Answers & Solutions for JEE (Main)-2022 (Online) Phase-2
(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and -1 mark for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and -1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Three masses \( M = 100 \text{ kg}, \ m_1 = 10 \text{ kg} \) and \( m_2 = 20 \text{ kg} \) are arranged in a system as shown in figure. All the surfaces are frictionless and strings are inextensible and weightless. The pulleys are also weightless and frictionless. A force \( F \) is applied on the system so that the mass \( m_2 \) moves upward with an acceleration of 2 \( \text{ms}^{-2} \). The value of \( F \) is (Take \( g = 10 \text{ms}^{-2} \))

   (A) 3360 N  \hspace{1cm} (B) 3380 N  \hspace{1cm} (C) 3120 N  \hspace{1cm} (D) 3240 N

Answer (C)

Sol. In frame of block of mass \( M \) moving with acceleration \( a \)

\[
\begin{align*}
 m_1 a & \leftrightarrow m_1 \\
\Rightarrow & \ 2 \text{m/s}^2
\end{align*}
\]

\[
\begin{align*}
 m_1 a - T = 2m_1 & \Rightarrow 10a - T = 20
\end{align*}
\]

\[
\begin{align*}
 m_2 & \uparrow 2 \text{m/s}^2
\end{align*}
\]

\[
\begin{align*}
 T - m_2 g & = m_2 2 \Rightarrow T - 200 = 40 \Rightarrow T = 240 \hspace{1cm} \text{(i)}
\end{align*}
\]

\[
\begin{align*}
 \Rightarrow & \text{ From equation 1 and } 2 10a = 260 \text{ or } a = 26 \text{ m/s}^2
\end{align*}
\]

\[
\begin{align*}
 F = (M + m_2)a = 120 \times 26
= 3120 \text{ N}
\end{align*}
\]

2. A radio can tune to any station in 6 MHz to 10 MHz band. The value of corresponding wavelength bandwidth will be

   (A) 4 m  \hspace{1cm} (B) 20 m  \hspace{1cm} (C) 30 m  \hspace{1cm} (D) 50 m

Answer (B)

Sol. \( \nu_1 = 6 \times 10^6 \text{ Hz} \)

\[
\begin{align*}
 \Rightarrow \lambda_1 = \frac{3 \times 10^8}{6 \times 10^6} = 50 \text{ m}
\end{align*}
\]

\[
\begin{align*}
 \nu_2 = 10 \times 10^6 \text{ Hz}
\end{align*}
\]

\[
\begin{align*}
 \Rightarrow \lambda_2 = \frac{3 \times 10^8}{10 \times 10^6} = 30 \text{ m}
\end{align*}
\]

\[
\begin{align*}
 \Rightarrow \text{ Wavelength band with}
= |\lambda_1 - \lambda_2| = 20 \text{ m}
\end{align*}
\]

3. The disintegration rate of a certain radioactive sample at any instant is 4250 disintegrations per minute. 10 minutes later, the rate becomes 2250 disintegrations per minute. The approximate decay constant is (Take \( \log_{10}1.88 = 0.274 \))

   (A) 0.02 \text{ min}^{-1} \hspace{1cm} (B) 2.7 \text{ min}^{-1} \hspace{1cm} (C) 0.063 \text{ min}^{-1} \hspace{1cm} (D) 6.3 \text{ min}^{-1}

Answer (C)

Sol. \( A_0 = 4250 \)

\[
\begin{align*}
 A = 2250 = A_0 e^{-\lambda t}
\end{align*}
\]

\[
\begin{align*}
 \Rightarrow \frac{2250}{4250} = e^{-\lambda t}
\end{align*}
\]

\[
\begin{align*}
 \Rightarrow \lambda(10) = \ln \left( \frac{4250}{2250} \right)
\end{align*}
\]

\[
\begin{align*}
 \lambda(10) = 0.636
\end{align*}
\]

\[
\begin{align*}
 \lambda = 0.063
\end{align*}
\]
4. A parallel beam of light of wavelength 900 nm and intensity 100 W/m² is incident on a surface perpendicular to the beam. The number of photons crossing 1 cm⁻² area perpendicular to the beam in one second is
(A) $3 \times 10^{16}$  (B) $4.5 \times 10^{16}$
(C) $4.5 \times 10^{17}$  (D) $4.5 \times 10^{20}$
Answer (B)

Sol. $\lambda = 900$ nm
$I = 100$ W/m²
$A = 10^{-4}$
$\Rightarrow P = 10^{-2}$ W
$\Rightarrow$ Number of photons incident per second
$\frac{10^{-2} \lambda}{hc} = \frac{9 \times 10^{-11} \times 10^2}{6.63 \times 10^{-34} \times 3 \times 10^8} = 4.5 \times 10^{16}$

5. In Young's double slit experiment, the fringe width is 12 mm. If the entire arrangement is placed in water of refractive index $\frac{4}{3}$, then the fringe width becomes (in mm)
(A) 16  (B) 9
(C) 48  (D) 12
Answer (B)

Sol. $B = 12 \times 10^{-3}$
$\beta' = \frac{\beta}{\mu} = \frac{12 \times 10^{-3}}{\frac{4}{3}}$
$= 9 \times 10^{-3}$ m = 9 mm

6. The magnetic field of a plane electromagnetic wave is given by
$B = 2 \times 10^{-8} \sin(0.5 \times 10^{11} x + 1.5 \times 10^{11} t) \hat{T}$.
The amplitude of the electric field would be
(A) 6 V/m⁻¹ along x-axis
(B) 3 V/m⁻¹ along z-axis
(C) 6 V/m⁻¹ along z-axis
(D) $2 \times 10^{-8}$ V/m⁻¹ along z-axis
Answer (C)

Sol. Speed of light $c = \frac{\omega}{k} = \frac{1.5 \times 10^{11}}{0.5 \times 10^{3}} = 3 \times 10^8$ m/sec
So, $E_0 = B_0 c$
$= 2 \times 10^{-8} \times 3 \times 10^8$
$= 6$ V/m
Direction will be along z-axis

7. In a series $LR$ circuit $X_L = R$ and power factor of the circuit is $P_1$. When capacitor with capacitance $C$ such that $X_L = X_C$ is put in series, the power factor becomes $P_2$. The ratio $P_1: P_2$ is
(A) $\frac{1}{2}$  (B) $\frac{1}{\sqrt{2}}$
(C) $\sqrt{3}$  (D) $2 : 1$
Answer (B)

Sol. $P_1 = \cos \phi = \frac{1}{\sqrt{2}} (X_L = R)$
$P_2 = \cos \phi' = 1$ (will become resonance circuit)
So, $P_1: P_2 = \frac{1}{\sqrt{2}}$

8. A charge particle is moving in a uniform field $\hat{E} = 2 \hat{i} + 3 \hat{j}$ T. If it has an acceleration of $(\alpha \hat{i} - 4 \hat{j})$ m/s², then the value of $\alpha$ will be
(A) 3  (B) 6
(C) 12  (D) 2
Answer (B)

As magnetic force is perpendicular to magnetic field
So, $\vec{F} \cdot \vec{B}$ must be 0
So, $2\alpha - 12 = 0$
$\alpha = 6$

9. $B_x$ and $B_Y$ are the magnetic field at the centre of two coils $X$ and $Y$ respectively each carrying equal current. If coil $X$ has 200 turns and 20 cm radius and coil $Y$ has 400 turns and 20 cm radius, the ratio of $B_x$ and $B_Y$ is
(A) 1 : 1  (B) 1 : 2
(C) 2 : 1  (D) 4 : 1
Answer (B)
12. When a particle executes Simple Harmonic Motion, the nature of graph of velocity as a function of displacement will be:

(A) Circular  (B) Elliptical 
(C) Sinusoidal  (D) Straight line

Answer (B)

Sol. Let $x = A \sin \omega t$

$\Rightarrow v = A \omega \cos \omega t$

$\Rightarrow v = \pm \omega \sqrt{A^2 - x^2}$

$\Rightarrow \frac{v^2}{\omega^2} + x^2 = A^2$

$\Rightarrow$ Ellipse

13. 7 mol of a certain monoatomic ideal gas undergoes a temperature increase of 40 K at constant pressure. The increase in the internal energy of the gas in this process is:

(Given $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$)

(A) 5810 J  (B) 3486 J

(C) 11620 J  (D) 6972 J

Answer (B)

Sol. $\Delta U = nC_v \Delta T$

$= 7 \times \frac{3R}{2} \times 40$

$= 3486 \text{ J}$

14. A monoatomic gas at pressure $P$ and volume $V$ is suddenly compressed to one eighth of its original volume. The final pressure at constant entropy will be:

(A) $P$  (B) $8P$

(C) $32P$  (D) $64P$

Answer (C)

Sol. $PV^\gamma = \text{constant}$

$\Rightarrow PV^\gamma = (P') \left(\frac{V}{8}\right) ^\gamma$ where $\gamma = 5/3$

$\Rightarrow P' = 32P$
15. A water drop of radius 1 cm is broken into 729 equal droplets. If surface tension of water is 75 dyne/cm, then the gain in surface energy upto first decimal place will be:

(Given \( \pi = 3.14 \))

(A) \( 8.5 \times 10^{-4} \) J  
(B) \( 8.2 \times 10^{-4} \) J  
(C) \( 7.5 \times 10^{-4} \) J  
(D) \( 5.3 \times 10^{-4} \) J

Answer (C)

Sol. \( 729 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3 \)
\[ \Rightarrow R = 9r \quad \text{(1)} \]
\[ \Delta U = S \times \Delta A \quad \text{(2)} \]
\[ \Rightarrow \Delta U = S \times \{ -4\pi R^2 + 729 \times 4\pi r^2 \} \]
\[ = S \times 4\pi \{ 729r^2 - 81r^2 \} \]
\[ = 7.5 \times 10^{-4} \text{ J} \]

16. The percentage decrease in the weight of a rocket, when taken to a height of 32 km above the surface of earth will be:

(Radius of earth = 6400 km)

(A) 1%  
(B) 3%  
(C) 4%  
(D) 0.5%

Answer (A)

Sol. \( g = \frac{GM}{r^2} \)
\[ \Rightarrow \frac{\Delta g}{g} = \frac{2 \Delta r}{r} \]
\[ \Rightarrow \frac{\Delta g}{g} \times 100 = 2 \times \frac{32}{6400} \times 100\% = 1\% \]
\[ \Rightarrow \% \text{ decrease in weight} = 1\% \]

17. As per the given figure, two blocks each of mass 250 g are connected to a spring of spring constant 2 Nm\(^{-1}\). If both are given velocity \( v \) in opposite directions, then maximum elongation of the spring is:

(A) \( \frac{v}{\sqrt{2}} \)  
(B) \( \frac{v}{2} \)  
(C) \( \frac{v}{4} \)  
(D) \( \frac{v}{\sqrt{2}} \)

Answer (B)

Sol. \( t_a = \frac{u \sin \theta}{g} \)
\[ \Rightarrow \frac{u_1 \sin(30^\circ)}{g} = \frac{u_2 \sin(45^\circ)}{g} \]
\[ \Rightarrow \frac{u_1}{u_2} = \frac{1}{2} = \frac{\sqrt{2}}{1} \]
20. A screw gauge of pitch 0.5 mm is used to measure the diameter of uniform wire of length 6.8 cm, the main scale reading is 1.5 mm and circular scale reading is 7. The calculated curved surface area of wire to appropriate significant figures is:

[Screw gauge has 50 divisions on its circular scale]

(A) 6.8 cm²  (B) 3.4 cm²  (C) 3.9 cm²  (D) 2.4 cm²

Answer (B)

Sol. Least count = \( \frac{0.5}{50} \) mm = 0.01 mm

∴ Diameter, \( d = 1.5 \text{ mm} + 7 \times 0.01 \)

∴ Surface area = \( (2\pi r) \times l \)

= \( \pi dl \)

= \( 3.142 \times \frac{1.57}{10} \times 6.8 \text{ cm}^2 \)

= 3.354 cm² = 3.4 cm²

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. If the initial velocity in horizontal direction of a projectile is unit vector \( \hat{i} \) and the equation of trajectory is \( y = 5x(1 - x) \). The y component vector of the initial velocity is \( \hat{j} \).

(Take \( g = 10 \text{ m/s}^2 \))

Answer (5)

Sol. \( y = 5x - 5x^2 \)

\( y = x \tan \theta - \frac{1}{2} \frac{gx^2}{v^2} \)

\( \tan \theta = 5 = \frac{u_y}{u_x} \)

\( \Rightarrow u_y = 5 \)

2. A disc of mass 1 kg and radius \( R \) is free to rotate about a horizontal axis passing through its centre and perpendicular to the plane of disc. A body of same mass as that of disc of fixed at the highest point of the disc. Now the system is released, when the body comes to the lowest position, it angular speed will be \( 4 \sqrt{\frac{x}{3R}} \text{ rad s}^{-1} \) where \( x = \ldots \).

(\( g = 10 \text{ ms}^{-2} \))

Answer (5)

Sol.

Loss in P.E = Gain in K.E.

\[ 2mgR = \frac{1}{2} mR^2 + mR^2 w^2 \]

\[ 2mg R = \frac{1}{2} \times \frac{3}{2} mR^2 w^2 \]

\[ w^2 = \frac{8g}{3R} \]

\[ w = \sqrt{\frac{8g}{3R}} = 4\sqrt{\frac{g}{2 \times 3R}} \]

\[ \Rightarrow x = \frac{g}{2} = 5 \]

3. In an experiment of determine the Young’s modulus of wire of a length exactly 1 m, the extension in the length of the wire is measured as 0.4 mm with an uncertainty of \( \pm 0.02 \text{ mm} \) when a load of 1 kg is applied. The diameter of the wire is measured as 0.4 mm with an uncertainty of \( \pm 0.02 \text{ mm} \) when a load of 1 kg is applied. The diameter of the wire is measured as 0.4 mm with an uncertainty of \( \pm 0.01 \text{ mm} \). The error in the measurement of Young’s modulus (\( \Delta Y \)) is found to be \( x \times 10^{10} \text{ Nm}^{-2} \). The value of \( x \) is \( \ldots \).

(Take \( g = 10 \text{ ms}^{-2} \))

Answer (2)
5. A composite parallel plate capacitor is made up of two different dielectric materials with different thickness (t1 and t2) as shown in figure. The two different dielectric materials are separated by a conducting foil F. The voltage of the conducting foil is ___ V.

![Image](image_url)

Answer (60)

**Sol.**

\[
\frac{C_1}{C_2} = \frac{t_1}{t_2} = \frac{3}{2}
\]

6. Resistances are connected in a meter bridge circuit as shown in the figure. The balancing length \( l_1 \) is 40 cm. Now an unknown resistance \( x \) is connected in series with \( P \) and new balancing length is found to be 80 cm measured from the same end. Then the value of \( x \) will be _________ \( \Omega \).

![Image](image_url)

Answer (20)
7. The effective current \( I \) in the given circuit at very high frequencies will be ______ A.

\[
\begin{aligned}
\frac{P}{40} &= \frac{Q}{60} \quad \ldots(1) \\
\frac{P + x}{80} &= \frac{Q}{20} \quad \ldots(2)
\end{aligned}
\]

\[
\begin{aligned}
P &= 20 \\
\frac{P}{P + x} &= \frac{80}{40} \times \frac{20}{60}
\end{aligned}
\]

\[
\frac{4}{4 + x} = \frac{1}{3}
\]

\[
x = 20
\]

8. The graph between \( \frac{1}{u} \) and \( \frac{1}{v} \) for a thin convex lens in order to determine its focal length is plotted as shown in the figure. The refractive index of lens is 1.5 and its both the surfaces have same radius of curvature \( R \). The value of \( R \) will be ______ cm.

\[
\begin{aligned}
\frac{1}{\lambda} &= \frac{1}{\lambda_1} \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_3} \right) \\
\frac{1}{\lambda_3} &= \frac{1}{\lambda_2} \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)
\end{aligned}
\]

\[
\therefore \lambda_3 - \lambda_2 = a\lambda
\]

\[
a = 5
\]

9. In the hydrogen spectrum, \( \lambda \) be the wavelength of first transition line of Lyman series. The wavelength difference will be “\( a\lambda \)” between the wavelength of 3rd transition line of Paschen series and that of 2nd transition line of Balmer series where \( a = _____ \).

\[
\begin{aligned}
\frac{1}{\lambda_1} &= R_H \left( \frac{1}{\lambda_2^2} - \frac{1}{\lambda_3^2} \right) \\
\frac{1}{\lambda_3^2} &= \frac{1}{\lambda_2^2} \left( \frac{1}{\lambda_1^2} - \frac{1}{\lambda_2^2} \right)
\end{aligned}
\]

10. In the circuit shown below, maximum Zener diode current will be ______ mA.

\[
\begin{aligned}
i_z &= \frac{60}{4 \times 10^{-3}} = 15 \times 10^{-3} = 15 \text{ mA} \\
i_L &= \frac{60}{10 \times 10^{-3}} = 6 \text{ mA}
\end{aligned}
\]

\[
l_z = i_z - i_L = 9 \text{ mA}
\]
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Match List-I with List-II.

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) BrF$_5$</td>
<td>(I) bent</td>
</tr>
<tr>
<td>(B) [CrF$_6$]$^{3-}$</td>
<td>(II) square pyramidal</td>
</tr>
<tr>
<td>(C) O$_3$</td>
<td>(III) trigonal bipyramidal</td>
</tr>
<tr>
<td>(D) PCl$_5$</td>
<td>(IV) octahedral</td>
</tr>
</tbody>
</table>

Choose the correct answer from the options given below:

(A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(B) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
(C) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)
(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Answer (B)

Sol. (A) 2SO$_2$(g) + O$_2$(g) $\xrightarrow{V_2O_5}$ 2SO$_3$
(B) 4NH$_3$(g) + 5O$_2$(g) $\xrightarrow{Pt(s)-Rh(s)}$ 4NO(g) + 6H$_2$O(g)
(C) N$_2$(g) + 3H$_2$(g) $\xrightarrow{Fe(s)}$ 2NH$_3$(g)
(D) Vegetable oil(l) + H$_2$ $\xrightarrow{Ni(s)}$ Vegetable ghee(s)

2. Given two statements below:

Statement I: In Cl$_2$ molecule the covalent radius is double of the atomic radius of chlorine.

Statement II: Radius of anionic species is always greater than their parent atomic radius.

Choose the most appropriate answer from options given below:

(A) Both Statement I and Statement II are correct.
(B) Both Statement I and Statement II are incorrect.
(C) Statement I is correct but Statement II is incorrect.
(D) Statement I is incorrect but Statement II is correct.

Answer (C)

Sol. • Covalent radius is not double of atomic radius.
• Radius of anionic species is always greater than their parent atomic radius as nuclear charge decreases in anionic counterpart.
4. Refining using liquation method is the most suitable for metals with:
   (A) Low melting point
   (B) High boiling point
   (C) High electrical conductivity
   (D) Less tendency to be soluble in melts than impurities

Answer (A)

Sol. Refining using liquation method is the most suitable for metals with low melting point.

5. Which of the following can be used to prevent the decomposition of H$_2$O$_2$?
   (A) Urea
   (B) Formaldehyde
   (C) Formic acid
   (D) Ethanol

Answer (A)

Sol. Urea is used as a stabilizer for the storage of H$_2$O$_2$.

6. Reaction of BeCl$_2$ with LiAlH$_4$ gives :
   (A) AlCl$_3$
   (B) BeH$_2$
   (C) LiH
   (D) LiCl
   (E) BeAlH$_4$

Choose the correct answer from options given below :
   (A) (A), (D) and (E)
   (B) (A), (B) and (D)
   (C) (D) and (E)
   (D) (B), (C) and (D)

Answer (B)

Sol. 2BeCl$_2$ + LiAlH$_4$ → 2BeH$_2$ + LiCl + AlCl$_3$

7. Borazine, also known as inorganic benzene, can be prepared by the reaction of 3-equivalents of “X” with 6-equivalents of “Y”. “X” and “Y”, respectively are:
   (A) B(OH)$_3$ and NH$_3$
   (B) B$_2$H$_6$ and NH$_3$
   (C) B$_2$H$_6$ and HN$_3$
   (D) NH$_3$ and B$_2$O$_3$

Answer (B)

Sol. 3B$_2$H$_6$ + 6NH$_3$ → 2B$_3$N$_3$H$_6$

8. Which of the given reactions is not an example of disproportionation reaction?
   (A) 2H$_2$O$_2$ → 2H$_2$O + O$_2$
   (B) 2NO + H$_2$O → HNO$_3$ + HNO$_2$
   (C) MnO$_4^-$ + 4H$^+$ + 3e$^-$ → MnO$_2$ + 2H$_2$O
   (D) 3MnO$_4^{2-}$ + 4H$^+$ → 2MnO$_4^-$ + MnO$_2$ + 2H$_2$O

Answer (C)

Sol. MnO$_4^-$ + 4H$^+$ + 3e$^-$ → MnO$_2$ + 2H$_2$O

The above reaction involves the reduction of MnO$_4^-$ to MnO$_2$.

9. The dark purple colour of KMnO$_4$ disappears in the titration with oxalic acid in acidic medium. The overall change in the oxidation number of manganese in the reaction is :
   (A) 5
   (B) 1
   (C) 7
   (D) 2

Answer (A)

Sol. 2KMnO$_4$ + 5H$_2$C$_2$O$_4$ + 3H$_2$SO$_4$ →

K$_2$SO$_4$ + 2MnSO$_4$ + 10CO$_2$ + 8H$_2$O

Change is oxidation state Mn is 5.

10. Cl + CH$_4$ → A + B

A and B in the above atmospheric reaction step are:
   (A) C$_2$H$_6$ and Cl$_2$
   (B) CHCl$_2$ and H$_2$
   (C) CH$_3$ and HCl
   (D) C$_2$H$_6$ and HCl

Answer (C)

Sol. Cl + CH$_4$ → CH$_3$ + HCl

11. Which technique among the following, is most appropriate in separation of a mixture of 100 mg of p-nitrophenol and picric acid?
   (A) Steam distillation
   (B) 2-5 ft long column of silica gel
   (C) Sublimation
   (D) Preparative TLC (Thin Layer Chromatography)

Answer (D)
Sol. Thin layer chromatography is a technique used to isolate non-volatile mixtures.

Hence, mixture of p-nitrophenol and Picric acid is separated by TLC.

12. The difference in the reaction of phenol with bromine in chloroform and bromine in water medium is due to:
   (A) Hyperconjugation in substrate
   (B) Polarity of solvent
   (C) Free radical formation
   (D) Electromeric effect of substrate

Answer (B)

Sol. Phenol gives different products with bromine in chloroform and water medium due to the polarity difference between chloroform and water acting as solvent.

13. Which of the following compounds is not aromatic?
   (A) ![Aromatic Ring](image)
   (B) ![Aromatic Ring](image)
   (C) ![Non-Aromatic](image)
   (D) ![Aromatic Ring](image)

Answer (C)

Sol. ![Non-Aromatic Reason](image)

hence it is not aromatic.

14. The products formed in the following reaction, A and B are

![Reaction](image)

Answer (C)

15. Which reactant will give the following alcohol on reaction with one mole of phenyl magnesium bromide (PhMgBr) followed by acidic hydrolysis?

![Reaction](image)

(A) CH₃ – C ≡ N
   (B) Ph – C ≡ N
   (C) CH₃ – C = O – Ph
   (D) Ph – C – CH₃

Answer (D)

Sol. Ph – C – CH₃
   (i) PhMgBr
   (ii) H₂O
   Ph – C – OH
16. The major product of the following reaction is

\[ \text{O}_2\text{N} - \text{OCH}_3 \xrightarrow{(i) \ Na/\text{liq.NH}_3} \ \text{NaOCH}_3 \] 
\[ \xrightarrow{(ii) \ CH}_3\text{CH}_2\text{OH} \] 

(A) (B) (C) (D)

Answer (A)

**Sol.**

When \( G = \text{EDG} \)       
When \( G = \text{EWG} \)

\[ \text{EDG} \rightarrow \text{Electron donating group} \] 
\[ \text{EWG} \rightarrow \text{Electron withdrawing group} \]

17. The correct stability order of the following diazonium salt is

\[ \text{(A)} \quad (\text{B}) \quad (\text{C}) \quad (\text{D}) \]

(A) (A) > (B) > (C) > (D)  
(B) (A) > (C) > (D) > (B)  
(C) (C) > (A) > (D) > (B)  
(D) (C) > (D) > (B) > (A)

Answer (B)

**Sol.** Diazonium salt containing aryl group directly linked to electron donating group is most stable due to resonance. The +M effect stabilizes the intermediate whereas Electron withdrawing group on benzene destabilizes the intermediate at para position.

[Diagram showing the diazonium salt and resonance stabilization]

Order will be A > C > D > B.

18. Stearic acid and polyethylene glycol react to form which one of the following soap/s detergents?

(A) Cationic detergent  
(B) Soap  
(C) Anionic detergent  
(D) Non-ionic detergent

Answer (D)

**Sol.**

\[ \text{CH}_3(\text{CH}_2)_10\text{COOH} + \text{HO(\text{CH}_2\text{CH}_2\text{O})}_n\text{CH}_2\text{CH}_3\text{OH} \]

\[ \xrightarrow{-\text{H}_2\text{O}} \]

\[ \text{CH}_3(\text{CH}_2)_10\text{COO(\text{CH}_2\text{CH}_2\text{O})}_n\text{CH}_2\text{CH}_3\text{OH} \]

The product do not contain any ion in their constitution hence it is a non-ionic detergent.

19. Which one of the following is a reducing sugar?

(A) (B)

Answer (B)
Answer (A)

Sol.

The sugar gives +ve Tollens’s test hence it’s a reducing sugar.

20. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Experimental reaction of CH$_3$Cl with aniline and anhydrous AlCl$_3$ does not give o and p-methylaniline.

Reason (R) : The –NH$_2$ group of aniline becomes deactivating because of salt formation with anhydrous AlCl$_3$ and hence yields m-methyl aniline.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

Answer (C)

Sol.

Aniline does not undergo Friedel Craft reaction because the reagent AlCl$_3$ being electron deficient acts as a Lewis acid.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Chlorophyll extracted from the crushed green leaves was dissolved in water to make 2 L solution of Mg of concentration 48 ppm. The number of atoms of Mg in this solution is x $\times$ 10$^{20}$ atoms. The value of x is ________. (Nearest integer)

(Given : Atomic mass of Mg is 24 g mol$^{-1}$; $N_A = 6.02 \times 10^{23}$ mol$^{-1}$)

Answer (24)

Sol. In 2L $\rightarrow$ 96 mg of Mg

Number of atoms of Mg = $\frac{96 \times 10^{-3}}{24} \times N_A$

= $4 \times 10^{-3} \times 6 \times 10^{23}$

= $24 \times 10^{20}$

x = 24

2. A mixture of hydrogen and oxygen contains 40% hydrogen by mass when the pressure is 2.2 bar. The partial pressure of hydrogen is _______ bar. (Nearest integer)

Answer (2)
Sol. 40% w/w hydrogen gas is given in mixture of H₂ and oxygen.

Wt. of H₂ = 40 g
Wt. of O₂ = 60 g

\[ \chi_{H_2} = \frac{n_{H_2}}{n_{H_2} + n_{O_2}} \]

\[ = \frac{40}{40 + 60} = \frac{2}{2 + \frac{32}{2}} = \frac{20}{20 + 1.875} = \frac{20}{21.875} = 0.914 \]

\[ P_{H_2} = \chi_{H_2} \times P_T \]

\[ = 0.914 \times 2.2 \]

\[ = 2.01 = 2 \text{ bar} \]

3. The wavelength of an electron and a neutron will become equal when the velocity of the electron is x times the velocity of neutron. The value of x is ______. (Nearest integer)

(Mass of electron is $9.1 \times 10^{-31}$ kg and mass of neutron is $1.6 \times 10^{-27}$ kg)

Answer (1758)

Sol. \[ \lambda_e = \frac{h}{m_e \times V_e}, \quad \lambda_N = \frac{h}{m_N \times V_N} \]

\[ \frac{\lambda_e}{\lambda_N} = \frac{m_N \times V_N}{m_e \times V_e} \]

\[ = \frac{1}{m_e} = \frac{1}{m_N} \times \frac{V_e}{V_N} \]

\[ m_N = \frac{V_e}{V_N} = x \]

\[ x = \frac{1.6 \times 10^{-27}}{9.1 \times 10^{-31}} \]

\[ = 0.17582 \times 10^4 \]

\[ = 1758 \]

4. 2.4 g coal is burnt in a bomb calorimeter in excess of oxygen at 298 K and 1 atm pressure. The temperature of the calorimeter rises from 298 K to 300 K. The enthalpy change during the combustion of coal is $-x$ kJ mol⁻¹. The value of x is ______. (Nearest integer)

(Given : Heat capacity of bomb calorimeter is 20.0 kJ K⁻¹. Assume coal to be pure carbon)

Answer (200)

Sol. \[ Q (\text{Heat evolved}) = -\frac{C_{\text{system}} \Delta T}{n} \]

\[ n_{\text{coal}} = \frac{2.4}{12} \]

\[ Q = -\frac{20(300-298)}{0.2} \]

\[ Q = -200 \text{ kJ / mol} \]

x = 200

5. When 800 mL of 0.5 M nitric acid is heated in a beaker, its volume is reduced to half and 11.5 g of nitric acid is evaporated. The molarity of the remaining nitric acid solution is $x \times 10^{-2}$ M. (Nearest integer)

(Molar mass of nitric acid is 63 g mol⁻¹)

Answer (54)

Sol. moles of HNO₃ = 800 × 0.5

Moles of HNO₃ = 400 × 10⁻³

= 0.4 moles

Weight of HNO₃ = 0.4 × 63 g

= 25.2 g

Remaining acid = 25.2 – 11.5

= 13.7 g

\[ M = \frac{13.7 \times 1000}{400 \times 63} = \frac{137}{252} = 0.54 \]

\[ = 54 \times 10^{-2} \]
6. At 298 K, the equilibrium constant is \(2 \times 10^{15}\) for the reaction:

\[
\text{Cu(s) + 2Ag}^+ (aq) \rightleftharpoons \text{Cu}^{2+}(aq) + 2\text{Ag(s)}
\]

The equilibrium constant for the reaction

\[
\frac{1}{2}\text{Cu}^{2+}(aq) + \text{Ag(s)} \rightleftharpoons \frac{1}{2}\text{Cu(s)} + \text{Ag}^+(aq)
\]

is \(x \times 10^{-8}\). The value of \(x\) is ________ (Nearest integer)

Answer (2)

Sol. Cu(s) + 2Ag^+(aq) \rightleftharpoons Cu^{2+}(aq) + 2Ag(s)

\[
k = 2 \times 10^{15}
\]

\[
\frac{1}{2}\text{Cu(s)} + \text{Ag}^+(aq) \rightleftharpoons \frac{1}{2}\text{Cu}^{2+}(aq) + \text{Ag(s)}
\]

\[
K' = \frac{1}{(K)^{\frac{1}{2}}} = \frac{1}{\left(2 \times 10^{15}\right)^{\frac{1}{2}}}
\]

\[
= 2.23 \times 10^{-8}
\]

\(x = 2\)

7. The amount of charge in F(Faraday) required to obtain one mole of iron from \(\text{Fe}_3\text{O}_4\) is _________. (Nearest integer)

Answer (3)

Sol. For \(\text{Fe}_3\text{O}_4\),

\[
x = \frac{+8}{3}
\]

where \(x\) is oxidation state of Fe.

\[
\text{Fe}_3\text{O}_4 + 8\text{H}^+ + 8\text{e}^- \rightarrow 3\text{Fe} + 4\text{H}_2\text{O}
\]

Charge required = \(\frac{8}{3} \times F = \frac{8F}{3} = 3F\)

8. For a reaction \(A \rightarrow 2B + C\) the half lives are 100 s and 50 s when the concentration of reactant A is 0.5 and 1.0 mol L\(^{-1}\) respectively. The order of the reaction is ________. (Nearest integer)

Answer (2)

Sol. \(t_{\frac{1}{2}} \propto \frac{1}{(a_0)^{n-1}}\)

\[
t_{\frac{1}{2}} = 100 \text{ sec} \quad a_0 = 0.5
\]

\[
t_{\frac{1}{2}} = 50 \text{ sec} \quad a_0 = 1
\]

\[
\frac{100}{50} = \left(\frac{1}{0.5}\right)^{n-1}
\]

\((2) = (2)^{n-1}\)

\(n - 1 = 1\)

\(n = 2\)

9. The difference between spin only magnetic moment value of \([\text{Co(H}_2\text{O})_6]\)\(\text{Cl}_2\) and \([\text{Cr(H}_2\text{O})_6]\)\(\text{Cl}_3\) is _________.

Answer (0)

Sol. \( \text{Co} \rightarrow 4s^2 3d^6 \)

\(\text{H}_2\text{O}\) is weak field ligand.

\(\text{Co}^{2+} \rightarrow 3d^7\)

\[
n = 3 \quad \mu_1 = \sqrt{n(n+2)}
\]

\[
= \sqrt{15} \quad \text{B.M.}
\]

\(\text{Cr} \rightarrow 4s^1 3d^5\)

\(\text{Co}^{3+} \rightarrow 3d^6\)

\[
n = 3 \quad \mu_2 = \sqrt{15} \quad \text{B.M.}
\]

\[
\mu_1 - \mu_2 = 0
\]

10. In the presence of sunlight, benzene reacts with \(\text{Cl}_2\) to give product X. The number of hydrogens in X is _________.

Answer (6)

Sol. \[
\text{Benzene} + 3\text{Cl}_2 \xrightarrow{\text{Sunlight}} \text{Product X}
\]

Total number of hydrogens are 6.
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Let \( f: \mathbb{R} \to \mathbb{R} \) be a continuous function such that \( f(3x) - f(x) = x \). If \( f(8) = 7 \), then \( f(14) \) is equal to

(A) 4  (B) 10  (C) 11  (D) 16

Answer (B)

Sol. \( f(3x) - f(x) = x \) \(...(1)\)

\[ x \to \frac{x}{3} \]

\[ f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3} \] \(...(2)\)

Again \( x \to \frac{x}{3} \)

\[ f\left(\frac{x}{3}\right) - f\left(\frac{x}{9}\right) = \frac{x}{3^2} \] \(...(3)\)

Similarly

\[ f\left(\frac{x}{3^{n-2}}\right) - f\left(\frac{x}{3^{n-1}}\right) = \frac{x}{3^{n-1}} \] \(...(n)\)

Adding all these and applying \( n \to \infty \)

\[ \lim_{n \to \infty} \left( f(3x) - f\left(\frac{x}{3^{n-1}}\right) \right) = x \left(1 + \frac{1}{3} + \frac{1}{3^2} + \ldots \right) \]

\[ f(3x) - f(0) = \frac{3x}{2} \]

Putting \( x = \frac{8}{3} \)

\[ f(8) - f(0) = 4 \]

\[ \Rightarrow f(0) = 3 \]

Putting \( x = \frac{14}{3} \)

\[ f(14) - 3 = 7 \Rightarrow f(14) = 0 \]

2. Let \( O \) be the origin and \( A \) be the point \( z_1 = 1 + 2i \). If \( B \) is the point \( z_2 \), \( \text{Re}(z_2) < 0 \), such that \( OAB \) is a right angled isosceles triangle with \( OB \) as hypotenuse, then which of the following is NOT true?

(A) \( \arg z_2 = \pi - \tan^{-1}3 \)

(B) \( \arg (z_1 - 2z_2) = -\tan^{-1}\frac{4}{3} \)

(C) \( |z_2| = \sqrt{10} \)

(D) \( |2z_1 - z_2| = 5 \)

Answer (D)

Sol. \( B(z_2) \)

\[ \begin{align*}
\frac{z_2 - 0}{(1 + 2i) - 0} &= \frac{|OB|}{\sqrt{10}} e^{\frac{i\pi}{4}} \\
\Rightarrow \frac{z_2}{1 + 2i} &= \sqrt{2} e^{\frac{i\pi}{4}} \\
\end{align*} \]

OR \( z_2 = (1 + 2i)(1 + i) \)

\[ = -1 + 3i \]

\( \arg z_2 = \pi - \tan^{-1}3 \)

\( |z_2| = \sqrt{10} \)

\( z_1 - 2z_2 = (1 + 2i) + 2 - 6i = 3 - 4i \)

\( \arg(z_1 - 2z_2) = -\tan^{-1}\frac{4}{3} \)

\( |2z_1 - z_2| = |2 + 4i + 1 - 3i| = |3 + i| \)

\[ = \sqrt{10} \]
3. If the system of linear equations.
\[
\begin{align*}
8x + y + 4z &= -2 \\
x + y + z &= 0 \\
\lambda x - 3y &= \mu
\end{align*}
\]
has infinitely many solutions, then the distance of the point \(\left(\lambda, \mu, \frac{1}{2}\right)\) from the plane \(8x + y + 4z + 2 = 0\) is
\[
(A) \quad 3\sqrt{5} \\
(B) \quad 4 \\
(C) \quad \frac{26}{9} \\
(D) \quad \frac{10}{3}
\]
Answer (D)

Sol. \(\Delta = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix} = 8(3) - 1(-\lambda) + 4(-3 - \lambda) = 24 + \lambda - 12 - 4\lambda = 12 - 3\lambda.
\]
So for \(\lambda = 4\), it is having infinitely many solutions.
\[
\Delta_x = \begin{vmatrix} -2 & 1 & 4 \\ 0 & 1 & 1 \\ \mu & -3 & 0 \end{vmatrix} = -2(3) - 1(-\mu) + 4(-\mu) = -6 - 3\mu = 0.
\]
For \(\mu = -2\)

Distance of \((4, -2, \frac{1}{2})\) from \(8x + y + 4z + 2 = 0\)
\[
= \frac{32 - 2 - 2 + 2}{\sqrt{64 + 16}} = \frac{10}{\sqrt{80}} = \frac{5}{4} \text{ units}
\]

4. Let \(A\) be a \(2 \times 2\) matrix with \(\text{det}(A) = -1\) and \(\text{det}((A + I) (\text{Adj} (A) + I)) = 4\). Then the sum of the diagonal elements of \(A\) can be
\[
(A) \quad -1 \\
(B) \quad 2 \\
(C) \quad 1 \\
(D) \quad -\sqrt{2}
\]
Answer (B)

Sol. \(\text{det}((A + I)(\text{Adj} (A) + I)) = 4\)
\[
\Rightarrow |A + I||\text{Adj} (A) + I| = 4
\]
\[
\Rightarrow |A + I|^2 = 4
\]
\[
\Rightarrow |A + I| = 2 \\
\Rightarrow |A + \text{Adj} (A)| = 4
\]

5. The odd natural number \(a\), such that the area of the region bounded by \(y = 1, y = 3, x = 0, x = y^a\) is \(\frac{364}{3}\), is equal to
\[
(A) \quad 3 \\
(B) \quad 5 \\
(C) \quad 7 \\
(D) \quad 9
\]
Answer (B)

Sol. \(a\) is an odd natural number and
\[
\left[ y^a dy \right]_1^3 = \frac{364}{3}
\]
\[
\Rightarrow \left[ \frac{1}{a+1}(y^{a+1}) \right]_1^3 = \frac{364}{3}
\]
\[
\Rightarrow \frac{3^{a+1} - 1}{a+1} = \frac{364}{3}
\]
Solving with \((-)\) sign,
\[
\frac{3^{a+1} - 1}{a+1} = -\frac{364}{3}, \text{No } a \text{ exist}
\]
\[
\therefore (a = 5)
\]
Solving with \((+)\) sign,
\[
\frac{3^{a+1} - 1}{a+1} = \frac{364}{3}, \text{No } a \text{ exist}
\]
\[
\therefore (a = 5)
\]

6. Consider two G.Ps. \(2, 2^2, 2^3, \ldots\) and \(4, 4^2, 4^3, \ldots\) of 60 and \(n\) terms respectively. If the geometric mean of all the 60 + \(n\) terms is \(\frac{225}{8}\), then \(\sum_{k=1}^{n} k(n-k)\) is equal to
\[
(A) \quad 560 \\
(B) \quad 1540 \\
(C) \quad 1330 \\
(D) \quad 2600
\]
Answer (C)

Sol. Given G.P’s \(2, 2^2, 2^3, \ldots 60 \text{ terms}\)
\[
4, 4^2, \ldots n \text{ terms}
\]
\[
\text{G.M} = 2^{\frac{225}{8}}
\]
Now, \(\text{G.M} = 2^{\frac{225}{8}}\)
\[
\left(2^2 \cdot 4^2 \cdot \ldots \cdot n^2\right)^{\frac{1}{60+n}} = 2^{\frac{225}{8}}
\]
If the function 
\[ f(x) = \begin{cases} \frac{\log_e(1-x+x^2) + \log_e(1+x+x^2)}{\sec x - \cos x}, & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \setminus \{0\} \\ k, & x = 0 \end{cases} \]
is continuous at \( x = 0 \), then \( k \) is equal to

(A) 1
(B) -1
(C) e
(D) 0

**Answer (A)**

**Sol.** 
\[ f(x) = \begin{cases} \frac{\log_e(1-x+x^2) + \log_e(1+x+x^2)}{\sec x - \cos x}, & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \setminus \{0\} \\ k, & x = 0 \end{cases} \]

for continuity at \( x = 0 \)

\[ \lim_{x \to 0} f(x) = k \]

\[ k = \lim_{x \to 0} \frac{\log_e(1-x+x^2) + \log_e(1+x+x^2)}{\sec x - \cos x} \]

\[ = \lim_{x \to 0} \frac{\cos x \log_e(x^4 + x^2 + 1)}{\sin^2 x} \]

\[ = \lim_{x \to 0} \frac{\log_e(x^4 + x^2 + 1)}{x^2} \]

\[ = \lim_{x \to 0} \frac{\ln (1+x^2 + x^4)}{x^2 + x^4} \cdot \frac{x^2 + x^4}{x^2} \]

\[ = 1 \]

8. If 

\[ f(x) = \begin{cases} x + a, & x \leq 0 \\ |x - 4|, & x > 0 \end{cases} \]

and 

\[ g(x) = \begin{cases} x + 1, & x < 0 \\ (x - 4)^2 + b, & x \geq 0 \end{cases} \]

are continuous on \( R \), then \( (gof)(2) + (fog)(-2) \) is equal to

(A) -10
(B) 10
(C) 8
(D) -8

**Answer (D)**

**Sol.** 
\[ f(x) = \begin{cases} x + a, & x \leq 0 \\ |x - 4|, & x > 0 \end{cases} \]

and 

\[ g(x) = \begin{cases} x + 1, & x < 0 \\ (x - 4)^2 + b, & x \geq 0 \end{cases} \]

\[ \because \ f(x) \text{ and } g(x) \text{ are continuous on } R \]

\[ \therefore \ (gof)(2) + (fog)(-2) \]

\( a = 4 \) and \( b = 1 - 16 = -15 \)

\[ = g(2) + f(-1) \]

\[ = -11 + 3 = -8 \]

9. Let 

\[ f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \leq 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases} \]

Then the set of all values of \( b, \) for which \( f(x) \) has maximum value at \( x = 1, \) is

(A) \((-6, -2)\)
(B) \((2, 6)\)
(C) \([-6, -2) \cup (2, 6]\)
(D) \([-\sqrt{6}, -2) \cup (2, \sqrt{6}]\)

**Answer (C)**

**Sol.** 
\[ f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \leq 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases} \]

**If \( f(x) \) has maximum value at \( x = 1 \) then \( f(1^+) \leq f(1) \)

\[ -2 + \log_2(b^2 - 4) \leq 1 - 1 + 10 - 7 \]

\[ \log_2(b^2 - 4) \leq 5 \]

\[ 0 < b^2 - 4 \leq 32 \]

(i) \( b^2 - 4 > 0 \Rightarrow b \in (-\infty, -2) \cup (2, \infty) \)

(ii) \( b^2 - 36 \leq 0 \Rightarrow b \in [-6, 6] \)

Intersection of above two sets

\( b \in [-6, -2) \cup (2, 6] \)
10. If \( a = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{2n}{n^2 + k^2} \) and 
\[
f(x) = \sqrt{\frac{1 - \cos x}{1 + \cos x}}, \quad x \in (0, 1),
\]
then 
\[
(A) \quad 2 \sqrt{2} f\left(\frac{a}{2}\right) = f\left(\frac{a}{2}\right) \\
(B) \quad f\left(\frac{a}{2}\right) = \sqrt{2}
\]
\[
(C) \quad \sqrt{2} f\left(\frac{a}{2}\right) = f\left(\frac{a}{2}\right) \\
(D) \quad f\left(\frac{a}{2}\right) = \sqrt{2} f\left(\frac{a}{2}\right)
\]
Answer (C)

Solution of differential equation can be written as 
\[
y \cdot \sec^2 x = \int \sin x \cdot \sec^2 x \, dx = \int \sec x \cdot \tan x \, dx
\]
\[
y \sec^2 x = \sec x + C
\]
\[
y\left(\frac{\pi}{3}\right) = 0, \quad 0 = \sec \frac{\pi}{3} + C \Rightarrow \quad C = -2
\]
\[
y = \frac{\sec x - 2}{\sec^2 x} = \cos x - 2 \cos^2 x
\]
\[
= \frac{1}{8} - 2 \left(\cos x - \frac{1}{4}\right)^2
\]
\[
y_{max} = \frac{1}{8}
\]

11. If 
\[
\frac{dy}{dx} + 2y \tan x = \sin x, \quad 0 < x < \frac{\pi}{2}
\]
and \( y\left(\frac{\pi}{3}\right) = 0 \),
then the maximum value of \( y(x) \) is:

\[
(A) \quad \frac{1}{8} \\
(B) \quad \frac{3}{4} \\
(C) \quad \frac{1}{4} \\
(D) \quad \frac{3}{8}
\]
Answer (A)

12. A point \( P \) moves so that the sum of squares of its distances from the points \((1, 2)\) and \((-2, 1)\) is 14. Let \( f(x, y) = 0 \) be the locus of \( P \), which intersects the \( x \)-axis at the points \( A, B \) and the \( y \)-axis at the points \( C, D \). Then the area of the quadrilateral \( ACBD \) is equal to

\[
(A) \quad \frac{9}{2} \\
(B) \quad \frac{3\sqrt{17}}{2} \\
(C) \quad \frac{3\sqrt{17}}{4} \\
(D) \quad 9
\]
Answer (B)

Sol. Let point \( P : (h, k) \)
\[
(h - 1)^2 + (k - 2)^2 + (h + 2)^2 + (k - 1)^2 = 14
\]
\[
2h^2 + 2k^2 + 2h - 6k - 4 = 0
\]
Locus of \( P : x^2 + y^2 + x - 3y - 2 = 0 \)
Intersection with \( x \)-axis,
\[
x^2 + x - 2 = 0
\]
\[
\Rightarrow \quad x = -2, \quad 1
\]
Intersection with \( y \)-axis,
\[
y^2 - 3y - 2 = 0
\]
\[
\Rightarrow \quad y = \frac{3 \pm \sqrt{17}}{2}
\]
Area of the quadrilateral \( ACBD \) is
\[
= \frac{1}{2} \left(|x_1| + |x_2|)(|y_1| + |y_2|)
\]
\[
= \frac{1}{2} \times 3 \times \sqrt{17} = \frac{3\sqrt{17}}{2}
\]
13. Let the tangent drawn to the parabola \( y^2 = 24x \) at the point \((\alpha, \beta)\) is perpendicular to the line \(2x + 2y = 5\). Then the normal to the hyperbola \( \frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1 \) at the point \((\alpha + 4, \beta + 4)\) does NOT pass through the point
(A) \((25, 10)\)  
(B) \((20, 12)\)  
(C) \((30, 8)\)  
(D) \((15, 13)\)

Answer (D)

Sol. Any tangent to \( y^2 = 24x \) at \((\alpha, \beta)\) \(\beta y = 12(x + \alpha)\)
Slope \(\frac{\beta}{12}\) and perpendicular to \(2x + 2y = 5\)
\[\Rightarrow \frac{12}{\beta} = 1 \quad \Rightarrow \beta = 12, \alpha = 6\]

Hence hyperbola is \(\frac{x^2}{36} - \frac{y^2}{144} = 1\) and normal is drawn at \((10, 16)\)
Equation of normal \(\frac{36 \cdot x}{10} + \frac{144 \cdot y}{16} = 36 + 144\)
\[\Rightarrow \frac{x}{50} + \frac{y}{20} = 1\]
This does not pass though \((15, 13)\) out of given option

14. The length of the perpendicular from the point \((1, -2, 5)\) on the line passing through \((1, 2, 4)\) and parallel to the line \(x + y - z = 0 = x - 2y + 3z - 5\) is
(A) \(\sqrt{\frac{21}{2}}\)  
(B) \(\sqrt{\frac{9}{2}}\)  
(C) \(\sqrt{\frac{73}{2}}\)  
(D) \(1\)

Answer (A)

Sol. Given \(\vec{a} = (\alpha,-1,-1)\) and \(\vec{b} = (2,1,-\alpha)\)
\[\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & -1 \\ 2 & 1 & -\alpha \end{vmatrix} = (-\alpha + 1)\hat{i} + (\alpha^2 - 2)\hat{j} + (\alpha - 2)\hat{k}\]
Projection of \(\vec{c}\) on \(\vec{d} = -\hat{i} + 2\hat{j} - 2\hat{k}\)
\[= \frac{|\vec{c} \cdot \vec{d}|}{|\vec{d}|} = 30 \ \text{(Given)}\]
\[= \frac{|\alpha - 1 - 4 + 2\alpha^2 - 2\alpha + 4|}{\sqrt{1 + 4 + 4}} = 30\]
On solving \(\alpha = \frac{-13}{2}\) (Rejected as \(\alpha > 0\)) and \(\alpha = 7\)
16. The mean and variance of a binomial distribution
are \( \alpha \) and \( \frac{\alpha}{3} \) respectively. If \( P(X = 1) = \frac{4}{243} \),
then
\[ P(X = 4 \text{ or } 5) \text{ is equal to:} \]

(A) \( \frac{5}{9} \) \quad (B) \( \frac{64}{81} \)

(C) \( \frac{16}{27} \) \quad (D) \( \frac{145}{243} \)

**Answer (C)**

**Sol.** Given, mean = \( np = \alpha \).

and variance = \( npq = \frac{\alpha}{3} \)

\[ q = \frac{1}{3} \quad \text{and} \quad p = \frac{2}{3} \]

\[ P(X = 1) = npq^1(q^{n-1}) = \frac{4}{243} \]

\[ \Rightarrow n = 6 \]

\[ P(X = 4 \text{ or } 5) = \binom{n}{4} q^4 p^2 + \binom{n}{5} q^5 p^1 = \frac{16}{27} \]

17. Let \( E_1, E_2, E_3 \) be three mutually exclusive events such that
\[ P(E_1) = \frac{2+3p}{6}, P(E_2) = \frac{2-p}{8} \]

and
\[ P(E_3) = \frac{1-p}{2}. \]

If the maximum and minimum values of \( p \) are \( p_1 \) and \( p_2 \),
then \( (p_1 + p_2) \) is equal to:

(A) \( \frac{2}{3} \) \quad (B) \( \frac{5}{3} \)

(C) \( \frac{5}{4} \) \quad (D) \( 1 \)

**Answer (B)**

**Sol.** \( 0 \leq \frac{2+3p}{6} \leq 1 \Rightarrow P \in \left[ -\frac{2}{3}, \frac{4}{3} \right] \)

\[ 0 \leq \frac{2-p}{8} \leq 1 \Rightarrow P \in [-6, 2] \]

\[ 0 \leq \frac{1-p}{2} \leq 1 \Rightarrow P \in [-1, 1] \]

\[ 0 < P(E_1) + P(E_2) + P(E_3) \leq 1 \]

18. \( S = \{ \theta \in [0, 2\pi] : 8^{2\sin^2 \theta} + 8^{2\cos^2 \theta} = 16 \} \). Then

\[ n(S) + \sum_{\theta \in S} \left( \sec \left( \frac{\pi}{4} + 2\theta \right) \cosec \left( \frac{\pi}{4} + 2\theta \right) \right) \text{ is equal to:} \]

(A) 0 \quad (B) -2 \quad (C) -4 \quad (D) 12

**Answer (C)**

**Sol.**

\[ n(S) + \sum_{\theta \in S} \left( \sec \left( \frac{\pi}{4} + 2\theta \right) \cosec \left( \frac{\pi}{4} + 2\theta \right) \right) \]

Now apply AM \( \geq \) GM for \( 8^{2\sin^2 \theta}, 8^{2\cos^2 \theta} \)

\[ \frac{8^{2\sin^2 \theta} + 8^{2\cos^2 \theta}}{2} \geq \left( 8^{2\sin^2 \theta \cdot 2\cos^2 \theta} \right)^{\frac{1}{2}} \]

\[ \geq 8 \]

\[ 8 \geq 8 \Rightarrow 8^{2\sin^2 \theta} = 8^{2\cos^2 \theta} \]

or \( \sin^2 \theta = \cos^2 \theta \)

\[ \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \]

\[ n(S) + \sum_{\theta \in S} \sec \left( \frac{\pi}{4} + 2\theta \right) \cosec \left( \frac{\pi}{4} + 2\theta \right) \]

\[ = 4 + \sum_{\theta \in S} \frac{2}{2 \sin \left( \frac{\pi}{2} + 2\theta \right) \cos \left( \frac{\pi}{4} + 2\theta \right)} \]

\[ = 4 + \sum_{\theta \in S} \frac{2}{\sin \left( \frac{\pi}{2} + 4\theta \right)} = 4 + 2 \sum_{\theta \in S} \cosec \left( \frac{\pi}{2} + 4\theta \right) \]

\[ = 4 + 2 \left[ \cosec \left( \frac{\pi}{2} + 2\pi \right) + \cosec \left( \frac{\pi}{2} + 3\pi \right) + \cosec \left( \frac{\pi}{2} + 5\pi \right) + \cosec \left( \frac{\pi}{2} + 7\pi \right) \right] \]
19. \[ \tan \left( 2 \tan^{-1} \frac{1}{5} + \sec^{-1} \frac{\sqrt{5}}{2} + 2 \tan^{-1} \frac{1}{8} \right) \] is equal to:

(A) 1
(B) \( \frac{5}{4} \)
(C) \( \frac{1}{4} \)
(D) \( \frac{5}{2} \)

Answer (B)

Sol. \[ \tan \left( 2 \tan^{-1} \frac{1}{5} + \sec^{-1} \frac{\sqrt{5}}{2} + 2 \tan^{-1} \frac{1}{8} \right) \]

\[ = \tan \left( 2 \tan^{-1} \left( \frac{1 + \frac{1}{5}}{1 - \frac{1}{5} \cdot \frac{1}{8}} \right) + \sec^{-1} \sqrt{\frac{5}{2}} \right) \]

\[ = \tan \left( 2 \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{2} \right) \]

\[ = \tan \left( \frac{\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{2}}{1 - \frac{3}{4} \cdot \frac{1}{2}} \right) \]

\[ = \tan \left( \frac{5}{8} \right) \]

\[ = \tan \left( \tan^{-1} \frac{5}{2} \right) = 2 \]

20. The statement \((\neg (p \iff q)) \land q\) is:

(A) a tautology
(B) a contradiction
(C) equivalent to \((p \Rightarrow q) \land q\)
(D) equivalent to \((p \Rightarrow q) \land p\)

Answer (D)

Sol. \((\neg (p \iff \neg q)) \land q\)

\[ = (p \iff q) \land q \]

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\[ \therefore (\neg (p \iff \neg q)) \land q \text{ is equivalent to } (p \Rightarrow q) \land p. \]

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a Numerical Value. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. If for some \(p, q, r \in \mathbb{R}\), not all have same sign, one of the roots of the equation \((p^2 + q^2) x^2 - 2q(p + r)x + q^2 + r^2 = 0\) is also a root of the equation \(x^2 + 2x - 8 = 0\), then \(\frac{q^2 + r^2}{p^2}\) is equal to__________.

Answer (272)

Sol. Let roots of \((p^2 + q^2) x^2 - 2q(p + r)x + q^2 + r^2 = 0\) be \(\alpha, \beta\)

\[ \alpha + \beta > 0 \text{ and } \alpha \beta > 0 \]

Also, it has a common root with \(x^2 + 2x - 8 = 0\)

\[ \therefore \text{ The common root between above two equations is 4.} \]

\[ \Rightarrow 16(p^2 + q^2) - 8q(p + r) + q^2 + r^2 = 0 \]

\[ \Rightarrow (16p^2 - 8pq + q^2) + (16q^2 - 8qr + r^2) = 0 \]

\[ \Rightarrow (4p - q)^2 + (4q - r)^2 = 0 \]

\[ \Rightarrow q = 4p \text{ and } r = 16p \]

\[ \therefore \frac{q^2 + r^2}{p^2} = \frac{16p^2 + 256p^2}{p^2} = 272 \]

2. The number of 5-digit natural numbers, such that the product of their digits is 36, is __________.

Answer (180)

Sol. Factors of 36 = \(2^2 \cdot 3^2 \cdot 1\)

Five-digit combinations can be

\((1, 2, 2, 3, 3), (1, 4, 3, 3, 1), (1, 9, 2, 2, 1)\)
The elements in the 11th set will have 1 + (10)2 = 21 term
Also upto 10th set total 3 × k type terms will be 1 + 3 + 5 + ....... + 19 = 100 – term
∴ Set 11 = {3 × 101, 3 × 102,.....3 × 121}
∴ Sum of elements = 3 × (101 + 102 +...+121)
\[
= \frac{3 \times 222 \times 21}{2} = 6993
\]
4. The number of distinct real roots of the equation
\[
x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0
\]
is ________. 

5. If the coefficients of \(x\) and \(x^3\) – in the expansion of \((1 + x)^p\) \((1 - x)^q\), \(p, \ q \leq 15\), are –3 and –5 respectively, then coefficient of \(x^3\) is equal to ________.

SOL. Given series
\[
\{3 \times 1\} , \{3 \times 2, 3 \times 3, 3 \times 4\}, \{3 \times 5, 3 \times 6, 3 \times 7, 3 \times 8, 3 \times 9\},......
\]
∴ 11th set will have 1 + (10)2 = 21 term
Also upto 10th set total 3 × k type terms will be 1 + 3 + 5 + ....... +19 = 100 – term
∴ Sum of elements = 3 × (101 + 102 +...+121)
\[
= \frac{3 \times 222 \times 21}{2} = 6993
\]
6. If \(n(2n+1)\) \(\binom{1}{0}(1- x^n)\) \(2n+1\) \(dx = 1177\) \(\binom{1}{0}(1- x^n)\) \(2n+1\) \(dx\),

then \(n \in \mathbb{N}\) is equal to ________.

SOL. \(\int_0^1(1-x^n)^{2n+1} \ dx = \int_0^1(1-x^n)^{2n+1} \ dx\)
\[
= \left[ (1-x^n)^{2n+1}.x \right]_0^1 - \int_0^1 x.x(2n+1)(1-x^n)^{2n} \cdot -nx^{n-1}\ dx
\]
\[
= n(2n+1)\int_0^1 (1-(1-x^n))(1-x^n)^{2n}\ dx
\]
\[
= n(2n+1)\int_0^1 (1-x^n)^{2n}\ dx - n(2n+1)\int_0^1 (1-x^n)^{2n}\ dx
\]
\(= (1+n(2n+1))\int_0^1 (1-x^n)^{2n+1}\ dx = n(2n+1)\int_0^1 (1-x^n)^{2n}\ dx\)
\[
= (2n^2 + n + 1)\int_0^1 (1-x^n)^{2n+1}\ dx = 1177\int_0^1 (1-x^n)^{2n+1}\ dx
\]
∴ \(2n^2 + n + 1 = 1177\)
\[
2n^2 + n - 1176 = 0
\]
∴ \(n = 24\) or \(-\frac{49}{2}\)
∴ \(n = 24\)

7. Let a curve \(y = y(x)\) pass through the point (3, 3) and the area of the region under this curve, above the \(x\)-axis and between the abscissae 3 and \(x(> 3)\) be \(\left(\frac{Y}{X}\right)^3\). If this curve also passes through the point \((\alpha, 6\sqrt{10})\) in the first quadrant, then \(\alpha\) is equal to ________.

Answer (6)
8. The equations of the sides AB, BC and CA of a triangle ABC are \(2x + y = 0, x + py = 15a\) and \(x - y = 3\) respectively. If its orthocentre is \((2, a), -\frac{1}{2} < a < 2\), then \(p\) is equal to _______.

Answer (3)

Sol.

![Diagram of triangle ABC with orthocentre at (2, a)]

Slope of \(AH = \frac{a+2}{1}\)

Sloe of \(BC = -\frac{1}{p}\)

\[p = a + 2 \quad \text{...(i)}\]

Coordinate of \(C = \left(\frac{18p-30}{p+1}, \frac{15p-33}{p+1}\right)\)

Slope of \(HC\)

\[\frac{15p-33}{p+1} - a = \frac{15p-33-(p-2)(p+1)}{18p-30-2} = \frac{15p-33-2p-2}{18p-30-2p-2}\]

\[= \frac{16p-p^2-31}{16p-32} \times 2 = -1\]

\[p^2 - 8p + 15 = 0\]

\[p = 3 \text{ or } 5\]

But if \(p = 5\) then \(a = 3\) not acceptable

\[p = 3\]

9. Let the function \(f(x) = 2x^2 - \log_e x, x > 0\), be decreasing in \((0, a)\) and increasing in \((a, 4)\). A tangent to the parabola \(y^2 = 4ax\) at a point \(P\) on it passes through the point \((8a, 8a - 1)\) but does not pass through the point \(\left(\frac{1}{a}, 0\right)\). If the equation of the normal at \(P\) is \(\frac{x}{\alpha} + \frac{y}{\beta} = 1\), then \(\alpha + \beta\) is equal to _______.

Answer (45)
Sol. \( \delta'(x) = \frac{4x^2 - 1}{x} \) so \( f(x) \) is decreasing in \( (0, \frac{1}{2}) \) and increasing in \( \left( \frac{1}{2}, \infty \right) \) \( \Rightarrow a = \frac{1}{2} \)

Tangent at \( y^2 = 2x \Rightarrow y = mx + \frac{1}{2m} \)

It is passing through (4, 3)

\[ 3 = 4m + \frac{1}{2m} \Rightarrow m = \frac{1}{2} \text{ or } \frac{1}{4} \]

So tangent may be

\[ y = \frac{1}{2}x + 1 \text{ or } y = \frac{1}{4}x + 2 \]

But \( y = \frac{1}{2}x + 1 \) passes through (–2, 0) so rejected.

Equation of Normal

\[ y = -4x - 2 \left( \frac{1}{2} \right) (-4) - \frac{1}{2} (-4)^3 \]

or \( y = -4x + 4 + 32 \)

or \( \frac{x}{9} + \frac{y}{36} = 1 \)

10. Let \( Q \) and \( R \) be two points on the line \( \frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2} \) at a distance \( \sqrt{26} \) from the point \( P(4, 2, 7) \). Then the square of the area of the triangle \( PQR \) is ________.

Answer (153)

Sol. \( L : \frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2} \)

Let \( T(2t - 1, 3t - 2, 2t + 1) \)

\( \therefore PT \perp QR \)

\( \therefore 2(2t - 5) + 3(3t - 4) + 2(2t - 6) = 0 \)

\[ 17t = 34 \quad \therefore t = 2 \]

So \( T(3, 4, 5) \)

\( \therefore PT = \sqrt{1^2 + 4^2 + 4^2} = 3 \)

\( \therefore QT = \sqrt{26 - 9} = \sqrt{17} \)

\( \therefore \) Area of \( \triangle PQR = \frac{1}{2} \times 2\sqrt{17} \times 3 = 3\sqrt{17} \)

\( \therefore \) Square of ar(\( \triangle PQR \)) = 153.