Date: 11/03/2023

Set-2

Question Paper Code

65/5/2

Corporate Office: Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Time: 3 hrs.

MATHEMATICS

Max. Marks: 80

Class-XII

(CBSE 2022-23)

Answers & Solutions

GENERAL INSTRUCTIONS

Read the following instructions very carefully and follow them:

(i) This question paper contains **38** questions. **All** questions are compulsory.

(ii) Question paper is divided into **FIVE** sections – Section **A**, **B**, **C**, **D** and **E**.

(iii) **In Section – A :** Question Numbers **1** to **18** are Multiple Choice Questions (MCQ) type and question number 19 & 20 are Assertion-Reason based questions of **1** mark each.

(iv) **In Section – B :** Question Number **21** to **25** are Very Short Answer (VSA) type Questions carrying **2** marks each.

(v) **In Section – C :** Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.

(vi) **In Section – D :** Question Number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.

(vii) **In Section – E :** Question Number **36** to **38** are case study based questions carrying **4** marks each where 2 VSA type questions are of **1** mark each and 1 SA type question is of **2** marks. Internal choice is provided in 2 marks question in each case-study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section – B, 3 questions in Section – C, 2 questions in Section – D and 2 questions in Section – E.

(ix) Use of calculators is **NOT** allowed.

SECTION - A

(Multiple Choice Questions)

Each question carries 1 mark.

Select the correct option out of the four given options:

1. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Answer (a)

Sol. We know that principal value of $\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$$\therefore \sin\left(\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right)=\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)=\sin\frac{\pi}{2}=1$$

2. Let A = {3, 5}. Then number of reflexive relations on A is

(a) 2 (b) 4

(c) 0 (d) 8

Answer (b)

Sol. Given A = {3, 5} $\Rightarrow$ n(A) = 2

Number of reflexive relations $=2^{n^2-n}$

$=2^{2^2-2}=2^{4-2}=2^2=4$

3. If $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}, B=\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}$ and A = B^2, then x equals

(a) ±1 (b) –1

(c) 1 (d) 2

Answer (c)

Sol. Given $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}$

$$\therefore B^2=\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}=\begin{bmatrix}x^2 & 0\\ x+1 & 1\end{bmatrix}$$

As A = B^2

$\Rightarrow x^2 = 1, x + 1 = 2$

$\Rightarrow x = 1$

4. If A = [a_{ij}] is a square matrix of order 2 such that $a_{ij}=\begin{cases}1, & \text{when } i\neq j\\ 0, & \text{when } i=j\end{cases}$, then A^2 is

(a) $\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$

(c) $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Answer (d)

Sol. For square matrix of order 2 if $a_{ij}=\begin{cases}1 & i\neq j\\ 0 & i=j\end{cases}$

$\Rightarrow a_{11} = 0,\ a_{12} = 1,\ a_{21} = 1,\ a_{22} = 0$

$\Rightarrow A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

$\Rightarrow A^2=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

5. The value of the determinant $\begin{vmatrix}6 & 0 & -1\\ 2 & 1 & 4\\ 1 & 1 & 3\end{vmatrix}$ is

(a) 10
(b) 8
(c) 7
(d) –7

Answer (d)

Sol. Given $\Delta=\begin{vmatrix}6 & 0 & -1\\ 2 & 1 & 4\\ 1 & 1 & 3\end{vmatrix}$

Expanding along row 1 we get

$\Delta = 6(3 - 4) - 0(6 - 4) - 1(2 - 1)$

$= 6(-1) -1(1) = - 6 -1 = -7$

6. The function f(x) = [x], where [x] denotes the greatest integer less than or equal to x, is continuous at

(a) $x = 1$
(b) $x = 1.5$
(c) $x = -2$
(d) $x = 4$

Answer (b)

Sol. f(x) = [x] is defined as

$$f(x)=\begin{cases}\vdots & \\ -2 & x\in[-2,-1)\\ -1 & x\in[-1,0)\\ 0 & x\in[0,1)\\ 1 & x\in[1,2)\\ 2 & x\in[2,3)\\ 3 & x\in[3,4)\\ \vdots & \end{cases}$$

$\therefore$ f(x) is discontinuous $\forall x\in Z$

Hence f(x) is continuous at x = 1.5

7. The derivative of x^{2x} w.r.t. x is

(a) x^{2x-1} (b) $2x^{2x} \log x$

(c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$

Answer (c)

Sol. Let $y = x^{2x}$

Taking log both sides we get

$\ln y = \ln x^{2x} = 2x \ln x$

differentiate both sides w.r.t. x we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x}{x} + 2\ln x$$

$$\Rightarrow \frac{dy}{dx} = x^{2x}\left(2\left(1+\ln x\right)\right) = 2x^{2x}\left(1+\log x\right)$$

8. The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is

(a) $(-1, \infty)$ (b) $(-2, -1)$

(c) $(-\infty, -2)$ (d) $[-1, 1]$

Answer (b)

Sol. For f(x) to be decreasing $f'(x) < 0$

$\Rightarrow f'(x) = 6x^2 + 18x + 12 < 0$

$= 6(x^2 + 3x + 2) < 0$

$= 6(x + 1)(x + 2) < 0$

$\Rightarrow x \in (-2, -1)$

9. The function $f(x) = x\,|\,x\,|$, $x \in R$ is differentiable

(a) only at $x = 0$ (b) only at $x = 1$

(c) in R (d) in $R - \{0\}$

Answer (c)

Sol. Given $f(x) = x\,|\,x\,|$

$$\Rightarrow f(x) = \begin{cases} x^2 & , \ x \geq 0 \\ -x^2 & , \ x < 0 \end{cases}$$

as $f(x)$ is a polynomial $\forall x \in R - \{0\}$

$f(x)$ is continuous as well as differentiable

Now examining at $x = 0$, we get

For continuity $\lim\limits_{x \to 0^-} f(x) = f(0) = \lim\limits_{x \to 0^+} f(x)$

$$\Rightarrow \lim_{x \to 0^-} x^2 = 0 = \lim_{x \to 0^+} x^2 = f(0)$$

$\therefore$ f(x) is continuous at x = 0

LHD at x = 0; $\lim_{h\to 0^-} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0} \frac{(-h)^2}{h} = 0$

RHD at x = 0; $\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{h^2}{h} = 0$

$\therefore$ f(x) is differentiable at x = 0

$\therefore$ f(x) is differentiable $\forall x \in R$

10. $\int \frac{\sec x}{\sec x - \tan x} dx$ equals

(a) sec x – tan x + c

(b) sec x + tan x + c

(c) tan x – sec x + c

(d) –(sec x + tan x) + c

Answer (b)

Sol. Let $I = \int \frac{\sec x}{\sec x - \tan x} dx$

$\Rightarrow I = \int \frac{\sec x(\sec x + \tan x)dx}{(\sec x - \tan x)(\sec x + \tan x)}$

$\Rightarrow I = \int \frac{\sec^2 x + \sec x \cdot \tan x}{\sec^2 x - \tan^2 x} dx$

as $\sec^2 x - \tan^2 x = 1$ we get

$\Rightarrow I = \int \sec^2 x\, dx + \int \sec x \tan x\, dx$

= tan x + sec x + c

11. The value of $\int_0^{\frac{\pi}{4}} (\sin 2x) dx$ is

(a) 0

(b) 1

(c) $\frac{1}{2}$

(d) $-\frac{1}{2}$

Answer (c)

Sol. Let $I = \int_0^{\pi/4} \sin 2x\, dx$

Put $2x = t \Rightarrow 2dx = dt$

$\Rightarrow I = \frac{1}{2}\int_0^{\pi/2} \sin t\ dt$

$= \frac{1}{2}[-\cos t]_0^{\pi/2} = \frac{1}{2}(-0+1) = \frac{1}{2}$

12. The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is

(a) 2 (b) 3

(c) 5 (d) 0

Answer (b*)

Sol. $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 3\left(\frac{dy}{dx}\right)^2 \cdot \frac{d^2y}{dx^2}$

$\therefore$ order = 2 & degree = 1

$\therefore$ sum = 2 + 1 = 3

* Please note in question only expression is given not the equation.

13. Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if

(a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(c) $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

Answer (b)

Sol. If two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear.

Then $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \lambda$

14. A unit vector $\hat{a}$ makes equal but acute angles on the co-ordinate axes. The projection of the vector $\hat{a}$ on the vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ is

(a) $\frac{11}{15}$ (b) $\frac{11}{5\sqrt{3}}$

(c) $\frac{4}{5}$ (d) $\frac{3}{5\sqrt{3}}$

Answer (a)

Sol. Let $\hat{a} = l\hat{i} + m\hat{j} + n\hat{k}$

$l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$

$\because\ \alpha = \beta = \gamma.$

$\Rightarrow l = m = n$

$\because\ l^2 + m^2 = n^2 = 1$

$\Rightarrow 3l^2 = 1$

$\Rightarrow l = \pm\frac{1}{\sqrt{3}}$

$\because\ \alpha, \beta, \gamma > 0$

$\Rightarrow l = m = n = \frac{1}{\sqrt{3}}$

$\hat{a} = \left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right) = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

As we know that projection of a vector $\vec{a}_1$ on vector $\vec{b}_1$ is $\vec{a}_1 \cdot \hat{b}_1$

$\Rightarrow$ Projection of vector $\hat{a}$ on vector $\vec{b} = \hat{a} \cdot \hat{b}$

$= \frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k}) \cdot \frac{(5\hat{i}+7\hat{j}-\hat{k})}{\sqrt{25+49+1}}$

$= \frac{5+7-1}{\sqrt{3} \cdot 5\sqrt{3}}$

$= \frac{11}{15}$

15. The angle between the lines 2x = 3y = – z and 6x = –y = –4z is

(a) 0°

(b) 30°

(c) 45°

(d) 90°

Answer (d)

Sol. $L_1 : 2x = 3y = -z$ $\qquad L_2 : 6x = -y = -4z$

$L_1 : \frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{(-1)}$ $\qquad L_2 : \frac{x}{\frac{1}{6}} = \frac{y}{(-1)} = \frac{z}{\left(-\frac{1}{4}\right)}$

Direction ratios of L_1 and L_2 are $\frac{1}{2}, \frac{1}{3}, -1$ and $\frac{1}{6}, -1, -\frac{1}{4}$ respectively if the direction ratios of lines L_1 and L_2 are a_1, b_1, c_1 and a_2, b_2, c_2 respectively, then angle between the lines is given by

$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{(a_1^2+b_1^2+c_1^2)(a_2^2+b_2^2+c_2^2)}}$

(where θ is angle between the lines)

Here $a_1a_2 + b_1b_2 + c_1c_2 = \frac{1}{2} \times \frac{1}{6} + \frac{1}{3}(-1) + (-1)\left(-\frac{1}{4}\right)$

$= \frac{1}{12} - \frac{1}{3} + \frac{1}{4} = 0$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = 90°$

16. If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

(b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$

(d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Answer (a)

Sol. If a lines makes angles of α, β and γ with the x, y and z axes respectively, then its direction cosines are $\cos\alpha$, $\cos\beta$, $\cos\gamma$

$\Rightarrow$ Direction cosines of the line are cos90°, cos135°, cos45° or $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

17. If for any two events A and B, P(A) = $\frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then P(B/A) is equal to

(a) $\frac{1}{10}$

(b) $\frac{1}{8}$

(c) $\frac{7}{8}$

(d) $\frac{17}{20}$

Answer (c)

Sol. $P(A) = \frac{4}{5}, P(A \cap B) = \frac{7}{10}$

As we know that $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$$\Rightarrow \quad P\left(\frac{B}{A}\right) = \frac{\frac{7}{10}}{\frac{4}{5}} = \frac{7 \times 5}{10 \times 4} = \frac{7}{8}$$

18. If A and B are two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then P(B′/A) is

(a) $\frac{1}{4}$

(b) $\frac{1}{8}$

(c) $\frac{3}{4}$

(d) 1

Answer (c)

Sol. $P(A) = \frac{1}{3},\ P(B) = \frac{1}{4}$

$\because$ A and B are independent events

$\Rightarrow \quad P(A \cap B) = P(A) \cdot P(B)$

$\because \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \quad P\left(\frac{B'}{A}\right) = \frac{P(B' \cap A)}{P(A)}$

$$= \frac{P(A) - P(A \cap B)}{P(A)} \quad (\because \ B' \cap A = A - A \cap B)$$

$$= \frac{P(A) - P(A) \cdot P(B)}{P(A)} = 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$$

Assertion – Reason Based Questions

In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A)

(c) (A) is true and (R) is false

(d) (A) is false, but (R) is true

19. Assertion (A) : $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx = 3$

Reason (R) : $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$

Answer (a)

Sol. $I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$ (let) …(i)

$\Rightarrow\ I = \int_2^8 \frac{\sqrt{10-(2+8-x)}}{\sqrt{2+8-x}+\sqrt{10-(2+8-x)}}dx \left[\text{using}: \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$

$\Rightarrow\ I = \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}}\,dx$ …(ii)

Adding eq. (i) and (ii)

$\Rightarrow\ I + I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx + \int_2^8 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}dx$

$\Rightarrow\ 2I = \int_2^8 \frac{\sqrt{10-x}+\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}\,dx$

$\Rightarrow\ 2I = \int_2^8 1\cdot\,dx$

$\Rightarrow\ 2I = x\Big|_2^8$

$\Rightarrow$ 2I = 8–2

$\Rightarrow$ 2I = 6

$\Rightarrow$ I = 3

$\therefore$ Assertion (A) is correct and Reason (R) is the correct explanation of (A)

$\therefore$ Option (a) is correct

20. Assertion (A) : Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$

Reason (R) : Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E\cap F)}{P(E)}$

Answer (a)

Sol. Consider an event of tossing two coins

Let event E : getting two heads ; E = {HH}

Let event F : getting atleast one head; F = {HH, HT, TH}

$E \cap F$ = {HH}

Sample space (S) = {HH, HT, TH, HT}

n(S) = 4

n(E) = 1, n(F) = 3, n(E $\cap$ F) = 1

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{3}{4} \quad \ldots(i)$$

$$P(E \cap F) = \frac{(E \cap F)}{n(S)} \quad \ldots(ii)$$

P(getting two heads given that atleast one head comes up)

$$= \left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{1/4}{3/4} = \frac{1}{3} \quad \text{(using (i) and (ii))}$$

$\therefore$ Assertion (A) is correct and Reason (R) is the correct explanation of (A)

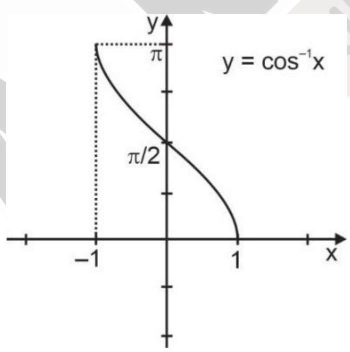
$\therefore$ Option (a) is correct.

SECTION - B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. Draw the graph of the principal branch of the function f(x) = $\cos^{-1}$ x.

Sol.



22. **(a)** If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

Sol. (a) Given : $|\vec{a}| = 3$, $|\vec{b}| = \frac{2}{3}$, $|\vec{a} \times \vec{b}| = 1$

Now, let θ be the angle between $\vec{a}$ and $\vec{b}$

Using formulae : $|\vec{a}\times\vec{b}| = |\vec{a}|\cdot|\vec{b}|\cdot\sin\theta$

$1 = 3 \times \frac{2}{3} \times \sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

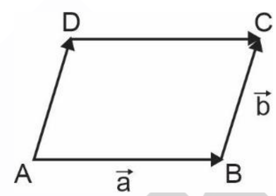
$\Rightarrow \theta = \frac{\pi}{6}$

Hence, $\vec{a}\times\vec{b}$ is unit vector if the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$.

OR

(b) Given : $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$



Area of parallelogram ABCD = $|\vec{a}\times\vec{b}|$

Now, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$

$= \hat{i}(-1\times1-(-7)\times3) - \hat{j}(1\times1-3\times2) + \hat{k}(-7\times1-2\times(-1))$

$= \hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2)$

$= 20\hat{i} + 5\hat{j} - 5\hat{k}$

Magnitude of $\vec{a}\times\vec{b} = \sqrt{(20)^2+(5)^2+(-5)^2}$

$|\vec{a}\times\vec{b}| = \sqrt{400+25+25}$

$= \sqrt{450}$

$= \sqrt{25\times9\times2}$

$= 5\times3\sqrt{2}$

$= 15\sqrt{2}$

Therefore, required area is $15\sqrt{2}$ square units.

23. (a) If $f(x) = \begin{cases} x^2, & \text{if } x \ge 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at x = 1.

OR

(b) Find the value(s) of 'λ', if the function

$f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \ne 0 \\ 1 & , \text{if } x = 0 \end{cases}$ is continuous at x = 0.

Sol. (a) Given $f(x) = \begin{cases} x^2 & ; x \geq 1 \\ x & ; x < 1 \end{cases}$

Now, f(x) is differentiable at x = 1 if LHD = RHD

LHD:

$$\lim_{h\to 0} \frac{f(x)-f(x-h)}{h}$$

$$= \lim_{h\to 0} \frac{f(1)-f(1-h)}{h}$$

$$= \lim_{h\to 0} \frac{(1)^2-(1-h)}{h}$$

$$= \lim_{h\to 0} \frac{1-1+h}{h}$$

$$= \lim_{h\to 0} \frac{h}{h} = 1$$

RHD:

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0} \frac{(1+h)^2-(1)^2}{h}$$

$$= \lim_{h\to 0} \frac{1+h^2+2h-1}{h}$$

$$= \lim_{h\to 0} \frac{h(h+2)}{h}$$

$$= \lim_{h\to 0} h+2 = 2$$

Since LHD $\neq$ RHD

f(x) is not differentiable at x = 1

OR

(b) Given : $f(x) = \begin{cases} \dfrac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1 & , \text{ if } x = 0 \end{cases}$ is continuous at x = 0

f is continuous at x = 0

If LHL = RHL = f(0)

i.e., $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = f(0)$

LHL at $x \to 0$

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$$

$$= \lim_{h\to 0} \frac{\sin^2 \lambda(0-h)}{(0-h)^2}$$

$$= \lim_{h\to 0} \frac{\sin^2(-\lambda h)}{h^2}$$

$$= \lim_{h\to 0} \frac{(-\sin\lambda h)^2}{h^2} \qquad \text{(as } \sin(-x) = -\sin x\text{)}$$

$$= \lim_{h\to 0} \frac{\sin^2 \lambda h}{h^2}$$

$$= \lim_{h\to 0} \left(\frac{\sin\lambda h}{\lambda h}\right)^2 \times \lambda^2$$

$$= (1)^2 \times \lambda^2 = \lambda^2 \qquad \left(\text{as } \lim_{x\to 0} \frac{\sin kx}{kx} = 1\right)$$

RHL at $x \to 0$

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$$

$$= \lim_{h\to 0} \frac{(\sin\lambda h)^2}{h^2}$$

$$= \lim_{h\to 0} \left(\frac{\sin\lambda h}{\lambda h}\right)^2 \times \lambda^2$$

$$= (1)^2 \times \lambda^2 = \lambda^2 \qquad \left(\text{as } \lim_{x\to 0} \frac{\sin kx}{kx} = 1\right)$$

And f(0) = 1

Now LHL = RHL = f(0)

$\lambda^2 = \lambda^2 = 1$

Hence, $\lambda = 1$

$\Rightarrow \lambda^2 = \pm 1$

Hence, $\lambda = \pm 1$

24. Sketch the region bounded by the lines 2x + y = 8, y = 2, y = 4 and the y-axis. Hence, obtain its area using integration.

Sol. We have,

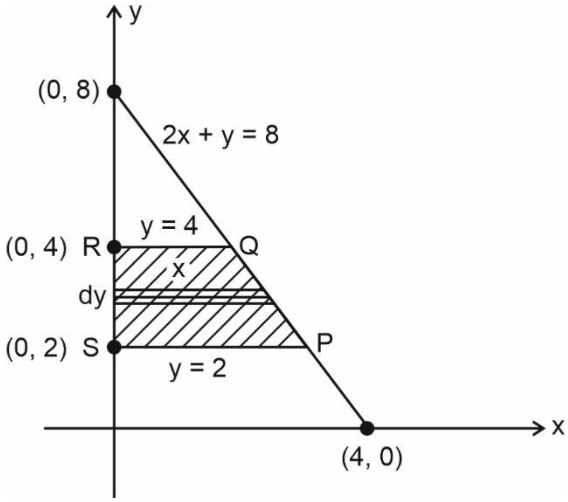
$2x + y = 8 \Rightarrow y = 8 - 2x$

When $x = 0$, $y = 8 - 2(0) = 8$

When $x = 4$, $y = 8 - 2(4) = 0$

Thus the line $2x + y = 8$ passes through (0, 8) and (4, 0) y = 2 and y = 4 are the lines parallel to x-axis passing through (0, 2) and (0, 4) respectively.

Now, we plot these points and sketch the region bounded by given lines and y-axis



Required area = area of region PQRSP

$= \text{area between the line } x = \dfrac{8-y}{2} \text{ and the y-axis between } y = 2 \text{ and } y = 4$

$$= \int_2^4 x\,dy$$

$$= \int_2^4 \left(4 - \frac{y}{2}\right) dy$$

$$= 4y - \frac{y^2}{4}\Bigg|_2^4$$

$$= \left(4(4) - \frac{(4)^2}{4}\right) - \left(4(2) - \frac{(2)^2}{4}\right)$$

$= (16 – 4) – (8 – 1)$

$= 12 – 7 = 5$

Hence, the required area is 5 sq. units.

25. Find the angle between the following two lines:

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right);$

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

Sol. Angle between two vectors

$\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$

And $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is given by

$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$$

Given, the pair of lines is

$$\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$$

So, $\vec{a}_1 = 2\hat{i} - 5\hat{j} + \hat{k}$

$\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$$\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$$

So, $\vec{a}_2 = 7\hat{i} - 6\hat{k}$

$\vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

Now, $\vec{b}_1 \cdot \vec{b}_2 = \left(3\hat{i} + 2\hat{j} + 6\hat{k}\right) \cdot \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$= (3 \times 1) + (2 \times 2) + (6 \times 2)$

$= 3 + 4 + 12$

$= 19$

Magnitude of $\vec{b}_1 = \sqrt{3^2 + 2^2 + 6^2}$

$$\left|\vec{b}_1\right| = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$$

Magnitude of $\vec{b}_2 = \sqrt{1^2 + 2^2 + 2^2}$

$$\left|\vec{b}_2\right| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

Now, $\cos\theta = \left|\dfrac{\vec{b}_1 \cdot \vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}\right|$

$$\cos\theta = \left|\frac{19}{7 \times 3}\right|$$

$$\cos\theta = \frac{19}{21}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

Therefore, the angle between the given vectors is $\cos^{-1}\left(\dfrac{19}{21}\right)$.

SECTION - C

This section comprises of Short Answer (SA) type questions of 3 marks each.

26. Using determinants, find the area of ΔPQR with vertices P(3, 1), Q(9, 3) and R(5, 7). Also, find the equation of line PQ using determinants.

Sol. Given $P \equiv (3, 1)$, $Q \equiv (9, 3)$ and $R \equiv (5, 7)$

Then area of ΔPQR will be

$$= \left|\frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ 5 & 7 & 1 \end{vmatrix}\right|$$

$$= \left|\frac{1}{2}[3(3-7)-1(9-5)+1(63-15)]\right|$$

$$= \left|\frac{1}{2}[-12-4+48]\right| = 16 \text{ sq. units}$$

Now equation of line PQ will be

$$\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow x(1-3) - y(3-9) + 1(9-9) = 0$

$\Rightarrow -2x + 6y = 0 \Rightarrow \boxed{x - 3y = 0}$

27. **(a)** Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

OR

(b) If y = tan x + sec x, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

Sol. **(a)** Let $y = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

and $z = \sin^{-1}(2x\sqrt{1-x^2})$

Now, Put $x = \sin\theta$

then, $y = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) = \theta$

$z = \sin^{-1}(2\sin\theta\cos\theta)$

$= 2\theta$

and $\frac{dy}{d\theta} = 1$, and $\frac{dz}{d\theta} = 2$

$\therefore \boxed{\frac{dy}{dz} = \frac{1}{2}}$

OR

(b) Given y = tan x + sec x

Differentiate with respect to x

$$\frac{dy}{dx} = \sec^2 x + \sec x \tan x$$

$$= \frac{1+\sin x}{\cos^2 x} = \frac{1+\sin x}{1-\sin^2 x}$$

$$= \frac{1}{1-\sin x}$$

again differentiate w. r. t. x

$$\frac{d^2y}{dx^2} = \frac{-(-\cos x)}{(1-\sin x)^2} = \frac{\cos x}{(1-\sin x)^2}$$

Hence proved

28. (a) Evaluate : $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos 2x}{1+\cos 2x}\, dx$

OR

(b) Find : $\int e^{x^2}(x^5+2x^3)\, dx$

Sol. (a) Let $= \displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos 2x}{1+\cos 2x}$

Since $f(x) = \dfrac{\cos 2x}{1+\cos 2x}$ is an even function

$$\therefore I = 2\int_{0}^{\frac{\pi}{4}} \frac{\cos 2x + 1 - 1}{1+\cos 2x}$$

$$= 2\left[\int_{0}^{\frac{\pi}{4}} \left(1 - \frac{1}{1+2\cos^2 x - 1}\right) dx\right]$$

$$= 2\left[\int_{0}^{\frac{\pi}{4}} \left(1 - \frac{1}{2}\sec^2 x\right) dx\right]$$

$$= 2\left[x - \frac{1}{2}\tan x\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\left(\frac{\pi}{4} - \frac{1}{2}\right) - (0)\right]$$

$$= \left(\frac{\pi}{2} - 1\right)$$

OR

(b) $I = \int e^{x^2}(x^5 + 2x^3)\, dx$

$= \int 2x^3 e^{x^2} dx + \int e^{x^2} x^5 dx \quad \ldots (1)$

Let

$I_1 = \int 2x^3 e^{x^2} dx$

$$= 2e^{x^2}\frac{x^4}{4} - \int 2x \cdot e^{x^2}\frac{2x^4}{4} dx$$

$$= \frac{e^{x^2}\cdot x^4}{2} - \int x^5 e^{x^2} dx + C$$

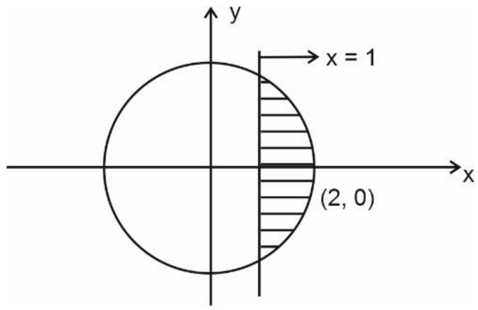
Put in eq^n (1)

$$I = \frac{e^{x^2}\cdot x^4}{2} - \int e^{x^2} x^5 dx + \int e^{x^2} x^5 dx + C$$

$$= \frac{e^{x^2} x^4}{2} + C$$

29. Find the area of the minor segment of the circle $x^2 + y^2 = 4$ cut off by the line $x = 1$, using integration.

Sol. Given circle $x^2 + y^2 = 4$ and line $x = 1$



$\therefore$ Area of required (shaded) region

$$= 2\int_1^2 y\,dx$$

$$= 2\int_1^2 \sqrt{4-x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_1^2$$

$$= 2\left[\left(2\cdot\frac{\pi}{2}\right)-\left(\frac{1}{2}\sqrt{3}+2\cdot\frac{\pi}{6}\right)\right]$$

$$= 2\left[\frac{2\pi}{3}-\frac{1}{2}\sqrt{3}\right]$$

$$= \left(\frac{4\pi}{3}-\sqrt{3}\right) \text{ square. unit}$$

30. Find the distance between the lines :

$$\vec{r} = (\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k});$$

$$\vec{r} = (3\hat{i}+3\hat{j}-5\hat{k})+\mu(4\hat{i}+6\hat{j}+12\hat{k})$$

Sol. Given lines, $\vec{r} = (\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r} = (3\hat{i}+3\hat{j}-5\hat{k})+\mu(4\hat{i}+6\hat{j}+12\hat{k})$

Here let $\vec{a}_1 = \hat{i}+2\hat{j}-4\hat{k}, \vec{b}_1 = 2\hat{i}+3\hat{j}+6\hat{k}$

and $\vec{a}_2 = 3\hat{i}+3\hat{j}-5\hat{k}, \vec{b}_2 = 4\hat{i}+6\hat{j}+12\hat{k}$

$\because \frac{2}{4}=\frac{3}{6}=\frac{6}{12}$ $\quad\therefore$ $\vec{b}_1 \,||\, \vec{b}_2$

$\therefore$ Both lines are parallel to each other

$\therefore$ Distance between these two lines

$$d = \left|\frac{\vec{b}_1\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$$

$$= \left|\frac{(2\hat{i}+3\hat{j}+6\hat{k})\times(2\hat{i}+\hat{j}-\hat{k})}{\sqrt{49}}\right|$$

and $(2\hat{i}+3\hat{j}+6\hat{k})\times(2\hat{i}+\hat{j}-\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$

$= -9\hat{i}+14\hat{j}-4\hat{k}$

$\therefore \quad d = \left|\dfrac{\sqrt{81+196+16}}{7}\right| = \dfrac{\sqrt{293}}{7}$

31. (a) Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\dfrac{x+3}{5}=\dfrac{y-1}{2}=\dfrac{z+4}{3}$.

OR

(b) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, if $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{c}|=2$.

Sol. (a) Given, $P \equiv (0, 2, 3)$

and line $L: \dfrac{x+3}{5}=\dfrac{y-1}{2}=\dfrac{z+4}{3}=t$

Let $Q \equiv (5t-3, 2t+1, 3t-4)$ be the foot of perpendicular drawn from P.

Then PQ perpendicular to the line L.

$\therefore \quad 5(5t-3-0)+2(2t+1-2)+3(3t-4-3)=0$

$\Rightarrow \quad 25t+4t+9t-15-2-21=0$

$\Rightarrow \quad 38t=38$

$\therefore \quad \boxed{t=1}$

$\therefore \quad Q \equiv (2, 3, -1)$

OR

(b) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors which satisfy $\vec{a}+\vec{b}+\vec{c}=0$ ...(i)

and $\vec{\mu}=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ with

$|\vec{a}|=3, |\vec{b}|=4$ and $|\vec{c}|=2$

Take dot product of (i) with $\vec{a}$ we get

$\vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$ ...(ii)

Similarly, with $\vec{b}$ and $\vec{c}$ we get

$\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}+\vec{b}\cdot\vec{c}=0$ ...(iii)

and $\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}\cdot\vec{c}=0$ ...(iv)

and we also know that $\vec{a}\cdot\vec{a}=|\vec{a}|^2$ and $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$

Adding (ii), (iii) and (iv)

$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\therefore \quad \mu=\dfrac{-1}{2}[9+16+4]=\dfrac{-29}{2}$

SECTION - D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x}dx$

Sol. $I = \int_0^{\pi} \frac{x}{1+\sin x}dx$ ... (i)

$I = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)} = \int_0^{\pi} \frac{\pi - x}{1+\sin x}$ ... (ii) $\left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$

(i) + (ii) $\Rightarrow 2I = \int_0^{\pi} \frac{\pi}{1+\sin x}dx$

$\Rightarrow I = \frac{\pi}{2}\int_0^{\pi} \frac{1}{1+\sin x}dx$

$= 2\times\frac{\pi}{2}\int_0^{\pi/2} \frac{1}{1+\sin x}dx$ $\left(\because\ f(\pi - x) = f(x)\right)$

$= \pi\int_0^{\pi/2} \frac{1}{1+\sin\left(\frac{\pi}{2}-x\right)}dx$ $\left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$

$= \pi\int_0^{\pi/2} \frac{1}{1+\cos x}dx = \pi\int_0^{\pi/2} \frac{1}{2\cos^2\frac{x}{2}}dx$

$= \frac{\pi}{2}\int_0^{\pi/2} \sec^2\frac{x}{2}dx$

$= \frac{\pi}{2}\cdot 2\cdot \tan\frac{x}{2}\Big]_0^{\pi/2}$

$= \pi\ (1 - 0) = \pi$

Ans : π

33. (a) The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

OR

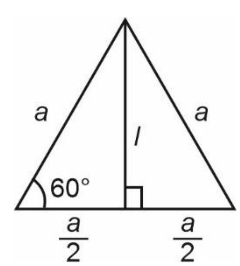
(b) Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

Sol. (a) Let length of median be l and side of triangle be a.

$l = a\sin 60° = \frac{\sqrt{3}}{2}a$

$a = \frac{2l}{\sqrt{3}}$

$\frac{da}{dt} = \frac{2}{\sqrt{3}}\frac{dl}{dt} = \frac{2}{\sqrt{3}}\times 2\sqrt{3} = 4$ cm/s.



Ans : 4 cm/s.

OR

(b) Let the two numbers be x and y.

$x + y = 5$ ... (i)

Let $z = x^3 + y^3$

$z = x^3 + (5 - x)^3$

$\frac{dz}{dx} = 3x^2 + 3(5 - x)^2(-1)$

$= 3x^2 - 3(x^2 - 10x + 25)$

$\frac{dz}{dx} = 15(2x - 5)$

$\frac{dz}{dx} = 0 \Rightarrow 2x - 5 = 0 \Rightarrow x = \frac{5}{2}$

$\frac{d^2z}{dx^2} = 2 > 0$

$\therefore$ z is least for $x = \frac{5}{2}$

Also $y = 5 - x = 5 - \frac{5}{2} = \frac{5}{2}$

Sum of squares = $x^2 + y^2$

$= \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 = \frac{25}{2} = 12.5$

Ans : 12.5

34. (a) In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly?

OR

(b) A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

Sol. (a) Let $E_1 \to$ Student knows the answer.

$E_2 \to$ Student guesses the answer.

A $\to$ The answer is correct

$P(E_1) = \frac{3}{5}$, $P\left(A/E_1\right) = 1$

$P(E_2) = \frac{2}{5}$, $P\left(A/E_2\right) = \frac{1}{3}$

Required probability $= P\left(\frac{E_1}{A}\right)$

$$= \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{3}{5}\times 1}{\frac{3}{5}\times 1 + \frac{2}{5}\times\frac{1}{3}}$$

$$= \frac{3}{3+\frac{2}{3}} = \frac{9}{11}$$

Ans. $= \frac{9}{11}$

OR

(b) Let E_1 : Ticket has a prize of ₹ 8

E_2 : Ticket has a prize of ₹ 4

E_3 : Ticket has a prize of ₹ 2

$x_1 = 8$, $P(x = x_1) = P_1 = \frac{2}{10}$

$x_2 = 4$, $P(x = x_2) = P_2 = \frac{5}{10}$

$x_3 = 2$, $P(x = x_3) = P_3 = \frac{3}{10}$

Mean value of the prize = $\sum_{i=1}^{3} P_i x_i$

$$= \frac{2}{10}\times 8 + \frac{5}{10}\times 4 + \frac{3}{10}\times 2$$

$$= \frac{42}{10} = 4.2$$

Ans. ₹ 4.2

35. Solve the following Linear Programming Problem graphically :

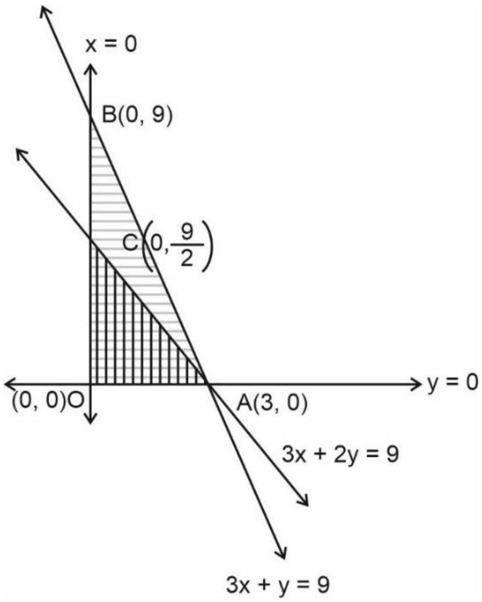
Maximize : $P = 70x + 40y$

subject to : $3x + 2y \le 9$,

$3x + y \le 9$,

$x \ge 0$, $y \ge 0$

Sol.



ΔOAC is the required bounded region.

P = 70x + 40y

At O(0, 0), P = 0

At (A(3, 0), P = 70 × 3 + 40 × 0

= 210

At $C\left(0, \frac{9}{2}\right), P = 70 \times 0 + 40 \times \frac{9}{2} = 180$

$\therefore$ Maximum of P = 210, which occurs when x = 3 and y = 0

Ans. : 210 when x = 3, y = 0

SECTION - E

This section comprises of 3 case study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub-parts (I) and (II) of marks 2 each.

Case Study-I

36. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹250.

Based on the above information, answer the following questions:

(I) Convert the given above situation into a matrix equation of the form AX = B.

(II) Find |A|.

(III) Find A^{-1}.

OR

(III) Determine $P = A^2 - 5A$.

Sol. Let the price of 1 pen, 1 bag and 1 instrument box are x, y and z respectively.

$\Rightarrow$ $5x + 3y + z = 160$

$2x + y + 3z = 190$

$x + 2y + 4z = 250$

(I) $\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$

(II) $|A| = \begin{vmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix}$

$= 5(4-6) - 3(8-3) + 1(4-1)$

$= -10 - 15 + 3 = -22$

(III) $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

Let matrix formed by co-factors = C

$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$, where $C_{ij} = (-1)^{i+j} M_{ij}$

$C_{11} = (-1)^2 (4 \times 1 - 2 \times 3) = -2,$

$C_{12} = (-1)^3 (4 \times 2 - 1 \times 3) = -5$

$C_{13} = (-1)^4 (2\times 2 - 1\times 1) = 3,$

$C_{21} = (-1)^3 (3\times 4 - 2\times 1) = -10$

$C_{22} = (-1)^4 (5\times 4 - 1\times 1) = 19$

$C_{23} = (-1)^5 (5\times 2 - 1\times 3) = -7$

$C_{31} = (-1)^4 (3\times 3 - 1\times 1) = 8$

$C_{32} = (-1)^5 (5\times 3 - 1\times 2) = -13$

$C_{33} = (-1)^6 (5\times 1 - 2\times 3) = -1$

$\Rightarrow$ $C = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}$

$\Rightarrow$ $A^{-1} = \dfrac{\text{adj}(A)}{|A|} = \dfrac{C^T}{|A|} = \left(-\dfrac{1}{22}\right)\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{11} & \frac{5}{11} & -\frac{4}{11} \\ \frac{5}{22} & -\frac{19}{22} & \frac{13}{22} \\ -\frac{3}{22} & \frac{7}{22} & \frac{1}{22} \end{bmatrix}$

OR

(III) $P = A^2 - 5A$

$A^2 = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix}$

$\Rightarrow P = A^2 - 5A$

$$= \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$$

Case Study-II

37. An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let B = $\{b_1, b_2, b_3\}$ and G = $\{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, answer the following questions :

(I) How many relations are possible from B to G ?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?

(III) Let R : B $\rightarrow$ B be defined by R = {(x, y) : x and y are students of the same sex}. Check if R is an equivalence relation.

OR

(III) A function f : B $\rightarrow$ G be defined by f = $\{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective. Justify your answer.

Sol. Given B = $\{b_1, b_2, b_3\}$ and G = $\{g_1, g_2\}$

i.e. n(B) = 3 and n(G) = 2

as we know if n(A) = m and n(B) = n

Number of relations A $\rightarrow$ B = 2^{mn}

(I) Number of relations B to G = $2^{3\times2} = 2^6 = 64$

(II) Number of functions from B to G = $n^m = 2^3 = 8$

(III) R : B $\rightarrow$ B will be

$R = \{(b_1, b_1), (b_1, b_2), (b_1, b_3), (b_2, b_1), (b_2, b_2), (b_2, b_3), (b_3, b_1), (b_3, b_2), (b_3, b_3)\}$

as b_1, b_2, b_3 are all boys

as $\forall (b_i, b_i)$ are present $\quad \therefore$ R is reflexive

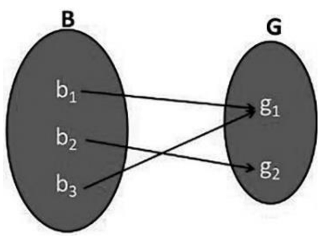
$\forall (b_i, b_j)$ there exist (b_j, b_i) $\quad \therefore$ R is symmetric

and as $\forall (b_i, b_j), (b_j, b_k)$ there is (b_i, b_k) $\quad \therefore$ R is transitive

Hence R is an equivalence relation

OR

(III) $f : B \to G$ has mapping diagram as below:



Clearly range of f is (g_1, g_2) = codomain of f.

but element g_1 has two pre-images

$\therefore$ f is onto but not one-one hence f is not bijective.

Case Study-III

38. An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if F(x, y) is a homogeneous function of degree zero, whereas a function F(x, y) is a homogeneous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution y = vx and then separate the variables.

Based on the above, answer the following questions :

(I) Show that $(x^2 - y^2)\,dx + 2xy\,dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solve the above equation to find its general solution.

Sol. (I) Based on the information given, we substitute y = vx in the equation $(x^2 - y^2)\,dx + 2xy\,dy = 0$

i.e., $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \quad \left(\text{as } y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}\right)$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{(v^2 - 1)x^2}{2vx^2}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$$

$$\Rightarrow \quad \frac{2v\,dv}{1 + v^2} + \frac{dx}{x} = 0 \qquad \ldots(i)$$

$\therefore$ The above equation, on substituting y = vx is reduced to a variable separable hence given equation is a differential equation of type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solving (i) further, we get

$$\int \frac{2v\,dv}{1+v^2} + \int \frac{dx}{x} = 0$$

$\Rightarrow$ $\ln(1 + v^2) + \ln x = \ln c$

$\Rightarrow$ $x\left(1+\left(\frac{y}{x}\right)^2\right) = c$

$\Rightarrow$ $x^2 + y^2 - cx = 0$

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