24/06/2022
Evening

Corporate Office: Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Memory Based Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Find radius ratio of proton, deuteron and alpha particle in same magnetic field having equal kinetic energy

(1) $1:1:\sqrt{2}$ (2) $1:\sqrt{2}:1$

(3) $1:2:\sqrt{2}$ (4) $\sqrt{2}:1:2$

Answer (2)

Sol. $R = \dfrac{\sqrt{2mK}}{qB}$

or $R \propto \dfrac{\sqrt{m}}{q}$

So, $R_1 : R_2 : R_3 = \dfrac{\sqrt{m_1}}{q_1} : \dfrac{\sqrt{m_2}}{q_2} : \dfrac{\sqrt{m_3}}{q_3}$

$= 1:\sqrt{2}:1$

2. Two travelling wave in opposite direction create standing wave having equation. $y = 10\cos(\pi x)\sin\left(\dfrac{2\pi t}{T}\right)$, where x and t are in m and sec.

Amplitude at $x = \dfrac{4}{3}$ m

(1) 5 (2) 10

(3) 12 (4) 11

Answer (1)

Sol. $y = 10\cos(\pi x)\sin\left(\dfrac{2\pi t}{T}\right)$

$\therefore$ $\text{Amp} = 10\cos(\pi x)$

at $x = \dfrac{4}{3}$ m

$\text{Amp} = \left|10\cos\left(\pi \times \dfrac{4}{3}\right)\right|$

$= 10 \times \left(\dfrac{1}{2}\right)$

= 5 units

3. A particle of mass 5 kg is projected vertically upward with velocity 20 m/s. If air resistance for motion is 10 N. Ratio of time of ascent to that of time of descent, is

(1) $\sqrt{2}:\sqrt{3}$

(2) $\sqrt{3}:\sqrt{2}$

(3) 2 : 3

(4) 3 : 2

Answer (1)

Sol. $a_{\text{upward}} = \left[-10 + \left(-\dfrac{10}{5}\right)\right]\hat{j} = -12\hat{j}$ m/s^2

$a_{\text{downwards}} = \left[-10 + \dfrac{10}{5}\right]\hat{j} = -8\hat{j}$ m/s^2

$d_{\text{upwards}} = d_{\text{downwards}}$

$\Rightarrow \dfrac{1}{2}(12)t_{\text{up}}^2 = \dfrac{1}{2}8(t_{\text{down}})^2$

$\Rightarrow \dfrac{t_{\text{up}}}{t_{\text{down}}} = \dfrac{\sqrt{2}}{\sqrt{3}}$

4. A mass m suspended with the help of string is moving in vertical circular motion. The tension in the string is

(1) Same through out

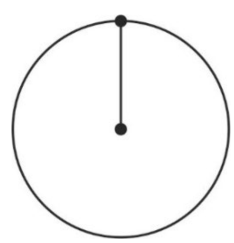
(2) Maximum at top

(3) Minimum at top

(4) Maximum at horizon level

Answer (3)

Sol. $T_{\min} = \dfrac{mv^2}{R} - mg$



At the top most position.

5. Parallel plate capacitor charged to 60 mC plate loses charge at 1.8 × 10^{-8} C/s. Magnitude of displacement current is

(1) 1.8 × 10^{-8} C/s (2) 3.6 × 10^{-8} C/s

(3) 5 × 10^{-8} C/s (4) 6 × 10^{-8} C/s

Answer (1)

Sol. In a parallel plate capacitor discharging

$I = I_D$

$\Rightarrow \quad I_D = \dfrac{dq}{dt}$

$= 1.8 \times 10^{-8}$ C/s

$\Rightarrow \quad I_D = 1.8 \times 10^{-8}$ A

6. A charged particle ($-q$, m) revolves around a cylinder of charge density ρ and radius R. The kinetic energy of particle is

(1) $\dfrac{\rho R^2 q}{4\varepsilon_0}$ (2) $\dfrac{\rho R^2 q}{2\varepsilon_0}$

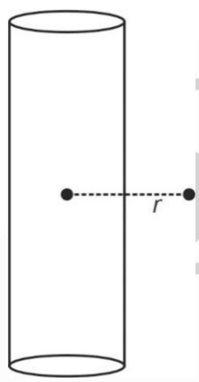
(3) $\dfrac{\rho R^2 q}{8\varepsilon_0}$ (4) $\dfrac{\rho R^2 q}{16\varepsilon_0}$

Answer (1)

Sol. $E = \dfrac{\lambda}{2\pi\varepsilon_0 \times r}$

$= \dfrac{\rho \times \pi R^2 \times l}{l \times 2\pi\varepsilon_0 r}$

$= \dfrac{\rho R^2}{2\varepsilon_0 r}$



$\therefore \quad KE = \dfrac{1}{2}mv^2$

$= \dfrac{1}{2}\left(\dfrac{mv^2}{r}\right) \times r$

$= \dfrac{1}{2} \times \left(q \times \dfrac{\rho R^2}{2\varepsilon_0 r}\right) \times r$

$= \left(\dfrac{\rho R^2 q}{4\varepsilon_0}\right)$

7. If the ratio of intensities of two waves is $\dfrac{9}{4}$, then the ratio of maximum to minimum intensity, is

(1) 20 : 1 (2) 25 : 1

(3) 16 : 1 (4) 64 : 1

Answer (2)

Sol. $\dfrac{I_1}{I_2} = \dfrac{9}{4}$

$\Rightarrow \quad \dfrac{A_1}{A_2} = \dfrac{3}{2}$ or $\dfrac{A_1 + A_2}{A_2 - A_2} = \dfrac{5}{1}$

$\Rightarrow \quad \dfrac{I_{max}}{I_{min}} = \dfrac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \dfrac{25}{1}$

8. Potential energy of two atoms is $U = \dfrac{A}{r^{10}} - \dfrac{B}{r^5}$. Find r for which potential energy is minimum.

(1) $2\left(\dfrac{A}{B}\right)^{\frac{1}{5}}$

(2) $\left(\dfrac{A}{B}\right)^{\frac{1}{5}}$

(3) $\left(\dfrac{2A}{B}\right)^{\frac{1}{5}}$

(4) $\left(\dfrac{A}{2B}\right)^{\frac{1}{5}}$

Answer (3)

Sol. $U = \dfrac{A}{r^{10}} - \dfrac{B}{r^5}$

If r is position for U (potential energy) to be minimum then at r

$\left.\dfrac{dU}{dr}\right|_{\text{at } r} = 0$

or $\dfrac{10A}{r^{11}} = \dfrac{5B}{r^6}$

$\Rightarrow \quad r = \left(\dfrac{2A}{B}\right)^{\frac{1}{5}}$

9. 

Two point charge particles with equal charge $Q = 10^{-5}$C are kept on a table top. The particles have mass m = 10 gm each and coefficient of friction between particles and table is 0.25. If particle are held at equilibrium so that l is equal to its minimum value then l is

(1) 2 m

(2) 6 m

(3) 10 m

(4) 21 m

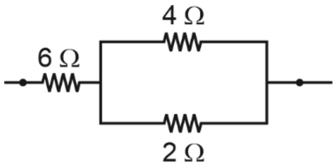
Answer (2)

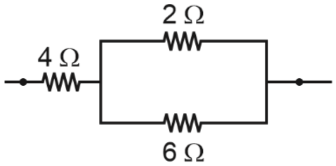
Sol. $\dfrac{kQ^2}{l^2} = \mu mg$

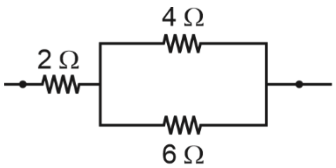
$l = \sqrt{\dfrac{9 \times 10^9 \times 10^{-10}}{0.25 \times 10^{-2} \times 10}}$

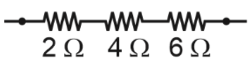
$= \dfrac{3}{0.5} = 6$ m

10. For which combination of 2, 4, 6 Ω resistances equivalent resistance is $\frac{22}{3}\Omega$

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. In 1st combination $R_{eq}=\left(\frac{4\times 2}{4+2}+6\right)\Omega=\frac{22}{3}\Omega$

In 2nd combination $R_{eq}=\left(\frac{6\times 2}{6+2}+4\right)\Omega=\frac{11}{2}\Omega$

In 3rd combination $R_{eq}=\left(\frac{6\times 4}{6+4}+2\right)\Omega=\frac{22}{5}\Omega$

In 4th combination R_{eq} = (2 + 4 + 6) Ω = 12 Ω

11. 1.5 kg hammer strikes the nail of mass 100 gm with speed of 60 m/sec. If heat given to nail is 1/4th of energy of hammer, find change in temperature of nail (s_{nail} = 2250 J/kg°C)

(1) 1.5°C (2) 2°C

(3) 2.5°C (4) 3°C

Answer (4)

Sol. $E=\frac{1}{2}\times(1.5)\times(60)^2$

$\therefore\ Q=\frac{1}{4}\times E$

$\Rightarrow\ m\times s\times\Delta T=\frac{1}{4}\times\frac{1}{2}\times 1.5\times(3600)$

$\Rightarrow\ 0.1\times s\times\Delta T=900\times 1.5\times\frac{1}{2}$

$\Rightarrow\ \Delta T=\frac{450\times 1.5}{0.1\times s}=\frac{6750}{s}$

$=\frac{6750}{2250}$

= 3°C

12. A bulb of power rating 200 W has efficiency of 3.5%. What is the peak value of magnetic field at a distance of 4 m?

(1) 7.5 × 10^{-8} T

(2) 1.1 × 10^{-8} T

(3) 1.5 × 10^{-8} T

(4) 1.7 × 10^{-8} T

Answer (4)

Sol. $P_{eff}=200\times\left(\frac{3.5}{100}\right)=7\text{ W}$

$I=\frac{P_{eff}}{4\pi r^2}=\frac{B_0^2}{2\mu_0}\times C$

$\Rightarrow\ B_0=\sqrt{2\mu_0\times\frac{P_{eff}}{4\pi r^2}\times\frac{1}{C}}$

$=\sqrt{\frac{2\times 10^{-7}\times 7}{4^2\times 3\times 10^8}}$

= 1.7 × 10^{-8}

13. Two massless springs with spring constants $2K$ and $9K$ having 50 g and 100 g attached at free end, both have same V_{max}. Then find ratio of amplitude of vibrations.

(1) 2 : 3

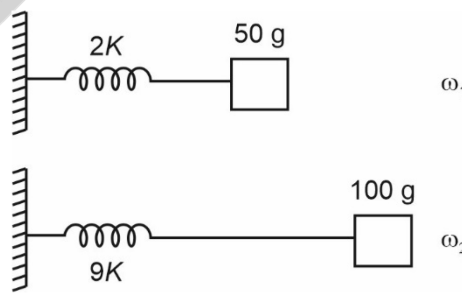
(2) 1 : 2

(3) 3 : 2

(4) 2 : 1

Answer (3)

Sol.



$\because\ \omega_1 A_1=\omega_2 A_2$

$\Rightarrow\ \frac{A_1}{A_2}=\frac{\omega_2}{\omega_1}$

$=\sqrt{\frac{9K}{100}\times\frac{50}{2K}}$

$=\sqrt{\frac{9}{4}}=\frac{3}{2}$

14. **Statement A:** An AC current can be created with 0 reactance.

Statement B: An AC circuit without power is not possible.

(1) Only statement A is true

(2) Only statement B is true

(3) Statement A is true and B is false

(4) Statement A is false and B is true

Answer (3)

Sol. An ideal circuit with inductor and capacitor with $X_L = X_C$ can have zero reactance

$\Rightarrow$ Statement A is true

In an AC circuit if only reactance part of impedance is there that is resistance is not available then average power consumed by the circuit can be zero.

$\Rightarrow$ Statement B is false

15. It is given that a flywheel starting from rest covers 5 rad in first second, then the angle covered in next second is (consider constant angular acceleration)

(1) 10 rad (2) 15 rad

(3) 20 rad (4) 25 rad

Answer (2)

Sol. $\theta_{1st} = 5 = \frac{1}{2}\alpha(1)^2$

$\theta_{2nd} = \theta_2 - \theta_{1st}$

$= \frac{1}{2}\alpha(2)^2 - \frac{1}{2}\alpha(1)^2$

$= \frac{1}{2}\alpha(3) = 15 \text{ rad}$

16. A coil of 1000 turns and area 1 m^2 is rotated about its diameter with constant angular velocity of 1 rotation/sec in a uniform and perpendicular magnetic field of 0.07 tesla. The maximum emf induced in the coil approximately is

(1) 200 volts (2) 400 volts

(3) 220 volts (4) 440 volts

Answer (4)

Sol. $\phi = NBA\cos\theta$

So, Emf $= \frac{-d\phi}{dt} = NBA\omega\sin\theta$

So Emf$_{max}$ = $NAB\omega$

= 1000 × 1 × 0.07 × 2π

= 440 volts.

17. Ratio of masses initially in radioactive substance is 2 : 1. Total mass of substance is 10^{-2} kg. Half lives of A and B is 4 sec and 8 sec respectively. Find the ratio of decayed mass of A to that of B after 16 sec.

(1) $\frac{11}{4}$ (2) $\frac{8}{3}$

(3) $\frac{5}{2}$ (4) $\frac{9}{5}$

Answer (3)

Sol. $\frac{N_{01}}{N_{02}} = \frac{2}{1}$

No. of half lives of $A = \frac{16}{4} = 4$

No. of half lives of $B = \frac{16}{8} = 2$

$N_1 = \frac{2}{2^4} = \frac{1}{8}$

$N_2 = \frac{1}{2^2} = \frac{1}{4}$

$\therefore$ ratio of decayed $= \frac{2 - \frac{1}{8}}{1 - \frac{1}{4}} = \frac{15 \times 4}{8 \times 3}$

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If distance between sun and earth is R. How many years will the earth taken to complete one revolution if distance is 3R.

Answer (5.20)

$3\sqrt{3}$ Years

Sol. $\because\ T^2 \propto R^3$

$\Rightarrow \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$

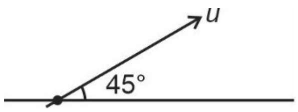
$\Rightarrow \frac{T_2}{1} = \left(\frac{3}{1}\right)^{\frac{3}{2}} = 3\sqrt{3}$

$\Rightarrow T_2 = 3\sqrt{3}$ years $\simeq 5.20$ years

22. A projectile is fired at an angle 45° with horizontal. After 2 seconds the projectile has velocity 20 m/s. Find range

Answer (80 m)

Sol.



$(u\cos45°)^2 + (u\sin45° - 20)^2 = (20)^2$

$\Rightarrow \quad u = 20\sqrt{2}$ m/s

$\therefore \quad R = \dfrac{u^2 \times \sin(2\theta)}{g}$

$= \dfrac{20 \times 20 \times 2}{10}$

= 80 m

23. A carnot engine absorbs heat 500 kcal at 727°C and rejects it at 127°C. Find work done (in kcal) by carnot engine.

Answer (300)

Sol. $\eta = \left(1 - \dfrac{400}{1000}\right) = 0.6$

$\therefore W = 500 \times 0.6$

$= 300$ kcal

24. If the energy of capacitor increases by 44% then find percentage change in charge stored in capacitor.

Answer (20)

Sol. $U = \dfrac{1}{2}\dfrac{q^2}{C}$

$U_i = \dfrac{q_i^2}{2C}$ & $U_f = \dfrac{q_f^2}{2C}$

$U_f = 1.44U_i$

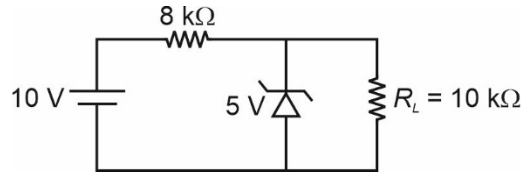
$\dfrac{q_f^2}{2C} = 1.44\dfrac{q_i^2}{2C}$

$\Rightarrow q_f = 1.2q_i$

So $\dfrac{q_f - q_i}{q_i} = 0.2$

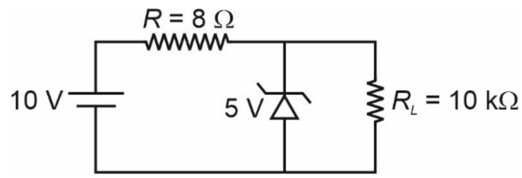
So % change in q = 20%

25. In the circuit shown, current through Zener diode (in mA) is



Answer (0.125)

Sol.



$i_L = \dfrac{5}{10000} = 0.5$ mA

$i_R = \dfrac{5}{8000} = 0.625$ mA

$\therefore i_z = i_R - i_L = 0.625 - 0.5$

$= 0.125$ mA

26. Q is the heat supplied to a system containing monoatomic gas. During the process if work done by the gas is $\dfrac{Q}{4}$ then molar specific heat during the process is xR. Value of x is equal to ______.

Answer (2.0)

Sol. $\Delta U = Q - \dfrac{Q}{4}$

$\Rightarrow nC_v\Delta T = \dfrac{3Q}{4}$

$\Rightarrow \; n \times \left(\dfrac{3R}{2}\right)\Delta T = \dfrac{3Q}{4}$

$\Rightarrow \; \dfrac{Q}{n\Delta T} = \dfrac{3}{2}R \times \dfrac{4}{3}$

$\therefore \; C = \dfrac{Q}{n\Delta T} = 2R$

$\therefore x = 2$

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. H_2 gas is produced in the preparation of

(1) Na_2CO_3

(2) NaOH

(3) Na metal

(4) $NaHCO_3$

Answer (2)

Sol. H_2 gas is produced in the preparation of NaOH by electrolysis of aq. NaCl.

$$NaCl \longrightarrow Na^+ + Cl^-$$

$$H_2O \rightleftharpoons H^+ + OH^-$$

$$\text{Cathode}: \; 2H^+ + 2e^- \longrightarrow H_2$$

$$\text{Anode}: \; 2Cl^- \longrightarrow Cl_2 + 2e^-$$

2. Among the following which one has highest melting point?

Ag, Hg, Ga, Ba

(1) Hg

(2) Ga

(3) Ag

(4) Ba

Answer (3)

Sol. Hg is liquid at room temperature. Ga has very low melting point around 30°C.

Melting point of Ba is 727°C and melting point of Hg is around 960°C.

Hence, highest melting point among given metals is of Ag.

3. The CFSE is maximum for

(1) $[Mo(H_2O)_6]^{3+}$

(2) $[Cr(H_2O)_6]^{3+}$

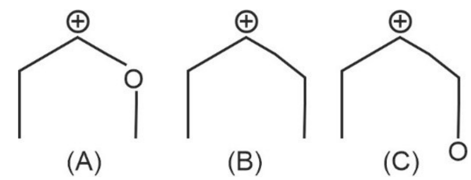
(3) $[Mn(H_2O)_6]^{3+}$

(4) $[Fe(H_2O)_6]^{3+}$

Answer (1)

Sol. Magnitude of CFSE value will be maximum for $[Mo(H_2O)_6]^{3+}$ as Mo belongs to 4*d* series of transition elements and has a d^3 configurarion.

4. Stability of carbocation



(1) A > B > C

(2) B > C > A

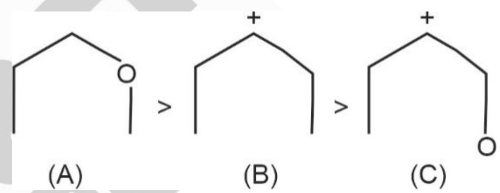
(3) A > C > B

(4) C > A > B

Answer (1)

Sol. Stability of carbocation ∝ +R effect of the substituent attached to the carbocation

So the correct order is



5. Which gas does not do global warming effect?

(1) CH_4

(2) N_2

(3) O_3

(4) H_2O

Answer (2)

Sol. Methane, ozone and water vapour show greenhouse effect, hence cause global warming.

N_2 does not cause global warming.

6. Which of the following use in fire extinguisher?

(1) Baking Soda

(2) Washing Soda

(3) Caustic Soda

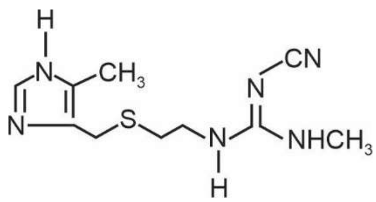
(4) Soda ash

Answer (1)

Sol. Among the following compounds.

Sodium hydrogen carbonate, $NaHCO_3$, Baking soda is used in fire extinguisher.

7. Name of given structure



(1) Ranitidine

(2) Cimetidine

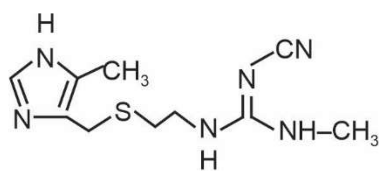
(3) Histamine

(4) Terfenadine

Answer (2)

Sol. The name of the given below structure is

Cimetidine



8. Which metal burns with green flame, blue from centre?

(1) Fe^{2+}

(2) Cu^{2+}

(3) Fe^{3+}

(4) Al^{3+}

Answer (2)

Sol. When a platinum wire dipped in salt paste (Cu^{2+}) and conc. HCl is taken near the mouth of flame having oxidising zone, the colour imparted by Cu^{2+} is green with blue centre.

9. PCl_5 exists but NCl_5 does not, which of the following statement correctly explain the above?

(1) N-does not have vacant *d*-orbital

(2) P-does not have 2*d* orbitals

(3) Back-bonding in NCl_5 is not possible

(4) N-atom is more E.N. atom, so does not form 5 bonds

Answer (1)

Sol. NCl_5 does not exist as N does not have vacant *d*-orbitals, whereas PCl_5 exists as P has 3d orbitals available for bonding

10. If K = Kinetic Energy of H-atom

P = Potential Energy of H-atom

T = Total Energy of H-atom

As total energy increase then which of the following option is correct?

(1) K and P increase but T decrease

(2) P and T increase but K decrease

(3) All increase

(4) All decrease

Answer (2)

Sol. Potential Energy = $\frac{-Ke^2}{r}$

Total Energy = $\frac{-Ke^2}{2r}$

Kinetic Energy = $\frac{Ke^2}{2r}$

As n increases, distance from nucleus increase and both potential energy and total energy increases.

KE decreases.

11. Number of peptide linkage in given sequence

ALA – VAL – GLY – LYS

(1) 5

(2) 4

(3) 3

(4) 6

Answer (3)

Sol. Number of peptide linkages in the given below sequence is 3

ALA – VAL - GLY - LYS

12. Which of the following option represents correct decreasing order of bond order of the following compounds?

$C_2^{2-}, N_2^{2-}, O_2^{2-}$

(1) $C_2^{2-} > N_2^{2-} > O_2^{2-}$

(2) $O_2^{2-} > N_2^{2-} > C_2^{2-}$

(3) $O_2^{2-} > C_2^{2-} > N_2^{2-}$

(4) $N_2^{2-} > O_2^{2-} > C_2^{2-}$

Answer (1)

Sol. Bond order of C_2^{2-} = 3

Bond order of N_2^{2-} = 2

Bond order of O_2^{2-} = 1

Correct order of bond order will be, $C_2^{2-} > N_2^{2-} > O_2^{2-}$

13. **Statement-1 :** π-bond makes the compound unstable

Statement-2 : Bond strength of C=C (double bond) is more than C–C (single bond)

(1) SI and S2 both are correct and S2 is correct explanation of S1

(2) S1 and S2 both are correct and S2 is not the correct explanation of S1

(3) S1 is false and S2 is correct

(4) S1 is correct and S2 is false

Answer (3)

Sol. Presence of multiple bond provides stability as it leads to decrease in energy.

C=C consist of both σ and π overlap thus strength of C=C is more than C–C.

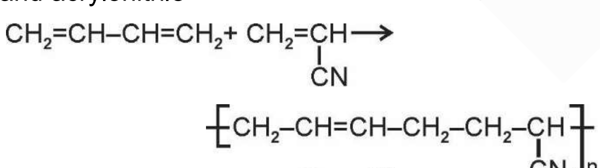
14. Which of the following polymer is not a condensation polymer?

(1) Buna-N (2) Silicone

(3) Nylon-6,6 (4) Dacron

Answer (1)

Sol. Buna-N is an addition copolymer of 1,3-butadiene and acrylonitrile



Buna-N

15. Which of the following sequence of reagent can perform the below conversion?

$$CH_3-CH_2-CH_2-OH \xrightarrow{?} CH_3-CH_2-CH_2-CH_2-NH_2$$

(1) $SOCl_2$, KCN, H_2/Pd

(2) $SOCl_2$, AgCN, H_2/Pd

(3) PCl_5, AgCN, H_2/Pd

(4) Red P/HI, KCN, H_2/Pd

Answer (1)

Sol. $CH_3-CH_2-CH_2-OH \xrightarrow{SOCl_2} CH_3-CH_2-CH_2-Cl$

$\downarrow$ KCN

$CH_3-CH_2-CH_2-C\equiv N \xrightarrow{H_2/Pd} CH_3-CH_2-CH_2-CH_2-NH_2$

16. Find the cell constant of a given cell.

Given: Resistance 20 Ω, molar conductivity = 0.154×10^{-3} Scm^2 mol^{-1}

Concentration = 0.1 M

(1) 3.08×10^{-7} cm^{-1}

(2) 30.8×10^{-7} cm^{-1}

(3) 0.308×10^{-9} cm^{-1}

(4) 4.08×10^{-5} cm^{-1}

Answer (1)

Sol. $\Lambda_M = \dfrac{1000\,K}{M}$

$\Lambda_M = 0.154 \times 10^{-3}$ Scm^2 mol^{-1}

Molarity (M) = 0.1 M

$K = \dfrac{0.154 \times 10^{-3} \times 0.1}{1000} = 0.154 \times 10^{-7}$ Scm^{-1}

and $K = \dfrac{x}{R}$

R = 20 Ω

x = $0.154 \times 20 \times 10^{-7}$

x = 3.08×10^{-7} cm^{-1}

17. Which of the ore does not have sulphide in them?

(1) Baryte

(2) Copper pyrites

(3) Galena

(4) Zinc blende

Answer (1)

Sol. The formula of ores is,

Copper pyrite = $CuFeS_2$

Zinc blende = ZnS

Galena = PbS

Baryte = $BaSO_4$

18. Which one is not the enamel of teeth?

(1) Ca^{2+} (2) P^{+3}

(3) $F^{\ominus}$ (4) P^{+5}

Answer (2)

Sol. The F^- ions make the enamel on teeth much harder by converting hydroxyapatite, $[3(Ca_3(PO_4)_2 \cdot Ca(OH)_2]$, the enamel on the surface into much harder fluorapatite, $[3(Ca_3(PO_4)_2] \cdot CaF_2]$.

So, P^{+3} is not present in enamel.

19. How many half-life are required to complete 90% of the reaction? [Given the reaction is first order]

(1) 3.32 (2) 2.07

(3) 1.44 (4) 4.02

Answer (1)

Sol. $\left(\dfrac{\ln 2}{t_{1/2}}\right)^{t_{90\%}} = \ln\left(\dfrac{100}{10}\right) = \ln(10)$

$$t_{90\%} \times \frac{0.693}{t_{1/2}} = 2.303$$

$$t_{90\%} = \frac{t_{1/2}}{0.3} = t_{1/2} \times \frac{10}{3} = 3.33t_{1/2}$$

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Energy of one particle with wavelength is 300 nm. The energy of Na number of particle is x × 10^5J. Find x.

Answer (4)

Sol. Energy of Na particles will be

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8 \times 6.023 \times 10^{23}}{300 \times 10^{-9}}$$

$= 0.397 \times 10^6$

$\approx 4 \times 10^5$ J

x = 4 (in nearest integer)

22. Volume occupied by a 3 g of gas A at 300 K is same as volume occupied by 0.2 g of H_2 at 200 K. Find the molar mass of A.

Answer (45)

Sol. $n_A T_A = n_{H_2} T_{H_2}$ {Assume constant pressure}

$$\frac{3}{M_A}(300) = \frac{0.2}{2}(200)$$

$$M_A = \frac{900}{0.1 \times 200}$$

$$= \frac{9000}{200}$$

= 45 g/mol

23. For the equilibrium $A(g) \rightleftharpoons B(g)$, ΔH is –40 kJ/mole. If the ratio of the activation energy of the forward (E_{a_f}) and backward (E_{a_b}) is 2/3, then find the value of E_{a_f}.

Answer (80)

Sol. $A(g) \rightleftharpoons B(g)$

Since, $\Delta H = E_{a_f} - E_{a_b}$ ...(i)

and, $\dfrac{E_{a_f}}{E_{a_b}} = \dfrac{2}{3}$

$$E_{a_f} = \frac{2E_{a_b}}{3}$$

Putting in (i),

$$-40 = \frac{-E_{a_b}}{3}$$

E_{a_b} = 120 kJ/mole

Hence, E_{a_f} = – 40 + 120 = 80 kJ/mole

24.

25.

26.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Find the area between the curves $y^2 = 2x$ and $x + y = 4$

(1) 10

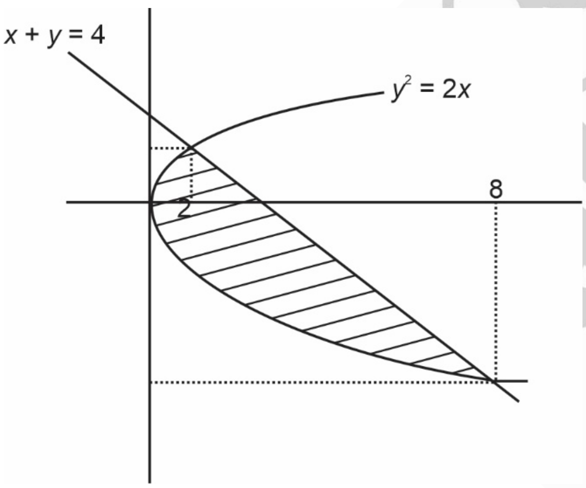
(2) $\frac{11}{3}$

(3) 18

(4) $\frac{14}{3}$

Answer (3)

Sol. $x + y = 4$



Solving the two curves, we get $(4 - x)^2 = 2x$

$\Rightarrow\ x^2 - 10x + 16 = 0$

$\Rightarrow\ x = 2$ and $x = 8$

Correspondingly $y = 2$ and $y = -4$

Required Area

$$= \int_{-4}^{2}\left[(4-y)-\frac{y^2}{2}\right]dy = 4y-\frac{y^2}{2}-\frac{y^3}{6}\Big|_{-4}^{2}$$

$$=\left(8-2-\frac{4}{3}\right)-\left(-16-8+\frac{32}{3}\right)$$

$= 18$

2. For given probability distribution

x	0	1	2	3	4
$P(x)$	k	$2k$	$3k$	$4k$	$5k$

Find $P\left(\frac{1<x<4}{x\le 2}\right)$

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$

(3) $\frac{2}{5}$ (4) $\frac{1}{2}$

Answer (4)

Sol. $\because\ \sum P(x)=1 \Rightarrow k+2k+3k+4k+5k=1$

$$\Rightarrow k=\frac{1}{15}$$

$$P\left(\frac{1<x<4}{x\le 2}\right)=\frac{P(1<x<4\cap x\le 2)}{P(x\le 2)}$$

$$=\frac{P(x=2)}{P(x=0)+P(x=1)+P(x=2)}$$

$$=\frac{3k}{k+2k+3k}=\frac{1}{2}$$

3. If $x > 0$, $y > 0$ and $x^2y^3 = 2^{15}$. Find the minimum value of $3x + 2y$.

(1) $40\left(\frac{2}{3}\right)^{\frac{1}{5}}$ (2) $40\left(\frac{2}{3}\right)^{\frac{2}{5}}$

(3) $20\left(\frac{2}{3}\right)^{\frac{1}{5}}$ (4) $20\left(\frac{2}{3}\right)^{\frac{2}{5}}$

Answer (1)

Sol. $3x+2y=2\left(\frac{3x}{2}\right)+3\left(\frac{2y}{3}\right)$

as x, $y > 0$ AM – GM inequality to be applied

$$\Rightarrow\ \frac{2\left(\frac{3x}{2}\right)+3\left(\frac{2y}{3}\right)}{5}\ge\left(\left(\frac{3x}{2}\right)^2\left(\frac{2y}{3}\right)^3\right)^{\frac{1}{5}}$$

$$\Rightarrow\ \frac{3x+2y}{5}\ge\left(\frac{2}{3}\cdot 2^{15}\right)^{\frac{1}{3}}\quad\left(\text{as } x^2y^3=2^{15}\right)$$

$$\Rightarrow\ 3x+2y\ge 40\left(\frac{2}{3}\right)^{\frac{1}{5}}$$

4. For hyperbola : $\frac{x^2}{a^2}-y^2=1$

Ellipse : $3x^2 + 4y^2 = 12$. If length of latus rectum is equal then find $12\left(e_E^2+e_H^2\right)$

(1) 36 (2) 48

(3) 42 (4) $\frac{41}{2}$

Answer (3)

Sol. H : $\frac{x^2}{a^2}-\frac{y^2}{1}=1$, length of latus rectum $=\frac{2.1}{a}=\frac{2}{a}$

E : $\frac{x^2}{4}-\frac{y^2}{3}=1$, length of latus rectum $=\frac{2(3)}{2}=3$

$\because\ \frac{2}{a}=3 \Rightarrow a=\frac{2}{3}$

$\because\ e_E^2=1-\frac{3}{4}=\frac{1}{4}$

and $e_H^2=1+\frac{1}{a^2}=1+\frac{9}{4}=\frac{13}{4}$

So, $12\left(e_E^2+e_H^2\right)=12\left(\frac{1}{4}+\frac{13}{4}\right)=42$

5.

6. Find the sum of roots of the equation $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$

(1) ln6 (2) ln3

(3) –ln3 (4) ln2

Answer (3)

Sol. $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$

$\Rightarrow (e^x - 2)(e^x + 2)(3e^x - 1)(2e^x - 1) = 0$

$\Rightarrow e^x = 2, \frac{1}{3}, \frac{1}{2}$

$\therefore$ x = ln2, –ln3, –ln2

$\therefore$ sum of roots = –ln3

7. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{\left(n^2+r^2\right)(n+r)}$ equals

(1) $\frac{1}{4}\ln 2+\frac{\pi}{8}$ (2) $\frac{1}{4}\ln 2-\frac{\pi}{8}$

(3) $-\frac{1}{4}\ln 2-\frac{\pi}{8}$ (4) $-\frac{1}{4}\ln 2+\frac{\pi}{8}$

Answer (1)

Sol. $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{\left(1+\frac{r^2}{n^2}\right)\left(1+\frac{r}{n}\right)}$

$=\int_0^1\frac{dx}{\left(1+x^2\right)(1+x)}$

$$\left\{\begin{array}{l}\text{let } \frac{1}{\left(1+x^2\right)(1+x)}=\frac{A}{1+x}+\frac{Bx+C}{1+x^2}\\ \Rightarrow 1 = A\left(1+x^2\right)+(Bx+C)(1+x)\\ \text{So } A=\frac{1}{2}, B=-\frac{1}{2}, C=\frac{1}{2}\end{array}\right\}$$

$=\frac{1}{2}\int_0^1\frac{dx}{1+x}-\frac{1}{2}\int_0^1\frac{xdx}{1+x^2}+\frac{1}{2}\int_0^1\frac{dx}{1+x^2}$

$=\left[\frac{1}{2}\ln(1+x)-\frac{1}{4}\ln\left(1+x^2\right)+\frac{1}{2}\tan^{-1}x\right]_0^1$

$=\frac{1}{2}\ln 2-\frac{1}{4}\ln 2+\frac{\pi}{8}=\frac{1}{4}\ln 2+\frac{\pi}{8}$

8. Find the number of solutions of $\cos\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)=\frac{\cos^2 2x}{4}$, in $(-2\pi, 2\pi)$

(1) 1 (2) 5

(3) 3 (4) 8

Answer (3)

Sol. $\cos^2\frac{\pi}{3}-\sin^2 x=\frac{\cos^2 2x}{4}$

$\Rightarrow\ 1-4\sin^2 x=\left(1-2\sin^2 x\right)^2$

$\Rightarrow\ 4\sin^4 x=0 \Rightarrow \sin x=0$

$\Rightarrow\ x=-\pi, 0, \pi$

9.

10. If $\log_{x+1}(x^2 - x + 6)^2 = 4$, find x.

(1) $\frac{4}{3}$

(2) $\frac{5}{3}$

(3) $\frac{1}{3}$

(4) $\frac{7}{3}$

Answer (2)

Sol. $\log_{(x+1)}(x^2 - x + 6)^2 = 4$

$\Rightarrow\ (x^2 - x + 6)^2 = (x + 1)^4$

$\Rightarrow\ (x^2 - x + 6)^2 = (x^2 + 2x + 1)^2$

$\Rightarrow\ (x^2 - x + 6 - x^2 - 2x - 1)(x^2 - x + 6 + x^2 + 2x + 1) = 0$

$\Rightarrow\ (-3x + 5)(2x^2 + x + 7) = 0$

So, $x = \frac{5}{3}$ is only possible real solution.

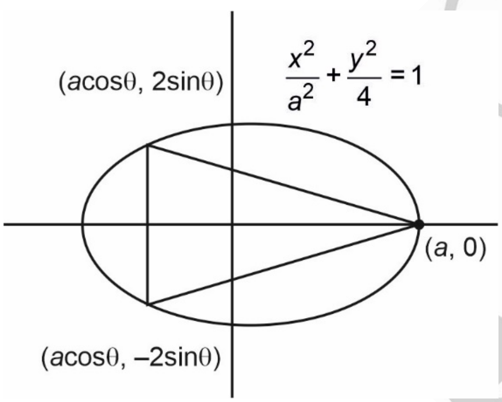
11. Let a triangle of maximum area be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{4} = 1\ (a > 2)$, such that one of its vertices coincides with one end of major axis, is $6\sqrt{3}$ sq. units. Find its eccentricity.

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{2}$ (4) $\frac{1}{3}$

Answer (2)

Sol.



$$\Delta = \frac{1}{2}(4\sin\theta)(a - a\cos\theta)$$

$$\Rightarrow\ \Delta = 2a(\sin\theta - \sin\theta\cos\theta)$$

$$\Rightarrow\ \Delta = a(2\sin\theta - \sin 2\theta)$$

$$\Rightarrow\ \Delta' = a(2\cos\theta - 2\cos 2\theta) = 0$$

So, $\theta = \frac{2\pi}{3}$

Hence $\Delta_{max} = a\left(2\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}a}{2} = 6\sqrt{3}$

So, $a = 4$

Now, $e = \sqrt{1 - \frac{4}{16}} = \frac{\sqrt{3}}{2}$

12.

13. $\int\limits_{-\pi/2}^{\pi/2} \frac{dx}{(1+e^x)\left(\sin^6 x + \cos^6 x\right)}$ equals

(1) π

(2) $\frac{3\pi}{4}$

(3) $\frac{5\pi}{4}$

(4) $\frac{\pi}{3}$

Answer (1)

Sol. Let $I = \int\limits_{-\pi/2}^{\pi/2} \frac{dx}{(1+e^x)\left(\sin^6 x + \cos^6 x\right)}$ ...(i)

$$I = \int\limits_{-\pi/2}^{\pi/2} \frac{dx}{(1+e^{-x})\left(\sin^6 x + \cos^6 x\right)} \quad \text{...(ii)}$$

Using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Adding eqns. (i) and (ii), we get

$$2I = \int\limits_{-\pi/2}^{\pi/2} \frac{dx}{\sin^6 x + \cos^6 x}$$

$$\Rightarrow\ I = \int\limits_{0}^{\pi/2} \frac{dx}{\sin^6 x + \cos^6 x}$$

$$\Rightarrow\ I = \int\limits_{0}^{\pi/2} \frac{dx}{1 - 3\sin^2 x\cos^2 x} = \int\limits_{0}^{\pi/2} \frac{4dx}{4 - 3\sin^2 2x}$$

$$\Rightarrow\ I = 2\int\limits_{0}^{\pi/4} \frac{4\sec^2 2x}{4\sec^2 2x - 3\tan^2 2x}dx$$

Let $\tan 2x = t$

$\Rightarrow\ 2\sec^2 2x\,dx = dt$

$$\Rightarrow\ I = \int\limits_{0}^{\infty} \frac{4dt}{4+t^2} = 2\tan^{-1}\left(\frac{t}{2}\right)\Bigg|_0^{\infty} = \pi$$

14. Let $f(x)=\begin{cases}\dfrac{\sin\{x\}}{\{x\}} & x\in(-2,-1)\\ \max.([|x|],2x) & |x|<1\\ -1 & \text{otherwise}\end{cases}$

If $f(x)$ is not differentiable at m points and not continuous at n points, then (m, n) equals (where { } represents fractional part function and [ ] represents greatest integer function respectively).

(1) (2, 3) (2) (3, 3)

(3) (2, 4) (4) (4, 3)

Answer (4)

Sol. $f(x)=\begin{cases}\dfrac{\sin(x+2)}{x+2} & \text{if } x\in(-2,-1)\\ 0 & \text{if } x\in(-1,0)\\ 2x & \text{if } x\in(0,1)\\ -1 & \text{otherwise}\end{cases}$

$f(x)$ is discontinuous at = x = –2, –1, 1

$f(x)$ is not differentiable at = x = –2, –1, 0 and 1

15. If $\Delta_r=\begin{vmatrix}2^{r-1} & \dfrac{(r+1)!}{1+\frac{1}{r}} & 4r^3-2nr\\ a & b & c\\ 2^n-1 & (n+1)!-1 & n^3(n+1)\end{vmatrix}$

then, $\sum_{r=1}^{n}\Delta_r=$

(1) abc (2) $(n + 3)!$

(3) 0 (4) $an! + b2^n + c$

Answer (3)

Sol. $\sum_{r=1}^{n}\Delta_r=\begin{vmatrix}\sum_{r=1}^{n}2^{r-1} & \sum_{r=1}^{n}((r+1)!-r!) & \sum_{r=1}^{n}(4r^3-2nr)\\ a & b & c\\ 2^n-1 & (n+1)!-1 & n^3(n+1)\end{vmatrix}$

$=\begin{vmatrix}2^n-1 & (n+1)!-1 & n^3(n+1)\\ a & b & c\\ 2^n-1 & (n+1)!-1 & n^3(n+1)\end{vmatrix}=0$

16. If slope of normal of a curve $y = y(x)$ is $\dfrac{x^2}{xy-x^2y^2-1}$, and it passes through (1, 1) then find $ey(e)$.

(1) $\dfrac{1+\tan 1}{1-\tan 1}$

(2) 1

(3) tan 1

(4) $\dfrac{1-\tan 1}{1+\tan 1}$

Answer (1)

Sol. $\because\ -\dfrac{dx}{dy}=\dfrac{x^2}{xy-x^2y^2-1}$

$\Rightarrow\ -xydx + x^2y^2dx + dx = x^2dy$

$(x^2y^2 + 1)dx = x(xdy + ydx)$

$\Rightarrow\ \dfrac{dx}{x}=\dfrac{d(xy)}{1+(xy)^2}$

$\Rightarrow\ \ln x = \tan^{-1}xy + C$ passes through (1, 1)

So $C=-\dfrac{\pi}{4}$

$\Rightarrow\ xy=\tan\left(\dfrac{\pi}{4}+\ln x\right)$

Put $x = e$, we get $e\cdot y(e)=\tan\left(\dfrac{\pi}{4}+1\right)$

$=\dfrac{1+\tan 1}{1-\tan 1}$

17. Curve C has tangent at P such that it meets x-axis at Q. Y-axis bisects PQ, then curve C is a parabola passing through (3, 3) with

(1) focus $\left(\dfrac{4}{3},0\right)$

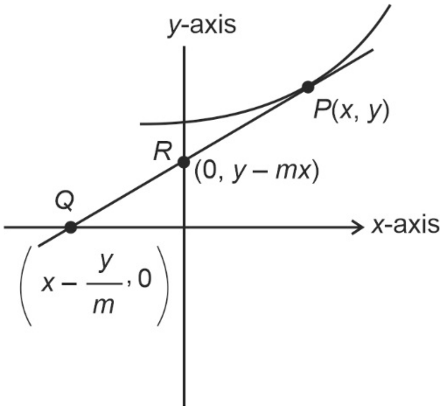
(2) focus $\left(0,\dfrac{3}{4}\right)$

(3) Latus Rectum = 3

(4) Latus Rectum = 6

Answer (3)

Sol. Let (x, y) be point on curve tangent at $P(x, y)$



$Y - y = m(X - x)$

Where $m = \frac{dy}{dx}$

For Q

Put $y = 0$

$\Rightarrow \quad X = x - \frac{y}{m}$

As y-axis bisects PQ,

$x + x - \frac{y}{m} = 0$

$\Rightarrow \quad \frac{dy}{y} = \frac{dx}{2x}$

$\ln y = \frac{1}{2}\ln x + \ln C$

$\Rightarrow \quad y^2 = Cx$

As it passes through (3, 3)

$\Rightarrow \quad C = 3$

$\Rightarrow \quad$ Parabola $y^2 = 3x$

So length of Latus Rectum = 3

18. If it is given that $x * y = x^2 + y^3$. Now if $(x * 1) * 1$ and $x * (1 * 1)$ both are equal, then find the value of $2\sin^{-1}\left[\frac{x^4 + x^2 - 2}{x^4 + x^2 + 2}\right]$

(1) $\frac{\pi}{6}$

(2) $\frac{\pi}{2}$

(3) $\frac{2\pi}{3}$

(4) $\frac{\pi}{3}$

Answer (4)

Sol. $\because \quad (x * 1) * 1 = (x^2 + 1) * 1 = (x^2 + 1)^2 + 1 = x^4 + 2x^2 + 2$

And $x * (1 * 1) = x * (1 + 1) = x * 2 = x^2 + 8$

$\therefore \quad x^4 + 2x^2 + 2 = x^2 + 8$

So $x^4 + x^2 - 6 = 0 \Rightarrow (x^2 + 3)(x^2 - 2) = 0$

Hence $x^2 = 2$

Now, $2\sin^{-1}\left(\frac{x^4 + x^2 - 2}{x^4 + x^2 + 2}\right) = 2\sin^{-1}\left(\frac{4 + 2 - 2}{4 + 2 + 2}\right)$

$= 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If the circle $(x - h)^2 + (y - k)^2 = r^2$ touches x-axis at (1, 0), $k > 0$ and $x + y = 0$ intersect the circle at two points making. Chord of length 2 units. Find $h + k + r$.

Answer (07)

Sol. Equation of circle touching x-axis at (1, 0) is

$(x - 1)^2 + y^2 + \lambda y = 0$, where λ is parameter

$(x-1)^2 + \left(y + \frac{\lambda}{2}\right)^2 = \frac{\lambda^2}{4}$,

on comparing with $(x - h)^2 + (y - k)^2 = r^2$

we get, $h = 1,\ k = -\frac{\lambda}{2},\ r = \frac{\lambda}{2}$

distance of chord $x + y = 0$ from centre $\left(1, -\frac{\lambda}{2}\right)$ is equal to

$\frac{\left|1 - \frac{\lambda}{2}\right|}{\sqrt{2}} = \sqrt{\frac{\lambda^2}{4} - 1}$

$\Rightarrow \quad \frac{\lambda^2}{4} + \lambda - 3 = 0$

$\Rightarrow \quad \lambda = -6, 2$ but 2 is not acceptable.

$\therefore \quad h = 1, k = 3, r = 3$

$\therefore \quad h + k + r = 7$

22. If the system of equations $x + y + \alpha z = 1$

$x + y + 3z = 1$, has unique solution

$x + 2z = 4$

if $\alpha \in \{1, 2, 3, \ldots\ldots\ldots 100\}$ and

HCF $(24, \alpha) = 1$, find sum of values of α.

Answer (1633)

Sol. For unique solution

$$\begin{vmatrix} 1 & 1 & \alpha \\ 1 & 1 & 3 \\ 1 & 0 & 2 \end{vmatrix} \neq 0 \Rightarrow \alpha \neq 3$$

If HCF $(24, \alpha) = 1$ then α should not be multiple of 2 OR 3.

Required sum =

$(1 + 2 + 3 + \ldots + 100) - (2 + 4 + 6 + \ldots + 100)$

$-(3 + 6 + 9 + \ldots + 99) + (6 + 12 + 18 + \ldots + 96)$

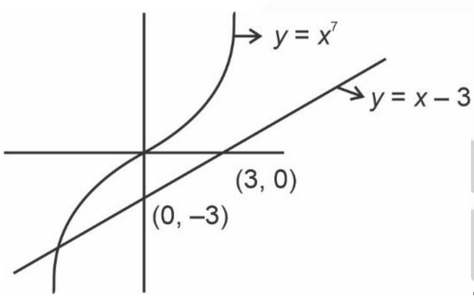
$$= \frac{100 \cdot 101}{2} - 2 \cdot \frac{50 \cdot 51}{2} - \frac{3 \cdot 33 \cdot 34}{2} + 6 \cdot \frac{16 \cdot 17}{2}$$

$= 1633$

23. Number of real solutions of the equation $x^7 - x + 3 = 0$ is

Answer (01)

Sol. $x^7 = x - 3$



We can observe that there exists only one point of intersection of these two curves, so number of real solution is 1.

24. Find the remainder when $1 + 3 + 3^2 + \ldots\ldots\ldots\ 3^{2021}$ is divided by 50.

Answer (04)

Sol. $1 + 3 + 3^2 + \ldots\ldots\ldots\ldots + 3^{2021} = \dfrac{3^{2022} - 1}{2}$

$$\because \quad \frac{3^{2022} - 1}{2} = \frac{(10-1)^{1011} - 1}{2}$$

$$= \frac{\left({}^{1011}C_0 \cdot 10^{1011} - {}^{1011}C_1 \cdot 10^{1010} + \ldots + {}^{1011}C_{1010} \cdot 10 - 1 - 1\right)}{2}$$

$= 50$ [Integer] $+ 1011.5 - 1$

$= 50$ [Integer] $+ 5054$

So, remainder will be 4.

25. Find the number of 7 digits numbers which are divisible by 11 and made up of 1, 2, 3, 4, 5, 7, 9

Answer (576)

Sol. Sum of all digits = 31

So clearly sum of digits at even places is 10 and sum of digits at odd places is 21 or vice versa.

Case I : 5, 7, 9 are used at even places

No. of numbers = $\underline{|3}.\underline{|4} = 144$

Case II : 1, 2, 7 are used at even places

No. of numbers $= \underline{|3}.\underline{|4} = 144$

Case III : 2, 3, 5 are used at even places

No. of numbers $= \underline{|3}.\underline{|4} = 144$

Case IV : 1, 4, 5 are used at even places

No. of numbers $= \underline{|3}.\underline{|4} = 144$

Total numbers = 576

26.

27.

28.

29.

30.

❑ ❑ ❑