IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each part (subject) has two sections.

   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.

   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Two identical balls A and B thrown with same velocity 'u' at two different angles with horizontal attained the same range R. If A and B reached the maximum height \( h_1 \) and \( h_2 \) respectively, then \( \frac{1}{2} \frac{R}{h_1 h_2} \).

Reason R: Product of said heights.

\[
\frac{1}{2} \sin \theta = \frac{1}{2} \cos \theta
\]

Choose the correct answer:

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Answer (A)

Sol.

\[
h_1 = \frac{u^2 \sin^2 \theta}{2g}
\]

\[
h_2 = \frac{u^2 \cos^2 \theta}{2g}
\]

\[
\therefore \sqrt{h_1 h_2} = \frac{u^2 \sin \theta \cos \theta}{2g}
\]

\[
= \frac{R}{4}
\]

\[
\Rightarrow R = 4 \sqrt{h_1 h_2}
\]

2. Two buses P and Q start from a point at the same time and move in a straight line and their positions are represented by \( X_P(t) = \alpha t + \beta t^2 \) and \( X_Q(t) = ft - \ell^2 \). At what time, both the buses have same velocity?

\[
\Rightarrow \frac{f - \alpha}{2(1 + \beta)}
\]

Answer (D)

Sol. \( X_P = \alpha t + \beta t^2 \)

\( X_Q = ft - \ell^2 \)

\[
\therefore V_P = \alpha + 2\beta t
\]

\[
V_Q = f - 2t
\]

\[
\therefore V_P = V_Q
\]

\[
\Rightarrow \alpha + 2\beta t = f - 2t
\]

\[
\Rightarrow t = \frac{f - \alpha}{2(1 + \beta)}
\]

3. A disc with a flat small bottom beaker placed on it at a distance \( R \) from its center is revolving about an axis passing through the center and perpendicular to its plane with an angular velocity \( \omega \). The coefficient of static friction between the bottom of the beaker and the surface of the disc is \( \mu \). The beaker will revolve with the disc if:

\[
\frac{2 \mu}{\omega^2} \leq R
\]

\[
\frac{\mu g}{2 \omega^2} \leq R
\]

\[
\frac{\mu g}{2 \omega^2} \geq R
\]

\[
\frac{\mu g}{2 \omega^2} \geq R
\]

Answer (B)

Sol. To move together

\[
\omega R \leq \mu g
\]

\[
\Rightarrow R \leq \frac{\mu g}{\omega^2}
\]

4. A solid metallic cube having total surface area 24 m² is uniformly heated. If its temperature is increased by 10°C, calculate the increase in volume of the cube.

(Given \( \alpha = 5.0 \times 10^{-4} \text{ °C}^{-1} \)).

\[
2.4 \times 10^6 \text{ cm}^3
\]

\[
1.2 \times 10^5 \text{ cm}^3
\]

\[
6.0 \times 10^4 \text{ cm}^3
\]

\[
4.8 \times 10^5 \text{ cm}^3
\]

Answer (B)
6. \( 6 \times l = 24 \)
\[ \Rightarrow \quad l = 2 \text{ m} \]
\[ \therefore \quad \Delta V \left( \frac{\Delta l}{l} \right) = 3 \times \frac{\Delta l}{l} \]
\[ \Rightarrow \quad \Delta V = 3 \times (\alpha \Delta T) \times V \]
\[ = 3 \times 5 \times 10^{-4} \times 10 \times (8) \]
\[ = 120 \times 10^{-3} \text{ m}^3 \]
\[ = 1.2 \times 10^5 \text{ cm}^3 \]

5. A copper block of mass 5.0 kg is heated to a temperature of 500°C and is placed on a large ice block. What is the maximum amount of ice that can melt?

[Specific heat of copper : 0.39 J g\(^{-1}\) °C\(^{-1}\) and latent heat of fusion of water : 335 J g\(^{-1}\)]

(A) 1.5 kg \hspace{1cm} (B) 5.8 kg

(C) 2.9 kg \hspace{1cm} (D) 3.8 kg

Answer (C)

Sol. \( m \Delta L = \Delta Q = ms \Delta T \)
\[ \Rightarrow \quad m = \frac{5 \times 0.39 \times 10^3 \times 500}{335} \]
\[ = 2.9 \text{ kg} \]

6. The ratio of specific heats \( \left( \frac{C_p}{C_v} \right) \) in terms of degree of freedom (f) is given by:

(A) \( 1 \frac{f}{3} \)

(B) \( 1 \frac{2}{f} \)

(C) \( 1 \frac{f}{2} \)

(D) \( 1 \frac{1}{f} \)

Answer (B)

Sol. \( \frac{C_p}{C_v} = \gamma \)

\[ C_v = \left( \frac{f}{2} \right)R \quad \text{and} \quad C_p - C_v = R \]
\[ \Rightarrow \quad \frac{C_p}{C} = 1 + \frac{f}{2} \frac{l}{2} = 1 + \frac{2}{f} \]

7. For a particle in uniform circular motion, the acceleration \( \dot{a} \) at any point \( P(R, \theta) \) on the circular path of radius \( R \) is (when \( \theta \) is measured from the positive x-axis and \( v \) is uniform speed):

(A) \( -\frac{v^2}{R} \sin \theta \hat{i} + \frac{v^2}{R} \cos \theta \hat{j} \)

(B) \( -\frac{v^2}{R} \cos \theta \hat{i} + \frac{v^2}{R} \sin \theta \hat{j} \)

(C) \( -\frac{v^2}{R} \cos \theta \hat{i} - \frac{v^2}{R} \sin \theta \hat{j} \)

(D) \( -\frac{v^2}{R} \hat{i} + \frac{v^2}{R} \hat{j} \)

Answer (C)

As the particle in uniform circular motion experiences only centripetal acceleration of magnitude \( \omega^2 R \) or \( \frac{v^2}{R} \) directed towards centre so from diagram.

\[ \ddot{a} = \frac{v^2}{R} \cos \theta (-\hat{i}) + \frac{v^2}{R} \sin \theta \hat{j} \]

8. Two metallic plates form a parallel plate capacitor. The distance between the plates is ‘d’. A metal sheet of thickness \( \frac{d}{2} \) and of area equal to area of each plate is introduced between the plates. What will be the ratio of the new capacitance to the original capacitance of the capacitor?

(A) 2 : 1 \hspace{1cm} (B) 1 : 2 \hspace{1cm} (C) 1 : 4 \hspace{1cm} (D) 4 : 1

Answer (A)

Sol. \( C_{eq} = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2k}} = \frac{\varepsilon_0 A}{d} = \frac{2\varepsilon_0 A}{d} \)

If \( C = \frac{\varepsilon_0 A}{d} \)
\[ \Rightarrow \quad C_{eq} = 2C \quad \text{or} \quad \frac{C_{new}}{C_{old}} = 2 \cdot \frac{2}{1} \]
9. Two cells of same emf but different internal resistances $r_1$ and $r_2$ are connected in series with a resistance $R$. The value of resistance $R$, for which the potential difference across second cell is zero, is:

(A) $r_2 - r_1$

(B) $r_1 - r_2$

(C) $r_1$

(D) $r_2$

Answer (A)

\[ I = \frac{2\varepsilon}{R + r_1 + r_2} \]

As per the question,

\[ \frac{2\varepsilon}{R + r_1 + r_2} \times r_2 - \varepsilon = 0 \]

$\Rightarrow R = r_2 - r_1$

10. Given below are two statements:

Statement-I : Susceptibilities of paramagnetic and ferromagnetic substances increase with decrease in temperature.

Statement-II : Diamagnetism is a result of orbital motions of electrons developing magnetic moments opposite to the applied magnetic field.

Choose the correct answer from the options given below:-

(A) Both Statement-I and Statement-II are true

(B) Both Statement-I and Statement-II are false

(C) Statement-I is true but Statement-II is false

(D) Statement-I is false but Statement-II is true

Answer (A)

Sol. Statement-I is true as susceptibility of ferromagnetic and paramagnetic materials is inversely related to temperature.

Statement-II is true as because of orbital motion of electrons the diamagnetic material is able to oppose external magnetic field.

11. A long solenoid carrying a current produces a magnetic field $B$ along its axis. If the current is doubled and the number of turns per cm is halved, the new value of magnetic field will be equal to

(A) $B$

(B) $2B$

(C) $4B$

(D) $\frac{B}{2}$

Answer (A)

Sol. $B = \mu_0 ni$

Now $i \rightarrow 2i$

And $n \rightarrow \frac{n}{2}$

$B' = \mu_0 \frac{n}{2} \times 2i = \mu_0 ni = B$

12. A sinusoidal voltage $V(t) = 210 \sin 3000t$ volt is applied to a series LCR circuit in which $L = 10$ mH, $C = 25 \mu F$ and $R = 100$ $\Omega$. The phase difference ($\phi$) between the applied voltage and resultant current will be:

(A) $\tan^{-1}(0.17)$

(B) $\tan^{-1}(9.46)$

(C) $\tan^{-1}(0.30)$

(D) $\tan^{-1}(13.33)$

Answer (A)

Sol. $X_L = 3000 \times 10 \times 10^{-3} = 30\Omega$

$X_C = \frac{1}{3000 \times 25} \times 10^6 = \frac{40}{3} \Omega$

So $X_L - X_C = 30 - \frac{40}{3} = \frac{50}{3} \Omega$

$\tan \theta = \frac{X_L - X_C}{R} = \frac{50/3}{100} = \frac{1}{6}$

So $\theta = \tan^{-1}(0.17)$

13. The electromagnetic waves travel in a medium at a speed of $2.0 \times 10^8$ m/s. The relative permeability of the medium is 1.0. The relative permittivity of the medium will be:

(A) 2.25

(B) 4.25

(C) 6.25

(D) 8.25

Answer (A)

Sol. $n = \frac{c}{v} = \frac{3}{2}$

$\sqrt{\varepsilon \mu} = n$

So $\varepsilon = \frac{9}{4} = 2.25$
14. The interference pattern is obtained with two coherent light sources of intensity ratio $4 : 1$. And the ratio $\frac{I_{\text{max}} + I_{\text{min}}}{I_{\text{max}} - I_{\text{min}}} = \frac{5}{x}$. Then, the value of $x$ will be equal to:

(A) 3  (B) 4  (C) 2  (D) 1

Answer (B)

Sol. $\frac{I_{\text{max}} + I_{\text{min}}}{I_{\text{max}} - I_{\text{min}}} = \frac{l_1 + l_2 + 2\sqrt{l_1 l_2} + l_1 + l_2 - 2\sqrt{l_1 l_2}}{l_1 + l_2 + 2\sqrt{l_1 l_2} - l_1 - l_2 + 2\sqrt{l_1 l_2}}$

$= \frac{2(l_1 + l_2)}{4\sqrt{l_1 l_2}}$

$= \frac{l_1 + 1}{2\sqrt{l_1 l_2}}$

So $x = 4$

15. A light whose electric field vectors are completely removed by using a good polaroid, allowed to incident on the surface of the prism at Brewster’s angle. Choose the most suitable option for the phenomenon related to the prism.

(A) Reflected and refracted rays will be perpendicular to each other.
(B) Wave will propagate along the surface of prism.
(C) No refraction, and there will be total reflection of light.
(D) No reflection, and there will be total transmission of light.

Answer (D)

Sol. When electric field vector is completely removed and incident on Brewster’s angle then only refraction takes place.

16. A proton, a neutron, an electron and an $\alpha$-particle have same energy. If $\lambda_p$, $\lambda_n$, $\lambda_e$ and $\lambda_\alpha$ are the de Broglie’s wavelengths of proton, neutron, electron and $\alpha$ particle respectively, then choose the correct relation from the following:

(A) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$
(B) $\lambda_\alpha < \lambda_n < \lambda_p < \lambda_e$
(C) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$
(D) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$

Answer (B)
18. Identify the logic operation performed by the given circuit:

(A) AND gate
(B) OR gate
(C) NOR gate
(D) NAND gate

Answer (A)

Sol. According to the circuit,

\[ Y = (A' + B')' \]

\[ \Rightarrow Y = AB \]

\[ \Rightarrow \text{AND gate} \]

19. Match List I with List II

<table>
<thead>
<tr>
<th>List I</th>
<th>List II</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Facsimile</td>
<td>I. Static Document Image</td>
</tr>
<tr>
<td>B. Guided media Channel</td>
<td>II. Local Broadcast Radio</td>
</tr>
<tr>
<td>C. Frequency Modulation</td>
<td>III. Rectangular wave</td>
</tr>
<tr>
<td>D. Digital Single</td>
<td>IV. Optical Fiber</td>
</tr>
</tbody>
</table>

Choose the correct answer from the following options:

(A) A-IV, B-III, C-II, D-I
(B) A-I, B-IV, C-II, D-III
(C) A-IV, B-II, C-III, D-I
(D) A-I, B-II, C-III, D-IV

Answer (B)

Sol. The correct match is:

<table>
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20. If \( n \) represents the actual number of deflections in a converted galvanometer of resistance \( G \) and shunt resistance \( S \). Then the total current \( I \) when its figure of merit is \( K \) will be

(A) \[ \frac{KS}{S+G} \]
(B) \[ \frac{(G+S)}{nKS} \]
(C) \[ \frac{nKS}{G+S} \]
(D) \[ \frac{nK(G+S)}{S} \]

Answer (D)

Sol. According to the information, current through galvanometer = \( nK \)

\[ i = \frac{S}{S+G} \]

\[ \Rightarrow i = \frac{nK(S+G)}{S} \]

**SECTION - B**

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. For \( z = a^2x^3y^2 \), where 'a' is a constant. If percentage error in measurement of 'x' and 'y' are 4% and 12%, respectively, then the percentage error for 'z' will be _______ %.

Answer (18)
Sol. % error in z = $3 \times 4 + \frac{1}{2} \times 12$

= 12 + 6 = 18%

2. A curved in a level road has a radius 75 m. The maximum speed of a car turning this curved road can be 30 m/s without skidding. If radius of curved road is changed to 48 m and the coefficient of friction between the tyres and the road remains same, then maximum allowed speed would be _______ m/s.

Answer (24)

Sol. \( v = \sqrt{\mu gr} \)

\[ \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} \]

\[ \Rightarrow \frac{30}{v_2} = \sqrt{\frac{75}{48}} = \sqrt{\frac{25}{16}} = \frac{5}{4} \]

\( \Rightarrow v_2 = 24 \text{ m/s} \)

3. A block of mass 200 g is kept stationary on a smooth inclined plane by applying a minimum horizontal force \( F = \sqrt{x} \text{ N} \) as shown in figure.

![Diagram](image)

The value of \( x = \) _______.

Answer (12)

Sol. \[ F \cos 60^\circ = mg \sin 60^\circ \]

\[ \Rightarrow F \times \frac{1}{2} = 0.2 \times 10 \times \frac{\sqrt{3}}{2} \]

\[ \Rightarrow F = 2\sqrt{3} \]

\[ \Rightarrow F = \sqrt{12} \text{ N} \]

\( \therefore \ x = 12 \)

4. Moment of Inertia (M.I.) of four bodies having same mass 'M' and radius '2R' are as follows:

\( I_1 \) = M.I. of solid sphere about its diameter

\( I_2 \) = M.I. of solid cylinder about its axis

\( I_3 \) = M.I. of solid circular disc about its diameter.

\( I_4 \) = M.I. of thin circular ring about its diameter

If \( 2(I_2 + I_3) + I_4 = x \cdot I_1 \) then the value of \( x \) will be _______.

Answer (5)

Sol. \[ 2\left(\frac{1}{2} \cdot \frac{1}{4} \right) \times M(2R)^2 + \frac{1}{2} M(2R)^2 = x \cdot \frac{2}{5} M(2R)^2 \]

\[ \Rightarrow 1 + \frac{1}{2} + \frac{1}{2} = x \cdot \frac{2}{5} \]

\[ \Rightarrow x = 5 \]

5. Two satellites \( S_1 \) and \( S_2 \) are revolving in circular orbits around a planet with radius \( R_1 = 3200 \text{ km} \) and \( R_2 = 800 \text{ km} \) respectively. The ratio of speed of satellite \( S_1 \) to the speed of satellite \( S_2 \) in their respective orbits would be \( \frac{1}{x} \) where \( x = \)

Answer (2)

Sol. \[ v = \sqrt{\frac{GM}{R}} \]

\[ \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{R_2}{R_1}} \]

\[ \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{3200}{800}} = 2 \]

\[ \Rightarrow \frac{v_1}{v_2} = \frac{1}{2} \]

\( x = 2 \)
6. When a gas filled in a closed vessel is heated by raising the temperature by 1ºC, its pressure increases by 0.4%. The initial temperature of the gas is ______ K.

Answer (250)
Sol. \( PV = nRT \)
So, \( \frac{dP}{P} \times 100 = \frac{dT}{T} \times 100 \)

\[ 0.4 = \frac{1}{T} \times 100 \]
\[ \Rightarrow T = 250 \text{ K} \]

7. 27 identical drops are charged at 22 V each. They combine to form a bigger drop. The potential of the bigger drop will be ______ V.

Answer (198)
Sol. Let the charge on one drop is \( q \) and its radius is \( r \).

So for one drop \( V = \frac{kq}{r} \)

For 27 drops merged new charge will be \( Q = 27q \)
and new radius is \( R = 3r \)

So new potential is \( V' = \frac{kQ}{R} = 9 \frac{kq}{r} = 9 \times 22 \text{ V} \)

\[ = 198 \text{ V} \]

8. The length of a given cylindrical wire is increased to double of its original length. The percentage increase in the resistance of the wire will be ______%.

Answer (300)
Sol. Volume is constant so on length doubled

Area is halved so

\[ R = \rho \frac{l}{A} \quad \text{and} \quad R' = \rho \frac{2l}{A} = 4 \rho \frac{l}{A} = 4R \]

So percentage increase will be

\[ R% = \frac{4R - R}{R} \times 100 = 300\% \]

9. In a series LCR circuit, the inductance, capacitance and resistance are \( L = 100 \text{ mH}, C = 100 \mu \text{F} \) and \( R = 10 \Omega \) respectively. They are connected to an AC source of voltage 220 V and frequency of 50 Hz. The approximate value of current in the circuit will be ______ A.

\[ Z = \sqrt{R^2 + \left( \frac{1}{2\pi fC} - \frac{1}{\sqrt{L}} \right)^2} \]

\[ = \sqrt{10^2 + \left( \frac{100}{10\pi} - \frac{1}{\sqrt{100}} \right)^2} \Omega \]

\[ = 10 \Omega \]

\[ \Rightarrow \text{Current} = \frac{220}{10} A = 22 \text{ A} \]

10. In an experiment of CE configuration of \( n-p-n \) transistor, the transfer characteristics are observed as given in figure.

If the input resistance is 200 \( \Omega \) and output resistance is 60 \( \Omega \), the voltage gain in this experiment will be ______.

Answer (15)
Sol. Voltage gain \( = \frac{I_C}{I_B} \frac{R_o}{R_i} \)

\[ = \frac{(10 \text{ mA}) (60 \Omega)}{(200 \mu \text{A}) (200 \Omega)} \]

\[ \Rightarrow \text{Voltage gain} = 15 \]
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. The minimum energy that must be possessed by photons in order to produce the photoelectric effect with platinum metal is [Given: The threshold frequency of platinum is $1.3 \times 10^{15}$ s$^{-1}$ and $h = 6.6 \times 10^{-34}$ Js.]

   (A) $3.21 \times 10^{-14}$ J  
   (B) $6.24 \times 10^{-16}$ J  
   (C) $8.58 \times 10^{-19}$ J  
   (D) $9.76 \times 10^{-20}$ J

Answer (C)

Sol.: The minimum energy possessed by photons will be equal to the work function of the metal. Hence,

$$w_0 = h\nu_0 = 6.6 \times 10^{-34} \times 1.3 \times 10^{15} = 8.58 \times 10^{-19} \text{ J}$$

2. At 25°C and 1 atm pressure, the enthalpy of combustion of benzene (I) and acetylene (g) are $-3268$ kJ mol$^{-1}$ and $-1300$ kJ mol$^{-1}$, respectively. The change in enthalpy for the reaction $3\text{C}_2\text{H}_2(\text{g}) \rightarrow \text{C}_6\text{H}_6(\text{l})$, is

   (A) $+324$ kJ mol$^{-1}$  
   (B) $+632$ kJ mol$^{-1}$  
   (C) $-632$ kJ mol$^{-1}$  
   (D) $-732$ kJ mol$^{-1}$

Answer (C)

Sol.: I. $\text{C}_6\text{H}_6(\ell) + \frac{15}{2} \text{O}_2(\text{g}) \rightarrow 6\text{CO}_2(\text{g}) + 3\text{H}_2\text{O}(\text{g})$

$$\Delta H_1 = -3268 \text{ kJ/mol}$$

II. $\text{C}_2\text{H}_2(\text{g}) + \frac{5}{2} \text{O}_2(\text{g}) \rightarrow 2\text{CO}_2(\text{g}) + \text{H}_2\text{O}(\text{g})$

$$\Delta H_2 = -1300 \text{ kJ/mol}$$

III. $3\text{C}_2\text{H}_2(\text{g}) \rightarrow \text{C}_6\text{H}_6(\ell)$  

Applying Hess’s law of constant heat summation

$$\Delta H_3 = 3 \times \Delta H_2 - \Delta H_1$$

$$= 3 \times (-1300) - (-3268)$$

$$= -632 \text{ kJ/mol}$$

3. Solute A associates in water. When 0.7 g of solute A is dissolved in 42.0 g of water, it depresses the freezing point by 0.2°C. The percentage association of solute A in water is:

   [Given: Molar mass of A = 93 g mol$^{-1}$. Molal depression constant of water is 1.86 K kg mol$^{-1}$.]

   (A) 50%  
   (B) 60%  
   (C) 70%  
   (D) 80%

Answer (D)

Sol.: Since, $\Delta T_f = ikm$

$$m = \frac{0.7 \times 1000}{93} \times \frac{1.86}{42} \times \frac{0.7 \times 1000}{93 \times 42}$$

$$\alpha = \frac{1 - 1}{2} = 0.8$$

Hence, percentage association of solute A is 80%.

4. The $K_{sp}$ for bismuth sulphide ($\text{Bi}_2\text{S}_3$) is $1.08 \times 10^{-73}$. The solubility of $\text{Bi}_2\text{S}_3$ in mol L$^{-1}$ at 298 K is

   (A) $1.0 \times 10^{-15}$  
   (B) $2.7 \times 10^{-12}$  
   (C) $3.2 \times 10^{-10}$  
   (D) $4.2 \times 10^{-8}$

Answer (A)

Sol.: $\text{Bi}_2\text{S}_3 \rightleftharpoons 2\text{Bi}^{3+} + 3\text{S}^{2-}$

$$2s \quad 3s$$

$$K_{sp} = (2s)^2(3s)^3 = 108s^5$$

$$108s^5 = 108 \times 10^{-75}$$

$$s = 1.0 \times 10^{-15} \text{ mol/L}$$
5. Match List I with List II.

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<td>Diastase</td>
<td>Malt</td>
</tr>
<tr>
<td>Urease</td>
<td>Soyabeen</td>
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<tr>
<td>Pepsin</td>
<td>Stomach</td>
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</table>

Choose the correct answer from the options given below:
(A) A-II, B-III, C-I, D-IV
(B) A-II, B-III, C-IV, D-I
(C) A-III, B-II, C-IV, D-I
(D) A-III, B-II, C-I, D-IV

**Answer (B)**

**Sol.:**

<table>
<thead>
<tr>
<th>Enzyme</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zymase</td>
<td>Yeast</td>
</tr>
<tr>
<td>Diastase</td>
<td>Malt</td>
</tr>
<tr>
<td>Urease</td>
<td>Soyabeen</td>
</tr>
<tr>
<td>Pepsin</td>
<td>Stomach</td>
</tr>
</tbody>
</table>

Hence, A-II, B-III, C-IV, D-I

6. The correct order of electron gain enthalpies of Cl, F, Te and Po is
   (1) F < Cl < Te < Po
   (2) Te < Po < Cl < F
   (3) Cl < F < Te < Po
   (4) F < Cl < Te < Po

**Answer (B)**

**Sol.:** Te → −190 kJ mol⁻¹

Po → −174 kJ mol⁻¹

F → −339 kJ mol⁻¹

Cl → −349 kJ mol⁻¹

Hence, correct order is Cl > F > Te > Po

7. Given below are two statements.
   **Statement-I:** During electrolytic refining, blister copper deposits precious metals.
   **Statement-II:** In the process of obtaining pure copper by electrolysis method, copper blister is used to make the anode.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement-I and Statement-II are true.
(2) Both Statement-I and Statement-II are false.
(3) Statement-I is true but Statement-II is false.
(4) Statement-I is false but Statement-II is true

**Answer (A)**

**Sol.** Copper is refined using an electrolytic method. Anodes are of impure copper and pure copper strips are taken as cathode. Impurities from the blister copper deposit as anode mud which contains antimony, selenium, tellurium, silver, gold and platinum. Hence both statements are true

8. Given below are two statements one is labelled as Assertion A and the other is labelled as Reason R:
   **Assertion A:** The amphoteric nature of water is explained by using Lewis acid/base concept
   **Reason R:** Water acts as an acid with NH₃ and as a base with H₂S.

In the light of the above statements choose the correct answer from the options given below:
(1) Both A and R are true and R is the correct explanation of A.
(2) Both A and R are true but R is NOT the correct explanation of A.
(3) A is true but R is false.
(4) A is false but R is true.

**Answer (D)**

**Sol.** The amphoteric nature of water is explained by using Bronsted-Lowry acid base concept

\[ \text{H}_2\text{O} + \text{NH}_3 \rightleftharpoons \text{OH}^- + \text{NH}_4^+ \] (acid)

\[ \text{H}_2\text{O} + \text{H}_2\text{S} \rightleftharpoons \text{H}_3\text{O}^+ + \text{HS}^- \] (base)

Hence, A is false but R is true

9. The correct order of reduction potentials of the following pairs is
   (A) Cl₂/Cl⁻
   (B) I₂/I⁻
   (C) Ag⁺/Ag
   (D) Na⁺/Na
   (E) Li⁺/Li

Choose the correct answer from the options given below:
(1) A > C > B > D > E
(2) A > B > C > D > E
(3) A > C > B > E > D
(4) A > B > C > E > D

Answer (A)

Sol.

A > C > B > D > E

10. The number of bridged oxygen atoms present in compound B formed from the following reactions is

\[ \text{Pb(NO}_3\text{)}_2 \xrightarrow{673 K} \text{A + PbO + O}_2 \]

\[ \text{NO}_2 \xrightarrow{\text{Dimersise}} \text{N}_2\text{O}_4 \]

Hence, correct order is A > C > B > D > E

Answer (A)

Sol.

\[ 2\text{Pb(NO}_3\text{)}_2 \xrightarrow{673 K} 4\text{NO}_2 + 2\text{PbO + O}_2 \]

Hence no bridged oxygen atom is present in N$_2$O$_4$.

11. The metal ion (in gaseous state) with lowest spin-only magnetic moment value is

(A) V$^{2+}$
(B) Ni$^{2+}$
(C) Cr$^{2+}$
(D) Fe$^{2+}$

Answer (B)

<table>
<thead>
<tr>
<th>Sol.</th>
<th>Valence shell</th>
<th>Configuration</th>
<th>Unpaired electrons</th>
</tr>
</thead>
<tbody>
<tr>
<td>V$^{2+}$</td>
<td>→ $3d^34s^0$</td>
<td>$n = 3$</td>
<td></td>
</tr>
<tr>
<td>Ni$^{2+}$</td>
<td>→ $3d^94s^0$</td>
<td>$n = 2$</td>
<td></td>
</tr>
<tr>
<td>Cr$^{2+}$</td>
<td>→ $3d^44s^0$</td>
<td>$n = 4$</td>
<td></td>
</tr>
<tr>
<td>Fe$^{2+}$</td>
<td>→ $3d^44s^0$</td>
<td>$n = 4$</td>
<td></td>
</tr>
</tbody>
</table>

Since Ni$^{2+}$ has least number of unpaired electrons. Hence Ni$^{2+}$ will have lowest spin only magnetic moment Value.

12. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

**Assertion A:** Polluted water may have a value of BOD of the order of 17 ppm.

**Reason R:** BOD is a measure of oxygen required to oxidise both the bio-degradable and non-biodegradable organic material in water.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A.
(B) Both A and R are correct but R is NOT the correct explanation of A.
(C) A is correct but R is not correct.
(D) A is not correct but R is correct.

Answer (C)

Sol. Highly polluted water could have a BOD value of 17 ppm or more.

The amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water is called Biochemical Oxygen demand (BOD).

Hence A is correct but R is not correct.

13. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

**Assertion A:** A mixture contains benzoic acid and naphthalene. The pure benzoic acid can be separated out by the use of benzene.

**Reason R:** Benzoic acid is soluble in hot water.

In the light of the above statements, choose the **most appropriate** answer from the options given below.
(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Answer (D)

Sol. Since, both benzoic acid and naphthalene will dissolve in benzene. Hence assertion is wrong. Benzoic acid is almost insoluble in cold water but soluble in hot water. Hence Reason is true.

14. During halogen test, sodium fusion extract is boiled with concentrated HNO₃ to
(A) remove unreacted sodium
(B) decompose cyanide or sulphide of sodium
(C) extract halogen from organic compound
(D) maintain the pH of extract.

Answer (B)

Sol. During test for halogen, if nitrogen or sulphur is also present in the compound, then sodium fusion extract is first boiled with concentrated nitric acid to decompose cyanide or sulphide of sodium formed during Lassaigne's test.

15. Amongst the following, the major product of the given chemical reaction is

(A) Benzyl bromide
(B) Bromo benzene
(C) Cyclohexyl bromide
(D) Methyl bromide

Answer (B)

Sol.

Hence 'A' is bromobenzene.
17. Which of the following conditions or reaction sequence will NOT give acetophenone as the major product?

(A) (a) $C_6H_5COH + CH_2MgBr$
    (b) $Na_2Cr_2O_7, H^+$

(B) (a) $H_3C\text{CH} = \text{CH} + C_6H_5MgBr$
    (b) PCC, DCM

(C) $C_6H_5OC_2H_5 + 2CH_3MgBr$

(D) $C_6H_5COCl + CH_3MgBr + CdCl_2$

Answer (C)

Sol. C will not give acetophenone

18. The major product formed in the following reaction, is

$\text{CH}_3\text{OH} + \text{C}_6\text{H}_5\text{CH} = \text{CH}_2 \xrightarrow{H^+} \text{C}_6\text{H}_5\text{CH} = \text{CH} \text{CH}_3$

(A) $\text{C}_6\text{H}_5\text{CH} = \text{CH} \text{CH}_2\text{OH}$

Answer (D)

Sol.

19. Which of the following ketone will NOT give enamine on treatment with secondary amines? [where t-Bu is $-\text{C}(\text{CH}_3)_3$]

(A) \[\text{C}_6\text{H}_5\text{CH} = \text{CH} \text{CH}_3\]

(B) \[\text{C}_2\text{H}_5\text{CH} = \text{CH} \text{CH}_3\]

(C) \[\text{t-Bu} \text{CH} = \text{CH} \text{t-Bu}\]

(D) \[\text{C}_6\text{H}_5\text{CH} = \text{CH} \text{CH}_3\]

Answer (C)

Sol. In order to form enamine from the reaction of carbonyl compound with $2^\circ$ amine, the carbonyl compound must have $\alpha$-hydrogen.

In $\text{t-Bu} \text{CO}\text{t-Bu}$ No $\alpha$-hydrogen is present.

Along with this, due to steric crowding by t-Bu group, it is difficult for $2^\circ$ amine to attack on this compound.
20. An antiseptic Dettol is a mixture of two compounds ‘A’ and ‘B’ where A has 6π electrons and B has 2π electrons. What is ‘B’?

(A) Bithionol
(B) Terpineol
(C) Chloroxylenol
(D) Chloramphenicol

Answer (B)

Sol. Dettol is a mixture of chloroxylenol and terpineol. Chloroxylenol has 6π electrons and terpineol has 2π electrons.
Hence B is terpineol.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, −00.33, −00.30, 30.27, −27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A protein ‘A’ contains 0.30% of glycine (molecular weight 75). The minimum molar mass of the protein ‘A’ is _______ × 10^3 g mol⁻¹ [nearest integer]

Answer (25)

Sol. 0.3% glycine means
100 g protein ‘A’ contains 0.3 g glycine.
Since, molar mass of glycine is 75
75 g glycine will be present in \( \frac{100}{0.3} \times 75 \) g protein

Minimum molar mass of protein ‘A’ is 25 × 10^3 g/mol

2. A rigid nitrogen tank stored inside a laboratory has a pressure of 30 atm at 06:00 am when the temperature is 27°C. At 03:00 pm, when the temperature is 45°C, the pressure in the tank will be _______ atm. [nearest integer]

Answer (32)

Sol. Since
\[ P \propto T \]
Hence, \( \frac{P_1}{T_1} = \frac{P_2}{T_2} \)
(\( P_1 \) is pressure at 6 am) \( P_2 \) is pressure at 3 pm

3. Amongst BeF₂, BF₃, H₂O, NH₃, CCl₄ and HCl, the number of molecules with non-zero net dipole moment is ______.

Answer (3)

Sol. F − Be − F \( \mu = 0 \)
F
\( \mu = 0 \)
\( \mu \neq 0 \)
\( \mu \neq 0 \)
\( \mu = 0 \)
H − Cl \( \mu \neq 0 \)

4. At 345 K, the half life for the decomposition of a sample of a gaseous compound initially at 55.5 kPa was 340 s. When the pressure was 27.8 kPa, the half life was found to be 170 s. The order of the reaction is ______. [integer answer]

Answer (0)

Sol. \( t_{\frac{1}{2}} \propto \frac{1}{[P_0]^{n-1}} \)
\[ \left( \frac{t_{\frac{1}{2}}}{t_{\frac{1}{2}}} \right)_1 = \left[ \frac{[P_0]^{n-1}}{[P_0]^{n-1}} \right] \]
\[ \left( \frac{t_{\frac{1}{2}}}{t_{\frac{1}{2}}} \right)_2 = \left[ \frac{[P_0]^{n-1}}{[P_0]^{n-1}} \right] \]

340 \( \left( \frac{27.8}{55.5} \right)^{n-1} \)

\[ 2 = \left( \frac{1}{2} \right)^{n-1} \]
\[ 2 = (2)^{1-n} \]
1 − n = 1
n = 0
5. A solution of Fe₂(SO₄)₃ is electrolyzed for 'x' min with a current of 1.5 A to deposit 0.3482 g of Fe. The value of x is _______. [nearest integer]

Given : 1 F = 96500 C mol⁻¹
Atomic mass of Fe = 56 g mol⁻¹

Answer (20)

Sol. Fe³⁺ + 3e⁻ → Fe

Moles of Fe deposited = \(\frac{0.3482}{56} = 6.2 \times 10^{-3}\)

For 1 mole Fe, charge required is 3 F
For 6.2 \times 10^{-3} mole Fe, charge required is 3 \times 6.2 \times 10^{-3} F

Since, charge required = 18.6 \times 10^{-3} \times 96500 C
= 1794.9 C

And,
1.5 \times t = 1794.9
\(t = \frac{1794.9}{1.5 \times 60} = 20\) min

6. Consider the following reactions:

PCl₃ + H₂O → A + HCl

A + H₂O → B + HCl

The number of ionisable protons present in the product B is

Answer (2)

Sol. PCl₃ + H₂O → P(OH)Cl₂ + HCl

A

P(OH)Cl₂ + H₂O → P(OH)₂Cl + HCl

B

Hydrogen attached with oxygen are ionisable. Hence number of ionisable protons present in compound B are 2.

7. Amongst FeCl₃, 3H₂O, K₂[Fe(CN)₆] and [Co(NH₃)₆]Cl₃, the spin-only magnetic moment value of the inner-orbital complex that absorbs light at shortest wavelength is_______B.M. [nearest integer]

Answer (2)

Sol. [Fe(H₂O)₃]Cl₃ → Outer-orbital complex
K₂[Fe(CN)₆] → Inner-orbital complex

[Co(NH₃)₆]Cl₃ → Inner-orbital complex

Since CN⁻ is a strong field ligand than NH₃. Hence K₂[Fe(CN)₆] is the inner-orbital complex that absorbs light at shortest wavelength.

Fe(III) → valence shell configuration 3d⁵
Since CN⁻ will do pairing, so unpaired electron = 1
\[\mu = \sqrt{l(l+2)} = \sqrt{3} \text{ BM} = 2 \text{ BM}\]

8. The Novac polymer has mass of 963 g. The number of monomer units present in it are

Answer (9)

Sol. Novac is

\[\text{Molar mass of monomer is 107 g/mol}\]
\[n = \frac{963}{107} = 9\]

Number of monomer units present in it are 9.

9. How many of the given compounds will give a positive Biuret test?_______? Glycine, Glycylalanine, Tripeptide, Biuret.

Answer (2)

Sol. Since dipeptides and free amino acids do not give biuret test. Hence glycine and glycylalanine do not give this test.

10. The neutralization occurs when 10 mL of 0.1M acid ‘A’ is allowed to react with 30 mL of 0.05 M base M(OH)₂. The basicity of the acid ‘A’ is_______.

[M is a metal]

Answer (3)

Sol. Milieq of acid A = Milieq of base M(OH)₂
\[(M \times V \times n - \text{Factor})_A = (M \times V \times n - \text{Factor})_{M(OH)₂}\]

[n-Factor of M(OH)₂ = 2]
0.1 \times 10 \times n-\text{Factor} = 0.05 \times 30 \times 2
\(n-\text{Factor})_A = 3\)

Hence basicity of acid A is 3.
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Let \( A = \{x \in R : |x + 1| < 2\} \) and \( B = \{x \in R : |x - 1| \geq 2\} \). Then which one of the following statements is NOT true?
   (A) \( A - B = (-1, 1) \)
   (B) \( B - A = R - (-3, 1) \)
   (C) \( A \cap B = (-3, -1) \)
   (D) \( A \cup B = R - [1, 3) \)

   \[ \text{Answer (B)} \]

2. Let \( a, b \in R \) be such that the equation \( ax^2 - 2bx + 15 = 0 \) has a repeated root \( \alpha \). If \( \alpha \) and \( \beta \) are the roots of the equation \( x^2 - 2bx + 21 = 0 \), then \( \alpha^2 + \beta^2 \) is equal to
   (A) 37
   (B) 58
   (C) 68
   (D) 92

   \[ \text{Answer (B)} \]

Sol. \( ax^2 - 2bx + 15 = 0 \) has repeated root so \( b^2 = 15a \)

and \( \alpha = \frac{15}{b} \)

\[ \therefore \alpha \text{ is a root of } x^2 - 2bx + 21 = 0 \]

So \( \frac{225}{b^2} = 9 \Rightarrow b^2 = 25 \)

Now \( \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4b^2 - 42 = 100 - 42 = 58 \)

3. Let \( z_1 \) and \( z_2 \) be two complex numbers such that

\[ \overline{z_1} = iz_2 \] and \( \arg \left( \frac{z_1}{z_2} \right) = \pi \). Then

(A) \( \arg z_2 = \left( \frac{\pi}{4} \right) \)
   (B) \( \arg z_2 = \left( -\frac{3\pi}{4} \right) \)
   (C) \( \arg z_1 = \left( \frac{\pi}{4} \right) \)
   (D) \( \arg z_1 = \left( -\frac{3\pi}{4} \right) \)

   \[ \text{Answer (C)} \]

Sol. \( \frac{z_1}{z_2} = -i \Rightarrow z_1 = -iz_2 \)

\[ \Rightarrow \arg(z_1) = -\frac{\pi}{2} + \arg(z_2) \] \...(i)

Also \( \arg(z_1) = \arg(\overline{z_2}) = \pi \)

\[ \Rightarrow \arg(z_1) + \arg(z_2) = \pi \] \...(ii)

From (i) and (ii), we get

\[ \arg(z_1) = \frac{\pi}{4} \text{ and } \arg(z_2) = \frac{3\pi}{4} \]

4. The system of equations

\(-kx + 3y - 14z = 25\)

\(-15x + 4y - kz = 3\)

\(-4x + y + 3z = 4\)

is consistent for all \( k \) in the set

(A) \( R \)
   (B) \( R - (-11, 13) \)
   (C) \( R - (13) \)
   (D) \( R - (-11, 11) \)

   \[ \text{Answer (D)} \]

Sol. The system may be inconsistent if

\[
\begin{pmatrix}
-k & 3 & -14 \\
-15 & 4 & -k \\
-4 & 1 & 3
\end{pmatrix} = 0 \Rightarrow k = \pm 11
\]

Hence if system is consistent then \( k \in R - \{11, -11\} \)

5. \( \lim_{x \to \frac{\pi}{2}} \tan^2 x \left( \frac{1}{(2\sin^2 x + 3\sin x + 4)^\frac{1}{2}} - \left( \sin^2 x + 6\sin x + 2 \right)^\frac{1}{2} \right) \) is equal to

(A) \( \frac{1}{12} \)
   (B) \( -\frac{1}{18} \)
   (C) \( -\frac{1}{12} \)
   (D) \( \frac{1}{6} \)

   \[ \text{Answer (A)} \]

\[ \lim_{x \to \frac{\pi}{2}} \tan^2 x \left( \frac{1}{(2\sin^2 x + 3\sin x + 4)^\frac{1}{2}} - \left( \sin^2 x + 6\sin x + 2 \right)^\frac{1}{2} \right) \]
\[ \lim_{x \to \frac{\pi}{2}} \tan^2 x \left( \sqrt{2 \sin^2 x + 3 \sin x + 4 - \sqrt{\sin^2 x + 6 \sin x + 2}} \right) \]

\[ \lim_{x \to \frac{\pi}{2}} \frac{\tan^2 x (\sin^2 x - 3 \sin x + 2)}{\sqrt{2 \sin^2 x + 3 \sin x + 4 + \sqrt{\sin^2 x + 6 \sin x + 2}}} \]

\[ \frac{1}{6} \lim_{x \to \frac{\pi}{2}} \frac{(1 - \sin x)(2 - \sin x)}{\cos^2 x} \cdot \sin^2 x \]

\[ \frac{1}{6} \lim_{x \to \frac{\pi}{2}} \frac{(2 - \sin x) \sin^2 x}{1 + \sin x} \]

\[ = \frac{1}{12} \]

6. The area of the region enclosed between the parabolas \( y^2 = 2x - 1 \) and \( y^2 = 4x - 3 \) is

(A) \( \frac{1}{3} \)

(B) \( \frac{1}{6} \)

(C) \( \frac{2}{3} \)

(D) \( \frac{3}{4} \)

Answer (A)

Sol. Area of the shaded region

\[ \frac{1}{12} \]

7. The coefficient of \( x^{101} \) in the expression \( (5 + x)^{500} + x(5 + x)^{499} + x^2(5 + x)^{498} + \ldots + x^{500}, x > 0 \), is

(A) \( 501 C_{101} (5)^{399} \)

(B) \( 501 C_{101} (5)^{400} \)

(C) \( 501 C_{100} (5)^{400} \)

(D) \( 500 C_{101} (5)^{399} \)

Answer (A)

Sol. Coefficient of \( x^{101} \) in

\[ \frac{x^{500} (5 + x)^{501}}{x^{101}} - 1 \]

\[ = \text{Coefficient of } x^{101} \text{ in } \frac{1}{5} \left( (5 + x)^{501} - x^{501} \right) \]

\[ = \frac{1}{5} \cdot 5^{100} \]

\[ = 5^{100} \cdot 1 \cdot 5^{99} \]

8. The sum \( 1 + 2 \cdot 3 + 3 \cdot 3^2 + \ldots + 10 \cdot 3^9 \) is equal to

(A) \( \frac{2 \cdot 3^{12} + 10}{4} \)

(B) \( \frac{19 \cdot 3^{10} + 1}{4} \)

(C) \( 5 \cdot 3^{10} - 2 \)

(D) \( \frac{9 \cdot 3^{10} + 1}{2} \)

Answer (B)

Sol. Let \( S = 1.3^3 + 2.3^1 + 3.3^2 + \ldots + 10.3^9 \)

\[ 3S = 1.3^1 + 2.3^2 + \ldots + 10.3^{10} \]

\[ -2S = (1.3^3 + 1.3^1 + 1.3^2 + \ldots + 1.3^9) - 10.3^{10} \]

\[ \Rightarrow \frac{S}{2} = \frac{1}{2} \left[ 10^{10} - 3^1 - \frac{3^10 - 1}{3 - 1} \right] \]

\[ \Rightarrow S = \frac{19 \cdot 3^{10} + 1}{4} \]

9. Let \( P \) be the plane passing through the intersection of the planes

\[ \hat{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 5 \text{ and } \hat{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 3 \text{, and the point } (2, 1, -2) \text{. Let the position vectors of the points } X \text{ and } Y \text{ be } \hat{i} - 2\hat{j} + 4\hat{k} \text{ and } 5\hat{i} - \hat{j} + 2\hat{k} \text{ respectively. Then the points} \]

(A) \( X \) and \( Y \) are on the same side of \( P \)

(B) \( Y \) and \( X - \hat{r} \) are on the opposite sides of \( P \)

(C) \( X \) and \( Y \) are on the opposite sides of \( P \)

(D) \( X + \hat{r} \) and \( X - \hat{r} \) are on the same side of \( P \)

Answer (C)

Sol. Let the equation of required plane

\[ \pi : (x + 3y - z - 5) + \lambda (2x - y + z - 3) = 0 \]

\[ \Rightarrow (2, 1, -2) \text{ lies on it so, } 2 + \lambda (-2) = 0 \]

\[ \Rightarrow \lambda = 1 \]

Hence, \( \pi : 3x + 2y - 8 = 0 \)

\[ \Rightarrow \pi_x = -9, \pi_y = 5, \pi_{x+y} = 4 \]

\[ \pi_{x-y} = -22 \text{ and } \pi_{y-x} = 6 \]

Clearly \( X \) and \( Y \) are on opposite sides of plane \( \pi \)
10. A circle touches both the y-axis and the line $x + y = 0$. Then the locus of its center is

(A) $y = \sqrt{2}x$  
(B) $x = \sqrt{2}y$  
(C) $y^2 - x^2 = 2xy$  
(D) $x^2 - y^2 = 2xy$

Answer (D)

Sol. Let the centre be $(h, k)$

So, $|h| = \frac{|h+k|}{\sqrt{2}}$

$\Rightarrow 2h^2 = h^2 + k^2 + 2hk$

Locus will be $x^2 - y^2 = 2xy$

11. Water is being filled at the rate of $1 \text{ cm}^3/\text{sec}$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm$^2$/sec) at which the wet conical surface area of the vessel increases, is

(A) 5  
(B) $\frac{\sqrt{21}}{5}$  
(C) $\frac{\sqrt{26}}{5}$  
(D) $\frac{\sqrt{26}}{10}$

Answer (C)

Sol. $V = \frac{1}{3}\pi r^2 h$ and $h = \frac{35}{2} = \frac{7}{3}$

$\Rightarrow V = \frac{1}{75}\pi h^3$

$dV = \frac{1}{25}\pi h^2 \frac{dh}{dt} = 1$

$\Rightarrow \frac{dh}{dt} = \frac{25}{\pi h^2}$

Now, $S = \pi rl = \pi \left(\frac{h}{2}\right)\left(\sqrt{h^2 + \frac{h^2}{4}}\right) = \frac{\pi}{25}\sqrt{26}h^2$

$\Rightarrow \frac{dS}{dt} = \frac{2\sqrt{26}\pi h}{25} \cdot \frac{dh}{dt} = \frac{2\sqrt{26}}{h}$

$\Rightarrow \frac{dS}{dt} = \frac{\sqrt{26}}{5}$

12. If $b_n = \int_0^{\frac{x}{2}} \frac{\cos^2 nx - \cos^2 (n-1)x}{\sin x} dx$, $n \in \mathbb{N}$, then

(A) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference $-2$

(B) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference $2$

(C) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

(D) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference $-2$

Answer (D)

Sol. $b_n - b_{n-1} = \int_0^{\frac{x}{2}} \frac{\cos^2 nx - \cos^2 (n-1)x}{\sin x} dx = \int_0^{\frac{x}{2}} \left[-\sin(2n-1)x \cdot \sin x\right] dx$

$= \cos(2n-1)x \left|_0^{\frac{x}{2}}\right. = -\frac{1}{2n-1}$

$\Rightarrow b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in A.P. with common difference $-2$

13. If $y = y(x)$ is the solution of the differential equation $2x^2 \frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(\frac{e}{3}) = \frac{e}{3}$, then $y(1)$ is equal to

(A) $\frac{1}{3}$  
(B) $\frac{2}{3}$  
(C) $\frac{3}{2}$  
(D) 3

Answer (B)

Sol. $2x^2 \frac{dy}{dx} - 2xy + 3y^2 = 0$

$\Rightarrow 2x(xdy - ydx) + 3y^2 dx = 0$

$\Rightarrow 2 \left(xdy - ydx\right) + 3 \frac{dy}{y} = 0$

$\Rightarrow \frac{2x}{y} + 3 \ln x = C$

$\Rightarrow y(e) = \frac{e}{3} \Rightarrow -6 + 3 = C \Rightarrow C = -3$

Now, at $x = 1, \frac{2}{y} + 0 = -3$

$\Rightarrow y = \frac{2}{3}$
14. If the angle made by the tangent at the point \((x_0, y_0)\) on the curve \(x = 12(t + \sin t \cos t)\),
\[ y = 12(1 + \sin t)^2, \quad 0 < t < \frac{\pi}{2} \] with the positive x-axis is \(\frac{\pi}{3}\),
then \(y_0\) is equal to:
(A) \(6(3 + 2\sqrt{2})\) \quad (B) \(3(7 + 4\sqrt{3})\)
(C) \(27\) \quad (D) \(48\)
Answer (C)

Sol. \[ \frac{dy}{dx} = \frac{24(1 + \sin t) \cos t}{12(1 + \cos 2t)} = \frac{1 + \sin t}{\cos t} = \tan\left(\frac{\pi}{4} + \frac{t}{2}\right) \]
\[ \Rightarrow \quad t = \frac{\pi}{6} \]
So, \(y_0\) (at \(t = \frac{\pi}{6}\)) = \(12(1 + \sin \frac{\pi}{6})^2 = 27\)

15. The value of \(2\sin(12^\circ) - \sin(72^\circ)\) is:
(A) \(\frac{\sqrt{5}(1 - \sqrt{5})}{4}\)
(B) \(\frac{1 - \sqrt{5}}{8}\)
(C) \(\frac{\sqrt{5}(1 - \sqrt{5})}{2}\)
(D) \(\frac{\sqrt{5}(1 - \sqrt{5})}{4}\)
Answer (D)

Sol. 
\[ 2\sin 12^\circ - \sin 72^\circ = \sin 12^\circ + (-\cos 42^\circ \cdot \sin 30^\circ) \]
\[ = \sin 12^\circ - \cos 42^\circ \]
\[ = \sin 12^\circ - \cos 48^\circ \]
\[ = 2\sin 18^\circ \cdot \cos 30^\circ \]
\[ = -2 \left(\frac{\sqrt{5} - 1}{4}\right) \cdot \frac{\sqrt{3}}{2} \]
\[ = \frac{\sqrt{15} - \sqrt{5}}{4} \]

16. A biased die is marked with numbers 2, 4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark \(\frac{1}{n}\). If the die is thrown thrice, then the probability that the sum of the numbers obtained is 48, is:
(A) \(\frac{7}{211}\) \quad (B) \(\frac{7}{212}\)
(C) \(\frac{3}{210}\) \quad (D) \(\frac{13}{212}\)
Answer (D)

Sol. There are only two ways to get sum 48, which are (32, 8, 8) and (16, 16, 16)
So, required probability
\[ = \frac{3}{210} + \frac{1}{212} = \frac{13}{212}\]

17. The negation of the Boolean expression \((\neg q \land p) \iff ((\neg p) \lor q)\) is logically equivalent to:
(A) \(p \iff q\) \quad (B) \(q \iff p\)
(C) \(\neg (p \iff q)\) \quad (D) \(\neg (q \iff p)\)
Answer (C)

Sol. Let \(S : (\neg q \land p) \iff ((\neg p) \lor q)\)
\[ \Rightarrow \quad S : \neg ((\neg q) \land p) \lor ((\neg p) \lor q) \]
\[ \Rightarrow \quad S : (q \lor (\neg p)) \lor ((\neg p) \lor q) \]
\[ \Rightarrow \quad S : (\neg p) \lor q \]
\[ \Rightarrow \quad S : p \iff q \]
So, negation of \(S\) will be \(\neg (p \iff q)\)

18. If the line \(y = 4 + kx, k > 0\), is the tangent to the parabola \(y = x - x^2\) at the point \(P\) and \(V\) is the vertex of the parabola, then the slope of the line through \(P\) and \(V\) is:
(A) \(\frac{3}{2}\) \quad (B) \(\frac{26}{9}\)
(C) \(\frac{5}{2}\) \quad (D) \(\frac{23}{6}\)
Answer (C)

Sol. \[ \text{Line } y = kx + 4 \text{ touches the parabola } y = x - x^2. \]
So, \(kx + 4 = x - x^2 \Rightarrow x^2 + (k - 1)x + 4 = 0\) has only one root
\[(k - 1)^2 = 16 \Rightarrow k = 5 \text{ or } -3 \text{ but } k > 0\]
So, \(k = 5\).
And hence \(x^2 + 4x + 4 = 0 \Rightarrow x = -2\)
19. The value of \( \tan^{-1}\left(\frac{\cos \left(\frac{15\pi}{4}\right) - 1}{\sin \left(\frac{\pi}{4}\right)}\right) \) is equal to :

(A) \( \frac{-\pi}{4} \)  
(B) \( \frac{-\pi}{8} \)  
(C) \( \frac{-5\pi}{12} \)  
(D) \( \frac{-4\pi}{9} \)

Answer (B)

\[
\text{Sol. } \tan^{-1}\left(\frac{\cos \left(\frac{15\pi}{4}\right) - 1}{\sin \left(\frac{\pi}{4}\right)}\right) = \tan^{-1}\left(\frac{1 - \frac{\sqrt{2}}{2}}{\frac{1}{\sqrt{2}}}\right) = \tan^{-1}\left(1 - \sqrt{2}\right) = -\tan^{-1}\left(\sqrt{2} - 1\right) = -\frac{\pi}{8}
\]

20. The line \( y = x + 1 \) meets the ellipse \( \frac{x^2}{4} + \frac{y^2}{2} = 1 \) at two points \( P \) and \( Q \). If \( r \) is the radius of the circle with \( PQ \) as diameter then \( (3r)^2 \) is equal to :

(A) \( 20 \)  
(B) \( 12 \)  
(C) \( 11 \)  
(D) \( 8 \)

Answer (A)

\[
\text{Sol. Let point } (a, a + 1) \text{ as the point of intersection of line and ellipse.}
\]
\[
\text{So, } \frac{a^2}{4} + \frac{(a+1)^2}{2} = 1 \Rightarrow a^2 + 2(a^2 + 2a + 1) = 4 \\
\Rightarrow 3a^2 + 4a - 2 = 0 \\
\text{If roots of this equation are } \alpha \text{ and } \beta .
\]
\[
\text{So, } P(\alpha, \alpha + 1) \text{ and } Q(\beta, \beta + 1) \\
\text{Then, } \frac{5\alpha^2}{4} = (\alpha - \beta)^2 + (\alpha - \beta)^2 \\
\Rightarrow 9\beta^2 = \frac{9}{4}(2(\alpha - \beta)^2)
\]

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let \( A = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} \) and \( B = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix} \). Then the number of elements in the set \{ \( (n, m) : n, m \in \{ 1, 2, \ldots, 10 \} \text{ and } nA^n + mB^n = 0 \} \text{ is } \___.

Answer (1)

\[
\text{Sol. } A^2 = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}^2 = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} \\
B^2 = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}
\]

So, \( B^2 = B, K \in I \)
\[nA^n + mB^n = nA + mB \]
\[= \begin{pmatrix} 2n - 2n \\ n - n \end{pmatrix} + \begin{pmatrix} -m & 2m \\ -m & 2m \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]
So, \( 2n - m = 1, -n + m = 0, 2m - n = 1 \)
So, \( (m, n) = (1, 1) \)

2. Let \( f(x) = [2x^2 + 1] \) and \( g(x) = \begin{cases} 2x - 3, & x < 0 \\ 2x + 3, & x \geq 0 \end{cases} \)

where \([x]\) is the greatest integer \( \leq x \). Then, in the open interval \((-1, 1)\), the number of points where \( f \circ g \) is discontinuous is equal to \______.

Answer (62)
Sol. \( f(g(x)) = \begin{cases} 
\frac{[2(2x-3)^2]+1}{[2(2x+3)^2]+1}, & x < 0 \\
\frac{[2(2x+3)^2]+1}{[2(2x-3)^2]+1}, & x \geq 0
\end{cases} \)

The possible points where \( f(g(x)) \) may be discontinuous are
2 \((2x-3)^2 = l\) & \( x \in (-1, 0) \)
2 \((2x+3)^2 = l\) & \( x \in [0, 1) \)
\(2x-3 \in (-5, -3)\) & \(2x+3 \in [3, 5)\)
\(2(2x-3)^2 \in (18, 50)\) & \(2(2x+3)^2 \in (18, 50)\)
So, no. of points = 31 It is discontinuous at all points except \( x = 0 \) of no. points = 31
So, total = 62

3. The value of \( b > 3 \) for which \( \int_0^b \frac{1}{3(x^2-1)(x^2-4)} \, dx = \log_e \left( \frac{49}{40} \right) \), is equal to

Answer (6)

Sol. \( I = \int \frac{1}{3(x^2-1)(x^2-4)} \, dx = \frac{1}{3} \left( \frac{1}{x^2-4} - \frac{1}{x^2-1} \right) dx \)

\( = \frac{1}{3} \left( \frac{1}{4} \ln \frac{x-2}{x+2} - \frac{1}{2} \ln \frac{x-1}{x+1} \right) + C \)

\( 12I = \ln \left( \frac{b-2}{b+2} \right) - 2 \ln \left( \frac{b-1}{b+1} \right) - \left( \ln \frac{1}{5} - 2 \ln \frac{1}{2} \right) \)

\( = \ln \left( \frac{b-2}{b+2} \right) - 2 \ln \left( \frac{b-1}{b+1} \right) - \left( \ln \frac{1}{5} - 2 \ln \frac{1}{2} \right) \)

So, \( \frac{49}{40} = \frac{(b-2)(b+2)}{(b+1)^2} \cdot \frac{5}{4} \)

\( \Rightarrow b = 6 \)

4. If the sum of the co-efficients of all the positive even powers of \( x \) in the binomial expansion of \( (2x^3 + \frac{3}{x})^{10} \) is \( 5^{10} - \beta \cdot 3^9 \), the \( \beta \) is equal to ______.

Answer (83)

Sol. \( T_{r+1} = 10C_r (2x^3)^{10-r} \left( \frac{3}{x} \right)^r \)

\( = 10C_r \cdot 2^{10-r} \cdot 3^r \cdot x^{30-4r} \)

So, \( r \neq 8, 9, 10 \)

Sum of required Coeff. = \( \left( \frac{2 \cdot 1^3 + 3}{1^{10}} \right) \)

\( = 10C_8 2^{2 \cdot 3^8} + 10C_9 2^3 3^9 + 10C_{10} 2^0 3^{10} \)

\( = 5^{10} - 3^9 \left( \frac{10C_8 2^2}{3} + 10C_9 \cdot 2^1 + 10C_{10} \cdot 3 \right) \)

\( \beta = \frac{4}{3} \cdot 10C_8 + 20 + 3 = 83 \)

5. If the mean deviation about the mean of the numbers 1, 2, 3, …, \( n \), where \( n \) is odd, is \( \frac{5(n+1)}{n} \), then \( n \) is equal to __________.

Answer (21)

Sol. Mean = \( \frac{\frac{n(n+1)}{2}}{n} = 2 \cdot n \cdot \frac{n+1}{2} \)

\( \Rightarrow \frac{(n-1) + (n-3) + (n-5) + \ldots + 0}{5(n+1)} = 5(n+1) \)

\( \Rightarrow \left( \frac{n+1}{4} \right) \cdot (n-1) = 5(n+1) \)

So, \( n = 21 \)

6. Let \( \vec{b} = \hat{i} + \hat{j} + \lambda \hat{k}, \lambda \in \mathbb{R} \). If \( \vec{a} \) is a vector such that \( \vec{a} \times \vec{b} = 13\hat{i} - \hat{j} - 4\hat{k} \) and \( \vec{a} \cdot \vec{b} + 21 = 0 \), then \( (\vec{b} - \vec{a})(\hat{k} - \hat{j}) + (\vec{b} + \vec{a})(\hat{i} - \hat{k}) \) is equal to

Answer (14)

Sol. Let \( \vec{a} = \hat{x} + y\hat{y} + z\hat{k} \)

\( \Rightarrow \lambda y - z = 13, z - \lambda x = -1, x - y = -4 \)

and \( x + y + \lambda z = -21 \)

\( \Rightarrow \) clearly, \( \lambda = 3, x = -2, y = 2 \) and \( z = -7 \)
So, \( b - a = 3\hat{i} - \hat{j} + 10\hat{k} \)
and \( b + a = -\hat{i} + 3\hat{j} - 4\hat{k} \)
\[ (b - a) \cdot (\hat{k} - \hat{j}) + (b + a) \cdot (\hat{i} - \hat{k}) = 11 + 3 = 14 \]

7. The total number of three-digit numbers, with one digit repeated exactly two times, is ______.

Answer (243)

Sol. C-1: All digits are non-zero
\[ 9C_2 \cdot \frac{3!}{2} = 216 \]
C-2: One digit is 0
0, 0, x \( \Rightarrow 9C_1 \cdot 1 = 9 \)
0, x, x \( \Rightarrow 9C_1 \cdot 2 = 18 \)
Total = 216 + 27 = 243

8. Let \( f(x) = (x - 1)(x^3 - 2x - 3) + x - 3, x \in R \). If \( m \) and \( M \) are respectively the number of points of local minimum and local maximum of \( f \) in the interval \((0, 4)\), then \( m + M \) is equal to ______.

Answer (3)

Sol. \( f(x) = |(x - 1)(x^2 + x - 3)| + (x - 3) \)
\[ f(x) = \begin{cases} 
(x - 3)(x^2) & 3 \leq x \leq 4 \\
(x - 3)(2 - x^2) & 1 \leq x < 3 \\
(x - 3)(x^2) & 0 < x < 1 \\
3x^2 - 6x & 3 < x < 4 \\
-3x^2 + 6x + 2 & 1 < x < 3 \\
3x^2 - 6x & 0 < x < 1 
\end{cases} \]
\[ f'(x) = \begin{cases} 
3x^2 - 6x & 3 < x < 4 \\
-3x^2 + 6x + 2 & 1 < x < 3 \\
3x^2 - 6x & 0 < x < 1 
\end{cases} \]
\[ f'(3^+) > 0 \Rightarrow (3^-) < 0 \rightarrow \text{Minimum} \]
\[ f'(1^+) > 0 \Rightarrow (1^-) < 0 \rightarrow \text{Minimum} \]
\( x \in (1, 3) \) \( f'(x) = 0 \) at one point \( \rightarrow \text{Maximum} \)
\( x \in (3, 4) \) \( f'(x) \neq 0 \)
\( x \in (0, 1) \) \( f'(x) \neq 0 \)
So, 3 points

9. Let the eccentricity of the hyperbola \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) be \( \frac{5}{4} \). If the equation of the normal at the point \( \left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right) \) on the hyperbola is \( 8\sqrt{5}x + \beta y = \lambda \), then \( \lambda - \beta \) is equal to ______.

Answer (85)

Sol. \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \left( e = \frac{5}{4} \right) \)
\[ b^2 = a^2 \left( \frac{25}{16} - 1 \right) \Rightarrow b = \frac{3a}{4} \]
Also \( \left( \frac{8}{\sqrt{5}}, \frac{12}{5} \right) \) lies on the given hyperbola
\[ \frac{64}{5a^2} - \frac{144}{25(9a^2/16)} = 1 \Rightarrow a = \frac{8}{5} \text{ and } b = \frac{6}{5} \]
Equation of normal
\[ \frac{64}{25} \left( \frac{x}{\sqrt{5}} \right) + \frac{36}{25} \left( \frac{y}{12/5} \right) = 4 \]
\[ \Rightarrow \frac{8}{5\sqrt{5}} x + \frac{3}{5} y = 4 \]
\[ \Rightarrow 8\sqrt{5}x + 15y = 100 \]
So, \( \beta = 15 \) and \( \lambda = 100 \)
Gives \( \lambda - \beta = 85 \)

10. Let \( h \) be the line in \( xy \)-plane with \( x \) and \( y \) intercepts \( \frac{1}{8} \) and \( \frac{1}{4\sqrt{2}} \) respectively and \( l \) be the line in \( zx \)-plane with \( x \) and \( z \) intercepts \( -\frac{1}{8} \) and \( -\frac{1}{6\sqrt{3}} \) respectively. If \( d \) is the shortest distance between the line \( h \) and \( l \), then \( d^2 \) is equal to ______.

Answer (51)

Sol. \( \frac{x - 1}{8} = \frac{y}{4\sqrt{2}} = \frac{z}{0} \) \( \ldots (i) \)
Equation of \( L_2 \)
\[ \frac{x + 1}{8} = \frac{y}{0} = \frac{z}{8} \ldots (ii) \]
\( d = \frac{|(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})|}{|\vec{b} \times \vec{d}|} \)
\[ = \frac{1}{4} \cdot \frac{\left(4\sqrt{2}i + 4j + 3\sqrt{6}k\right)}{\sqrt{(4\sqrt{2})^2 + 4^2 + (3\sqrt{6})^2}} \]
\[ = \frac{\sqrt{2}}{\sqrt{32 + 16 + 54}} = \frac{1}{\sqrt{51}} \]
\( d^2 = 51 \)