Answers & Solutions

for

JEE (MAIN)-2021 (Online) Phase-4

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each part has two sections.

   (i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and -1 mark for wrong answer.

   (ii) Section-II : This section contains 10 questions. In Section-II, attempt any five questions out of 10. There will be no negative marking for Section-II. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and there is no negative marking for wrong answer.
PART – A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. If \( R_E \) be the radius of Earth, then the ratio between the acceleration due to gravity at a depth ‘\( r \)’ below and a height ‘\( r \)’ above the earth surface is:
   
   \[
   (1) \quad \frac{1 - \frac{r}{R_E}}{\frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}} \\
   (2) \quad \frac{1 + \frac{r}{R_E}}{\frac{r^2}{R_E^2} + \frac{r^3}{R_E^3}}
   \]
   
   \[
   (3) \quad \frac{1 + \frac{r}{R_E}}{\frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}} \\
   (4) \quad \frac{1 + \frac{r}{R_E}}{\frac{r^2}{R_E^2} + \frac{r^3}{R_E^3}}
   \]

   Answer (3)

   Sol. at a depth \( r \)
   
   \[
   Gm \times (R_E - r) = \frac{GM}{R_E^2} \left( 1 - \frac{r}{R_E} \right)
   \]

   at a height \( r \)
   
   \[
   Gm \left( \frac{R_E + r}{R_E} \right)^2 = \frac{GM}{R_E^2} \left( 1 + \frac{r}{R_E} \right)^2
   \]

   \[
   \therefore \quad \frac{a_1}{a_2} = \left( 1 - \frac{r}{R_E} \right) \left( 1 + \frac{r}{R_E} \right)^2
   \]

   \[
   = 1 + \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}
   \]

   2. Two thin metallic spherical shells of radii \( r_1 \) and \( r_2 \) \((r_1 < r_2)\) are placed with their centres coinciding. A material of thermal conductivity \( K \) is filled in the space between the shells. The inner shell is maintained at temperature \( \theta_1 \) and the outer shell at temperature \( \theta_2 (\theta_1 < \theta_2) \). The rate at which heat flows radially through the material is:
   
   \[
   (1) \quad \frac{K(\theta_2 - \theta_1)(r_2 - r_1)}{4\pi r_2 f_2} \\
   (2) \quad \frac{4\pi Kr_2 f_2 (\theta_2 - \theta_1)}{r_2 - r_1}
   \]

   \[
   (3) \quad \frac{\pi r_2 f_2 (\theta_2 - \theta_1)}{r_2 - r_1} \\
   (4) \quad \frac{K(\theta_2 - \theta_1)}{r_2 - r_1}
   \]

   Answer (2)

   Sol. \[
   \frac{dQ}{dt} = \frac{\theta_2 - \theta_1}{R_{th}}
   \]

   Now, \( R_{th} = \int_{r_1}^{r_2} \frac{dx}{K \times 4\pi x^2} \)

   \[
   = \frac{1}{4\pi K} \left( \frac{r_2 - r_1}{r_2 f_2} \right)
   \]

   ∴ \[
   \frac{dQ}{dt} = \frac{4\pi K r f_2 (\theta_2 - \theta_1)}{(r_2 - r_1)}
   \]

   3. A bob of mass ‘\( m \)’ suspended by a thread of length \( l \) undergoing simple harmonic oscillations with time period \( T \). If the bob is immersed in a liquid that has density \( \frac{1}{4} \) times that of the bob and the length of the thread is increased by \( 1/3^{rd} \) of the original length, then the time period of the simple harmonic oscillations will be:
   
   \[
   (1) \quad \frac{3}{4}T \\
   (2) \quad T \\
   (3) \quad \frac{4}{3}T \\
   (4) \quad \frac{3}{2}T
   \]

   Answer (3)

   Sol. \[
   T = 2\pi \sqrt{\frac{l}{g}}
   \]

   \[
   T' = 2\pi \sqrt{\frac{4l}{9g - \frac{4}{3}g}} = 2\pi \sqrt{\frac{4l}{3g}} = \frac{4}{3}T
   \]

   4. A mixture of hydrogen and oxygen has volume 500 cm\(^3\), temperature 300 K, pressure 400 kPa and mass 0.76 g. The ratio of masses of oxygen to hydrogen will be:
   
   \[
   (1) \quad 3 : 16 \\
   (2) \quad 16 : 3 \\
   (3) \quad 3 : 8 \\
   (4) \quad 8 : 3
   \]

   Answer (2)
Sol. \( PV = nRT \)

\[
\Rightarrow n = \frac{400 \times 10^3 \times 500 \times 10^{-6}}{R \times 300}
\]

\[
n = \frac{8}{100} = \frac{m_{H_2}}{2} + \frac{m_{O_2}}{32} \quad \text{(i)}
\]

and \( m_{H_2} + m_{O_2} = 0.76 \) \quad \text{(ii)}

\[
\therefore m_{H_2} = 0.12 \text{ g and } m_{O_2} = 0.64 \text{ g}
\]

So ratio \( = \frac{16}{3} \)

5. If velocity \([V]\), time \([T]\) and force \([F]\) are chosen as the base quantities, the dimensions of the mass will be:

(1) \([FT^{-1}V^{-1}]\)  
(2) \([FVT^{-1}]\)  
(3) \([FT^2V]\)  
(4) \([FTV^{-1}]\)  

Answer (4)

Sol. Mass, \( M = [V]^a [T]^b [F]^c \)

\[
\Rightarrow M = [LT^{-1}]^a [T]^b [MLT^{-2}]^c
\]

\[
a + c = 0 \Rightarrow a = -c = -1
\]

\[
\Rightarrow -a + b - 2c = 0
\]

\[
\Rightarrow b = a + 2c = -1 + 2 \times 1 = 1
\]

\( M = [FTV^{-1}] \)

6. For a body executing S.H.M.:

(a) Potential energy is always equal to its K.E.
(b) Average potential and kinetic energy over any given time interval are always equal.
(c) Sum of the kinetic and potential energy at any point of time is constant.
(d) Average K.E. in one time period is equal to average potential energy in one time period.

Choose the most appropriate option from the options given below:

(1) Only (b)  
(2) (b) and (c)  
(3) Only (c)  
(4) (c) and (d)  

Answer (4)

Sol. \( v = \omega \sqrt{A^2 - x^2} \)

\[
KE = \frac{1}{2} m \omega^2 (A^2 - x^2)
\]

\[
PE = \frac{1}{2} m \omega^2 x^2
\]

\[
E_{\text{Total}} = KE + PE
\]

\[
= \frac{1}{2} m \omega^2 A^2
\]

Also \( (KE)_{\text{avg}} = (PE)_{\text{avg}} \)

7. A free electron of 2.6 eV energy collides with a H\(^+\) ion. This results in the formation of a hydrogen atom in the first excited state and a photon is released. Find the frequency of the emitted photon. \((h = 6.6 \times 10^{-34} \text{ J s})\)

(1) \(0.19 \times 10^{15} \text{ MHz}\)  
(2) \(1.45 \times 10^9 \text{ MHz}\)  
(3) \(9.0 \times 10^{27} \text{ MHz}\)  
(4) \(1.45 \times 10^{16} \text{ MHz}\)

Answer (2)

Sol. \( hf = 2.6 + 13.6 \times \frac{1}{4} \)

\( = 6 \text{ eV} \)

\[
\Rightarrow f = \frac{6 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}
\]

\( = 1.45 \times 10^9 \text{ MHz}\)

8. Consider two separate ideal gases of electrons and protons having same number of particles. The temperature of both the gases are same. The ratio of the uncertainty in determining the position of an electron to that of a proton is proportional to:

(1) \(\frac{m_p}{m_e}\)  
(2) \(\frac{m_e}{m_p}\)  
(3) \(\left(\frac{m_p}{m_e}\right)^{3/2}\)  
(4) \(\sqrt[3]{\frac{m_e}{m_p}}\)

Answer (1)

Sol. \( (\Delta x) \Delta p \geq \frac{h}{4\pi} \)

\[
\Delta x \propto \frac{1}{\Delta p}
\]

\[
\Rightarrow \frac{\Delta x_e}{\Delta x_p} = \frac{\Delta p_p}{\Delta p_e} = \sqrt{\frac{m_p}{m_e}}
\]
9. A block moving horizontally on a smooth surface with a speed of 40 m/s splits into two parts with masses in the ratio of 1 : 2. If the smaller part moves at 60 m/s in the same direction, then the fractional change in kinetic energy is:

\[
\begin{align*}
\text{(1)} & \quad \frac{1}{3} \\
\text{(2)} & \quad \frac{2}{3} \\
\text{(3)} & \quad \frac{1}{4} \\
\text{(4)} & \quad \frac{1}{8} 
\end{align*}
\]

Answer (4)

Sol. \( m_2 = \left( \frac{m}{3} \right) \times 60 + \left( \frac{2m}{3} \right) V \)

\[ \Rightarrow \frac{2V}{3} = 20 \Rightarrow V = 30 \text{ m/s} \]

\[ k_f = \frac{1}{2} \left( \frac{m}{3} \right) (60)^2 + \frac{1}{2} \left( \frac{2m}{3} \right) (30)^2 \]

\[ \Delta k = \frac{k_f - k_i}{k_i} = \frac{1}{8} \]

10. Statement I :

To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect a capacitor across the output parallel to the load \( R_L \).

Statement II :

To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect an inductor in series with \( R_L \).

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both statement I and statement II are true

(2) Statement I is false but statement II is true

(3) Both statement I and statement II are false

(4) Statement I is true but statement II is false

Answer (1)

Sol. To get steady dc output from the pulsating voltage received from a full wave rectifier we can use capacitor.

Both statements are true.

11. The magnetic field vector of an electromagnetic wave is given by \( B = B_0 \left( \hat{i} + \frac{j}{\sqrt{2}} \right) \cos(kz - \omega t) \); where \( \hat{i}, \hat{j} \) represents unit vector along \( x \) and \( y \)-axis respectively. At \( t = 0 \) s, two electric charges \( q_1 \) of \( 4\pi \text{ coulomb} \) and \( q_2 \) of \( 2\pi \text{ coulomb} \) located at \( (0, 0, \frac{\pi}{k}) \) and \( (0, 0, \frac{3\pi}{k}) \), respectively, have the same velocity of \( 0.5\ c \hat{i} \), (where \( c \) is the velocity of light). The ratio of the force acting on charge \( q_1 \) to \( q_2 \) is:

\[ \begin{align*}
\text{(1)} & \quad \sqrt{2} : 1 \\
\text{(2)} & \quad 2 : 1 \\
\text{(3)} & \quad 2\sqrt{2} : 1 \\
\text{(4)} & \quad 1 : \sqrt{2} 
\end{align*} \]

Answer (2)

Sol. At given positions value of \( E \) and \( B \) is same

\[ \text{Force on } 1 = 4\pi (\vec{E} + \vec{V} \times \vec{B}) \]

\[ \text{Force on } 2 = 2\pi (\vec{E} + \vec{V} \times \vec{B}) \]

So \[ \frac{\text{Force on } 1}{\text{Force on } 2} = 2 : 1 \]

12. The equivalent resistance of the given circuit between the terminals A and B is:

\[ \begin{align*}
\text{(1)} & \quad 3 \Omega \\
\text{(2)} & \quad 1 \Omega \\
\text{(3)} & \quad \frac{9}{2} \Omega \\
\text{(4)} & \quad 0 \Omega 
\end{align*} \]

Answer (2)

Sol. On simplification

\[ \begin{align*}
\frac{1}{R_{AD}} &= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \\
R_{AB} &= 1 \Omega 
\end{align*} \]
13. A coil is placed in a magnetic field $\vec{B}$ as shown below:

A current is induced in the coil because $\vec{B}$ is:
(1) parallel to the plane of coil and increasing with time
(2) outward and decreasing with time
(3) outward and increasing with time
(4) parallel to the plane of coil and decreasing with time

Answer (2)

Sol.

As current is induced
So, $\vec{B}$ is decreasing and outward

14. Statement I: If three forces $\vec{F}_1, \vec{F}_2$, and $\vec{F}_3$ are represented by three sides of a triangle and $\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$, then these three forces are concurrent forces and satisfy the condition for equilibrium.

Statement II: A triangle made up of three forces $\vec{F}_1, \vec{F}_2$, and $\vec{F}_3$ as its sides taken in the same order, satisfy the condition for translatory equilibrium.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is false but statement II is true
(2) Both statement I and statement II are false
(3) Both statement I and statement II are true
(4) Statement I is true but statement II is false

Answer (3)

Sol. As, $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

So three forces are concurrent and object is in equilibrium.

Also if $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

then $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ lie along sides of triangle taken in order

15. Choose the incorrect statement:

(a) The electric lines of force entering into a Gaussian surface provide negative flux.

(b) A charge ‘q’ is placed at the centre of a cube. The flux through all the faces will be the same.

(c) In a uniform electric field net flux through a closed Gaussian surface containing no net charge, is zero.

(d) When electric field is parallel to a Gaussian surface, it provides a finite non-zero flux.

Choose the most appropriate answer from the options given below:

(1) (d) Only
(2) (c) and (d) Only
(3) (a) and (c) Only
(4) (b) and (d) Only

Answer (1)

Sol. We know, total flux through close surface, $\phi = \frac{Q_{\text{in}}}{\varepsilon_0}$

if $\phi = 0$, $Q_{\text{in}} = 0$

If electric field is parallel to surface then electric flux $= 0$

16. A system consists of two identical spheres each of mass 1.5 kg and radius 50 cm at the ends of a light rod. The distance between the centres of the two spheres is 5 m. What will be the moment of inertia of the system about an axis perpendicular to the rod passing through its midpoint?

(1) $1.875 \times 10^5$ kgm$^2$
(2) $19.05$ kgm$^2$
(3) $18.75$ kgm$^2$
(4) $1.905 \times 10^5$ kgm$^2$

Answer (2)
17. If \( V_A \) and \( V_B \) are the input voltages (either 5 V or 0 V) and \( V_O \) is the output voltage then the two gates represented in the following circuits (A) and (B) are:

\[
I = 2 \times \left[ \frac{2}{5} mR^2 + \frac{md^2}{4} \right]
\]

\[
I = 2 \left[ \frac{2}{5} \times 1.5 \times (0.5)^2 + 1.5 \times (2.5)^2 \right]
\]

\[
= 1.2 \times (0.5)^2 + 3 \times (2.5)^2
\]

\[
I = 18.75 + 0.30
\]

\[
I = 19.05
\]

18. A current of 1.5 A is flowing through a triangle, of side 9 cm each. The magnetic field at the centroid of the triangle is:

(A) \( 7 \times 10^{-7} \) T, outside the plane of triangle
(B) \( 3 \times 10^{-7} \) T, outside the plane of triangle
(C) \( 5 \times 10^{-7} \) T, inside the plane of triangle
(D) \( 3 \times 10^{-5} \) T, inside the plane of triangle

Answer (4)

Sol.

Field due to each wire will add up.

\[
B_{\text{net}} = 3B_0 \quad [B_0: \text{Field due to single wire}]
\]

\[
B_0 = \frac{\mu_0 I_0}{4\pi d} \times 2\sin 60^\circ
\]

\[
B_0 = \frac{\sqrt{3} \mu_0 I_0}{4\pi d}
\]

\[
d = \frac{a}{2\sqrt{3}}
\]

\[
B_0 = \frac{3\mu_0 I_0}{2\pi a}
\]

\[
B_{\text{net}} = \frac{9\mu_0 I_0}{2\pi a} = \frac{18 \times 10^{-7} \times 1.5}{9 \times 10^{-2}}
\]

\[
= 3 \times 10^{-5} \text{ T}
\]
19. **Statement I**: Two forces \( (\vec{P} + \vec{Q}) \) and \( (\vec{P} - \vec{Q}) \) where \( \vec{P} \perp \vec{Q} \), when act at an angle \( \theta_1 \) to each other, the magnitude of their resultant is \( \sqrt{3(P^2 + Q^2)} \), when they act at an angle \( \theta_2 \), the magnitude of their resultant becomes \( \sqrt{2(P^2 + Q^2)} \). This is possible only when \( \theta_1 < \theta_2 \).

**Statement II**: In the situation given above.

\( \theta_1 = 60^\circ \) and \( \theta_2 = 90^\circ \)

In the light of the above statements, choose the most appropriate answer from the options given below:

1. Both Statement I and Statement II are true.
2. Statement I is true but Statement II is false.
3. Statement I is false but Statement II is true.
4. Both Statement I and Statement II are false.

**Answer (1)**

\[
\begin{align*}
\vec{P} + \vec{Q} + \vec{P} - \vec{Q} &= \sqrt{3} \sqrt{P^2 + Q^2} \\
4P^2 &= 3(P^2 + Q^2) \\
P^2 &= 3Q^2 \\
\theta_1 &= 2\tan^{-1}\left(\frac{Q}{P}\right) \\
&= 60^\circ \\
\text{For case-II} \\
4P^2 &= 2(P^2 + Q^2) \\
P &= Q \\
\theta_2 &= 2\tan^{-1}\left(\frac{Q}{P}\right) = 90^\circ
\end{align*}
\]

20. Four identical hollow cylindrical columns of mild steel support a big structure of mass \(50 \times 10^3\) kg. The inner and outer radii of each column are 50 cm and 100 cm respectively. Assuming uniform local distribution, calculate the compression strain of each column.

\[\text{[use } Y = 2.0 \times 10^{11} \text{ Pa, } g = 9.8 \text{ m/s}^2]\]

\(\text{(1) } 3.60 \times 10^{-8} \quad \text{(2) } 1.87 \times 10^{-3} \quad \text{(3) } 7.07 \times 10^{-4} \quad \text{(4) } 2.60 \times 10^{-7}\]

**Answer (4)**

\[
\begin{align*}
\text{Compressive force on each column } &= \frac{mg}{4} \\
\text{Stress } &= \frac{F}{A} \\
\text{Strain } &= \frac{\text{mg}}{4\pi Y\left[r_2^2 - r_1^2\right]} \\
&= \frac{50 \times 10^3 \times 9.8}{4\pi \times 2 \times 10^{11}\left[r_2^2 - (0.50)^2\right]} \\
&= \frac{50 \times 10^3 \times 9.8}{6\pi \times 10^{11}} \\
&= \frac{5 \times 9.8 \times 10^{-7}}{6\pi} = 2.6 \times 10^{-7}
\end{align*}
\]

**SECTION - II**

**Numerical Value Type Questions**: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. At very high frequencies, the effective impedance of the given circuit will be _____ Ω.

**Answer (2)**
Sol. At high frequencies, $X_C \rightarrow 0 \& X_L \rightarrow \infty$

$$Z = 1 + \frac{2 \times 2}{2 + 2} = 2 \Omega$$

2. Cross-section view of a prism is the equilateral triangle ABC shown in the figure. The minimum deviation is observed using this prism when the angle of incidence is equal to the prism angle. The time taken by light to travel from P (midpoint of BC) to A is $5 \times 10^{-10}$ s. (Given, speed of light in vacuum = $3 \times 10^8$ m/s and $\cos 30^\circ = \frac{\sqrt{3}}{2}$)

Answer (5)

Sol. $\mu = \frac{\sin \left( \frac{A + \delta_m}{2} \right)}{\sin \frac{A}{2}}$

$$i = \frac{A + \delta_m}{2} = A \Rightarrow \delta_m = A$$

$$\mu = 2 \cos \frac{A}{2} = \sqrt{3}$$

$$t = \frac{PA}{v} = \frac{0.10 \times \sqrt{3}}{3 \times 10^8 \sqrt{3}} = 5 \times 10^{-10} \text{ sec}$$

3. A particle is moving with constant acceleration 'a'. Following graph shows $v^2$ versus $x$ (displacement) plot. The acceleration of the particle is _____ m/s$^2$.

Answer (1)

Sol. $v^2 = u^2 + 2ax$

$$a = \frac{1}{2} \left( \text{slope of } v^2 \text{ vs } x \text{ graph} \right)$$

$$a = \frac{1}{2} \left( \frac{40}{20} \right) = 1 \text{ m/s}^2$$

4. In a Young's double slit experiment, the slits are separated by 0.3 mm and the screen is 1.5 m away from the plane of slits. Distance between fourth bright fringes on both sides of central bright fringe is 2.4 cm. The frequency of light used is ____ $\times 10^{14}$ Hz.

Answer (5)

Sol. $\Delta y = 2 \cdot \frac{4 D \lambda}{d}$

$$2.4 \times 10^{-2} = \frac{8 \times 1.5 \times \lambda}{0.3 \times 10^{-3}}$$

$$\Rightarrow \lambda = 600 \text{ nm}$$

$$f = \frac{c}{\lambda} = 5 \times 10^{14} \text{ Hz}$$
5. The diameter of a spherical bob is measured using a vernier callipers. 9 divisions of the main scale, in the vernier callipers, are equal to 10 divisions of vernier scale. One main scale division is 1 mm. The main scale reading is 10 mm and 8th division of vernier scale was found to coincide exactly with one of the main scale division. If the given vernier callipers has positive zero error of 0.04 cm, then the radius of the bob is \( \text{____} \times 10^{-2} \) cm.

Answer (52)

Sol. Reading = 10 + 8 \times 0.1 = 10.8 mm
Diameter = Reading – Zero error
= 10.4 mm
Radius = 5.2 mm

6. A parallel plate capacitor of capacitance 200 \( \mu \)F is connected to a battery of 200 V. A dielectric slab of dielectric constant 2 is now inserted into the space between plates of capacitor while the battery remain connected. The change in the electrostatic energy in the capacitor will be \( \text{____} \) J.

Answer (4)

Sol. \( U_{\text{initial}} = \frac{1}{2} \times (200 \times 10^{-6}) \times (200)^2 \)
\( U_{\text{final}} = \frac{1}{2} \times (200 \times 10^{-6} \times 2) \times (200)^2 \)
\( \Delta U = \frac{1}{2} \times 200 \times 10^{-6} \times 4 \times 10^4 = 4\text{J} \)

7. A sample of gas with \( \gamma = 1.5 \) is taken through an adiabatic process in which the volume is compressed from 1200 cm\(^3\) to 300 cm\(^3\). If the initial pressure is 200 kPa. The absolute value of the work done by the gas in the process = \( \text{____} \) J.

Answer (480)

Sol. \( P_{\text{initial}} = 200 \text{ kPa} = P_1 \text{(say)} \)
\( P_{\text{final}} = P_2 \text{(Say)} = 1600 \text{ kPa} \)
\( W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{240 - 480}{1.5 - 1} = -480 \text{ J} \)

8. A resistor dissipates 192 J of energy in 1 s when a current of 4 A is passed through it. Now, when the current is doubled, the amount of thermal energy dissipated in 5 s is \( \text{____} \) J.

Answer (3840)

Sol. \( \frac{1}{2} R_1 t_1 = 192 \)
\( \frac{1}{2} R_2 t_2 = X \)
\( \frac{192}{X} = \frac{1}{20} \Rightarrow X = 3840 \text{ J} \)

9. A long solenoid with 1000 turns/m has a core material with relative permeability 500 and volume \( 10^3 \text{ cm}^3 \). If the core material is replaced by another material having relative permeability of 750 with same volume maintaining same current of 0.75 A in the solenoid, the fractional change in the magnetic moment of the core would be approximately \( \left( \frac{x}{499} \right) \).

Find the value of \( x \).

Answer (250)

Sol. \( M_1 = \frac{\chi_1}{\mu_1 - 1} \)
\( M_2 = \frac{\chi_2}{\mu_2 - 1} \)
\( \Delta M = \frac{M_1 - M_2}{M_2} = \frac{250}{499} \)

10. A bandwidth of 6 MHz is available for A.M. transmission. If the maximum audio signal frequency used for modulating the carrier wave is not to exceed 6 kHz. The number of stations that can be broadcasted within this band simultaneously without interfering with each other will be \( \text{____} \).

Answer (500)

Sol. Bandwidth = \( 6 \times 10^6 \text{ Hz} \)

Number of stations that can be broadcasted
\( = \frac{6 \times 10^6}{2 \times 6 \times 10^3} = 500 \)
SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. The Eu$^{2+}$ ion is a strong reducing agent in spite of its ground state electronic configuration (outermost):
   [Atomic number of Eu = 63]
   (1) 4f$^6$
   (2) 4f$^7$
   (3) 4f$^6$6s$^2$
   (4) 4f$^7$6s$^2$

   Answer (2)

   Sol. Outermost electronic configuration of Eu:
   
   Eu (63) = 4f$^7$6s$^2$
   
   Eu$^{2+}$ = 4f$^7$

2. Match List-I with List-II.

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Parameter)</td>
<td>(Unit)</td>
</tr>
<tr>
<td>(a) Cell constant</td>
<td>(i) S cm$^2$ mol$^{-1}$</td>
</tr>
<tr>
<td>(b) Molar conductivity</td>
<td>(ii) Dimensionless</td>
</tr>
<tr>
<td>(c) Conductivity</td>
<td>(iii) m$^{-1}$</td>
</tr>
<tr>
<td>(d) Degree of dissociation of electrolyte</td>
<td>(iv) Ω$^{-1}$ m$^{-1}$</td>
</tr>
</tbody>
</table>

   Choose the most appropriate answer from the options given below

   (1) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)
   (2) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)
   (3) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)
   (4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

   Answer (4)

   Sol. Parameter | Unit
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
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</tr>
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<td>Degree of dissociation of electrolyte</td>
<td>Dimensionless</td>
</tr>
</tbody>
</table>

3. Which of the following is NOT an example of fibrous protein?
   (1) Albumin
   (2) Collagen
   (3) Myosin
   (4) Keratin

   Answer (1)

   Sol. When the polypeptide chains run parallel and are held together by hydrogen and disulphide bonds, then fibre-like structure is formed. Such proteins are generally insoluble in water. Some common examples are keratin (present in hair, wool, silk) and myosin (present in muscles), etc.

   This structure results when the chains of polypeptides coil around to give a spherical shape. These are usually soluble in water. Insulin and albumins are the common examples of globular proteins.

4. Match List-I with List-II.

<table>
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<td>(Group in Qualitative analysis)</td>
</tr>
<tr>
<td>(a) Mn$^{2+}$</td>
<td>(i) Group-III</td>
</tr>
<tr>
<td>(b) As$^{3+}$</td>
<td>(ii) Group-IIA</td>
</tr>
<tr>
<td>(c) Cu$^{2+}$</td>
<td>(iii) Group-IV</td>
</tr>
<tr>
<td>(d) Al$^{3+}$</td>
<td>(iv) Group-IIIB</td>
</tr>
</tbody>
</table>

   Choose the most appropriate answer from the options given below

   (1) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)
   (2) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)
   (3) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)
   (4) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

   Answer (2)

   Sol. Metal ion | Group in qualitative analysis
<table>
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<td>Group-IIA</td>
</tr>
<tr>
<td>Al$^{3+}$</td>
<td>Group-III</td>
</tr>
</tbody>
</table>
5. The major product of the following reaction is

\[ \text{CH}_3\text{Cl} \stackrel{\text{NaOH, C}_2\text{H}_5\text{OH}}{\rightarrow} \text{Major product} \]

(1) \( \text{CH}_3\text{OH} \)
(2) \( \text{CH}_3\text{H}_2 \)
(3) \( \text{CH}_3\text{HO} \)
(4) \( \text{CH}_3\text{Cl} \)

Answer (1)

\[ \text{Sol.} \quad \text{CH}_3\text{Cl} \stackrel{\text{NaOH, C}_2\text{H}_5\text{OH}}{\rightarrow} \text{CH}_3\text{OH} \]

6. Which one of the following statements is incorrect?

(1) Bond dissociation enthalpy of \( \text{H}_2 \) is highest among diatomic gaseous molecules which contain a single bond
(2) Atomic hydrogen is produced when \( \text{H}_2 \) molecules at a high temperature are irradiated with UV radiation
(3) At around 2000 K, the dissociation of dihydrogen into its atoms is nearly 8.1%
(4) Dihydrogen is produced on reacting zinc with \( \text{HCl} \) as well as \( \text{NaOH}_{(aq)} \)

Answer (3)

\[ \text{Sol.} \quad \text{The H-H bond dissociation enthalpy is the highest for a single bond between two atoms of any element. It is because of this factor that the dissociation of dihydrogen into its atoms is only \( \sim 0.081\% \) around 2000 K. The atomic hydrogen is produced at a high temperature in an electric arc or under ultraviolet radiations.} \]

\[ \text{Zn} + 2\text{HCl} \rightarrow \text{ZnCl}_2 + \text{H}_2 \]

\[ \text{Zn} + 2\text{NaOH} \rightarrow \text{Na}_2\text{ZnO}_2 + \text{H}_2 \]

7. For the following sequence of reactions, the correct products are:

\[ 1. \text{Br}_2/\text{Fe}/\Delta \]
\[ 2. \text{Mg/dry ether} \]
\[ 3. \text{CH}_3\text{OH} \rightarrow \text{Products} \]

(1) \( \text{Br} \) + \( \text{Mg} \) + \( \text{CH}_3\text{OH} \)
(2) \( \text{Br} \) + \( \text{HMgBr} \)
(3) \( \text{H} \) + \( \text{Mg} \) + \( \text{CH}_3\text{OH} \)
(4) \( \text{Br} \) + \( \text{Mg} \) + \( \text{CH}_3\text{OH} \)

Answer (1)

\[ \text{Sol.} \quad \text{Br}_2/\text{Fe}/\Delta \rightarrow \text{MgBr} \rightarrow \text{MgBr} + \text{CH}_3\text{OH} \rightarrow \text{Br} + \text{Mg} + \text{CH}_3\text{OH} \]

8. For the reaction given below:

\[ \text{CHO} \stackrel{1. \text{NaOH, } \Delta}{\rightarrow} \text{Product} \]
\[ \text{CH}_3\text{OH} \]

The compound which is not formed as a product in the reaction is a:

(1) Dicarboxylic acid
(2) Monocarboxylic acid
(3) Compound with both alcohol and acid functional groups
(4) Diol
10. Identify correct A, B and C in the reaction sequence given below:

Answer (4)

Sol.

11. The major products A and B formed in the following reaction sequence are:

Answer (4)
12. The number of $S=O$ bonds present in sulphurous acid, peroxodisulphuric acid and pyrosulphuric acid, respectively are:

(1) 2, 3 and 4  
(2) 2, 4 and 3  
(3) 1, 4 and 4  
(4) 1, 4 and 3

Answer (3)

Sol. Sulphurous acid ($\text{H}_2\text{SO}_3$)

\[
\begin{align*}
\text{O} & \quad \text{O} \\
\text{H} & \quad \text{O} \\
\text{S} & \quad \text{O} \\
\text{O} & \quad \text{H}
\end{align*}
\]

Peroxodisulphuric acid ($\text{H}_2\text{S}_2\text{O}_8$)

\[
\begin{align*}
\text{O} & \quad \text{O} & \quad \text{O} \\
\text{S} & \quad \text{O} & \quad \text{O} \\
\text{O} & \quad \text{O} & \quad \text{O}
\end{align*}
\]

Pyrosulphuric acid ($\text{H}_2\text{S}_2\text{O}_7$)

\[
\begin{align*}
\text{O} & \quad \text{O} & \quad \text{O} \\
\text{S} & \quad \text{O} & \quad \text{O} \\
\text{O} & \quad \text{O} & \quad \text{O}
\end{align*}
\]

13. The structures of A and B formed in the following reaction are: [Ph = $-\text{C}_6\text{H}_5$]

\[
\begin{align*}
\text{Ph} & \quad \text{O} \\
\text{A} & \quad \text{B}
\end{align*}
\]
14. Which one of the following correctly represents the order of stability of oxides, $X_2O$; ($X$ = halogen)?

(1) $I > Cl > Br$  
(2) $Cl > I > Br$  
(3) $Br > Cl > I$  
(4) $Br > I > Cl$

**Answer (1)**

**Sol.** A combination of kinetic and thermodynamic factors lead to the generally decreasing order of stability of oxides formed by halogens, $I > Cl > Br$. The higher oxide of halogen tend to be more stable than lower one.

15. Which among the following is not a polyester?

(1) Dacron  
(2) Glyptal  
(3) PHBV  
(4) Novolac

**Answer (4)**

**Sol.** Polyesters

These are the polycondensation products of dicarboxylic acids and diols.

Novolac is not a polyester.

16. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

**Assertion (A)**: Lithium salts are hydrated.

**Reason (R)**: Lithium has higher polarising power than other alkali metal group members.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(2) (A) is correct but (R) is not correct
(3) (A) is not correct but (R) is correct
(4) Both (A) and (R) are correct but (R) is NOT the correct explanation of (A)

**Answer (4)**

**Sol.** Lithium has smallest size among the alkali metal. So charge to size ratio of lithium is very high. Due to this lithium salts are hydrated in nature. LiCl is deliquescent and crystallises as a hydrate, LiCl. $2H_2O$ whereas other alkali metal chlorides do not form hydrates. Polarising power is related to covalent character.

17. Arrange the following conformational isomers of n-butane in order of their increasing potential energy.

(1) II $<$ IV $<$ III $<$ I  
(2) I $<$ III $<$ IV $<$ II  
(3) II $<$ III $<$ IV $<$ I  
(4) I $<$ IV $<$ III $<$ II

**Answer (2)**
15

**Sol.** Energy in increasing order is,

Anti < Gauche < Partially Eclipsed < Eclipsed
I < III < IV < II

18. Spin only magnetic moment in BM of \([\text{Fe(CO)}_4(\text{C}_2\text{O}_4)]^+\) is:

(1) 5.92
(2) 1.73
(3) 1
(4) 0

Answer (2)

**Sol.**

\[
\text{Fe(CO)}_4(\text{C}_2\text{O}_4)]^+ \\
\]

oxidation state of iron in the given compound is

\[X + 0 - 2 = +1\]
\[x = +3\]

\[\text{Fe}(26) = [\text{Ar}] 4s^23d^6\]

\[\text{Fe}^{3+} = [\text{Ar}]3d^5\]

It is a low spin complex because of four strong CO ligand. So number of unpaired electron is 1.

\[
\mu = \sqrt{n(n+2)}
\]

\[
= \sqrt{1(1+2)}
\]

\[
= \sqrt{3} = 1.73 \text{ B.M.}
\]

19. In which one of the following sets all species show disproportionation reaction?

(1) \(\text{ClO}_4^-, \text{MnO}_4^-, \text{ClO}_2^-\) and \(\text{F}_2\)
(2) \(\text{ClO}_2^-, \text{F}_2, \text{MnO}_4^-\) and \(\text{Cr}_2\text{O}_7^{2-}\)
(3) \(\text{MnO}_4^-, \text{ClO}_2, \text{Cl}_2\) and \(\text{Mn}^{3+}\)
(4) \(\text{Cr}_2\text{O}_7^{2-}, \text{MnO}_4^-, \text{ClO}_2^-\) and \(\text{Cl}_2\)

Answer (Bonus)

**Sol.** \(\text{ClO}_4^-, \text{MnO}_4^-, \text{Cr}_2\text{O}_7^{2-}\) – Cl, Mn, Cr in these anions are present in highest oxidation state. These will not undergo disproportionation.

20. The deposition of X and Y on ground surfaces is referred as wet and dry depositions, respectively. X and Y are

(1) \(X = \text{SO}_2, Y = \text{Ammonium salts}\)
(2) \(X = \text{CO}_2, Y = \text{SO}_2\)
(3) \(X = \text{Ammonium salts, } Y = \text{SO}_2\)
(4) \(X = \text{Ammonium salts, } Y = \text{CO}_2\)

Answer (3)

**Sol.** Ammonium salts are also formed and can be seen as an atmospheric haze (aerosol of fine particles). Aerosol particles of oxides or ammonium salts in rain drops result in wet-deposition. \(\text{SO}_2\) is also absorbed directly on both solid and liquid ground surfaces and is thus deposited as dry-deposition.

SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 1.22 g of an organic acid is separately dissolved in 100 g of benzene \(K_b = 2.6 \text{ K kg mol}^{-1}\) and 100 g of acetone \(K_b = 1.7 \text{ K kg mol}^{-1}\). The acid is known to dimerize in benzene but remain as a monomer in acetone. The boiling point of the solution in acetone increases by 0.17°C. The increase in boiling point of solution in benzene in °C is \(x \times 10^{-2}\). The value of \(x\) is _______. (Nearest integer)

[Atomic mass : C = 12.0, H = 1.0, O = 16.0]

Answer (13)

**Sol.** \(K_b\) (benzene) = 2.6 K kg mol\(^{-1}\)

\(K_b\) (acetone) = 1.7 K kg mol\(^{-1}\)

In acetone

\[
0.17 = \frac{1 \times 1.7 \times 1.22 \times 1000}{M \times 100}
\]

\[M = 122 \text{ g/mol}\]

In benzene

\[
\Delta T_b = \frac{1}{2} \times \frac{1.22}{122} \times 2.6 \times 10
\]

\[= 0.13 \text{ °C}\]

\[= 13 \times 10^{-2} \text{ °C}\]
2. In the electrolytic refining of blister copper, the total number of main impurities, from the following, removed as anode mud is_____.

Pb, Sb, Se, Te, Ru, Ag, Au and Pt

Answer (6)

Sol. Impurities from the blister copper deposit as anode mud which contains antimony, selenium, tellurium, silver, gold and platinum.

3. For the reaction \( A \rightarrow B \), the rate constant \( k \) (in s\(^{-1}\)) is given by

\[
\log_{10} k = 20.35 - \frac{(2.47 \times 10^3)}{T}
\]

The energy of activation in kJ mol\(^{-1}\) is ________. (Nearest integer)

\[\text{[Given : } R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}\]

Answer (47)

Sol. \( \log_{10} K = 20.35 - \frac{(2.47 \times 10^3)}{T} \) \( \ldots (i) \)

\[
\log K = \log A - \frac{E_a}{2.303 RT} \quad \ldots (ii)
\]

Comparing (i) and (ii),

\[
\frac{E_a}{2.303 RT} = \frac{2.47 \times 10^3}{T}
\]

\[
E_a = 2.47 \times 10^3 \times 2.303 \times 8.314
\]

\[
= 47293.44 \text{ J mol}^{-1}
\]

\[
= 47.2934 \text{ kJ mol}^{-1}
\]

4. The empirical formula for a compound with a cubic close packed arrangement of anions and with cations occupying all the octahedral sites in \( A_x B \). The value of \( x \) is _______. (Integer answer)

Answer (1)

Sol. Cation\(_{(\text{octahedral site})} \) Anion\(_{(\text{ccp})} \)

\[
A \left(1 + 12 \times \frac{1}{4}\right) \quad B \left(8 \times \frac{1}{2} + 8 \times \frac{1}{8}\right)
\]

\( A_4 B_4 \)

Empirical formula = \( AB \)

\[\Rightarrow x = 1\]

5. The value of magnetic quantum number of the outermost electron of \( \text{Zn}^+ \) ion is _______. (Integer answer)

Answer (0)

Sol. \( \text{Zn}(30) = [\text{Ar}]4s^23d^{10} \)

\( \text{Zn}^+ = [\text{Ar}]4s^13d^{10} \)

Outermost electron is present in 4s

\( n = 4 \quad l = 0 \quad m_l = 0 \)

6. \( \text{CH}_4 \) is adsorbed on 1 g charcoal at 0ºC following the Freundlich adsorption isotherm. 10.0 mL of \( \text{CH}_4 \) is adsorbed at 100 mm of Hg, whereas 15.0 mL is adsorbed at 200 mm of Hg. The volume of \( \text{CH}_4 \) adsorbed at 300 mm of Hg is \( 10^x \) mL. The value of \( x \) is ______ \( \times 10^{-2} \). (Nearest integer)

\[\text{[Use } \log_{10} 2 = 0.3010, \log_{10} 3 = 0.4771]\]

Answer (128)

Sol. \( \frac{x}{m} = kp^n \)

\[
\frac{10}{1} = k(100)^n
\]

\[
\frac{15}{1} = k(200)^n
\]

\[
\frac{x}{1} = k(300)^n
\]

\[
\log 10 = \log k + \frac{1}{n} \log 100
\]

\[
\log 15 = \log k + \frac{1}{n} \log 200
\]

\[
\log \left(\frac{3}{2}\right) = \frac{1}{n} \log 2
\]

\[n = \frac{0.3010}{0.1761} = 1.7
\]

\[
\log y = \log k + \frac{1}{n} \log 300
\]

\[
\log y - \log 10 = \frac{1}{n} \log 3
\]
7. Sodium oxide reacts with water to produce sodium hydroxide. 20.0 g of sodium oxide is dissolved in 500 mL of water. Neglecting the change in volume, the concentration of the resulting NaOH solution is \[ \text{_____} \times 10^{-1} \text{ M}. \] (Nearest integer)

[Atomic mass : Na = 23.0, O = 16.0, H = 1.0]

Answer (13)

**Sol.**

\[
\text{Na}_2\text{O} + \text{H}_2\text{O} \rightarrow 2\text{NaOH}
\]

Moles

\[
\frac{20}{23 \times 2 + 16} = \frac{2 \times 20}{(23 \times 2 + 16)}
\]

Molarity

\[
\frac{0.645 \times 1000}{500} = 1.290 \text{ M}
\]

\[
= 12.90 \times 10^{-1} \text{ M}
\]

\[
= 13 \times 10^{-1} \text{ M}
\]

8. According to molecular orbital theory, the number of unpaired electron(s) in \( \text{O}_2^- \) is \[ \text{_____} \].

Answer (0)

**Sol.** Electronic configuration of \( \text{O}_2^- \) (according to MOT) is

\[
\sigma_1s^2, \sigma^*1s^2, \sigma_2s^2, \sigma^*2s^2, \text{ and } \pi^2\rho_y^2, \pi^*2\rho_y^2, \pi_2\rho_z^2, \pi^*2\rho_z^2
\]

Total unpaired electron in \( \text{O}_2^- \) is zero.

9. The pH of a solution obtained by mixing 50 mL of 1 M HCl and 30 mL of 1 M NaOH is \[ x \times 10^{-4} \]. The value of \( x \) is \[ \text{_____} \]. (Nearest integer)

[log 2.5 = 0.3979]
PART-C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. If \( e^{d^2} + e^{2x} \cdot e^{2\log_2 y} \), \( y(0) = 0 \), then for \( y = 1 \), the value of \( x \) lies in the interval

(1) \( \frac{1}{2}, 1 \)  
(2) \( 0, \frac{1}{2} \)  
(3) (1, 2)  
(4) (2, 3)

Answer (3)

Sol.

\[
\begin{align*}
\int \frac{e^{d^2} + e^{2x} \cdot e^{2\log_2 y}}{e^{2x} \cdot e^{2\log_2 y}} \, dy = \int dx \\
\Rightarrow (1 + 2x \ln 2) dy = dx
\end{align*}
\]

For LHS put \( y + 2^y = t \)

\[
\Rightarrow (1 + 2^y \ln 2) \, dy = dt
\]

\[
\ln (y + 2^y) = x + c
\]

\[
(0, 0)
\]

\[
\therefore \ln(y + 2^y) = x
\]

If \( y = 1 \)

\[
x = \ln 3
\]

\[
x \in (1, 2)
\]

2. If \( \alpha + \beta + \gamma = 2\pi \), then the system of equations

\[
x + (\cos \gamma)y + (\cos \beta)z = 0
\]

\[
(\cos \gamma)x + y + (\cos \alpha)z = 0
\]

\[
(\cos \beta)x + (\cos \alpha)y + z = 0
\]

has:

(1) a unique solution
(2) no solution
(3) infinitely many solutions
(4) exactly two solutions

Answer (3)

Sol. \( \Delta = \begin{vmatrix} 1 & \cos \gamma & \cos \beta \\ \cos \gamma & 1 & \cos \alpha \\ \cos \beta & \cos \alpha & 1 \end{vmatrix} \)

\[
= (1 - \cos^2 \alpha) - \cos \gamma (\cos \gamma - \cos \alpha \cos \beta) + \cos \beta (\cos \alpha \cos \gamma - \cos \beta)
\]

\[
= 1 - (\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) + 2 \cos \alpha \cos \beta \cos \gamma
\]

\[
= 1 - (1 - 2 \cos \alpha \cos \beta \cos \gamma) + 2 \cos \alpha \cos \beta \cos \gamma
\]

\[
= 0
\]

\[
\therefore \text{System has infinite solution}
\]

3. The locus of mid-points of the line segments joining \((-3, -5)\) and the points on the ellipse \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \) is

(1) \( 36x^2 + 16y^2 + 90x + 56y + 145 = 0 \)
(2) \( 9x^2 + 4y^2 + 18x + 8y + 145 = 0 \)
(3) \( 36x^2 + 16y^2 + 72x + 32y + 145 = 0 \)
(4) \( 36x^2 + 16y^2 + 108x + 80y + 145 = 0 \)

Answer (4)

Sol. Let general point on ellipse \( (2 \cos \theta, 3 \sin \theta) \)

Let mid-point of \((2 \cos \theta, 3 \sin \theta) \) & \((-3, -5)\) be \((h, k)\)

\[
\Rightarrow 2\cos \theta - 3 = 2h \quad \& \quad 3\sin \theta - 5 = 2k
\]

\[
\Rightarrow \cos \theta = \frac{2h + 3}{2} \quad \& \quad \sin \theta = \frac{2k + 5}{3}
\]

\[
\sin^2 \theta + \cos^2 \theta = 1
\]

\[
\Rightarrow \left( \frac{2y + 5}{3} \right)^2 + \left( \frac{2x + 3}{2} \right)^2 = 1
\]

\[
\Rightarrow 4(2y + 5)^2 + 9(2x + 3)^2 = 36
\]

\[
\Rightarrow 36x^2 + 16y^2 + 108x + 80y + 145 = 0
\]

4. The number of solutions of the equation \( 32 \tan^2 x + 32 \sec^2 x = 81, \ 0 \leq x \leq \frac{\pi}{4} \) is

(1) 3
(2) 1
(3) 2
(4) 0

Answer (2)
Sol. \[32\tan^2x + 32\tan^2x = 81\]
\[32\tan^2x(33) = 81\]
\[\tan^2x = \frac{81}{33} = \frac{27}{11}\]

If \(x \in \left[0, \frac{\pi}{4}\right]\) then \(32\tan^2x \in [1, 32]\) & is always increasing.

Hence one solution only.

5. Negation of the statement \((p \lor r) \implies (q \lor r)\) is
   (1) \(~p \land q \land \sim r\)  
   (2) \(p \land \sim q \land \sim r\)  
   (3) \(~p \land q \land r\)  
   (4) \(p \land q \land r\)

Answer (2)

Sol. \(~(p \lor r) \implies (q \lor r)\)
\[= (p \lor r) \land \sim (q \lor r)\]

\[
\therefore \quad p \land \sim q \land \sim r
\]

6. An angle of intersection of the curves, \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\) and \(x^2 + y^2 = ab, a > b\) is
   (1) \(\tan^{-1}\left(\frac{a + b}{\sqrt{ab}}\right)\) 
   (2) \(\tan^{-1}\left(\frac{a - b}{2\sqrt{ab}}\right)\)  
   (3) \(\tan^{-1}\left(\frac{a - b}{\sqrt{ab}}\right)\)  
   (4) \(\tan^{-1}(2\sqrt{ab})\)

Answer (3)

Sol. Let point of intersection be \((x_1, y_1)\)
\[x_1^2 + y_1^2 = ab = 0\]
\[b^2x_1^2 + a^2y_1^2 - a^2b^2 = 0\]
\[x_1^2 = \frac{a^2b^2}{a + b}, \quad y_1^2 = \frac{ab^2}{a + b}\]
\[\Rightarrow \quad x_1 = a\sqrt{\frac{b}{a + b}}, \quad y_1 = b\sqrt{\frac{a}{a + b}} \quad \ldots (i)\]

Tangent to ellipse \(\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1\)

Slope \(= m_1 = -\frac{b^2x_1}{a^2y_1}\)

Tangent to circle \(xx_1 + yy_1 = ab\)

Slope \(= m_2 = -\frac{x_1}{y_1}\)

\[\tan \theta = \frac{x_1y_1(a^2 - b^2)}{a^2b^2} = \frac{ab\sqrt{ab}(a^2 - b^2)}{a + b} \quad \text{(Using (i))}\]

\[\tan \theta = \frac{a - b}{\sqrt{ab}}\]

7. The distance of the point \((-1, 2, -2)\) from the line of intersection of the planes \(2x + 3y + 2z = 0\) and \(x - 2y + z = 0\) is
   (1) \(\frac{5}{2}\)  
   (2) \(\frac{\sqrt{34}}{2}\)  
   (3) \(\frac{\sqrt{42}}{2}\)  
   (4) \(\frac{1}{\sqrt{2}}\)

Answer (2)

Sol. \(P(-1, 2, -2)\)

DRs of line of intersection (LOI).
\[
\begin{vmatrix}
i & j & k \\
1 & 2 & 1 \\
3 & 2 & 1
\end{vmatrix} = 7i - 0j - 7k
\]

DRs \(= (1, 0, -1)\)

Equation of line of intersection
\[
\frac{x}{1} = \frac{y}{0} = \frac{z}{-1} = r
\]

\(M(r, 0, -r)\)

Direction ratios of PM \((r + 1, -2, -r + 2)\)

Apply PM perpendicular line \(1(r + 1) + 0(-2) - 1(-r + 2) = 0\)
\[2r - 1 = 0\]
\[r = \frac{1}{2}\]

\(M\left(\frac{1}{2}, 0, \frac{-1}{2}\right)\)

\(PM = \sqrt{\frac{9}{4} + 4 + \frac{9}{4}} = \sqrt{\frac{34}{2}}\)
8. Let \( A \) be the set of all points \((\alpha, \beta)\) such that the area of triangle formed by the points \((5, 6), (3, 2)\) and \((\alpha, \beta)\) is 12 square units. Then the least possible length of a line segment joining the origin to a point in \( A \), is  

\[
\begin{align*}
(1) \quad & \frac{8}{\sqrt{5}} \\
(2) \quad & \frac{16}{\sqrt{5}} \\
(3) \quad & \frac{4}{\sqrt{5}} \\
(4) \quad & \frac{12}{\sqrt{5}}
\end{align*}
\]

Answer (1)

Sol. 

\[
\begin{vmatrix}
\alpha & \beta & 1 \\
5 & 6 & 1 \\
3 & 2 & 1
\end{vmatrix} = 12
\]

\[
|4\alpha - 2\beta - 8| = 24
\]

\[
|2\alpha - \beta - 4| = 12
\]

Locus = \(2x - y - 4 = 12, \quad 2x - y - 4 = -12\)

\(2x - y - 16 = 0, \quad 2x - y + 8 = 0\)

Required length = minimum perpendicular distance from origin

\[
= \min \left\{ \frac{16}{\sqrt{5}}, \frac{8}{\sqrt{5}} \right\} = \frac{8}{\sqrt{5}}
\]

9. If \( \frac{dy}{dx} = x \left( \frac{y^2}{x^2} + \frac{\phi \left( \frac{y^2}{x^2} \right)}{\phi'(x)} \right) \), \( x > 0, \phi > 0, \) and \( y(1) = -1 \), then \( \phi \left( \frac{y^2}{x^2} \right) \) is equal to

\[
\begin{align*}
(1) \quad & 4\phi(2) \\
(2) \quad & 4\phi(1) \\
(3) \quad & 2\phi(1) \\
(4) \quad & \phi(1)
\end{align*}
\]

Answer (2)

Sol. Let \( \frac{y^2}{x^2} = u \)

\[
\Rightarrow y^2 = ux^2
\]

\[
2uy' = u'x^2 + 2ux \quad \text{... (i)}
\]

Given

\[
yy' = x \left( \frac{y^2}{x^2} + \frac{\phi \left( \frac{y^2}{x^2} \right)}{\phi'(x)} \right)
\]

\[
\Rightarrow yy' = x + \phi(u)
\]

\[
\Rightarrow \frac{1}{2} \left[ u'x^2 + 2ux \right] = x + \phi(u)
\]

\[
\frac{1}{2} u'x^2 = x \phi(u)
\]

\[
\Rightarrow xu' = \frac{2\phi(u)}{\phi'(u)}
\]

\[
x \frac{du}{dx} = \frac{2\phi(u)}{\phi'(u)}
\]

\[
\int \frac{\psi(u)du}{\phi(u)} = \int \frac{2dx}{x}
\]

In \( \phi(u) = 2 \ln x + \ln c \)

\( \phi(u) = cx^2 \)

\( \phi \left( \frac{y^2}{x^2} \right) = cx^2 \quad \text{... (ii)} \)

\( x = 1 \Rightarrow y = -1 \)

\( \therefore \phi(1) = c \)

\( \Rightarrow \phi \left( \frac{y^2}{x^2} \right) = \phi(1)x^2 \)

Put \( x = 2 \)

\( \phi \left( \frac{y^2}{4} \right) = 4\phi(1) \)

10. Let \( \vec{a}, \vec{b}, \vec{c} \) be three vectors mutually perpendicular to each other and have same magnitude. If a vector \( \vec{r} \) satisfies

\[
\vec{a} \times \left\{ \left( \vec{r} - \vec{b} \right) \times \vec{a} \right\} + \vec{b} \times \left\{ \left( \vec{r} - \vec{c} \right) \times \vec{b} \right\} + \vec{c} \times \left\{ \left( \vec{r} - \vec{a} \right) \times \vec{c} \right\} = \vec{0},
\]

then \( \vec{r} \) is equal to

\[
\begin{align*}
(1) \quad & \frac{1}{3} \left( \vec{a} + \vec{b} + \vec{c} \right) \\
(2) \quad & \frac{1}{2} \left( \vec{a} + \vec{b} + 2\vec{c} \right) \\
(3) \quad & \frac{1}{3} \left( 2\vec{a} + \vec{b} + \vec{c} \right) \\
(4) \quad & \frac{1}{3} \left( 2\vec{a} + 2\vec{b} - \vec{c} \right)
\end{align*}
\]

Answer (2)

Sol. \( |\vec{a}| = |\vec{b}| = |\vec{c}| = d \) (say)

\[
\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0
\]

\[
\vec{a} \times \left( \left( \vec{r} - \vec{b} \right) \times \vec{a} \right) + \vec{b} \times \left( \left( \vec{r} - \vec{c} \right) \times \vec{b} \right) + \vec{c} \times \left( \left( \vec{r} - \vec{a} \right) \times \vec{c} \right) = \vec{0}
\]
13. Let $f$ be any continuous function on $[0, 2]$ and twice differentiable on $(0, 2)$. If $f(0) = 0$, $f(1) = 1$ and $f(2) = 2$, then
   
   (1) $f''(x) > 0$ for all $x \in (0, 2)$
   (2) $f'(x) = 0$ for some $x \in [0, 2]$
   (3) $f''(x) = 0$ for some $x \in (0, 2)$
   (4) $f''(x) = 0$ for all $x \in (0, 2)$

Answer (3)

Sol. There exists a $C_1 \in (0, 1)$ such that

$$f'(C_1) = \frac{f(1) - f(0)}{1 - 0} = 1$$

and there exists a $C_2 \in (1, 2)$ such that

$$f'(C_2) = \frac{f(2) - f(1)}{2 - 1} = 1$$

Hence there exists a $C \in (C_1, C_2)$ such that

$$f''(C) = \frac{f''(C_1) - f''(C_2)}{C_1 - C_2} = 0$$

14. If $x = \lim_{x \to \pi/4} \frac{\tan^3 x - \tan x}{\cos x}$ and $\beta = \lim_{x \to 0} (\cos x)^{\cot x}$

are the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair $(a, b)$ is

(1) $(-1, -3)$
(2) $(-1, 3)$
(3) $(1, 3)$
(4) $(1, -3)$

Answer (3)

Sol. $\alpha = \lim_{x \to \pi/4} \frac{\tan x(-\cos 2x)}{\cos x \cdot \cos x + \frac{\pi}{4} \cdot \cos x - \frac{\pi}{4} \cdot \cos x} = -2 \lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{\cos x + \frac{\pi}{4}} = -4$

$\beta = \lim_{x \to 0} (\cos x)^{\cot x} = e^{\lim_{x \to 0} (\cos x - 1) \cdot \cot x} = e^0 = 1$

Quadratic equation having roots $\alpha, \beta$ is

$$x^2 + 3x - 4 = 0$$

Clearly $a = 1$, $b = 3$
15. The domain of the function
\[ f(x) = \sin^{-1}\left(\frac{3x^2 + x - 1}{(x - 1)^2}\right) + \cos^{-1}\left(\frac{x - 1}{x + 1}\right) \] is
(1) \([-1, 1]\) and \([-1, 1]\)
(2) \([-2, 1) \cup \{0\}]
(3) \([0, \infty)\) and \([-\infty, 0) \cup \left[\frac{1}{4}, \infty\right) \setminus \{1\}\)
(4) \([-2, 0) \cup \left[\frac{1}{4}, \frac{1}{2}\right]\]
Answer (3)

Sol. \[
\begin{align*}
3x^2 + x - 1 & \in [-1, 1] \text{ and } x - 1 \in [-1, 1] \\
\Rightarrow x & \in [-2, 1) \text{ and } x \in \left(-\infty, 0\right] \cup \left[\frac{1}{4}, \infty\right) \setminus \{1\}
\end{align*}
\]
Finally, \(x \in \{0\} \cup \left[\frac{1}{4}, \frac{1}{2}\right]\)

16. The mean and variance of 7 observations are 8 and 8 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is
(1) \(\frac{536}{25}\)
(2) \(\frac{134}{5}\)
(3) \(\frac{112}{5}\)
(4) \(\frac{92}{5}\)
Answer (1)

Sol. \[
\bar{x}_{\text{old}} = 8 \Rightarrow \sum x_{\text{old}} = 56
\]
\[
\frac{\sum x^2_{\text{old}}}{7} - (\bar{x}_{\text{old}})^2 = 16 \Rightarrow \sum x^2_{\text{old}} = 560
\]
Sum of remaining 5 observation
\[
\sum x = 56 - 14 = 42
\]
Sum of squares of 5 observation = 560 - 6^2 - 8^2 = 460
Variance = \(\frac{460}{5} - \left(\frac{42}{5}\right)^2 = \frac{536}{25}\)

17. The sum of the roots of the equation,
\[ x + 1 - 2 \log_2(3 + 2^x) + 2 \log_4(10 - 2^{-x}) = 0 \]
is
(1) \(\log_2 14\)
(2) \(\log_2 12\)
(3) \(\log_2 11\)
(4) \(\log_2 13\)
Answer (3)

Sol. \[
x + 1 - 2 \log_2(3 + 2^x) + 2 \log_4(10 - 2^{-x}) = 0
\]
\[
\Rightarrow x + 1 = \log_2 \left(\frac{(3 + 2^x)^2}{10 - 2^{-x}}\right)
\]
\[
\Rightarrow 2^{x+1} = \frac{9 + 6 \cdot 2^x + 2 \cdot 2^x}{10 - 2^{-x}}
\]
\[
\Rightarrow 20 \cdot 2^x - 2 = 9 + 6 \cdot 2^x + 2 \cdot 2^x
\]
\[
\Rightarrow (2^x)^2 - 14(2^x) + 11 = 0
\]
Let two roots are \(2^{x_1}\) and \(2^{x_2}\)
Then \(2^{x_1} \cdot 2^{x_2} = 11 \Rightarrow x_1 + x_2 = \log_2 11\)
Sum of roots = \(\log_2 11\)

18. Let \(f : N \rightarrow N\) be a function such that \(f(m + n) = f(m) + f(n)\) for every \(m, n \in N\). If \(f(6) = 18\), then \(f(2)\). \(f(3)\) is equal to
(1) 54
(2) 18
(3) 6
(4) 36
Answer (1)

Sol. \[
f(m + n) = f(m) + f(n), f : N \rightarrow N
\]
\[
\text{then } f(x) = kx
\]
\[
f(6) = 18 \Rightarrow 18 = k \times 6 \Rightarrow k = 3
\]
\[
f(x) = 3x
\]
\[
f(2).f(3) = 6 \times 9 = 54
\]

19. Let \(a_1, a_2, a_3, \ldots\) be an A.P. If \(\frac{a_1 + a_2 + \ldots + a_{10}}{a_1 + a_2 + \ldots + a_p} = \frac{100}{p^2}, p \neq 10\), then \(\frac{a_{11}}{a_{10}}\) is equal to
(1) \(\frac{19}{21}\)
(2) \(\frac{100}{121}\)
(3) \(\frac{21}{19}\)
(4) \(\frac{121}{100}\)
Answer (3)

Sol. \[
a_1, a_2, a_3, \ldots\ \text{are in A.P.}
\]
Let its common difference be \(d\).
\[
\frac{a_1 + a_2 + \ldots + a_{10}}{a_1 + a_2 + \ldots + a_p} = \frac{100}{p^2}
\]
\[
\Rightarrow \frac{a_1 + a_2 + \ldots + a_{10}}{a_1 + a_2 + \ldots + a_p} = \frac{100}{p^2}
\]
\[
\frac{10}{2} \{2a_1 + 9d\} = 100 \\
\Rightarrow \frac{p}{2} \{2a_1 + (p-1)d\} = p^2 \\
\Rightarrow \frac{2a_1 + 9d}{2a_1 + (p-1)d} = \frac{10}{p} \\
\Rightarrow 2pa_1 + 9pd = 20a_1 + 10(p-1)d \\
= (2p - 20) a_1 = (p - 10)d \\
\therefore 2a_1 = d \\
\Rightarrow a_{11} = a_1 + 10d = a_1 + 20a_1 = 21 \\
a_{10} = a_1 + 9d = a_1 + 18a_1 = 19
\]

20. If \( z \) is a complex number such that \( \frac{z - i}{z - 1} \) is purely imaginary, then the minimum value of \(|z - (3 + 3i)|\) is

\((1) \ 6\sqrt{2} \quad (2) \ 2\sqrt{2} \quad (3) \ 3\sqrt{2} \quad (4) \ 2\sqrt{2} - 1\)

Answer (2)

Sol. \( \frac{z - i}{z - 1} \) is purely imaginary number

\[
\Rightarrow \frac{z - i + z + i}{z - 1} = 0 \\
\Rightarrow (z - i)(\overline{z} - 1) + (z - 1)(\overline{z} + i) = 0 \\
\Rightarrow 2z\overline{z} - (z + \overline{z})i + (z - \overline{z}) = 0 \\
\Rightarrow x^2 + y^2 - x - y = 0, \text{ let } z = x + iy.
\]

Which a circle of centre \( \left( \frac{1}{2}, \frac{1}{2} \right) \) and radius \( \frac{1}{\sqrt{2}} \)

\( \Rightarrow \) Minimum value of \(|z - (3 + 3i)| = QR \)

\[
= \sqrt{\left(3 - \frac{1}{2}\right)^2 + \left(3 - \frac{1}{2}\right)^2} - \frac{1}{\sqrt{2}} \\
= \frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 2\sqrt{2}
\]
1. Let $L$ = length of tangent from $A$ to the circle
   & $R$ = radius of circle
   \[
   \angle PAB = \angle BPQ = 0
   \]
   \[
   \Rightarrow \text{area of } \triangle PAQ = 2 \cdot \frac{1}{2} \cdot L \sin \theta \cdot L \cos \theta = L^2 \cdot \sin \theta \cdot \cos \theta
   \]
   \[
   \text{area of } \triangle PBQ = 2 \cdot \frac{1}{2} \cdot R \sin \theta \cdot R \cos \theta = R^2 \cdot \sin \theta \cdot \cos \theta
   \]
   Hence:
   \[
   \frac{\text{area of } \triangle APQ}{\text{area of } \triangle BPQ} = \frac{L^2}{R^2}
   \]
   \[
   \text{Now, } L = \sqrt{S_A} = \sqrt{3^2 + 1^2 - 2 \times 3 \times 1 + 1} = 3
   \]
   \[
   \Rightarrow 8 \times \left( \frac{\text{area of } \triangle APQ}{\text{area of } \triangle BPQ} \right) = 8 \times \left( \frac{3}{2} \right)^2 = 18
   \]

4. If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \ldots$, then 160 $S$ is equal to

Answer (305)

Sol. $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \ldots$

Subtracting,

\[
\Rightarrow \frac{4S}{5} = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \ldots
\]

Where, $M = \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \ldots$

\[
\frac{1}{5}M = \frac{2}{5^3} + \frac{4}{5^4} + \frac{6}{5^5} + \ldots
\]

\[
4M = \frac{2}{5^2} + 2 \left( \frac{1}{5^3} + \frac{1}{5^4} + \ldots \right)
\]

\[
\Rightarrow M = \frac{1}{8}
\]

Putting in (i) gives

\[
S = \frac{61}{32}
\]

\[
160S = 305
\]
\[ l = \int \frac{t}{1 + t^3} \, dt \]

\[ \frac{t}{1 + t^3} = \frac{A}{1 + t} + \frac{Bt + C}{1 + t^2 - t} \]

\[ t = A(1 - t + t^2) + (1 + t)(Bt + C) \]

By comparing coeff of \( x, x^2 \) and constant term,

\[ A = -\frac{1}{3}, \quad B = \frac{1}{3}, \quad C = \frac{1}{3} \]

\[ l = -\frac{1}{3} \int \frac{1}{1 + t} \, dt + \frac{1}{3} \int \frac{t + 1}{t^2 - t + 1} \, dt \]

\[ l = -\frac{1}{3} \ln(1 + t) + \frac{1}{6} \left[ \frac{2t^2 - 1}{t^2 - t + 1} \right] + C \]

\[ l = -\frac{1}{3} \ln(\sec x + \tan x) + \frac{1}{6} \tan^{-1}\left(\frac{2t - 1}{\sqrt{3}}\right) + C \]

\[ \alpha = -\frac{1}{3}, \quad \beta = \frac{1}{6}, \quad \gamma = \frac{1}{\sqrt{3}} \]

\[ 18(\alpha + \beta + \gamma^2) = 3 \]

7. Suppose the line \( \frac{x - 2}{\alpha} = \frac{y - 2}{-5} = \frac{z + 2}{2} \) lies on the plane \( x + 3y - 2z + \beta = 0 \). Then \((\alpha + \beta)\) is equal to _______.

Answer (7)

Sol. Line \( \frac{x - 2}{\alpha} = \frac{y - 2}{-5} = \frac{z + 2}{2} \) lies in \( x + 3y - 2z + \beta = 0 \)

\[ \alpha = 19 \]

Point \((2, 2, -2)\) will be on plane

\( (1)(2) + 3(2) - 2(-2) + \beta = 0 \)

\( \beta = -12 \)

\( \alpha + \beta = 19 - 12 = 7 \)

8. A tangent line \( L \) is drawn at the point \((2, -4)\) on the parabola \( y^2 = 8x \). If the line \( L \) is also tangent to the circle \( x^2 + y^2 = a \), then \( a \) is equal to _______.

Answer (2)

Sol. Equation of tangent \( y = 4(x + x_t) \)

\( y(-4) = 4(x + 2) \)

\[ -y = x + 2 \]

\( \Rightarrow x + y + z = 0 \)

Length of perpendicular from centre \((0, 0)\) should be equal to radius

\( \text{Length of perpendicular} = \frac{2}{\sqrt{2}} = \sqrt{a} \Rightarrow a = 2 \)

9. The number of elements is the set

\[ \{ A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}; \quad a, b, d \in \{-1, 0, 1\} \text{ and } (I - A)^3 = I - A^3 \} \]

where \( I \) is \( 2 \times 2 \) identity matrix, is _______.

Answer (8)

Sol. \( (I - A)^3 = I - A^3 \)

\[ I - A^3 - 3A(I - A) = I - A^3 \]

\[ 3A(I - A) = 0 \quad \Rightarrow A(I - A) = 0 \]

\[ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \begin{bmatrix} 1-a & -b \\ 0 & 1-d \end{bmatrix} = 0 \]

\[ \begin{bmatrix} a(1-a) & -b(a-1+d) \\ 0 & d(1-d) \end{bmatrix} = 0 \]

\[ a(1-a) = 0, \quad d(1-d) = 0, \quad b(a-1+d) = 0 \]

So Total 8 cases
10. If the line \( y = mx \) bisects the area enclosed by the lines \( x = 0, \ y = 0, \ x = \frac{3}{2} \) and the curve \( y = 1 + 4x - x^2 \), then 12 \( m \) is equal to ______. 

**Answer (26)**

**Sol.**

If \( y = mx \) passes through \( B \) then

\[
\text{Area of } OAB = \frac{1}{2} \cdot 1 \cdot \frac{19}{2} = \frac{57}{4}
\]

Which means for bisection it should pass through a point below \( B \),

Let it be \( P \), so for bisection \( \text{ar}(\triangle OAP) = \frac{39}{8} \cdot \frac{1}{2} \)

So

\[
\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdot m = \frac{39}{8} \cdot \frac{1}{2}
\]

\[
m = \frac{39 \cdot 4}{8 \cdot 9} = \frac{26}{12}
\]