefore \sum_{i=1}^{10} S_i = 5 \left[ 20 \times \frac{10 \times 11}{2} - 9 \times 10 \right]$

$= 5 \times 1100 - 90$

$= 5 \times 1010 = 5050$
24. For the data

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_i$</td>
<td>4</td>
<td>24</td>
<td>28</td>
<td>$\alpha$</td>
<td>$\beta$</td>
</tr>
</tbody>
</table>

If mean of data is 5 and mean deviation about mean is $M$ and variance is $\sigma^2$ then \( \frac{3\alpha}{M + (\sigma^2)} \) is

Answer (08)

Sol. \( \bar{x} = \frac{4 \times 4 + 24 \times 2 + 28 \times 0 + 16 \times 2 + 8 \times 4}{80} = 5 \)

\( \therefore \ \alpha = 16 \)

Now \( M = \frac{4 \times 4 + 24 \times 2 + 28 \times 0 + 16 \times 2 + 8 \times 4}{80} = \frac{8}{5} \)

\( \sigma^2 = \frac{4 \times 4^2 + 24 \times 2^2 + 28 \times 0^2 + 16 \times 2^2 + 8 \times 4^2}{80} = \frac{22}{5} \)

\( \therefore \ \frac{3\alpha}{M + \sigma^2} = \frac{3 \times 16}{8 + \frac{22}{5}} = 08 \)